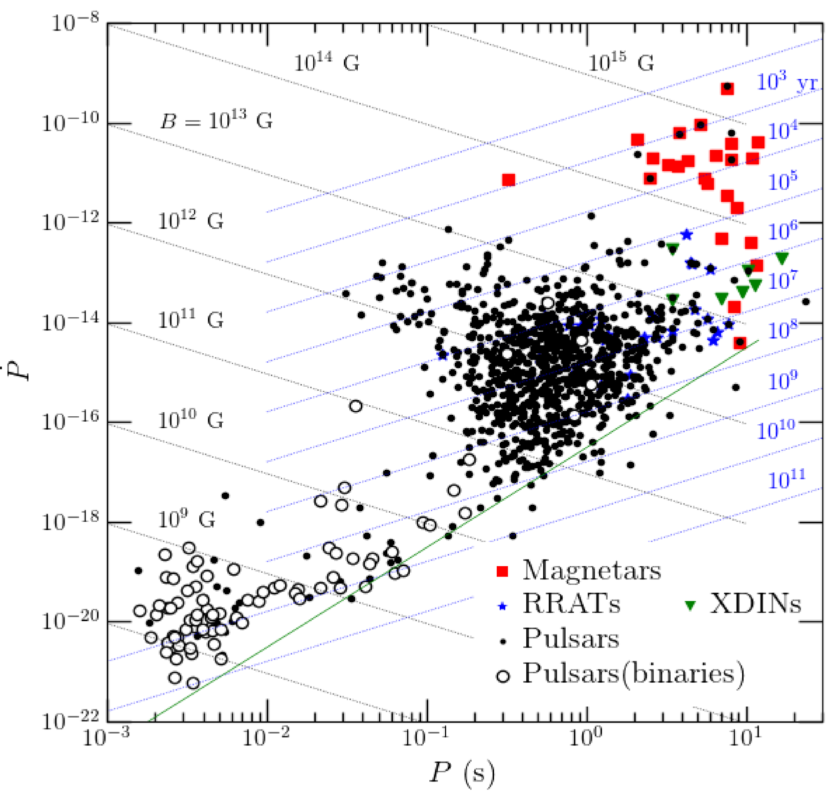
Self-Consistent Field Method for Structure of Rotating Neutron Stars with DFT-Rooted Equation of States

Nuclear Theory Group Kwon Hyukjin (M2)
Department of Physics, School of Science, Institute of Science Tokyo, Japan
Email : kwon.h.ab@m.titech.ac.jp, Supervisor : K. Sekizawa

1 Introduction

Neutron stars are fascinating laboratories for both astrophysics and nuclear physics. Traditional neutron star models, developed within the framework of general relativity, often employ simplified equation of states (EoS), such as the polytropic EoS. These models are useful for constructing well-defined stellar structures and studying their general properties. However, when comparing theoretical predictions with observational data, it is essential to use **realistic EoS based on nuclear matter**, rather than idealized models, in order to accurately describe the internal composition and physical behavior of neutron stars.



The density functional theory (DFT) framework is a powerful tool for studying not only finite nuclei but also the properties of nuclear matter. It provides a density-dependent energy functional known as the energy density functional (EDF) which can be conveniently extended to many-body systems. One of the most widely used and successful EDF models is the **Skyrme EDF**, which has been remarkably effective in describing a wide range of nuclear properties.

Most studies of neutron stars are based on the Tolman-Oppenheimer-Volkoff (TOV) equation, which describes hydrostatic equilibrium under the assumption of spherical symmetry. However, many observational results indicate that neutron stars often **rotate rapidly**. In this study, we introduce an approach to modeling rotating neutron stars using the **Komatsu-Eriguchi-Hachisu (KEH) method [Monthly Notice. Sup. 237, 355-379 (1989)]**, which numerically solves Einstein's field equations under the assumption of axial symmetry. We apply this method in combination with the Skyrme EDF to incorporate realistic nuclear interactions.

