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Recent progress on exotic nuclei in relativistic density functional theory with deformation and continuum effects

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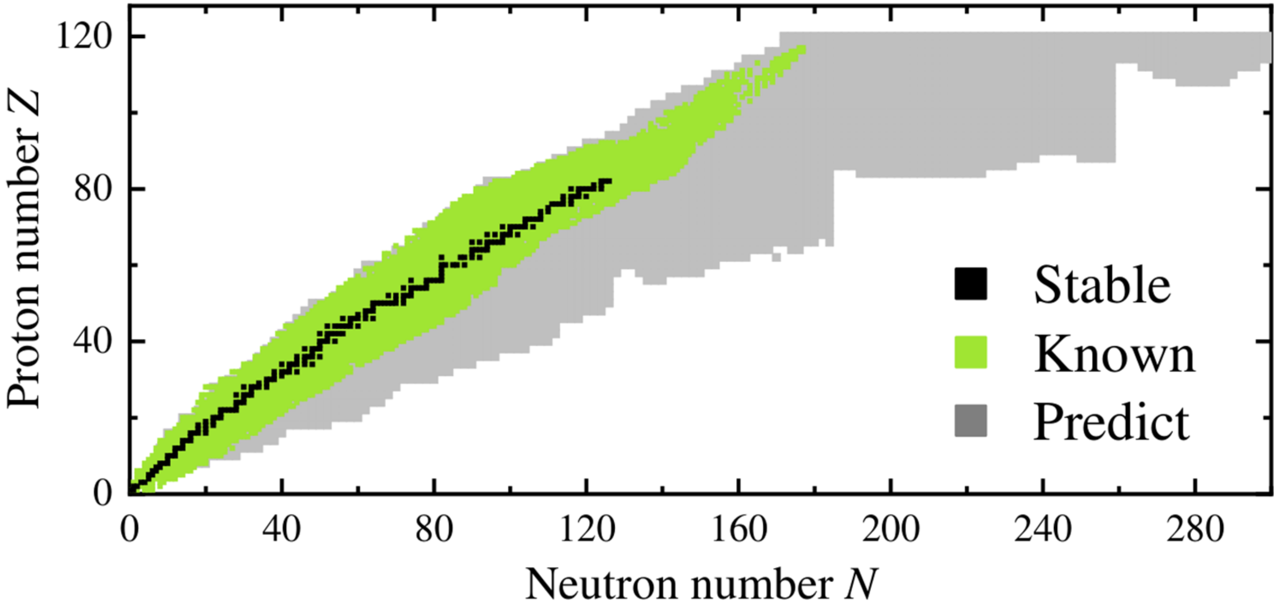
Outline

- **Introduction**
- Theoretical framework
- Achievements of DRHBc
- Recent progress on nuclear radii
- Summary

Exotic nuclei

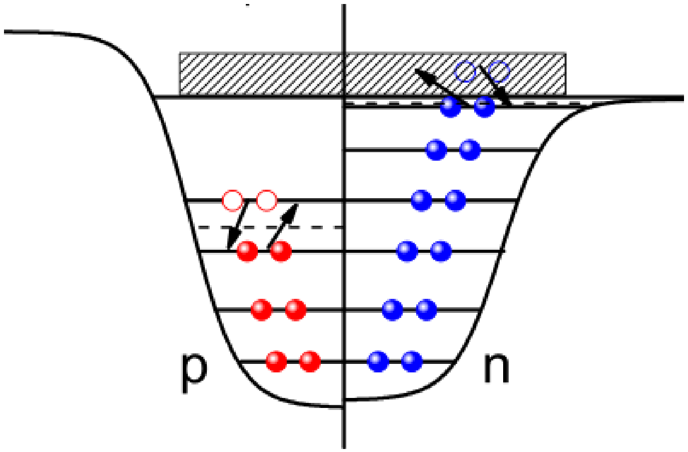
- The study of exotic nuclei far away from β-stability line is one of the most important frontiers in nuclear physics.
  - Neutron/proton halo   Tanihata *et al.*, PRL 55, 2676 (1985)
  - Change in magic number   Ozawa *et al.*, PRL 84, 5493 (2000)
  - Pygmy resonance   Adrich *et al.*, PRL 95, 132501 (2005)
  - ……



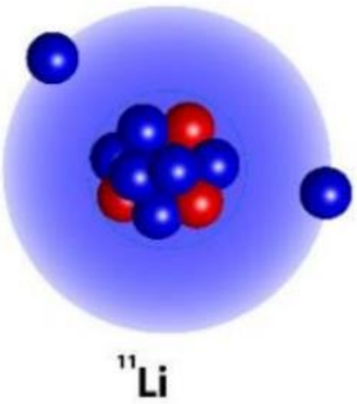
National Nuclear Data Center (NNDC)
DRHBc Mass Table Collaboration, ADNDT 106, 014316 (2022)

Continuum effect

- In halo nuclei, the system is weakly bound, and the Fermi energy is close to the continuum threshold:



Meng *et al.*, PPNP 57, 470 (2006)



From X.H. Wu

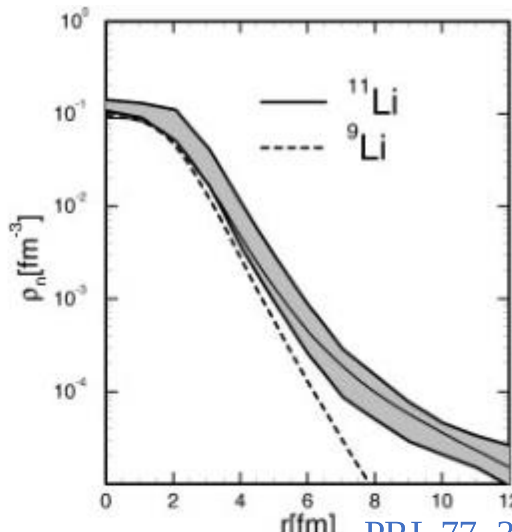
- ➢ The pairing interaction can scatter nucleons from bound states to the resonant states in the continuum.
  —— Effects of pairing correlations and continuum
- ➢ The density could become more diffuse.
  —— Asymptotic behaviors of nuclear density far away from the center.

RCHB theory

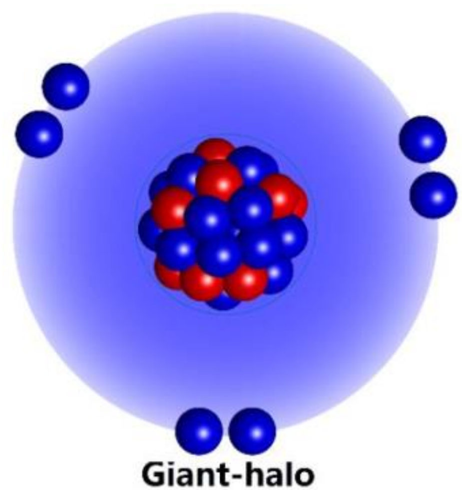
- Based on relativistic density functional theory, with pairing correlation and continuum effects properly considered, Meng and Ring developed the spherical relativistic continuum Hartree-Bogoliubov (RCHB) theory.

