

Study of resonant states in nuclei via breakup reactions

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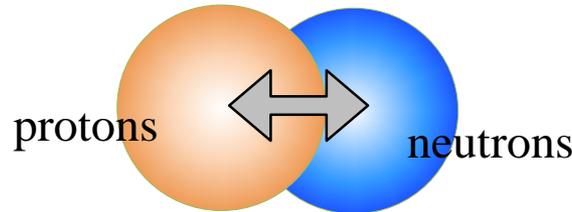
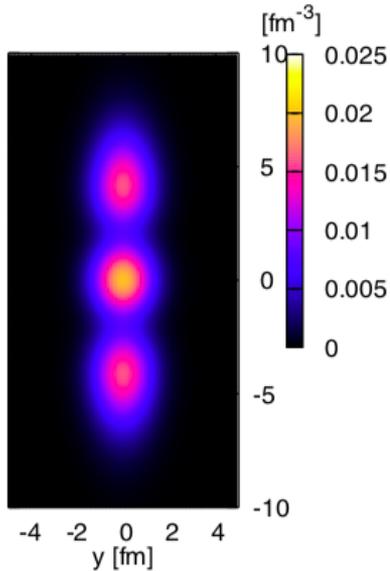
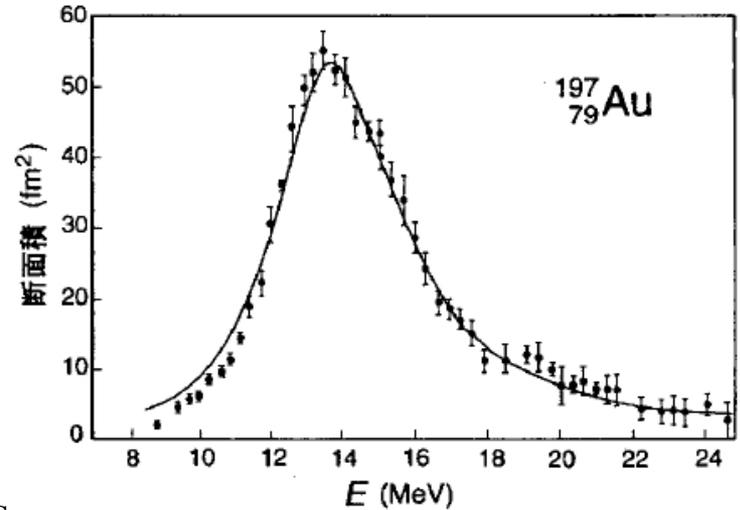
Few-Body systems in Hadron, Nuclei, Atomic and Molecular Physics

Introduction

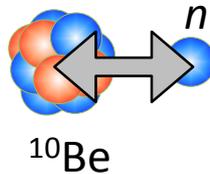
■ Resonance in Nuclei

✓ Giant dipole resonance

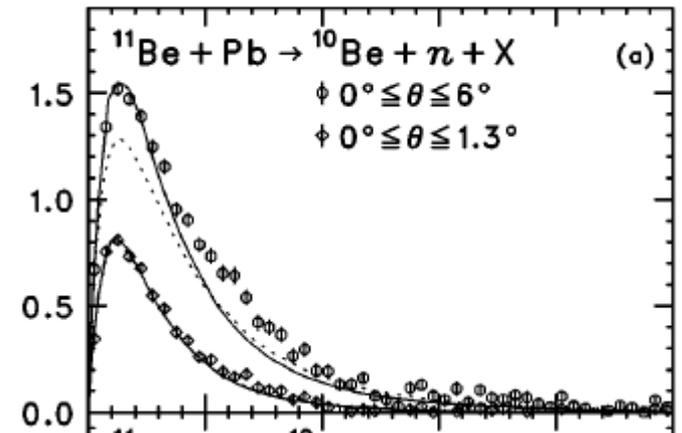
Corrective nuclear excitation



✓ Soft dipole resonance



N. Fukuda et al., PRC 70 054606 (2004)



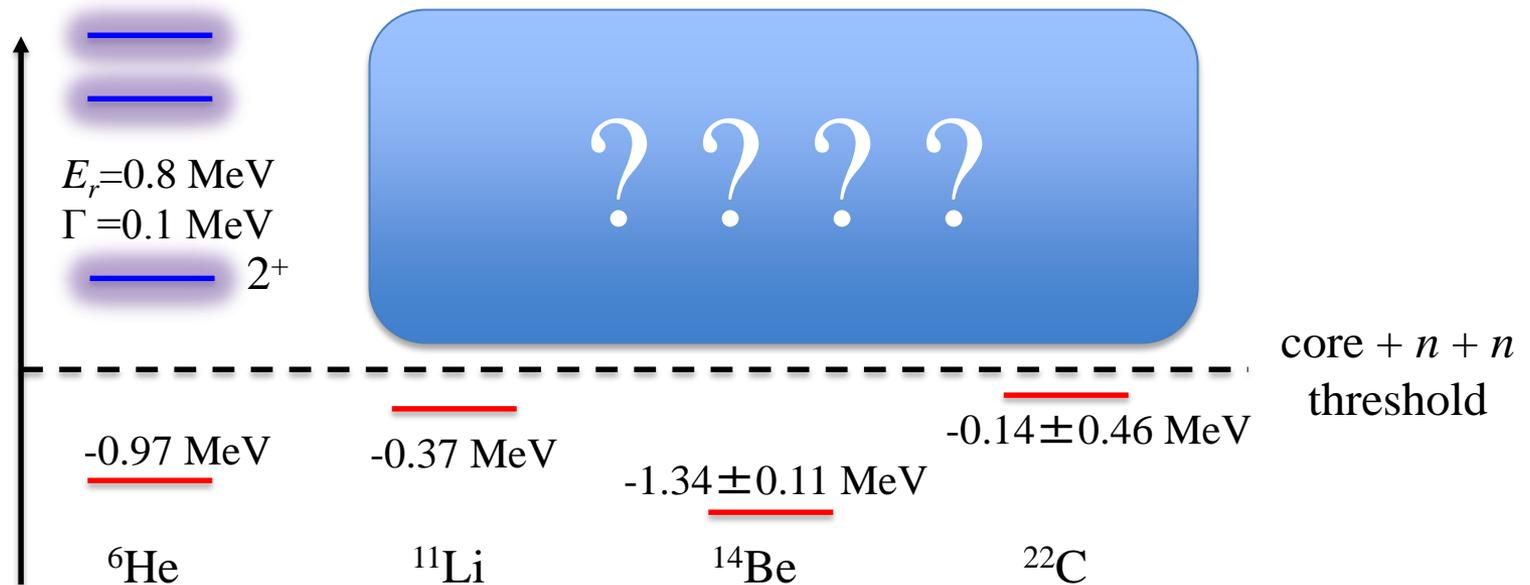
Y. Funaki, Phys. Rev. C **94**, 024344 (2016).

✓ Cluster resonance

Linear chain structure

Two-neutron halo nuclei

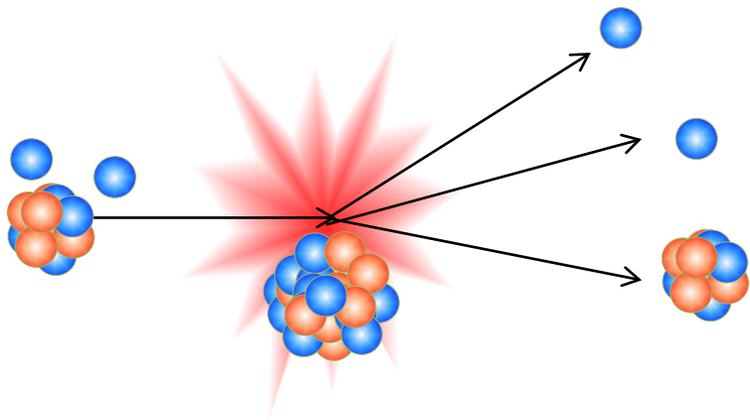
${}^6\text{He}$, ${}^{11}\text{Li}$, ${}^{14}\text{Be}$, and ${}^{22}\text{C}$ are known as two-neutron halo nuclei consisting of a core nucleus and two loosely bound neutrons.



- ✓ Excited bound states do not exist.
- ✓ Except for ${}^6\text{He}$, experimental information on resonances of two-neutron halo is very scarce.
- ✓ Existence of a resonance of ${}^{11}\text{Li}$ is a longstanding open question.

Breakup Reaction

- ◆ Breakup reactions are useful for investigating resonances.



Observable

➤ Breakup cross section for E_x

✓ *Resonance states*

→ *peak structure*

✓ *Excited modes*

- ◆ To extract properties of resonances, an accurate method for describing breakup processes is highly desired.

- ◆ Coulomb breakup with a heavy target (^{208}Pb , ^{209}Bi)

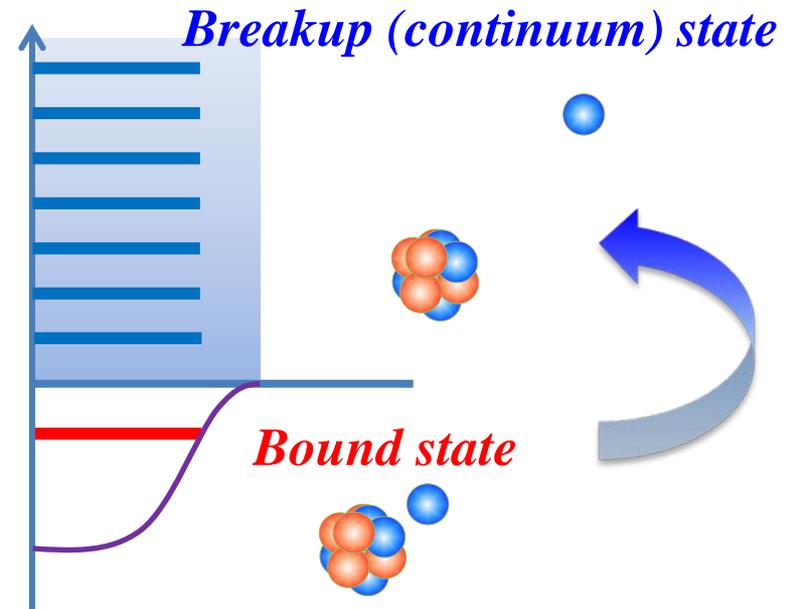
- Coulomb interaction

- ◆ Nuclear breakup with a light target (p , ^{12}C)

- Nuclear interaction

Continuum-Discretized Coupled-Channels

- Breakup is effects of *coupling to continuum states*.
- In CDCC, breakup continuum states are assumed as *a finite number of discretized states*.



