Effects of center-of-mass correction and nucleon anomalous moment on nuclear charge radii

Yusuke Tanimura (Soongsil Univ./Tohoku Univ.)

Collaborator: Myung-Ki Cheoun (Soongsil Univ.)

Yusuke Tanimura and Myung-Ki Cheoun, arXiv:2312.15983 [nucl-th] (2023).

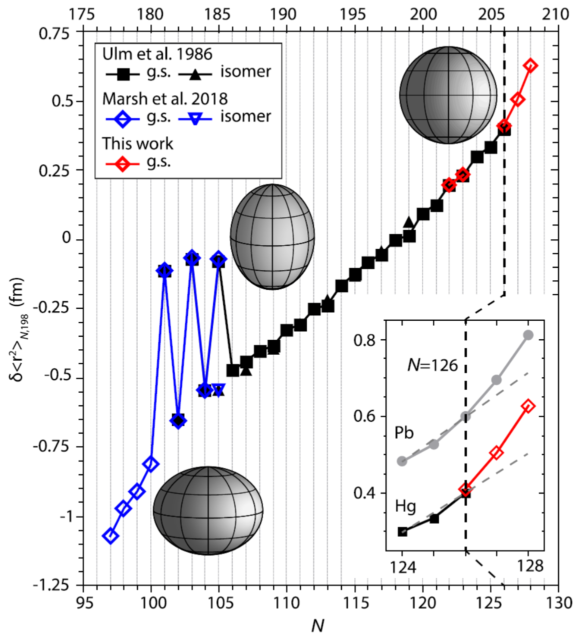
Soongsil University

OMEG
Origin of Matter and Evolution of Galaxies
Key Research Institutes in Universities

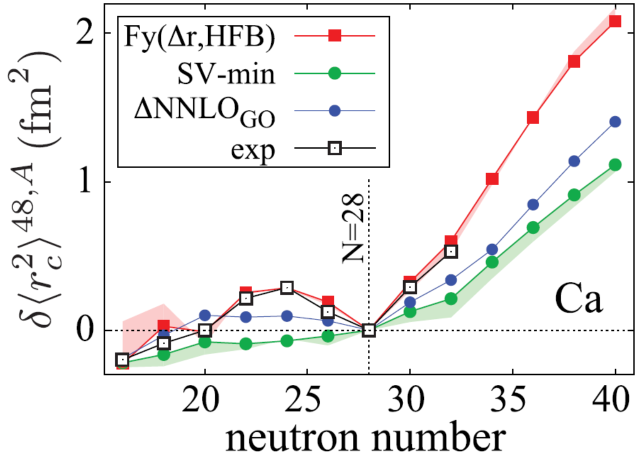
TOHOKU UNIVERSITY

Nuclear charge radius

- Measured accurately with electromagnetic (EM) probe
- Information of nuclear structure and nuclear force
  - ✓Shell effects
  - ✓Pairing correlation
  - ✓Deformation
  - ✓Symmetry energy?
- Theoretical interpretation of charge radii
  - ✓Many-body effects
  - ✓Nuclear force
  - ✓Internal EM structure of nucleon



"Odd-even shape staggering" in Hg isotopes
Goodacre et al., PRL**126**, 032502 ('21).



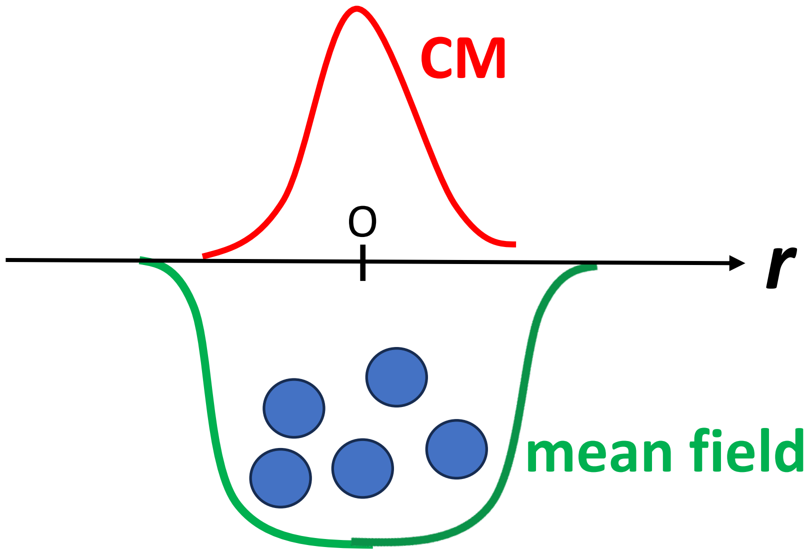
"Anomalous behaviors" of r_{ch} in Ca isotopes
due to continuum effects?
Miller et al., Nat. Phys. **15**, 432 ('19)

Charge radius with DFT

- Nuclear DFT (mean-field model)
  - Universal "energy density functional"
    - ✓binding energy
    - ✓nuclear size, EM moments, … (one-body observables)
  - Suitable for systematic study of charge radii

- **Translational symmetry breaking**
  - Nucleons are confined in a "mean-field potential" fixed in space
  - Center of mass (CM) of the system is localized around the potential

  → Fluctuation of CM motion gives spurious contribution to radius (and any observable)



Charge radius with DFT

- Nuclear DFT (mean-field model)
  - Universal "energy density functional"
    - ✓binding energy
    - ✓nuclear size, EM moments, … (one-body observables)
  - Suitable for systematic study of charge radii

In this work, we study the charge radii with

- ■ **Correction for center-of-mass motion of nucleus**
- ■ **Electromagnetic properties of nucleon (anomalous magnetic moment and finite size)**