Meng and Ring, PRL 77, 3963 (1996)
Meng, NPA 635, 3 (1998)

- RCHB was successfully applied in many studies on exotic nuclei:
  - Interpreting of the halo in ^{11}Li Meng and Ring, PRL 77, 3963 (1996)
  - Prediction of the giant halo Meng and Ring, PRL 80, 460 (1998)
  - First mass table including continuum effects Xia *et al.*, ADNDT 121,1 (2018)
  - …… Meng *et al.*, PPNP 57, 470 (2006)



PRL 77, 3963 (1996)



From X.H. Wu

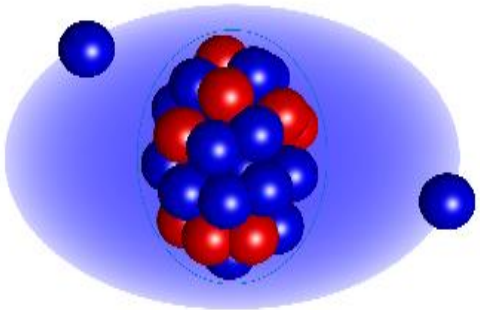
DRHBc theory

- For axially deformed halo nuclei, Zhou *et al*. developed the deformed relativistic Hartree-Bogoliubov theory in continuum (DRHBc), with the effects of deformation, pairing correlations and continuum taken into account simultaneously.

Zhou *et al*., PRC 82, 011301 (2010)

- The deformed halo nuclei ^{42,44}Mg were predicted by the DRHBc theory, and the shape decoupling between halo and core was revealed.

Zhou *et al*., PRC 82, 011301 (2010)
Li *et al*., PRC 85, 024312 (2012)



From X.H. Wu

In this talk

✓ The achievements of the DRHBc theory are briefly reviewed.

✓ Recent progresses on charge radii and neutron radii based on the DRHBc theory are reported.

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DRHBc theory

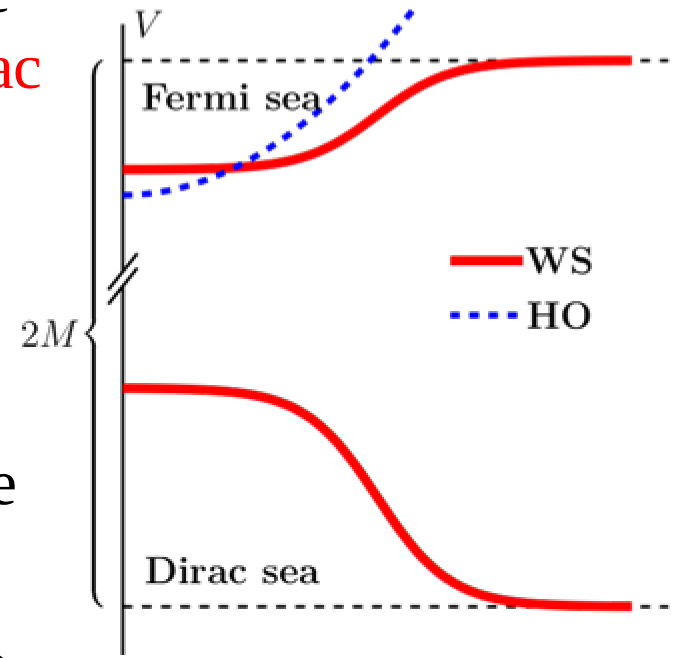
- Considering mean fields and pairing correlations self-consistently, the relativistic Hartree-Bogoliubov (RHB) equation for nucleons:

$$\begin{pmatrix} \hat{h}_D - \lambda_\tau & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h}_D^* + \lambda_\tau \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}$$

Kucharek and Ring, ZPA 339, 23 (1991)

- To properly consider continuum effect, the RHB equation is solved in a spherical Dirac Woods-Saxon (DWS) basis.

Zhou *et al*., PRC, 68, 034323 (2003)



- For the nuclei with axial deformation and spatial reflection symmetry, the densities and potentials are expanded with Legendre polynomials:

$$f(\boldsymbol{r}) = \sum_\lambda f_\lambda(r) P_\lambda(\cos\theta), \quad (\lambda = 0, 2, 4, \ldots, \lambda_{\max})$$

Zhou *et al*., PRC 82, 011301 (2010)

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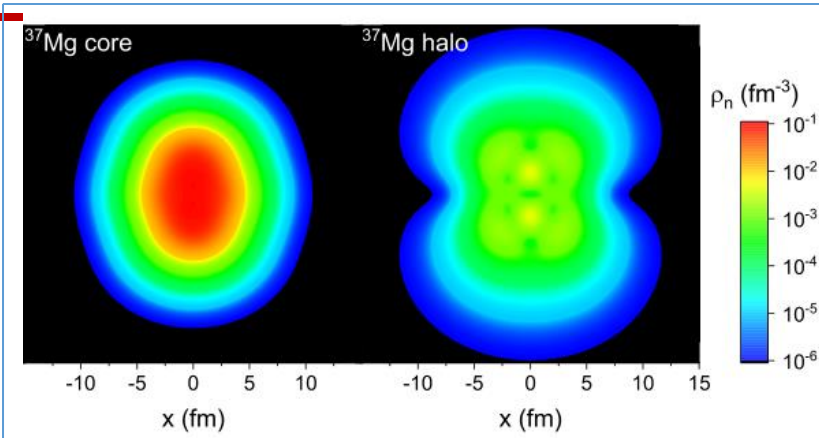
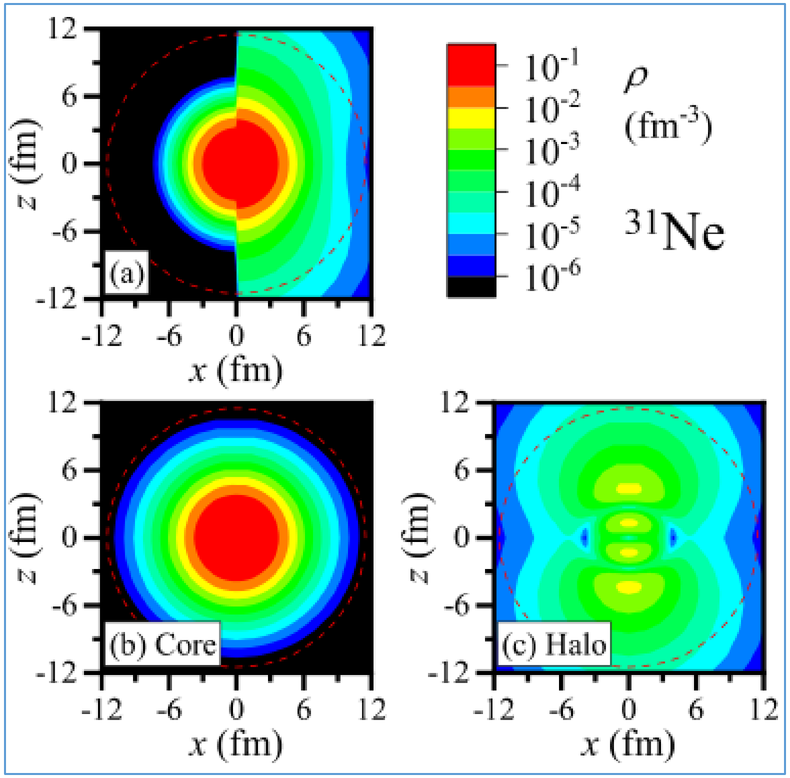
Halo structures

