

# Dynamical shape coexistence in $^{60}\text{Ca}$

Kouhei Washiyama (CCS, Univ. Tsukuba, Japan)



Collaborator: Kenichi Yoshida (RCNP, Osaka Univ., Japan)

Bohr-Mottelson collective model

Shape coexistence

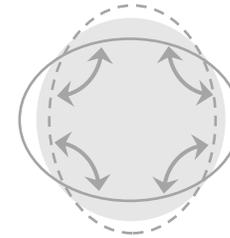
Excited  $0^+$  state—Dynamical shape coexistence



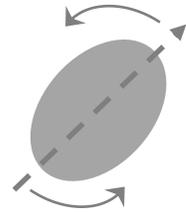
Five-dimensional collective Hamiltonian (5DCH) method

$$\mathcal{H} = T_{\text{vib}} + T_{\text{rot}} + V(\beta, \gamma) \quad \text{Vibrational mass}$$

$$T_{\text{vib}} = \frac{1}{2} D_{\beta\beta}(\beta, \gamma) \dot{\beta}^2 + D_{\beta\gamma}(\beta, \gamma) \dot{\beta} \dot{\gamma} + \frac{1}{2} D_{\gamma\gamma}(\beta, \gamma) \dot{\gamma}^2$$



$$T_{\text{rot}} = \frac{1}{2} \sum_{k=1}^3 \mathcal{J}_k(\beta, \gamma) \omega_k^2$$



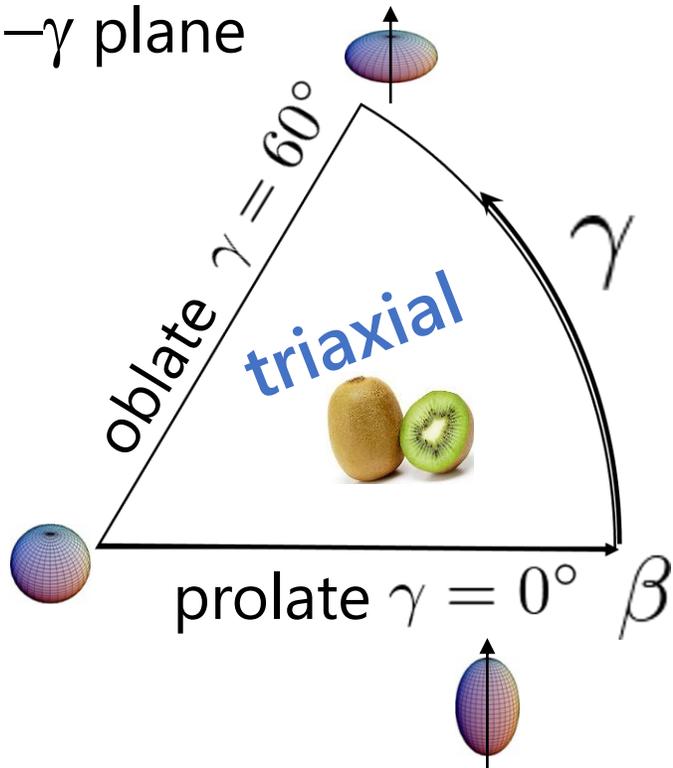
Moment of inertia

# Representation of $\beta, \gamma$ deformation parameters

Quadrupole deformation

$$Q_{2m} = \langle r^2 Y_{2m}(\hat{r}) \rangle = \int r^2 Y_{2m}(\hat{r}) \rho(\mathbf{r}) d^3 r$$