- *Average method*: ➤ G. H. Rawitscher, Phys. Rev. C 9 (1974), 2210

$$\phi_i = N_i \int_{E_i}^{E_{i-1}} \psi(E) dE \quad \psi(E) : \text{continuum w.f.}$$

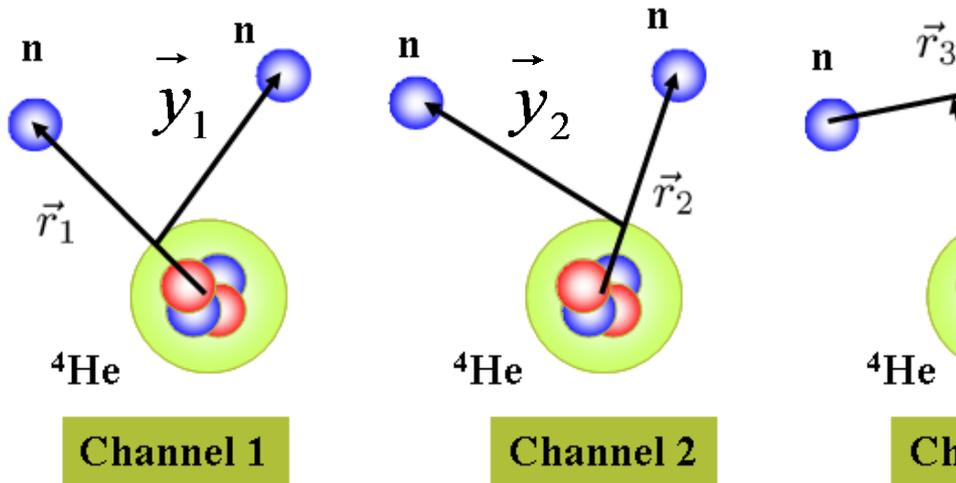
- *Pseudo-states method*: ➤ R. C. Johnson and P. C. Tandy, Nucl. Phys. A 235 (1974), 56

$$\phi_i = \sum_{j=1}^N C_j^{(i)} \varphi_j \quad \varphi_j : \text{Basis function}$$

Pseudo-state Discretization

□ Gaussian Expansion Method

E. Hiyama, Y. Kino, and M. K

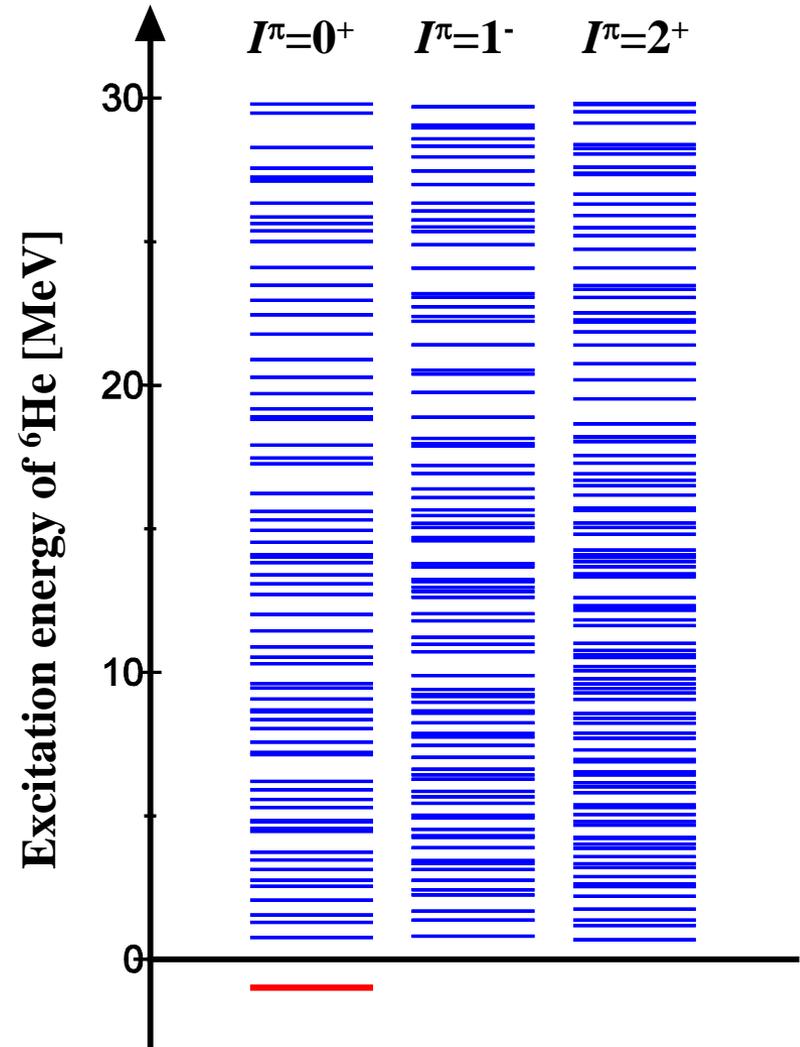


Three-body Hamiltonian

$$H = T_r + T_y + V_{nn} + V_{nc} + V_{n4He}$$

Gaussian basis functions

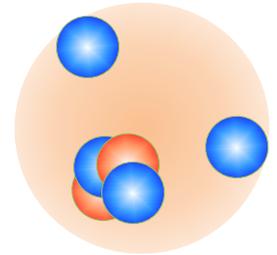
$$\phi_{lm}(r, y) = \sum_{c=1}^3 \sum_{nj\ell\lambda} A_{nj\ell\lambda}^{(c)} \varphi_{n\ell}(r_c) \varphi_{j\lambda}(y)$$



${}^6\text{He}$ Elastic Scattering

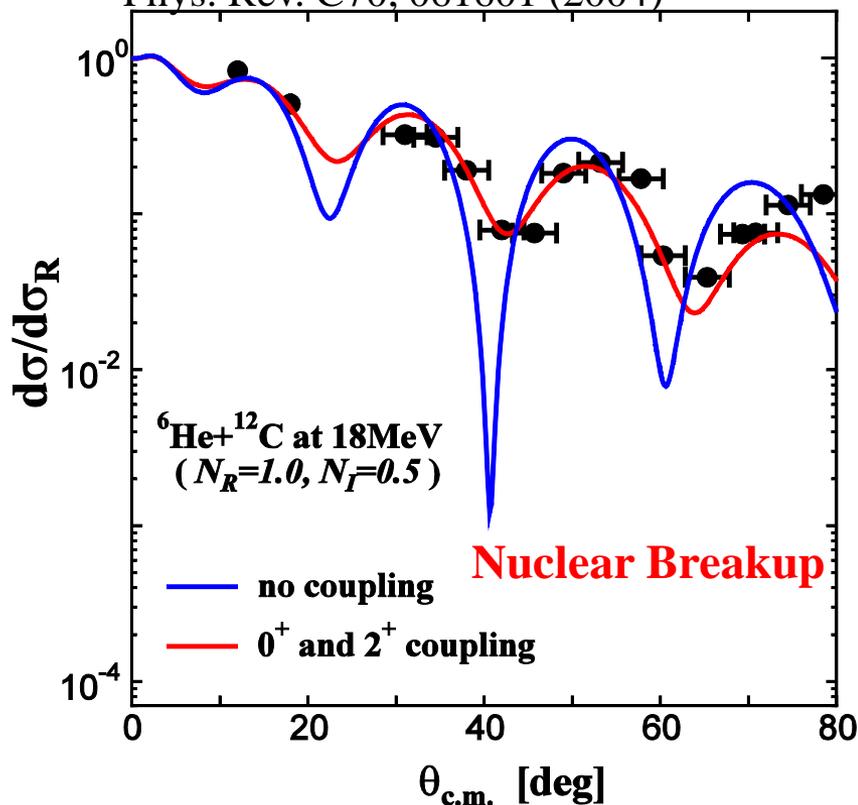
□ ${}^6\text{He}$

- A typical example of *two-neutron halo*
- Described by $n + n + {}^4\text{He}$ three-body model

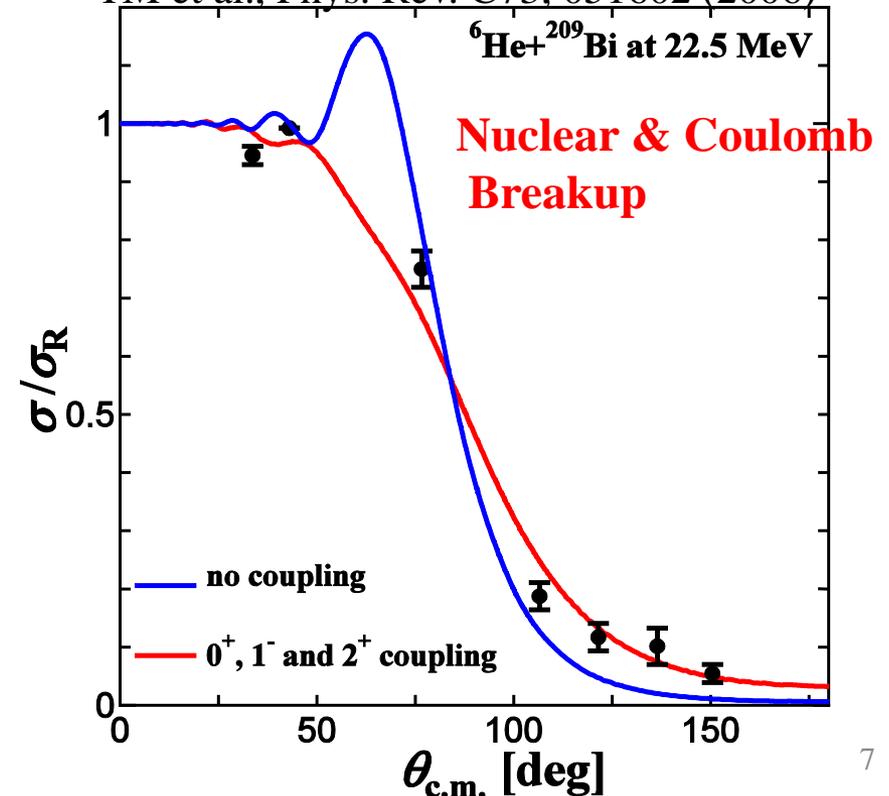


TM, Hiyama, et al.

