Nuclear DFT studies on light clusters

PRC **108**, 014318 (2023)
EPJ Web Conf. **311**, 00021 (2024)

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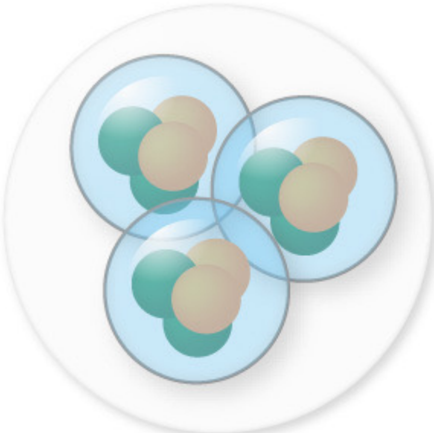
Shell model picture



α cluster model picture

Collaborators:

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Kenichi Yoshida (RCNP, Osaka U)

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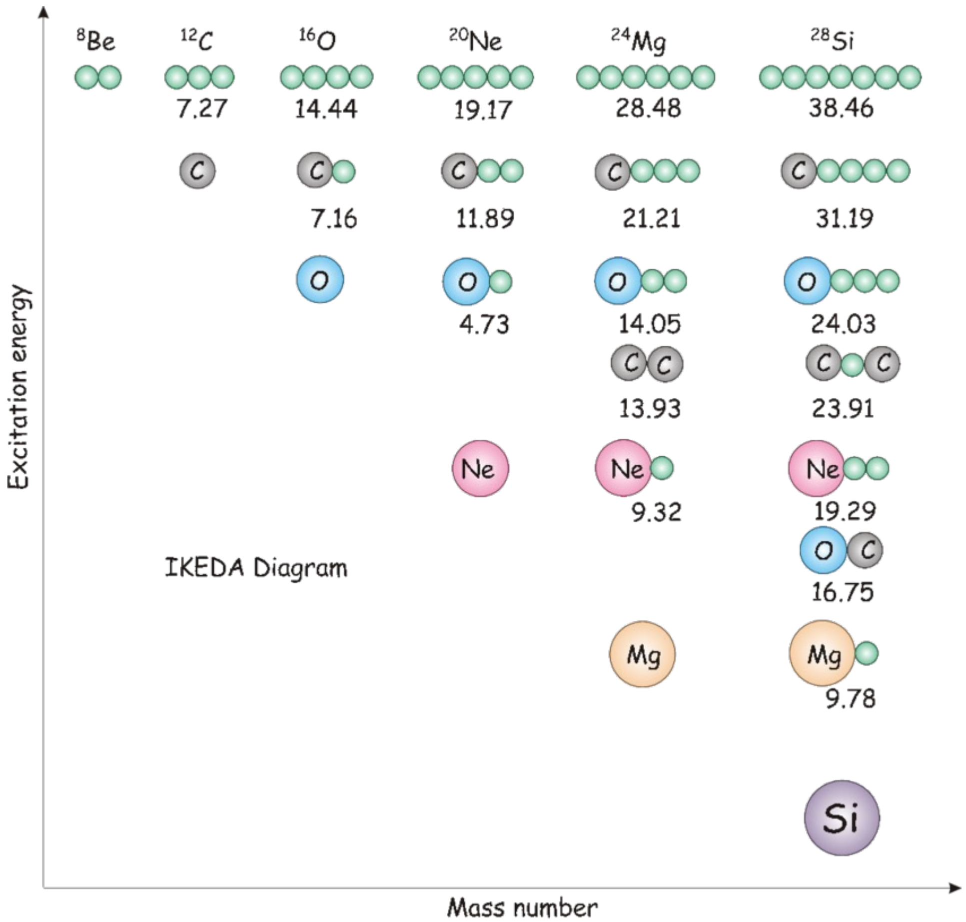
Alpha clusters in light nuclei



The *threshold rule*

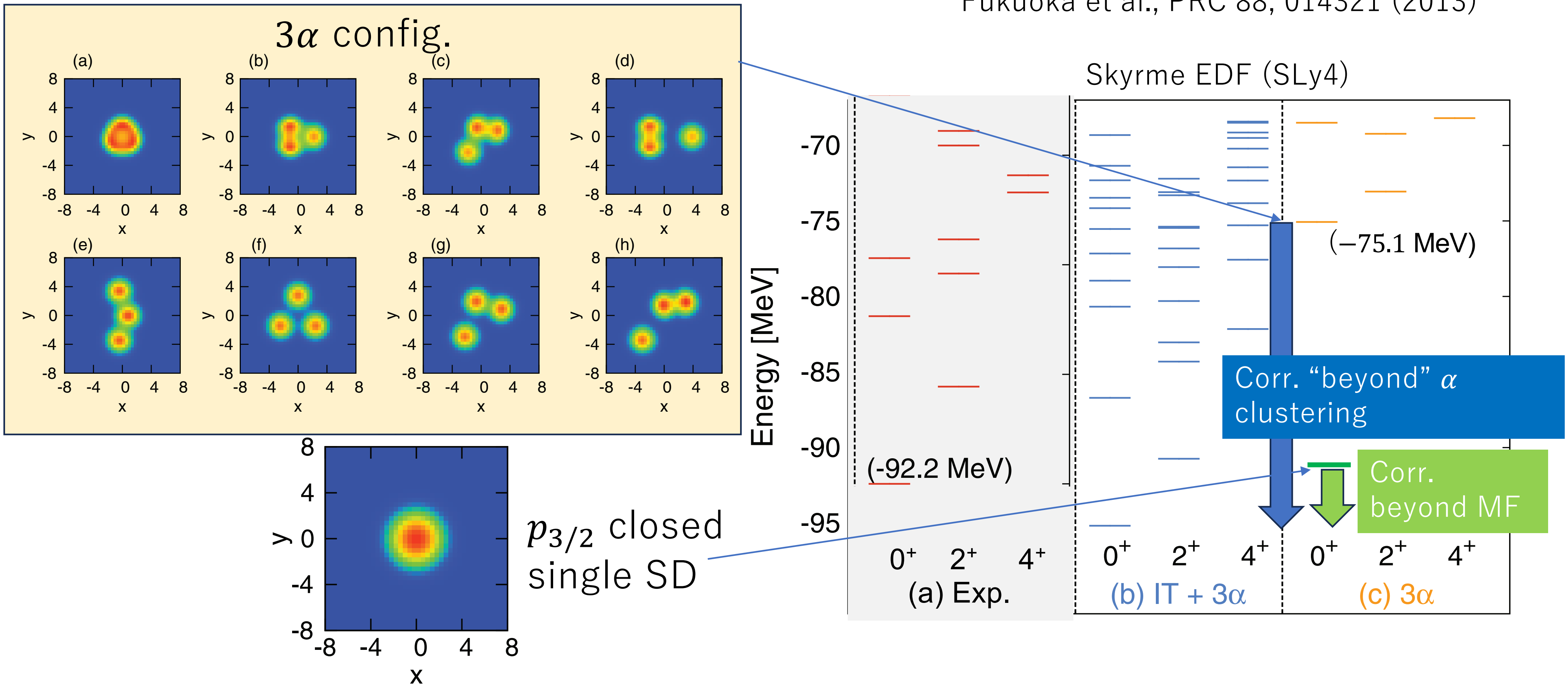
- Prominent clustering in states near the threshold
- Seeds of clustering in the ground state
- Significant impact on element synthesis (e.g., Hoyle state)

Ikeda, Takigawa, Horiuchi, PTPS Ext. Num. (1968) 464

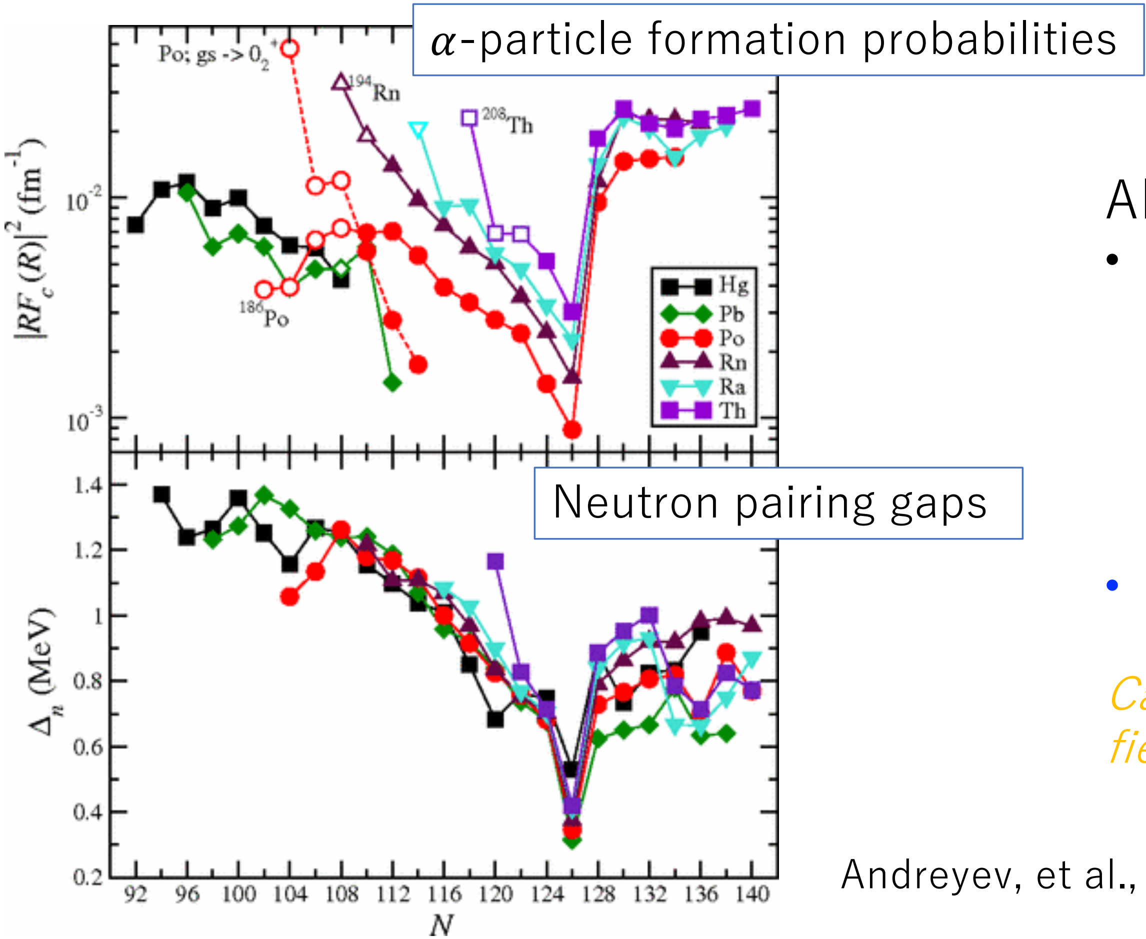
Figure from von Oertzen, Freer, Kanada-En'yo, PR **432**, 43 (2006)