$\beta$ - $\gamma$  plane



$\beta$  Size of deformation

$\gamma$  Degree of non-axial shape

$$\beta \cos \gamma \propto Q_{20}$$

$$\beta \sin \gamma \propto (Q_{22} + Q_{2-2})$$

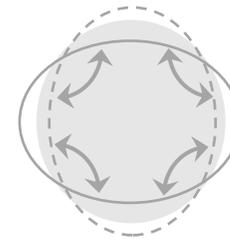
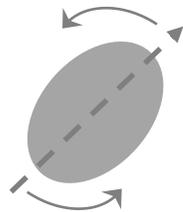
Five-dimensional collective Hamiltonian (5DCH) method

$$\mathcal{H} = T_{\text{vib}} + T_{\text{rot}} + V(\beta, \gamma) \quad \text{Vibrational mass}$$

$$T_{\text{vib}} = \frac{1}{2} D_{\beta\beta}(\beta, \gamma) \dot{\beta}^2 + D_{\beta\gamma}(\beta, \gamma) \dot{\beta} \dot{\gamma} + \frac{1}{2} D_{\gamma\gamma}(\beta, \gamma) \dot{\gamma}^2$$

$$T_{\text{rot}} = \frac{1}{2} \sum_{k=1}^3 \mathcal{J}_k(\beta, \gamma) \omega_k^2$$

Moment of inertia



$V(\beta, \gamma) \leftarrow E[\rho]$  Constrained DFT

$D_{\beta\beta}, D_{\beta\gamma}, D_{\gamma\gamma}, \mathcal{J}_k$  **DFT + Local QRPA (FAM)**

Inertia  $\leftrightarrow$  Quadrupole collectivity

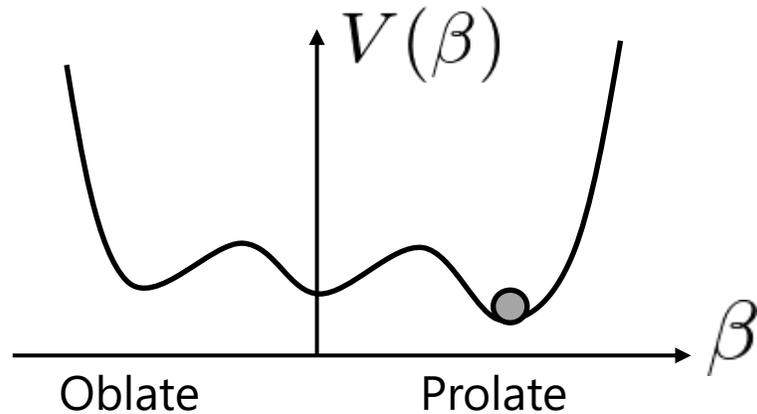
Quantize Hamiltonian

$$\hat{H} \Psi_{\alpha IM}(\beta, \gamma, \Omega) = E_{\alpha I} \Psi_{\alpha IM}(\beta, \gamma, \Omega)$$



$E_{\alpha I}$ , transition probability B(E2)

## Shape coexistence



## Signatures

- Low-lying  $I^\pi = 0^+$  states
  - Bands associated with  $0^+$  states
- + Theoretical interpretation