2 Neutron Star EoS from Skyrme EDF

☐ Skyrme Energy Density Functional

$$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\rm eff} + \mathcal{H}_{\rm fin} + \mathcal{H}_{\rm so} + \mathcal{H}_{\rm sg} + \mathcal{H}_{\rm Coul}$$ (t_i, x_i, α are fitted parameters)

$$\mathcal{H}_0 = \frac{1}{4}t_0\left[(2+x_0)\rho^2 - (2x_0+1)(\rho_p^2+\rho_n^2)\right]$$

$$\mathcal{H}_3 = \frac{1}{24}t_3\rho^\alpha\left[(2+x_3)\rho^2 - (2x_3+1)(\rho_p^2+\rho_n^2)\right]$$

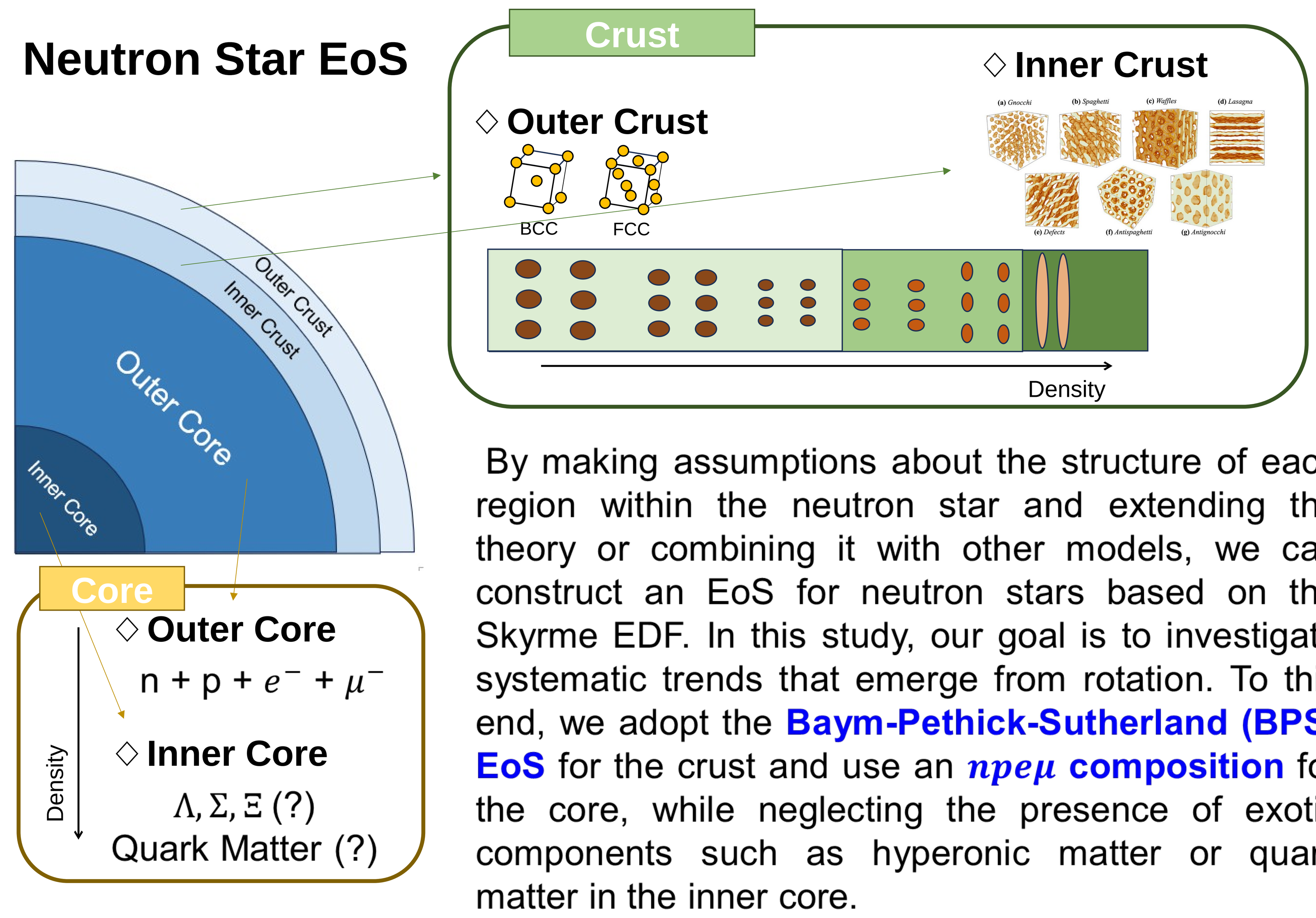
$$\mathcal{H}_{\rm eff} = \frac{1}{8}\left[t_1(2+x_1)+t_2(2+x_2)\right]\tau\rho + \frac{1}{8}\left[t_2(2x_2+1)-t_1(2x_1+1)\right](\tau_p\rho_p+\tau_n\rho_n)$$