^{12}C: $\mathbf{3\alpha}$ configurations vs a single SD

Fukuoka et al., PRC 88, 014321 (2013)



Alpha clusters in heavy nuclei



Andreyev, et al., PRL 110, 242502 (2013)

Alpha decay

- Existence of cluster components in the ground state

$$\Psi = \Psi_{\mathrm{sh}} + \Psi_{\mathrm{cluster}}$$

Varga, Lovas, Liotta, PRL 69, 37 (1992)

- Correlation with pairing

Can we see the effect in the mean-field wave functions?

Local α-removal strength function

$$S_\alpha^{(-)}(\boldsymbol{r},E) = \langle \Phi_0 | \hat{\alpha}^\dagger(\boldsymbol{r}) \delta(E - \hat{H}) \hat{\alpha}(\boldsymbol{r}) | \Phi_0 \rangle$$

$$\hat{\alpha}(\boldsymbol{r}) = \int \phi_0^r(\boldsymbol{x}_1)\phi_0^r(\boldsymbol{x}_2)\phi_0^r(\boldsymbol{x}_3)\phi_0^r(\boldsymbol{x}_4)$$

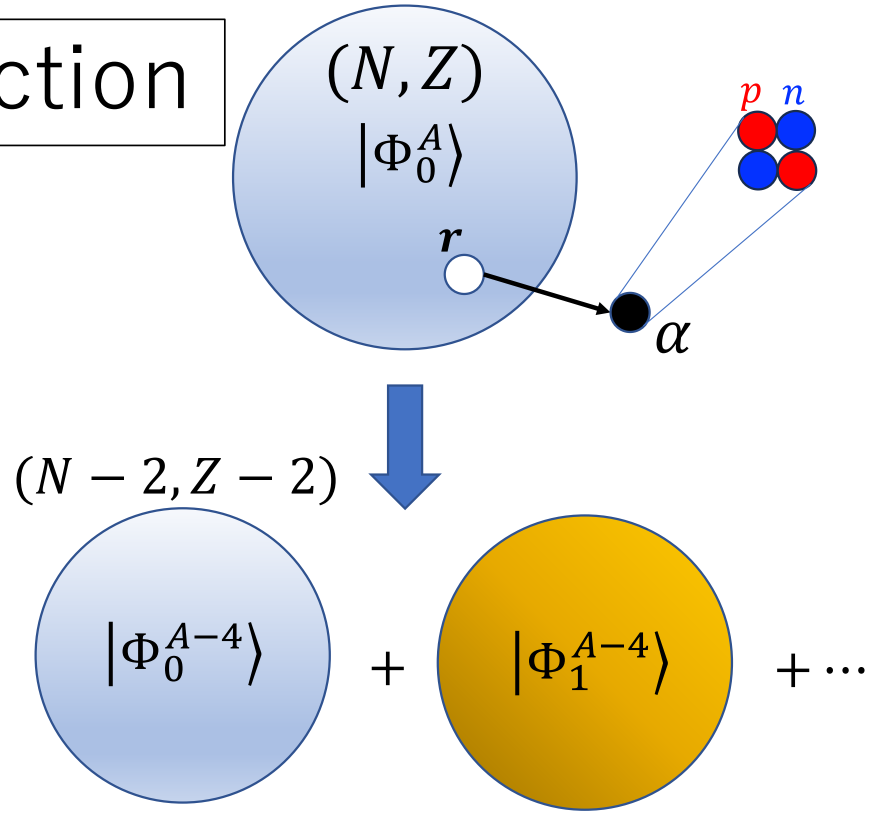
$$\times \hat{\psi}_{n\uparrow}(\boldsymbol{x}_1)\hat{\psi}_{n\downarrow}(\boldsymbol{x}_2)\hat{\psi}_{p\uparrow}(\boldsymbol{x}_3)\hat{\psi}_{p\downarrow}(\boldsymbol{x}_4)\, d\boldsymbol{x}_1 \cdots d\boldsymbol{x}_4$$

$$\phi_0^r(\boldsymbol{x}) = \phi_{0s_{1/2}}(\boldsymbol{x} - \boldsymbol{r}) \xrightarrow{\text{Point-}\alpha\text{ approx.}} \delta(\boldsymbol{x} - \boldsymbol{r})$$

$|\Phi_0\rangle$: Mean-field (HF+BCS) state

No correlations beyond mean field

Probed by α knockout reaction

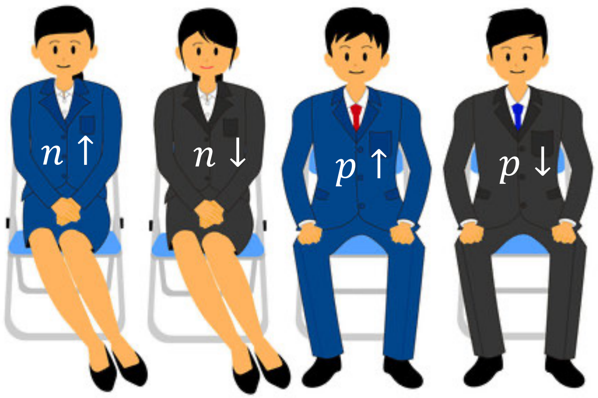


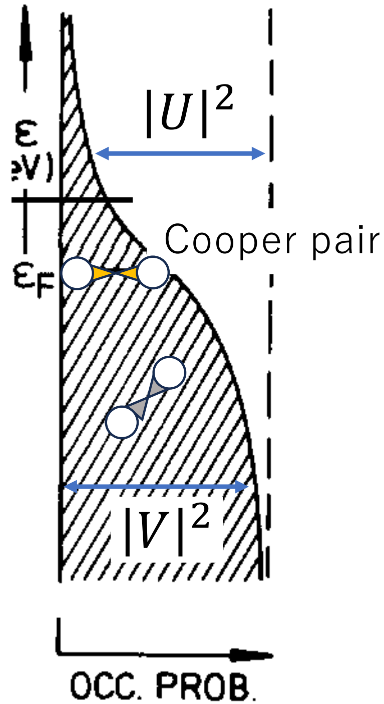
$$\hat{\alpha}(\boldsymbol{r})|\Phi_0^A\rangle = \sum_n C_n(\boldsymbol{r})|\Phi_n^{A-4}\rangle$$

$$S_\alpha(\boldsymbol{r},E) = \sum_n |C_n(\boldsymbol{r})|^2 \delta(E - E_n)$$

Local α-removal strength function

Looking for
occupied 4 seats
at the same location

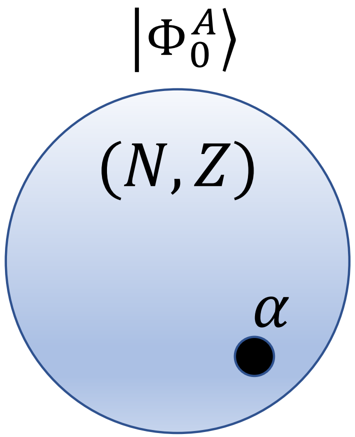




$$\boxed{S_\alpha^{(-)}(\boldsymbol{r},E)=\sum_{k,k'}F_k^{(n)}(\boldsymbol{r})F_{k'}^{(p)}(\boldsymbol{r})\delta(E-E_{kk'})}$$

$$F_k^{(q)}(\boldsymbol{r})=\begin{cases}\left|\kappa_q(\boldsymbol{r})\right|^2\ \ (=V^*U^T;\ \text{pair density}) & \text{for } k=0\\ \left|V_i^{(q)}(\boldsymbol{r}\uparrow)V_j^{(q)}(\boldsymbol{r}\downarrow)-V_j^{(q)}(\boldsymbol{r}\uparrow)V_i^{(q)}(\boldsymbol{r}\downarrow)\right|^2 & \text{for } k=(ij)_{2qp}\end{cases}$$

