Determination of ¹²C/¹³C Ratios in Orion IRc2 Acetylene Isotopologues Using TOPSEGI and Quantum Chemical Calculations

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Investigation Ref. Object Measurement technique Range cm^{-1} Year μm Planets Jupiter Cassini/CIRS mid infrared spectra $670 \sim 760$ $14.9 \sim 13.2$ 2006(1)ISO-SWS $666.7 \sim 714.3$ $15 \sim 14$ (2)Saturn 1997Uranus ISO (Infrared Space Observatory) 1998 $694.4 \sim 769.2$ $13 \sim 14.4$ (3)Voyager 2/IRIS Neptune 1991 $720 \sim 740$ $13.9 \sim 13.5$ (4)TEXES Satellites Titan 2017 $742.9 \sim 746.7$ $13.5 \sim 13.4$ (5)Comets Hyakutake Infrared Telescope Facility at Mauna Kea 1996 $3282 \sim 3288$ $3.047 \sim 3.041$ (6)Cryogenic echelle spectrometer (CSHELL) IRC +10216 $714.29 \sim 909.09$ $14 \sim 11$ Carbon star TEXES 2008(7)**Orion IRc2** SOFIA/EXES 2018 $750.19 \sim 771.60$ $13.33 \sim 12.96$ (8)

References— (1) Nixon et al. 2007; (2) de Graauw et al. 1997; (3) Encrenaz et al. 1998; (4) Bézard et al. 1991; (5) Bézard et al. 2022; (6) Brooke et al. 1996; (7) Fonfría et al. 2008; (8) Rangwala et al. 2018;

NOTE—We focus on the frequency ranges exhibiting infrared activity among various acetylene observation data.



Jupiter (Planet)



Saturn & Titan (Planet & Satellite)



Hyakutake (Comet)



Acetylene is among the simplest organic molecules found abundantly in outer space.

IRC +10216 (Carbon star)



Orion IRc2

- Pentsak, E. O., Murga, M. S., & Ananikov, V. P. 2024, ACS Earth and SpaceChemistry, 8, 798
- Nickerson, Sarah, et al. "The mid-infrared molecular inventory toward orion IRc2." The Astrophysical Journal 945.1 (2023): 26.
- https://images.nasa.gov/

- > Carbon, the primary element in acetylene is mainly formed in thermonuclear fusion reaction at the stars of intermediate masses (from $\sim 0.8 M_{\odot}$ to $8 M_{\odot}$).
- > 12C/13C is an important diagnostic tool for probing the Galactic chemical evolution or simply the **nucleosynthesis history** of the Galaxy. (AGB Star => $(12C(p,\gamma)13N(\beta+)13C))$
- > Observations have shown that the carbon star Y CVn has a 12C/13C ratio of 3.5 (13C \approx 22%)



- Pentsak, E. O., Murga, M. S., & Ananikov, V. P. 2024, ACS Earth and SpaceChemistry, 8, 798
- Woods, Paul M. "Carbon isotope measurements in the Solar System." arXiv preprint arXiv:0901.4513 (2009).
- Schöier, F. L., & Olofsson, H. (2000). The 12C/13C-ratio in cool carbon stars. arXiv preprint astro-ph/0005360.

- > IRc2 : The brightest source among the 'IRc' sources in the KL nebula, a region of the Orion Molecular
 - **Cloud. (Interesting point : Its nature remains unclear)**



- Two kinematic components (About velocities relative to the LSR)
 - Blue Clump (blueshifted)
 -7.1 ± 0.7 km/s
 - Red Clump (redshifted) +1.4 ± 0.5 km/s
- IRTF : NASA Infrared Telescope Facility
- 1990 1991 : 8 lines observed
- SOFIA : Stratospheric Observatory for Infrared Astronomy
- 2015 : 13 lines observed
- 2018 2020 : 87 lines observed

- Stahler, S. W., & Palla, F. (2008). The formation of stars. John Wiley & Sons.
- Nickerson, Sarah, et al. "The mid-infrared molecular inventory toward orion IRc2." The Astrophysical Journal 945.1 (2023): 26.
- Evans, Neal J., J. H. Lacy, and John S. Carr. "Infrared molecular spectroscopy toward the Orion IRc2 and IRc7 sources-A new probe of physical conditions and abundances in molecular clouds.
- Rangwala, Naseem, et al. "High spectral resolution SOFIA/EXES observations of C2H2 toward orion IRc2." The Astrophysical Journal 856.1 (2018): 9.

