

Hadron interactions from lattice QCD

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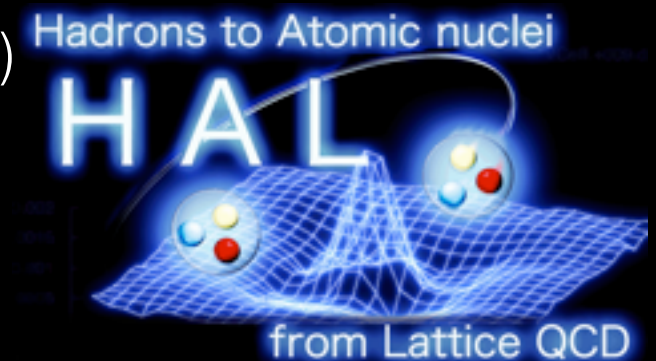
HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

S. Aoki, T. Aoyama, Y. Akahoshi, K. Sasaki, T. Miyamoto (YITP, Kyoto Univ.)

T. Doi, T. M. Doi, S. Gongyo, T. Hatsuda, T. Iritani, T. Sugiura (RIKEN)

Y. Ikeda, N. Ishii, K. Murano, H. Nemura (RCNP, Osaka Univ.)

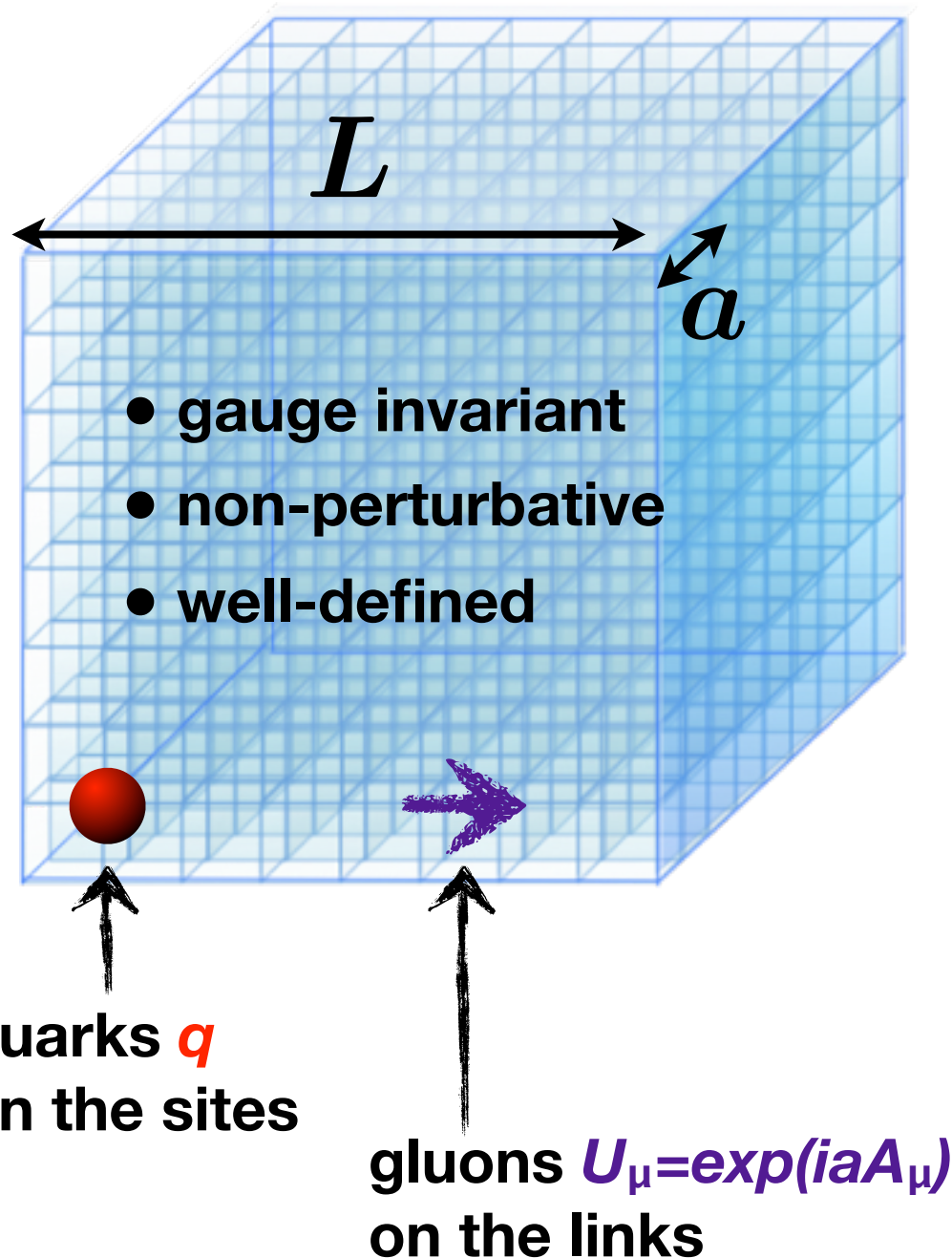
T. Inoue (Nihon Univ.)



Lattice QCD = 1st-principles calculation of QCD

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A_\mu^a) q - m \bar{q} q$$

★ strong coupling $\alpha_s = g^2/4\pi \sim 1$, non-Abelian --> non-trivial vacuum...



- path integral formulation

$$Z = \int [dU][d\bar{q}dq] \exp\left(-\int d\tau d^3x \mathcal{L}_E\right)$$

typically $(30-100)^4 \sim 10^{6-8}$ sites employed

Monte Carlo Simulations

input parameters

- quark mass m_q
- gauge coupling g

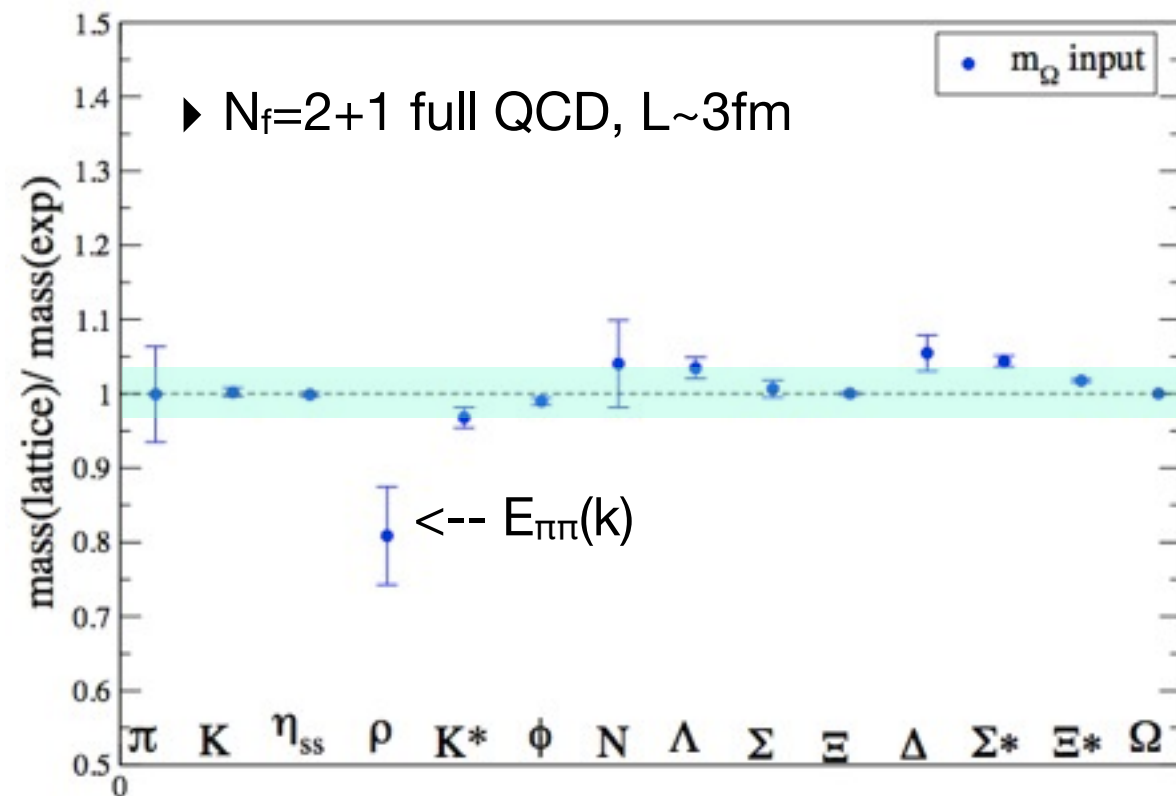
observables $\langle O(r,t) \rangle$

- ensemble average of O

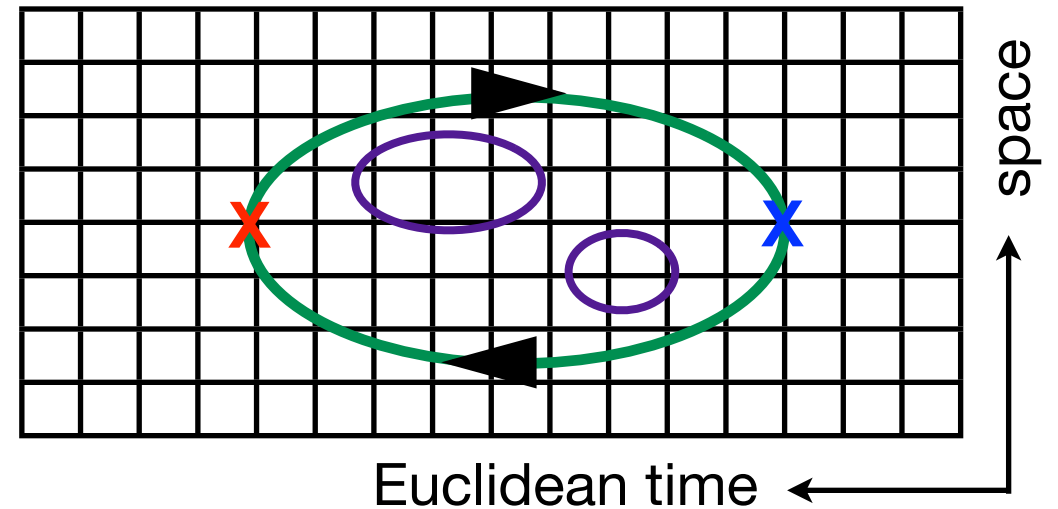


Single hadron spectroscopy from LQCD

★ Low-lying hadrons on physical point (physical m_q)

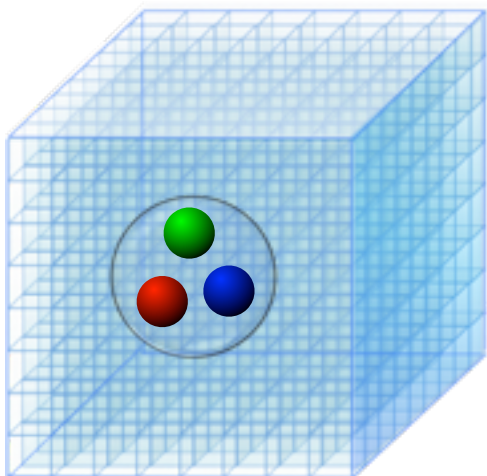
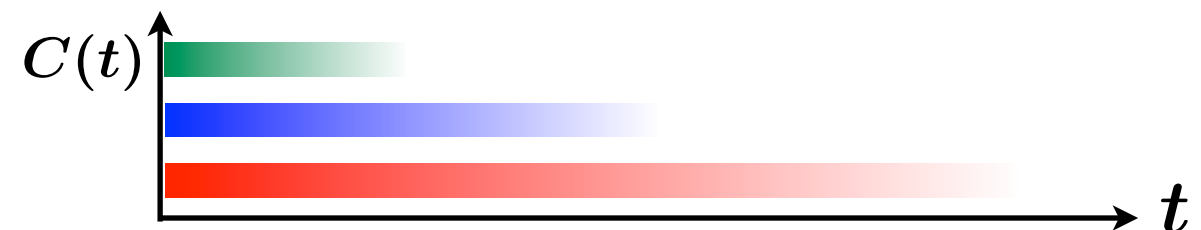


Aoki et al. (PACS-CS), PRD81 (2010).



$$C(t) = \sum_{\vec{x}} \langle 0 | \phi(\vec{x}, t) \phi(\vec{0}, 0)^\dagger | 0 \rangle$$

$$= A_1 e^{-M_1 t} + A_2 e^{-M_2 t} + A_3 e^{-M_3 t} + \dots$$



✓ a few % accuracy achieved for single hadrons

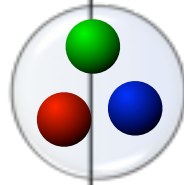
➡ **Next challenge in spectroscopy : hadron resonances**

Hadron resonances

● Particle data group

<http://www-pdg.lbl.gov/>

ρ	$1/2^+$ ****	$\Delta(1232)$	$3/2^+$ ****	Σ^+	$1/2^+$ ****	Ξ^0	$1/2^+$ ****	Λ_c^+	$1/2^+$ ****
n	$1/2^+$ ****	$\Delta(1600)$	$3/2^+$ ***	Σ^0	$1/2^+$ ****	Ξ^-	$1/2^+$ ****	Λ_c^0	$1/2^+$ ****
$N(1440)$	$1/2^+$ ****	$\Delta(1620)$	$1/2^-$ ****	$\Sigma(1385)$	$3/2^+$ ****	$\Xi(1530)$	$3/2^+$ ****	$\Lambda_c(2595)^+$	$1/2^-$ ***
$N(1520)$	$3/2^-$ ****	$\Delta(1700)$	$3/2^-$ ****	$\Sigma(1480)$	*	$\Xi(1620)$	*	$\Lambda_c(2625)^+$	$3/2^-$ ***
$N(1535)$	$1/2^-$ ****	$\Delta(1750)$	$1/2^+$ *	$\Sigma(1560)$	*	$\Xi(1690)$	*	$\Lambda_c(2765)^+$	*
$N(1650)$	$1/2^-$ ****	$\Delta(1900)$	$1/2^-$ **	$\Sigma(1580)$	$3/2^-$ *	$\Xi(1820)$	$3/2^-$ ***	$\Lambda_c(2880)^+$	$5/2^+$ ***
$N(1675)$	$5/2^-$ ****	$\Delta(1905)$	$5/2^+$ ****	$\Sigma(1620)$	$1/2^-$ *	$\Xi(1950)$	***	$\Sigma_c(2455)$	$1/2^+$ ****
$N(1680)$	$5/2^+$ ****	$\Delta(1910)$	$1/2^+$ ****	$\Sigma(1660)$	$1/2^+$ ***	$\Xi(2030)$	$\geq \frac{5}{2}^+$ ***	$\Sigma_c(2520)$	$3/2^+$ ***
$N(1700)$	$3/2^-$ ***	$\Delta(1920)$	$3/2^+$ ***	$\Sigma(1670)$	$3/2^-$ ****	$\Xi(2120)$	*	$\Sigma_c(2600)$	***
$N(1710)$	$1/2^+$ ****	$\Delta(1930)$	$5/2^-$ **	$\Sigma(1690)$	**	$\Xi(2250)$	**	Ξ_c^+	$1/2^+$ ***
$N(1720)$	$3/2^+$ ****	$\Delta(1940)$	$3/2^-$ **	$\Sigma(1730)$	$3/2^+$ *	$\Xi(2370)$	*	Ξ_c^0	$1/2^+$ ***
$N(1860)$	$5/2^+$ **	$\Delta(2000)$	$5/2^+$ **	$\Sigma(1770)$	$1/2^+$ *	$\Xi(2500)$	*	Ξ_c^-	$1/2^+$ ***
$N(1875)$	$3/2^-$ ***	$\Delta(2150)$	$1/2^-$ *	$\Sigma(1775)$	$5/2^-$ ****	Ω^-	$3/2^+$ ****	$\Xi_c(2645)$	$3/2^+$ ***
$N(1880)$	$1/2^+$ **	$\Delta(2200)$	$7/2^-$ **	$\Sigma(1840)$	$3/2^+$ **	$\Omega(2250)^-$	***	$\Xi_c(2790)$	$1/2^-$ ***
$N(1895)$	$1/2^-$ **	$\Delta(2300)$	$9/2^+$ **	$\Sigma(1880)$	$1/2^+$ **	$\Omega(2380)^-$	**	$\Xi_c(2815)$	$3/2^-$ ***
$N(1900)$	$3/2^+$ ***	$\Delta(2350)$	$5/2^-$ *	$\Sigma(1900)$	$1/2^-$ *	$\Omega(2470)^-$	**	$\Xi_c(2930)$	*
$N(1990)$	$7/2^+$ **	$\Delta(2390)$	$7/2^+$ *	$\Sigma(1915)$	$5/2^+$ ****			$\Xi_c(2970)$	***
$N(2000)$	$5/2^+$ **	$\Delta(2400)$	$9/2^-$ **	$\Sigma(1940)$	$3/2^+$ *			$\Xi_c(3055)$	***
$N(2040)$	$3/2^+$ *	$\Delta(2420)$	$11/2^+$ ****	$\Sigma(1940)$	$3/2^+$ *			$\Xi_c(3080)$	***
$N(2060)$	$5/2^-$ **	$\Delta(2750)$	$13/2^-$ ***	$\Sigma(2000)$	$1/2^-$ *			$\Xi_c(3123)$	*
$N(2100)$	$1/2^+$ *	$\Delta(2950)$	$15/2^+$ **	$\Sigma(2030)$	$7/2^+$ ****			Ω_c^0	$1/2^+$ ***
$N(2120)$	$3/2^-$ **			$\Sigma(2070)$	$5/2^+$ *			$\Omega_c(2770)^0$	$3/2^+$ ****
$N(2190)$	$7/2^-$ ****			$\Sigma(2080)$	$3/2^+$ **			Ξ_c^+	*
$N(2220)$	$9/2^+$ ****	Λ	$1/2^+$ ****	$\Sigma(2100)$	$7/2^-$ **			Λ_b^0	$1/2^+$ ***
$N(2250)$	$9/2^-$ ****	$\Lambda(1405)$	$1/2^-$ ****	$\Sigma(2250)$	***			$\Lambda_b(5912)^0$	$1/2^-$ ***
$N(2300)$	$1/2^+$ **	$\Lambda(1520)$	$3/2^-$ ****	$\Sigma(2455)$	**			$\Lambda_b(5920)^0$	$3/2^-$ ***
$N(2570)$	$5/2^-$ **	$\Lambda(1600)$	$1/2^+$ ***	$\Sigma(2620)$	**			Σ_b	$1/2^+$ ***
$N(2600)$	$11/2^-$ ***	$\Lambda(1670)$	$1/2^-$ ****	$\Sigma(3000)$	*			Σ_b^+	$3/2^+$ ***
$N(2700)$	$13/2^+$ **	$\Lambda(1690)$	$3/2^-$ ****	$\Sigma(3170)$	*			Ξ_b^+	$1/2^+$ ***
		$\Lambda(1710)$	$1/2^+$ *					Ξ_b^0	$1/2^+$ ***
		$\Lambda(1800)$	$1/2^-$ **					$\Xi_b(5935)^-$	$1/2^+$ ***
		$\Lambda(1810)$	$1/2^+$ ***					$\Xi_b(5945)^0$	$3/2^+$ ***
		$\Lambda(1820)$	$5/2^+$ ****					$\Xi_b^*(5955)^-$	$3/2^+$ ***
		$\Lambda(1830)$	$5/2^-$ ****					Ω_b^-	$1/2^+$ ***
		$\Lambda(1890)$	$3/2^+$ ****					$P_c(4380)^+$	*
		$\Lambda(2000)$	*					$P_c(4450)^+$	*
		$\Lambda(2020)$	$7/2^+$ *						
		$\Lambda(2050)$	$3/2^-$ *						
		$\Lambda(2100)$	$7/2^-$ ****						
		$\Lambda(2110)$	$5/2^+$ ***						
		$\Lambda(2325)$	$3/2^-$ *						
		$\Lambda(2350)$	$9/2^+$ ***						
		$\Lambda(2585)$	**						