Phys. Rev. C70, 061601 (2004)



TM et al., Phys. Rev. C73, 051602 (2006)

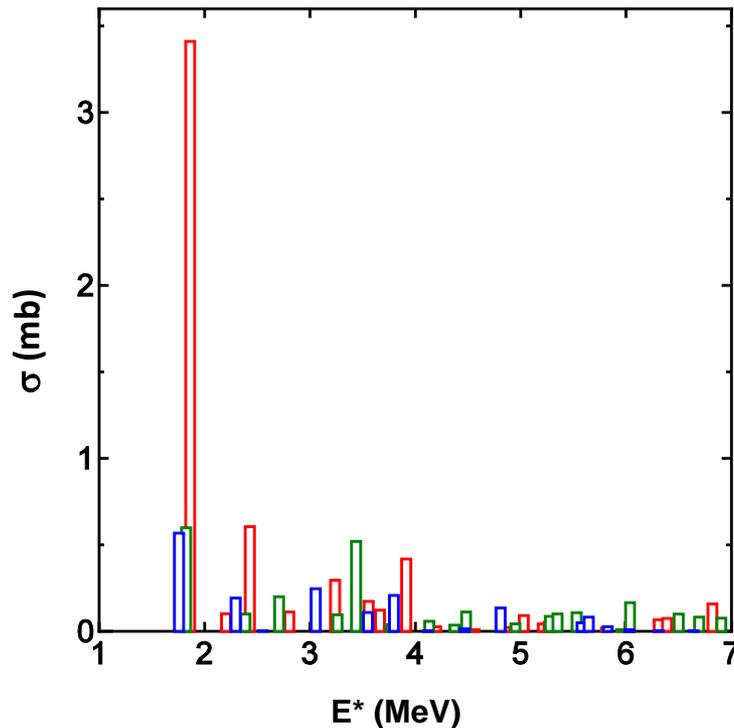


Breakup Cross Section

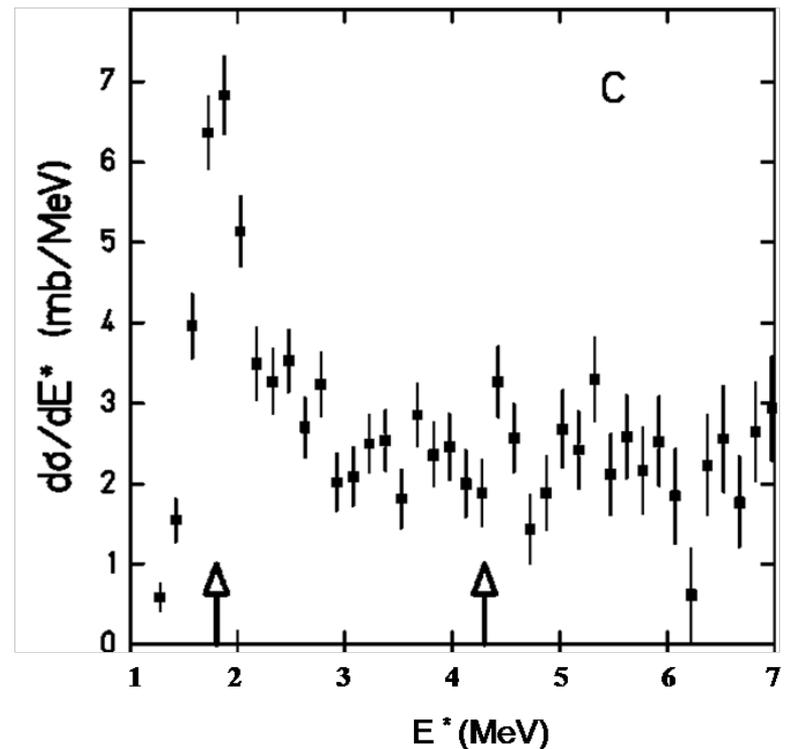
Breakup cross sections calculated by CDCC are **discrete** in the internal energy of the projectile.

${}^6\text{He}+{}^{12}\text{C}$ scattering at 240 MeV/nucl.

4-body CDCC calc.



PRC59, 1252(1999), T. Aumann *et al.*



*How to calculate the **continuous** breakup cross section*

Complex-Scaled Method

S. Aoyama, T. Myo, K. Kato, and K. Ikeda,
Prog. Theor. Phys. 116, 1 (2006)

Complex-scaling operator: U^θ

$$U^\theta f(r) = e^{i3/2\theta} f(re^{i\theta})$$

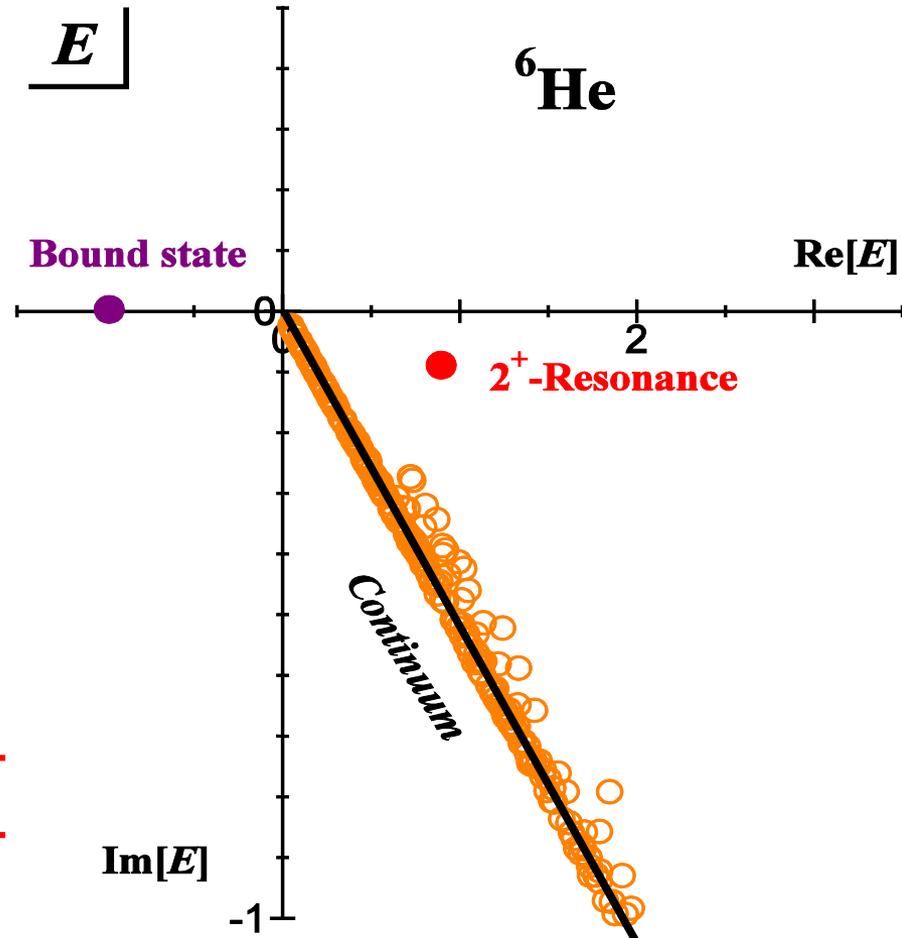
Coordinate: $r \rightarrow re^{i\theta}$

Momentum: $k \rightarrow ke^{-i\theta}$

Asymptotic form

$$e^{ikr} \longrightarrow e^{ikr \cos \theta} e^{-kr \sin \theta}$$

Useful for searching **many-body resonances**



Green's function with Complex-Scaling Method (CDCS Green's function)

$$G^{(-)}(E, \xi, \xi') = \frac{1}{E - H - i\epsilon} \approx \sum_{\nu} U^{-\theta} \frac{|\Phi_{\nu}^{\theta}\rangle \langle \tilde{\Phi}_{\nu}^{\theta}|}{E - E_{\nu}^{\theta}} U^{\theta}$$

New Smoothing Procedure with *CSM*

$$\frac{d\sigma}{dE} = \int T^\dagger(E')T(E')\delta(E - E')dE' = \frac{1}{\pi}\text{Im}\mathcal{R}(E)$$

$$T(E) = \langle \psi^{(-)}(E, \xi) \chi_C^{(-)}(\mathbf{R}) | V | \Psi^{(+)}(\xi, \mathbf{R}) \rangle$$

Response function

Final state of the projectile

$$\mathcal{R}(E) = \int d\xi d\xi' \langle \Psi^{(+)}(\xi, \mathbf{R}) | V^* | \chi_C^{(-)}(\mathbf{R}) \rangle_{\mathbf{R}} \mathcal{G}^{(-)}(E, \xi, \xi') \langle \chi_C^{(-)}(\mathbf{R}) | V | \Psi^{(+)}(\xi, \mathbf{R}) \rangle_{\mathbf{R}}$$

Green's function with Complex-Scaling Method (CDCS Green's function)

$$\mathcal{G}^{(-)}(E, \xi, \xi') = U^{-\theta} \frac{1}{E - H^\theta - i\epsilon} U^\theta \approx \sum_\nu U^{-\theta} \frac{|\Phi_\nu^\theta\rangle \langle \tilde{\Phi}_\nu^\theta|}{E - E_\nu^\theta} U^\theta$$