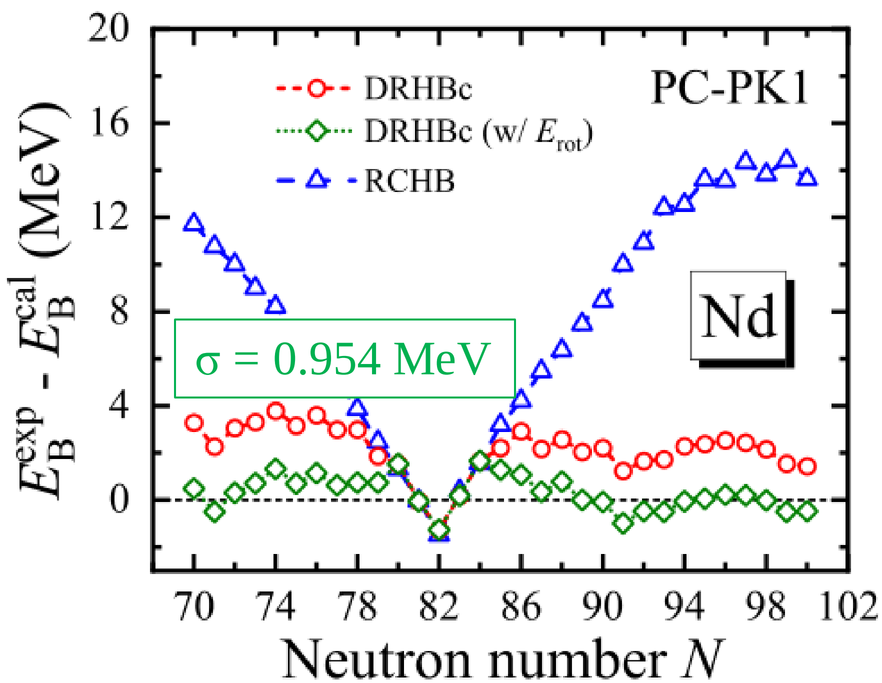
- The DRHBc theory has been applied in many studies on the interpretations and predictions of halo nuclei.
  - C — PLB 785, 530 (2018)
    NPA 1003, 122011 (2020)
  - B — PRL 126, 082501 (2021)
    PRC 103, 054315 (2021)
  - Ne — SCPMA 65, 262011 (2022)
    PLB 855, 138792 (2024)
  - Al — PRC 108, L041301 (2023)
    PRC 110, 014320 (2024)
  - Mg — PRC 82, 011301 (2010)
    PRC 85, 024312 (2012)
    PLB 844, 138112 (2023)
    PLB 849, 138422 (2024)
  - Na — PRC 107, L041303 (2023)



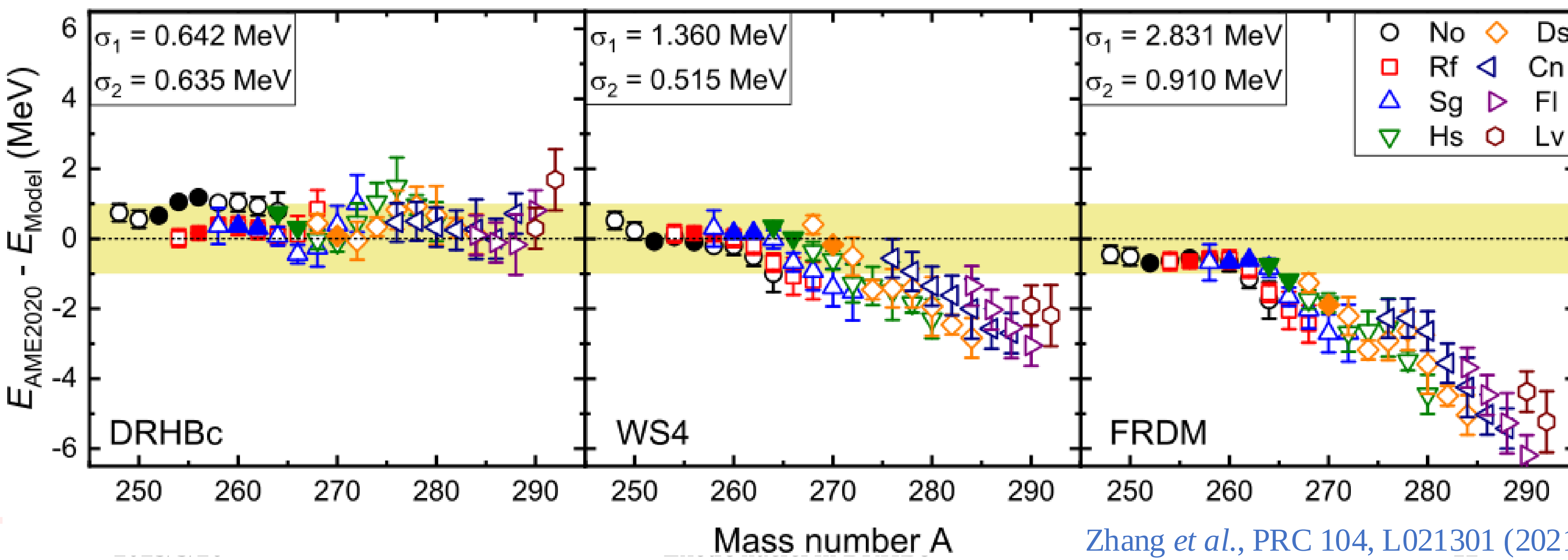


DRHBc for masses

- Taking several isotopic chains as examples, the experimental masses are well reproduced.
- The comparisons with mac-mic models illustrate the predictive power of microscopic DFT.