Total local $\boldsymbol{\alpha}$-removal strength

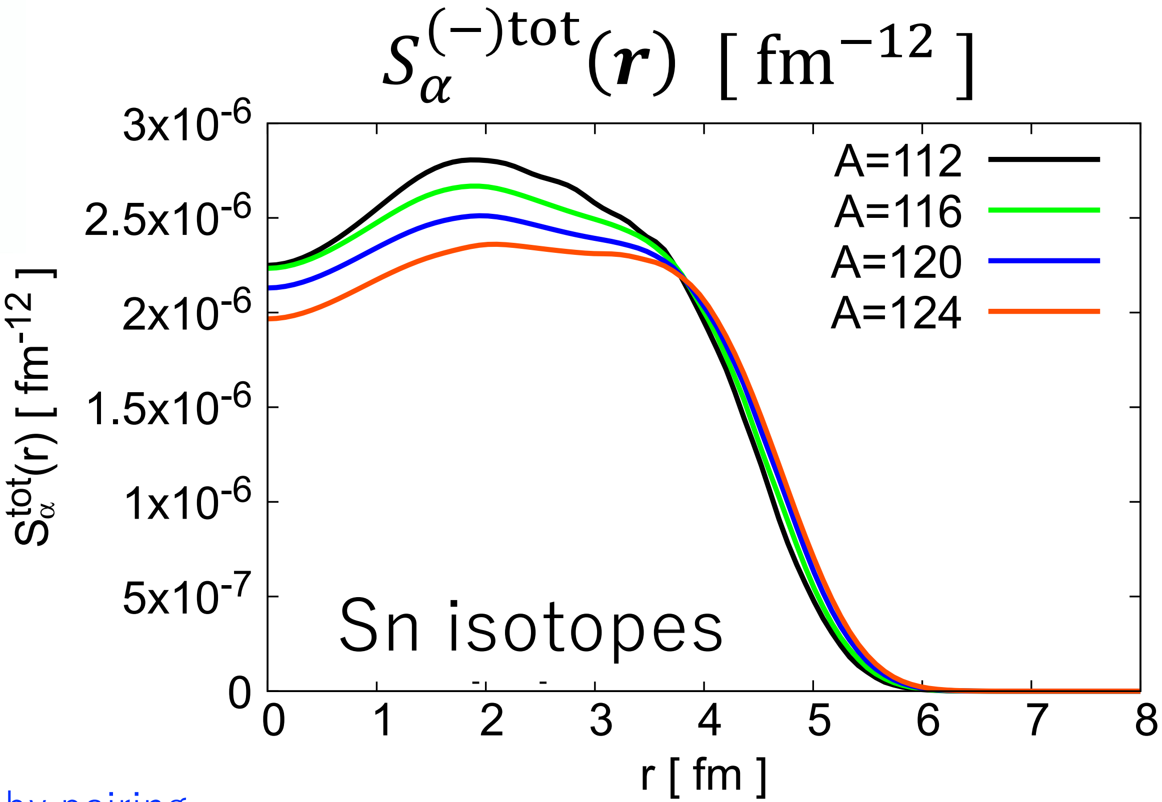


$$S_\alpha^{(-)\mathrm{tot}}(\boldsymbol{r}) \equiv \langle \Phi_0^A | \hat{\alpha}^\dagger(\boldsymbol{r}) \hat{\alpha}(\boldsymbol{r}) | \Phi_0^A \rangle$$

$$= \sum_n \left| \langle \Phi_n^{A-4} | \hat{\alpha}(\boldsymbol{r}) | \Phi_0^A \rangle \right|^2$$

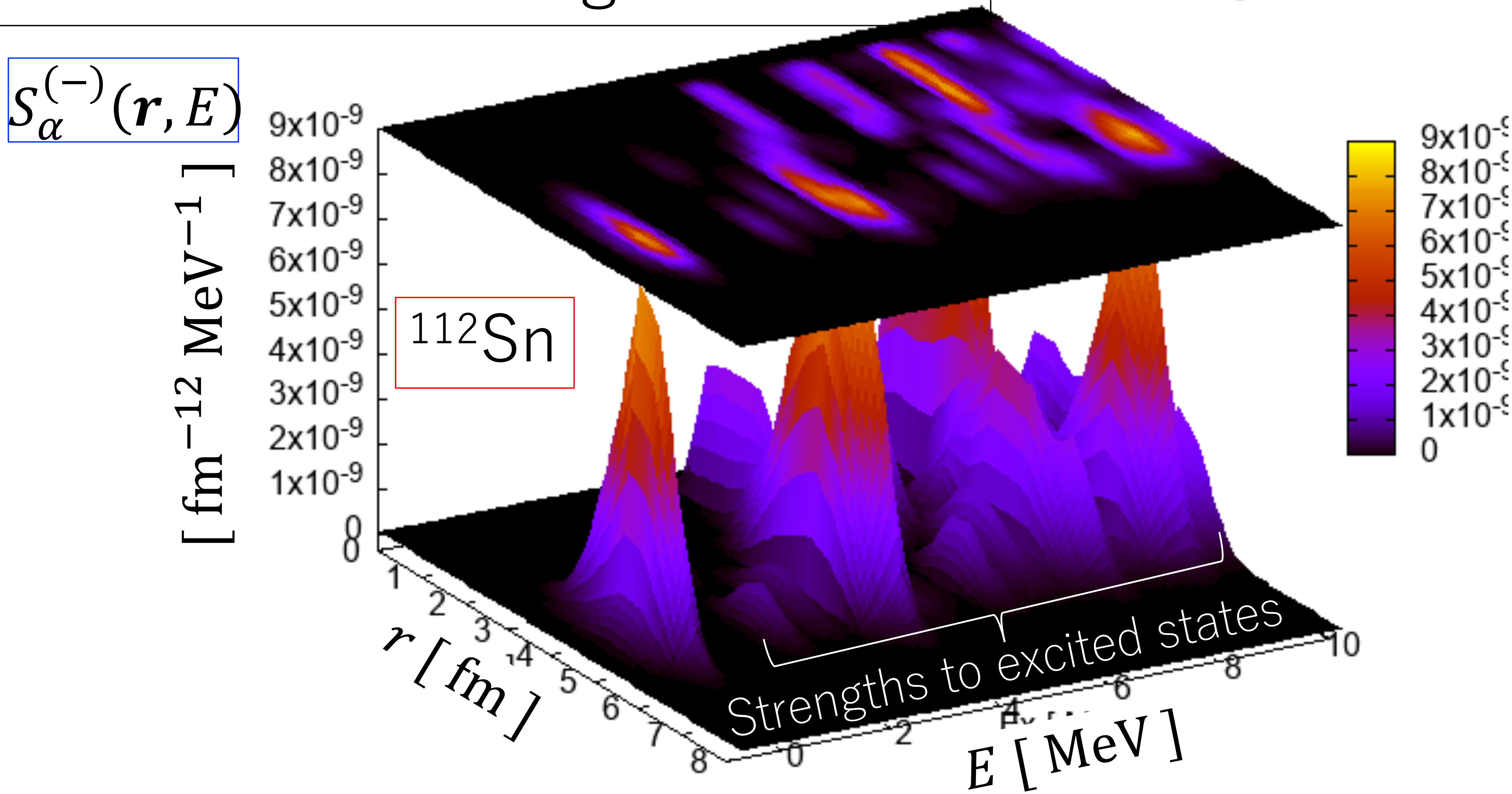
$$= \prod_{q=n,p} \left(\rho_\uparrow^q(\boldsymbol{r}) \rho_\downarrow^q(\boldsymbol{r}) + |\kappa^q(\boldsymbol{r})|^2 \right)$$