Mean-squared (MS) charge radius

$$\langle r^2 \rangle_{\mathrm{ch}} = \langle r^2 \rangle_p + \langle r^2 \rangle_\kappa + \left(0.588 + 0.011 \frac{N}{Z} \ \mathrm{fm}^2 \right)$$

Mean-squared (MS) charge radius

$$\left\langle r^2 \right\rangle_{\text{ch}} = \left\langle r^2 \right\rangle_p + \left\langle r^2 \right\rangle_\kappa + \left(0.588 + 0.011 \frac{N}{Z} \ \text{fm}^2 \right)$$

$$\left\langle r^2 \right\rangle_p = \frac{1}{Z} \left\langle \sum_{i \in p} \boldsymbol{r}_i^2 \right\rangle = \frac{1}{Z} \int d^3 r \ r^2 \rho_p(\boldsymbol{r})$$

← point-proton density

Mean-squared (MS) charge radius

$$\langle r^2 \rangle_{\rm ch} = \langle r^2 \rangle_p + \langle r^2 \rangle_\kappa + \left(0.588 + 0.011\frac{N}{Z}\ {\rm fm}^2\right)$$

$$\langle r^2 \rangle_p = \frac{1}{Z}\left\langle \sum_{i\in p} \boldsymbol{r}_i^2 \right\rangle = \frac{1}{Z}\int d^3r\ r^2 \rho_p(\boldsymbol{r})$$ ← point-proton density

$$\langle r^2 \rangle_\kappa = \frac{1}{Z}\int d^3r\ r^2 \rho_\kappa(\boldsymbol{r}),$$ ← **Contribution of the (point) anomalous magnetic moment to charge density**

where

$$\rho_\kappa(\boldsymbol{r}) \equiv \sum_{\tau=p,n} \kappa_\tau \frac{\hbar}{2mc} \boldsymbol{\nabla} \cdot \sum_{\alpha\in\tau} v_\alpha^2 \bar{\psi}_\alpha(\boldsymbol{r}) i\boldsymbol{\alpha}\psi_\alpha(\boldsymbol{r})$$

m: nucleon mass

$\kappa_p = 1.793$, $\kappa_n = -1.913$

Mean-squared (MS) charge radius

$$\langle r^2 \rangle_{\mathrm{ch}} = \langle r^2 \rangle_p + \langle r^2 \rangle_\kappa + \left(0.588 + 0.011 \frac{N}{Z} \ \mathrm{fm}^2 \right)$$

↑ **Finite-size of nucleons**
Independent of nuclear structure

$$\langle r^2 \rangle_p = \frac{1}{Z} \left\langle \sum_{i \in p} \boldsymbol{r}_i^2 \right\rangle = \frac{1}{Z} \int d^3 r \ r^2 \rho_p(\boldsymbol{r})$$

← point-proton density

$$\langle r^2 \rangle_\kappa = \frac{1}{Z} \int d^3 r \ r^2 \rho_\kappa(\boldsymbol{r}),$$

← **Contribution of the (point) anomalous magnetic moment to charge density**

where

$$\rho_\kappa(\boldsymbol{r}) \equiv \sum_{\tau = p,n} \kappa_\tau \frac{\hbar}{2mc} \boldsymbol{\nabla} \cdot \sum_{\alpha \in \tau} v_\alpha^2 \bar{\psi}_\alpha(\boldsymbol{r}) i \boldsymbol{\alpha} \psi_\alpha(\boldsymbol{r})$$

m: nucleon mass
$\kappa_p = 1.793$, $\kappa_n = -1.913$

Center-of-mass correction of $<r^2>_p$

Point-proton squared radius
w/o CM correction

$$Z\left\langle r^2\right\rangle_p = \left\langle \sum_{i\in p} \boldsymbol{r}_i^2 \right\rangle = \int d^3r\ r^2 \rho_p(\boldsymbol{r})$$

Center-of-mass correction of $<r^2>_p$

Point-proton squared radius
with CM correction

$$Z\left\langle r^2\right\rangle_{p,\mathrm{corr}} = \left\langle \sum_{i\in p}(\boldsymbol{r}_i - \boldsymbol{R}_G)^2 \right\rangle$$
$$= Z\left[\left\langle r^2\right\rangle_p + \Delta_p^{(\mathrm{CM1})} + \Delta_p^{(\mathrm{CM2})}\right]$$

$$\boldsymbol{R}_G = \frac{1}{A}\sum_{i=1}^{A}\boldsymbol{r}_i$$

1- and 2-body parts of the CM correction:

$$\Delta_p^{(\mathrm{CM1})} = -\frac{2}{AZ}\sum_{\alpha\in p} v_\alpha^2\langle\alpha|r^2|\alpha\rangle + \frac{1}{A^2}\sum_{\alpha} v_\alpha^2\langle\alpha|r^2|\alpha\rangle,$$

$$\Delta_p^{(\mathrm{CM2})} = +\frac{2}{AZ}\sum_{\alpha\beta\in p}(v_\alpha^2 v_\beta^2 - u_\alpha v_\alpha u_\beta v_\beta)|\langle\alpha|\boldsymbol{r}|\beta\rangle|^2 - \frac{1}{A^2}\sum_{\alpha\beta}(v_\alpha^2 v_\beta^2 - u_\alpha v_\alpha u_\beta v_\beta)|\langle\alpha|\boldsymbol{r}|\beta\rangle|^2,$$

