Global Analysis of Fragmentation Functions

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Introduction

• **Fragmentation Functions (FFs)** are a key issue in high energy hadron production processes.
  - **FFs** $D(z)$: probability to produce a hadron with momentum fraction $z$ from a parent proton
  - **Analysis of FFs with their uncertainties** must be important to search for new phenomena

• **Present Status (for light hadrons)**
    - parton $\rightarrow \pi^+ (\pi^-)$ distribution
    - Many ansatz for determination of distributions
  - **KKP**: B.A. Kniehl, G. Kramer, B. Potter, NPB582, 514 (2000)
    - parton $\rightarrow \pi^\pm$ distribution (no parton $\rightarrow \pi^+ (\pi^-)$ distribution)
    - 2nd moment is problematic. (AKK: Flavor decomposed distribution)

- Independent global analysis of FFs including new data
- Estimate their uncertainties (It’s new!!!)

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Cross Section of $e^+e^- \rightarrow h^\pm X$

- **Observable:**

$$\frac{1}{\sigma_{tot}} \frac{d\sigma^h}{dz}, \quad \sigma_{tot} = \sum_q \sigma_0^q \left( 1 + \frac{\alpha_s(Q^2)}{\pi} \right)$$

$$z \equiv \frac{2P_h \cdot q}{Q^2} = \frac{2E_h}{Q} : \text{scaling variable}$$

$$\frac{d\sigma^h}{dz} = \sum_i \int_z^1 \frac{d\xi}{\xi} C_i(\xi, Q^2, \mu_{F,R}^2) D_i^h \left( \frac{z}{\xi}, \mu_F^2 \right)$$

$Q^2 = \mu_{F,R}^2 = s : \text{CMS energy}$

- **Coefficient Function calculable in pQCD**

- **Fragmentation Function extracted from experiments**

- **DGLAP equation:**

$$\frac{\partial}{\partial \ln \mu^2} D_j(z, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \sum_i \int_z^1 \frac{d\xi}{\xi} P_{ij} \left( \frac{z}{\xi}, \alpha_s \right) D_i^h \left( \xi, \mu^2 \right)$$

$$P_{ij}(z, \alpha_s) = P_{ij}^{(0)}(z) + \frac{\alpha_s(\mu^2)}{2\pi} \Delta P_{ij}^{(1)}(z) + \ldots \quad P_{ij} : j \rightarrow i \text{ splitting function}$$
Ansatz (for $\pi^{\pm}$)

- Function form (most simplest form)

\[
D_{u,d}^{\pi^+}(z, \mu_0^2) = N_{u}^{\pi^+} z^{\alpha_u^+} (1 - z)^{\beta_u^+} \\
D_{u,d,s,\bar{s}}^{\pi^+}(z, \mu_0^2) = N_{u}^{\pi^+} z^{\alpha_{u,s}^+} (1 - z)^{\beta_{u,s}^+} \\
D_{c,\bar{c}}^{\pi^+}(z, m_c^2) = N_{c}^{\pi^+} z^{\alpha_c^+} (1 - z)^{\beta_c^+} \\
D_{b,\bar{b}}^{\pi^+}(z, m_b^2) = N_{b}^{\pi^+} z^{\alpha_b^+} (1 - z)^{\beta_b^+} \\
D_{g}^{\pi^+}(z, \mu_0^2) = N_{g}^{\pi^+} z^{\alpha_g^+} (1 - z)^{\beta_g^+}
\]

- Constraint condition
  - 2nd moment should be finite and less than 1

\[
N = M^{2nd} \frac{\Gamma(\alpha + \beta + 3)}{\Gamma(\alpha + 2)\Gamma(\beta + 1)} , \quad M^{2nd} = \int_0^1 zD(z)dz \\
\alpha_i > -2, \quad \beta_i > -1, \quad 0 < M_i^{2nd} \left( = \int_0^1 zD_i^h(z)dz \right) < 1
\]
Experimental Data: \( e^+e^- \rightarrow h^\pm X \)

- the number of Data: \( 264 \) \[ Q^2 \geq 1 \text{GeV}^2, \; z > 0.1(\sqrt{s} < M_Z), \; z > 0.05(\sqrt{s} \geq M_Z) \]