Enhancement by pairing



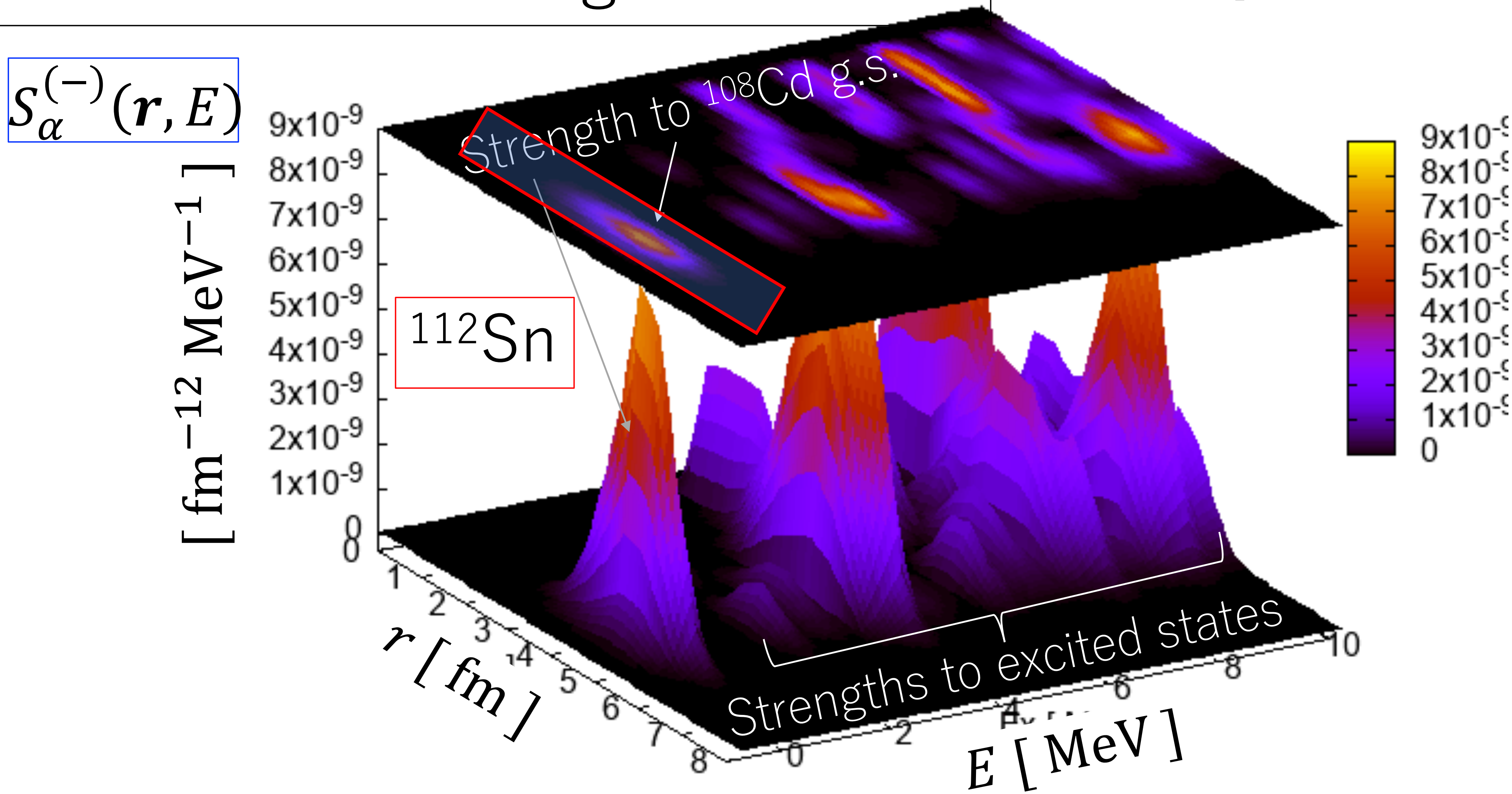
Local α-removal strength function

^{A}Sn → $^{A-4}$Cd



Local α-removal strength function

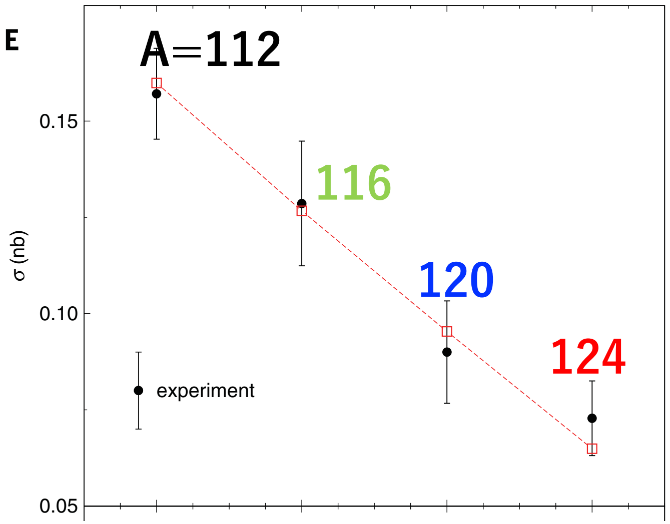
^{A}Sn → $^{A-4}$Cd



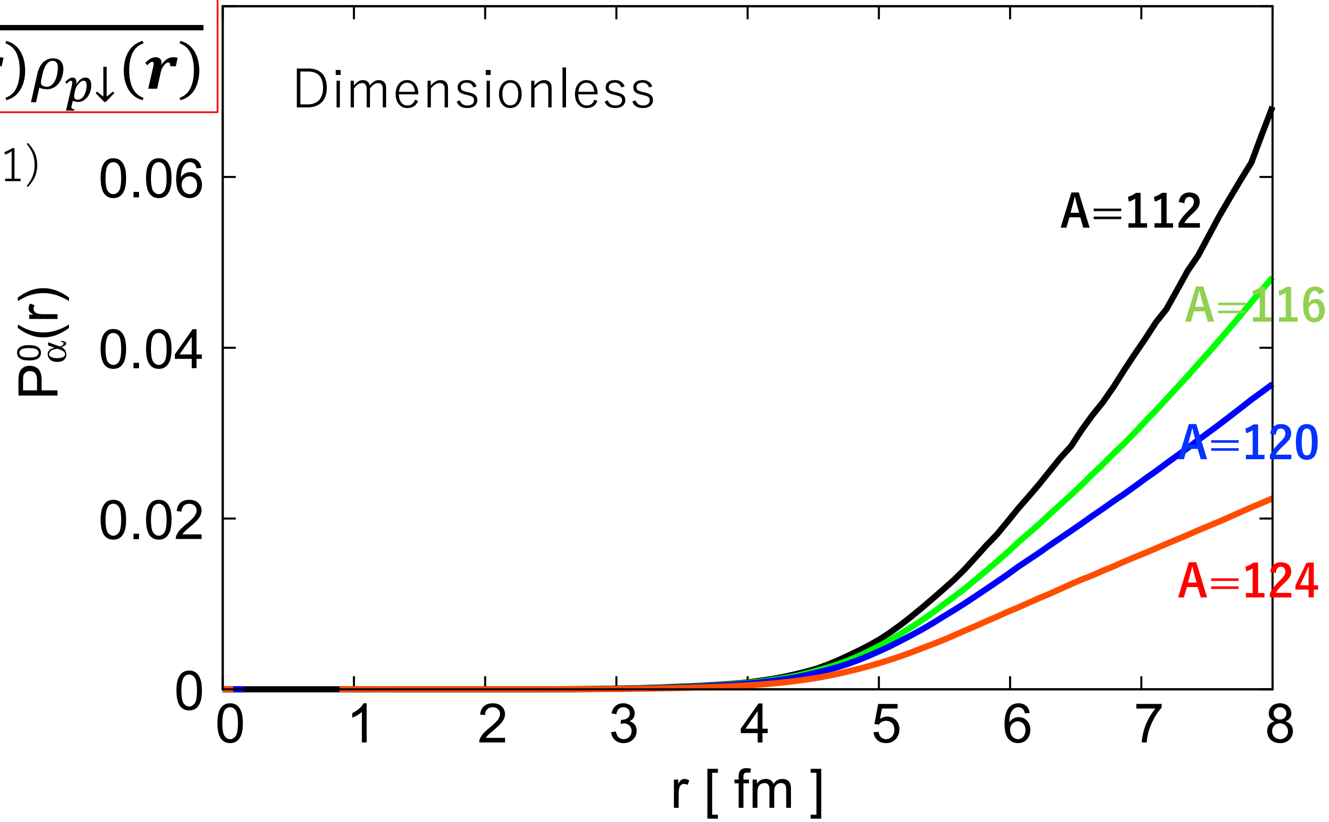
Normalized local α strength (g.s. → g.s.)

$$P_{\alpha}^{0}(\boldsymbol{r}) = \frac{S_{\alpha}^{0}(\boldsymbol{r})}{\rho_{n\uparrow}(\boldsymbol{r})\rho_{n\downarrow}(\boldsymbol{r})\rho_{p\uparrow}(\boldsymbol{r})\rho_{p\downarrow}(\boldsymbol{r})}$$

Tanaka et al. Science 371, 260 (2021)

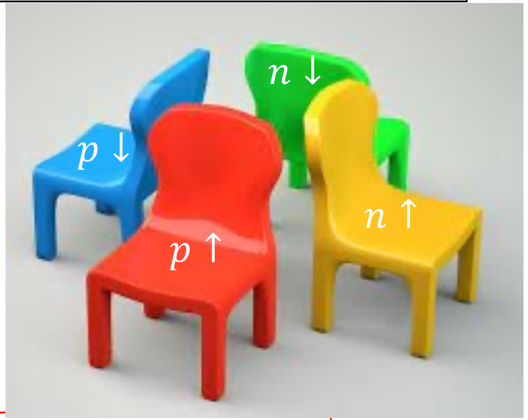


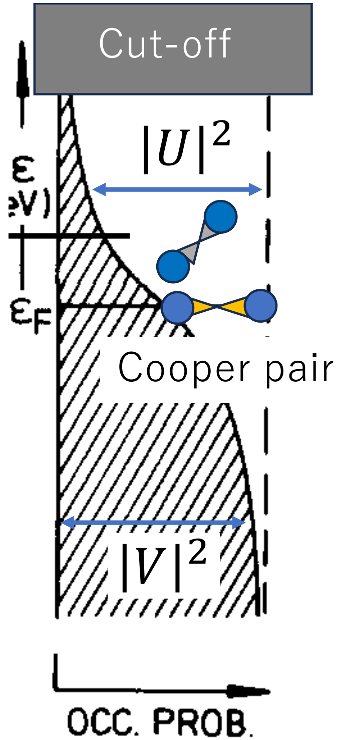
Consistent with exp.



Local α-addition strength function

Looking for
vacant 4 seats at
the same location





$$S_\alpha^{(+)}(\boldsymbol{r},E)=\sum_{k,k'}G_k^{(n)}(\boldsymbol{r})G_{k'}^{(p)}(\boldsymbol{r})\delta(E-E_{kk'})$$

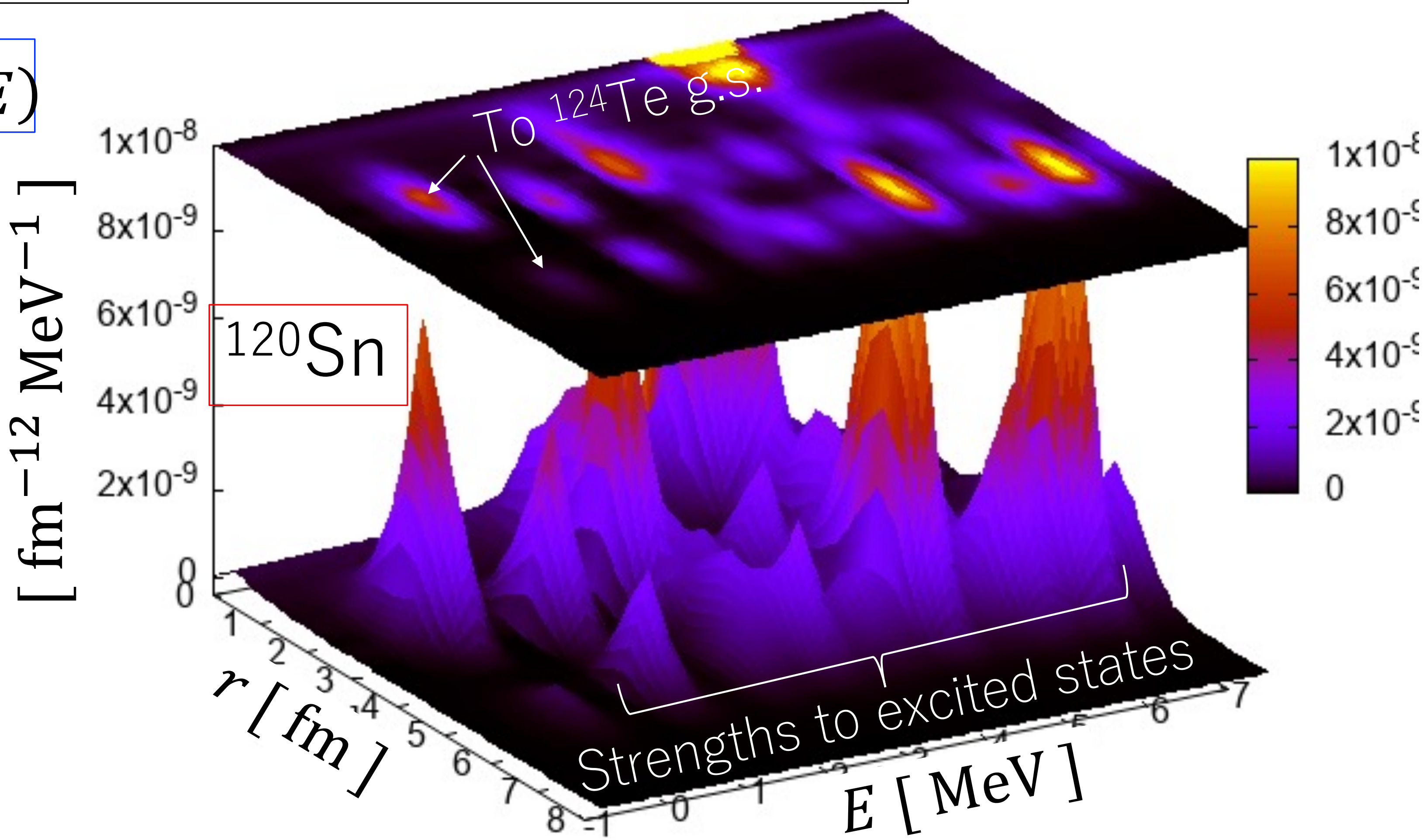
Replacement: $V_i^*(\boldsymbol{r}) \leftrightarrow U_i(\boldsymbol{r})$