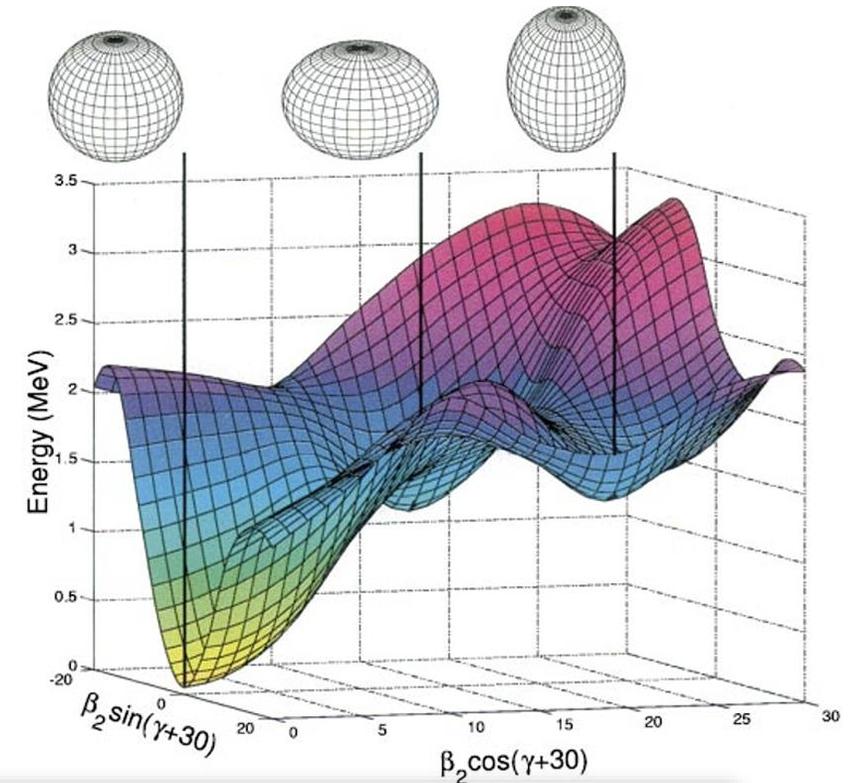
$^{16}\text{O}$ ,  $^{40}\text{Ca}$

$^{186}\text{Pb}$

Andreyev et al.,  
Nature (2000)

$^{74,76}\text{Kr}$

Clement et al.,  
PRC(2007)



In this talk,

**“Dynamical” shape coexistence**

**Candidate :  $^{60}\text{Ca}$**

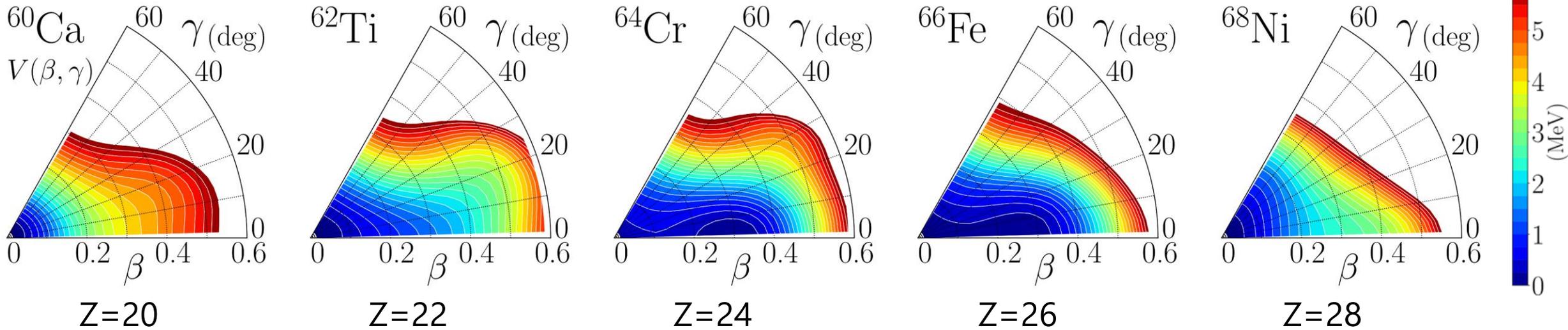
Heyde, Wood, RMP83,1467(2011)

Garrett, Zielinska, Clement, PPNP124, 103931(2022)

