Spin-triplet pairing in nuclear DFT

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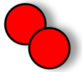
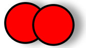
Pairing in nuclei





- Nucleons has spin and isospin degrees of freedom.
- Four channels in the nucleon pair

Nuclear force	space	spin	isospin	Two-nucleon exchange	observables	Local density In nuclear DFT
^{1}E (singlet-even)	symmetric (s-wave etc)	anti-symmetric (singlet)	symmetric (triplet)	anti-symmetric	standard pairing mass, level density	$\tilde{\rho}_k, \tilde{\tau}_k$
^{3}O (triplet-odd)	anti-symmetric (p-wave etc)	symmetric (triplet)	symmetric (triplet)	anti-symmetric	$^3P_2$ superfluidity in neutron matter	$\tilde{J}_k$
^{3}E (triplet-even)	symmetric (s-wave etc)	symmetric (triplet)	anti-symmetric (singlet) np-pair	anti-symmetric	isoscalar pairing β-decay half-life $\beta\beta$-decay half-life	$\tilde{s}_0, \tilde{\boldsymbol{T}}_0, \tilde{\boldsymbol{F}}_0$
^{1}O (singlet-odd)	anti-symmetric (p-wave etc)	anti-symmetric (singlet)	anti-symmetric (singlet) np-pair	anti-symmetric	??	$\tilde{\boldsymbol{j}}_0$

standard nuclear pairing: spin-singlet pair condensation
this talk focuses on: spin-triplet pair condensation

Spin-triplet pairing in electron and nuclear systems

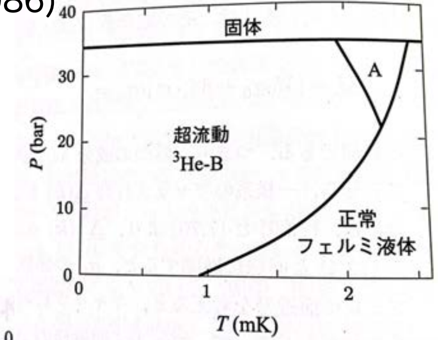
恒藤敏彦、現代物理学叢書 超伝導・超流動(岩波, 2001)
北孝文、統計力学から理解する超伝導理論 (サイエンス社, 2013)
玉垣良三、物理学最前線15 (1986)

Electron system (^{3}He superfluidity)

- A and B phases (ABM and BW)
- p-wave superfluidity
- order parameter

$$\Psi^{(3)} = \boldsymbol{A}(\boldsymbol{k}) \cdot i\boldsymbol{\sigma}\sigma_2 \qquad \hat{A}_\mu(\hat{\boldsymbol{k}}) = \sum_{m=-1}^{1} A_{\mu m} Y_{1m}(\hat{\boldsymbol{k}}) = \sum_j A_{\mu j}\hat{k}_j$$

$A_{\mu j}$: ^{3}P order parameter (3×3)



BW(Balian-Werthamer) (1963)

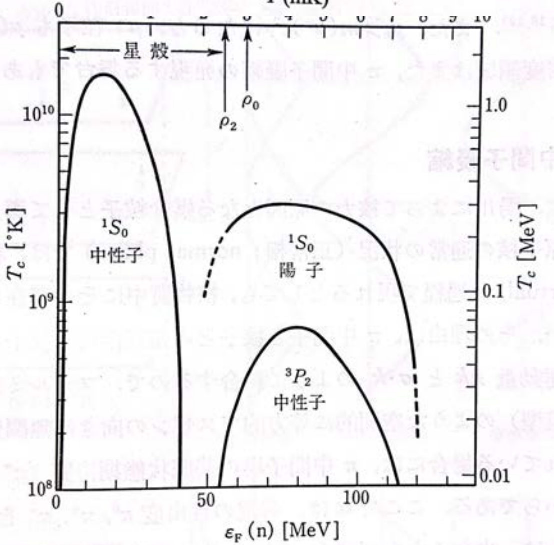
$$\boldsymbol{A}(\hat{\boldsymbol{k}}) = \hat{\boldsymbol{k}} \qquad A_{\mu j} = A\delta_{\mu j} \qquad \hat{\Psi} = A\begin{pmatrix} -\hat{k}_x + i\hat{k}_y & \hat{k}_z \\ \hat{k}_z & \hat{k}_x + i\hat{k}_y \end{pmatrix}$$