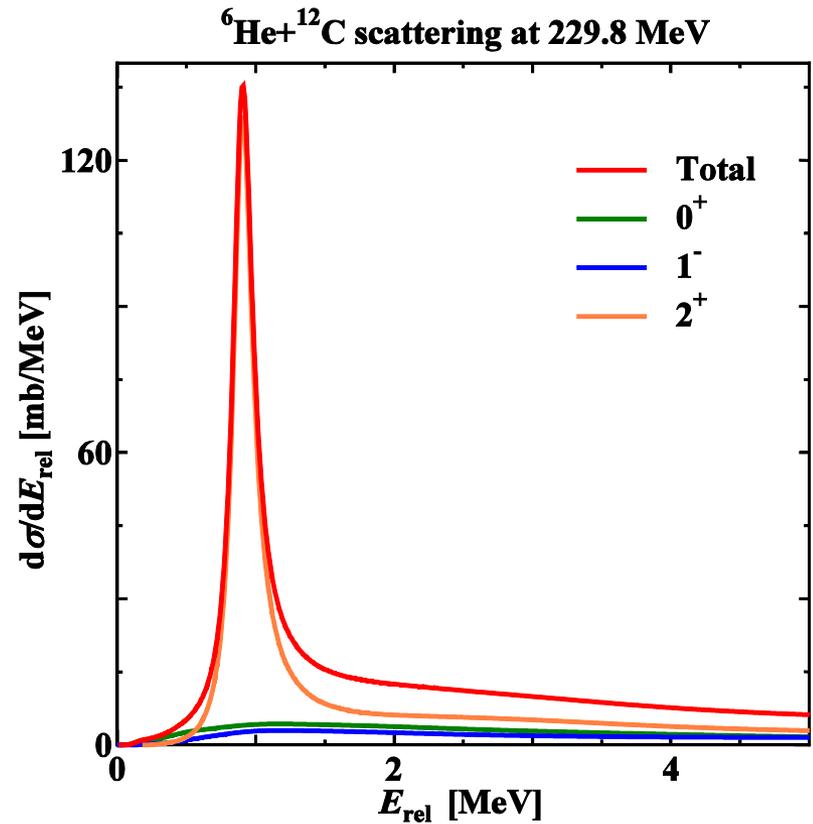
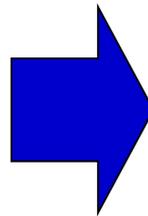
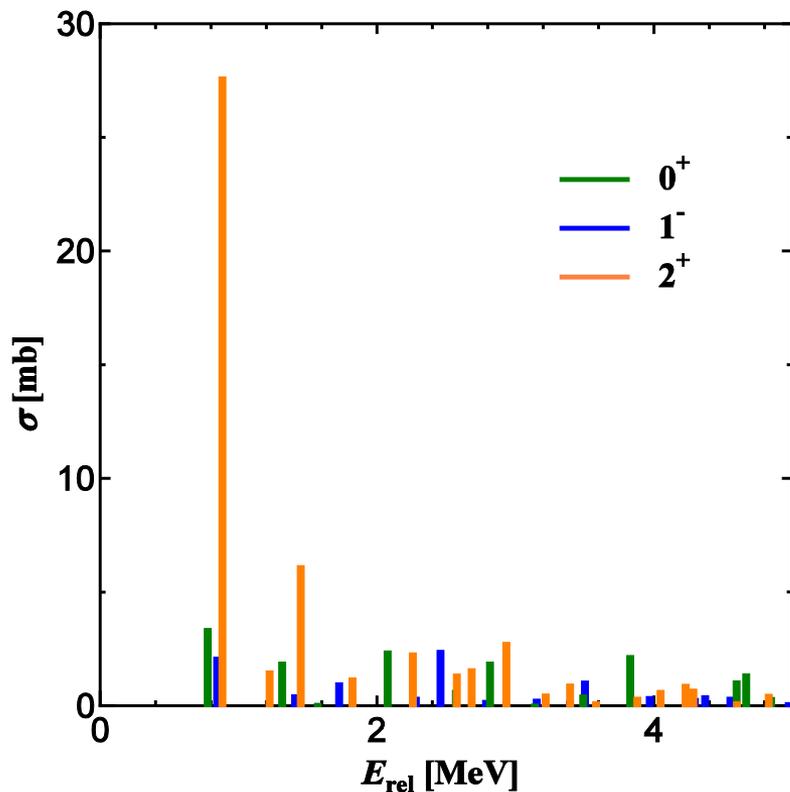
$$\mathcal{G}^{(-)}(E, \xi, \xi') \approx \sum_\nu \sum_{i,j} |\Phi_i\rangle \frac{\langle \Phi_i | U^{-\theta} |\Phi_\nu^\theta\rangle \langle \tilde{\Phi}_\nu^\theta | U^\theta | \Phi_j \rangle}{E - E_\nu^\theta} \langle \Phi_j |$$

$$\mathcal{R}(E) = \sum_\nu \sum_{i,j} \langle \Psi^{(+)} | V^* | \chi_C^{(-)} \Phi_i \rangle \frac{\langle \Phi_i | U^{-\theta} |\Phi_\nu^\theta\rangle \langle \tilde{\Phi}_\nu^\theta | U^\theta | \Phi_j \rangle}{E - E_\nu^\theta} \langle \Phi_j \chi_C^{(-)} | V | \Psi^{(+)} \rangle$$

T-matrix calculated by CDCC

Differential Breakup Cross Section

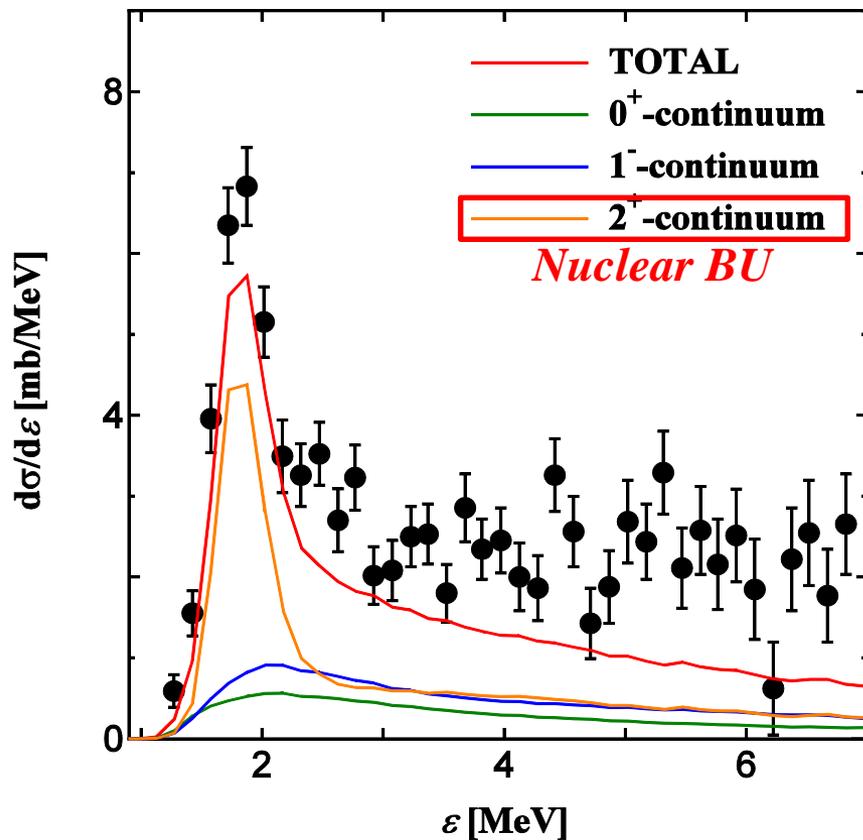
$$\frac{d\sigma}{dE} = \frac{1}{\pi} \text{Im} \sum_{\nu} \sum_{i,j} T_i^{\text{CDCC}\dagger} \frac{\langle \Phi_i | U^{-\theta} | \Phi_{\nu}^{\theta} \rangle \langle \tilde{\Phi}_{\nu}^{\theta} | U^{\theta} | \Phi_j \rangle}{E - E_{\nu}^{\theta}} T_j^{\text{CDCC}}$$



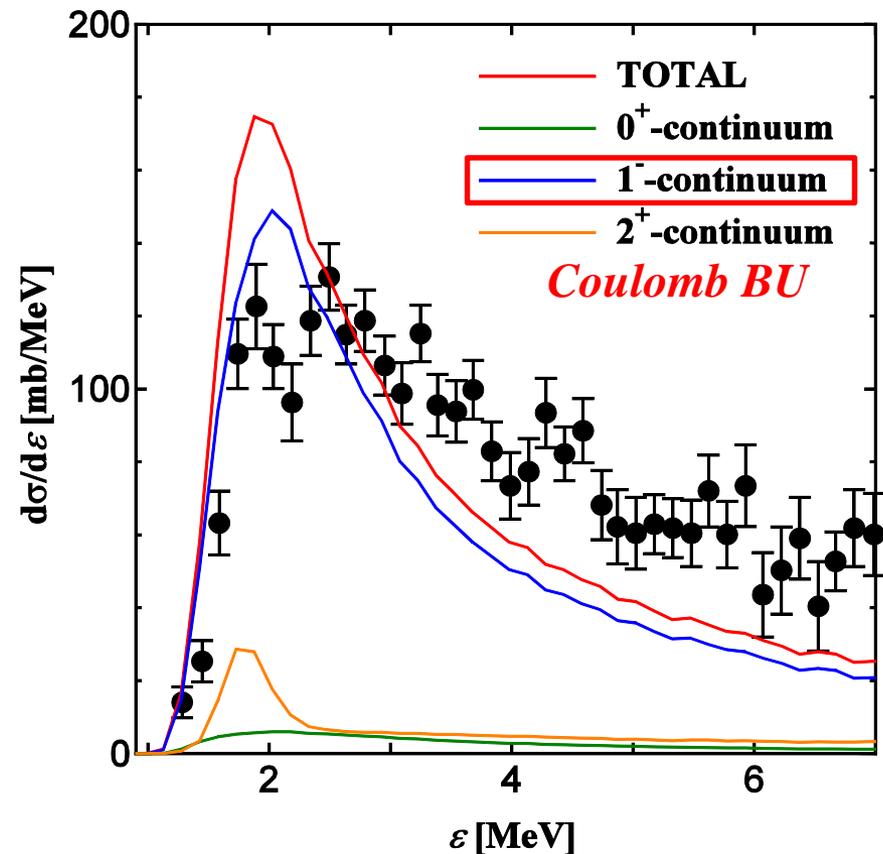
${}^6\text{He}$ Breakup Reaction

TM, Yahiro, and Kato, PRC82, 051602 (2010)