$$\mathcal{H}_{\rm fin} = \frac{1}{32}\left[3t_1(2+x_1)-t_2(2+x_2)\right](\nabla\rho)^2 - \frac{1}{32}\left[3t_1(2x_1+1)+t_2(2x_2+1)\right]\left[(\nabla\rho_p)^2+(\nabla\rho_n)^2\right]$$

$$\mathcal{H}_{\rm so} = \frac{1}{2}W_0\left[\boldsymbol{J}\cdot\nabla\rho + \boldsymbol{J}_p\nabla\rho_p + \boldsymbol{J}_n\nabla\rho_n\right]$$

$$\mathcal{H}_{\rm so} = -\frac{1}{16}(t_1x_1+t_2x_2)J^2 + \frac{1}{16}(t_1-t_2)\left[J_p^2+J_n^2\right]$$

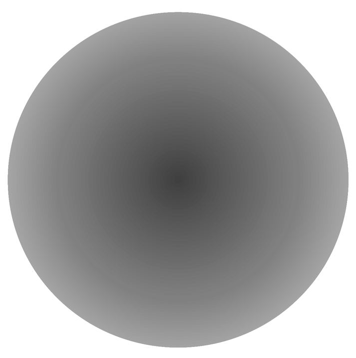
☐ Neutron Star EoS



By making assumptions about the structure of each region within the neutron star and extending the theory or combining it with other models, we can construct an EoS for neutron stars based on the Skyrme EDF. In this study, our goal is to investigate systematic trends that emerge from rotation. To this end, we adopt the **Baym-Pethick-Sutherland (BPS) EoS** for the crust and use an $\boldsymbol{npe\mu}$ **composition** for the core, while neglecting the presence of exotic components such as hyperonic matter or quark matter in the inner core.

3 Method for Rotating Neutron Star Structure

① Initial Condition from TOV equation



Metric

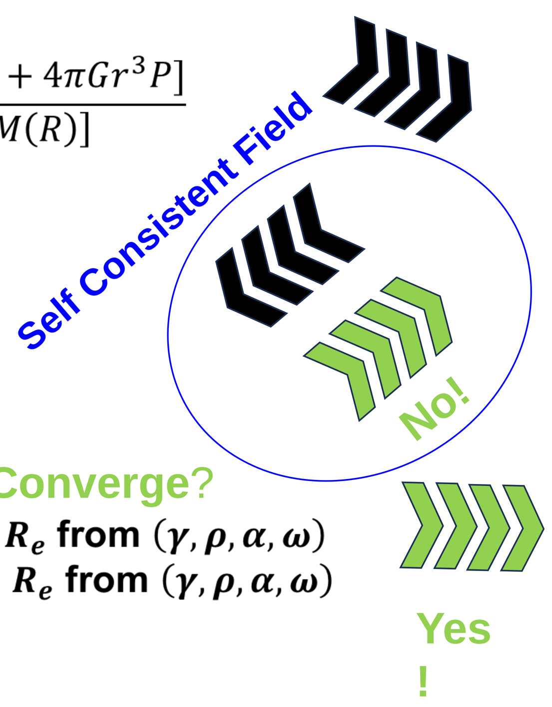
$$ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + r^2d\theta^2 - r^2\sin^2\theta\, d\phi^2$$

Einstein Equation

$$\frac{dP(r)}{dr} = -\frac{(\rho+P)[GM(r)+4\pi Gr^3P]}{r[r-2GM(R)]}$$

Initial Metric Potential $(\gamma, \rho, \alpha, \omega)$

Where, $\gamma = \nu + \lambda,\ \rho = \nu - \lambda,\ \alpha = \frac{\gamma-\rho}{2},\ \omega = 0$