α, β: canonical s.p. states,
v_α, u_α: occupation amplitudes

Center-of-mass correction of <r^2>$_p$

Point-proton squared radius with CM correction

$$Z\left\langle r^2\right\rangle_{p,\mathrm{corr}} = \left\langle \sum_{i\in p}(\boldsymbol{r}_i - \boldsymbol{R}_G)^2 \right\rangle$$

$$= Z\left[\left\langle r^2\right\rangle_p + \Delta_p^{(\mathrm{CM1})} + \Delta_p^{(\mathrm{CM2})}\right]$$

$$\boldsymbol{R}_G = \frac{1}{A}\sum_{i=1}^{A}\boldsymbol{r}_i$$

1- and 2-body parts of the CM correction:

$$\Delta_p^{(\mathrm{CM1})} = -\frac{2}{AZ}\sum_{\alpha\in p} v_\alpha^2\langle\alpha|r^2|\alpha\rangle + \frac{1}{A^2}\sum_{\alpha} v_\alpha^2\langle\alpha|r^2|\alpha\rangle,$$

$$\Delta_p^{(\mathrm{CM2})} = +\frac{2}{AZ}\sum_{\alpha\beta\in p}(v_\alpha^2 v_\beta^2 - u_\alpha v_\alpha u_\beta v_\beta)|\langle\alpha|\boldsymbol{r}|\beta\rangle|^2 - \frac{1}{A^2}\sum_{\alpha\beta}(v_\alpha^2 v_\beta^2 - u_\alpha v_\alpha u_\beta v_\beta)|\langle\alpha|\boldsymbol{r}|\beta\rangle|^2,$$

The CM correction has been neglected, or taken into account with
- **CM1 only**
- **harmonic-oscillator approximation**

$$\langle r^2\rangle_{\mathrm{ch}} = \langle r^2\rangle_p + \Delta_p^{(\mathrm{CM1})} + \Delta_p^{(\mathrm{CM2})} + \langle r^2\rangle_\kappa + \left(0.588 + 0.011\frac{N}{Z}\ \mathrm{fm}^2\right)$$

CM correction

Anomalous magnetic moment

Nucleon finite-size effect (independent of nuclear structure)

1. Decomposition of the correction terms $\langle r^2\rangle_\kappa$, $\Delta^{(CM1)}$, and $\Delta^{(CM2)}$

2. Comparison of RMS charge radii to experimental data

for isotope chains

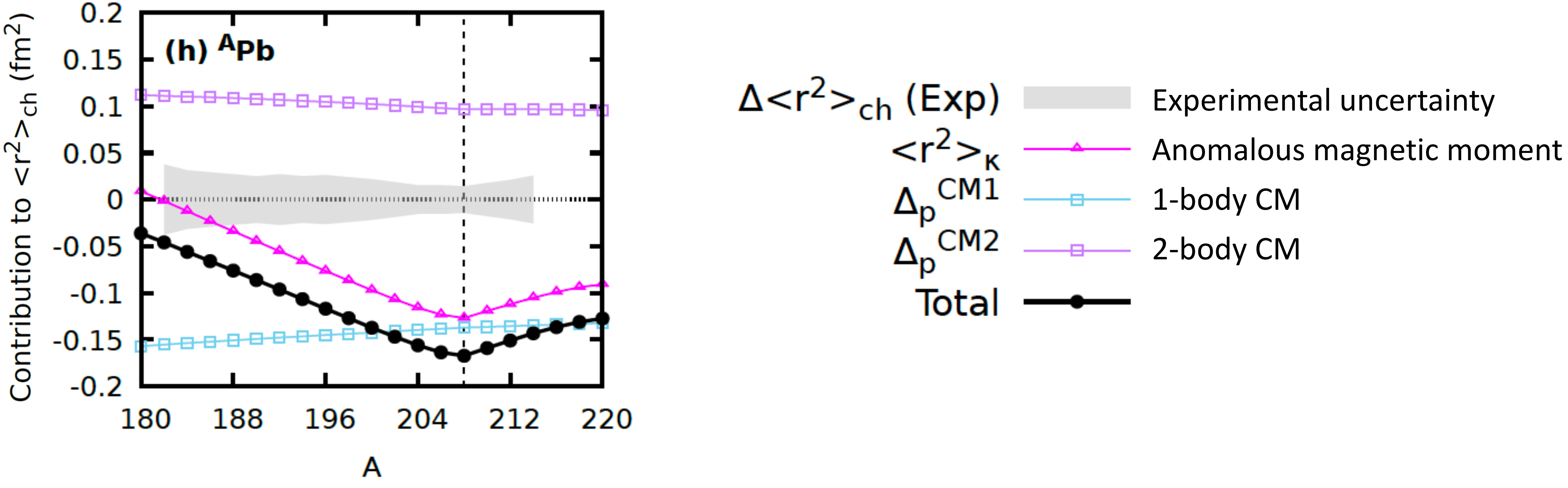
$^{4-8}$He, $^{10-22}$C, $^{12-28}$O, $^{36-56}$Ca, $^{50-80}$Ni, $^{78-112}$Zr, $^{100-134}$Sn, $^{180-220}$Pb

Model:

- Relativistic Hartree-Bogoliubov (RHB) model
    - Meson-exchange interaction
    - DD-ME2 + Gogny D1S for pairing

Lalazissis et al., PRC**71**, 024312 (2005).