${}^6\text{He}+{}^{12}\text{C}@240\text{ MeV/nucl.}$



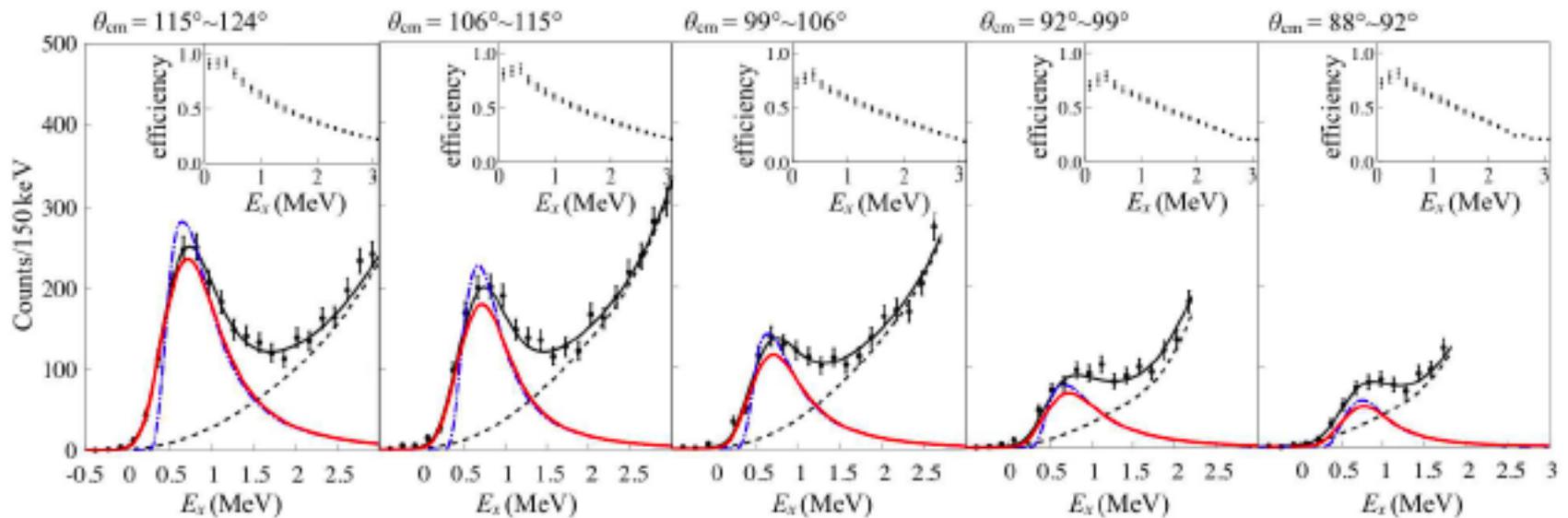
${}^6\text{He}+{}^{208}\text{Pb}@240\text{ MeV/nucl.}$



Recent experiment of ^{11}Li

- ◆ Measurement of the $^{11}\text{Li}(p, p')$ reaction at 6 MeV/nucleon with **high static and high resolution** has been performed, and a low-lying excited state of ^{11}Li has clearly been identified.

J. Tanaka et al., Phys. Lett. B774, 268 (2017).

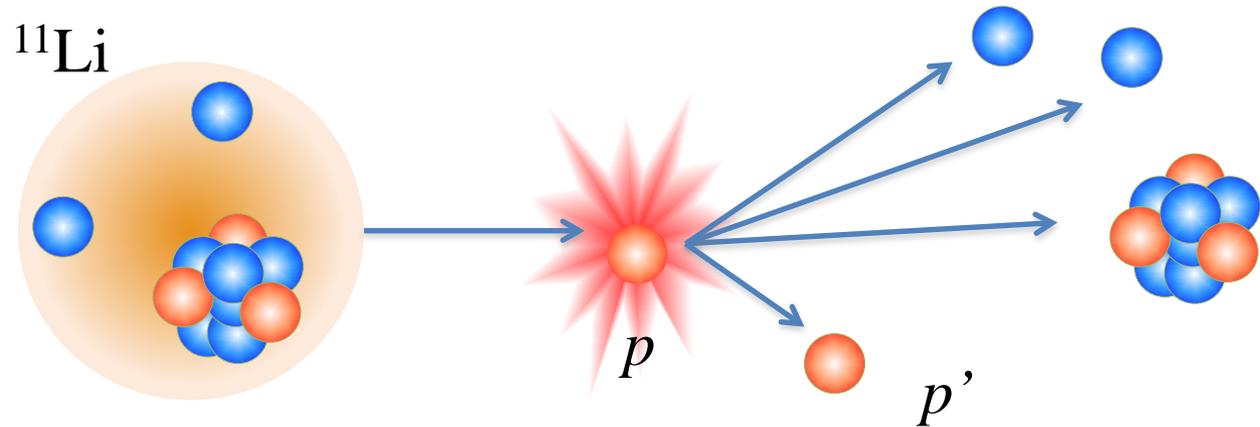


- ◆ The purpose of this work is to analyse the reaction with **a microscopic reaction model**.

$^{11}\text{Li}(p, p')$ @ 6MeV/nucl

^{11}Li

- ✓ Borromean
- ✓ $S_{2n} \sim 0.37$ MeV



^{11}Li Breakup

$\rightarrow ^{11}\text{Li} + p = n + n + ^9\text{Li} + p$

\rightarrow Continuum-Discretized
Coupled-Channels (CDCC)

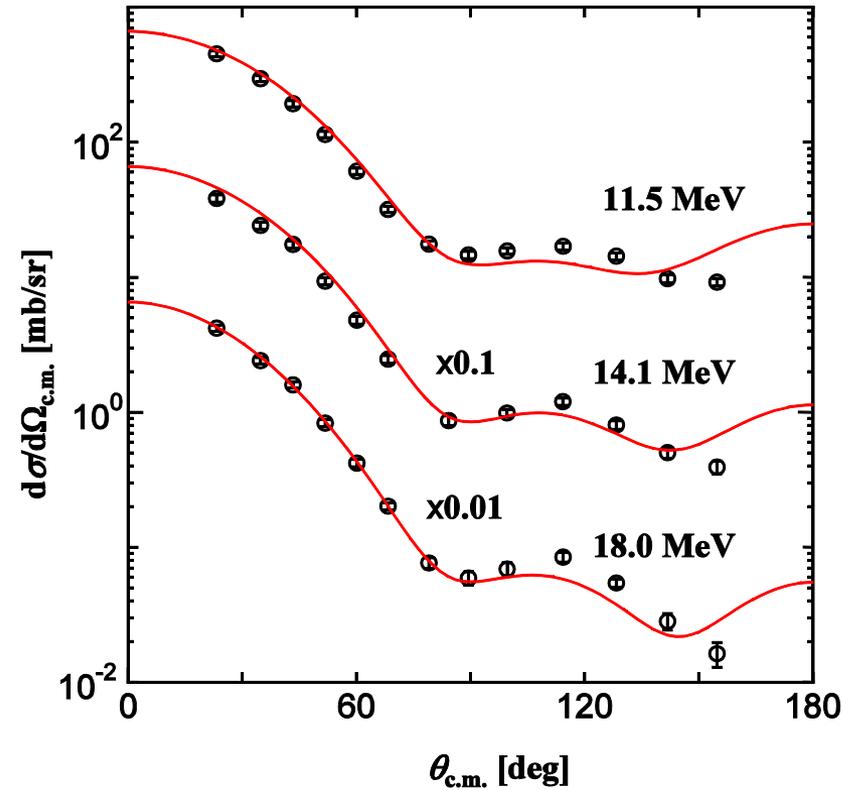
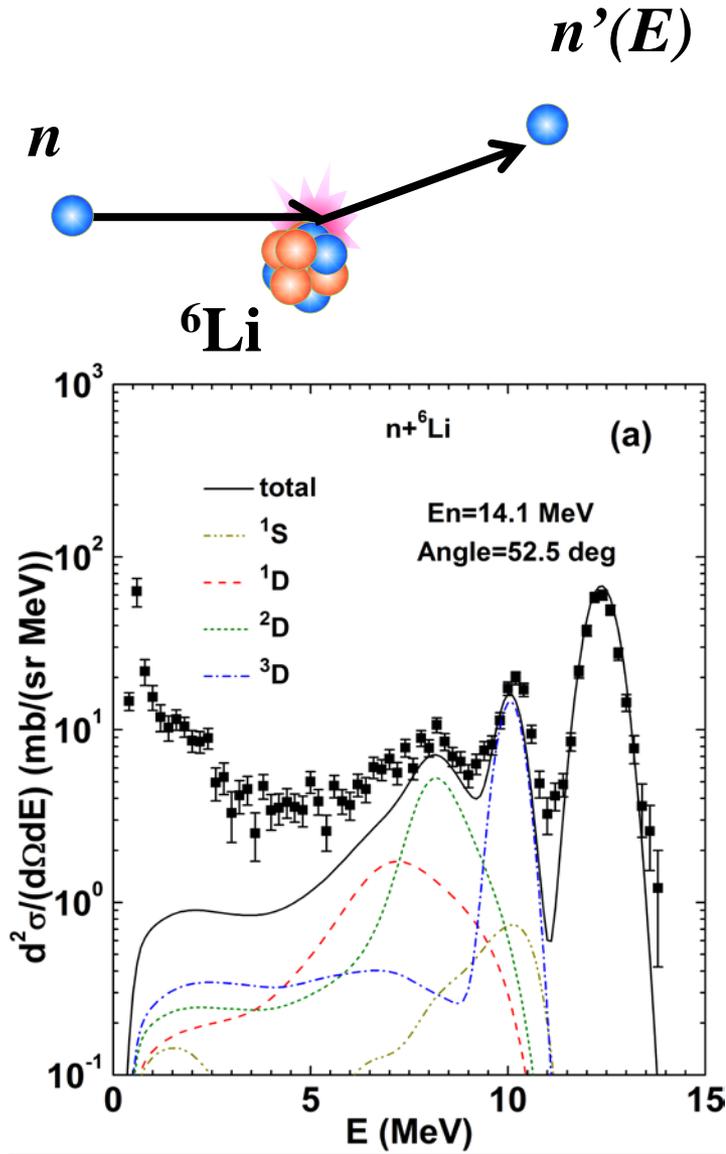
Yahiro, Ogata, TM, Minomo,
Prog. Theor. Exp. Phys. 2012, 01A206 (2012).

Coupling Potential of $^{11}\text{Li}-p$

\rightarrow Folding with JLM interaction
Jeukenne, Lejeune, Mahaux,
Phys. Rev. C16, 80 (1977).