- Infrared absorption spectra
 - The motion corresponding to a normal mode must be accompanied by a change of electric dipole moment.
 - Infrared active : Change of electric dipole moment.
 - Infrared inactive : No change of electric dipole moment.
 - for example(Infrared active) : ${}^{12}C_2H_2(\nu_3,\nu_5)$, ${}^{13}CCH_2(\nu_1 \sim \nu_5)$
 - Symmetric isotopologues of acetylene(${}^{12}C_2H_2$), two nuclear spin isomers (para(even) : ortho(odd) = 1:3) exist.
 - Normal modes of Acetylene : $3 \times 4 5 = 7 \Rightarrow 5$ (doubly degenerate)



Label	Normal mode	Description	_
ν_1		CH symmetric stretch	(Infrared inactive)
$ u_2 $		${\bf CC}$ symmetric stretch	(Infrared inactive)
$ u_3$		${\bf CH}$ antisymmetric stretch	(Infrared active)
$ u_4$		Symmetric bend	(Infrared inactive, doubly degenerate)
$ u_5 $		Antisymmetric bend	(Infrared active, doubly degenerate)
	Nouse des of A	a a tu d a u a	-

Normal modes of Acetylene

- McQuarrie, D. A. (2008). Quantum chemistry. University Science Books.
- Atkins, P. W., De Paula, J., & Keeler, J. (2023). Atkins' physical chemistry. Oxford university press.
- Herman, Michel. "The acetylene ground state saga." Molecular Physics 105.17-18 (2007): 2217-2241.

- C₂H₂ has **no permanent dipole moment**, it cannot be observed via rotational transitions at radio wavelengths like CO or HCN.
- \sim C₂H₂ can only be studied in the mid-infrared (MIR; 5~28 μ m), where its ro-vibrational transitions.
- > Its v_5 ro-vibration band at 13.7 μ m is the strongest



• Rangwala, Naseem, et al. "High spectral resolution SOFIA/EXES observations of C2H2 toward orion IRc2." The Astrophysical Journal 856.1 (2018): 9.

J state

 $\nu' = 1$

v'' = 0

Symbol	Description				
Ŝ	Ro-Vibration terms				
Ĝ	Vibrational terms				
$ ilde{F}$	Rotational terms				
<u></u>	Spectral branches (D, P, O)				

 ν_P, ν_R, ν_O Spectral branches(P, R, Q)

$$\tilde{S} = \tilde{G} + \tilde{F}$$
, $\begin{cases} \tilde{G} = \left(\nu + \frac{1}{2}\right)\tilde{\nu} \\ \tilde{F} = \tilde{B}J(J+1) \end{cases}$

♦ ν_i (vibrational quantum number)= 0,1,2, ... $\tilde{\nu}_{P} = \tilde{S}(\nu_{0} + 1, \tilde{\nu}, \tilde{B}, J - 1) - \tilde{S}(\nu_{0}, \tilde{\nu}, \tilde{B}, J) = \tilde{\nu} - 2\tilde{B}J$ $\tilde{\nu}_{O} = \tilde{S}(\nu_{O} + 1, \tilde{\nu}, \tilde{B}, J) - \tilde{S}(\nu_{O}, \tilde{\nu}, \tilde{B}, J) = \tilde{\nu}$







Partition function

$$Q_{rot} = \sum_{J_S=0}^{\infty} (2J_S + 1) \exp\left(-\frac{E_J}{kT}\right)$$

 $[\]tilde{\nu}_R = \tilde{S}(\nu_0 + 1, \tilde{\nu}, \tilde{B}, J + 1) - \tilde{S}(\nu_0, \tilde{\nu}, \tilde{B}, J) = \tilde{\nu} + 2\tilde{B}(J + 1)$

Atkins, P. W., De Paula, J., & Keeler, J. (2023). Atkins' physical chemistry. Oxford university press.

Chang, R., & Thoman Jr, J. W. (2014). Physical chemistry for the chemical sciences. Royal Society of Chemistry.

Quantum chemical calculations : Centrifugal distortion constant (linear rotor)



Atkins, P. W., De Paula, J., & Keeler, J. (2023). Atkins' physical chemistry. Oxford university press.