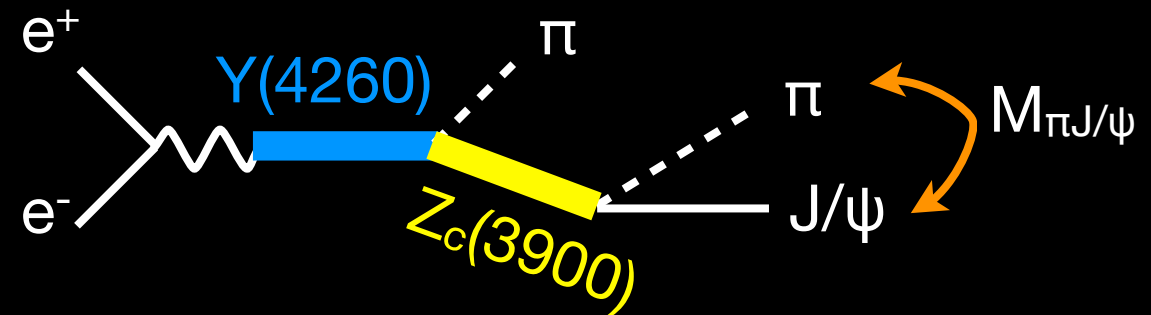


LIGHT UNFLAVORED (S = C = B = 0)		STRANGE (S = ±1, C = B = 0)		CHARMED, STRANGE (C = S = ±1)		cc			
J^P	J^P	J^P	J^P	J^P	J^P	J^P	J^P		
π^+	$1^-(0^-)$	$\rho(1690)$	$1^+(3^-)$	K^+	$1/2(0^-)$	D_s^+	$0^-(0^-)$	$\eta_c(1S)$	$0^+(0^-)$
π^0	$1^-(0^-)$	$\rho(1700)$	$1^+(1^-)$	K^0	$1/2(0^-)$	D_s^0	$0^-(0^-)$	$J/\psi(1S)$	$0^-(1^-)$
π^-	$1^-(0^-)$	$\omega(1700)$	$1^-(2^+)$	K_s^0	$1/2(0^-)$	$D_{s1}^0(2317)^0$	$0^-(0^+)$	$\chi_{c0}(1P)$	$0^+(0^+)$
η	$0^+(0^+)$	$\eta(1760)$	$0^+(0^+)$	K_L^0	$1/2(0^-)$	$D_{s1}^0(2460)^0$	$0^-(1^+)$	$\chi_{c1}(1P)$	$0^+(1^+)$
$\eta(500)$	$0^+(0^+)$	$\eta(1800)$	$1^-(0^+)$	$K^*(892)$	$1/2(1^-)$	$D_{s1}^0(2536)^0$	$0^-(1^+)$	$\chi_{c2}(1P)$	$0^+(2^+)$
$\eta(770)$	$1^-(1^-)$	$\eta(1820)$	$0^+(2^+)$	$K^*(1270)$	$1/2(1^+)$	$D_{s1}^0(2573)^0$	$0^-(2^+)$	$\eta_c(2S)$	$0^+(0^-)$
$\omega(782)$	$0^-(1^-)$	$\eta(1870)$	$0^-(2^+)$	$K^*(1400)$	$1/2(1^+)$	$D_{s1}^0(2700)^0$	$0^-(1^-)$	$\psi(2S)$	$0^-(1^-)$
$\eta'(958)$	$0^+(0^+)$	$X(1835)$	$?^-(0^+)$	$K^*(1430)$	$1/2(1^-)$	$D_{s1}^0(2860)^0$	$0^-(1^-)$	$\psi(3770)$	$0^-(1^-)$
$\phi(980)$	$1^-(0^+)$	$X(1840)$	$?^7(7^{77})$	$K^*(1450)$	$1/2(0^+)$	$D_{s1}^0(2860)^0$	$0^-(3^-)$	$\psi(3823)$	$?^7(2^-)$
$\phi(1020)$	$0^-(1^-)$	$a_1(1420)$	$1^-(1^+)$	$K_1^*(1430)$	$1/2(2^+)$	$D_{s1}^0(3040)^0$	$0^-(2^+)$	$X(3872)$	$0^+(1^+)$
$a_1(1170)$	$0^-(1^+)$	$\phi_2(1850)$	$0^-(3^-)$	$K^*(1460)$	$1/2(0^-)$	BOTTOM (B = ±1)		$X(3915)$	$0^+(0/2^+)$
$a_1(1235)$	$1^-(1^+)$	$\eta_0(1870)$	$0^-(2^+)$	$K_1^*(1460)$	$1/2(0^-)$	B^-	$1/2(0^-)$	$X_{c0}(2P)$	$0^+(2^+)$
$a_1(1290)$	$1^-(1^+)$	$\eta_2(1880)$	$1^-(2^+)$	$K_2^*(1580)$	$1/2(2^-)$	B^0	$1/2(0^-)$	$X(3940)$	$?^7(7^{77})$
$\phi(1270)$	$0^+(2^+)$	$\rho(1900)$	$1^+(1^-)$	$K_2^*(1630)$	$1/2(2^+)$	$B^+/\bar{B}^0$ ADMIXTURE		$X(4020)$	$1^7(1^+)$
$f_1(1285)$	$0^+(1^+)$	$\phi(1910)$	$0^+(2^+)$	$K_2^*(1650)$	$1/2(1^+)$	$B^+/\bar{B}^0/B^0/\bar{B}^0$ baryon ADMIXTURE		$\psi(4040)$	$0^-(1^-)$
$\eta(1295)$	$0^+(0^+)$	$a_2(1950)$	$1^-(0^+)$	$K^*(1680)$	$1/2(1^-)$	V_{cb} and V_{cb} CKM Matrix Elements		$X(4050)^{\pm}$	$?^7(1^{\pm})$
$\eta(1300)$	$1^-(0^+)$	$\phi(1950)$	$0^+(2^+)$	$K^*(1770)$	$1/2(2^-)$	B^+	$1/2(1^-)$	$X(4140)$	$0^-(7^{7+})$
$a_2(1320)$	$1^-(2^+)$	$\rho_2(1990)$	$1^+(3^-)$	$K_1^*(1780)$	$1/2(3^-)$	B^*		$\psi(4160)$	$0^-(1^-)$
$\phi(1370)$	$0^-(0^+)$	$\phi(2010)$	$0^+(2^+)$	$K_2^*(1820)$	$1/2(2^-)$	$B_1^*(5721)^0$	$1/2(1^+)$	$X(4200)^0$	$?^7(1^+)$
$a_1(1380)$	$?^7(1^-)$	$\phi(2020)$	$0^+(0^+)$	$K^*(1830)$	$1/2(0^-)$	$B_1^*(5732)$	$?^7(1^+)$	$X(4230)$	$?^7(1^-)$
$\tau_1(1400)$	$1^-(1^+)$	$a_4(2040)$	$1^-(4^+)$	$K_1^*(1850)$	$1/2(0^+)$	$B_2^*(5747)^+$	$1/2(2^+)$	$X(4240)^0$	$?^7(0^-)$
$\eta(1405)$	$0^+(0^+)$	$\phi(2050)$	$0^+(4^+)$	$K_2^*(1980)$	$1/2(2^+)$	$B_2^*(5747)^0$	$1/2(2^+)$	$X(4250)^0$	$?^7(1^+)$
$f_1(1420)$	$0^+(1^+)$	$\eta_2(2100)$	$1^-(2^+)$	$K_2^*(2045)$	$1/2(4^+)$	$B_3^*(5840)^+$	$1/2(1^+)$	$X(4260)$	$?^7(1^-)$
$\omega(1420)$	$0^-(1^-)$	$\phi(2100)$	$0^+(0^+)$	$K_2^*(2250)$	$1/2(2^-)$	$B_3^*(5840)^0$	$1/2(1^+)$	$X(4300)$	$?^7(1^-)$
$\phi(1430)$	$0^+(2^+)$	$\phi(2150)$	$0^+(2^+)$	$K_2^*(2320)$	$1/2(3^+)$	$B_4^*(5900)^+$	$1/2(1^+)$	$X(4360)$	$?^7(1^-)$
$a_2(1450)$	$1^-(0^+)$	$\rho(2150)$	$1^-(1^-)$	$K_2^*(2380)$	$1/2(5^-)$	$B_4^*(5970)^+$	$1/2(1^+)$	$\psi(4415)$	$0^-(1^-)$
$\rho(1450)$	$1^-(1^-)$	$\phi(2170)$	$0^-(1^-)$	$K_4^*(2500)$	$1/2(4^-)$	$B_4^*(5970)^0$	$1/2(1^+)$	$X(4430)^{\pm}$	$?^7(1^{\pm})$
$\phi(1475)$	$0^+(0^+)$	$\phi(2200)$	$0^+(0^+)$	$K(3100)$	$?^7(7^{77})$	BOTTOM, STRANGE (B = ±1, S = ±1)		$X(4660)$	$?^7(1^-)$
$\phi(1500)$	$0^+(0^+)$	$f_2(2220)$	$0^+(2^+)$	CHARMED (C = ±1)				bb	
$f_1(1510)$	$0^+(1^+)$	$\eta(2225)$	$0^+(0^+)$	D^+	$1/2(0^-)$	B_c^0	$0^-(0^-)$	$\eta_b(1S)$	$0^-(0^-)$
$\rho_1^0(1525)$	$0^+(2^+)$	$\eta(2250)$	$0^+(0^+)$	D^0	$1/2(0^-)$	B_c^+	$0^-(0^-)$	$J/\psi(1S)$	$0^-(1^-)$
$\phi(1565)$	$0^+(2^+)$	$\rho_2(2250)$	$1^-(3^-)$	$D^{*+}(2007)^0$	$1/2(1^-)$	B_c^+	$0^-(1^-)$	$\chi_{b0}(1P)$	$0^+(0^+)$
$\rho(1570)$	$1^-(1^-)$	$\phi(2300)$	$0^+(2^+)$	$D^{*0}(2010)^0$	$1/2(1^-)$	$B_{cb}(5830)^0$	$0^-(1^+)$	$\chi_{b1}(1P)$	$0^+(1^+)$
$a_1(1595)$	$0^-(1^+)$	$\phi(2300)$	$0^+(4^+)$	$D_2^{*+}(2400)^0$	$1/2(0^+)$	$B_{cb}^0(5850)$	$?^7(1^+)$	$\chi_{b2}(1P)$	$0^+(2^+)$
$\tau_1(1600)$	$1^-(1^+)$	$\phi(2330)$	$0^+(0^+)$	$D_2^{*0}(2400)^0$	$1/2(0^+)$	BOTTOM, CHARMED (B = C = ±1)		$\eta_b(2S)$	$0^+(0^-)$
$a_1(1640)$	$1^-(1^+)$	$\phi(2340)$	$0^+(2^+)$	$D_1(2420)^0$	$1/2(1^+)$	B_c^+	$0^-(0^-)$	$T(2S)$	$0^-(1^-)$
$\phi(1640)$	$0^+(2^+)$	$\rho_2(2350)$	$1^-(5^-)$	$D_1(2420)^0$	$1/2(1^+)$	B_c^0	$0^-(0^-)$	$T(1D)$	$0^-(2^-)$
$\eta_0(1645)$	$0^+(2^+)$	$a_4(2450)$	$1^-(6^+)$	$D_1(2430)^0$	$1/2(1^+)$	B_c^+	$0^-(0^-)$	$\chi_{b0}(2P)$	$0^+(0^+)$
$\omega(1650)$	$0^-(1^-)$	$\phi(2510)$	$0^+(6^+)$	$D_1(2430)^0$	$1/2(1^+)$	B_c^0	$0^-(0^-)$	$\chi_{b1}(2P)$	$0^+(1^+)$
$\omega(1670)$	$0^-(3^-)$	OTHER LIGHT		$D_2^+(2460)^0$	$1/2(2^+)$	B_c^+	$0^-(0^-)$	$\chi_{b2}(2P)$	$0^+(2^+)$
$\tau_2(1670)$	$1^-(2^+)$	Further States		$D_2^0(2460)^0$	$1/2(2^+)$	B_c^0	$0^-(0^-)$	$T(3S)$	$0^-(1^-)$
$\phi(1680)$	$0^-(1^-)$			$D_2^+(2550)^0$	$1/2(2^+)$	B_c^+	$0^-(0^-)$	$\chi_{b1}(3P)$	$0^+(1^+)$
				$D_2^0(2550)^0$	$1/2(2^+)$	B_c^0	$0^-(0^-)$	$T(4S)$	$0^-(1^-)$
				$D_2^+(2600)$	$1/2(2^+)$			$X(10610)^{\pm}$	$1^{\pm}(1^{\pm})$
				$D_2^0(2640)^0$	$1/2(2^+)$			$X(10610)^0$	$1^0(1^0)$
				$D(2740)^0$	$1/2(1^+)$			$X(10650)^{\pm}$	$?^7(1^{\pm})$
				$D(2750)$	$1/2(3^-)$			$T(10060)$	$0^-(1^-)$
				$D(3000)^0$	$1/2(1^+)$			$T(11020)$	$0^-(1^-)$

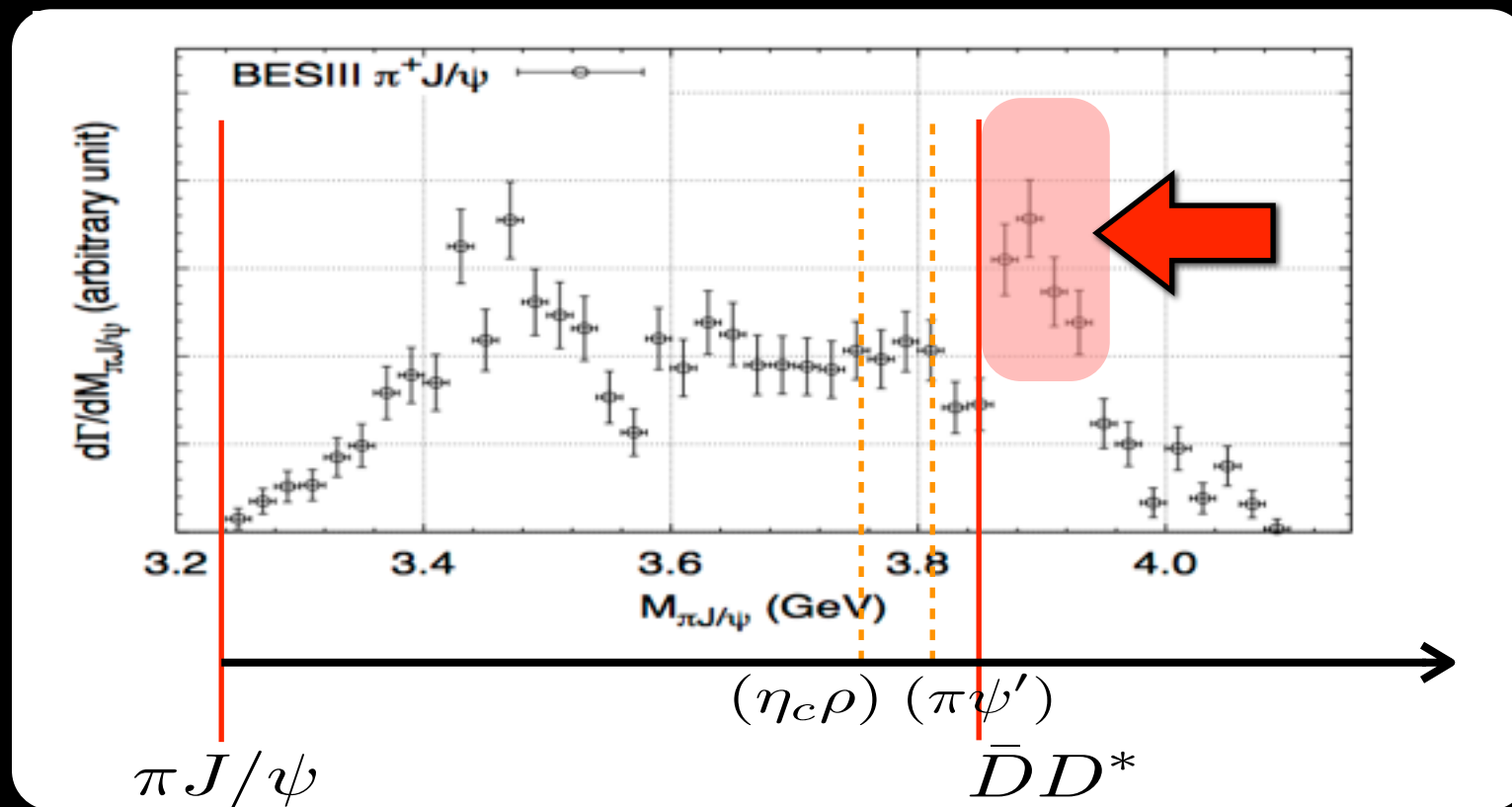
Tetraquark candidate $Z_c(3900)$

- Expt. observations**

$Y(4260)$ 3-body decay

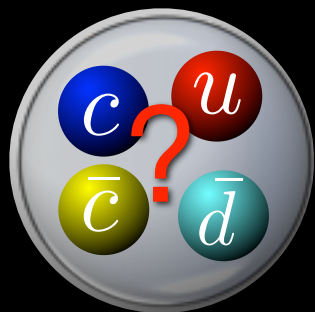


$$e^+ + e^- \rightarrow Y(4260) \rightarrow \pi + Z_c(3900) \rightarrow \pi^{+/-} + J/\psi$$



BESIII Coll., PRL110 (2013).

see also Belle Coll., PRL110 (2013).



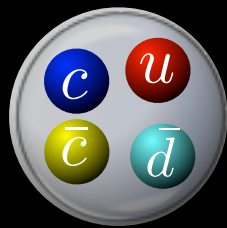
- peak in $\pi^{+/-} J/\psi$ invariant mass (minimal quark content $cc^{\text{bar}} ud^{\text{bar}}$ $\leftrightarrow$ tetraquark?)
- $M \sim 3900$, $\Gamma \sim 60$ MeV (Breit-Wigner, Flatte) $\rightarrow$ just above $D^{\text{bar}} D^*$ threshold
- $J^{\text{PC}}=1^{+-}$ is most probable $\leftrightarrow$ couple to s-wave meson-meson states

Tetraquark candidate $Z_c(3900)$

★ **structure of $Z_c(3900)$** studied by models

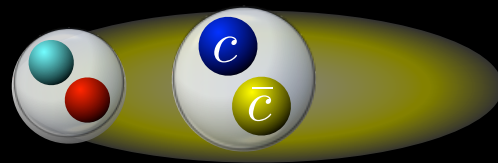
genuine resonance

tetraquark



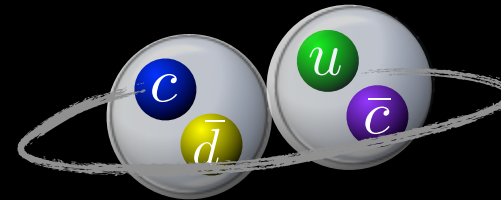
Maiani et al.('13)

$J/\psi + \pi$ atom



Voloshin('08)