ABM(Anderson-Brinkman-Morel)(1972)

$$\boldsymbol{A}(\hat{\boldsymbol{k}}) = A(\hat{k}_x + i\hat{k}_y)\hat{\boldsymbol{z}} \qquad A_{\mu j} = A\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & i & 0 \end{pmatrix}$$

Nuclear system ($^3P_2$ superfluidity in neutron star)

- neutron $^3P_2$ superfluidity in $2/3\rho_0 \leq \rho \leq 3\rho_0$
- five order parameters in $^3P_2$ superfluidity $\Delta^*_{JM}(k) = (-1)^M \Delta_{J-M}(k)$
- not many discussions in finite nuclei (Oishi EPJA57,180(2021))



Spin-singlet and triplet pairing in nuclear DFT

pair density matrix $\hat{\tilde{\rho}}(\boldsymbol{r}st, \boldsymbol{r}'s't') = -2s' \langle \Psi | \hat{a}_{\boldsymbol{r}'-s't'}, \hat{a}_{\boldsymbol{r}st} | \Psi \rangle$ (spin-singlet: s=s')

non-local pair density (spin-singlet condensation) $\tilde{\rho}_t(\boldsymbol{r}, \boldsymbol{r}') = \sum_s \hat{\tilde{\rho}}(\boldsymbol{r}st, \boldsymbol{r}'st)$

non-local spin pair density (spin-triplet condensation) $\tilde{\boldsymbol{s}}_t(\boldsymbol{r}, \boldsymbol{r}') = \sum_{ss'} \tilde{\rho}(\boldsymbol{r}st, \boldsymbol{r}'s't) \hat{\boldsymbol{\sigma}}_{s's}$

$$\tilde{\rho}_t(\boldsymbol{r}, \boldsymbol{r}') = \tilde{\rho}_t(\boldsymbol{r}', \boldsymbol{r}) \qquad \tilde{\boldsymbol{s}}_t(\boldsymbol{r}, \boldsymbol{r}') = -\tilde{\boldsymbol{s}}_t(\boldsymbol{r}', \boldsymbol{r})$$

Local densities $\tilde{\rho}_t(\boldsymbol{r}) = \tilde{\rho}_t(\boldsymbol{r}, \boldsymbol{r})$ order parameter for spin-singlet pairing

$$\begin{aligned}
\tilde{\boldsymbol{s}}_t(\boldsymbol{r}, \boldsymbol{r}') &= \tilde{\boldsymbol{s}}_t\left(\boldsymbol{R} + \frac{\boldsymbol{r}_{\mathrm{rel}}}{2}, \boldsymbol{R} - \frac{\boldsymbol{r}_{\mathrm{rel}}}{2}\right) \\
&= \tilde{\boldsymbol{s}}_t(\boldsymbol{R}, \boldsymbol{R}) + \boldsymbol{r}_{\mathrm{rel}} \cdot \left[\frac{\partial}{\partial \boldsymbol{r}_{\mathrm{rel}}} \otimes \boldsymbol{s}_t \left(\boldsymbol{R} + \frac{\boldsymbol{r}_{\mathrm{rel}}}{2}, \boldsymbol{R} - \frac{\boldsymbol{r}_{\mathrm{rel}}}{2}\right)\Big|_{\boldsymbol{r}_{\mathrm{rel}}=0}\right] + \mathcal{O}(|\boldsymbol{r}_{\mathrm{rel}}|^2) \\
&= \frac{1}{2} \boldsymbol{r}_{\mathrm{rel}} \cdot (\boldsymbol{\nabla} - \boldsymbol{\nabla}') \otimes \tilde{\boldsymbol{s}}_t(\boldsymbol{r}, \boldsymbol{r}')\Big|_{\boldsymbol{r}=\boldsymbol{r}'=\boldsymbol{R}} + \mathcal{O}(|\boldsymbol{r}_{\mathrm{rel}}|^2) \\
&= i \boldsymbol{r}_{\mathrm{rel}} \cdot \tilde{\mathsf{J}}_t(\boldsymbol{R}) + \mathcal{O}(|\boldsymbol{r}_{\mathrm{rel}}|^2)
\end{aligned}$$

spin-current (tensor) pair density $\tilde{\mathsf{J}}_t(\boldsymbol{r}) = \frac{1}{2i} \left[(\boldsymbol{\nabla} - \boldsymbol{\nabla}') \otimes \tilde{\boldsymbol{s}}_t(\boldsymbol{r}, \boldsymbol{r}')\right]_{\boldsymbol{r}=\boldsymbol{r}'}$ ← nine order parameters