Quantum chemical calculations

Proton [#]	14 - 13 - 12 - 11 - 9 - 8 - 7 - 6 - 5 - 4 - 3 - 2 - 1		2He 2H	42He	ŝLi	3Li	10B 9Be	6 ² С ¹¹ 5 ¹ В ¹⁰ Ве	Ĵ⁴N Ĵ³C	160 155N 14℃	170	²⁰ 0Ne ¹⁹ F ¹⁸ O	210Ne	24Mc	25AI	2흡Si 23AI 2출Mg	214Si			312Si
	0 -	Ó	i	2	3	4	5	6	Ż N€	8 eut	9 roi	10 n [:	11 #]	12	13	14	15	16	17	18
>	>	ls R	501 	to	pe _	es	: 1	NN 2	NC	C	(ł	าล	lf-	-li	fe)				
		1	H	:	1	. 2	.3	2	У											
		1(^D B	le	:	1	.5	1	×	: 1	10	6	y							
		1	4 6]		57	'0	0	y											
		2 1	6 31	41	:	7	.1	7	×	(]	10)5	y							
		3 1	2 0 4	Si	:	1	53	3 y	y											

 All quantum chemical calculations were performed using the Gaussian16

Num.	Structure	Method	ν_1	ν_2	ν_3	1/4	ν_5	$\tilde{\mathbf{B}}$
[#]		FSF = 1.0000	$[\mathrm{cm}^{-1}]$	$[\mathrm{cm}^{-1}]$	[cm ⁻¹]	$[\mathrm{cm}^{-1}]$	$[\mathrm{cm}^{-1}]$	$[\mathrm{cm}^{-1}]$
		HF	3719.692	2246.893	3607.484	793.610	882.237	1.2113
		B3LYP	3542.390	2087.886	3442.297	534.175	774.627	1.1766
0	.H — — .H	PBE0	3563.447	2102.532	3460.432	575.732	780.424	1.1770
		MP2	3568.687	2001.497	3480.443	374.534	747.863	1.1585
		HF	3705.880	2211.601	3601.439	787.455	880.998	1.1824
		B3LYP	3529.920	2054.704	3436.511	530.081	773.484	1.1485
1	-H	PBE0	3550.771	2069.187	3454.626	571.308	779.278	1.1489
		MP2	3557.340	1969.037	3474.553	371.702	746.750	1.1307
		HF	3670.580	2082.710	2843.301	657.380	837.488	1.0184
		B3LYP	3498.078	1946.382	2695.795	466.499	697.315	0.9901
2		PBE0	3517.957	1958.769	2712.453	499.230	707.655	0.9903
		MP2	3528.849	1883.762	2696.072	332.028	662.589	0.9762
3		HF	3691.056	2175.212	3596.685	781.435	879.596	1.1529
	10 117 117 10	B3LYP	3516.414	2020.534	3431.992	526.002	772.308	1.1197
		PBE0	3537.079	2034.844	3450.073	566.906	778.088	1.1201
		MP2	3544.806	1935.681	3470.024	368.870	745.624	1.1022
		HF	3650.710	2053.231	2836.946	655.611	831.742	0.9935
4		B3LYP	3479.841	1917.850	2690.606	463.780	694.732	0.9659
a		PBE0	3499.486	1930.164	2707.181	496.565	704.675	0.9661
		MP2	3511.667	1854.562	2692.251	329.755	660.872	0.9522
		HF	3668.247	2067.710	2803.612	648.627	837.200	1.0018
5		B3LYP	3496.169	1930.915	2659.920	460.335	697.020	0.9740
		PBE0	3515.974	1943.344	2676.210	492.517	707.511	0.9742
		MP2	3527.424	1866.274	2663.404	327.934	661.778	0.9601
		HF	2997.782	1972.156	2648.723	647.765	662.808	0.8692
6		B3LYP	2833.916	1846.155	2527.438	445.952	568.754	0.8457
		PBE0	2853.342	1857.424	2540.753	480.778	573.011	0.8458
		MP2	2817.727	1793.148	2555.446	312.166	549.103	0.8346
		HF	3648.706	2036.737	2797.652	646.773	831.499	0.9764
7	(m)	B3LYP	3478.197	1901.036	2655.100	457.617	694.411	0.9491
		PBE0	3497.776	1913.382	2671.303	489.848	704.507	0.9493
		MP2	3510.435	1835.879	2659.907	325.662	660.044	0.9355
		HF	2965.050	1955.340	2640.730	644.736	656.746	0.8542
8	6 2 6	B3LYP	2804.642	1829.336	2519.795	441.000	567.229	0.8310
~		PBE0	2823.697	1840.609	2533.075	475.411	571.500	0.8311
		MP2	2791.728	1774.858	2547.684	308.749	547.595	0.8201
		HF	2931.432	1937.423	2633.996	644.163	648.181	0.8387
9	(p) (2) (p) (p)	B3LYP	2774.495	1811.482	2513.385	436.132	565.592	0.8160
~		PBE0	2793.182	1822.756	2526.626	470.176	569.825	0.8160
		MP9	2764 762	1755 570	9541 937	305 350	546 050	0.8050

