Equation of state of nuclear matter and nuclei in laboratories and in neutron-star crusts

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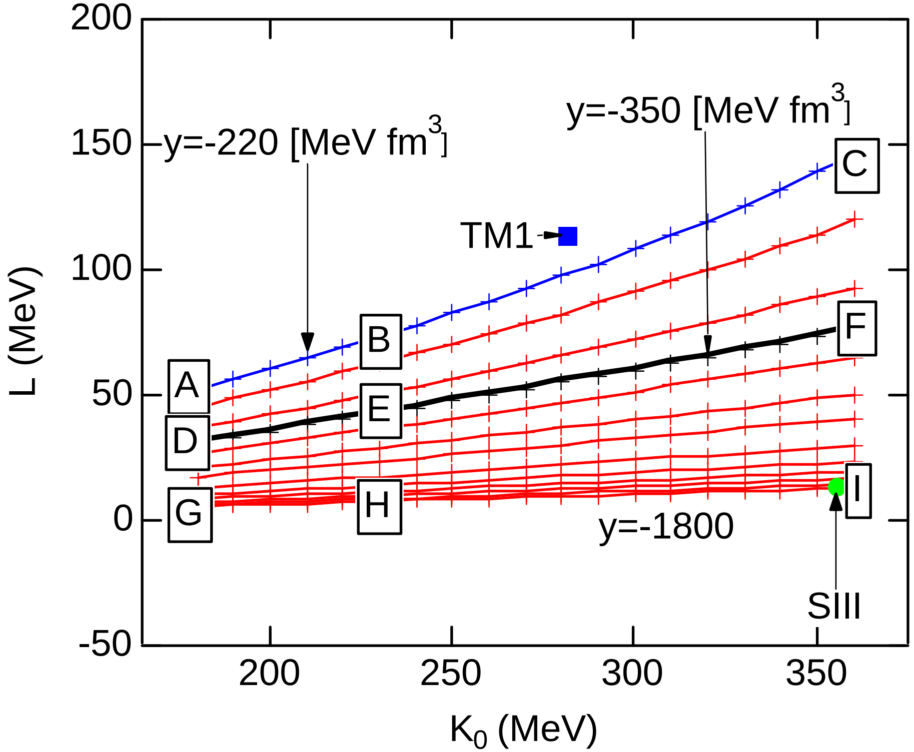
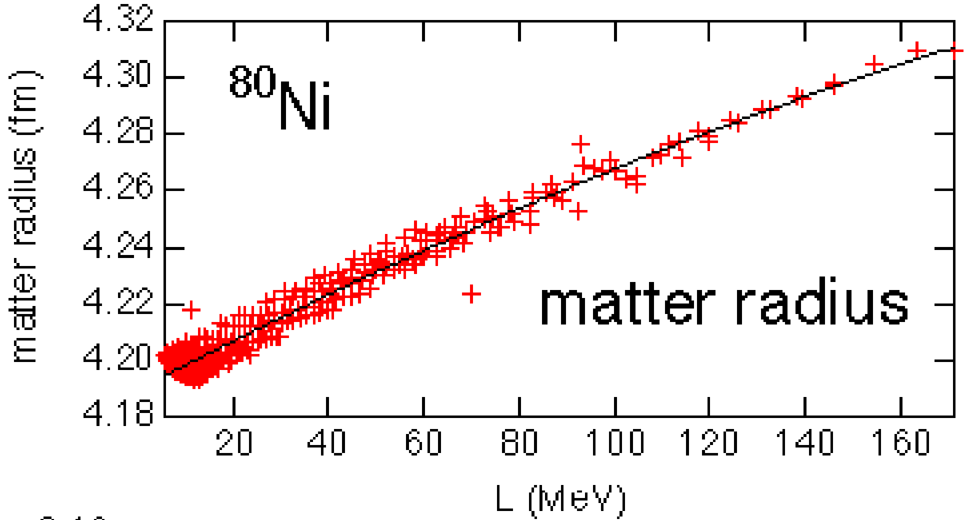
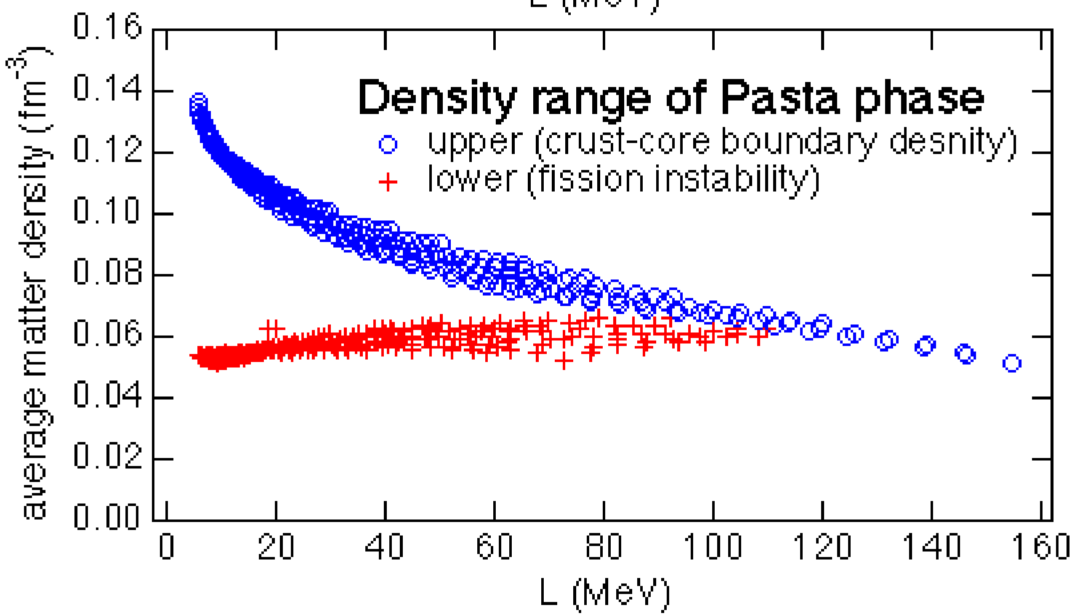
LARGEST UNCERTAINTY