Pan *et al.*, PRC 106, 014316 (2022)

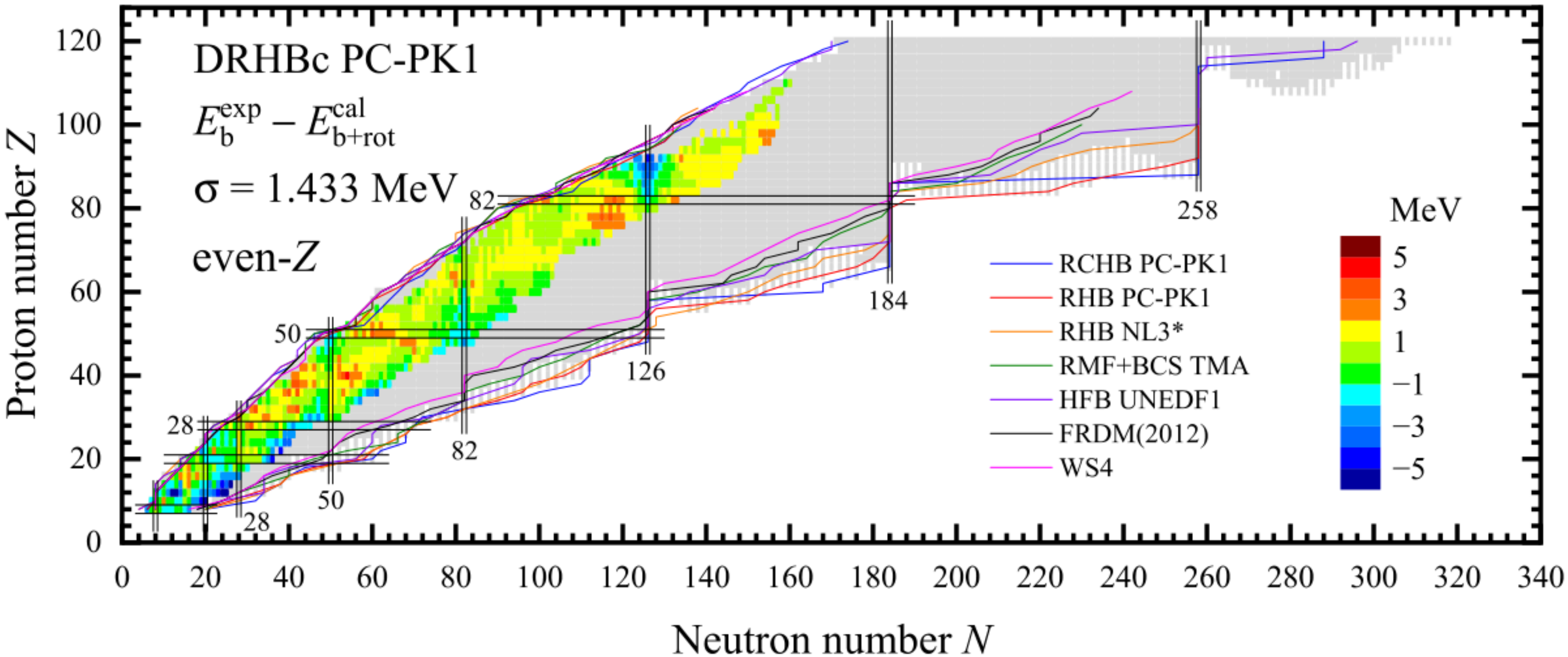


Zhang *et al.*, PRC 104, L021301 (2021)

DRHBc mass table

https://drhbctable.jcnp.org/

- In 2018, the DRHBc Mass Table Collaboration was established, aiming to provide a microscopic mass table that simultaneously includes the effects of deformation and continuum.
- The even-*Z* part of the DRHBc mass table has been completed.



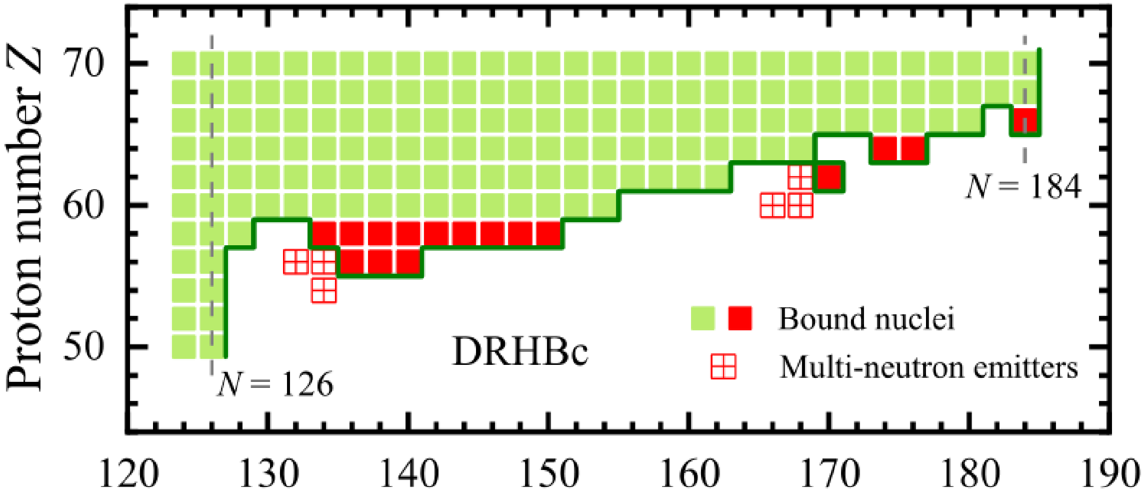
Zhang *et al.*, (DRHBc Mass Table Collaboration) PRC 102, 024314 (2020)
Pan *et al.*, (DRHBc Mass Table Collaboration) PRC 106, 014316 (2022)
Zhang *et al.*, (DRHBc Mass Table Collaboration) ADNDT 144, 101488 (2022)
Guo *et al.*, (DRHBc Mass Table Collaboration) ADNDT 158, 101661 (2024)