② Update Metric Potential using Integral Form

Metric

$$ds^2 = -e^{\gamma+\rho}dt^2 + e^{2\alpha}(dr^2 + r^2d\theta^2) + e^{\gamma-\rho}r^2\sin^2\theta\,(d\phi - \omega dt)^2$$

Einstein Equation

$$\Delta\left[\rho e^{\gamma/2}\right] = S_\rho(r,\mu)$$

$$\left(\Delta + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\mu\frac{\partial}{\partial\mu}\right)\gamma e^{\gamma/2} = S_\gamma(r,\mu)$$

$$\left(\Delta + \frac{2}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\mu\frac{\partial}{\partial\mu}\right)\omega e^{(\gamma-2\rho)/2} = S_\omega(r,\mu)$$

Detail of source term $S_\rho, S_\gamma, S_\omega$
[Monthly Notice. Sup. 237, 355-379 (1989)]

$$\rho = -\frac{1}{4\pi}e^{-\frac{\gamma}{2}}\int_0^\infty dr'\int_{-1}^{1}d\mu'\int_0^{2\pi}d\phi'\, r'^2 S_\rho(r',\mu')\frac{1}{|r-r'|}$$

$$r\sin\theta\,\gamma = \frac{1}{2\pi}e^{-\frac{\gamma}{2}}\int_0^\infty dr'\int_0^{2\pi}d\theta'\, r'^2 S_\gamma(r',\theta')\log|r-r'|$$

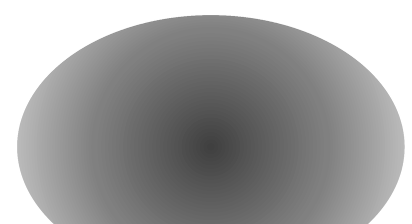
$$r\sin\theta\cos\phi\omega = -\frac{1}{4\pi}e^{-\frac{2\rho-\gamma}{2}}\int_0^\infty dr'\int_0^{\pi}d\theta'\int_0^{2\pi}d\phi'\, r'^3\sin^2\theta'\cos\phi' S_\omega(r',\theta')\frac{1}{|r-r'|}$$