To analyze low-lying states in  $N=40$  nuclei

To analyze an emergence of dynamical shape coexistence in  $^{60}\text{Ca}$

## Constrained DFT in $\beta$ - $\gamma$ plane



## Shape transition in mean-field level

Spherical  $\rightarrow$  Prolate  $\rightarrow$  Spherical

$$0 < \beta < 0.6$$

$$0 < \gamma < 60^\circ$$

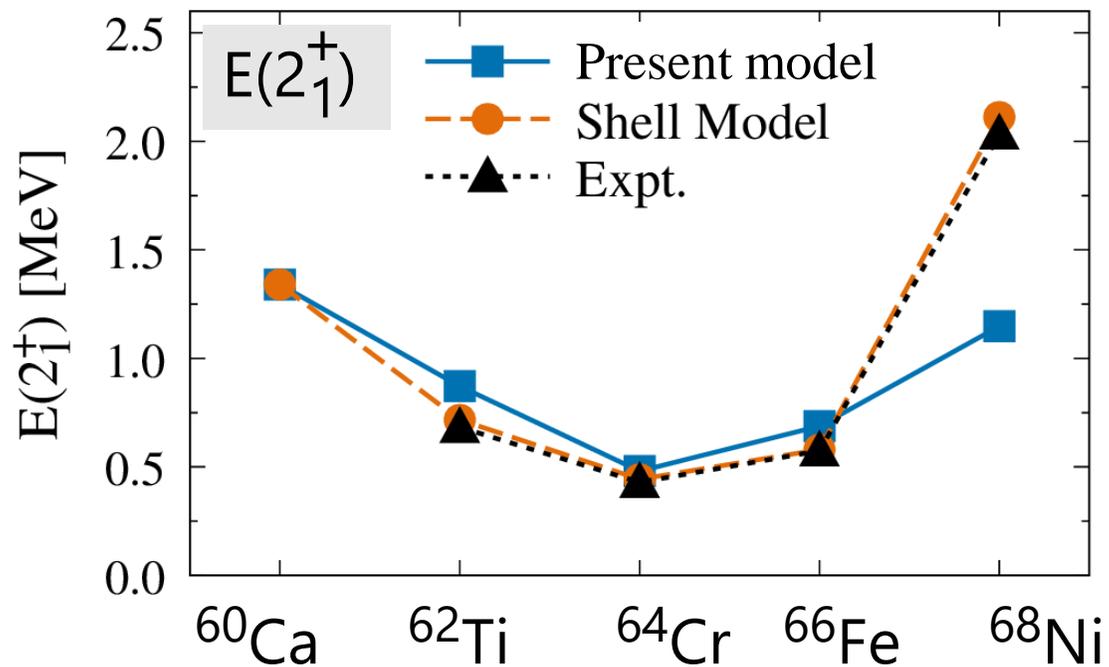
$\sim 100$  mesh points

SkM\*

Volume pairing

Washiyama & Yoshida, in preparation

# Systematics on the $2_1^+$ energy in N=40



Shell model : Lenzi et al., PRC82,054301 (2010)

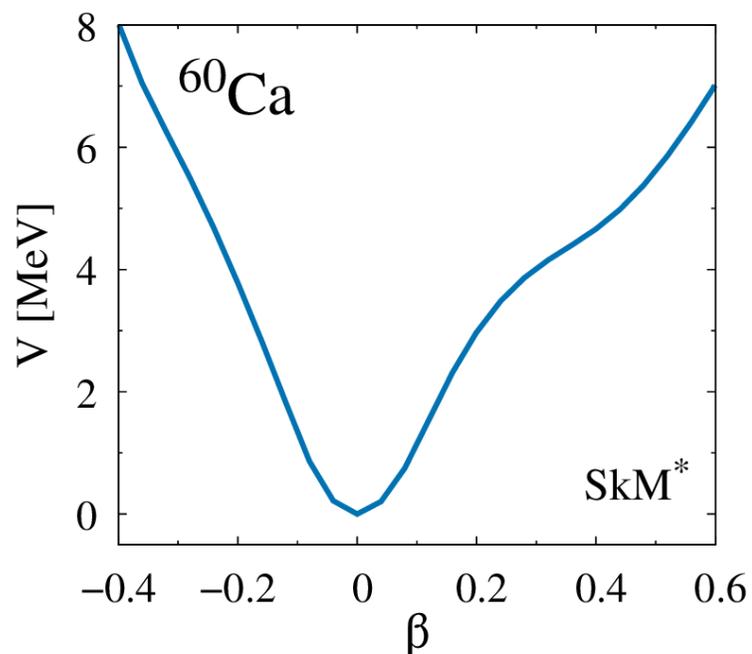
Expt.: Cortés et al., PLB 800, 135071 (2020) +NNDC