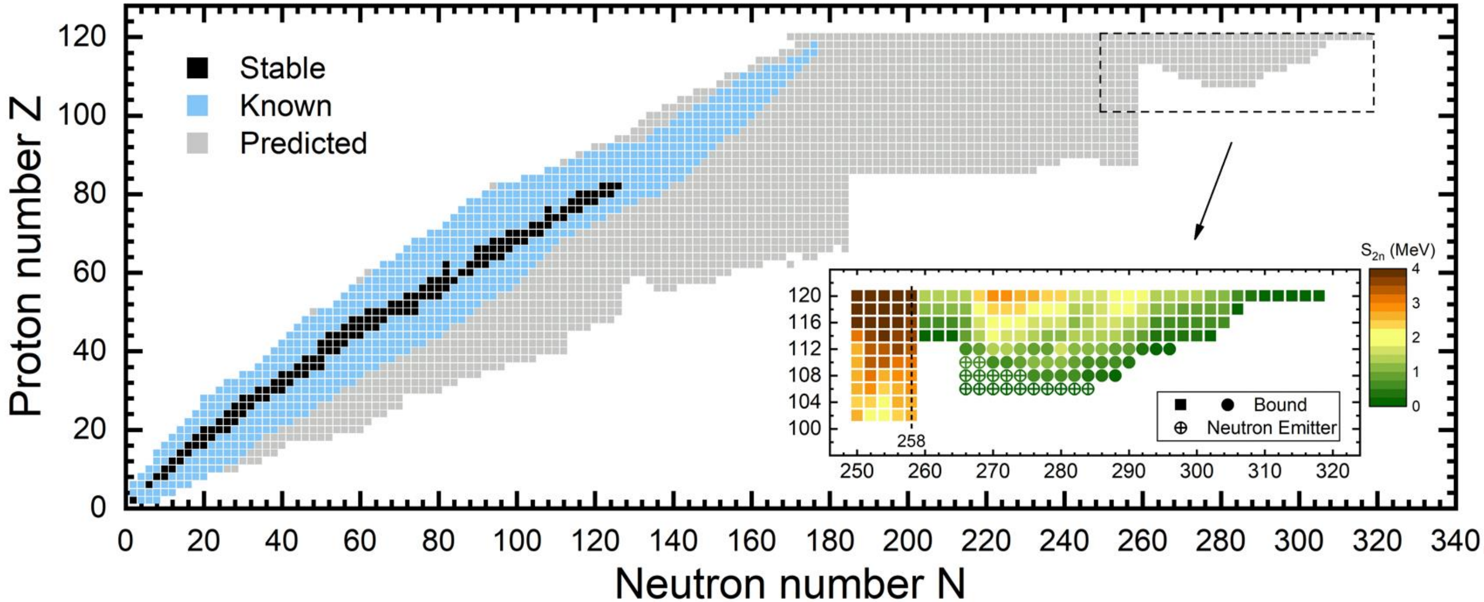
Applications of DRHBc

https://drhbctable.jcnp.org/

- Possible "peninsulas" in the nuclear landscape

Zhang *et al.*, PRC 104, L021301 (2021)
He *et al.*, CPC 45, 101001 (2021)
Pan *et al.*, PRC 104, 024331 (2021)
He *et al.*, PRC 110, 014301 (2024)





Applications of DRHBc

https://drhbctable.jcnp.org/

- Possible "peninsulas" in the nuclear landscape

  Zhang *et al.*, PRC 104, L021301 (2021) Pan *et al.*, PRC 104, 024331 (2021)
  He *et al.*, CPC 45, 101001 (2021) He *et al.*, PRC 110, 014301 (2024)

- Proton emission and α-decay

  Xiao *et al.*, PLB 845, 138160 (2023) Choi *et al.*, PRC 109, 054310 (2024)

- Angular momentum projection and rotating deformed halo nuclei

  Sun *et al.*, Sci. Bull., 66 2072 (2021) Sun *et al.*, PRC, 104 064319 (2021)

- Dynamical correlations with collective Hamiltonian

  Sun *et al.*, CPC 46 064103 (2022) Zhang *et al.*, PRC 108, 024310 (2023)

- Solar *r*-process simulation

  Choi *et al.*, arXiv:2411.19470 (2024) Pan *et al.*, arXiv:2503.09324 (2025)

- Extension to triaxiality and nuclear magnetism

  Zhang *et al.*, PRC 108, L041301 (2023) Pan *et al.*, PLB 855, 138792 (2025)

- More interesting studies ……

  Pan *et al.*, IJMPE 28, 1950082 (2019) Zhang *et al.*, PRC 100, 034312 (2019)
  In *et al.*, IJMPE 30, 2150009 (2021) Choi *et al.*, PRC 105, 024306 (2022)
  Kim *et al.*, PRC 105, 034340 (2022) Guo *et al.*, PRC 108, 014319 (2023)
  Mun *et al.*, PLB 847, 138298 (2023) Zheng *et al.*, CPC 48, 014107 (2024)
  Mun *et al.*, PRC 110, 014314 (2024) Zhang et al., CPC 48, 104105 (2024)
  Huang *et al.*, PRC 111, 034314 (2025) ……

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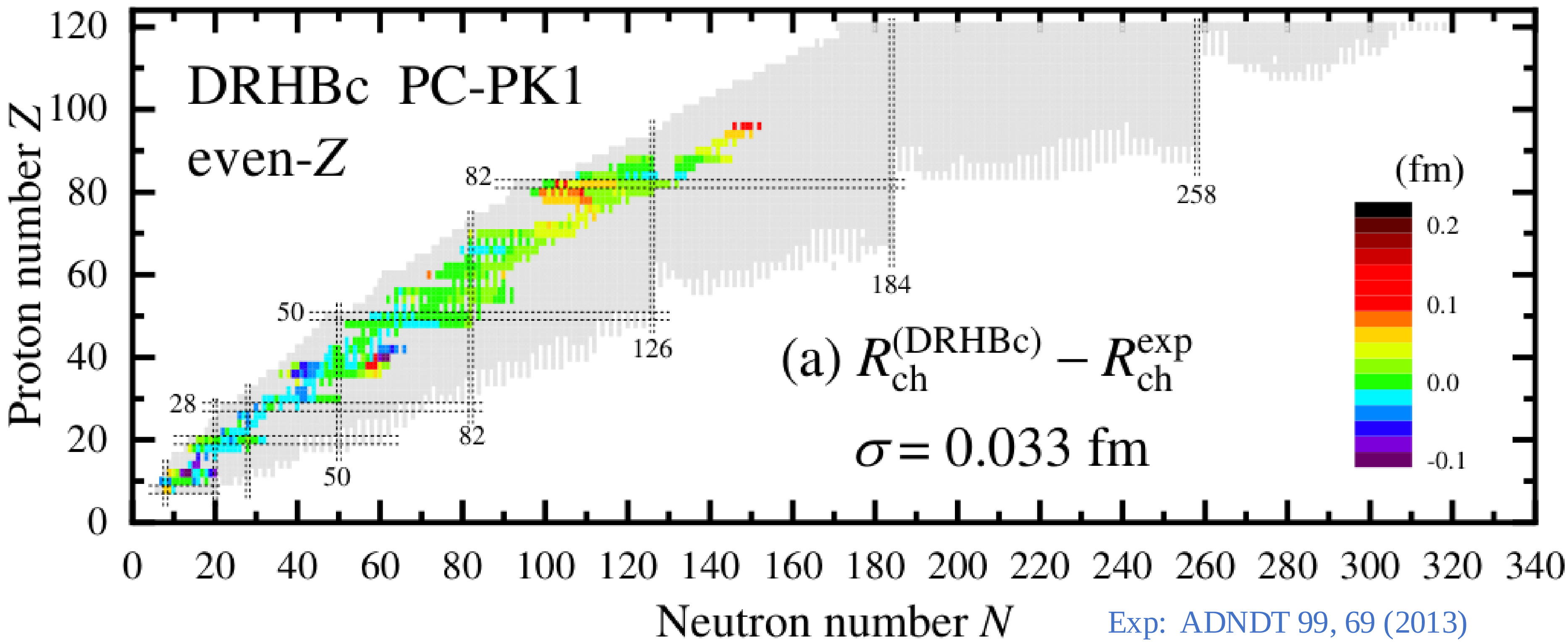
Charge radii