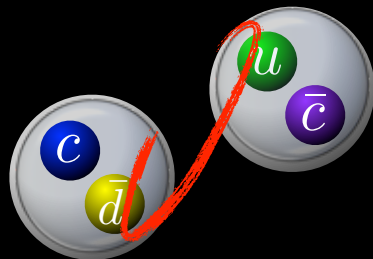
$D^{\text{bar}}D^*$ molecule



Nieves et al.('11) + many others

kinematical origin

$D^{\text{bar}}D^*$ threshold effect

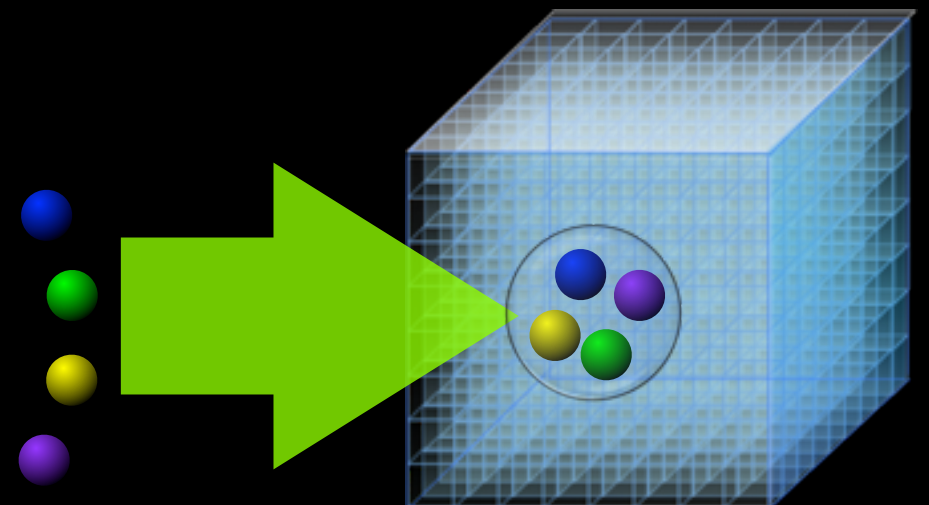


Chen et al.('14), Swanson('15)

conclusion not achieved

→ **poor information on interactions**

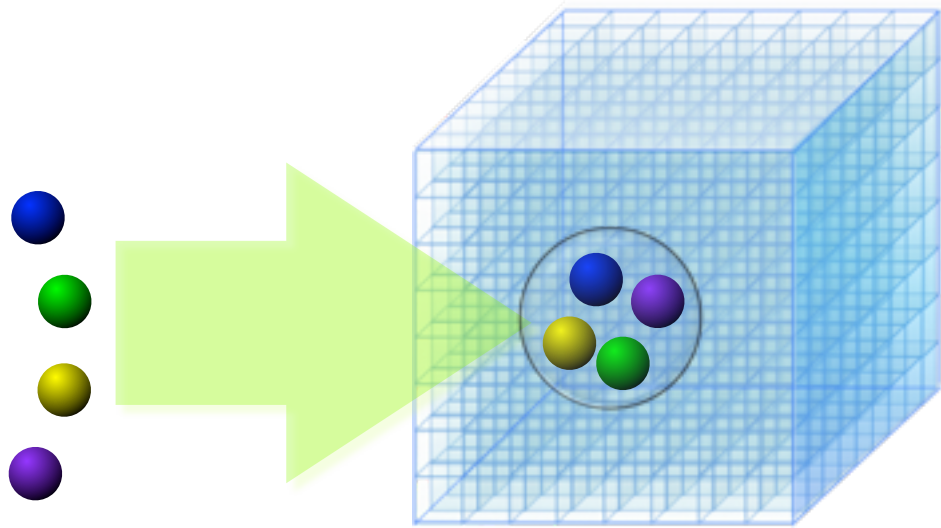
★ LQCD simulations for $Z_c(3900)$



$Z_c(3900)$ on the lattice

◆ Conventional approach: temporal correlation

➡ identify all relevant $W_n(L)$ ($n=0,1,2,3,\dots$)

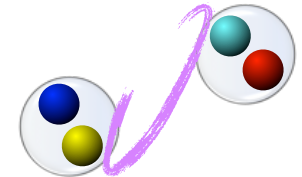


$$\langle 0 | \Phi(x) \Phi^\dagger(0) | 0 \rangle = A_1 e^{-W_1 \tau} + A_2 e^{-W_2 \tau} + \dots$$

($W_1, W_2, \dots$ are eigen-energies)

e.g., 4-quark operator

$$\Phi(x) = \bar{q}(x) \bar{q}(x) q(x) q(x)$$



✓ No positive evidence for $Z_c(3900)$ in $J^{PC} \equiv 1^{+-}$

(observed spectrum consistent with scat. states)

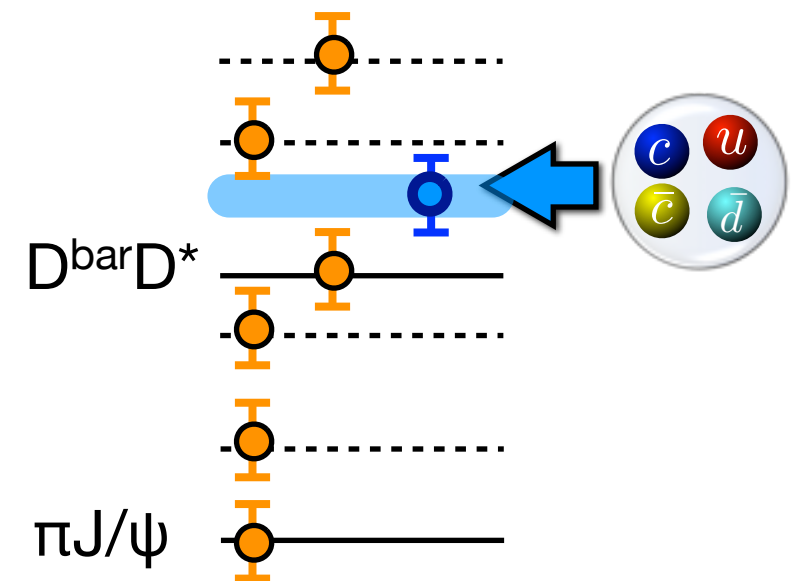
S. Prelovsek et al., PLB 727 (2013), PRD91 (2015).

S.-H. Lee et al., PoS Lattice2014 (2014).

★ Why is the peak observed in expt.?

► (broad) resonance? threshold effect?

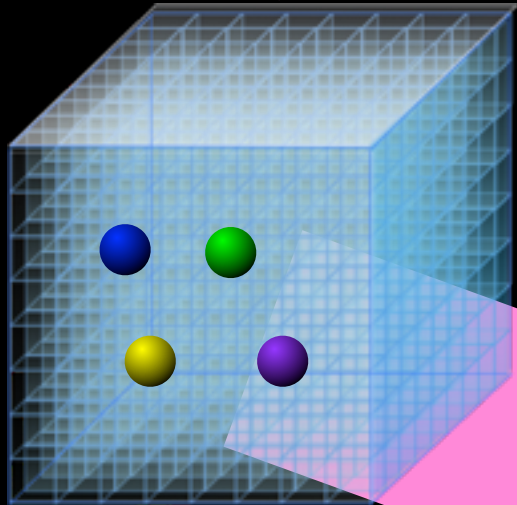
★ How can we find resonance in LQCD data?



variational method

Strategy for studies of resonances from LQCD

lattice QCD



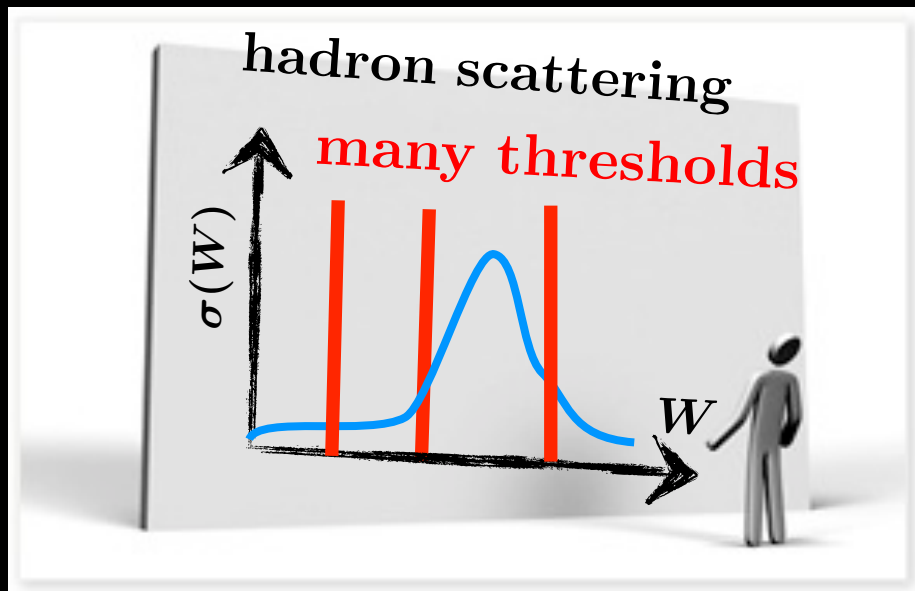
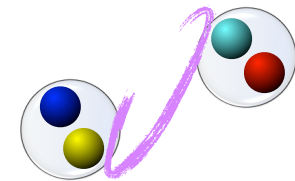
Conventional approach

$$\langle 0 | \Phi(x) \Phi^\dagger(0) | 0 \rangle = A_1 e^{-W_1 \tau} + A_2 e^{-W_2 \tau} + \dots$$

($W_1, W_2, \dots$ are eigen-energies)

e.g., 4-quark operator

$$\Phi(x) = \bar{q}(x)\bar{q}(x)q(x)q(x)$$



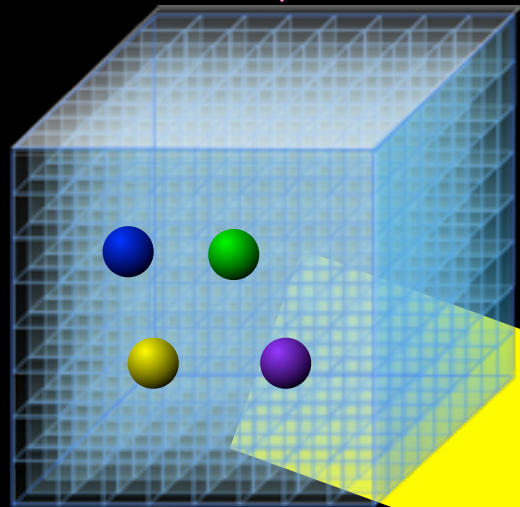
hadron resonances

- ★ Resonance energy does NOT correspond to eigen-energy
- ★ Resonances are embedded into coupled-channel scattering states
- ➔ Resonance energy is determined from **pole of coupled-channel S-matrix**

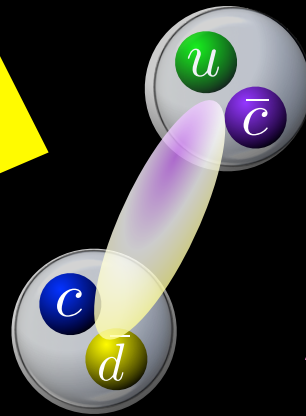
Strategy for studies of resonances from LQCD

lattice QCD

(Resonance search through scattering observable)



hadron interactions
(faithful to S-matrix)



scattering theory



hadron resonances

contents

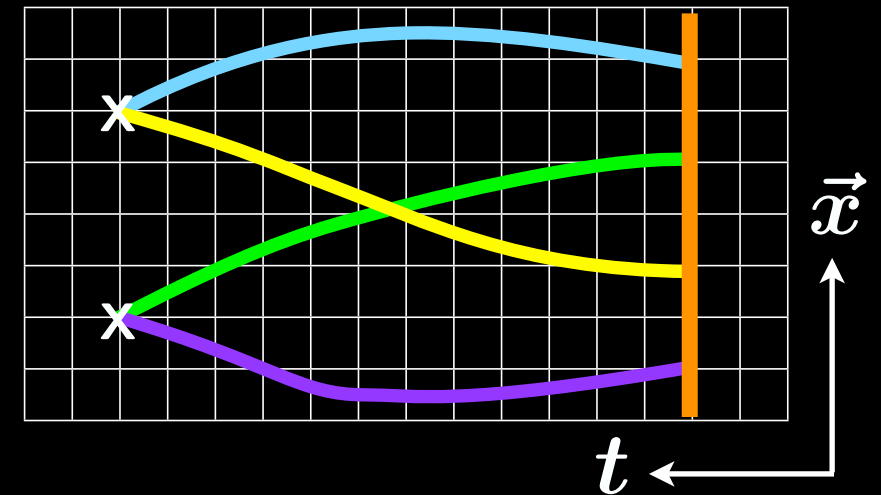
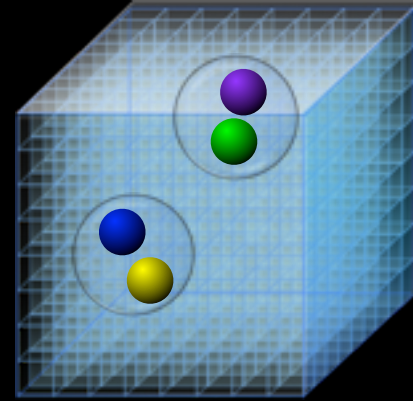
- hadron interactions & HAL QCD method
- tetraquark candidate $Z_c(3900)$
- dibaryon systems
- summary

Hadronic interactions from LQCD

hadronic correlation function

$$C_{(2)}(\vec{r}, t) \equiv \langle 0 | \phi_1(\vec{r}, t) \phi_2(\vec{0}, t) \mathcal{J}^\dagger(t=0) | 0 \rangle$$

$$= \sum_n A_n \psi_n(\vec{r}) e^{-W_n t}$$



- Energy eigenvalue $W_n(L)$
- NBS (Nambu-Bethe-Salpeter) wave function $\psi_n(r)$ ($\rightarrow \sin(k_n r + \delta(k_n)) / k_n r$)

C.D. Lee et al., NPB619 (2001).

HAL QCD Method (derive potential as representation of S-matrix)

► $\psi_n(r) \rightarrow$ **2PI kernel** ($\psi = \varphi + G_0 U \psi$)

$$(\nabla^2 + k_n^2) \psi_n(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r}, \vec{r}') \psi_n(\vec{r}')$$

$$U(\vec{r}, \vec{r}') \equiv \sum_{n < n_{th}} (E_n - H_0) \psi_n(\vec{r}) \bar{\psi}_n(\vec{r}')$$

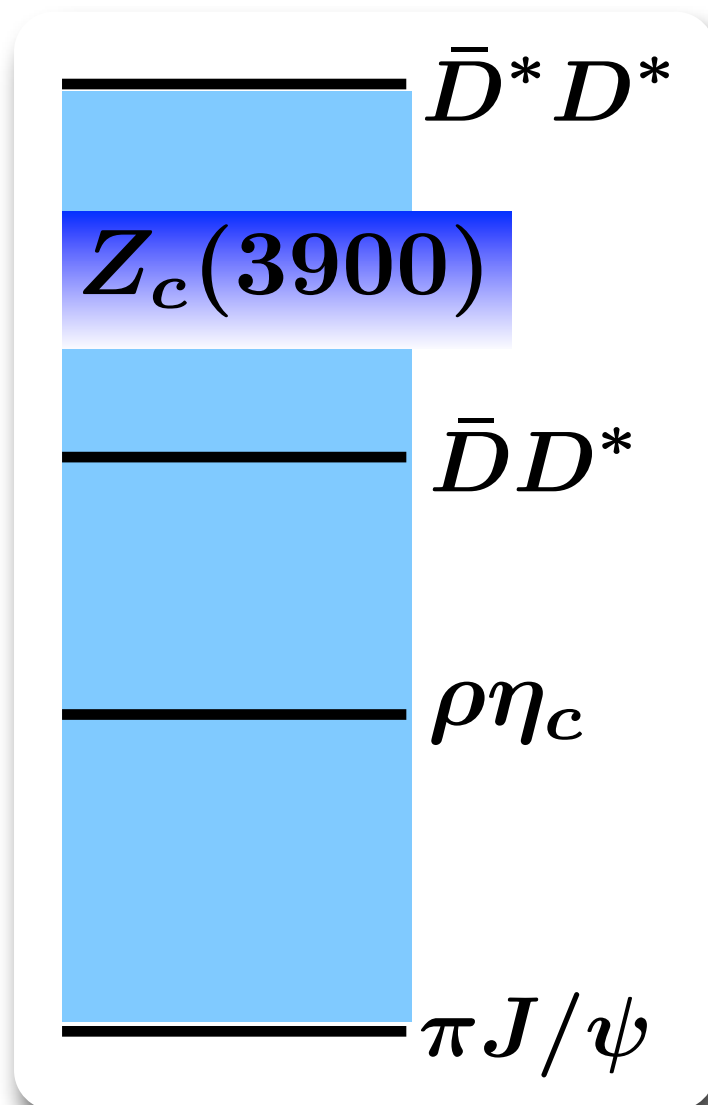
$\rightarrow$ phase shift, resonance pole, ...

Ishii, Aoki, Hatsuda, PRL 99, 022001 (2007).

Ishii et al. [HAL QCD], PLB 712, 437 (2012).

Results on $Z_c(3900)$ in $I^G(J^{PC})=1^+(1^{+-})$

[Y. Ikeda et al., \[HAL QCD\], PRL117, 242001 \(2016\).](#)