L : densty-derivative coefficient of symmetry energy







Macroscopic properties of nuclei in labs. and in neutron stars

=> We adopt a macroscopic model.

1. Masses and radii os stable nuclei => We construct about 200 EOS's systematically.

2. Focus on saturation parameters of nearly symmetric nuclear matter

   and identify the allowed region of these EOS parameter values.

3. Calculate masses and radii of unstable nuclei in labs. => dominant EOS parameter

4. Calculate nuclei in neutron-star crusts. => dominant EOS parameter

   proton number, average proton fraction of spherical nuclei

   pasta-nucleus phase : lower and upper boundary density

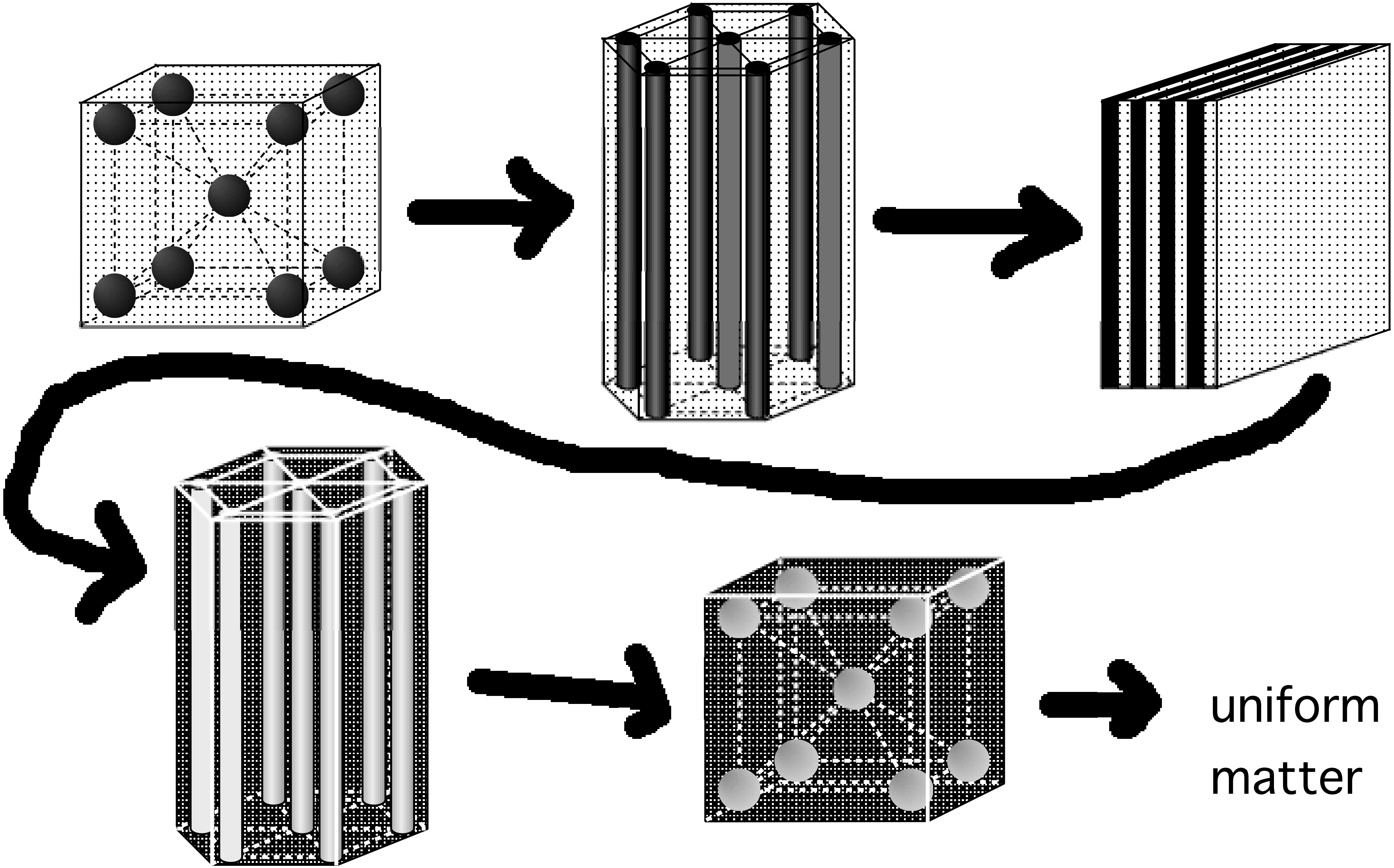
Energy per nucleon of nearly symmetric nuclear matter

$$w(n,\ x) \approx w_0 + \frac{K_0}{18n_0^2}(n-n_0)^2 + (1-2x)^2\left[S_0 + \frac{L}{3n_0}(n-n_0)\right]$$

n_0 : nuclear density, w_0 :saturation density, K_0 : incompressibility

S_0 : symmetry energy at $n=n_0$, L: its density derivative coefficient

spherical nuclei and pasta nuclei



K.Oyamatsu, NPA561, 431 (1993)

What dominates the existence of pasta nuclei in crusts?