Richardson, Phys. Rev. D **5**, 1883 (1972).
Masaki et al Phys. Rev. Res. **2**, 013193 (2020)

Symmetry property of spin-current pair density

Rohozinski et al. Phys. Rev. C **81**, 014313 (2010)

spin-current pair density $\tilde{\mathsf{J}}_t(\boldsymbol{r}) = \frac{1}{2i}[(\boldsymbol{\nabla} - \boldsymbol{\nabla}') \otimes \tilde{\boldsymbol{s}}_t(\boldsymbol{r}, \boldsymbol{r}')]_{\boldsymbol{r}=\boldsymbol{r}'}$ (3 spatial x 3 spin)

1 pseudoscalar ($^3P_0$)

$$\tilde{J}_t(\boldsymbol{r}) = \sum_i \tilde{\mathsf{J}}_{tii}(\boldsymbol{r})$$

3 vectors ($^3P_1$)

$$\tilde{\boldsymbol{J}}_{tk}(\boldsymbol{r}) = \sum_{ij} \varepsilon_{ijk} \tilde{\mathsf{J}}_{tij}(\boldsymbol{r})$$

5 pseudotensors ($^3P_2$)

$$\underline{\tilde{\mathsf{J}}}_{tij}(\boldsymbol{r}) = \frac{1}{2}\tilde{\mathsf{J}}_{tij}(\boldsymbol{r}) + \frac{1}{2}\tilde{\mathsf{J}}_{tji}(\boldsymbol{r}) - \frac{1}{3}\tilde{J}_t(\boldsymbol{r})\delta_{ij}$$

spin-current pair density has the same spatial symmetry as the particle-hole spin-current (tensor) density has.

We consider spherical finite nuclei (with parity symmetry)

$$\tilde{J}_t(\boldsymbol{r}) = 0 \qquad \tilde{\boldsymbol{J}}_t(\boldsymbol{r}) = \tilde{J}_{tr}(\boldsymbol{r})\boldsymbol{e}_r \qquad \underline{\tilde{\mathsf{J}}}_t(\boldsymbol{r}) = 0$$

radial component only: spin and relative momentum are orthogonal to radial direction

In the case of axial symmetry (with z-simplex symmetry) (r, ϕ, z)

$$\tilde{J}_t(\boldsymbol{r}) = 0 \qquad \tilde{\boldsymbol{J}}_t(\boldsymbol{r}) = \tilde{J}_{tr}(\boldsymbol{r})\boldsymbol{e}_r + \tilde{J}_{tz}(\boldsymbol{r})\boldsymbol{e}_z \qquad \underline{\tilde{\mathsf{J}}}_{tr\phi}(\boldsymbol{r}), \quad \underline{\tilde{\mathsf{J}}}_{tz\phi}(\boldsymbol{r}) \neq 0$$

- $^3P_1$ channel is active in spherical nuclei.
- Is there spin-triplet pair condensation in finite nuclei ?
- What is the property of the spin-current pair density?

