Extracting information about polarized parton densities from experiment

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Workshop on RHIC Spin Physics
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data probing $\Delta g$ are rolling in now …

**PHENIX, STAR**
**HERMES, COMPASS, SMC**

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**What do they imply?**

**How to analyze properly?**

**Potential problems?**
main objective

(as far as possible) a model independent determination of

\[ \Delta g(x, \mu) = \frac{1}{4\pi x P^+} \int dy^- e^{iy^- x P^+} \langle P, S | F_{a+}^+ j(0, y^-, \bar{0}) \mathcal{F} \bar{F}^+ + j(0) | P, S \rangle \bigg|_{\mu} \]

features:

- interpretation as diff. of number operators only in \( A^+=0 \) gauge

- all \( n \geq 2 \) moments, \( \int x^{n-1} \ldots dx \), give local operators but there is no gauge-invariant local gluonic operator for \( n=1 \)

- in \( A^+=0 \) gauge the \( 1^{st} \) moment also collapses into a local operator and has the interpretation as gluon contribution to the spin “sum rule”
requirements

essential to have an adequate and proven theoretical framework:

- if possible, **NLO corrections** should be used
  - scale dependence
  - K-factors
- reliability of pQCD framework
- theoretical uncertainties
- comparison with **unpol. cross sections**
  - benchmark (pdf's ≈ known)
  - foundation for $A_{LL}$
- map out where pQCD applies
- relevance of resummations
- reliable statements about $\int \Delta g(x, \mu) \, dx$ require good $x$ coverage
- unbiased analysis procedure w/o unnecessary assumptions
complications

- information on pdfs inside complicated convolutions and always in a sum over many partonic subprocesses
- no 1:1 correspondence between data and particular values of $x$
- no process is only sensitive to $\Delta g \rightarrow \text{"global analysis"}$

prospects of learning from data depends on our ability to efficiently evaluate $\Delta \sigma = \sum_{abc} \Delta f_a \otimes \Delta f_b \otimes \Delta \hat{\sigma}_{ab \rightarrow cX} \otimes D_c$

- cross sections beyond LO are numerically very time-consuming
- $\chi^2$ analysis of data requires typically 1000's of evalutions of $\Delta \sigma$

- tempting to use approximations - but how reliable are they?
K-factor myth

often it is *assumed* that NLO corrections drop out in $A_{LL}$ or that $K=\text{const}$

assumptions about $K$ factors can be quite misleading
data ↔ x-range probed

how to figure out which range in $x$ is probed?

difficult!

easy example:

$x$-range spreads out significantly

depends on unknown $\Delta g$

complication: possible oscillations obscure $\langle x \rangle$

$pp \rightarrow \pi X$

$d\Delta \sigma / dp_T \ d\log_{10} x$

$p_T = 2.5 \text{ GeV}$
data ↔ x-range probed

a closer look:
diff. subprocesses populate different x-ranges

\[ \Delta g(\mu) = 0.24 \]
\[ x \approx 0.04 \pm 0.23 \]

**gg vs. qg** interplay explains all:

- large pos. \( \Delta g \) → pronounced gg peak
- small pos. \( \Delta g \) → double peak
- not too large neg. \( \Delta g \) → oscillations

estimates of \( x^\pm dx \) very difficult w/o knowing \( \Delta g \)
can one assume that $\Delta \Sigma$ is known?

**NO**, it is misleading to extract only $\Delta g$ w/o refitting the quarks:

considerable variation of quark singlet at input

constraint from DIS

- $x \Delta \Sigma$
- $x \Delta g$
MC based methods

$\Delta g$ extraction through signal/background separation based on MC

e.g., $lp \to HX \quad A_{LL} = \frac{\sum \Delta f \otimes d\Delta \hat{\sigma} \otimes D_c}{\sum f \otimes d\hat{\sigma} \otimes D_c} \times \frac{\sigma_{\gamma g} \Delta g \Delta \hat{\sigma}_{\gamma g}}{\sigma_{\text{tot}} g \hat{\sigma}_{\gamma g}} + A_{LL}^{\text{backgr.}}$

“fractions” from MC

MC crucial to model experiment but cannot replace a full global analysis:

- requires kind of “mean-value” theorem as

\[ \Delta g \otimes d\Delta \hat{\sigma} \otimes D_c \neq \sigma_{\gamma g} \Delta g \Delta \hat{\sigma}_{\gamma g} \]

(\text{also note that } \langle \hat{A}_{\gamma g}(x) \rangle \neq \hat{A}_{\gamma g}(\langle x \rangle))

- $MC$ hadronization not compatible with collinear pQCD which defines pdfs

- $MC$ neither LO nor NLO (parton showers, ...)

\[ \text{in general, expect: } \Delta g(MC) \neq \Delta g(\text{pQCD analysis}) \]
global analysis

unpolarized pdfs: CTEQ, MRST
- gluon constrained by scaling-violations
- 2nd moment constrained (mom. sum)
- pp data only for fine-tuning pdfs