○ Yes! There are pasta nuclei.

- Compressible liquid-drop model
  - FPS interaction (fitted to FP EOS)
    - Lorenz, Ravenhall and Pethick, PRL70(1993)
  - Uncertainties in μ_p and surface tension
    - Watanabe, Iida and Sato, NPA676(2000)
- Thomas-Fermi model
  - oya1-4 (fitted to masses and sizes of stable nuclei and FP EOS)
    - K.Oyamatsu, NPA561(1993)
  - DBHF EOS
    - Sumiyoshi, Oyamatsu and Toki, NPA595(1995)

× No! No pasta nuclei can exist.

- Compressible liquid-drop model (SKM interaction)
  - SKM interaction (Lorenz, Ravenhall and Pethick, PRL70(1993))
  - SLy4, SLy7 (Douchin, Haensel and Meyer, NPA665(2000))
- Relativistic Thomas-Fermi model (Euler-Lagrange Eq.)
  - RMF Lagrangian (Cheng, Yao and Dai, PRC55(1997))

The subnuclear density EOS dominates the existence.
=>Let's study empirically allowed EOS systematically!

Step 1

Generate all empirically allowed EOS's systematically

Adopted macroscopic model

Energy per cell

$$W = \int_{cell} d\mathbf{r} \left[\varepsilon_0(n_n, n_p) + m_n n_n + m_p n_p \right] + \int_{cell} d\mathbf{r}\, F_0 \left| \nabla n_n \right|^2 + (\text{electron kinetic energy}) + (\text{Coulomb})$$

n_n (n_p) : local neutron (proton) density, $n=n_n+n_p$: total density

$\varepsilon_0(n_n, n_p)$: EOS of uniform nuclear matter (energy density)

F_0 : surafce energy parameter

Parametrization of the EOS (energy density)

$$\varepsilon_0(n_n, n_p) = \frac{3}{5}\left(3\pi^2\right)^{2/3}\left(\frac{\hbar^2}{2m_n} n_n^{5/3} + \frac{\hbar^2}{2m_p} n_p^{5/3}\right) + \left[1 - \left(1 - 2Y_p\right)^2\right] v_s(n) + \left(1 - 2Y_p\right)^2 v_n(n)$$

Fermi kinetic energy density — potential energy density

potential energy densities of symmetric and neutron matter

$$v_s(n) = a_1 n^2 + \frac{a_2 n^3}{1 + a_3 n} \qquad v_n(n) = b_1 n^2 + \frac{b_2 n^3}{1 + b_3 n}$$

★$a_1 \sim b_2$ and F_0 : masses and radii of stable nuclei (b_3=1.59 fm^3, a fit to FP EOS)

★very flexible function form

a_3 can vary K_0 widely. (better than Skyrme)

The function can be fitted to SIII and TM1 EOS very well.

Simplified Thomas-Fermi calculation

energy minimization with respect to parameters of $n_n(r)$ and $n_p(r)$ (and lattice constant)

neutron (proton) density distribution n_n (n_p)

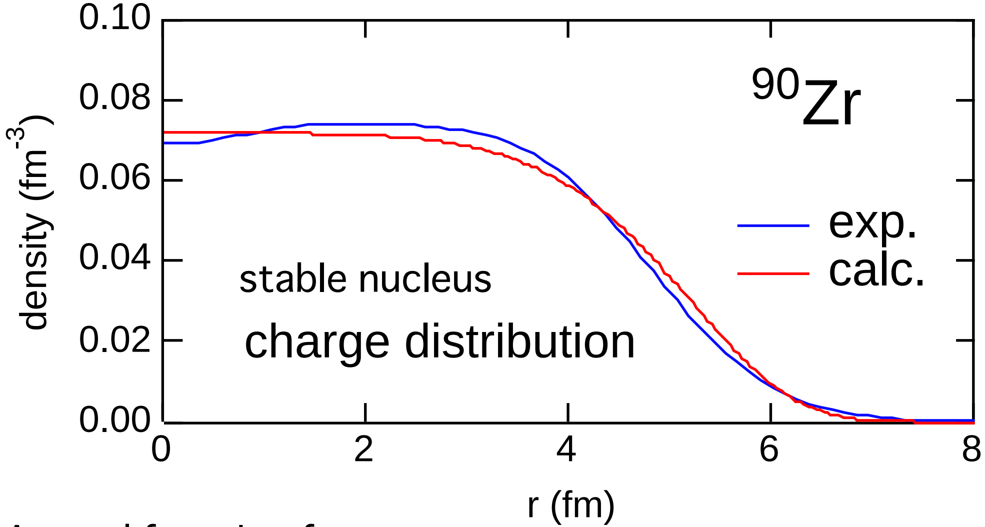
$$n_i(r) = \begin{cases} \left(n_i^{in} - n_i^{out}\right)\left[1 - \left(\frac{r}{R_i}\right)^{t_i}\right]^3 + n_i^{out} & r < R_i \\ n_i^{out} & r < R_i \end{cases}$$

R_n (R_p) : neutron (proton) radius parameter

t_n (t_p) : neutron (proton) surface thickness parameter

n_i^{in} : central desnity

n_n^{out} : neutron gas density ($n_p^{out}=0$)



A good function form
The n and p distributions are independent.
=> neutron skin
The empirical information is limited: radius and thickness.
The gradient term in Euler Eq. is continuous.
The density is zero beyond the classical turning point.