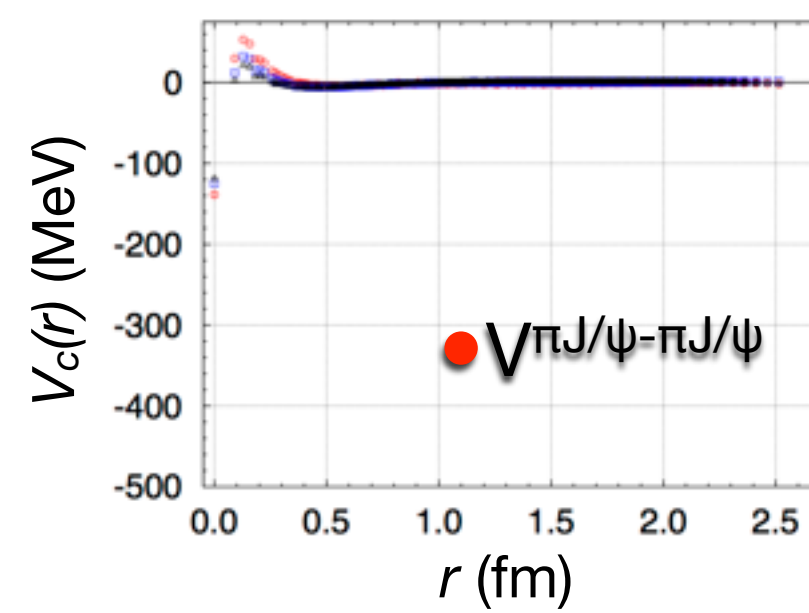
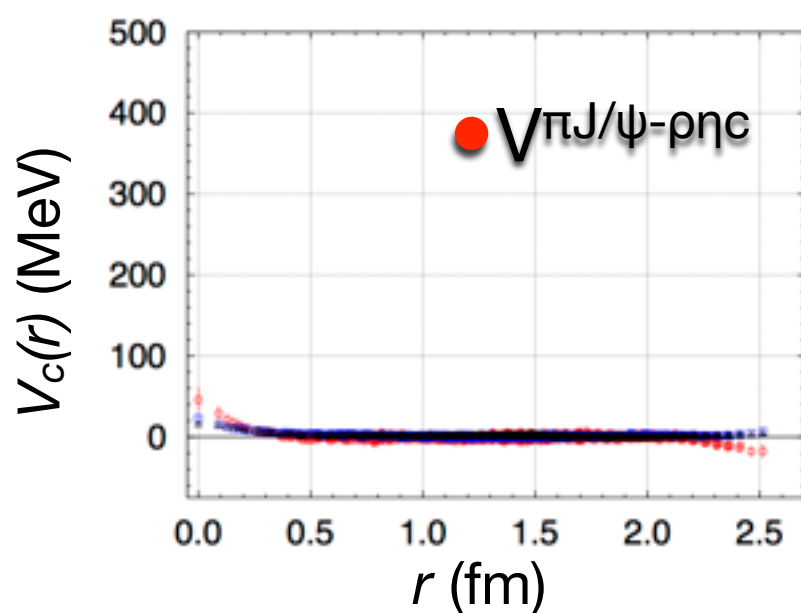
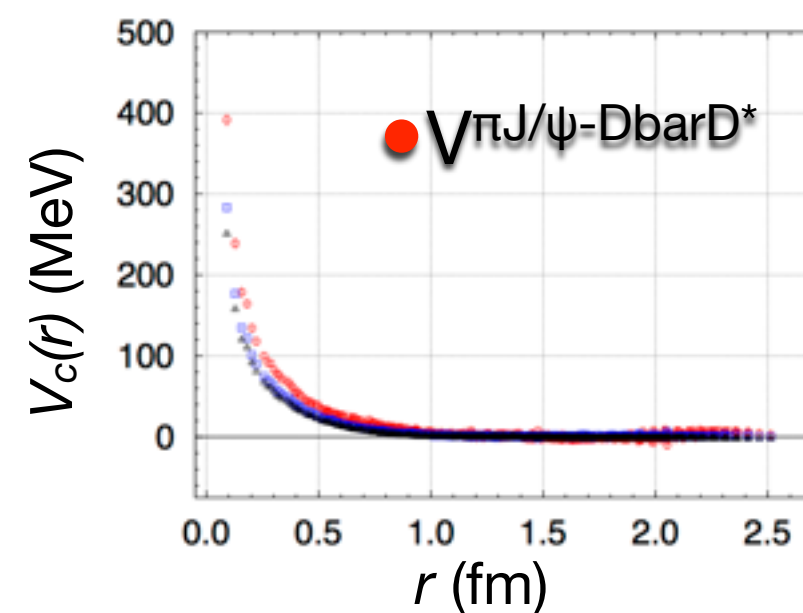
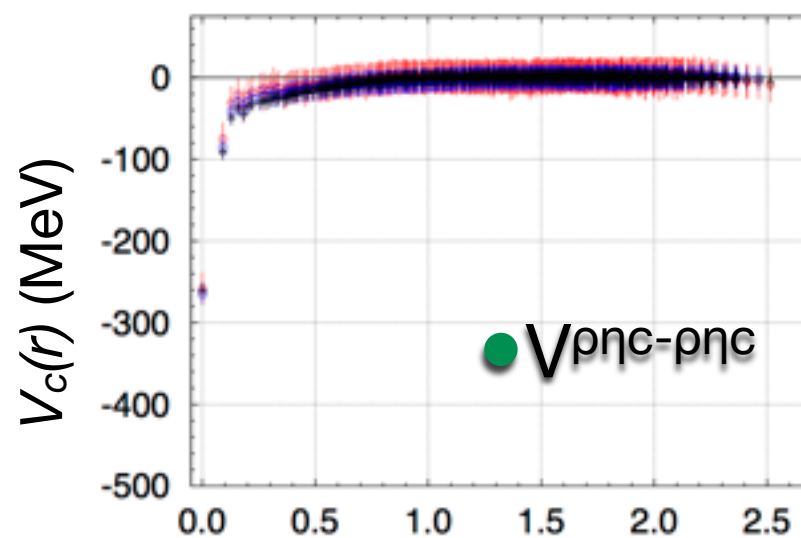
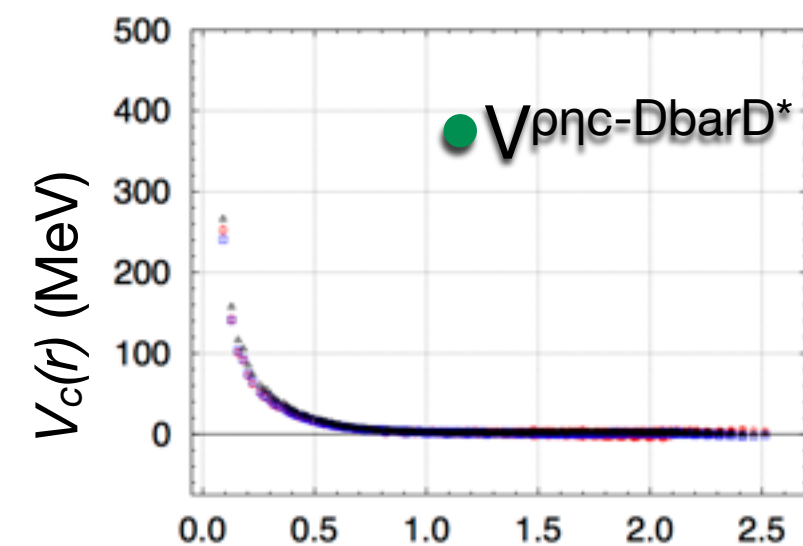
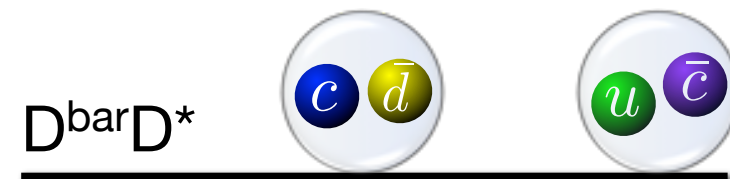
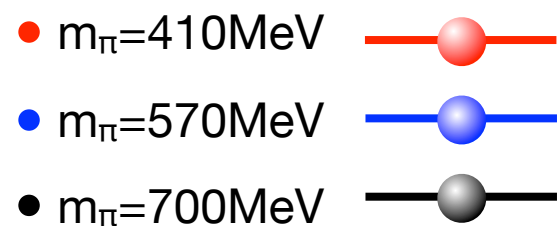
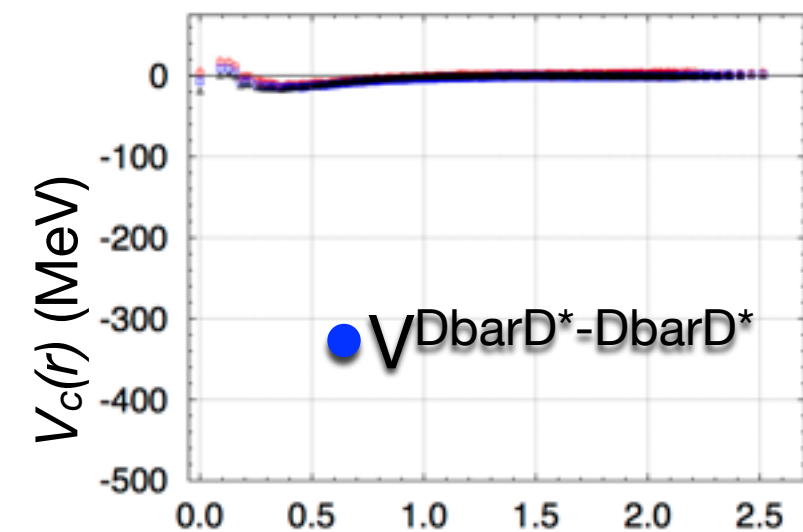


light meson mass (MeV)
 $m_\pi = 411(1), 572(1), 701(1)$
 $m_\rho = 896(8), 1000(5), 1097(4)$

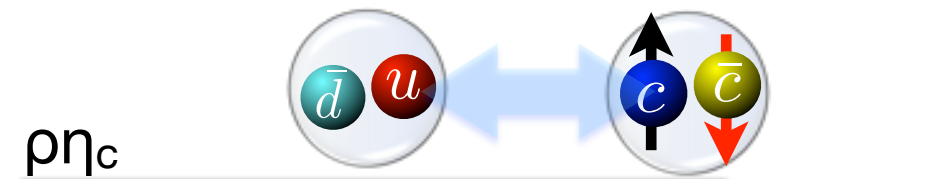
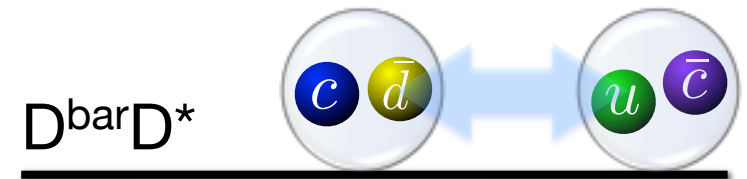
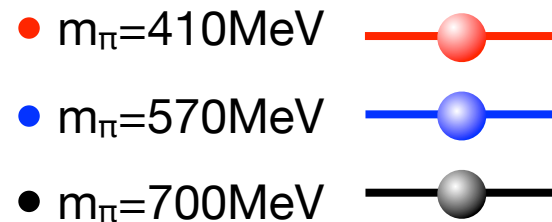
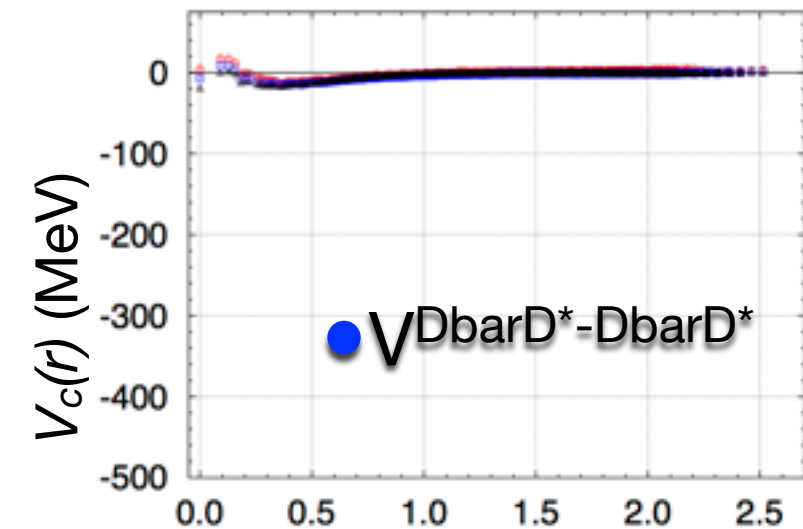
charm meson mass (MeV)
 $m_{\eta_c} = 2988(1), 3005(1), 3024(1)$
 $m_{J/\psi} = 3097(1), 3118(1), 3143(1)$
 $m_D = 1903(1), 1947(1), 2000(1)$
 $m_{D^*} = 2056(3), 2101(2), 2159(2)$

- s-wave coupled-channel ($\pi J/\psi$ - $\rho \eta_c$ - $D^{\text{bar}} D^*$) potential
- 2-body observable
- comparison w/ expt. data

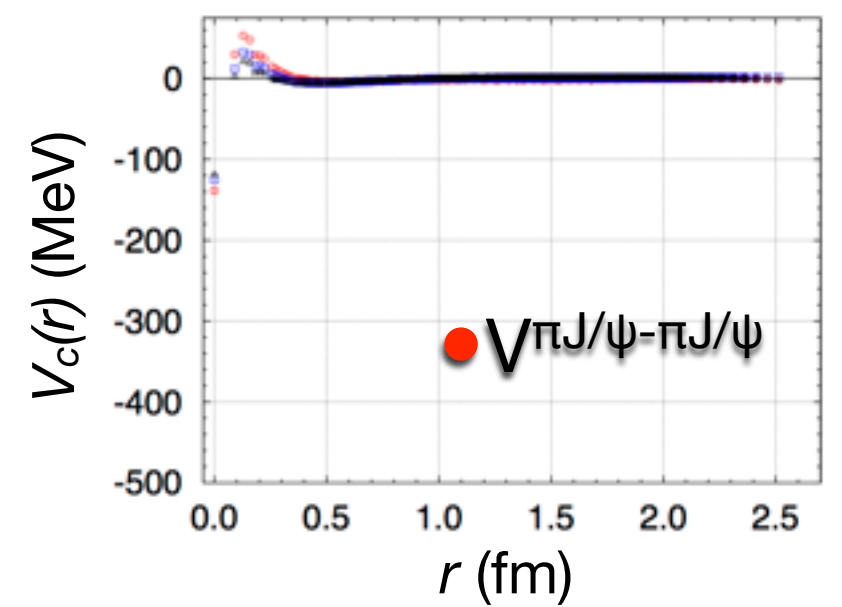
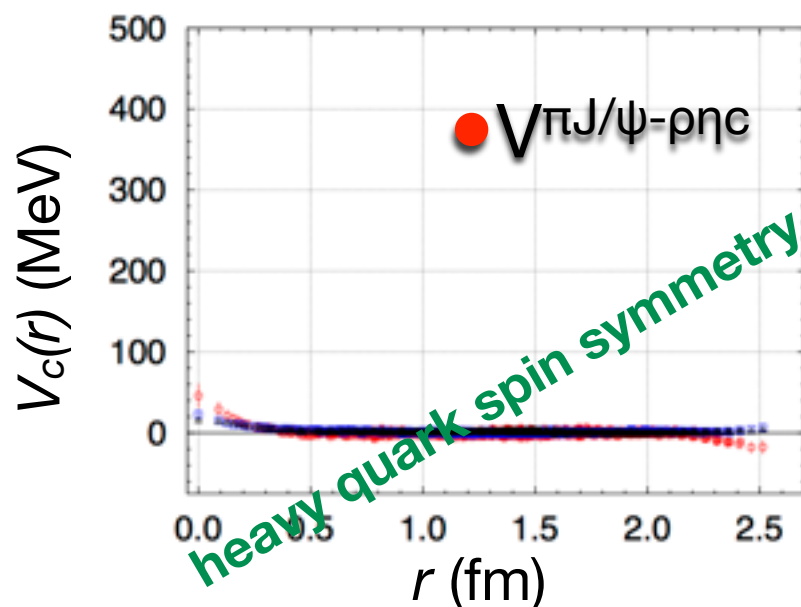
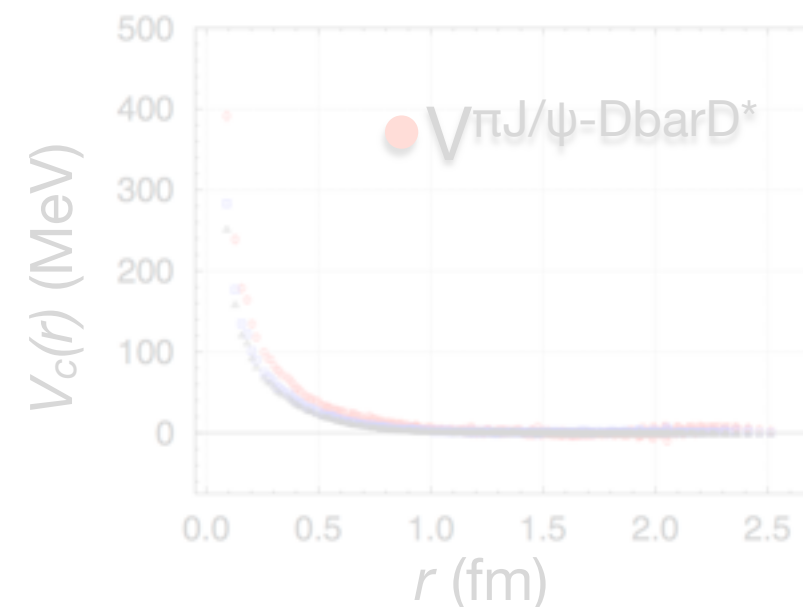
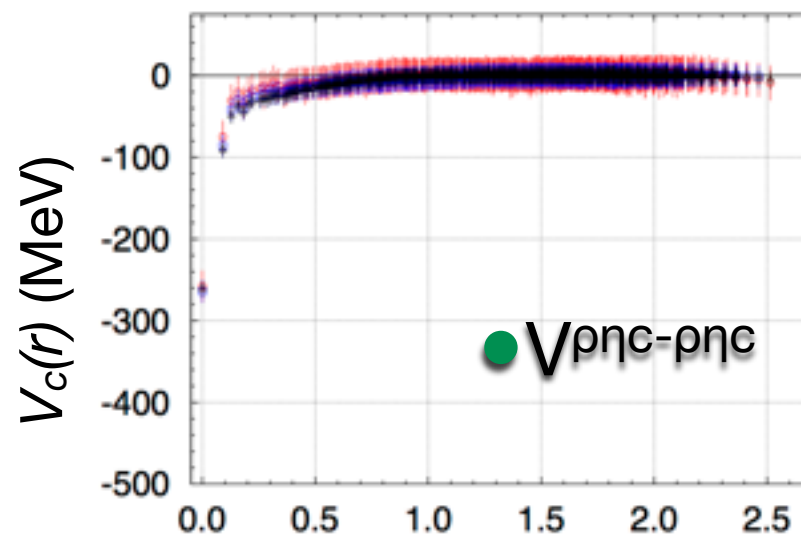
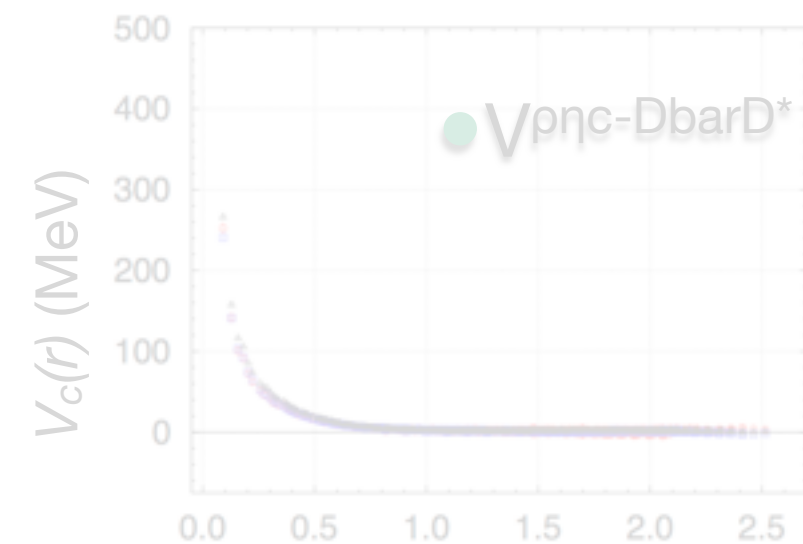
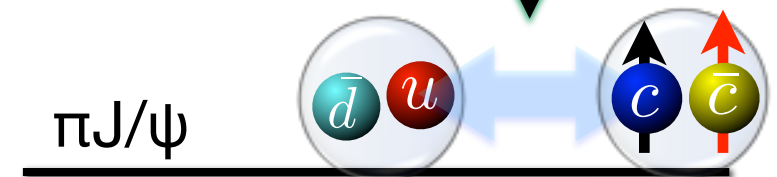
3x3 potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$)



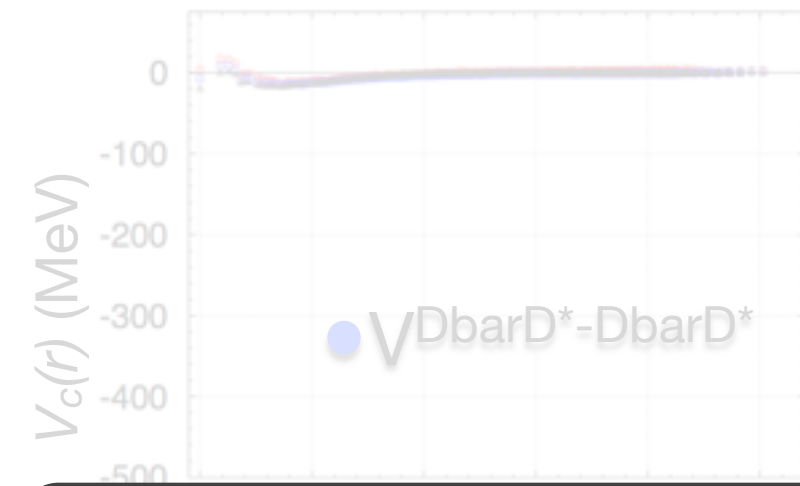
3x3 potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$)