1. Decomposition of corrections for Pb isotopes

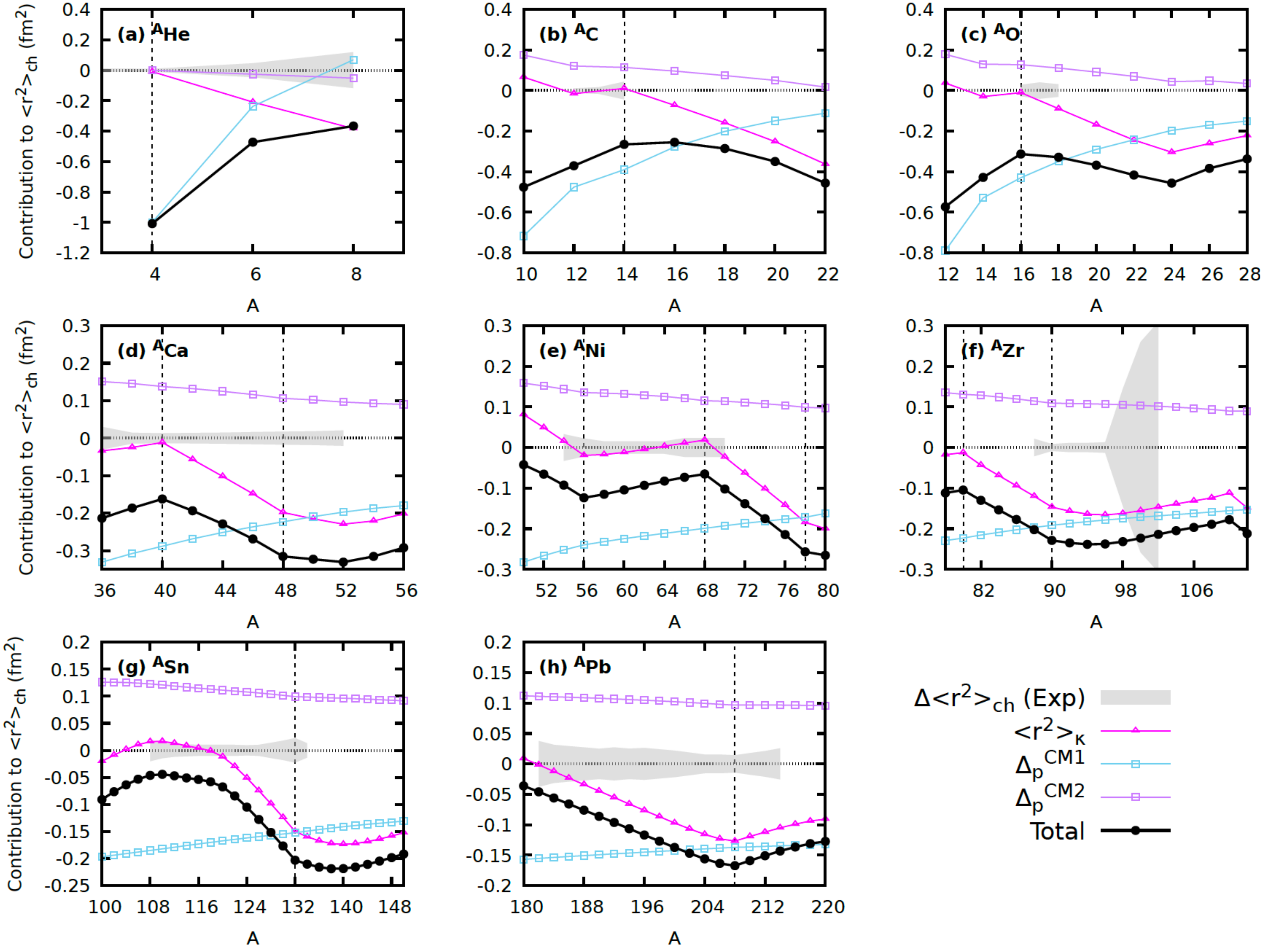


- **$<r^2>_\kappa$, $\Delta^{(CM1)}_p$, and $\Delta^{(CM2)}_p$ are all of the same order and larger than the experimental uncertainty**

→ None of them should be neglected

Decomposition

- **$\langle r^2 \rangle_\kappa$ is sensitive to the shell structure while $\Delta^{(CMi)}$ are rather smooth**
- 2b-1b ratio
- $\Delta_p^{(CM2)}/\Delta_p^{(CM1)} \approx 0$ for light nuclei
- $\Delta_p^{(CM2)}/\Delta_p^{(CM1)} \rightarrow -1$ for heavy nuclei



Tanimura and Cheoun,
arXiv:2312.15983 [nucl-th] (2023).