$$G_k^{(q)}(\boldsymbol{r})=\begin{cases}\left|\kappa_q(\boldsymbol{r})\right|^2 & \text{for } k=0\\ \left|U_i^{(q)}(\boldsymbol{r}\uparrow)U_j^{(q)}(\boldsymbol{r}\downarrow)-U_j^{(q)}(\boldsymbol{r}\uparrow)U_i^{(q)}(r\downarrow)\right|^2 & \text{for } k=(ij)_{2qp}\end{cases}$$

Local α-addition strength function

$^{A}\mathrm{Sn} \rightarrow {}^{A+4}\mathrm{Te}$

$S_{\alpha}^{(+)}(\boldsymbol{r}, E)$

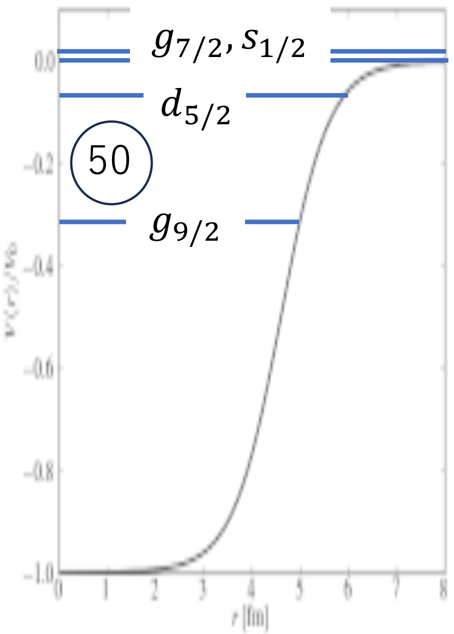


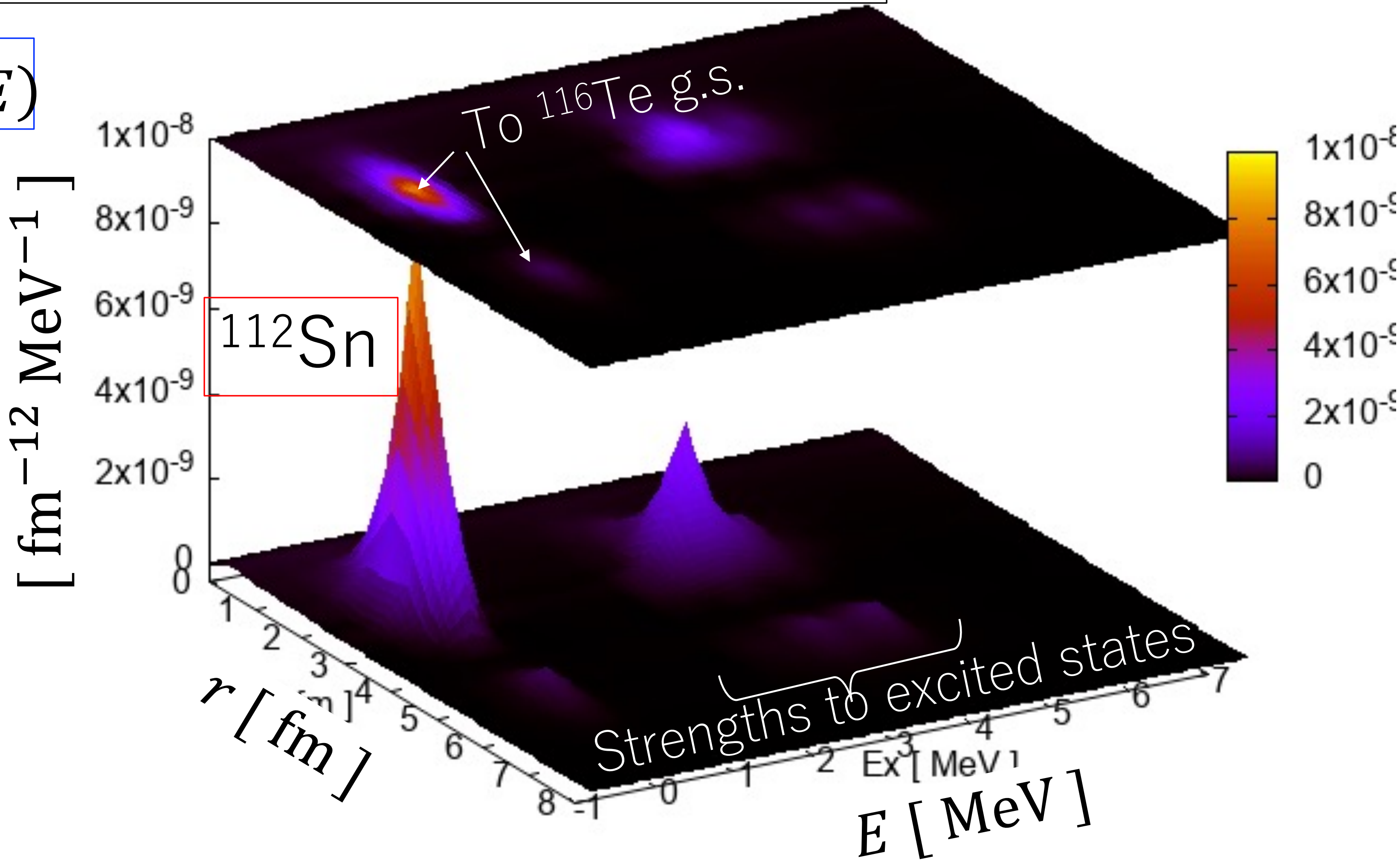
Probed by α transfer reaction

Local α-addition strength function

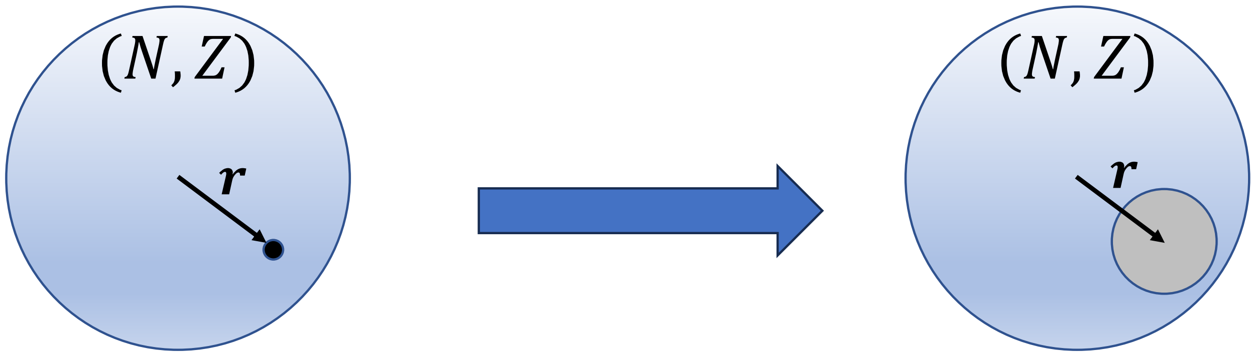
$^{A}\mathrm{Sn} \rightarrow {}^{A+4}\mathrm{Te}$

$$S_{\alpha}^{(+)}(\boldsymbol{r}, E)$$





Finite-size effect of $\boldsymbol{\alpha}$ particle



$$\phi_0^{\boldsymbol{r}}(\boldsymbol{x}) \equiv \left(\frac{\pi}{\nu}\right)^{\frac{3}{4}} \mathrm{e}^{-\frac{\nu(\boldsymbol{x}-\boldsymbol{r})^2}{2}}$$

$$\boxed{\hat{\alpha}(\boldsymbol{r}) = \hat{\psi}_{n\uparrow}(\boldsymbol{r})\hat{\psi}_{n\downarrow}(\boldsymbol{r})\hat{\psi}_{p\uparrow}(\boldsymbol{r})\hat{\psi}_{p\downarrow}(\boldsymbol{r})}$$

$$\boxed{\hat{\alpha}(\boldsymbol{r}) = \int \phi_0^{\boldsymbol{r}}(\boldsymbol{x}_1)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_2)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_3)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_4)\hat{\psi}_{n\uparrow}(\boldsymbol{x}_1)\hat{\psi}_{n\downarrow}(\boldsymbol{x})\hat{\psi}_{p\uparrow}(\boldsymbol{x}_3)\hat{\psi}_{p\downarrow}(\boldsymbol{x}_4)d\boldsymbol{x}_1 \cdots d\boldsymbol{x}_4}$$