Good on  $2_1^+$  energy

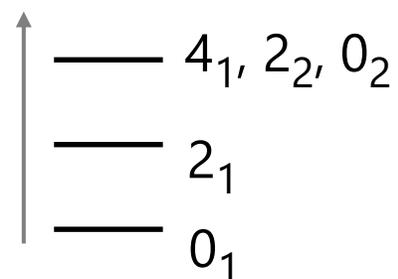
Large deviation in  $^{68}\text{Ni}$

(good description by shell model)

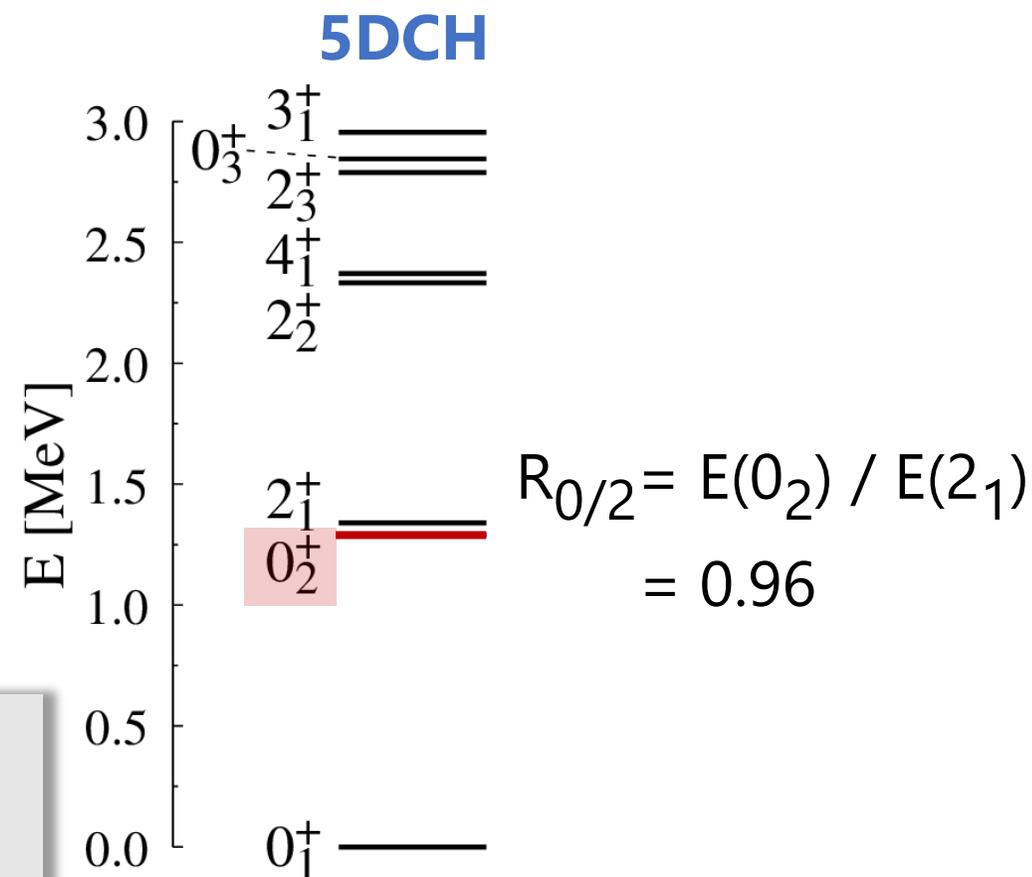
## Potential



cf. Spherical harmonic vibrator limit



## Low-lying spectra in $^{60}\text{Ca}$



**What is the origin of low  $R_{0/2}$  ?