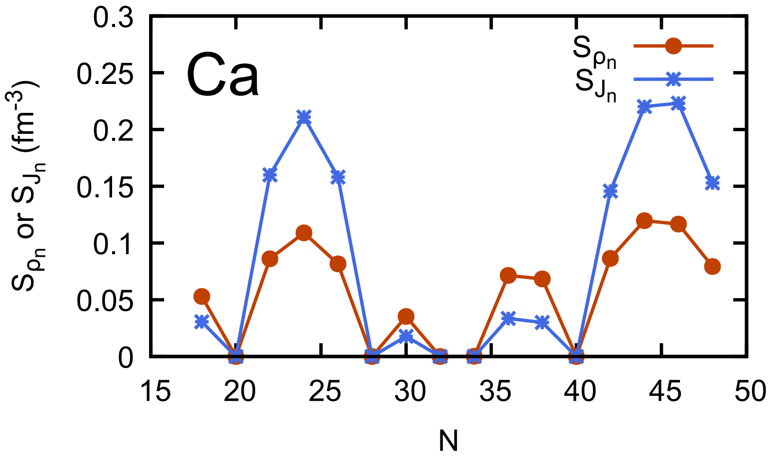
Spin-triplet pair condensate

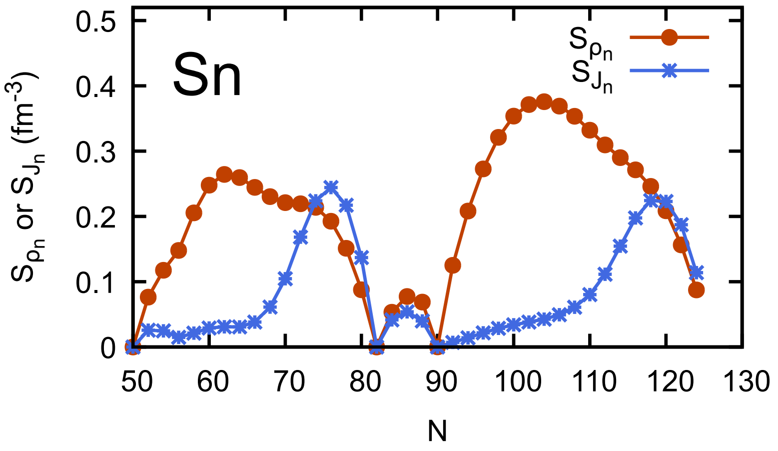
spin-singlet and spin-triplet condensation relevant quantities (proportional to pair EDFs)

$$S_{\rho_n} = \int d\boldsymbol{r} |\tilde{\rho}_n(\boldsymbol{r})|^2 \qquad S_{J_n} = R^2 \int d\boldsymbol{r} |\tilde{\boldsymbol{J}}_n(\boldsymbol{r})|^2$$

R^2=10fm^2

Ca and Sn isotopes: only neutron is super





- pair EDF: spin-singlet only (volume-type pairing)
- spin-singlet pair EDF induces spin-triplet pair condensate
- spin-triplet pair EDF has different orbital dependence (large at high-j intruder)

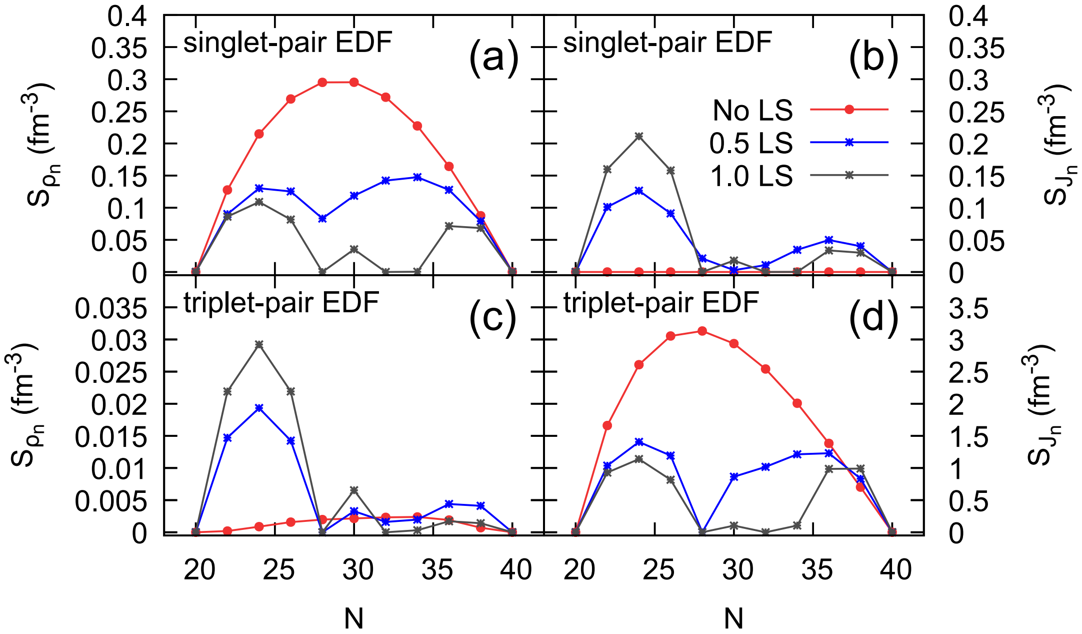
Spin-singlet/triplet condensations and EDFs