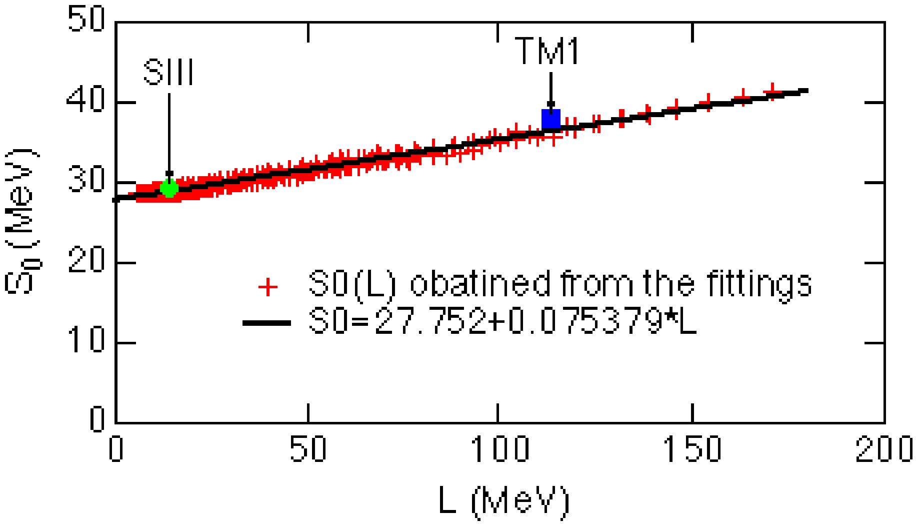
The values of parameters $a_1 \sim b_3$(EOS) and F_0 are determined
to fit masses and radii of stable nuclei.

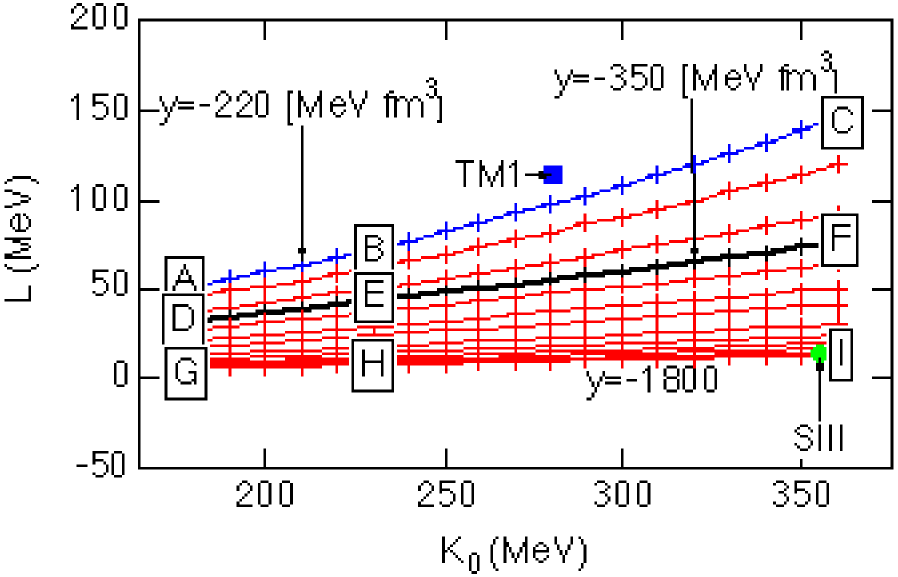
=> about 200 sets of empirical EOS+F_0

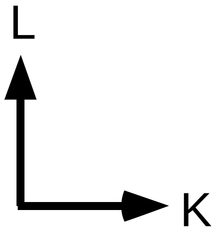
EOS parameter values obtained from stable nuclei