2. Absolute values of charge radii

+(0.8)^2: used for parameter fit

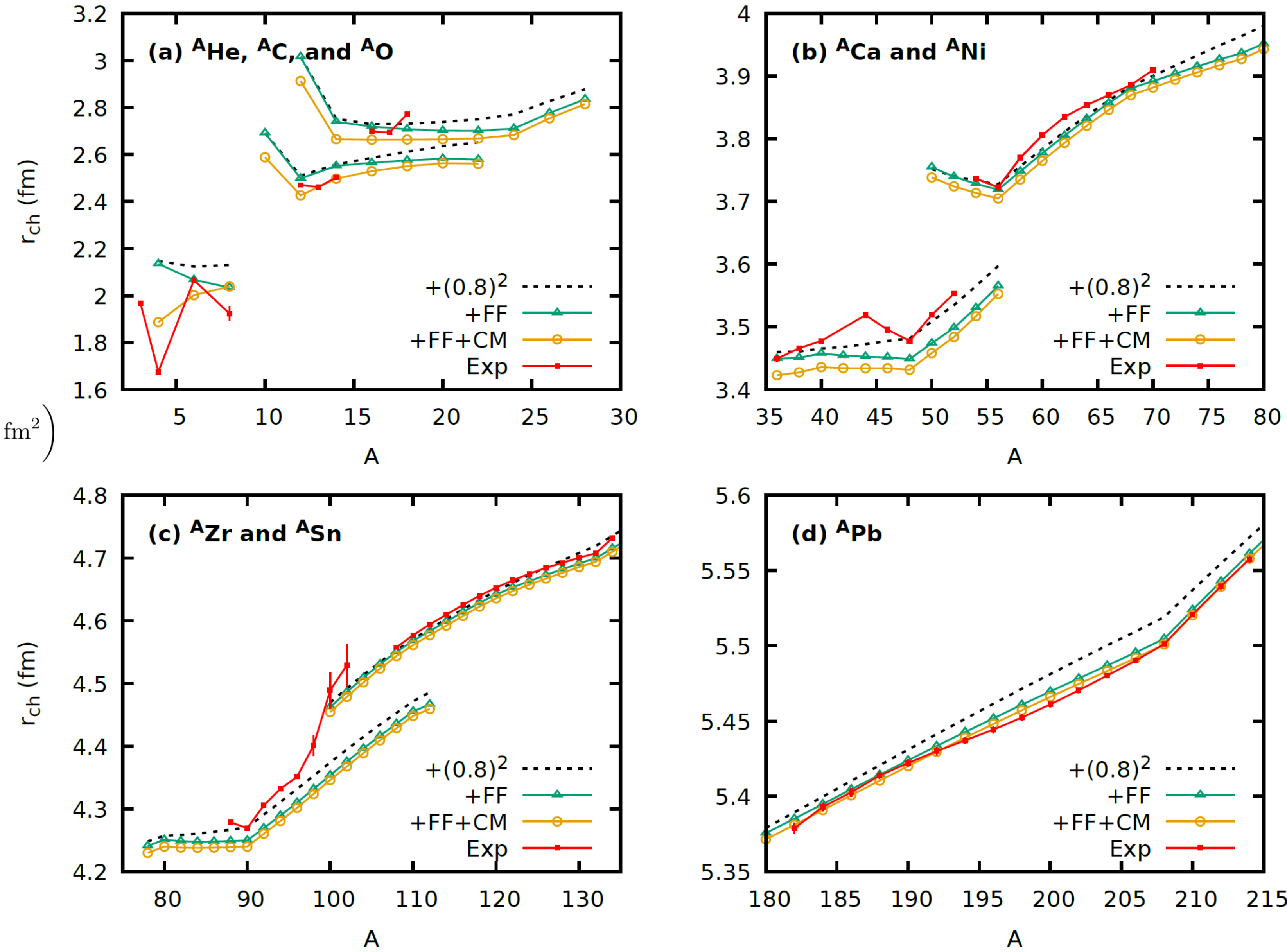
$$\langle r^2 \rangle_{\mathrm{ch}} = \langle r^2 \rangle_p + (0.8\ \mathrm{fm})^2$$

+FF: with nucleon EM form factors

$$\langle r^2 \rangle_{\mathrm{ch}} = \langle r^2 \rangle_p + \langle r^2 \rangle_\kappa + \left(0.588 + 0.011\frac{N}{Z}\ \mathrm{fm}^2\right)$$

+FF+CM: with CM correction

$$\langle r^2 \rangle_{\mathrm{ch}} = \langle r^2 \rangle_p + \Delta_p^{(\mathrm{CM1})} + \Delta_p^{(\mathrm{CM2})} + \langle r^2 \rangle_\kappa + \left(0.588 + 0.011\frac{N}{Z}\ \mathrm{fm}^2\right)$$