**

# Discussion: Role of inertial functions in the $0_2$ state

$$\mathcal{H} = T_{\text{vib}} + T_{\text{rot}} + V(\beta, \gamma)$$

Test a constant mass  $D$

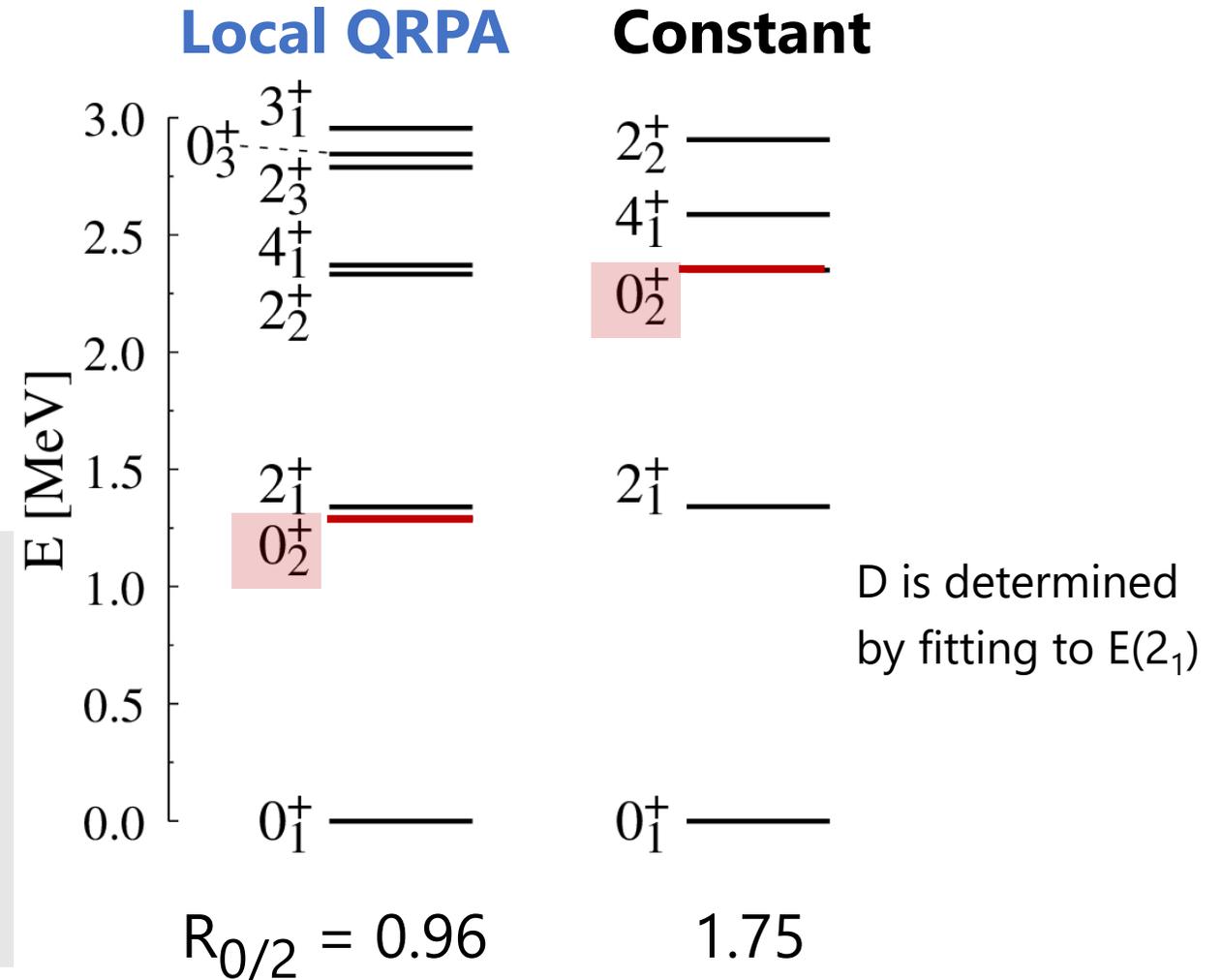
$$T_{\text{vib}} = \frac{1}{2} D_{\beta\beta}(\beta, \gamma) \dot{\beta}^2 + \dots$$