S_0:symmetry energy

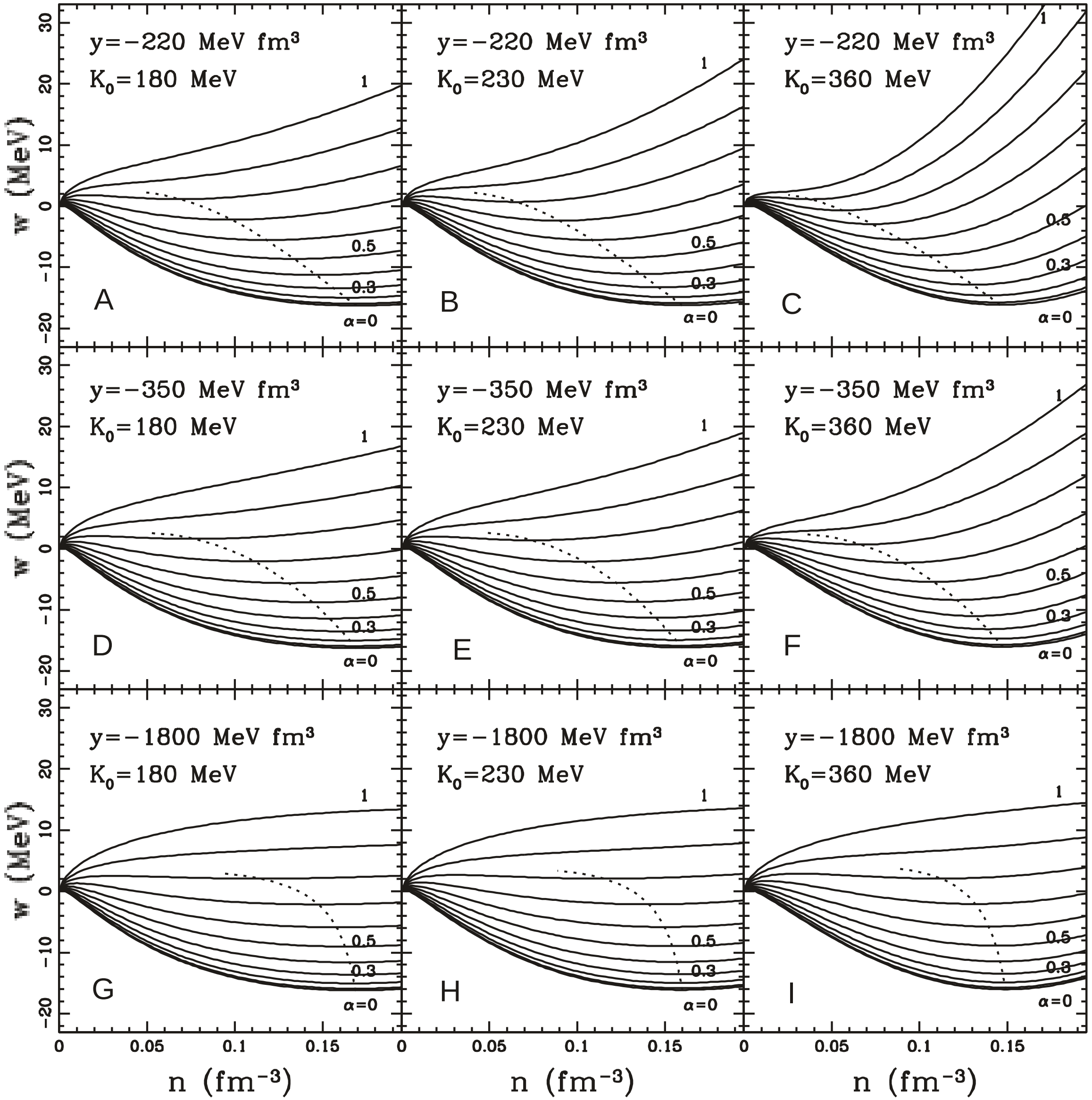
L : density symmetry coefficient







9 representative EOS A-I

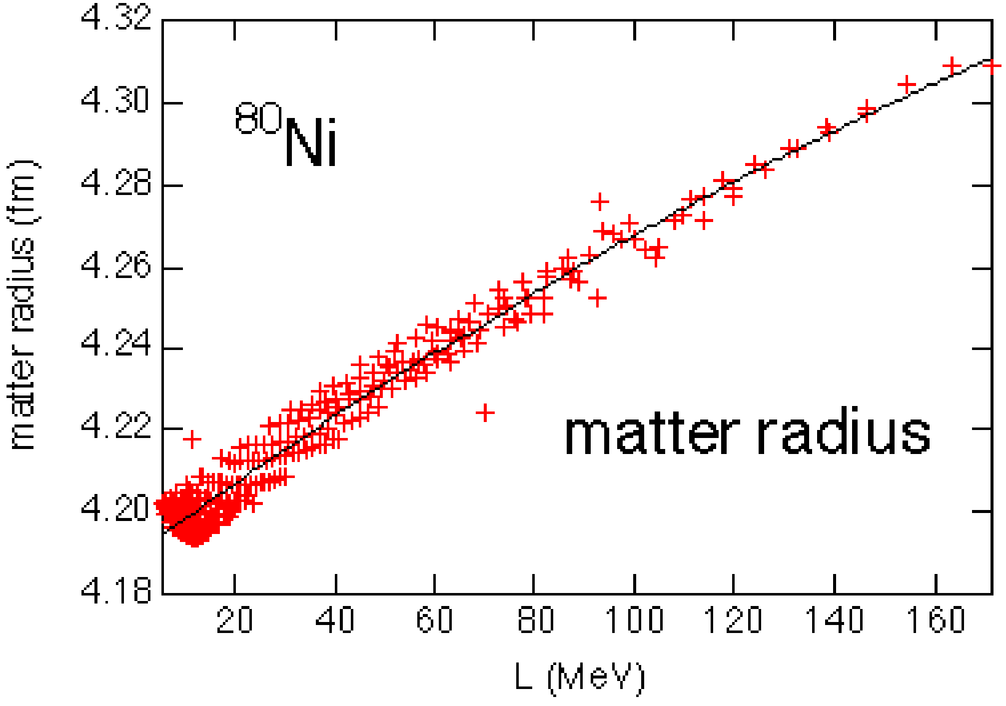


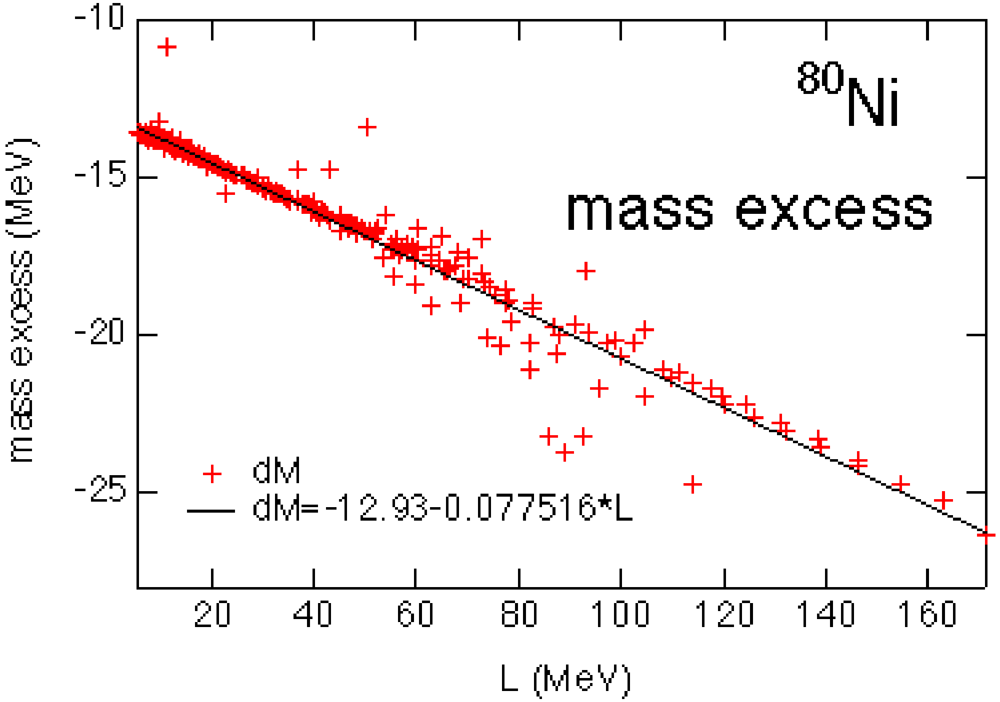