Finite-size effect of $\boldsymbol{\alpha}$ particle (BCS case)

$$\kappa^{\alpha}(\boldsymbol{r}) \equiv \int \phi_0^{\boldsymbol{r}}(\boldsymbol{x}_1)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_2)\langle\psi_\uparrow(\boldsymbol{x}_1)\psi_\downarrow(\boldsymbol{x}_2)\rangle d\boldsymbol{x}_1 d\boldsymbol{x}_2 = \sum_{i>0} u_i v_i \langle\phi_i|P_\alpha(\boldsymbol{r})|\phi_i\rangle$$

$$\sum_{\substack{i>j \\ \epsilon_i=\epsilon,\epsilon_j=\epsilon'}} F_{ij}(\boldsymbol{r}) = \frac{1}{2}\sum_{\substack{i,j \\ \epsilon_i=\epsilon,\epsilon_j=\epsilon'}} F_{ij}(\boldsymbol{r}) = \frac{1}{2}\sum_{\epsilon_i=\epsilon} v_i^2\langle\phi_i|P_\alpha(\boldsymbol{r})P(\epsilon')P_\alpha(\boldsymbol{r})|\phi_i\rangle$$

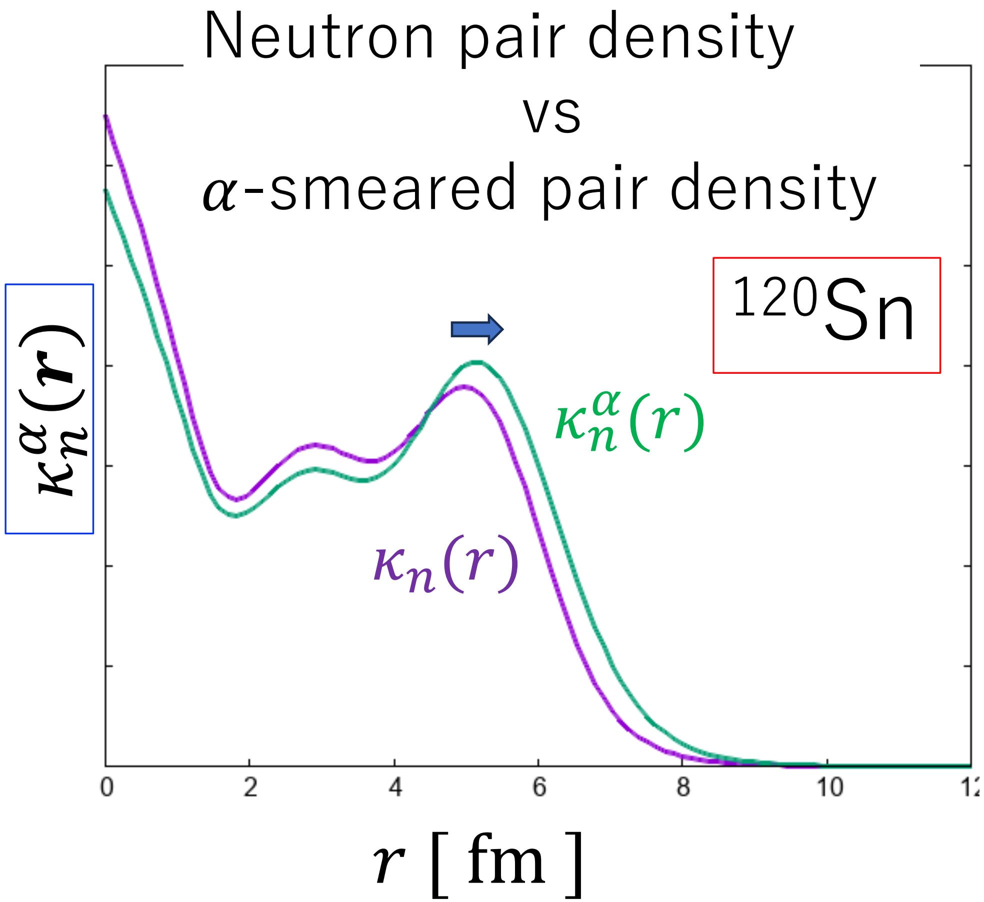
Sum over the states with the same single-particle energy

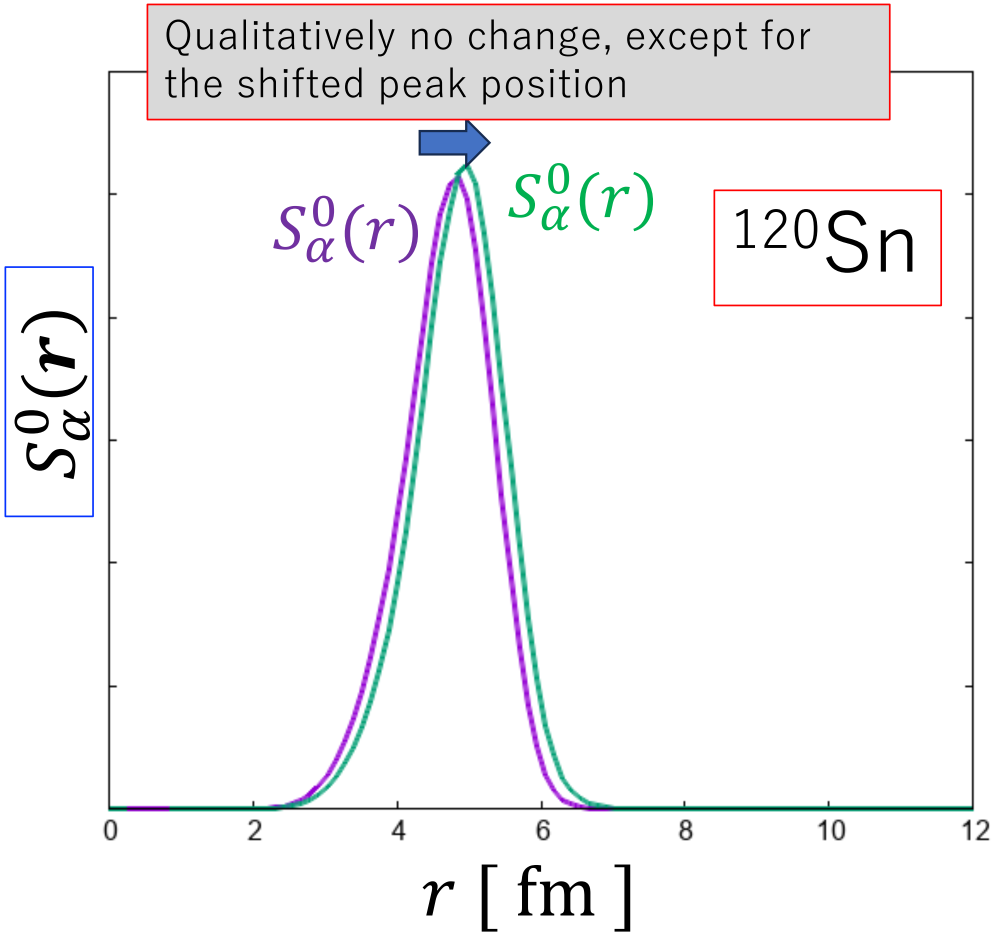
$$P(\epsilon') \equiv \sum_{\epsilon_j=\epsilon'} v_j^2|\phi_j\rangle\langle\phi_j|$$

$$\boxed{P_\alpha(\boldsymbol{r}) \equiv \sum_{\sigma=\uparrow,\downarrow} |\phi_{0\sigma}^{\boldsymbol{r}}\rangle\langle\phi_{0\sigma}^{\boldsymbol{r}}|}$$

$$\phi_{0\sigma}^{\boldsymbol{r}}(\boldsymbol{x}) \equiv \left(\frac{\pi}{\nu}\right)^{\frac{3}{4}} \mathrm{e}^{-\frac{\nu}{2}(\boldsymbol{x}-\boldsymbol{r})^2}\chi_\sigma$$