Ca isotopes

singlet condensation — triplet condensation

spin-singlet pair EDF

spin-triplet pair EDF



- Spin-orbit EDF induces the spin-triplet pair condensation from the spin-singlet pair EDF.

$$\tilde{\rho}(r) = -\frac{1}{4\pi r^2}\sum_{nlj}(2j+1)u_1(nlj,r)u_2(nlj,r) \qquad \tilde{J}(r) = -\frac{1}{4\pi r^3}\sum_{nlj}(2j+1)\underbrace{\left[j(j+1)-l(l+1)-\frac{3}{4}\right]}_{2\langle \boldsymbol{l}\cdot\boldsymbol{s}\rangle}u_1(nlj,r)u_2(nlj,r)$$

- Coupling constant of spin-triplet pair EDF is adjusted to provide the same pairing energy in ^{44}Ca.
- Spin-triplet pair EDF alone (not realistic) shows similar pairing property.

Pairing gap and pairing energy

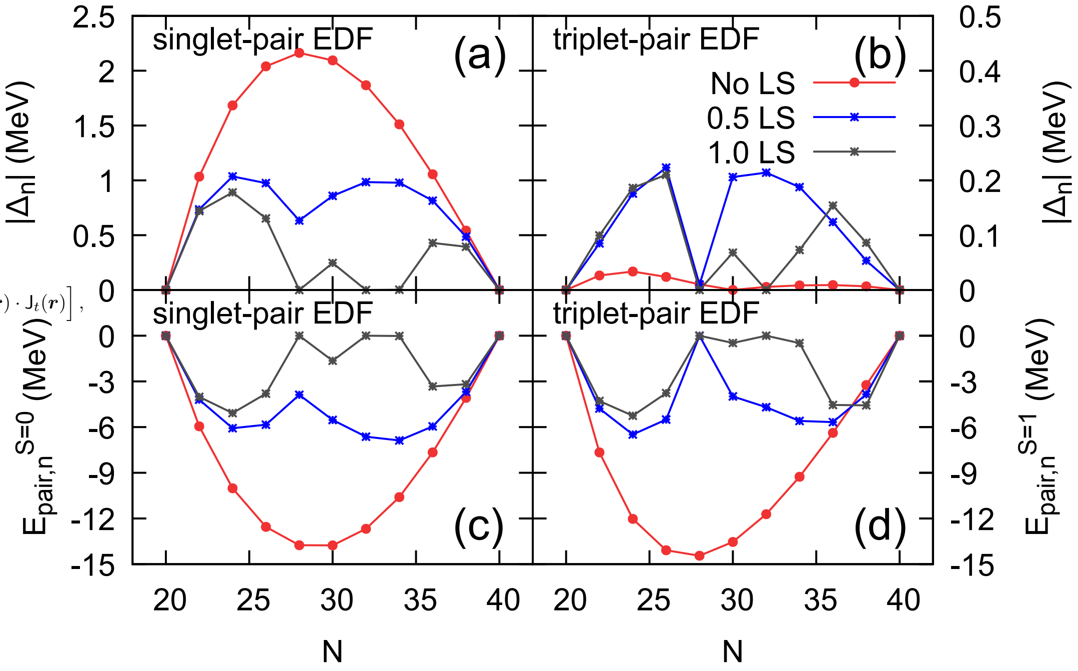
spin-singlet pair EDF spin-triplet pair EDF

pairing gap

$$\Delta_t = \frac{\int d\boldsymbol{r} \sum_{ss'\mu} \phi_2^{(t)*}(\mu, \boldsymbol{r}s') \tilde{h}_{s's}^{(t)}(\boldsymbol{r}) \phi_2^{(t)}(\mu, \boldsymbol{r}s)}{\int d\boldsymbol{r} \sum_{s\mu} |\phi_2^{(t)}(\mu, \boldsymbol{r}s)|^2}$$