Collaborator: Prof. J. Meng

- Nuclear charge radius is one of the most important properties in atomic nuclei, reflecting significant information.

JPG 42, 055108 (2015)

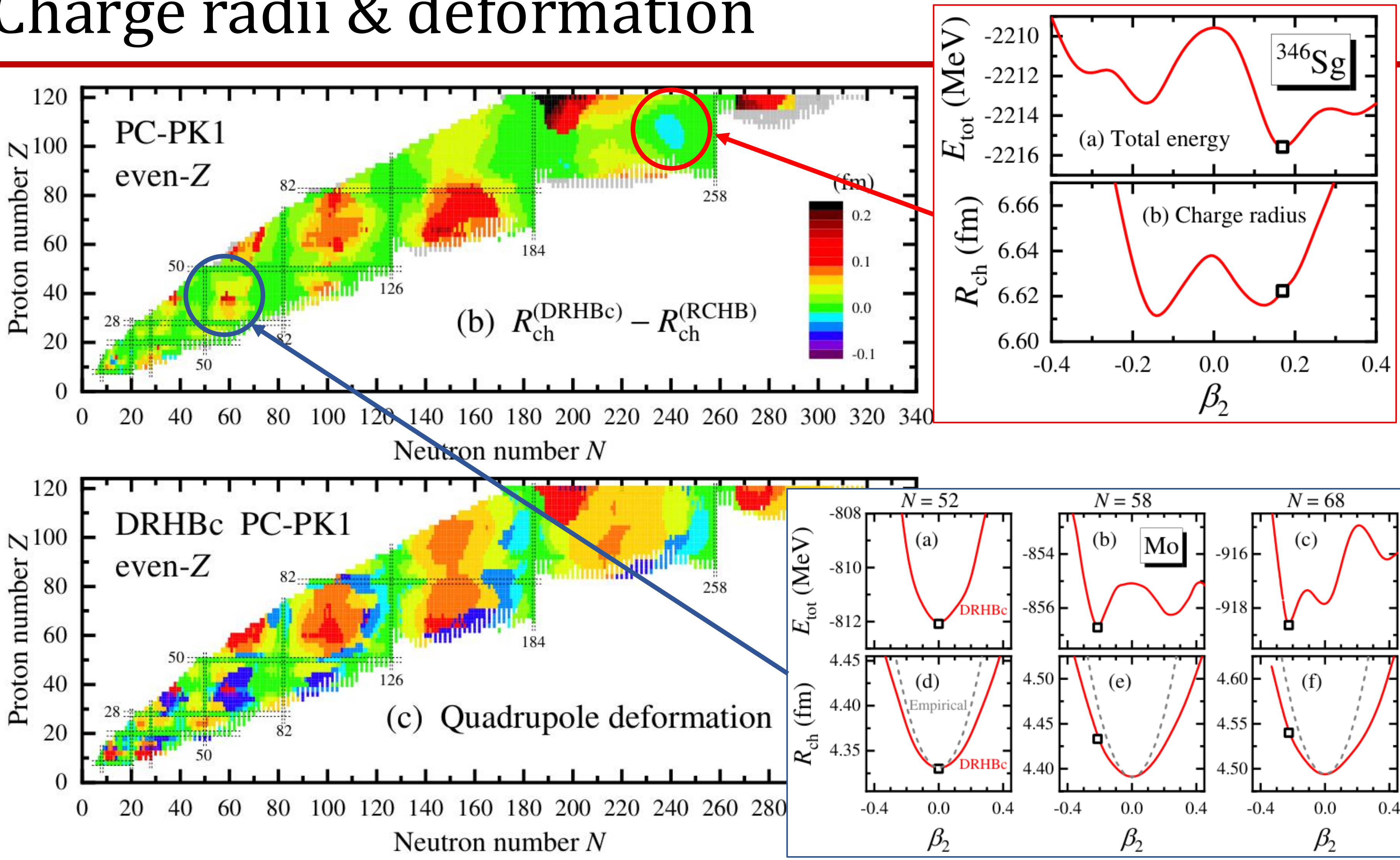


Exp: ADNDT 99, 69 (2013)
ADNDT 140, 101440 (2021)

➢ DRHBc reproduces the R_{ch} data with σ = 0.033 fm, smaller than the RCHB result σ = 0.036 fm.

ADNDT 121, 1 (2018)

Charge radii & deformation



■ For most nuclei, the deformation effect increases R_{ch}.