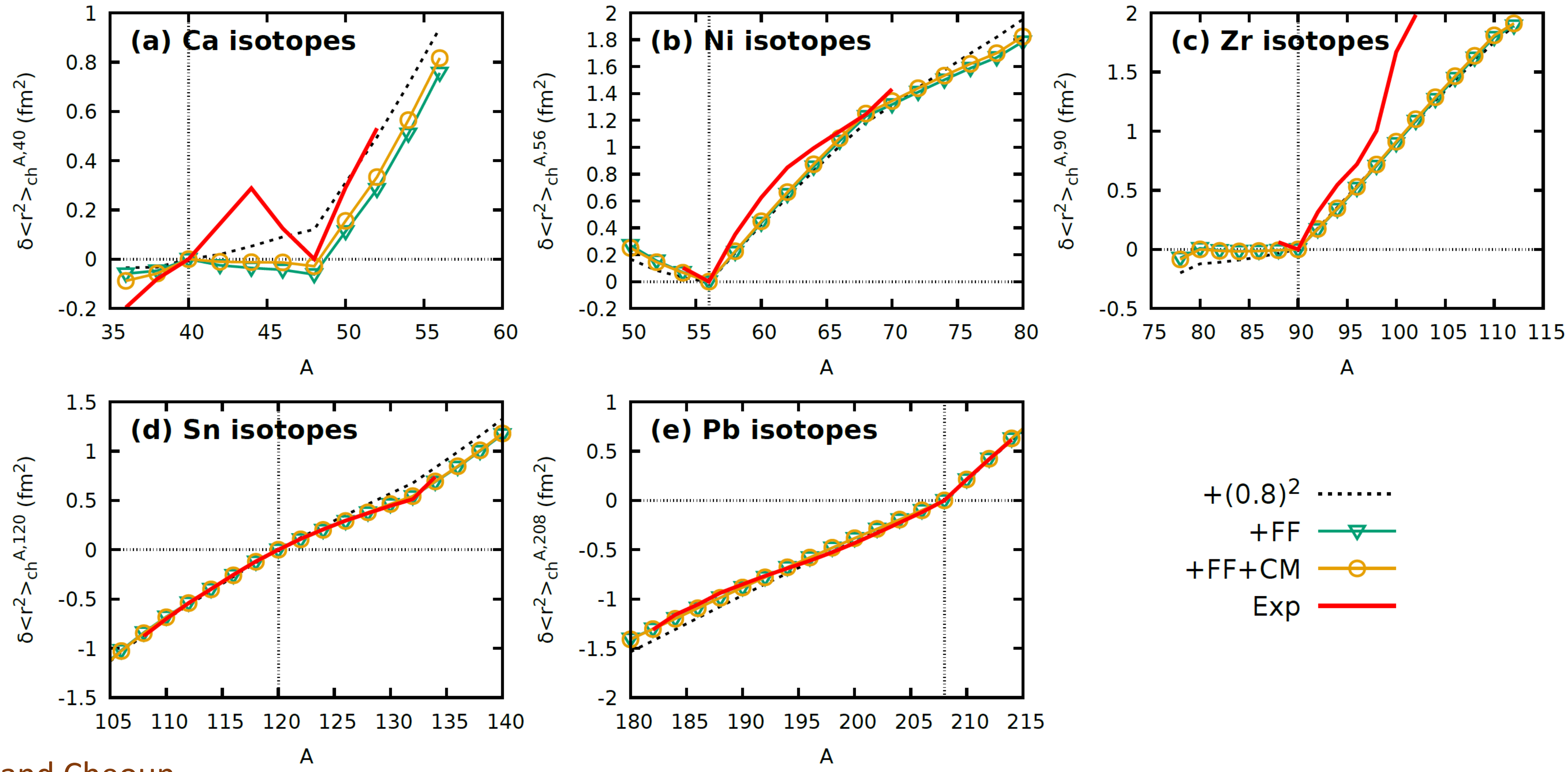


Sizable impacts of the corrections from light to heavy nuclei

Tanimura and Cheoun, arXiv:2312.15983 [nucl-th] (2023).

Isotopic shifts

$$\delta \left\langle r^2 \right\rangle_{\text{ch}}^{A,A'} = \left\langle r^2 \right\rangle_{\text{ch}} (A) - \left\langle r^2 \right\rangle_{\text{ch}} (A')$$



Tanimura and Cheoun,
arXiv:2312.15983 [nucl-th] (2023).

Comparison with harmonic-oscillator model

Tanimura and Cheoun, arXiv:2312.15983 [nucl-th] (2023).

$$\langle r^2\rangle_p = \frac{3}{4}\frac{\hbar}{m\omega}\left[f_2(Z)^{1/3} + \frac{1}{3f_2(Z)^{1/3}}\right]$$

$$\Delta_p^{(\mathrm{CM1})} = \frac{3}{4}\frac{\hbar}{m\omega}\left\{\frac{Z}{A^2}\left(1-\frac{2A}{Z}\right)\left[f_2(Z)^{1/3}+\frac{1}{3}f_2(Z)^{-1/3}\right]+\frac{N}{A^2}\left[f_2(N)^{1/3}+\frac{1}{3}f_2(N)^{-1/3}\right]\right\}$$

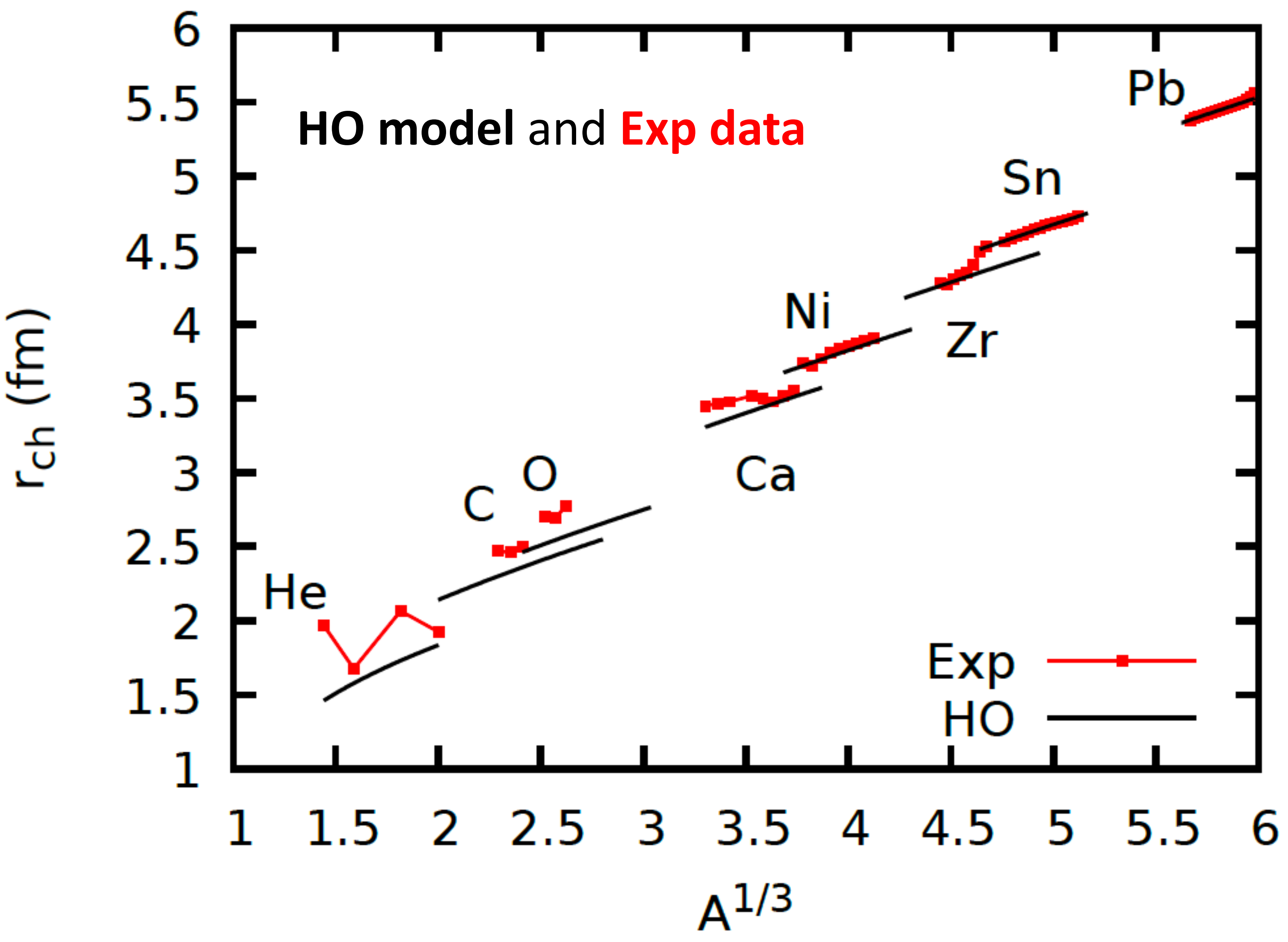
$$\Delta_p^{(\mathrm{CM2})} = -\frac{3}{4}\frac{\hbar}{m\omega}\left\{\frac{Z}{A^2}\left(1-\frac{2A}{Z}\right)\left[f_2(Z)^{1/3}+\frac{1}{3}f_2(Z)^{-1/3}-2\right]\right.$$
$$\left.+\frac{N}{A^2}\left[f_2(N)^{1/3}+\frac{1}{3}f_2(N)^{-1/3}-2\right]\right\}$$