Step 2

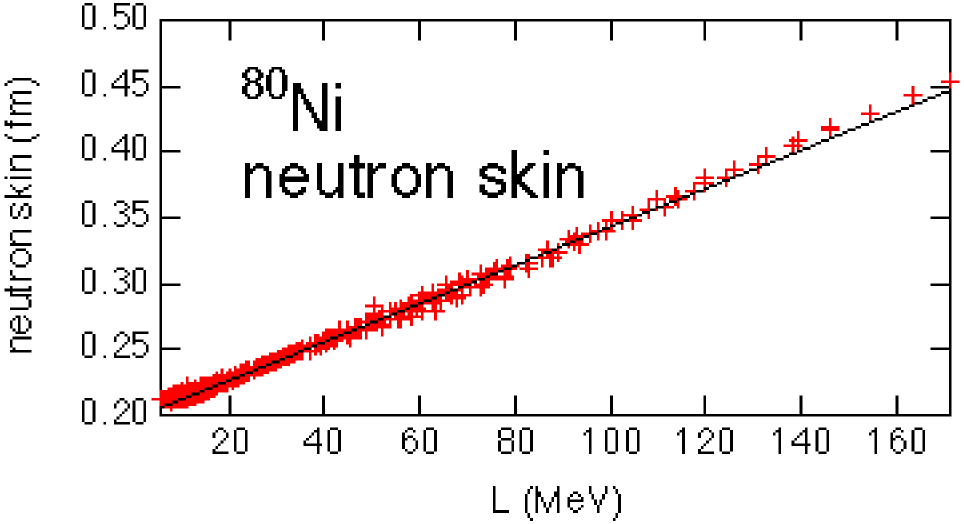
Calculations with the 200 EOS's

Neutron-rich unstable nuclei in labs

The mass, radius and neutron skin are dependent on L.