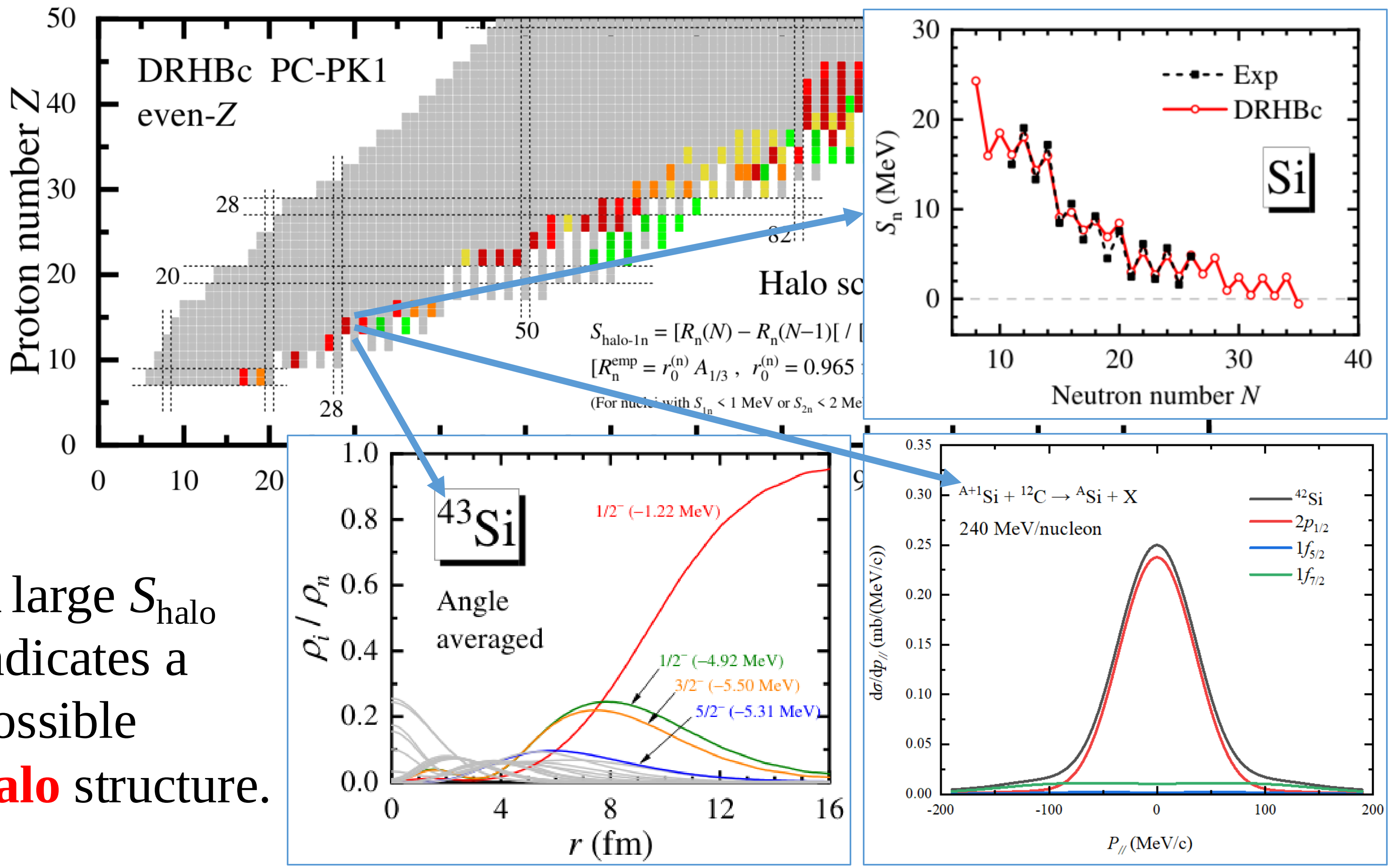
■ Near $Z = 100$, $N = 240$, deformation effect decreases R_{ch}.

Neutron radii & Halo scales

Collaborators: Prof. S.S. Zhang
Ms. J.L. An

$$S_{\text{halo}} = \frac{\Delta R_n^{\text{exp(cal)}}}{\Delta R_n^{\text{emp}}} = \frac{R_n^{\text{exp(cal)}}(N+m) - R_n^{\text{exp(cal)}}(N)}{R_n^{\text{emp}}(N+m) - R_n^{\text{emp}}(N)}$$

Equation taken from PRC 108, L041301 (2023)



- A large S_{halo} indicates a possible **halo** structure.

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Summary and outlook

- The DRHBc theory has been successfully applied in many studies on exotic nuclei.

- Charge radii and their correlations with deformations in even-Z nuclei are investigated based on DRHBc.

- Neutron halo scales of even-Z nuclei are studied, and a large S_{halo} indicates a possible halo structure.

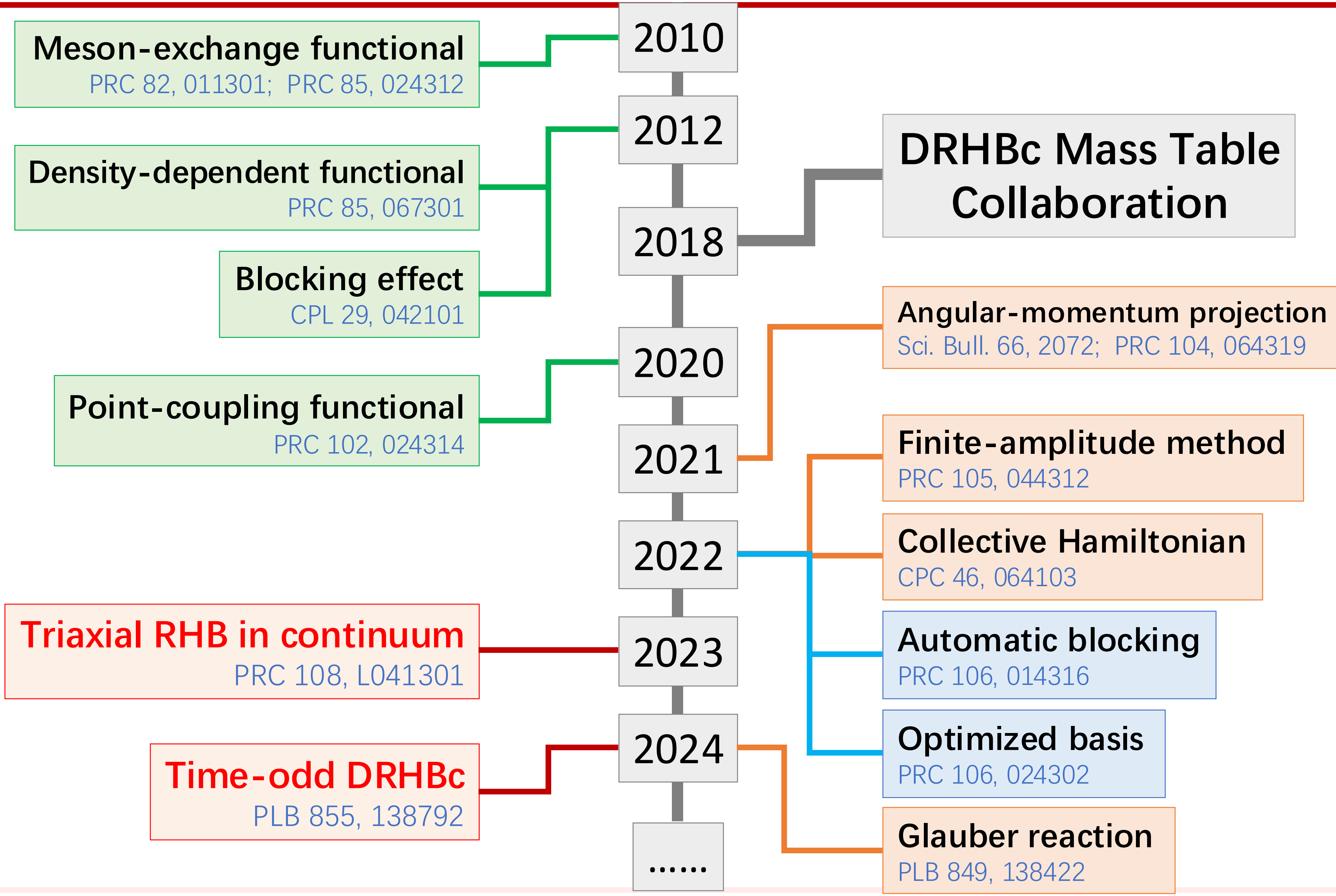
- Based on the DRHBc theory and its extensions, more studies on exotic nuclei are in progress.