New Metric Potential $(\gamma, \rho, \alpha, \omega)$

③ Calculate Equilibrium Condition

$$\ln H + \nu + \frac{1}{2}(1+v^2) + \int j(\Omega)d\Omega = C$$

④ Calculate Physical Quantities

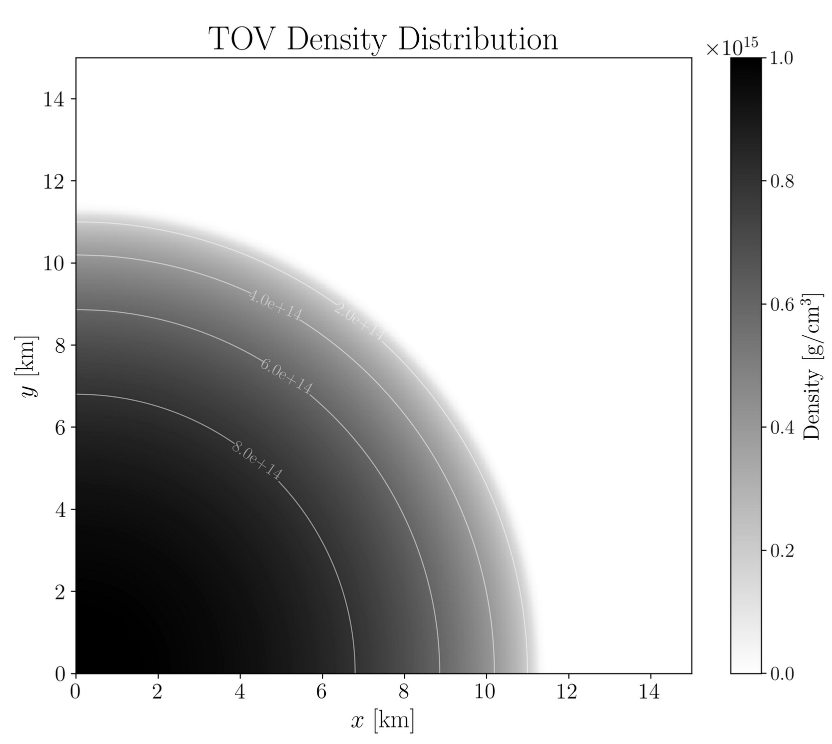


M = Mass, J = Angular momentum, T = Rotational Energy…

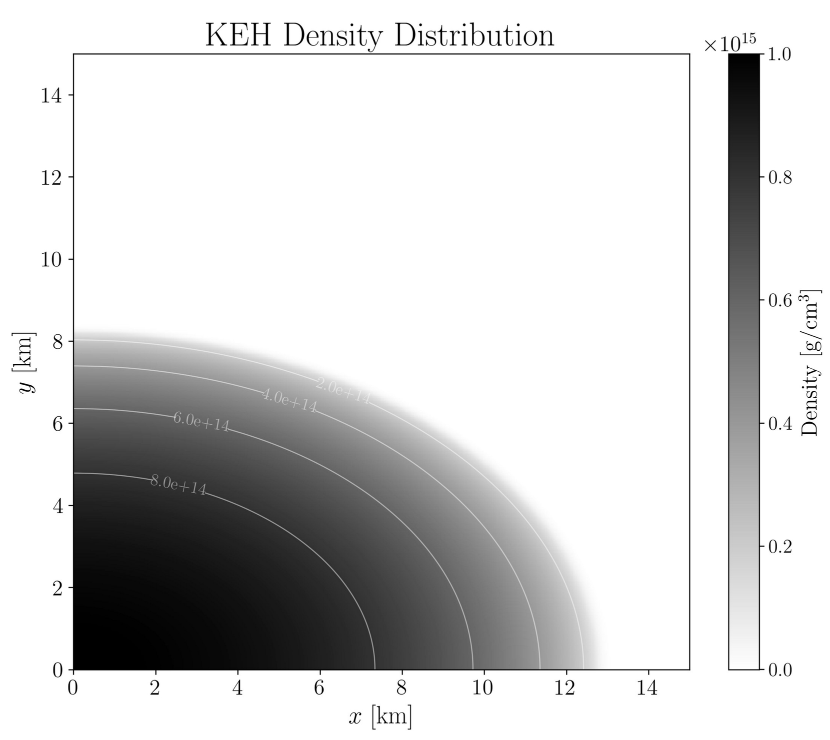
4 Numerical Results

☐ Density Distribution of Rotating Neutron Star

EOS type : SLy4, Central Density : $1.0 \times 10^{15}\,\mathrm{g/cm^3}$

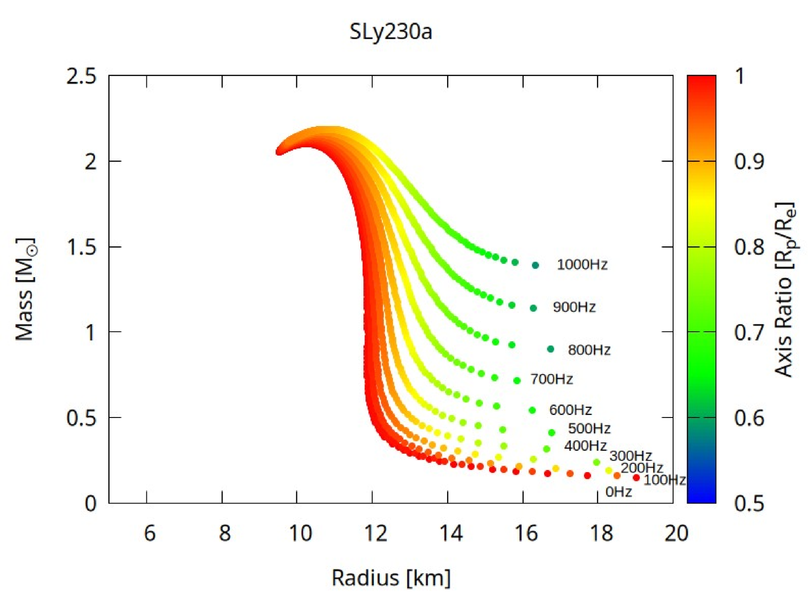


Mass: $1.43M_\odot$
Equator Radius : 11.74 km
Polar Radius : 11.74 km
Rotational Frequency : 0 Hz