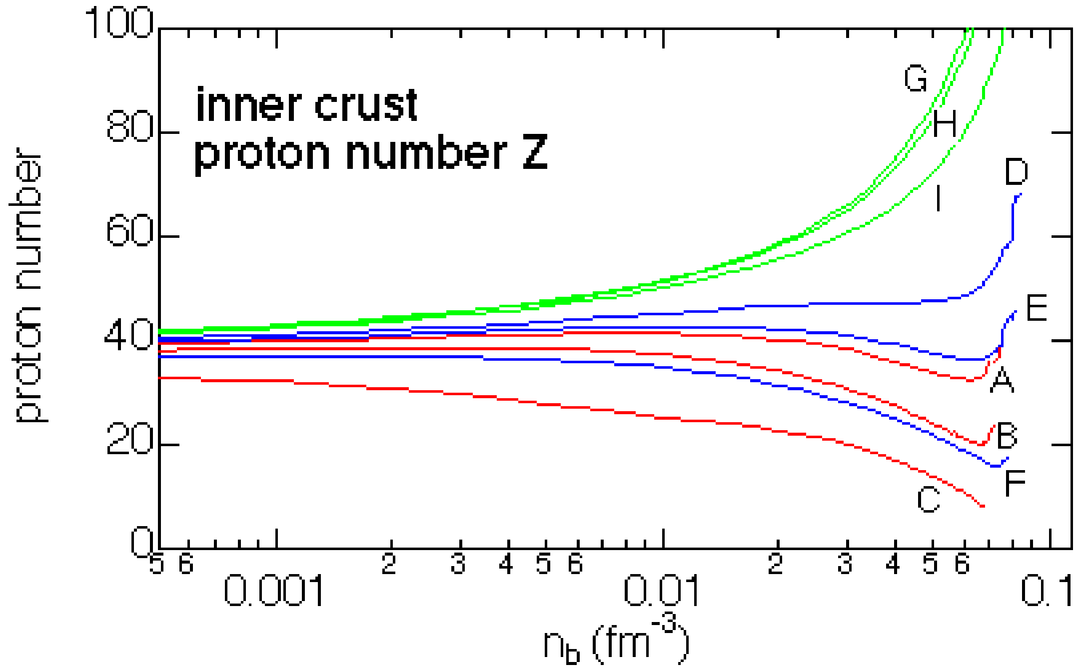


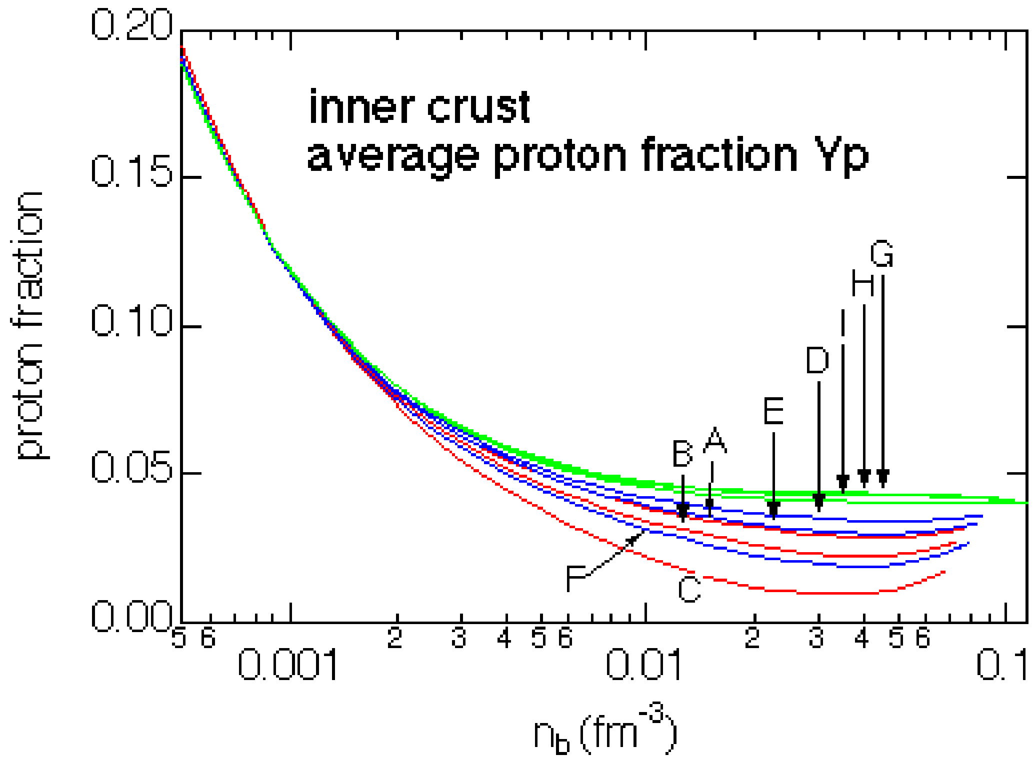
Step 3 (Calculations with 200 EOS's)

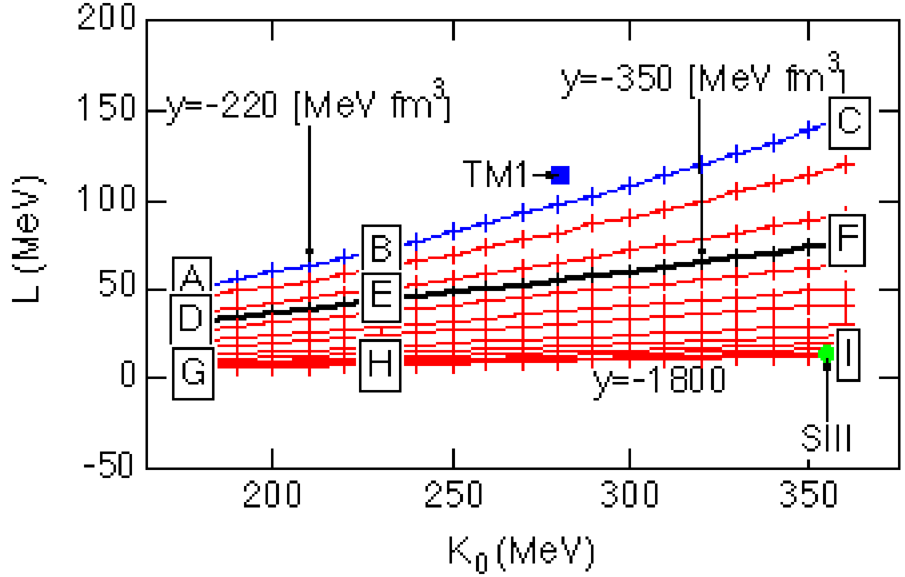
Nuclei in neutron star crusts

Density region of pasta nuclei

Z and Y_p are small for large L.
For large L, S(n) is small at $n<n_0$.