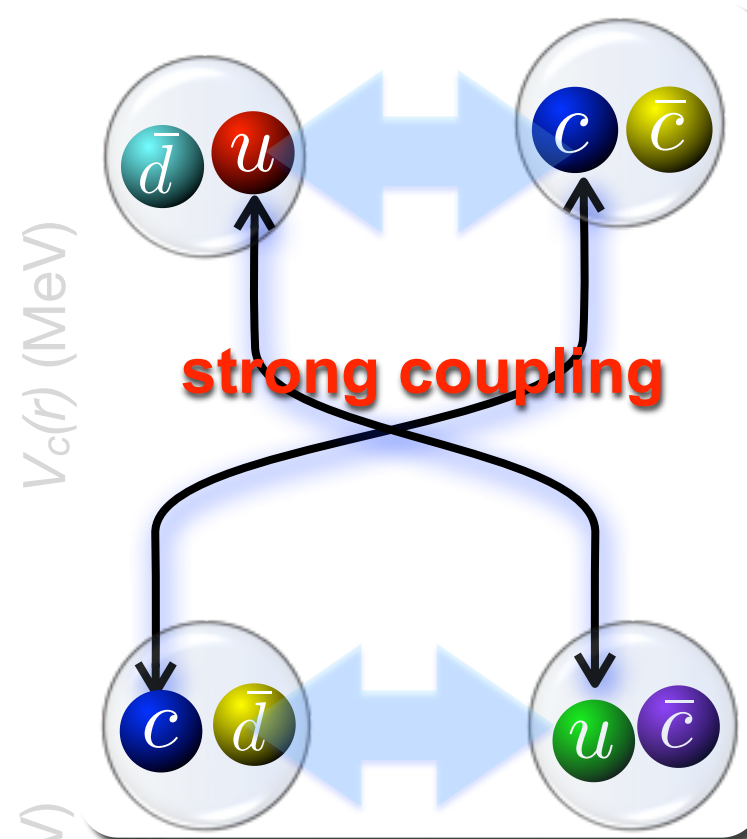
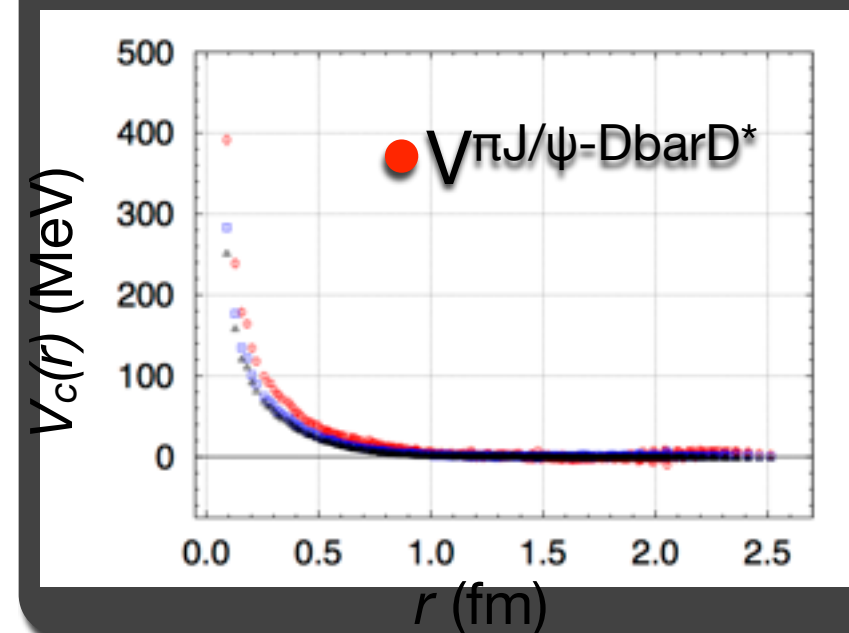
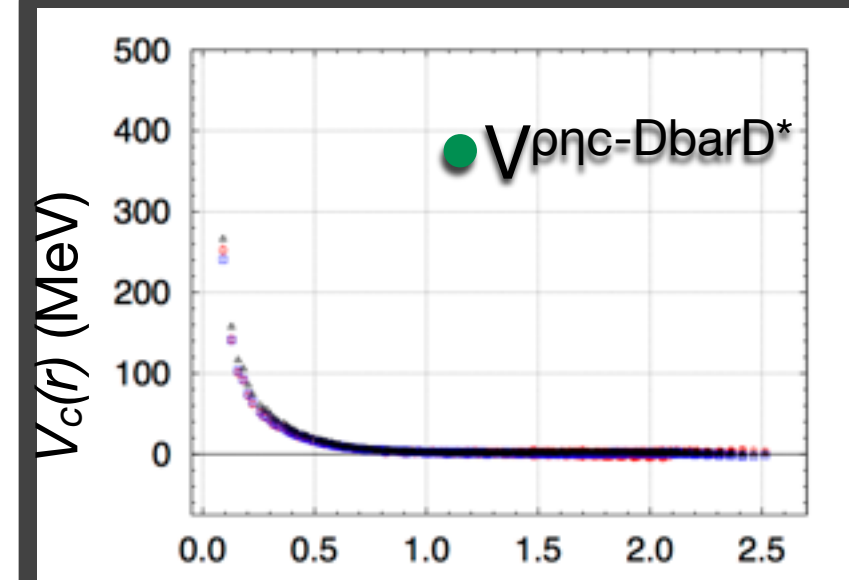
heavy quark spin symmetry



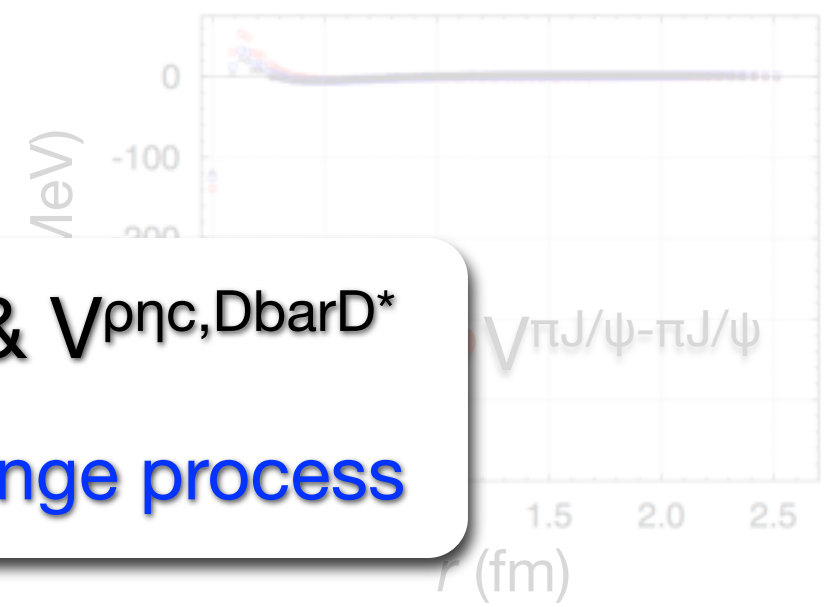
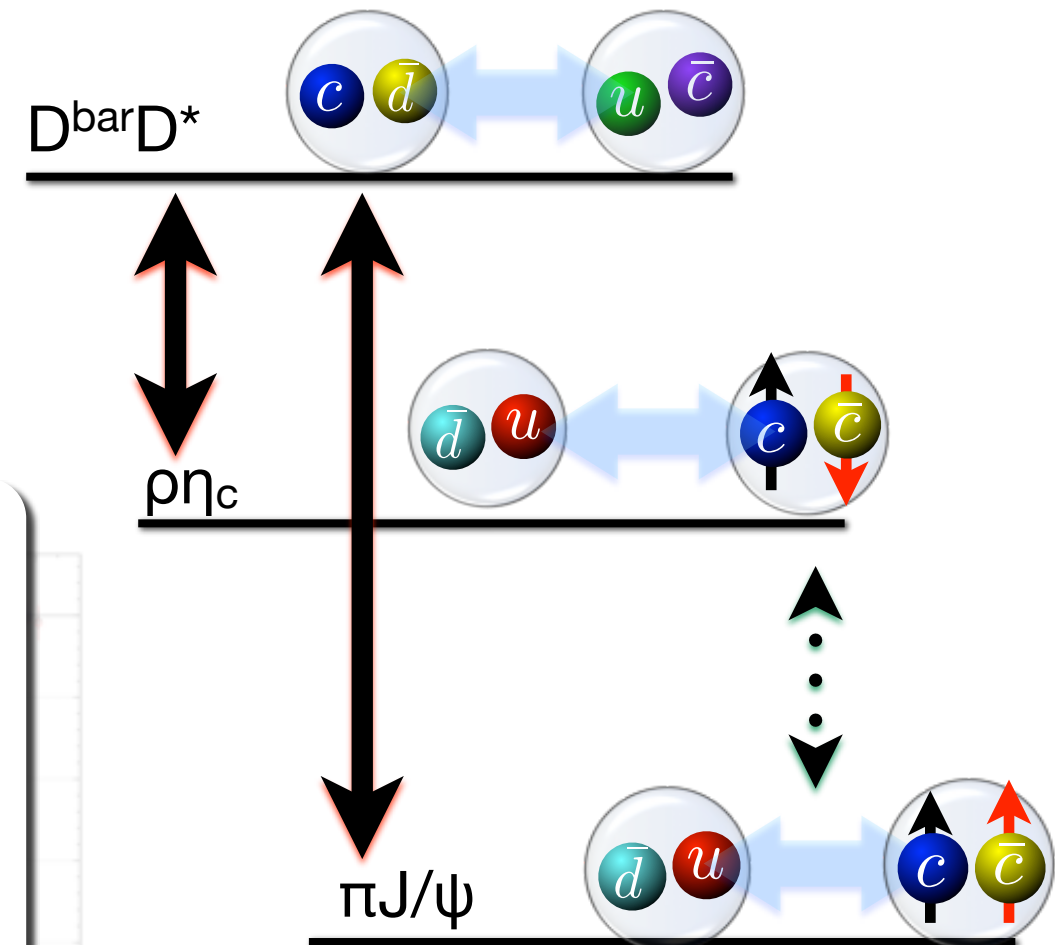
3x3 potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$)



- $m_\pi=410\text{MeV}$ (red line)
- $m_\pi=570\text{MeV}$ (blue line)
- $m_\pi=700\text{MeV}$ (black line)

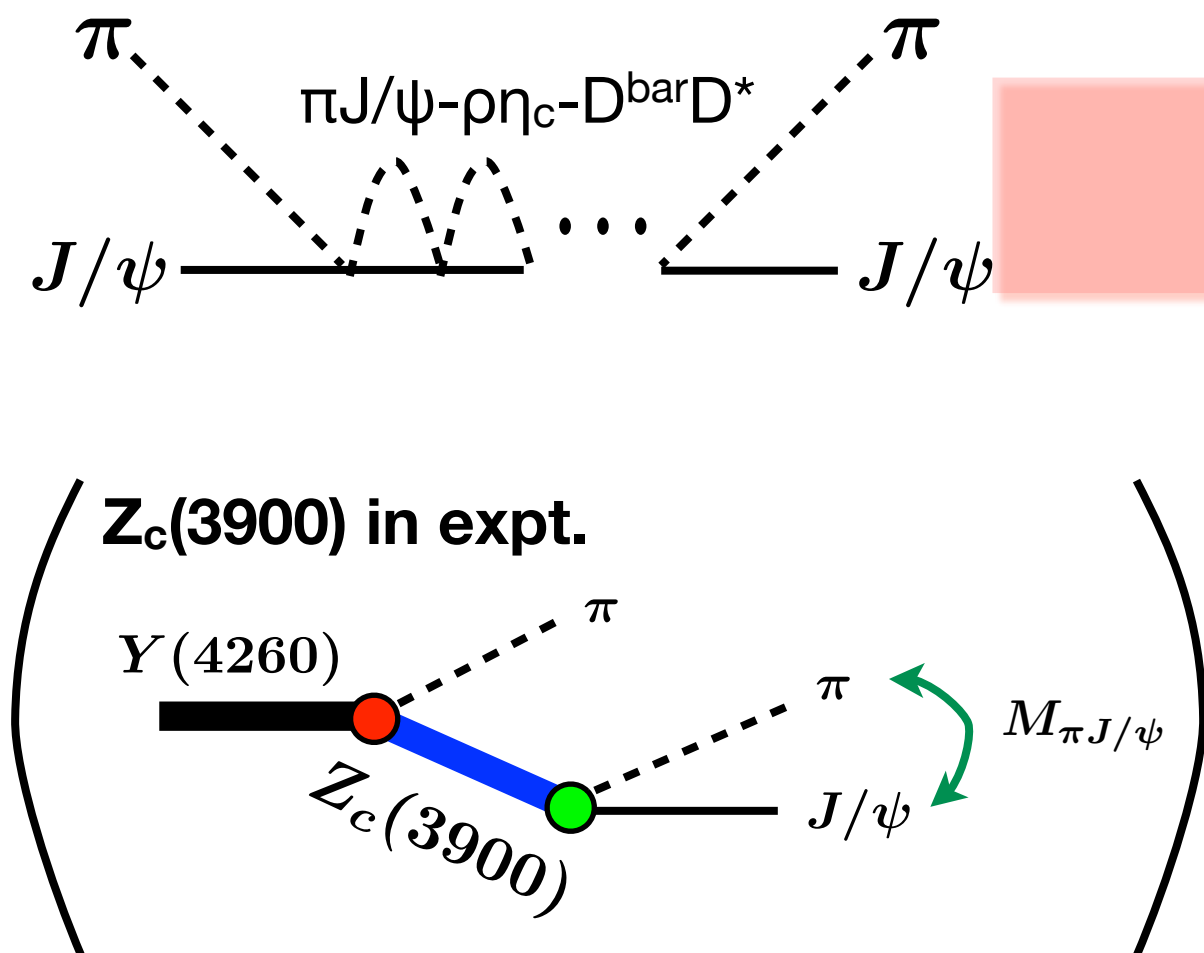


- strong $V_{\pi J/\psi, D^{\text{bar}}D^*}$ & $V_{\rho\eta_c, D^{\text{bar}}D^*}$
- ➔ charm quark exchange process

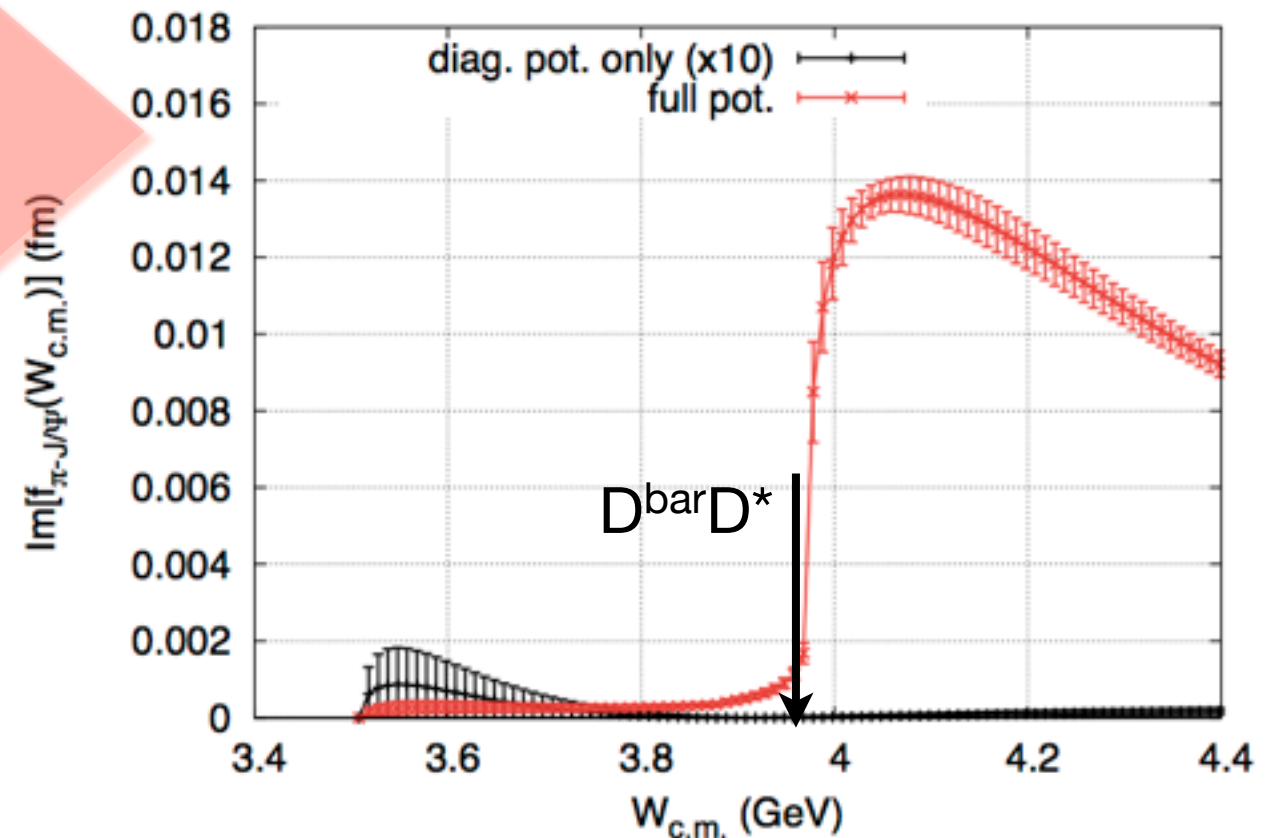


Mass spectra of $\pi J/\psi$ (2-body scattering)

★ 2-body scattering (the most ideal to understand $Z_c(3900)$)



● $\pi J/\psi$ invariant mass ($m_\pi = 410 \text{ MeV}$)



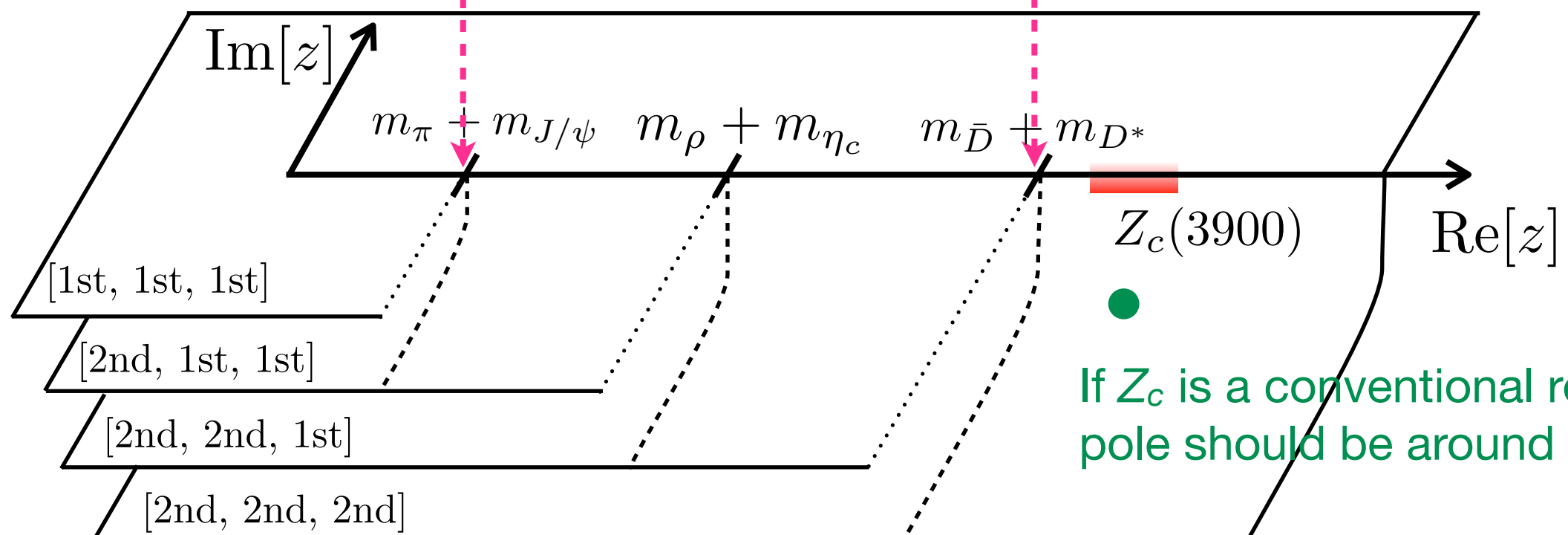
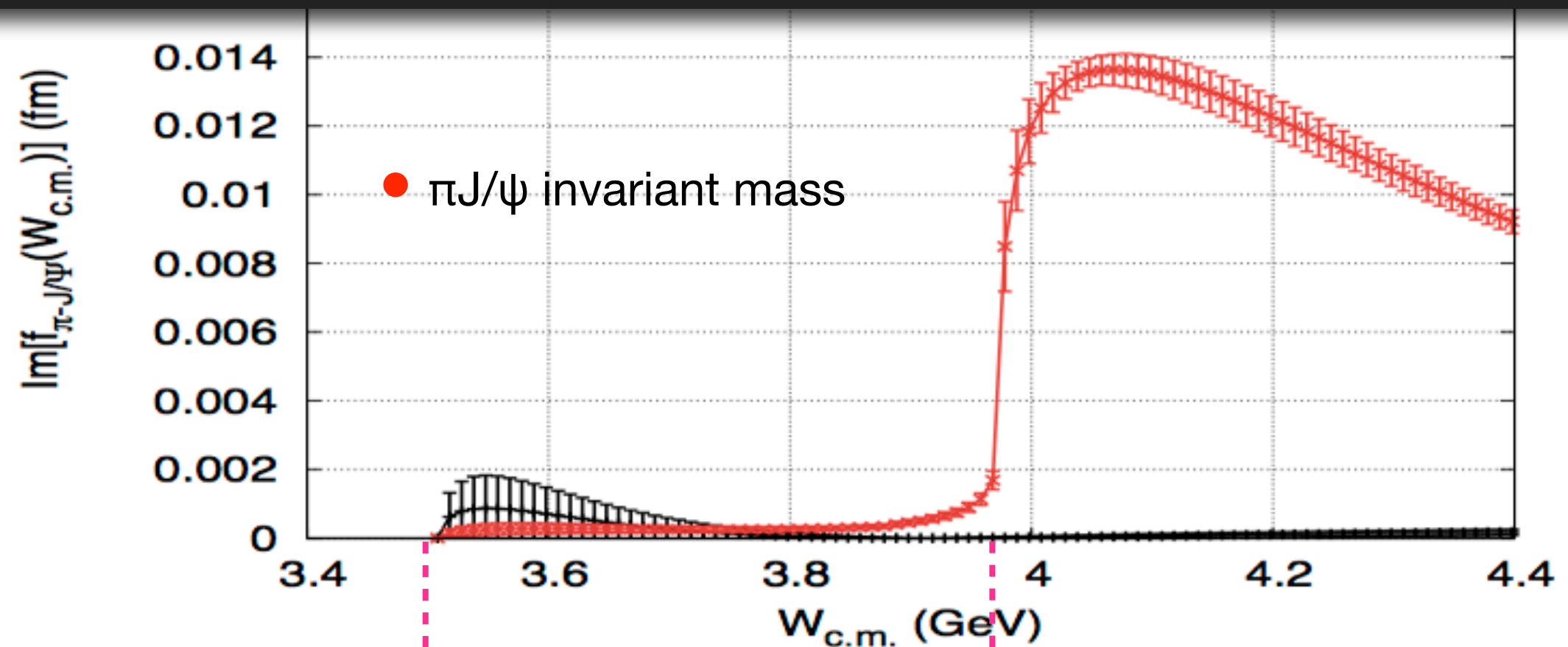
✓ Enhancement just above $D^{\text{bar}} D^*$ threshold