where

$$f_\nu(N_p) = \sqrt{\left(\frac{3N_p}{\nu}\right)^2 - \frac{1}{27}} + \frac{3N_p}{\nu} \quad \text{and} \quad \frac{3}{4}\frac{\hbar}{m\omega} = 0.793A^{1/3}\ \mathrm{fm}^2$$

Comparison with harmonic-oscillator model

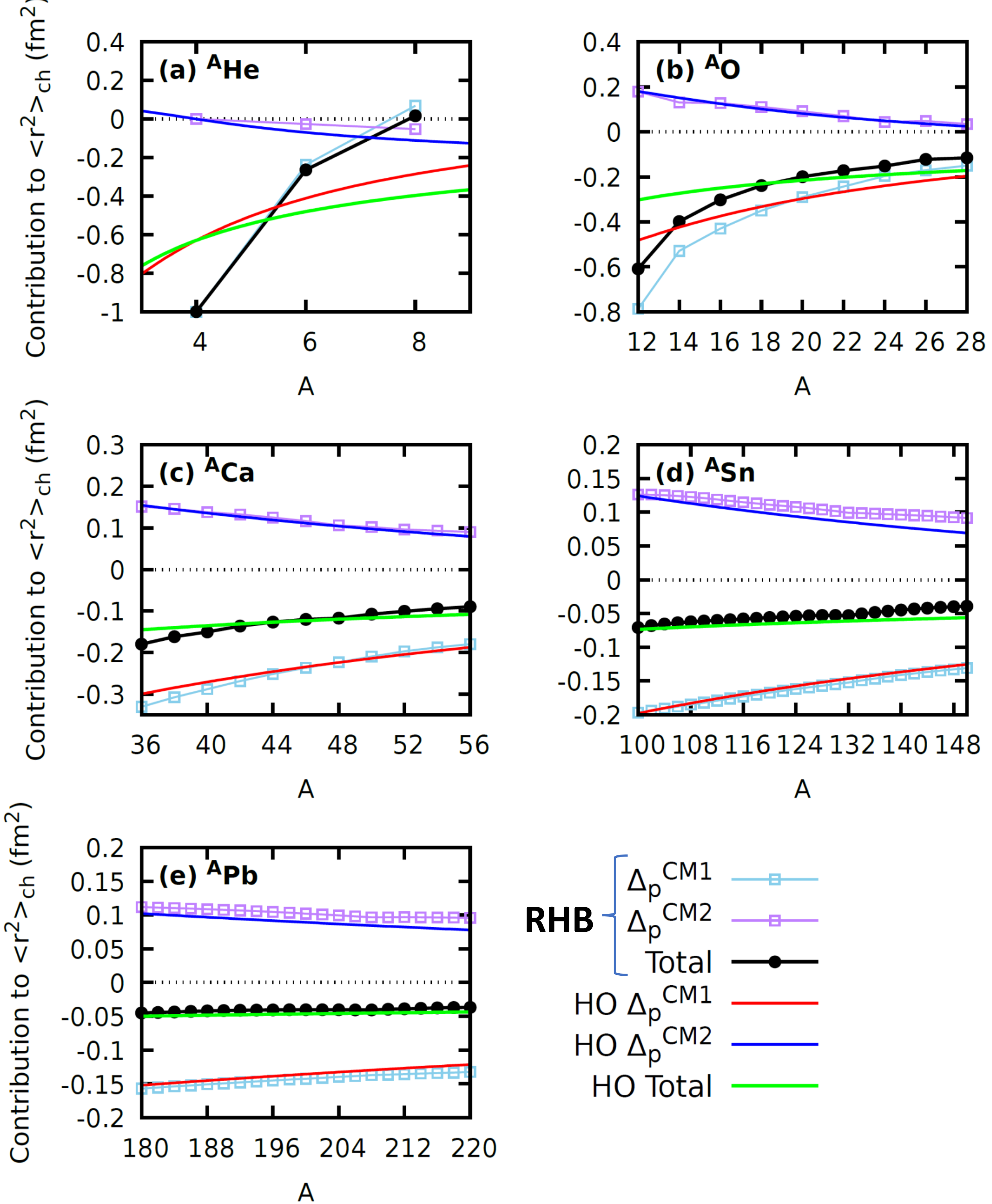
Tanimura and Cheoun, arXiv:2312.15983 [nucl-th] (2023).