K-factor approx., etc. often reasonable

polarized pdfs: completely different situation!
- gluon largely unconstrained by existing DIS data
- no momentum sum rule; pol. pdfs can have nodes
- pp data determine Δg and other aspects of pdfs

full NLO global analysis mandatory; approximations often misleading
possible framework

**Task**: handle *exact* NLO expressions in global $\chi^2$ analysis

$\rightarrow$ computing time becomes excessive

**Idea**: use Mellin $n$-moments to get rid of slow multi-convolutions

$$h^n \equiv \int_0^1 dx \, x^{n-1} \, h(x) \quad \text{convolutions factorize} \quad (g \otimes h)^n = g^n h^n$$

**Example**: $pp \rightarrow \pi X$

$$d\Delta \sigma = \sum_{abc} \int \Delta f_a \, \Delta f_b \, d\Delta \tilde{\sigma}_{ab \rightarrow cX} \, D_c \, dx_a dx_b dz_c$$

express pdfs by their Mellin inverses

$$\frac{1}{2\pi i} \int_{C_n} dn \, x_a^{-n} \, \Delta f_a^n$$

$$\frac{1}{2\pi i} \int_{C_m} dm \, x_b^{-m} \, \Delta f_b^m$$

$$= \frac{1}{(2\pi i)^2} \sum_{abc} \int_{C_n} dn \int_{C_m} dm \, \Delta f_a^n \, \Delta f_b^m \int x_a^{-n} x_b^{-m} \, d\Delta \tilde{\sigma}_{ab \rightarrow cX} \, D_c \, dx_a dx_b dz_c \equiv d\Delta \tilde{\sigma}_{ab \rightarrow cX}(n,m)$$

can be *pre-calculated* on grids!
possible framework

applicability: any process; tested for $pp\rightarrow\gamma X$, $pp\rightarrow\pi X$, $pp\rightarrow\text{jet} X$, ...

precision: $64 \times 64$ grids sufficient for less than 0.5% deviation

performance:

"before": typ. NLO code $O(1-2 \text{ min/pt.})$
too slow for fitting w/o approximations

"after Mellin tune-up": bullet-train performance
100 evaluations of x-sec takes a few seconds
ideal tool for multidim. fitting beyond LO
**example: “GRSV”+pp data**

\[ A_{LL} \] for several trial gluons:

\[
\begin{align*}
A^\pi_{LL} & \quad \text{NLO} \\
\Delta g = g & \quad \text{DIS on } \text{ly: } \Delta g \text{ unconstrained} \\
\Delta g = 0.7 & \quad + \text{pp data: large pos. } \Delta g \text{ disfavored} \\
\Delta g = 0.6 & \\
\Delta g = 0.45 & \\
\Delta g = 0.3 & \\
\Delta g = g & \\
\Delta g = 0 & \\
\Delta g = -1.05 & \\
\end{align*}
\]

\[ \chi^2 \quad (\text{similar for STAR jets}) \]

- DIS only: \( \Delta g \) unconstrained
- + pp data: large pos. \( \Delta g \) disfavored

**long to-do list:**

- DIS/SIDIS sets not up-to-date
- pdf uncertainties
latest AAC analysis

**latest AAC analysis**

Hirai, Kumano, Saito

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**again:**

- DIS only: $\Delta g$ unconstrained
  - + pp data: large pos. $\Delta g$ disfavored

**other features:**

- no SIDIS so far; SU(3) sea
- pdf uncertainties: Hessian method

\[
[d\Delta f]^2 = \Delta \chi^2 \sum_{ij} \frac{\partial \Delta f}{\partial a_i} H_{ij}^{-1} \frac{\partial \Delta f}{\partial a_j}
\]

use $\Delta \chi^2 = 12.64$ for 1-$\sigma$ errors

- find tension in deuteron data
  - positive $\Delta g$ at large-$x$ ?
  - but: tension gone in latest data sets !!

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**effect of positive $\Delta g$**
analysis by Sassot et al.

- full NLO global analysis of DIS + SIDIS data (plus $A_{LL}$ for pions)

find:
- from DIS/SIDIS alone: $\Delta g \gtrsim 0$
- current pp data: no effect
- $\delta \Delta g$: Hessian method not applicable

other features:
- full flavor separation
- pdf uncert.: Lagrange multiplier

$$\Phi(\lambda_i, a_j) = \chi^2(a_j) + \sum_i \lambda_i O_i(a_j)$$

use $\Delta \chi^2 = 1$ or 2 (5) % variation
have to understand better what is going on in these fits!

- what drives the parameters

- small details may change results \( \rightarrow \) larger theor. errors?

- proposal: "default analysis" under well defined conditions to check methods/codes of different groups
  - successfully done for CTEQ/MRST/GRV/ ... some problems/bugs found and removed
  - not systematically done in polarized case yet

there are many other issues which need to be revisited...
- considerable uncertainty in unpolarized gluon density for $x \gtrsim 0.4$
  
  difficult to relate $\Delta g$ and $\Delta g/g$

- RHIC pp data of any help here? possible!
  but first we have to understand fragmentation much better
KKP frag. fcts. seem to work well for RHIC even at low scales (scale roughly set by $p_T$)
but at similar scales (and $z$) they fail to describe, e.g. HERMES multiplicities.