Finite-size effect of $\boldsymbol{\alpha}$ particle

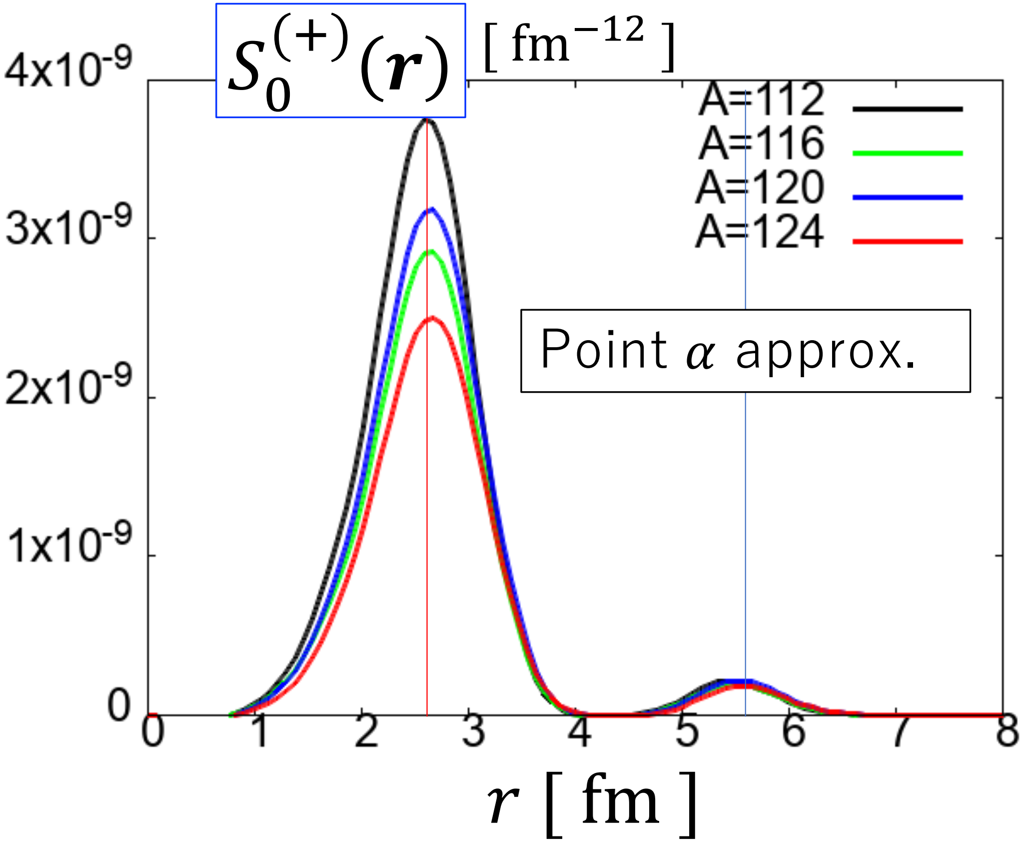


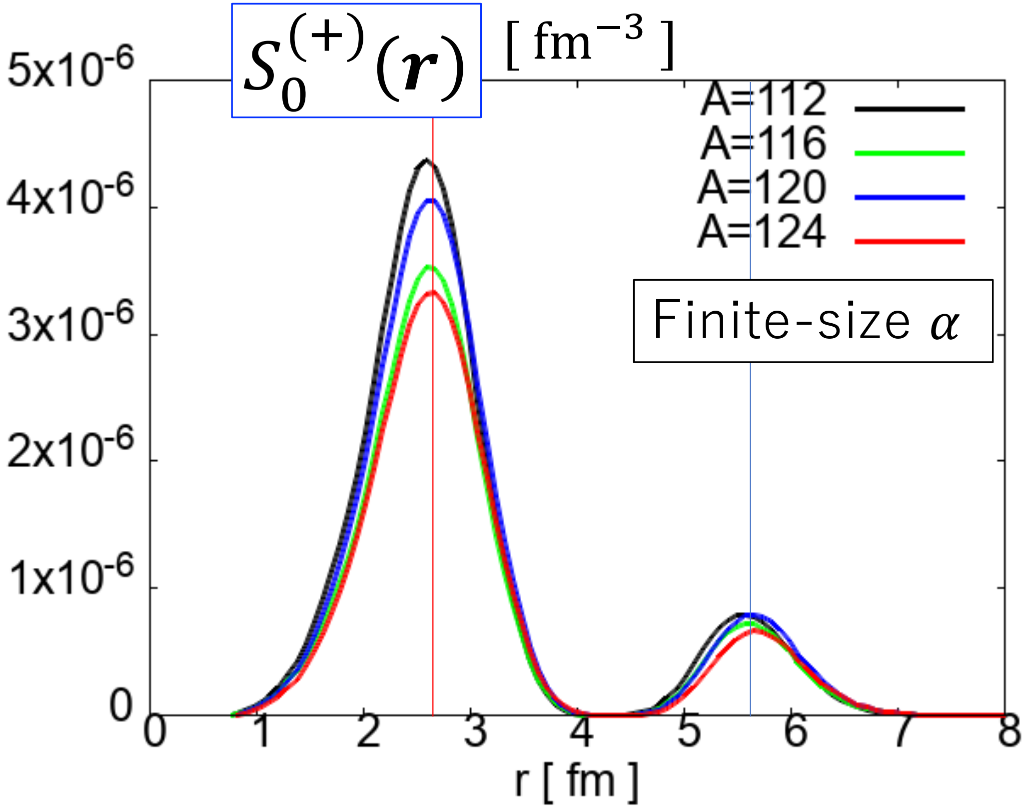


Local α-addition strength (Sn(g.s.) →Te(g.s.))

$^{112-124}$Sn

$$S_0^{(+)}(\boldsymbol{r}) \equiv \int_0^{\epsilon} S_\alpha^{(+)}(\boldsymbol{r},E)dE = \left|\langle \Phi_0^{A+4} | \hat{\alpha}^\dagger(\boldsymbol{r}) | \Phi_0^{A} \rangle\right|^2$$





Local α-add. str.: ^{124}Sn(g.s.) → ^{128}Te($E_x = 5$ MeV)

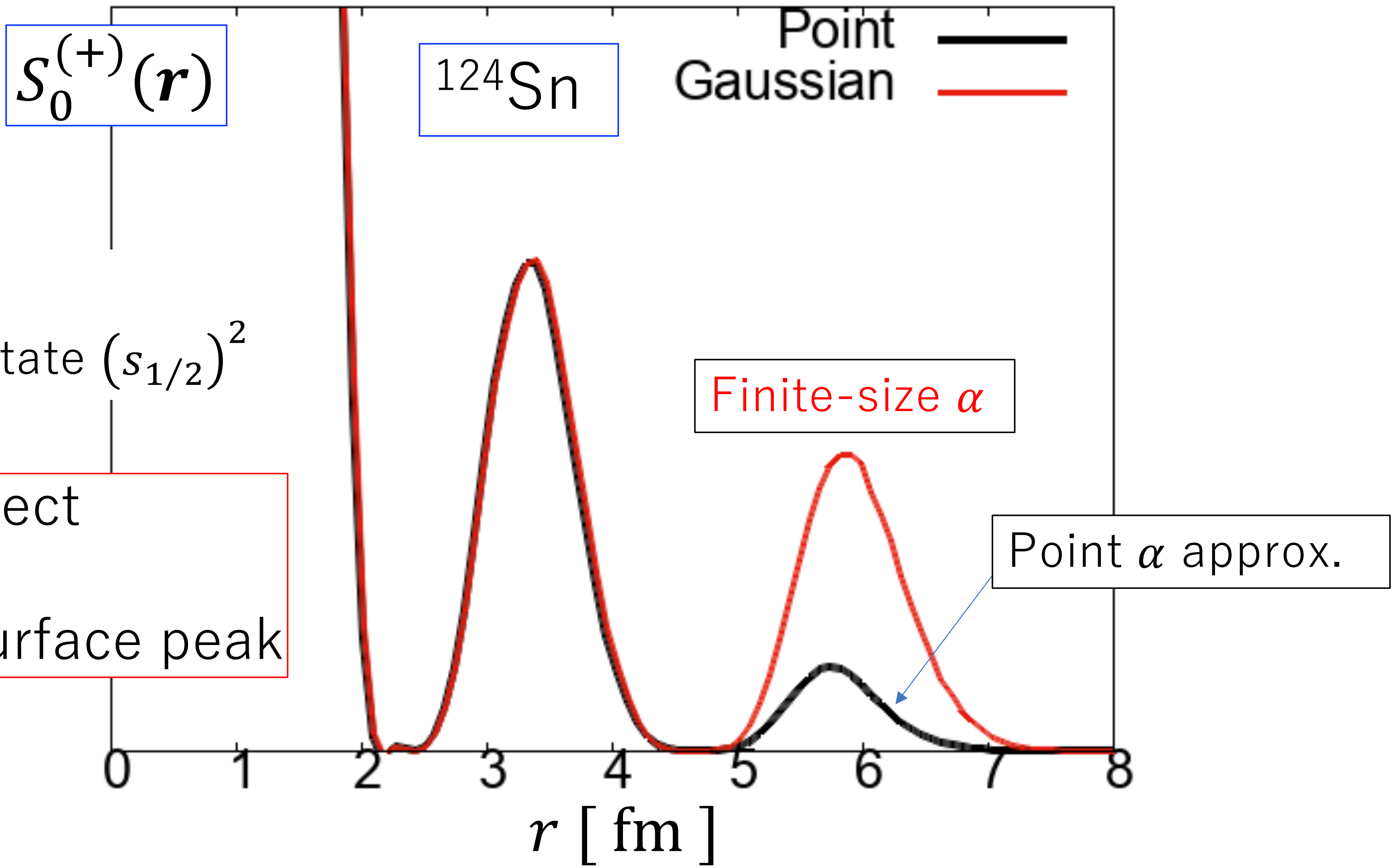
Neutron g.s.