CDCC with JLM

Analysis of ${}^6\text{Li}(n, n')$



◆ ${}^6\text{Li} = d + \alpha$

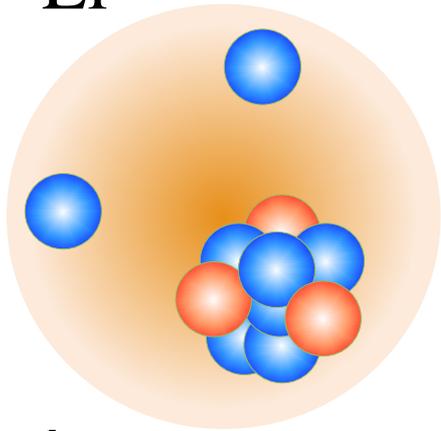
◆ CDCC with JLM

✓ Elastic cross section

✓ DDX

^{11}Li Three-body model

^{11}Li

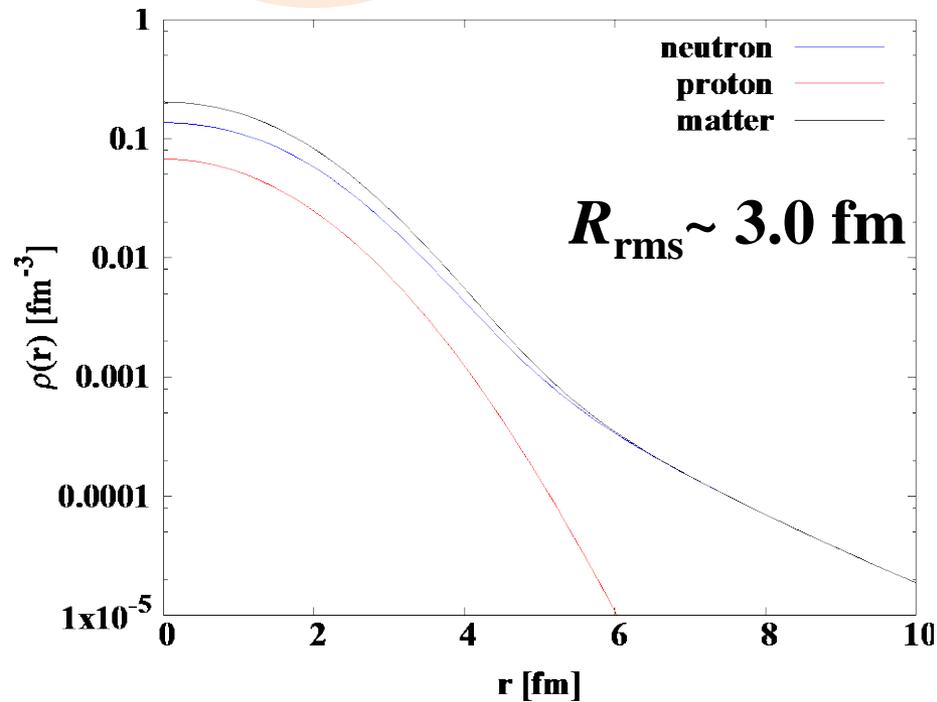


$^{11}\text{Li} : n + n + ^9\text{Li}$ (spinless)

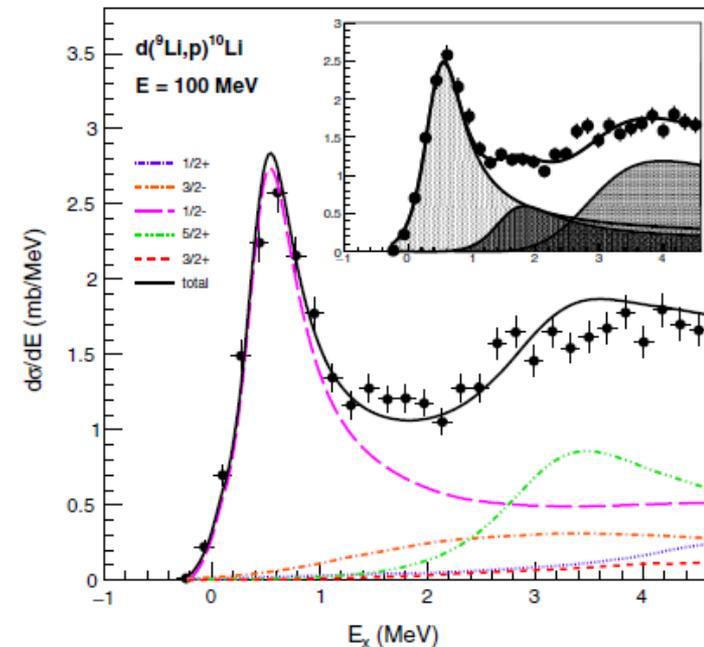
V_{NN} : Minnesota interaction

V_{NC} : Esbensen and Bertsch, NPA 542, 310

V_{NNC} : to optimize $S_{2n} \sim 0.37$ MeV



^{10}Li resonance ~ 0.5 MeV

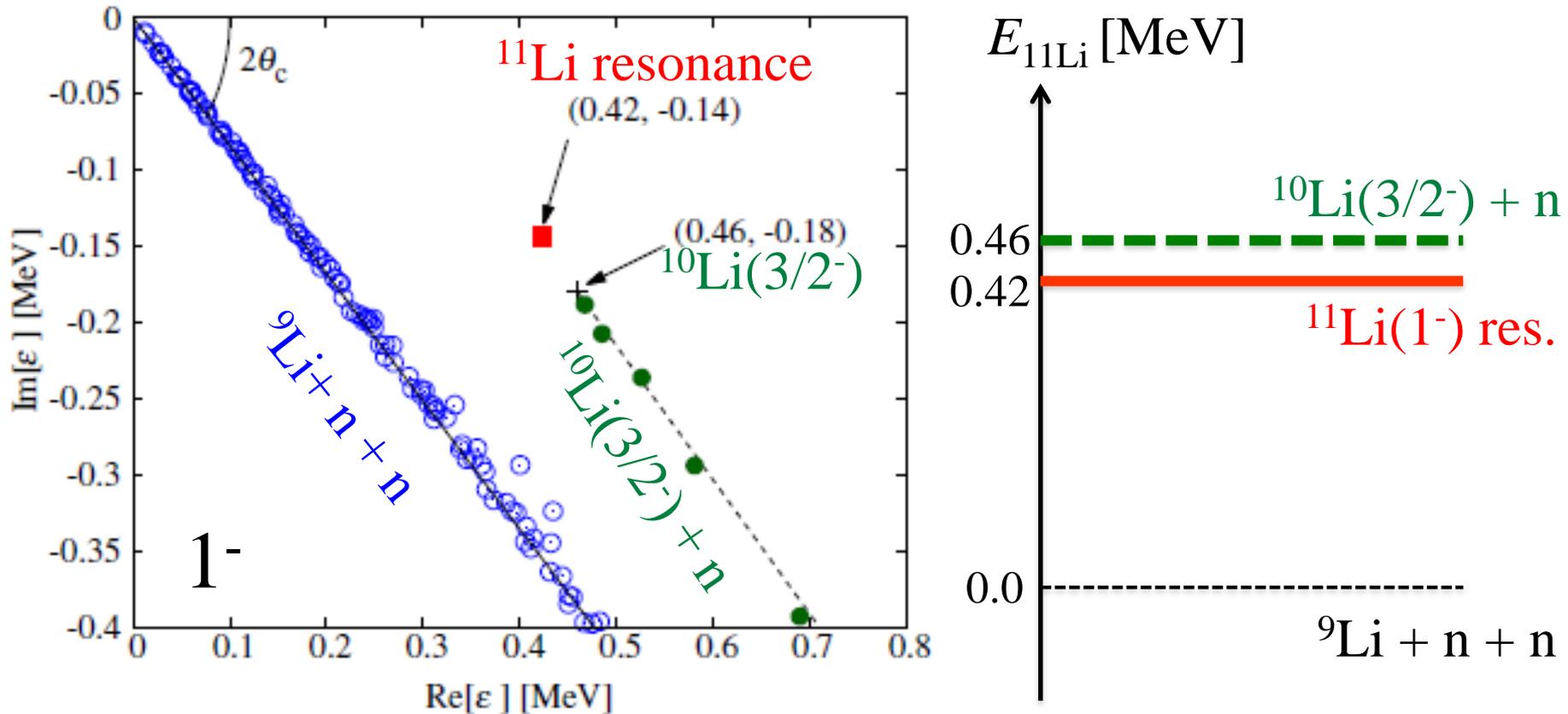


^{11}Li Resonance

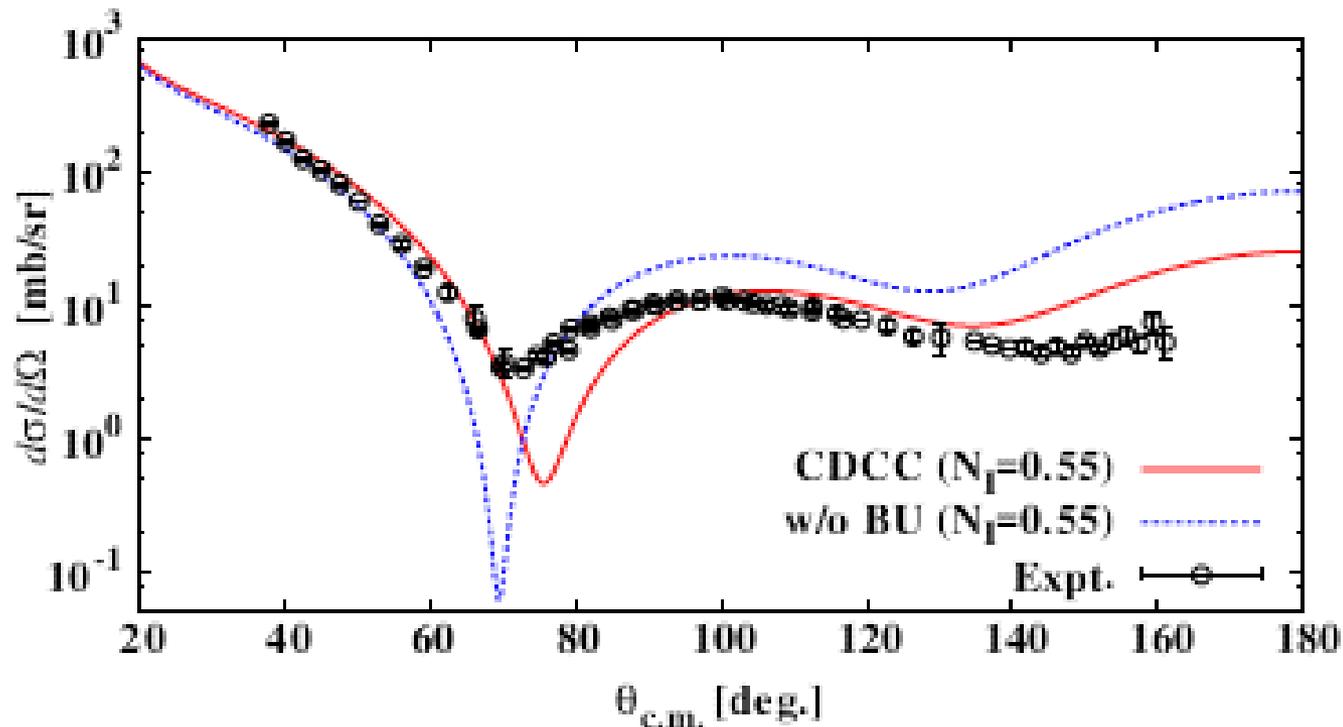
■ Complex-scaling method:

[Aoyama, Myo, Kato, and Ikeda, Prog. Theor. Phys. 116, 1 \(2006\)](#)

- ^{11}Li resonance: S. N. Ershov et al., Phys. Rev. C 70, 054608 (2004).
E. C. Pinilla et al., Phys. Rev. C 85, 054610 (2012).

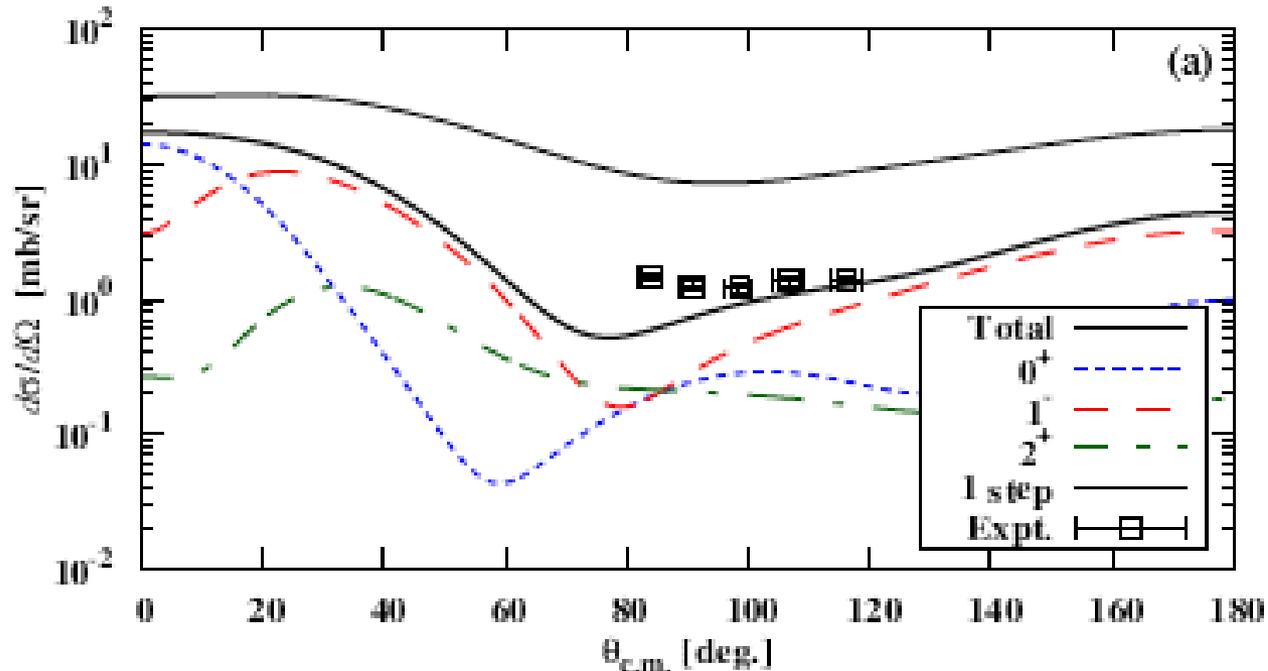


Elastic Cross Section



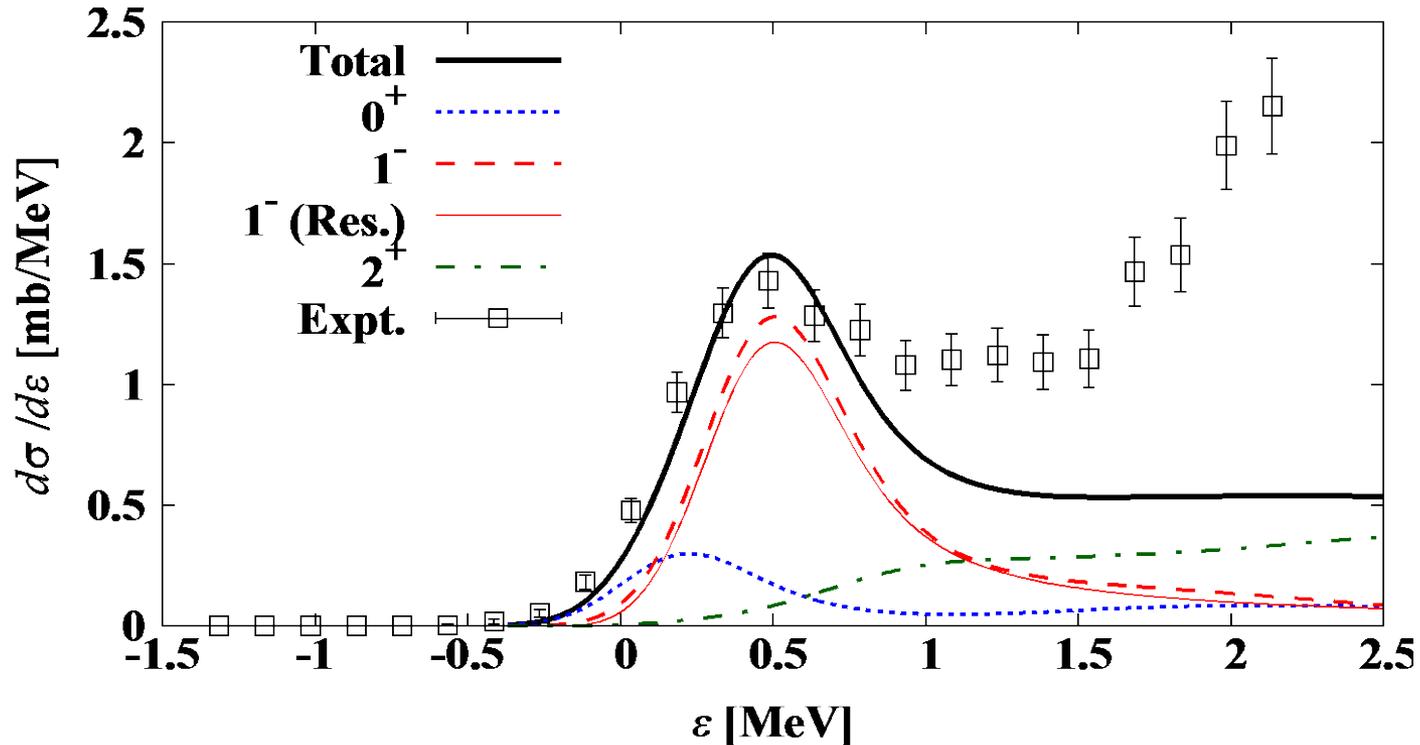
- ◆ The four-body CDCC calculation reproduces the experimental data reasonably well.
- ◆ Breakup effects are significant.

Breakup Cross Section



- ◆ The breakup cross section is integrated up to 1.5 MeV for excited energy of ^{11}Li so as to cover well the low-lying peak structure.
- ◆ The 1-step calculation w/o coupling effects is over estimated the experimental data.

Energy Spectrum

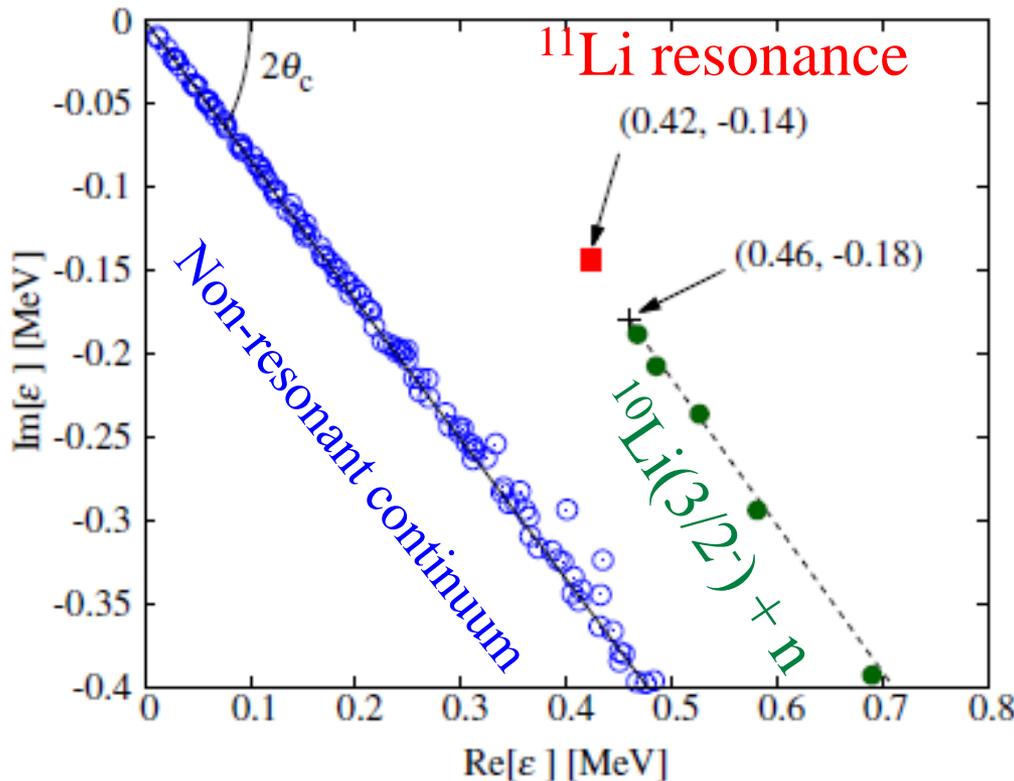


- ◆ The contribution of the dipole resonance dominates the low-lying peak.
- ◆ The width of the low-lying peak is reproduced by taking into account non-resonant components.