"Kretzer" does a better job here!

**in addition:** de Florian, Navarro, Sassot

flavor sep. strongly depends on fragmentation fcts. - weird!

[data taken from A. Hillenbrand's thesis; plot by Rodolfo Sassot]
to-do list:

theory:

up-to-date pdf analysis; understand what drives parameters

global analysis of fragmentation functions + uncertainties

started! K. Sudoh's talk; also de Florian, Sassot, MS, Vogelsang

calculate missing processes for RHIC: hh/hj/jj-correlations, ...

work in progress: Jäger, Owens, MS, Vogelsang

experiment:

\( \Delta g \): data at larger \( p_T \); other processes (photons, of course)

forward/central 2-hadron (or other) correlations

fragmentation: incl. spectra for \( \pi^+ \), \( \pi^- \), kaons, eta, ...
extra slides
$\Delta g$ “semantics”

we always talk about things like “large $\Delta g$” - but what does it mean?

we should probably compare with $\frac{1}{2}$ - the spin of the proton

pion data: $-0.9 \lesssim \Delta g(\text{input}) \lesssim 0.5$

still pretty large!!

"anomaly inspired" $\Delta g$'s are really large: $\Delta g(\text{input}) \gtrsim 1.5$

(but they are now excluded)

beware of pQCD evolution: small $\Delta g$ can turn asymptotically into large $\Delta g$

however, only $J_g = \Delta g + L_g$ is gauge-invariant

$J_i$
### avail. NLO results

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Dom. partonic process</th>
<th>probes</th>
<th>LO Feynman diagram</th>
</tr>
</thead>
</table>
| $\bar{p}p \rightarrow \pi + X$  
[61, 62] | $\bar{g}g \rightarrow gg$  
$\bar{g}g \rightarrow gg$ | $\Delta g$ | |
| $\bar{p}p \rightarrow j_{\text{et}}(s) + X$  
[71, 72] | $\bar{g}g \rightarrow gg$  
$\bar{g}g \rightarrow gg$ | $\Delta g$ | (as above) |
| $\bar{p}p \rightarrow \gamma + X$  
$\bar{p}p \rightarrow \gamma + \text{jet} + X$  
$\bar{p}p \rightarrow \gamma\gamma + X$  
[67, 73, 74, 75, 76] | $\bar{g}g \rightarrow \gamma q$  
$\bar{g}g \rightarrow \gamma q$  
$\bar{q}\bar{q} \rightarrow \gamma\gamma$ | $\Delta g$  
$\Delta g$  
$\Delta q, \Delta \bar{q}$ | |
| $\bar{p}p \rightarrow DX, BX$  
[77] | $\bar{g}g \rightarrow c\bar{c}, b\bar{b}$ | $\Delta g$ | |
| $\bar{p}p \rightarrow \mu^+\mu^- X$  
(Drell-Yan)  
[78, 79, 80] | $\bar{q}\bar{q} \rightarrow \gamma^* \rightarrow \mu^+\mu^-$ | $\Delta q, \Delta \bar{q}$ | |
| $\bar{p}p \rightarrow (Z^0, W^\pm) X$  
$\bar{p}p \rightarrow (Z^0, W^\pm) X$  
[78] | $\bar{q}\bar{q} \rightarrow Z^0, \bar{q}'\bar{q}' \rightarrow W^\pm$  
$q\bar{q} \rightarrow W^\pm, q'\bar{q}' \rightarrow W^\pm$ | $\Delta q, \Delta \bar{q}$ | |

**RHIC spin:**

$$\Delta g, \Delta \bar{q}$$

**NLO for polarized lepton-proton scattering (COMPASS, HERMES):**

only photoproduction of single-hadrons and heavy flavors available

Jäger, MS, Vogelsang

Bojak, MS
Mellin technique

\[ d\Delta\tilde{\sigma}_{ab\rightarrow cX}(n,m) \]

- contains all time-consuming integrations
- calculated once and forever before the fit
- stored in large n×m grids

\[ \int_{C_n} dn \int_{C_m} dm \]

- fast numerical Mellin inverse in complex n,m plane
- exponential fall-off of \( x^{-n}, x^{-m} \)
- along contour optimal
- integration = summation in n,m

\[ \Delta f^n_a \Delta f^m_b \]

- Mellin moments of ansatz for pdfs in \( x \)-space,
  e.g., \( f_a(x,\mu_0) = N x^{\alpha} (1-x)^{\beta} \)
- parameters determined in standard \( \chi^2 \) analysis
indicators that $\Delta g$ is not large and positive at $x$ around $0.03 \div 0.2$...

...but it is still a very long way to the entire 1$^{st}$ moment.

Small-$x$ range requires 500 GeV RHIC data AND a polarized ep-collider like eRHIC.