- TOPSEGI : Python-based software package developed to improve the accuracy of molecular temperature and column density estimations in interstellar environments
- Lee, M., Park, J., Oh, S., Cheoun, M.K., Park, S.Y., 2025(arXiv:2501.15824).
- The package is implemented in Python 3 and is freely available on GitHub at https://github.com/BrownNo28/ISM.
- Chojeong, in central Korea, is my hometown !



- > Originates from a dialect in central Korea, meaning "dust(ほこり)" or "tiny particles".
- This name symbolizes the focus on analyzing molecular components, which, like dust in the universe, are the fundamental building blocks of the ISM.



M42 (Orion Nebula)

- Rotation Diagram (Traditional)
 Single-parameter fitting
- Assuming LTE(Local Thermodynamic Equilibrium)

$$\ln \frac{N_j}{g_j} = -\frac{1}{T_{ex}} \frac{E_l}{k} + \ln \frac{N}{Q_R(T_{ex})}$$
$$y = ax + b$$

- Analysis Tool : Rotation Diagram
 Structure : (0). C₂H₂, symmetric(odd)
- WLR(y = ax + b)32 Branch P $\overline{\mathbf{A}}$ Branch Q 10(N/Jdl) Φ Branch R 0 29 Φ $a = (-5.71 \pm 0.38) \times 10^{-3}$ $\diamond b = (3.22 \pm 0.01) \times 10^{1}$ $T[K] = (1.75 \pm 0.12) \times 10^2$ 28- $N[cm^{-2}] = (1.50 \pm 0.15) \times 10^{16}$ 200 400 600 800 0 $E_l/k_b[K]$

Quadtree : 2D-optimization algorithm(This work)
 Two-parameter fitting

$$\chi^{2} = \sum_{\text{obs }l} \left(\frac{N_{l}^{\text{theo}} - N_{l}^{\text{obs}}}{\sigma_{l}^{\text{obs}}} \right)^{2}; N_{l}^{\text{theo}} = N \frac{g_{l}}{Q(T)} \exp\left(-\frac{E_{l}}{k_{B}T}\right)$$

$$N_i^{\text{theo}} = N_i^{\text{theo}}(N, T)$$
 Quadtree Node_{max}(*i*





Goldsmith, P. F., & Langer, W. D. 1999, The AstrophysicalJournal, 517, 209

• Nickerson, Sarah, et al. "The mid-infrared molecular inventory toward orion IRc2." The Astrophysical Journal 945.1 (2023): 26.











Species	Source	Estimated Temperature	Estimated Column Density	Method
-		T (K)	N (cm ⁻²)	
		Blue Clump		
C ₂ H ₂ (even, Para)	Nickerson et al. (2023)	145 ± 9	$(1.23 \pm 0.15) \times 10^{16}$	RD
	This work	161^{+3}_{-3}	$(0.983^{+0.019}_{-0.018}) \times 10^{16}$	χ^2
C ₂ H ₂ (odd, Ortho)	Nickerson et al. (2023)	175 ± 12	$(1.50 \pm 0.15) \times 10^{16}$	RD
	This work	192^{+3}_{-3}	$(1.31^{+0.015}_{-0.014}) \times 10^{16}$	χ^2
¹³ CCH ₂	Nickerson et al. (2023)	91 ± 9	$(2.56 \pm 0.18) \times 10^{15}$	RD
	This work	87.1 ^{+12.3} -10.4	$(2.45^{+0.20}_{-0.19}) \times 10^{15}$	χ^2
		Red Clump		
C ₂ H ₂ (even, Para)	Nickerson et al. (2023)	158 ± 16	$(3.09 \pm 0.57) \times 10^{15}$	RD
	This work	170^{+11}_{-10}	$(2.43^{+0.18}_{-0.18}) \times 10^{15}$	χ^2
C ₂ H ₂ (odd, Ortho)	Nickerson et al. (2023)	229 ± 27	$(3.58 \pm 0.71) \times 10^{15}$	RD
	This work	270^{+21}_{-19}	$(2.65^{+0.17}_{-0.16}) \times 10^{15}$	χ^2
¹³ CCH ₂	Nickerson et al. (2023)	64 ± 6	$(6.74 \pm 0.64) \times 10^{14}$	RD
	This work	$64.4^{+70.7}_{-23.4}$	$(6.69^{+1.98}_{-1.66}) \times 10^{14}$	χ^2