➡ effect of strong $V^{\pi J/\psi, D^{\text{bar}} D^*}$ (black $\rightarrow V^{\pi J/\psi, D^{\text{bar}} D^*} = 0$)

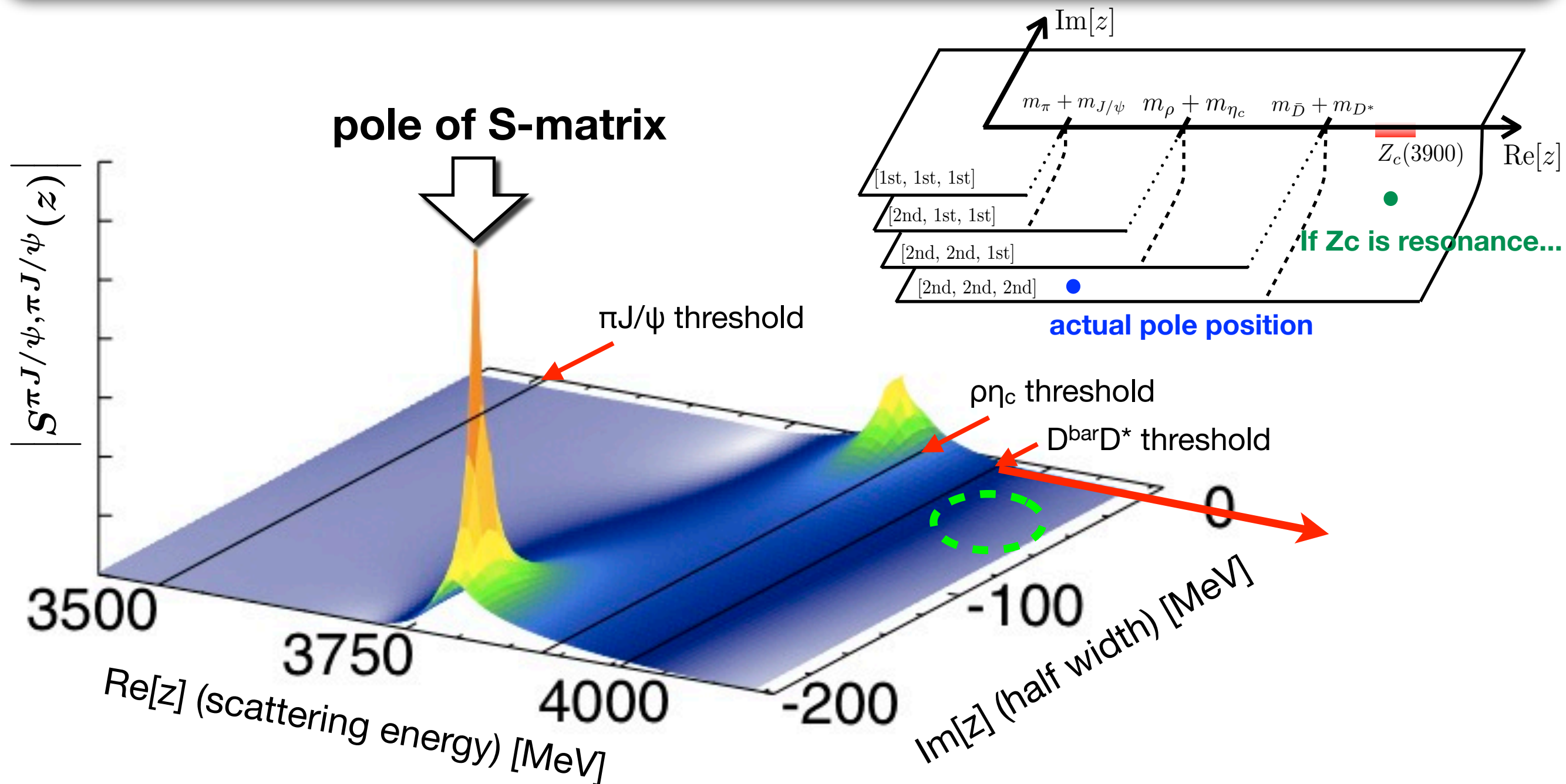
● line shape **not Breit-Wigner**

✓ Is $Z_c(3900)$ a conventional resonance? $\rightarrow$ pole of S-matrix

Pole of S-matrix on complex energy plane



Pole of S-matrix ($\pi J/\psi$:2nd, $\rho \eta_c$:2nd, $D^{\text{bar}} D^*$:2nd)

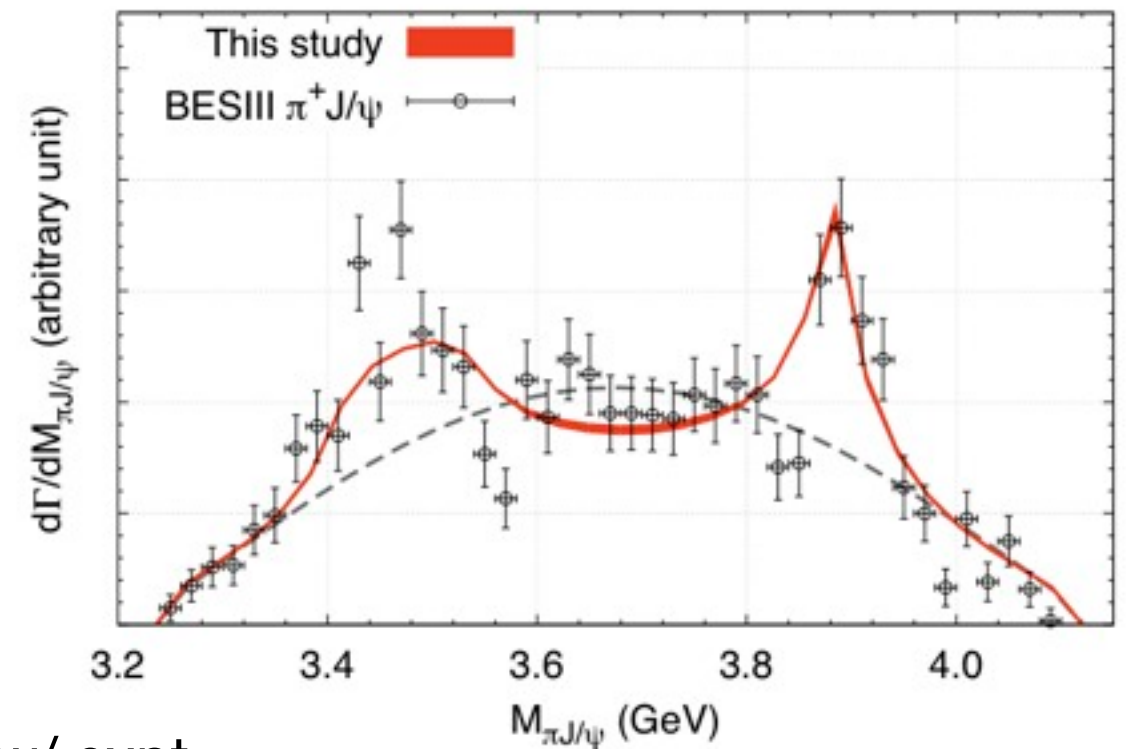
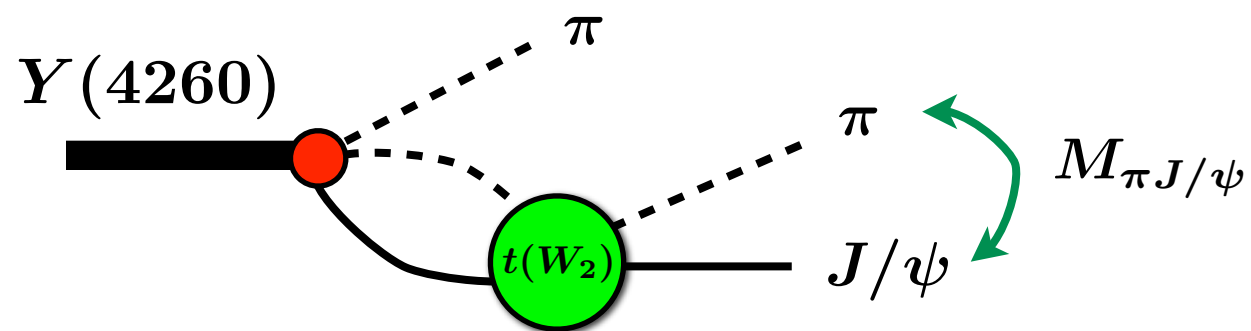


- Pole corresponding to “virtual state”
- Pole contribution to scat. observable is **small** (far from scat. axis)
- $Z_c(3900)$ is not a resonance but **“threshold cusp” induced by strong $V^{\pi J/\psi, D^{\text{bar}} D^*}$**

Invariant mass of 3-body decay

Ikeda [HAL QCD], J. Phys. G45, 024002 (2018).

$Z_c(3900)$ in expt.

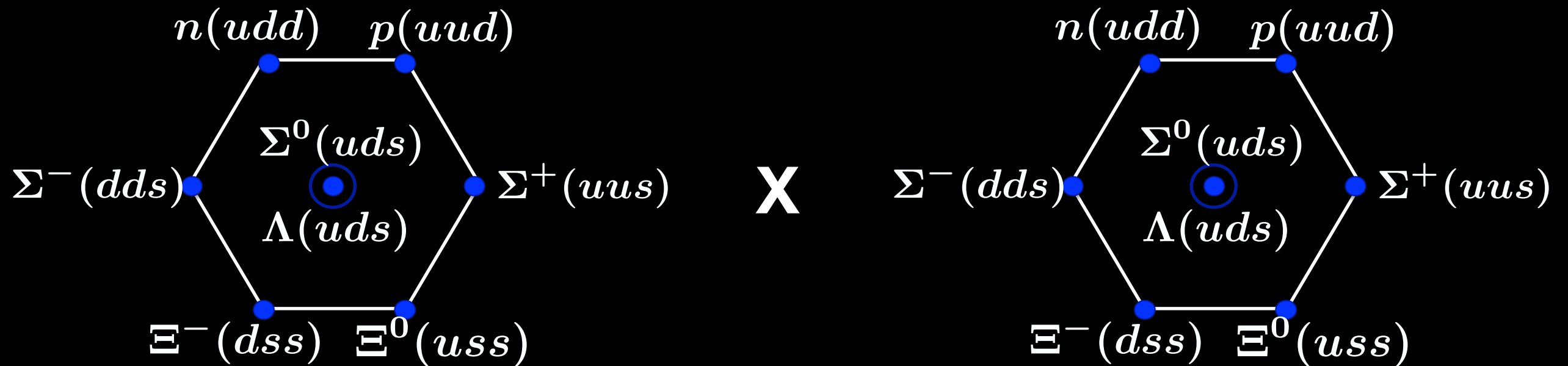


- ➔ physical hadron masses employed to compare w/ expt.
- ➔ t-matrix for subsystem obtained from $V^{\text{LQCD}}(r)$

- Expt. data well reproduced **with cusp scenario**

conclusion: $Z_c(3900)$ is threshold cusp caused by strong $V^{\pi J/\psi}, D\bar{u}D^*$

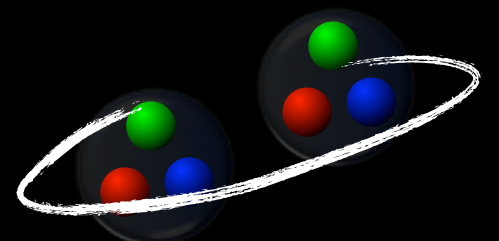
Octet BB forces & H-dibaryon



$$= (27 \oplus 8 \oplus 1)_{\text{sym.}} \oplus (10^* \oplus 10 \oplus 8)_{\text{anti-sym.}}$$

$\uparrow$
 $NN (^1S_0)$

$\uparrow$
 $NN (^1S_0) : \text{deuteron}$



H-dibaryon ($\Lambda\Lambda$ - ΞN - $\Sigma\Sigma$)? Jaffe (1977)

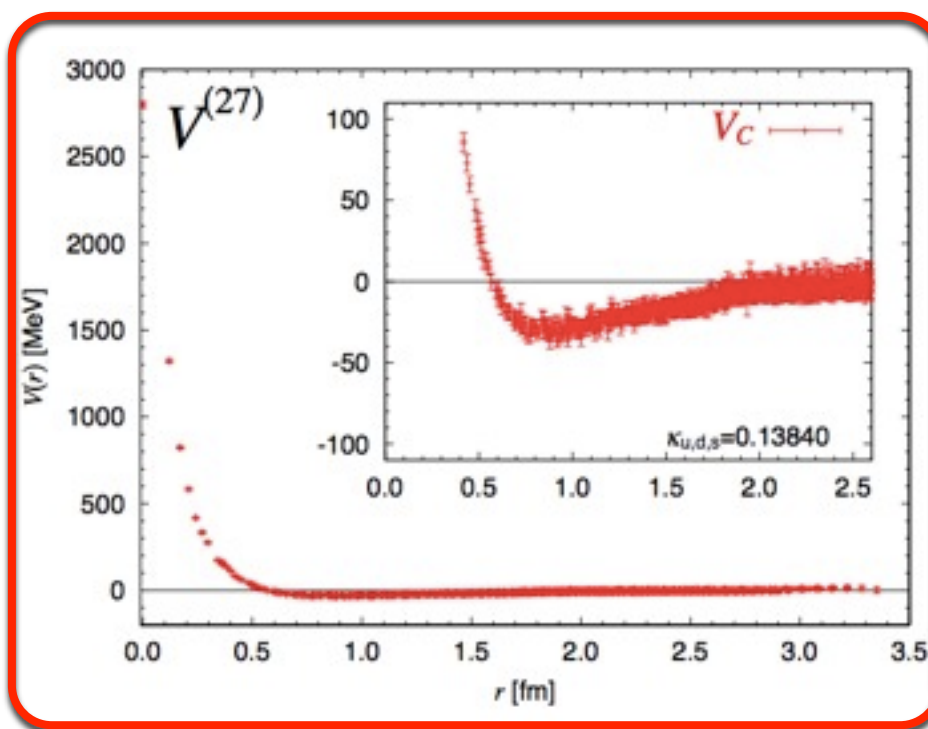
Generalized BB forces in flavor SU(3) limit

❖ Full QCD in $SU(3)_F$ limit : $m_\pi \sim 0.47 \text{ GeV}$, $L = 3.9 \text{ fm}$

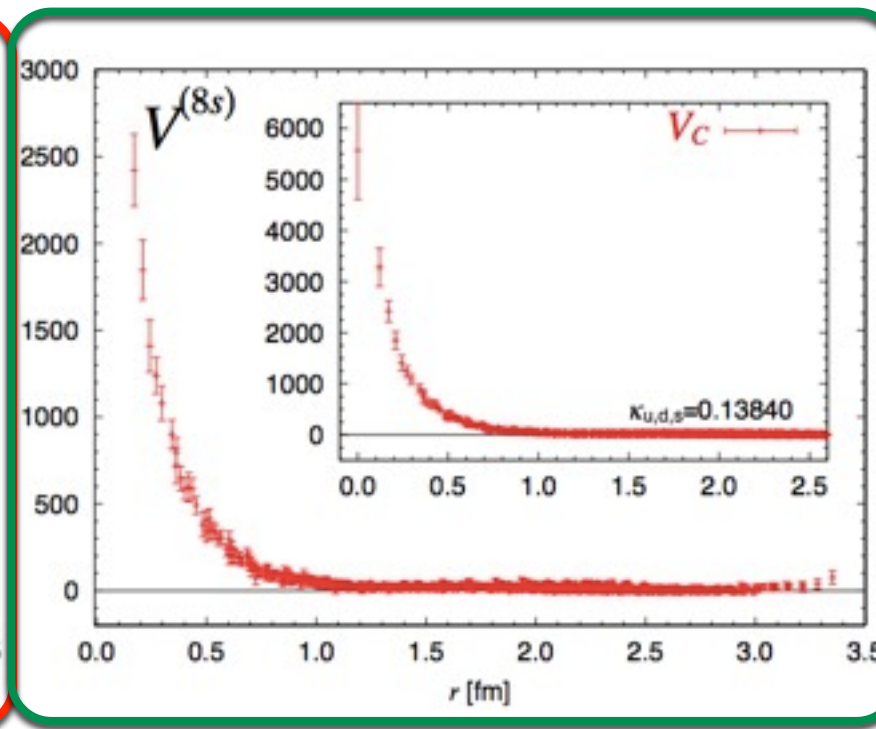
[Inoue et al. \(HAL QCD\), PRL106 \(2011\), NPA881 \(2012\).](#)

★ potentials in flavor symmetric channels, $27 + 8_s + 1$

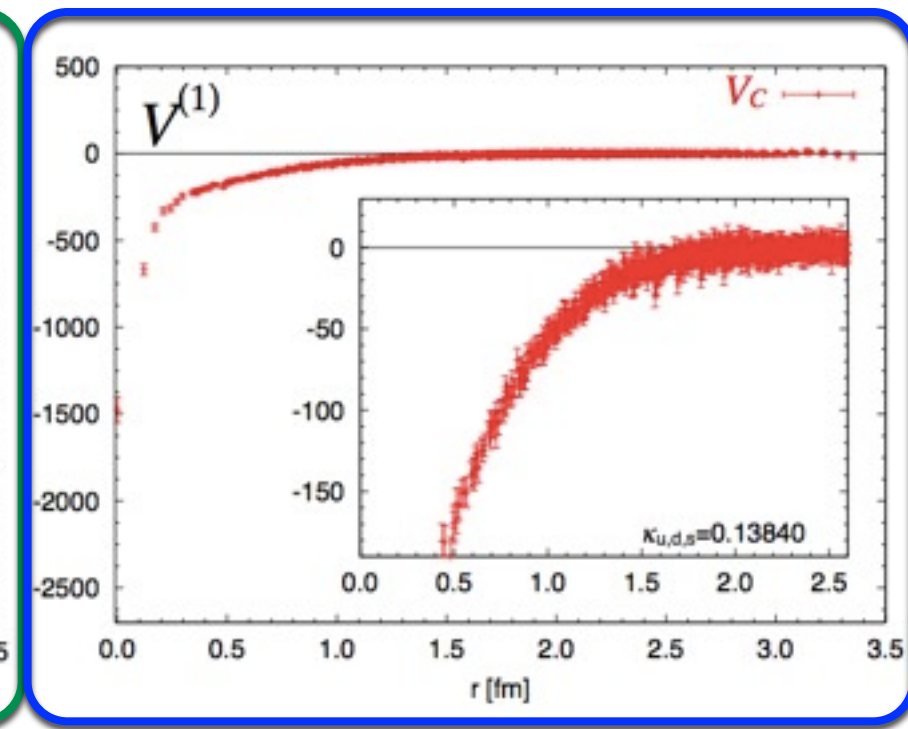
NN 1S_0 channels
(partially Pauli blocked)



8_s channel
(Pauli forbidden)



H-dibaryon channel
(Pauli allowed)



H-dibaryon?