<table>
<thead>
<tr>
<th>Experiment</th>
<th>( \sqrt{s} ) (GeV)</th>
<th>データ数</th>
</tr>
</thead>
<tbody>
<tr>
<td>TASSO</td>
<td>12, 14, 22, 30, 34, 44</td>
<td>29</td>
</tr>
<tr>
<td>TCP</td>
<td>29</td>
<td>18</td>
</tr>
<tr>
<td>HRS</td>
<td>29</td>
<td>2</td>
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<tr>
<td>TOPAZ</td>
<td>58</td>
<td>4</td>
</tr>
<tr>
<td>SLD</td>
<td>91.2</td>
<td>29</td>
</tr>
<tr>
<td>SLD [light quark]</td>
<td>91.2</td>
<td>29</td>
</tr>
<tr>
<td>SLD [c quark]</td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>SLD [b quark]</td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>ALEPH</td>
<td>91.2</td>
<td>22</td>
</tr>
<tr>
<td>OPAL</td>
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<td>22</td>
</tr>
<tr>
<td>DELPHI</td>
<td>91.2</td>
<td>17</td>
</tr>
<tr>
<td>DELPHI [light quark]</td>
<td></td>
<td>17</td>
</tr>
<tr>
<td>DELPHI [b quark]</td>
<td></td>
<td>17</td>
</tr>
</tbody>
</table>

We do not include the data of charged hadron and jet production in order to reduce ambiguities of both theory and experiments.
\( \chi^2 \) Analysis

- **Input parameters and results**

  \[ \mu_0^2 = 1 \text{ GeV}^2 \]
  \[ \Lambda_{QCD}^{n_f=4} = 0.220 \text{ (LO)}, \ 0.323 \text{ (NLO)} \]
  \[ \alpha_s \text{ varies with a change of } n_f \]
  \[ m_c = 1.43 \text{ GeV}, \ m_b = 4.3 \text{ GeV} \]

  - **Total** \( \chi^2 = 453.19[\text{LO}], \ 433.49[\text{NLO}] \)

- **Uncertainty estimation**: Hessian method

  \[ \Delta \chi^2 \equiv \chi^2(\hat{\alpha} + \delta \alpha) - \chi^2(\hat{\alpha}) = \sum_{i,j} H_{ij} \delta a_i \delta a_j, \quad H_{ij} = \frac{\partial^2 \chi^2(\hat{\alpha})}{\partial a_i \partial a_j} \]

  \[ \left[ \delta D(z) \right]^2 = \Delta \chi^2 \sum_{i,j} \frac{\partial D(z, \hat{\alpha})}{\partial a_i} H_{ij}^{-1} \frac{\partial D(z, \hat{\alpha})}{\partial a_j} \]

  - **N=14, \( \Delta \chi^2=15.94 \):** \( \int_0^{\Delta \chi^2} K(N,s)ds = 0.683 \ [K(N,s): \chi^2 \text{ distribution}] \)
Optimized Distribution ($\pi^+$)

\[ Q^2_0 = 1 \text{ GeV}^2 \text{ (for } u, d, s, g) \]
\[ Q^2_0 = (1.43)^2 \text{ GeV}^2 \text{ (for } c) \]
\[ Q^2_0 = (4.3)^2 \text{ GeV}^2 \text{ (for } b) \]

---

K. Sudoh (KEK)
$Q_0^2 = 1 \text{ GeV}^2$ (for $u,d,s,g$)

$Q_0^2 = (1.43)^2 \text{ GeV}^2$ (for $c$)

$Q_0^2 = (4.3)^2 \text{ GeV}^2$ (for $b$)
Comparison with Kretzer (\(\pi^+\))

- **LO results**

  \[Q_0^2 = 1\ \text{GeV}^2 \ (\text{for } u,d,s,g)\]
  \[Q_0^2 = (1.43)^2\ \text{GeV}^2 \ (\text{for } c)\]
  \[Q_0^2 = (4.3)^2\ \text{GeV}^2 \ (\text{for } b)\]

  Our results

  initial scale: \(Q^2 = 0.26\ \text{GeV}^2\)
  → evolved to \(Q^2 = 1\ \text{GeV}^2\)
  Mass threshold is different

  Kretzer

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Comparison with KKP ($\pi^+$)

- **LO results** *(Initial scales and mass thresholds are different.)*

\[
\begin{align*}
Q_0^2 &= 1 \text{ GeV}^2 \text{ (for } u,d,s,g) \\
Q_0^2 &= (1.43)^2 \text{ GeV}^2 \text{ (for } c) \\
Q_0^2 &= (4.3)^2 \text{ GeV}^2 \text{ (for } b)
\end{align*}
\]

\[i \to (\pi^+ + \pi^-)/2 \text{ distribution!!}\]

\[i.e. \ D_u^{\pi^\pm} = (D_u^{\pi^+} + D_u^{\pi^-})/2\]