Mass: $1.65M_\odot$
Equator Radius : 13.56 km
Polar Radius : 10.08 km
Rotational Frequency : 1000 Hz

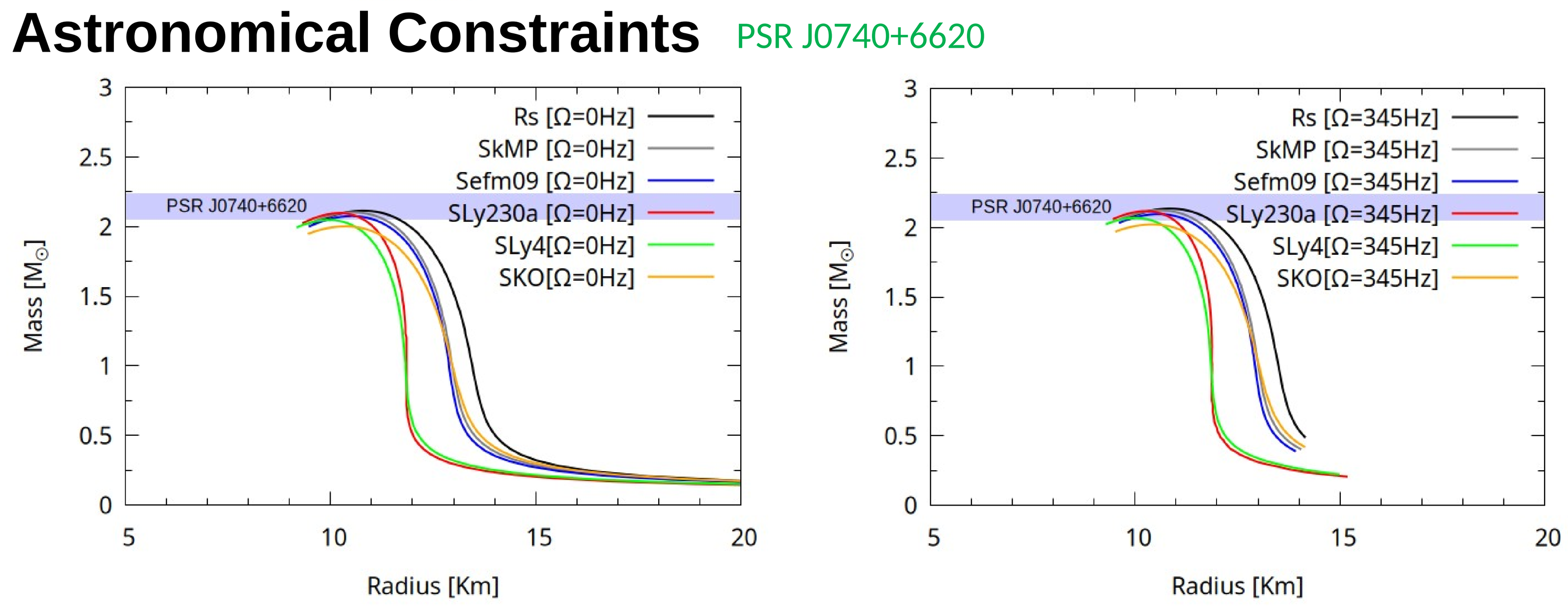
☐ M-R Relation at Each Rotation Speed



When considering the rotational effects of neutron stars, it can be observed that both their **mass and radius increase** as the rotational velocity rises. Additionally, for higher angular velocities relative to the central density, neutron stars exhibit significant **deformation**. This deformation affects not only the shape of the neutron star but also the distribution of density within its interior.

☐ Astronomical Constraints

PSR J0740+6620



5 Conclusion

When rotation frequency is high ($\Omega \gtrsim 500\mathrm{Hz}$), rotation effects can not be neglected. By taking rotational effects into account, we can achieve more accurate calculations of the internal structure of neutron stars, which in turn enables **more precise studies** of dense nuclear matter based on astronomical observations.