◆ origin of repulsive core <--> Pauli principle

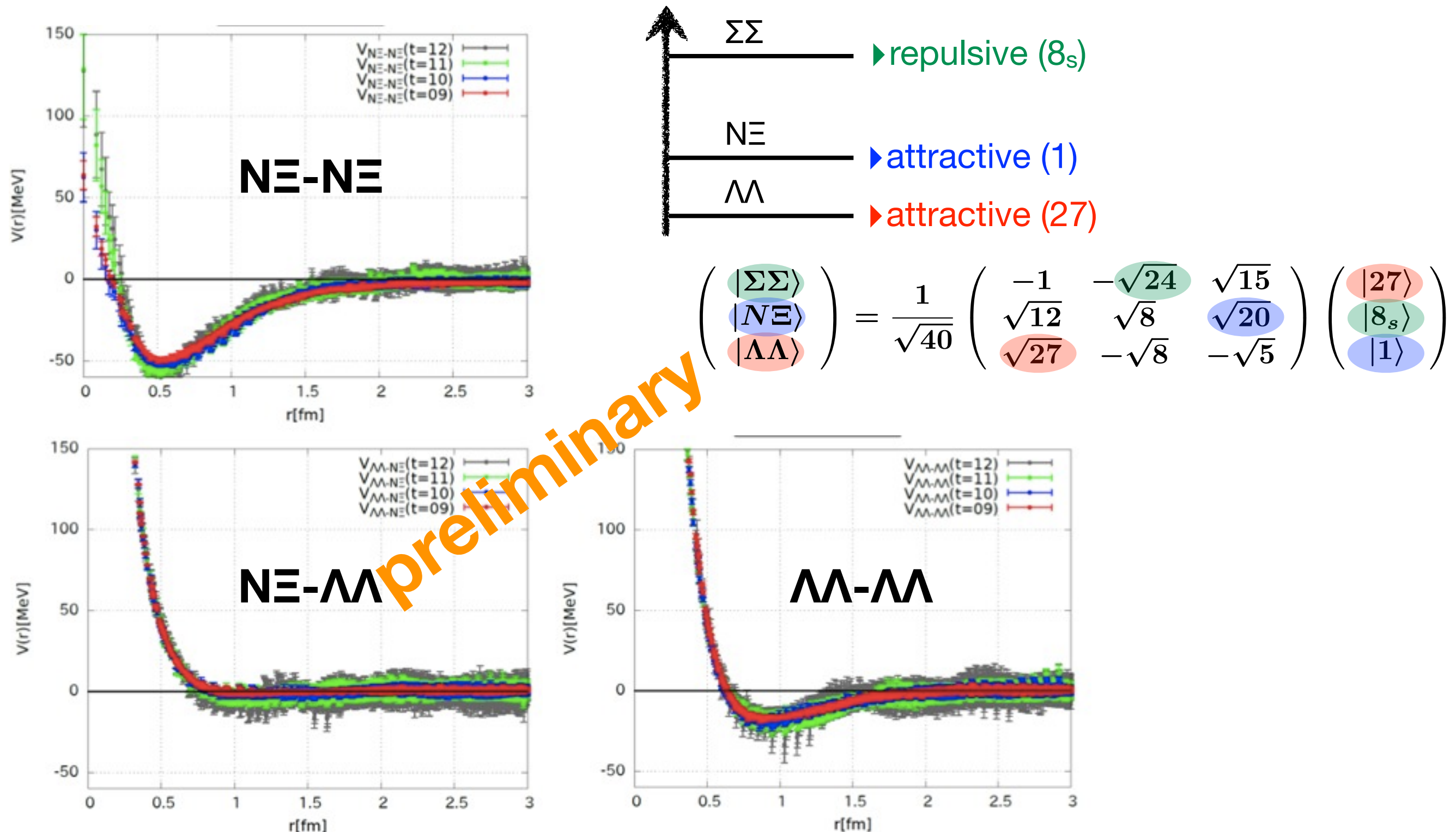
(+ gluon exchange)

[see, Oka & Yazaki, NPA464 \(1987\)](#)



Fate of H-dibaryon @ almost physical point

❖ $N_f=2+1$ full QCD, $m_\pi \sim 0.146 \text{ GeV}$ (almost physical), $L \sim 8.1 \text{ fm}$ (large volume)



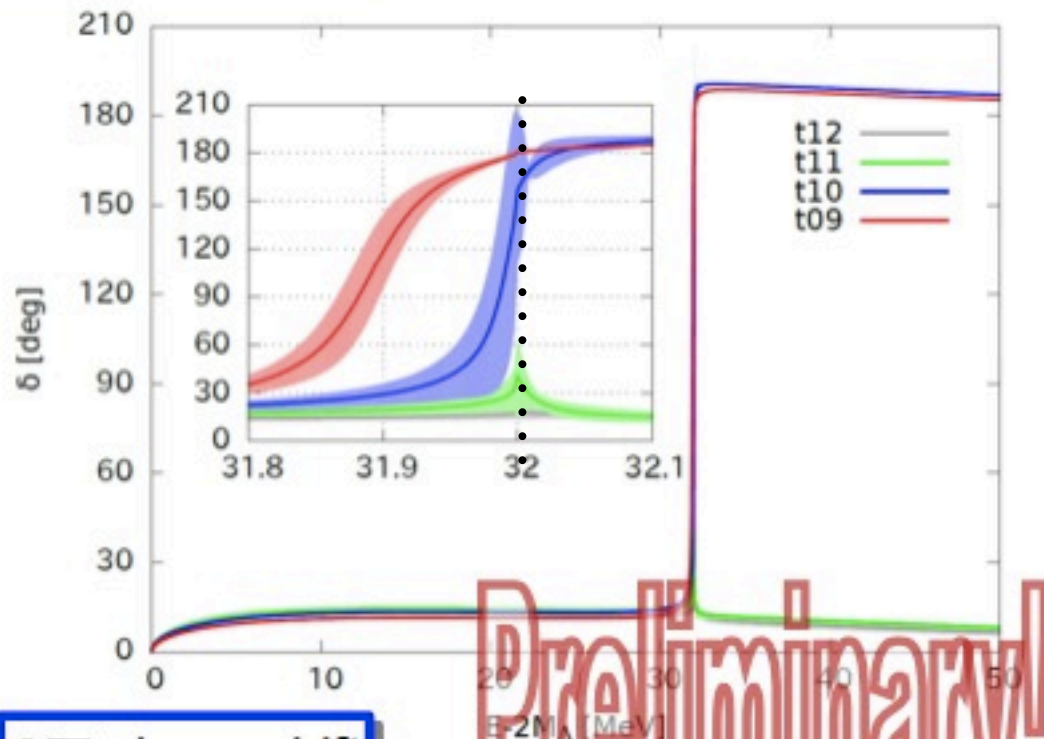
Sasaki et al. [HAL QCD], in preparation.

Fate of H-dibaryon @ almost physical point

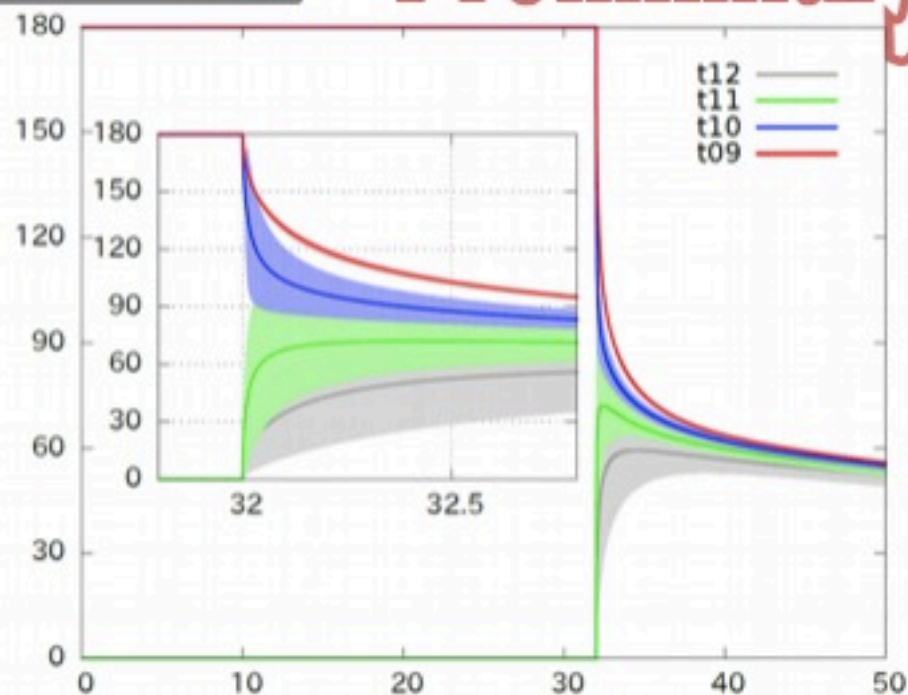
★ $\Lambda\Lambda$ and ΞN phase shifts

$$S(k) = \begin{pmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}$$

$\Lambda\Lambda$ phase shift



ΞN phase shift

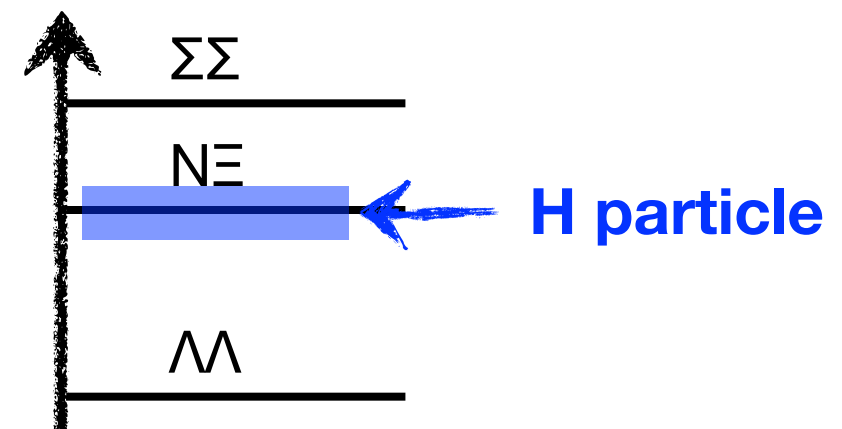


Original prediction of H-dibaryon

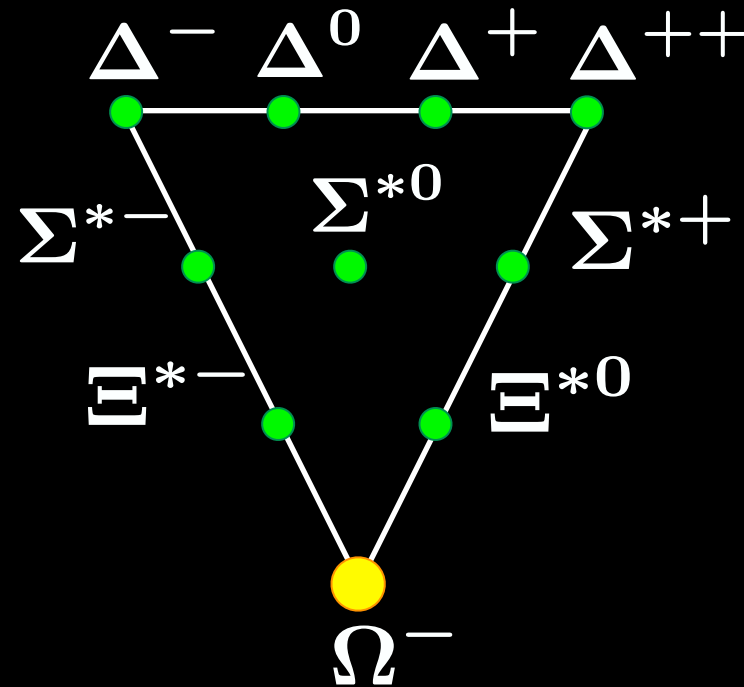
Jaffe (1977) based on quark model,
“Perhaps a Stable Dihyperon”

Answer from QCD for H-dibaryon

“Perhaps near threshold Dihyperon”



Extension to decuplet baryon Ω

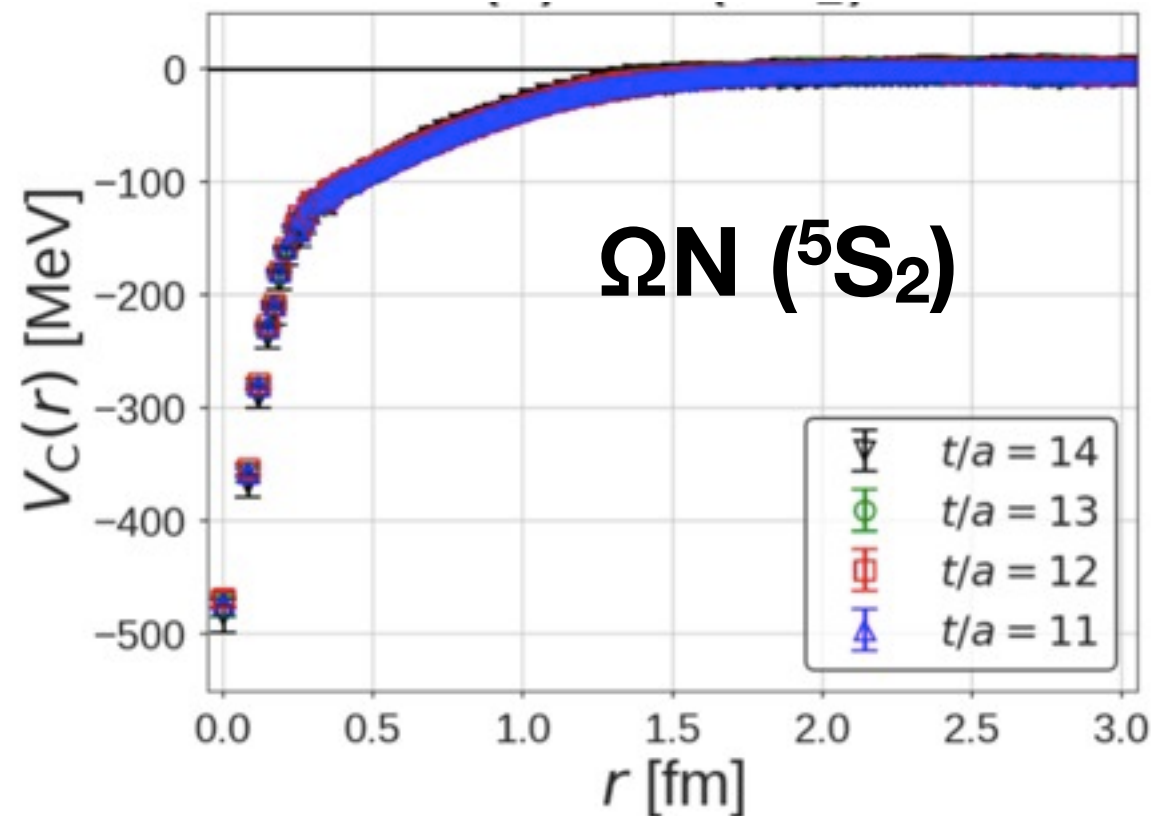


- $\Omega N (^5S_2) : 8 \times 10 = 35 + 8 + 10 + 27$
- $\Omega\Omega (^1S_0) : 10 \times 10 = 28 + 27 + 35 + 10^*$

Both are Pauli allowed states

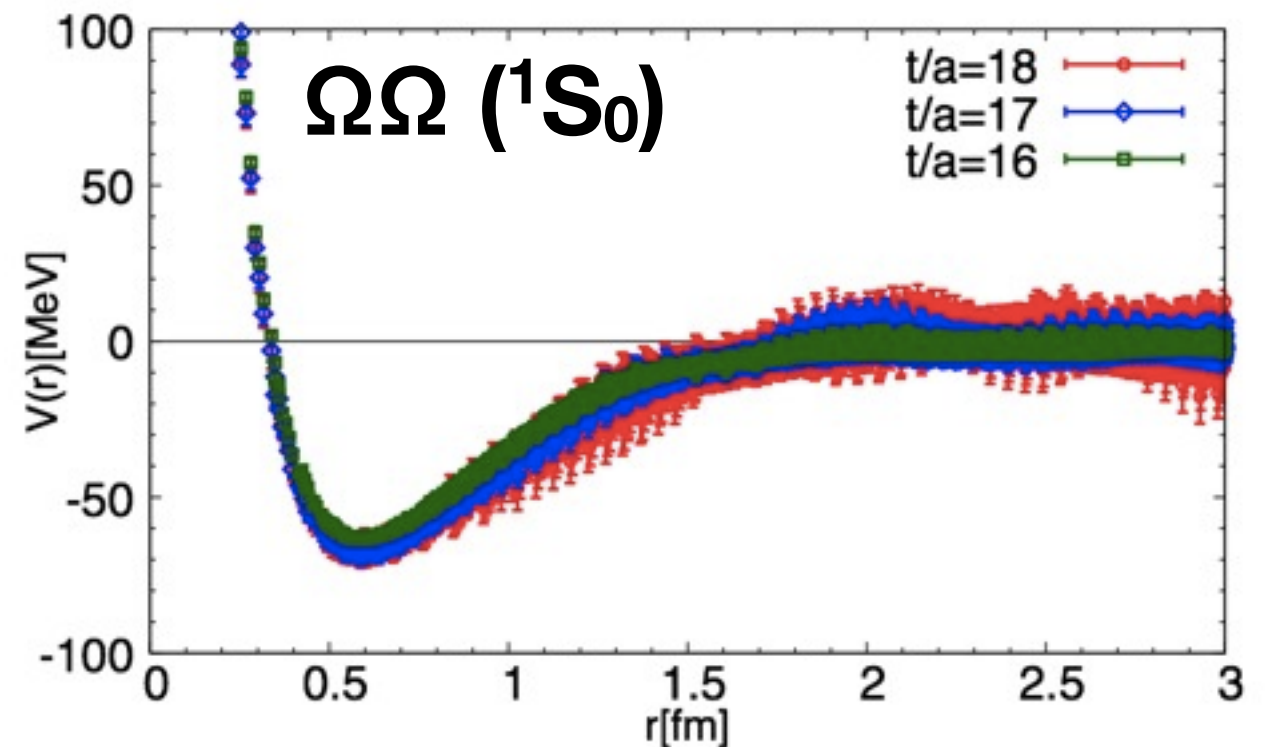
Ω -dibaryon systems @ almost physical point

[Iritani et al. \[HAL QCD\], PLB792 \(2019\).](#)



- entirely attractive

[Gongyo, Sasaki et al. \[HAL QCD\], PRL120 \(2018\).](#)