Comparison with harmonic-oscillator model

HO model is

- not good for very light and/or near-dripline nuclei
- nearly satisfactory for Ca and heavier nuclei



Comparison with harmonic-oscillator model

HO model is

- not good for very light and/or near-dripline nuclei
- nearly satisfactory for Ca and heavier nuclei

Connection with symmetry-restoration technique

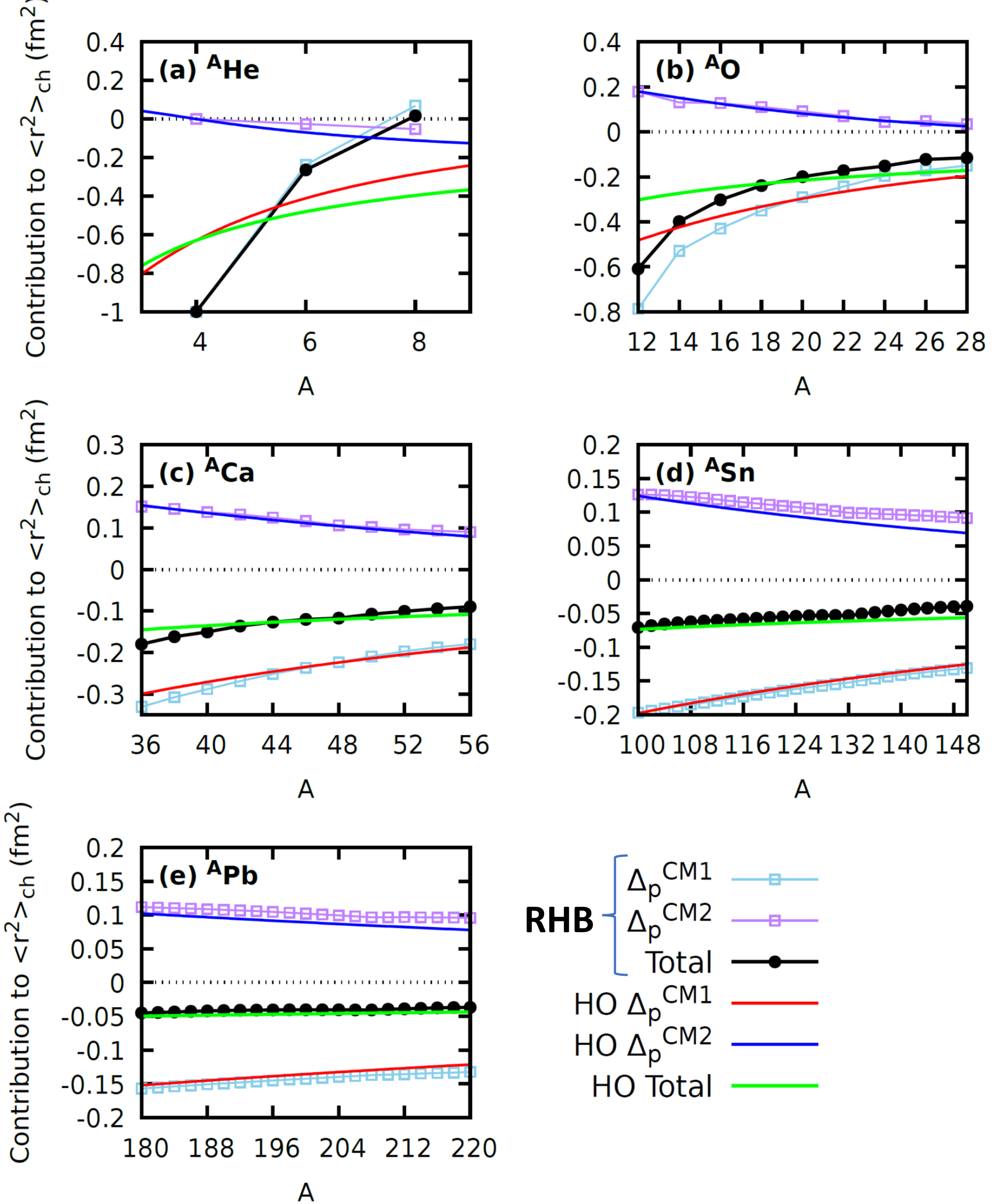
HO approximation to our approach yields

$$\Delta_p^{(\mathrm{CM1})} + \Delta_p^{(\mathrm{CM2})} = -\frac{9\hbar^2}{4\langle \boldsymbol{P}_{\mathrm{CM}}^2 \rangle}$$

cf. approximate momentum projection method modifying the charge radius as

$$r_{ch}^2 \to r_{ch}^2 - \frac{9\hbar^2}{4\langle \boldsymbol{P}_{cm}{}^2 \rangle}$$

Schmidt and Reinhard, NPA**530**, 283 (1991)
Reinhard and Nazarewicz, PRC**103**, 054310 (2021)



Summary

- Charge radius with important corrections
  - **CM correction (for symmetry-breaking in mean-field model)**
  - **EM structure of nucleon (directly modifies ρ_{ch})**
- Sizable impacts on charge radii

- Matter radius and neutron skin?
- Refit of parameters in EDF?