The increase in uncertainty arises from the limited number of observational data points (four) used in the fitting process.

> ^{1}H ^{12}C ^{12}C ^{1}H ^{1}H ^{13}C ^{12}C ^{1}H

^{12}C	_	$2(N(\text{Ortho} - C_2H_2) + N(\text{Para} - C_2H_2))$
13C		$N(^{13}CCH_2)$

Source \ Clump	Blue Clump	Method	Red Clump	Method
Nickerson et al. (2023) (¹² C/ ¹³ C)	21.3 ± 2.2	RD	19.8 ± 3.4	RD
This Work (¹² C/ ¹³ C)	$18.72^{+1.54}_{-1.46}$	χ^2	$15.07^{+1.61}_{-1.60}$	$\chi^2 ({}^{12}C) + RD ({}^{13}C)$

The RD method overestimates the 12C/13C ratio.

Result (example : pseudo data of 13C2H2)



Future work : HNC

	Η	HITRANonline							
		Home	Data Ace	cess D					
	Lir	Line-by-Line Search							
	2. 9	2. Select Isotopologues							
	Sele	Select isotopologues for the following molecules the							
	ID	ID Formula AFGL Code Abundance							
	2 1	$H^{12}C^{14}N$	124	0.985114					
	2	H ¹³ C ¹⁴ N	134	0.011068					
	☑ 3	H ¹² C ¹⁵ N	125	0.003622					
ID	Formula								
1	¹ H ¹² C ¹⁴ N								
2		1	H ¹³ C	¹⁴ N					
3		1	H ¹² C	¹⁵ N					
4		2	D ¹² C	¹⁴ N					
5		2	D ¹³ C	¹⁴ N					
6		1	H ¹³ C	¹⁵ N					
7		2	D ¹² C	¹⁵ N					
8		2	D ¹³ C	¹⁵ N					