$$\rightarrow \frac{1}{2} D \dot{\beta}^2 + \dots$$

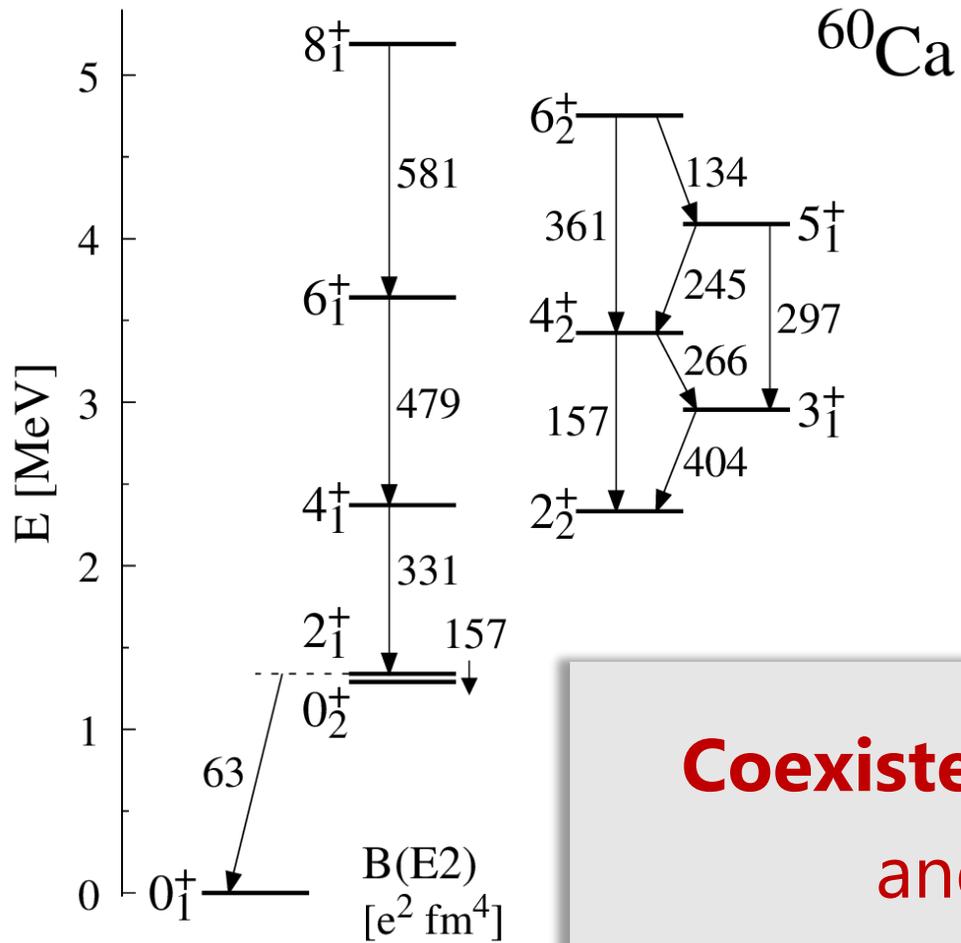
Ignore  $\beta$ - $\gamma$  dependence of  
Local QRPA inertial functions

Low  $R_{0/2}$  ~~←~~ potential  
 ← inertial functions  
 = **dynamical correlations**