Wave Function of ^{11}Li Resonance

- Probability of ^{10}Li resonance in ^{11}Li

$$\alpha_n^\theta = 2 \underbrace{\langle \Phi_n^\theta | \phi_{res}^\theta \rangle}_{^{10}\text{Li res. w.f.}} \langle \phi_{res}^\theta | \Phi_n^\theta \rangle$$



- Non-resonant continuum

$$\text{Re}[\alpha_n] \sim 0.0$$

- $^{10}\text{Li}(3/2^-) + n$ continuum

$$\text{Re}[\alpha_n] \sim 0.9$$

- ^{11}Li resonance

$$\text{Re}[\alpha_n] \sim 0.9$$

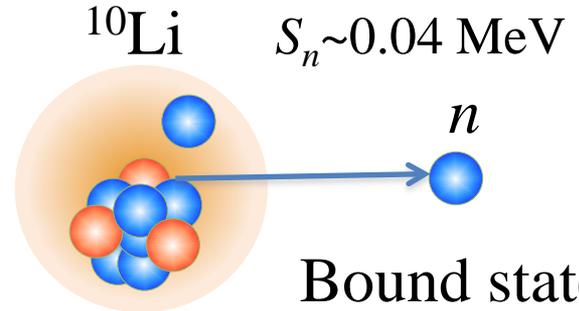
Borromean Feshbach Resonance

$$\psi_{res}({}^{11}\text{Li}) = \left[\phi_{res}({}^{10}\text{Li}) \otimes \chi_n(\ell=0) \right]$$

0.42 MeV

0.46 MeV

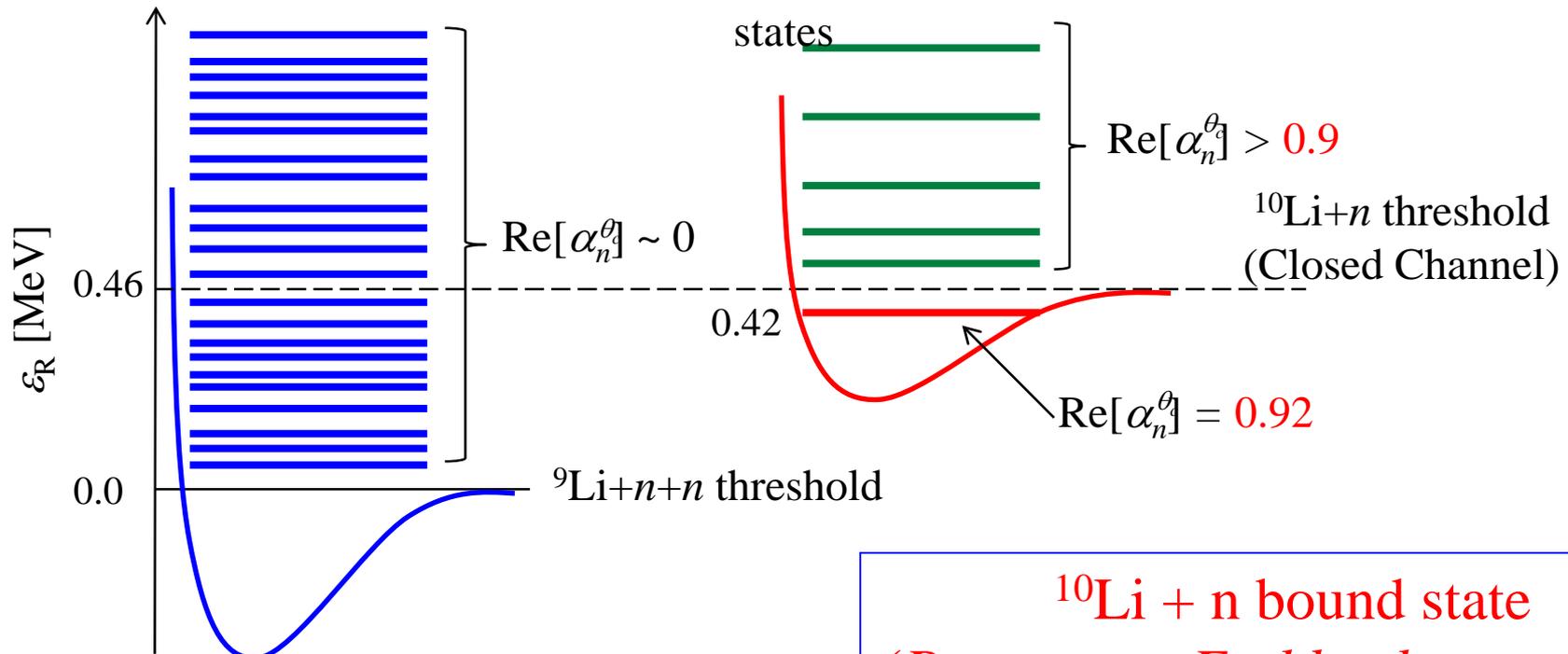
-0.04 MeV



${}^9\text{Li}+n+n$ nonresonant continuum

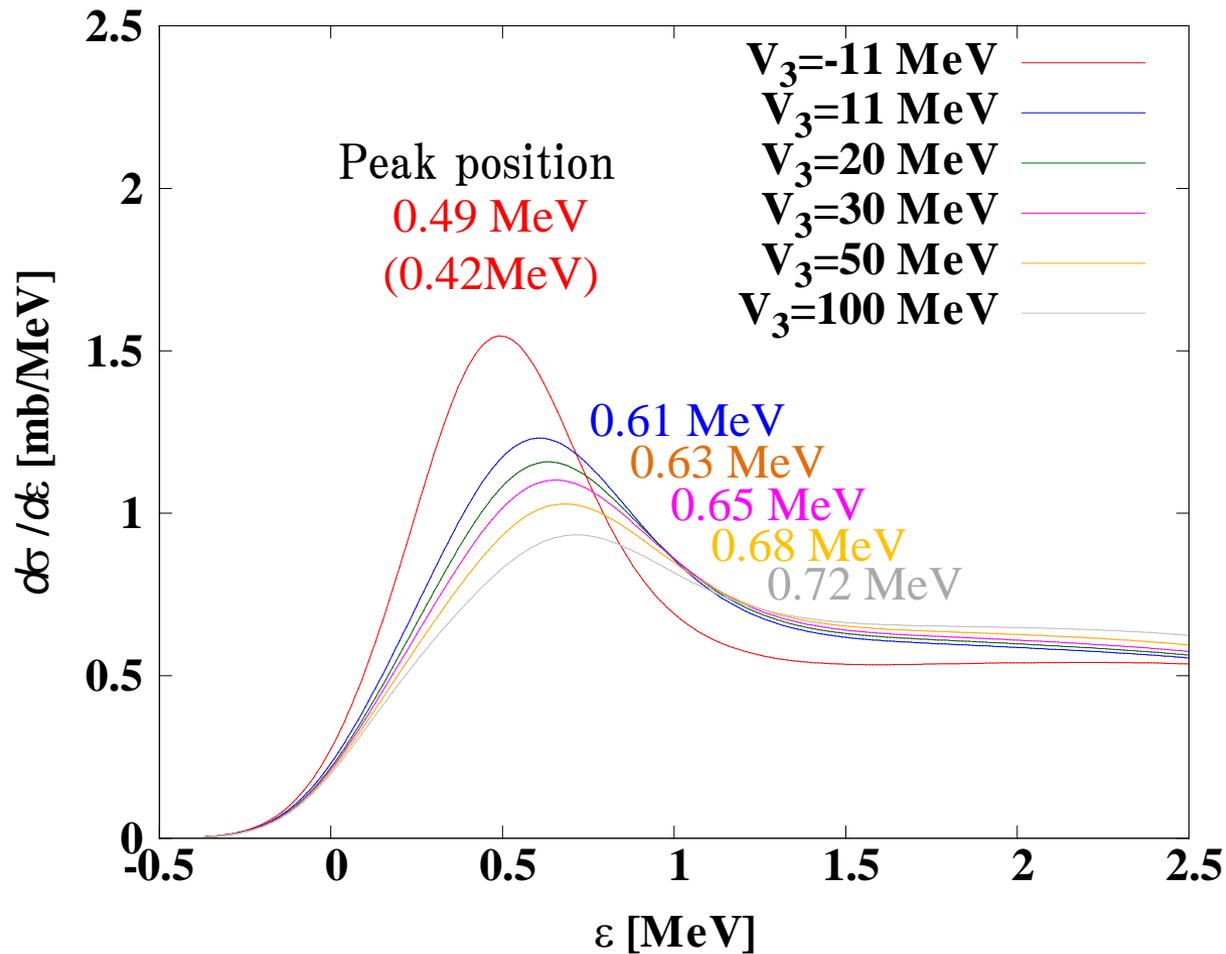
${}^{10}\text{Li}+n$ bound and continuum states

(s-wave)



${}^{10}\text{Li} + n$ bound state
(Borromean Feshbach resonance)

Resonance effect on $d\sigma/d\varepsilon$



Summary of ^{11}Li

- ◆ We found a dipole resonance in ^{11}Li at 0.42 MeV with the width of 0.28 MeV.
- ◆ The three-body model of ^{11}Li with the resonance has been validated by the good agreement between the results of four-body CDCC and the recent measurement of $^{11}\text{Li}(p,p')$.
- ◆ The resonance of ^{11}Li is interpreted as a bound state of $^{10}\text{Li} + n$ system, that is, a *Borromean Feshbach resonance*.

<http://arxiv.org/abs/1711.07209>

Perspective

- ✓ Three-body calculation of ^{11}Li with ^9Li spin