Thank you for your attention!

DRHBc Mass Table Collaboration

https://drhbctable.jcnp.org/

Development of DRHBc



Lagrangian

- The starting point of the relativistic density functional theory is a meson-exchange (ME) or point-coupling (PC) Lagrangian.

- A PC Lagrangian: $\mathcal{L} = \mathcal{L}^{\text{free}} + \mathcal{L}^{\text{4f}} + \mathcal{L}^{\text{der}} + \mathcal{L}^{\text{hot}} + \mathcal{L}^{\text{em}},$

$$
\begin{aligned}
\mathcal{L}^{\text{free}} &= \bar{\psi}(i\gamma_\mu\partial^\mu - M)\psi, \\
\mathcal{L}^{\text{4f}} &= -\frac{1}{2}\alpha_S(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi) \\
&\quad - \frac{1}{2}\alpha_{TS}(\bar{\psi}\vec{\tau}\psi)(\bar{\psi}\vec{\tau}\psi) - \frac{1}{2}\alpha_{TV}(\bar{\psi}\vec{\tau}\gamma_\mu\psi)(\bar{\psi}\vec{\tau}\gamma^\mu\psi), \\
\mathcal{L}^{\text{der}} &= -\frac{1}{2}\delta_S\partial_\nu(\bar{\psi}\psi)\partial^\nu(\bar{\psi}\psi) - \frac{1}{2}\delta_V\partial_\nu(\bar{\psi}\gamma_\mu\psi)\partial^\nu(\bar{\psi}\gamma^\mu\psi) \\
&\quad - \frac{1}{2}\delta_{TS}\partial_\nu(\bar{\psi}\vec{\tau}\psi)\partial^\nu(\bar{\psi}\vec{\tau}\psi) - \frac{1}{2}\delta_{TV}\partial_\nu(\bar{\psi}\vec{\tau}\gamma_\mu\psi)\partial^\nu(\bar{\psi}\vec{\tau}\gamma^\mu\psi), \\
\mathcal{L}^{\text{hot}} &= -\frac{1}{3}\beta_S(\bar{\psi}\psi)^3 - \frac{1}{4}\gamma_S(\bar{\psi}\psi)^4 - \frac{1}{4}\gamma_V[(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)]^2, \\
\mathcal{L}^{\text{em}} &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\bar{\psi}\gamma^\mu\frac{1-\tau_3}{2}A_\mu\psi.
\end{aligned}
$$

  - Parameters α, β, γ, and δ are the coupling constants.

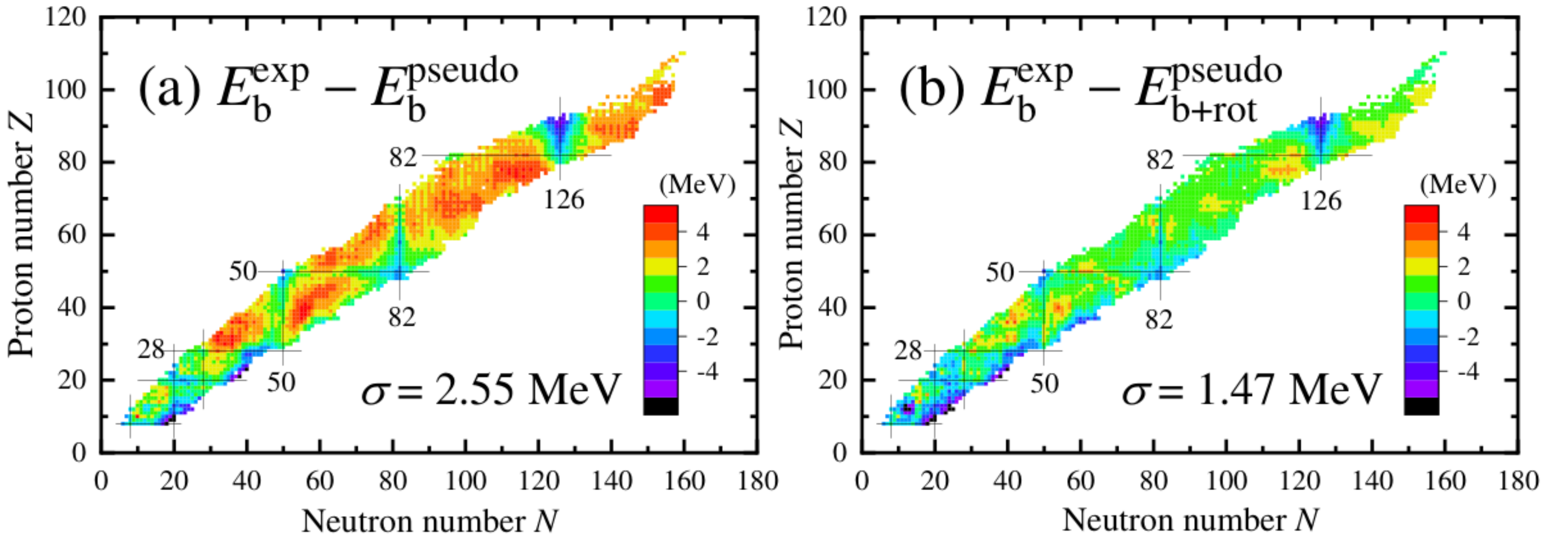
Meng (*Ed.*), Relativistic Density Functional For Nuclear Structure (2016)

"Pseudo" DRHBc mass table

- Binding energies of odd-Z nuclei are estimated by

$$E_B(Z,N) = \frac{1}{2}[E_b(Z+1,N) + E_b(Z-1,N)] - \frac{1}{2}[\delta_p(Z+1,N) + \delta_p(Z-1,N)]$$

with odd-even mass differences δ_p approximated by average pairing gaps.



- A "pseudo" DRHBc mass table for all nuclei is established with the precision close to the "real" mass table, and then applied in r-process simulations.

Pan *et al.*, arXiv:2503.09324 (2025)