## Low-lying spectra in $^{60}\text{Ca}$

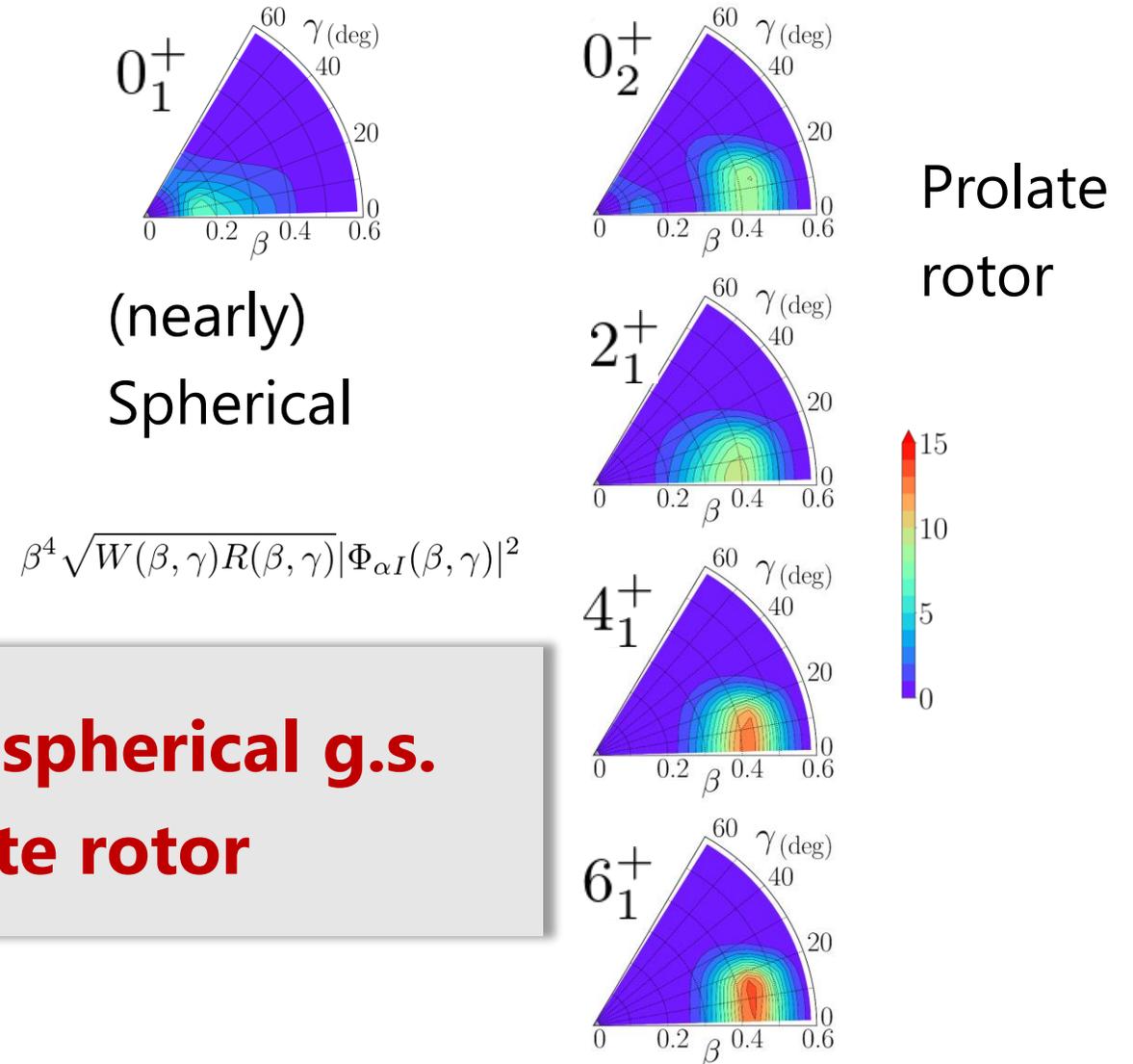


## Band structure in $^{60}\text{Ca}$



**Coexistence of spherical g.s. and prolate rotor**

## Collective wave functions



Low-lying spectra in N=40 neutron-rich nuclei

Dynamical shape coexistence in  $^{60}\text{Ca}$

Potential: Spherical-vibrator-like potential

Dynamical correlations in the kinetic energies

→ generate low  $0_2^+$  state

→ spherical-like ground state and prolate-rotor  $0_2^+$  band

This research used computational resources of Wisteria/BDEC-01 Odyssey (Univ. Tokyo), provided by the Multidisciplinary Cooperative Research Program in CCS, Univ. Tsukuba.