**Our results**

\[
\begin{align*}
Q_0^2 &= 2 \text{ GeV}^2 \text{ (for } u,d,s,g) \\
Q_0^2 &= (2.99)^2 \text{ GeV}^2 \text{ (for } c) \\
Q_0^2 &= (9.46)^2 \text{ GeV}^2 \text{ (for } b)
\end{align*}
\]

**KKP**
Comparison with Kretzer ($\pi^+$)

- **NLO results**

  \[ Q_0^2 = 1 \text{ GeV}^2 \quad \text{(for } u,d,s,g) \]
  \[ Q_0^2 = (1.4)^2 \text{ GeV}^2 \quad \text{(for } c) \]
  \[ Q_0^2 = (4.5)^2 \text{ GeV}^2 \quad \text{(for } b) \]

  **Our results**

  initial scale: $Q^2 = 0.4 \text{ GeV}^2$

  \[ \rightarrow \text{evolved to } Q^2 = 1 \text{ GeV}^2 \]

  Mass threshold is different

K. Sudoh (KEK)
Comparison with KKP ($\pi^+$)

- **NLO results** (Initial scales and mass thresholds are different.)

Our results

\[ Q_0^2 = 1 \text{ GeV}^2 \text{ (for } u,d,s,g) \]
\[ Q_0^2 = (1.43)^2 \text{ GeV}^2 \text{ (for } c) \]
\[ Q_0^2 = (4.3)^2 \text{ GeV}^2 \text{ (for } b) \]

KKP

\[ Q_0^2 = 2 \text{ GeV}^2 \text{ (for } u,d,s,g) \]
\[ Q_0^2 = (2.99)^2 \text{ GeV}^2 \text{ (for } c) \]
\[ Q_0^2 = (9.46)^2 \text{ GeV}^2 \text{ (for } b) \]

\[ i \rightarrow (\pi^+ + \pi^-)/2 \text{ distribution!!} \]

\[ i.e. D_u^{\pi^+} = (D_u^{\pi^+} + D_u^{\pi^-})/2 \]
FFs with Uncertainties ($\pi^+$)

- gluon
  - $Q = 1$ GeV
- u quark
  - $Q = 1.43$ GeV
- d quark
- c quark
  - $Q = 1.43$ GeV
- b quark
  - $Q = 4.3$ GeV

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Comparison with Data

- Comparison with inclusive X-section data at high energy
Comparison with Data (2)

- \((\text{Data-Theory})/\text{Theory}\)

![Graphs showing comparison between data and theory for different Q values and particle types.](image)
FFs for $K^{\pm}$

- Function form:

\[
D_{u}^{K^+}(z, \mu_0^2) = N_{u}^{K^+} z^{\alpha_u^{K^+}} (1-z)^{\beta_u^{K^+}}
\]
\[
D_{\bar{u}}^{K^+}(z, \mu_0^2) = N_{\bar{u}}^{K^+} z^{\alpha_{\bar{u}}^{K^+}} (1-z)^{\beta_{\bar{u}}^{K^+}}
\]
\[
D_{c,\bar{c}}^{K^+}(z, m_c^2) = N_{c}^{K^+} z^{\alpha_c^{K^+}} (1-z)^{\beta_c^{K^+}}
\]
\[
D_{b,\bar{b}}^{K^+}(z, m_b^2) = N_{b}^{K^+} z^{\alpha_b^{K^+}} (1-z)^{\beta_b^{K^+}}
\]
\[
D_{g}^{K^+}(z, \mu_0^2) = N_{g}^{K^+} z^{\alpha_g^{K^+}} (1-z)^{\beta_g^{K^+}}
\]

NLO results
Summary

• Global analysis of FFs was done for independent parametrization
  – Determine function forms in LO, NLO analyses
  – Large correlation between gluon and disfavored distributions
  – In particular, determination of gluon distribution is difficult

• Uncertainties of FFs were estimated
  – Large error bands in low $x$ region (especially for gluon)
  – Uncertainties could be small by doing NLO analysis

Outlook:

• It is under analysis for other hadrons (coming soon!!)
• Application for other processes of high energy hadron production