Estimate of density region of pasta nuclei

lower boundary

stability against fission of spherical nuclei

In the liquid drop model, (Coulomb self energy)=2*(surface energy)

==> (volume fraction of nucleus) = 1/8

upper boundary (core-crust boundary)

instability against forming proton clusters

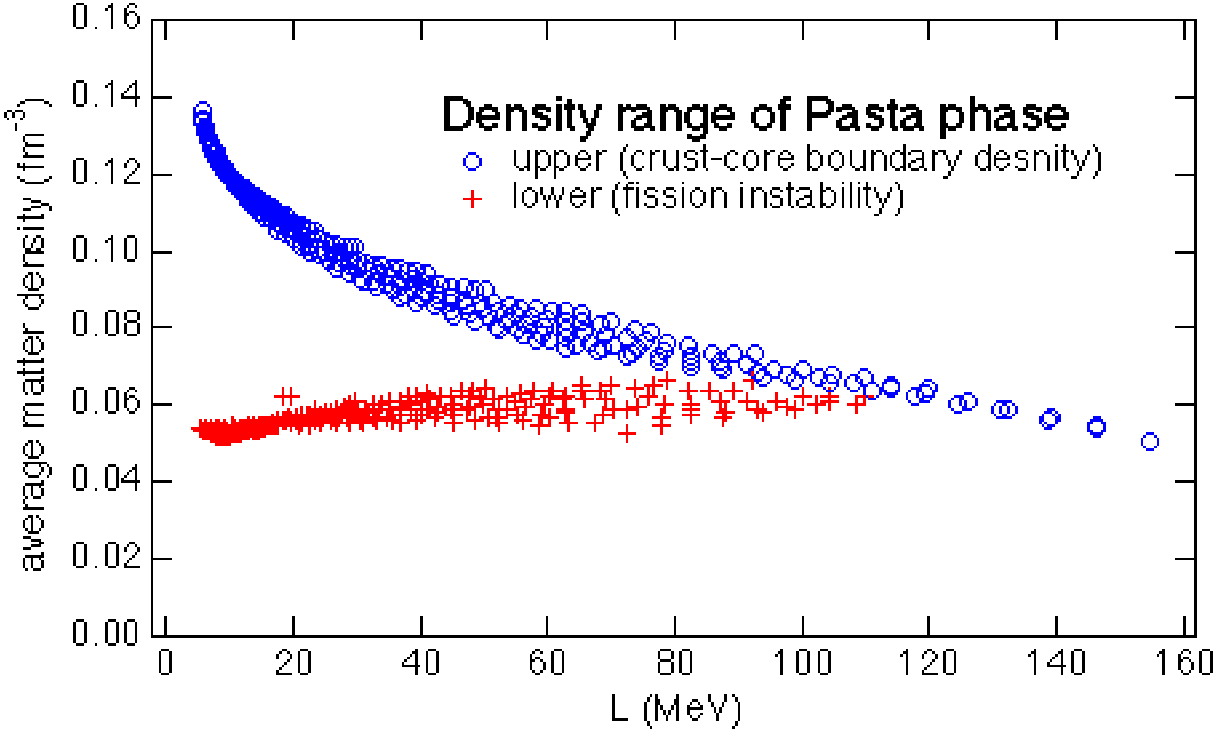
$$v(Q) = v_0 + 2(4\pi e^2 \beta)^{1/2} - \beta k_{\mathrm{TF}}^2 > 0 \qquad Q^2 = \left(\frac{4\pi e^2}{\beta}\right)^{1/2} - k_{\mathrm{TF}}^2$$

$$v(Q) \approx v_0 = \frac{\partial \mu_{\mathrm{p}}}{\partial n_{\mathrm{p}}} - \frac{(\partial \mu_{\mathrm{p}}/\partial n_{\mathrm{n}})^2}{\partial \mu_{\mathrm{n}}/\partial n_{\mathrm{n}}} = \left(\frac{\partial \mu_{\mathrm{p}}}{\partial n_{\mathrm{p}}}\right)_{\mu_{\mathrm{n}},\mu_{\mathrm{e}}}$$

$$\beta = D_{\mathrm{pp}} + 2D_{\mathrm{np}}\zeta + D_{\mathrm{nn}}\zeta^2, \qquad \zeta = -\frac{\partial \mu_{\mathrm{p}}/\partial n_{\mathrm{n}}}{\partial \mu_{\mathrm{n}}/\partial n_{\mathrm{n}}},$$

$$k_{\mathrm{TF}}^2 = \frac{4\pi e^2}{\partial \mu_{\mathrm{e}}/\partial n_{\mathrm{e}}} = \frac{4\alpha}{\pi}(3\pi^2 n_{\mathrm{e}})^{1/3}.$$

The upper bound (core-crust boundary density) is clearly dependent on L while the lower is almost constant.



Summary

1. The values of L and K_0 cannot be determined from masses and radii of stable nuclei.

2. Radii and masses of unstable nuclei have appreciable sensitivity to L.

3. The core-crust boundary density is dependent on L.

4. The existence of the pasta phase is dominated by L.
   The pasta phase exists if L < 100 MeV.

5. The present uncertainty in L is too large.