Ta	ble	2.	Observe	$1 \nu_2$	band	HNC	Transitions	and	Inferred	Parameter
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Transition	Wavenumber	E_l/k_b	g_l	$v_{\rm LSR}$	$ au_0$	N_l
	(cm^{-1})	(K)		$(\rm km~s^{-1})$		$\times 10^{14} \rm cm^{-2}$
P3e	453.64798	26.1	42	$-6.8 {\pm} 0.3$	$0.031{\pm}0.003$	1.09 ± 0.20
P5e	447.5964	65.3	66	$-7.0 {\pm} 0.2$	$0.041{\pm}0.001$	$0.94{\pm}0.05$
P6e	444.57022	91.4	78	$-5.3 {\pm} 0.4$	$0.029 {\pm} 0.003$	$0.86 {\pm} 0.19$
P7e	441.54388	121.8	90	$-9.1 {\pm} 0.3$	$0.031{\pm}0.003$	$0.65 {\pm} 0.11$
P8e	438.51747	156.6	102	$-7.9 {\pm} 0.3$	$0.028 {\pm} 0.002$	$0.64{\pm}0.08$
Q1e	462.74319	4.4	18	$-8.9 {\pm} 0.2$	$0.057 {\pm} 0.003$	$0.46 {\pm} 0.04$
Q2e	462.78519	13.1	30	$-8.1{\pm}0.2$	$0.071{\pm}0.004$	0.71 ± 0.10
Q3e	462.84818	26.1	42	$-8.9 {\pm} 0.1$	$0.080 {\pm} 0.003$	0.73 ± 0.06
Q4e	462.93214	43.5	54	$-8.4{\pm}0.2$	$0.080 {\pm} 0.003$	0.72 ± 0.05
Q5e	463.03705	65.3	66	$-8.3 {\pm} 0.1$	$0.082{\pm}0.002$	$0.69 {\pm} 0.04$
Q6e	463.16287	91.4	78	$-8.4{\pm}0.2$	$0.078 {\pm} 0.003$	$0.81 {\pm} 0.04$
Q8e	463.47714	156.6	102	$-7.7 {\pm} 0.2$	$0.055 {\pm} 0.003$	$0.54 {\pm} 0.06$
Q9e	463.66551	195.8	114	$-7.7 {\pm} 0.3$	$0.037 {\pm} 0.002$	$0.30 {\pm} 0.04$
Q11e	464.10446	287.1	138	$-8.1 {\pm} 0.5$	$0.022{\pm}0.005$	$0.13 {\pm} 0.07$
Q12e	464.35494	339.3	150	$-5.9{\pm}0.6$	$0.013 {\pm} 0.002$	0.09 ± 0.02
R0e	465.74576	0.0	6	$-8.3 {\pm} 0.3$	$0.037 {\pm} 0.003$	$0.17 {\pm} 0.03$
R1e	468.76863	4.4	18	$-8.0 {\pm} 0.2$	$0.069 {\pm} 0.003$	$0.79 {\pm} 0.05$
R2e	471.7907	13.1	30	$-8.1{\pm}0.2$	$0.076 {\pm} 0.003$	$0.92 {\pm} 0.08$
R3e	474.8119	26.1	42	$-8.0 {\pm} 0.2$	$0.081{\pm}0.003$	1.01 ± 0.08
R5e	480.85136	65.3	66	$-8.2{\pm}0.2$	$0.071{\pm}0.003$	$0.81 {\pm} 0.04$
m R7e	486.88636	121.8	90	$-7.7 {\pm} 0.3$	$0.048 {\pm} 0.003$	$0.78 {\pm} 0.07$
R8e	489.902	156.6	102	$-6.2{\pm}0.3$	$0.042{\pm}0.003$	$0.51 {\pm} 0.08$
R9e	492.91629	195.8	114	$-7.8{\pm}0.3$	$0.043 {\pm} 0.002$	0.75 ± 0.06
R10e	495.92916	239.3	126	$-5.9{\pm}0.5$	$0.026{\pm}0.004$	$0.30 {\pm} 0.07$

Table 3. Observed ν_2 band H¹³CN Transitions and Inferred Parameters

Transition	Wavenumber	E_l/k_b	g_l	$v_{\rm LSR}$	$ au_0$	N_l
	(cm^{-1})	(K)		$(\mathrm{km}~\mathrm{s}^{-1})$		$\times 10^{14} \mathrm{cm}^{-2}$
R1e	711.72312	4.1	36	-7.5 ± 0.4	$0.059 {\pm} 0.005$	$2.46 {\pm} 0.27$
R6e	726.098321	87.0	156	-6.5 ± 0.2	$0.083 {\pm} 0.005$	$5.30 {\pm} 0.44$
R8e	731.839651	149.2	204	$-5.7 {\pm} 0.3$	$0.054{\pm}0.005$	3.05 ± 0.40

https://hitran.org/

 Nickerson, Sarah, et al. "The first mid-infrared detection of HNC in the interstellar medium: Probing the extreme environment toward the orion hot core." The Astrophysical Journal 907.1 (2021): 51.

- > Acetylene is among the simplest organic molecules found abundantly in outer space.
- > 12C/13C is an important diagnostic tool for probing the Galactic chemical evolution or simply the nucleosynthesis history of the Galaxy. (AGB Star => $(12C(p,\gamma)13N(\beta+)13C))$
- The rotational diagram (RD) method overestimates the 13C/12C ratio, while the χ² method reduces uncertainties for more accurate and reliable results.

Source \ Clump	Blue Clump	Method	Red Clump	Method
Nickerson et al. (2023) (¹² C/ ¹³ C)	21.3 ± 2.2	RD	19.8 ± 3.4	RD
This Work (12C/13C)	$18.72^{+1.54}_{-1.46}$	χ^2	$15.07^{+1.61}_{-1.60}$	$\chi^2 ({}^{12}C) + RD ({}^{13}C)$

- > The RD method **overestimates** the 12C/13C ratio.
- TOPSEGI (Python package) developed in this study facilitates efficient χ2 fitting and isotopic ratio analysis.
- Future work will expand to include the analysis of molecules and isotopologues in space, such as HCN.



Thank you for your attention