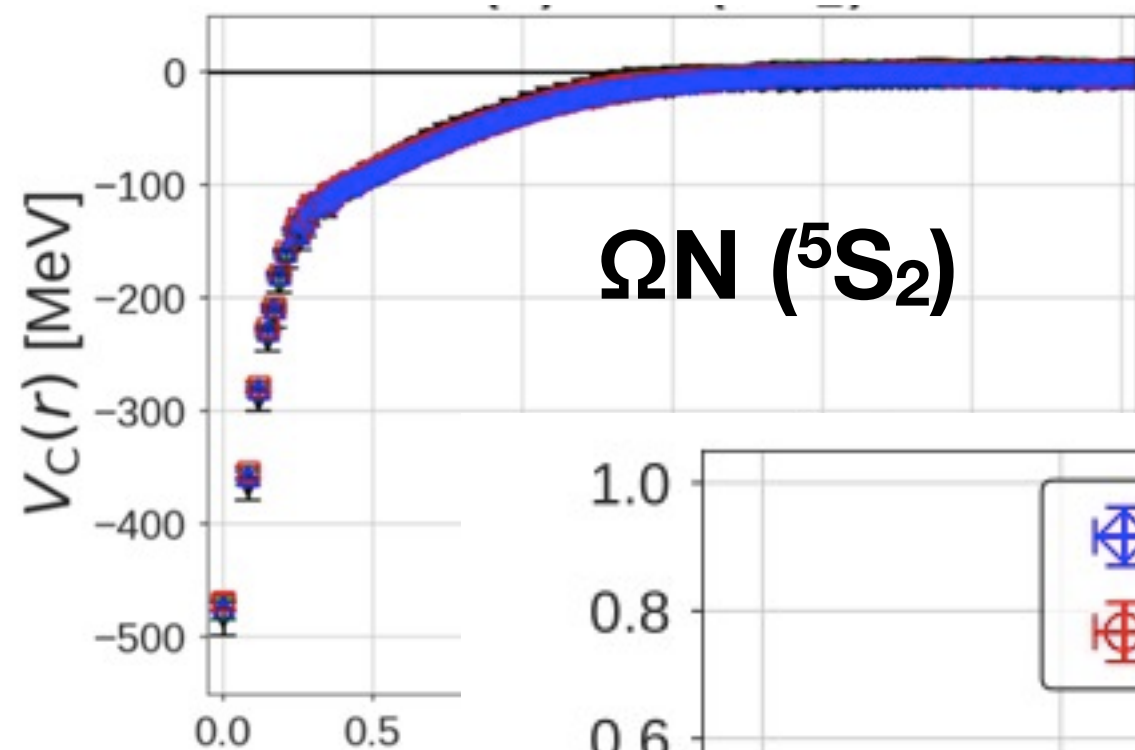


- **repulsive core** + attractive pocket
 - ▶ repulsive force by gluon exchange

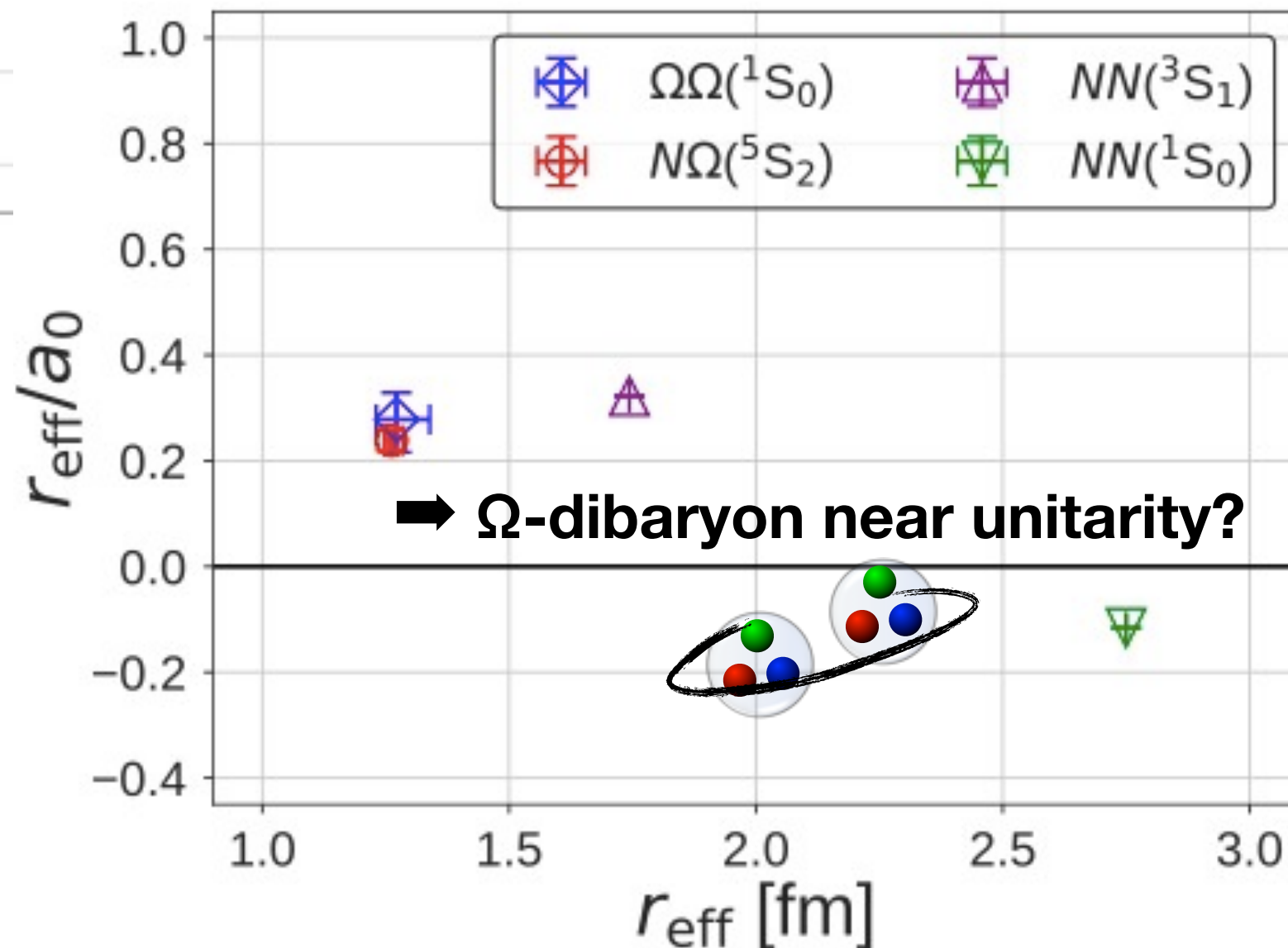
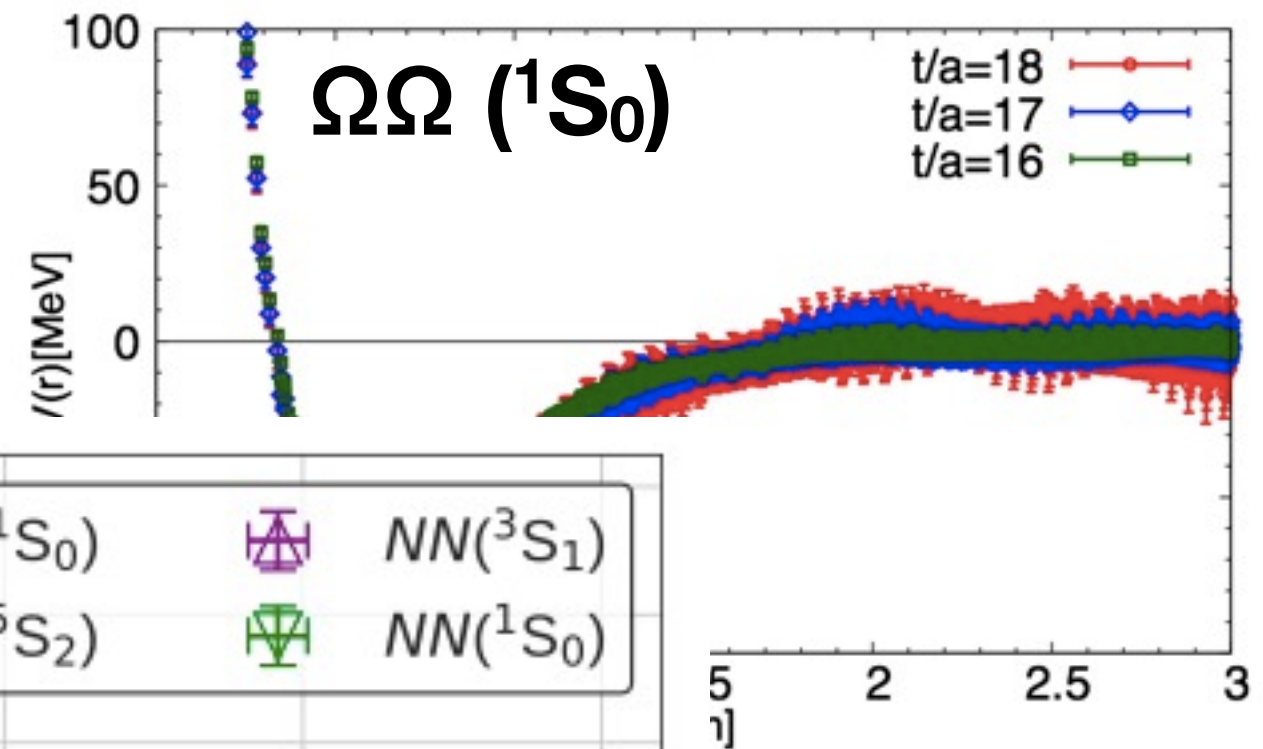
[Oka, Yazaki \(1980\)](#)

Ω -dibaryon systems @ almost physical point

Iritani et al. [HAL QCD], PLB792 (2019).

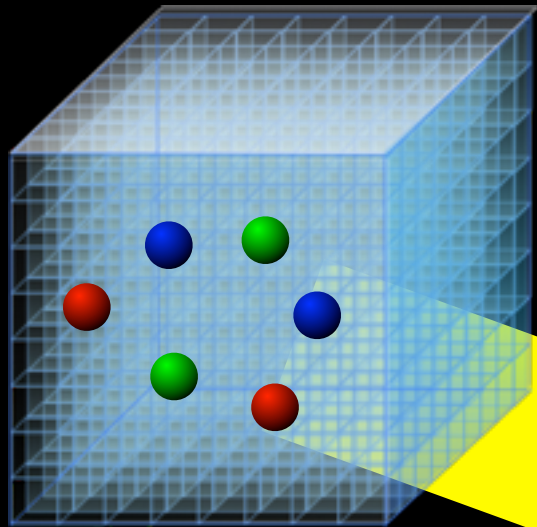


Gongyo, Sasaki et al. [HAL QCD], PRL120 (2018).



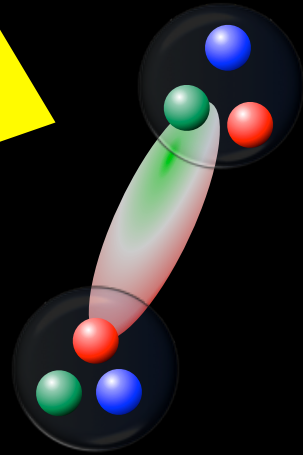
Summary

lattice QCD



- HAL QCD method

hadronic interactions



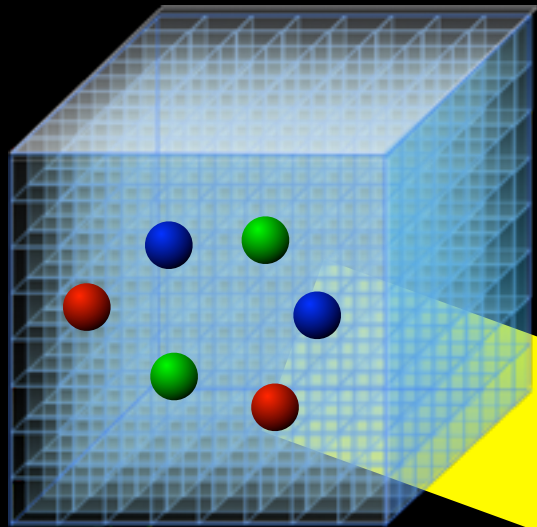
hadron resonances



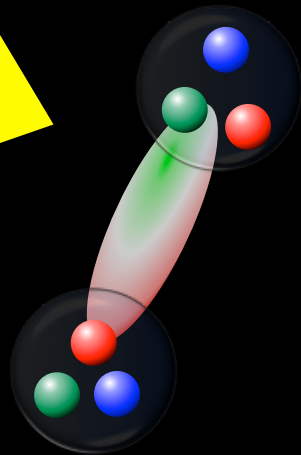
- $Z_c(3900)$ is threshold cusp induced by strong $V^{D\bar{D}^*, \pi J/\psi}$
- H particle is very close to $N\Xi$ threshold
- Ω -dibaryons are similar to deuteron

Future: from quarks to hadrons, nuclei & neutron stars

lattice QCD



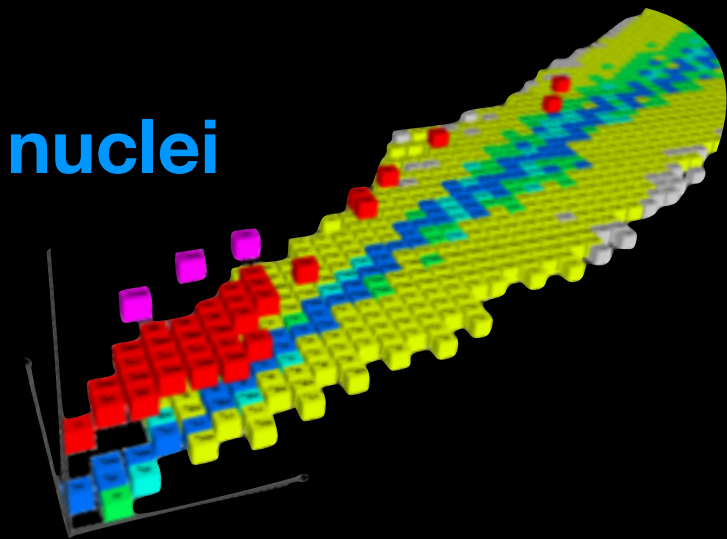
hadronic
interactions



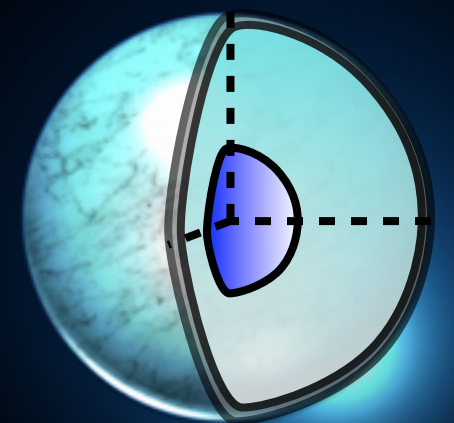
hadron resonances



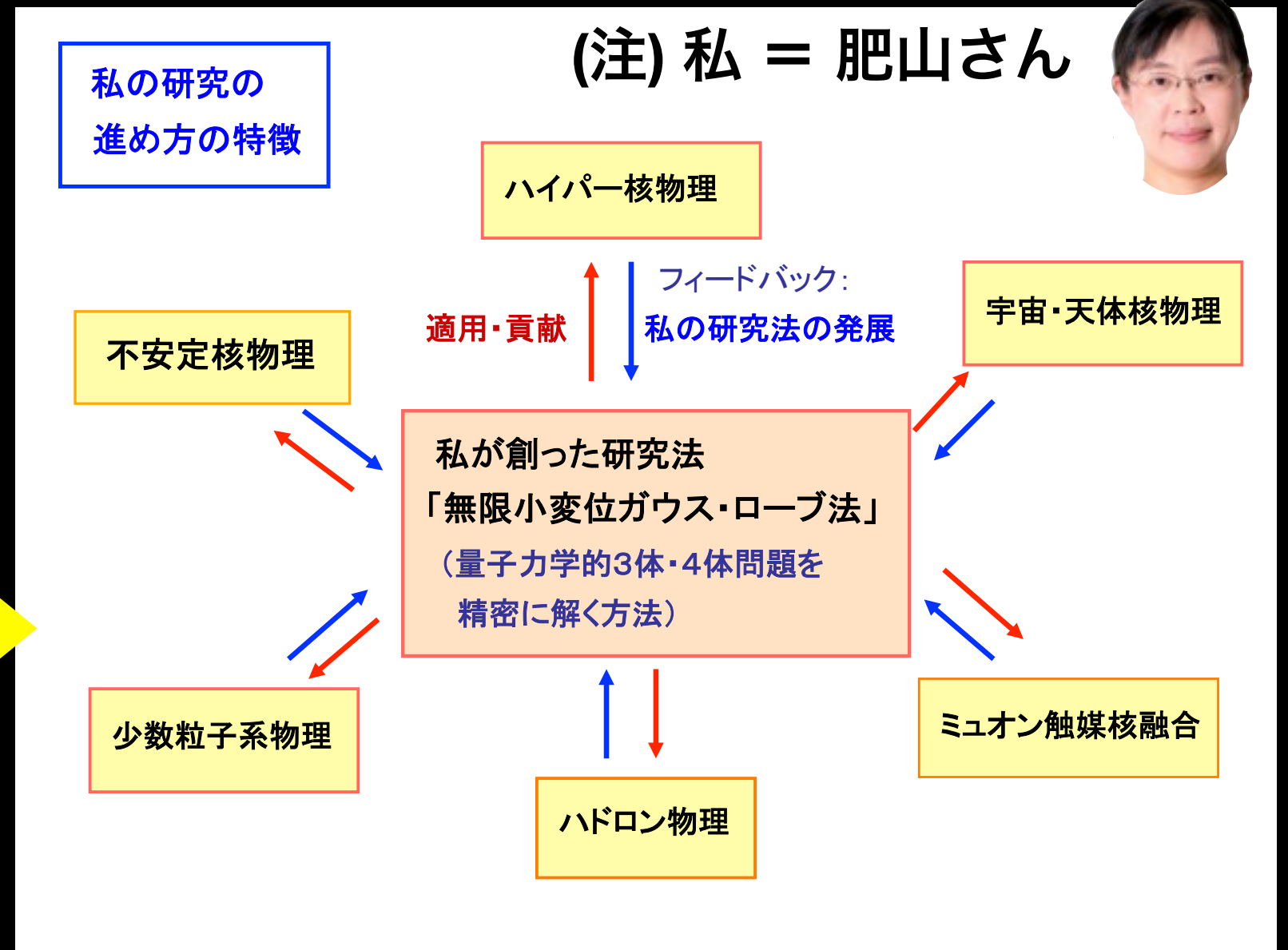
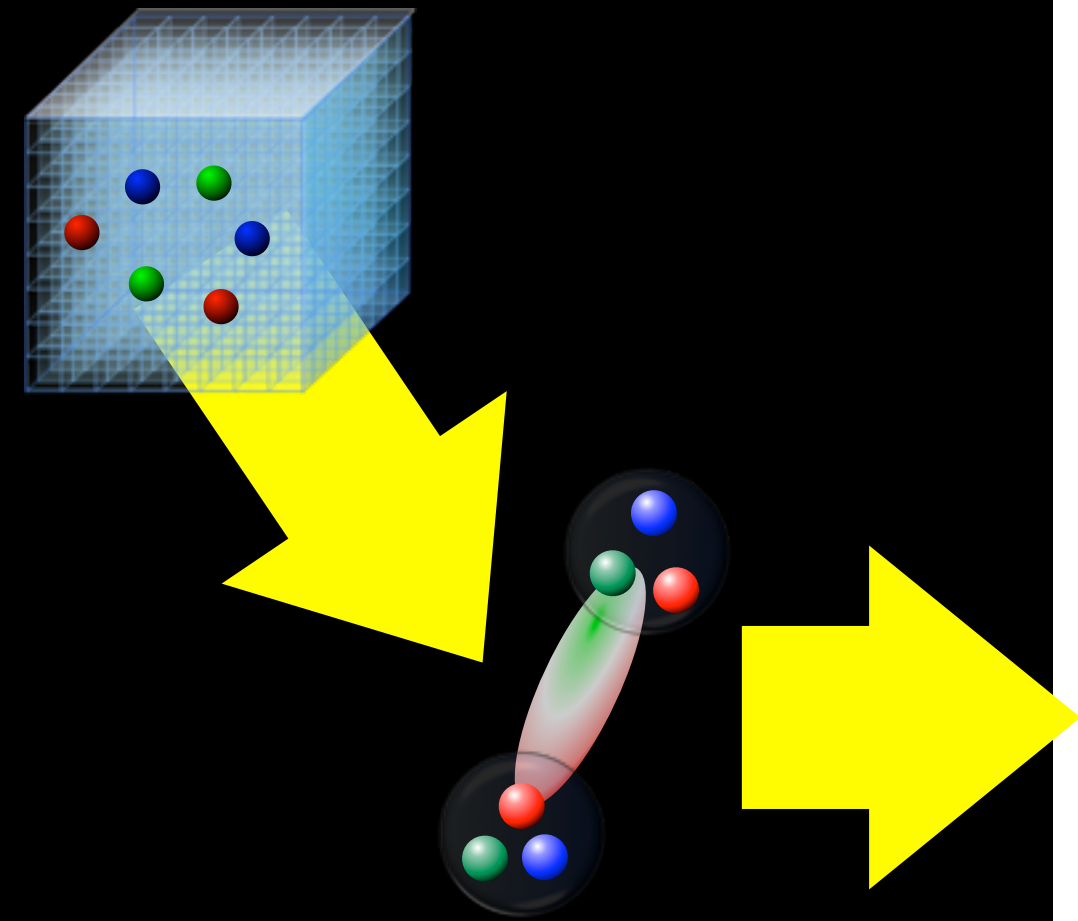
nuclei



EOS of
neutron stars



Collaboration w/ Prof. Emiko Hiyama



肥山さん、文部科学大臣表彰「科学技術賞」の受賞
おめでとうございます!!