$$= \frac{1}{N_t} \int d\boldsymbol{r} \left[\tilde{U}_t(\boldsymbol{r}) \rho_t(\boldsymbol{r}) + \tilde{M}_t(\boldsymbol{r}) \tau_t(\boldsymbol{r}) + \tilde{\mathsf{B}}_t(\boldsymbol{r}) \cdot \mathsf{J}_t(\boldsymbol{r}) \right],$$

pairing energy

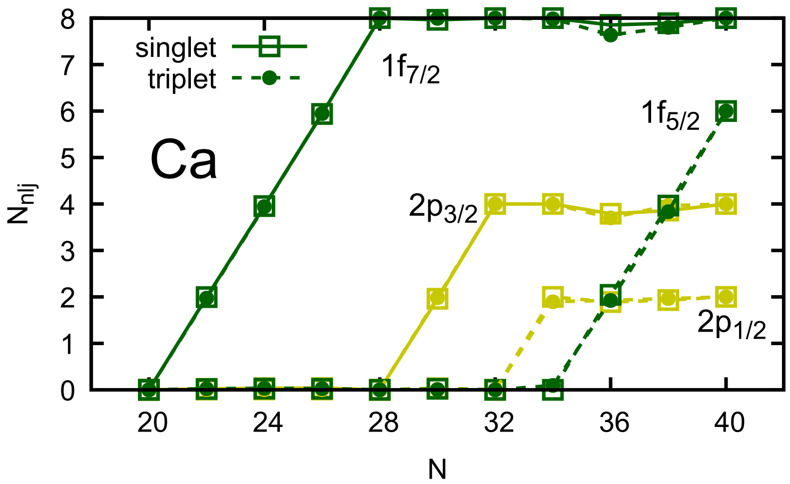


- Pairing gap: an extended definition of density-averaged gap
- Pairing energy is similar (coupling constant is fitted at ^{44}Ca) in singlet-pair and triplet pair EDFs.
- Pairing gap does not correspond to OES when spin-triplet pair EDF is used

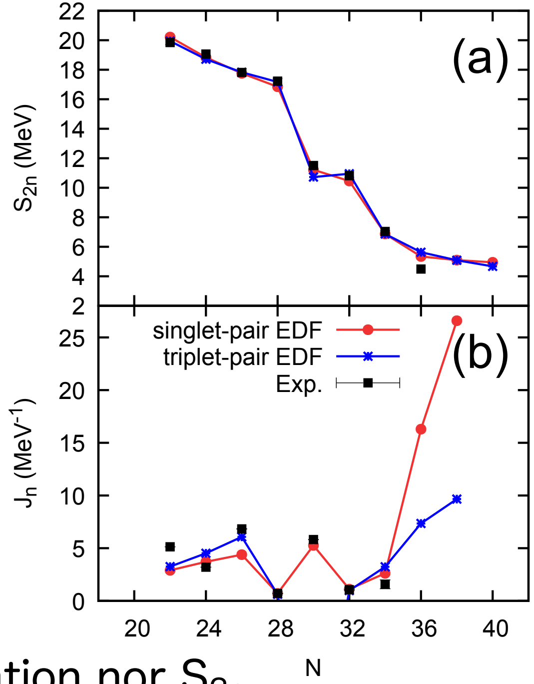
Observables

What (ground-state) observable is relevant to spin-triplet pair condensation?

Occupation probabilities



two-neutron separation energy (S_{2n})
and pairing rotational MOI



- Spin-triplet pair EDF does not affect s.p. occupation nor S_{2n}.
- Some difference in pairing rotational MOI: $\Delta S_{2n}=S_{2n}(N,Z) - S_{2n}(N+2,Z) = 4/J_n$.

Summary

- Spin-triplet like-particle (triplet-odd) pairing in finite nuclei
  - Spin-current pair density is an order parameter for spin-triplet pairing within the LDA.
  - Spin-triplet pair condensation is induced by the spin-singlet pair EDF with spin-orbit EDF.
  - Spin-singlet and spin-triplet pair has different orbital dependence.
  - Spin-triplet pair EDF induces spin-orbit splitting and spin-singlet pair condensation.
  - Related observables
  - NH, Tomohiro Oishi, and Kenichi Yoshida, arXiv:2308.02617.