Proton two-particle state $(s_{1/2})^2$

Finite α effect

↓

Enhancement of surface peak



α reduced width

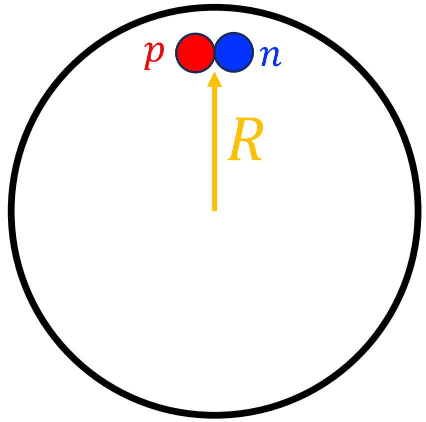
$$\hat{\alpha}(\boldsymbol{r}) = \int \phi_0^{\boldsymbol{r}}(\boldsymbol{x}_1)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_2)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_3)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_4)\hat{\psi}_{n\uparrow}(\boldsymbol{x}_1)\hat{\psi}_{n\downarrow}(\boldsymbol{x})\hat{\psi}_{p\uparrow}(\boldsymbol{x}_3)\hat{\psi}_{p\downarrow}(\boldsymbol{x}_4)d\boldsymbol{x}_1 \cdots d\boldsymbol{x}_4$$

$$= \int \Phi_{\mathrm{CM}}^{\boldsymbol{r}}(\boldsymbol{R})\ \hat{\alpha}^{\boldsymbol{R}}\ d\boldsymbol{R}$$

$$\hat{\alpha}^{\boldsymbol{R}} = \int \phi_{rel}^{\boldsymbol{r}}(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \boldsymbol{\xi}_3)\delta\left(\boldsymbol{R} - \frac{1}{4}\sum_{k=1}^{4}\boldsymbol{x}_k\right)\hat{\psi}_{n\uparrow}(\boldsymbol{x}_1)\hat{\psi}_{n\downarrow}(\boldsymbol{x})\hat{\psi}_{p\uparrow}(\boldsymbol{x}_3)\hat{\psi}_{p\downarrow}(\boldsymbol{x}_4)d\boldsymbol{x}_1 \cdots d\boldsymbol{x}_4$$

$$\boxed{\mathcal{Y}_{mn}(\boldsymbol{r}) \equiv \langle\Phi_m^{A-4}|\hat{\alpha}^{\boldsymbol{r}}|\Phi_n^A\rangle \approx \frac{\langle\Phi_m^{A-4}|\hat{\alpha}(\boldsymbol{r})|\Phi_n^A\rangle}{\int \Phi_{\mathrm{CM}}^{\boldsymbol{r}}(\boldsymbol{R})\, d\boldsymbol{R}} = \left(\frac{\nu}{\pi}\right)^{3/4}\langle\Phi_m^{A-4}|\hat{\alpha}(\boldsymbol{r})|\Phi_n^A\rangle}$$

Deuterons in nuclei

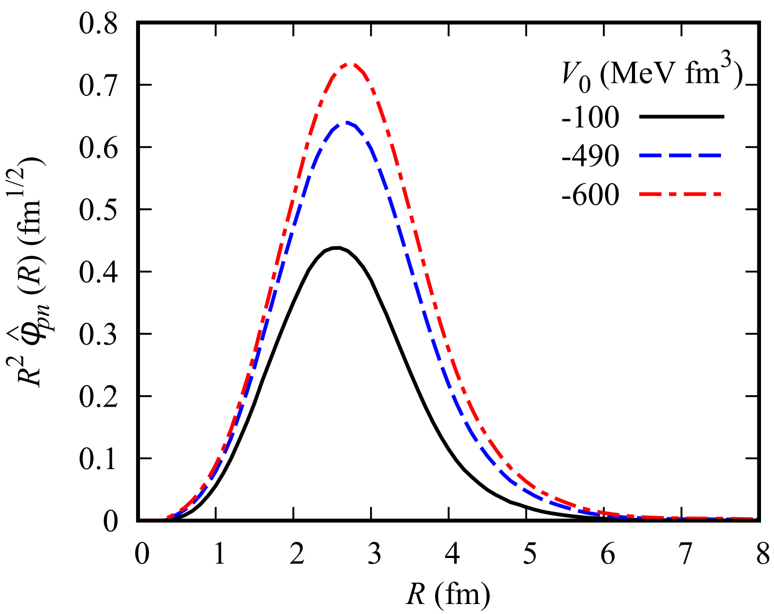


- QRPA calculation by Kenichi Yoshida

$$\hat{\varphi}_{pn}(R) \equiv \langle {}^{14}\mathrm{N} | \hat{d}(R) | {}^{16}\mathrm{O} \rangle$$

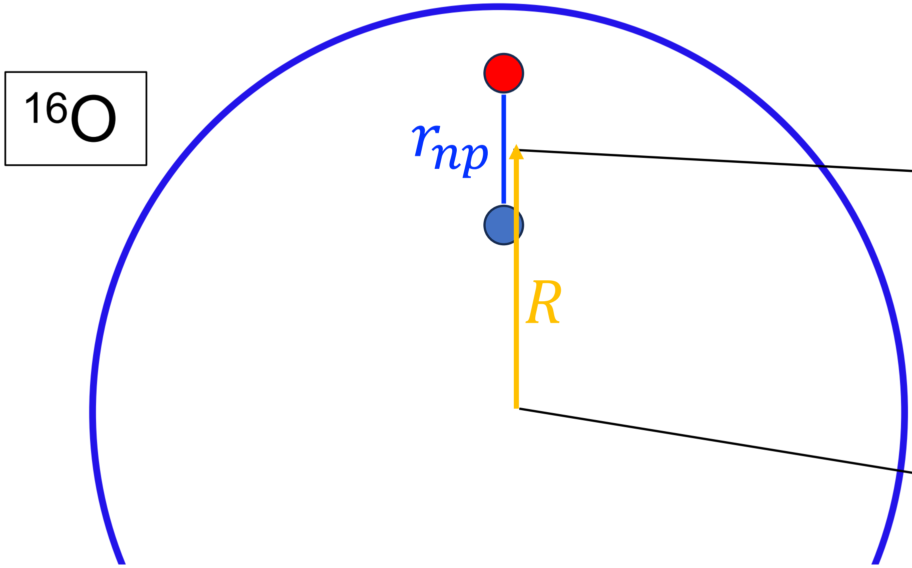
$$\hat{d}(R) = \left[\psi_n(R)\psi_p(R)\right]^{T=0}_{S=1}$$

Chazono et al., PRC 103, 024609 (2021)

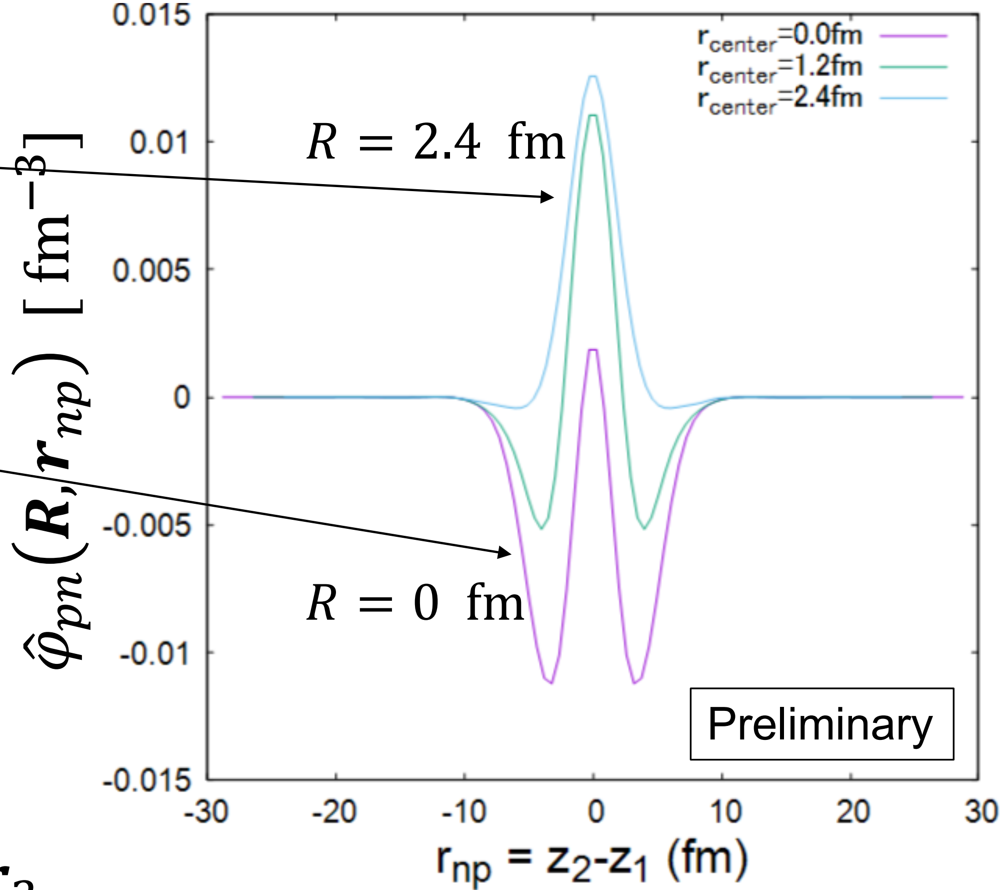


- Finite-size effect ($r_n \neq r_p$)?

Deuterons in nuclei



Ochi (M2, Master Thesis, 2025)



$$\hat{\varphi}_{pn}(\boldsymbol{R},\boldsymbol{r}_{np}) \equiv \langle {}^{14}\mathrm{N} | \hat{d}(\boldsymbol{R},\boldsymbol{r}_{np}) | {}^{16}\mathrm{O} \rangle$$

$$\hat{d}(\boldsymbol{R},\boldsymbol{r}_{np}) = [\psi_n(\boldsymbol{r}_1)\psi_p(\boldsymbol{r}_2)]^{T=0}_{S=1}$$

$$\boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2), \qquad \boldsymbol{r}_{np} = \boldsymbol{r}_1 - \boldsymbol{r}_2$$

Summary

- Local α-removal strength function: $S_{\alpha}^{(-)}(\boldsymbol{r}, E)$
  - HF+BCS calculation
  - $^{112\text{-}124}$Sn: g.s. → g.s.
    - Consistent with α-knockout experiment
    - Sensitive to pairing correlations
    - Finite-α effect: Peak shift to larger r
- Local α-addition strength function: $S_{\alpha}^{(+)}(\boldsymbol{r}, E)$
    - Strong isotopic dependence due to bound/unbound orbitals
    - Finite-α effect: Enhancement of surface peak
- Deuterons in nuclei
    - Correlations beyond mean field (QRPA)
    - Position-dependent deuteron-like states

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筑波大学計算科学研究センター 学際共同利用プログラム

Oakforest-PACS + Wisteria-O (Univ. of Tokyo)

ERATO

Three-nucleon forces project

Research Director: Kimiko Sekiguchi
(Professor, School of Science, Tokyo Institute of Technology)
Research Term: Oct 2023 - Mar 2029
Grant Number: JPMJER2304

TOMOE