相対論的重イオン衝突
初期時空発展とスケーリング

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based on Fukushima, Gelis, NPA874, 108 (2012) [arXiv:1106.1396]

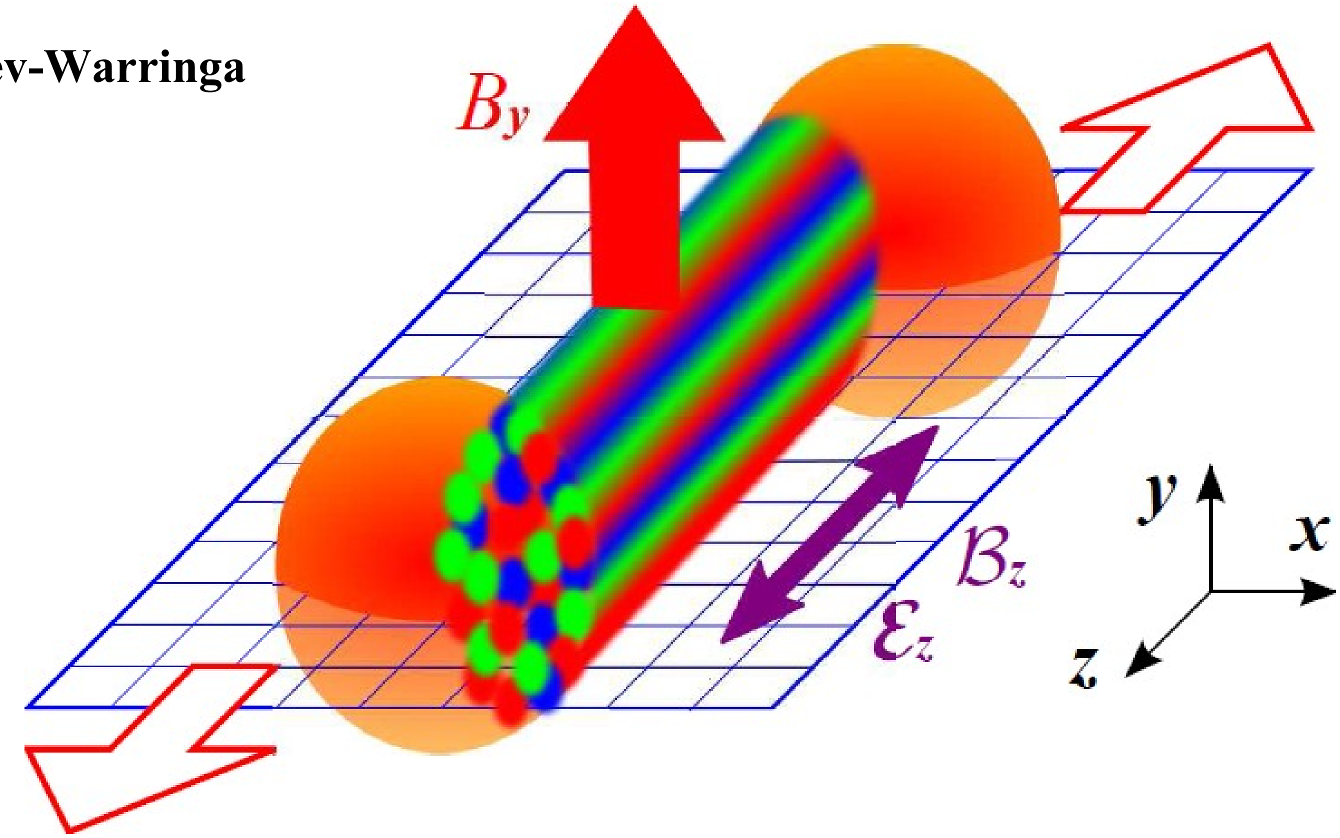
Heavy-Ion Collision

What is the initial condition in the HIC?

Negative longitudinal pressure

Topological charge density → Chiral magnetic effect

KF-Kharzeev-Warringa



Quantum Fluctuations at High Energy

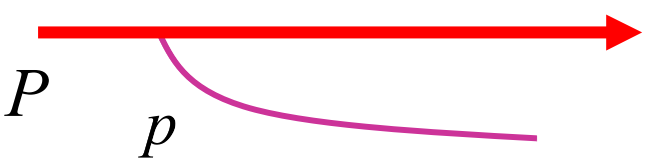
Frames with infinite P^+

$$x \approx \frac{p^+}{P^+}$$

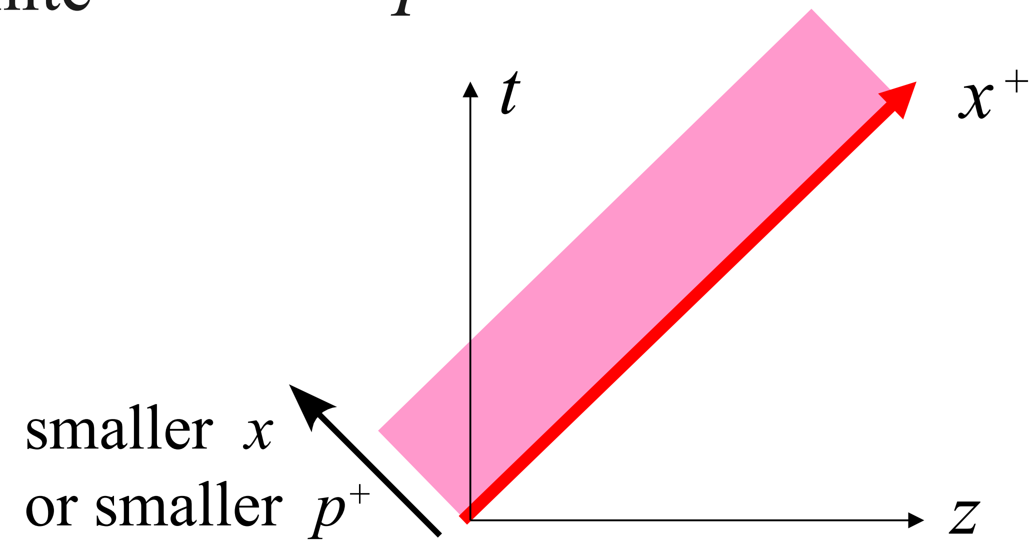
$E \approx P^z$ and P^+ are infinite

x^-: Lorentz contract

x^+: time dilation



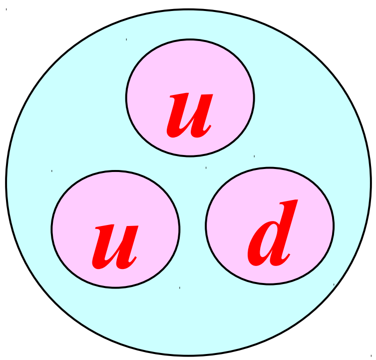
$$\Delta E \sim p^- = \frac{p_\perp^2}{2xP^+}$$



Partons live long in the IMFs → Parton Picture
(Quantum fluctuations ~ Real particles)

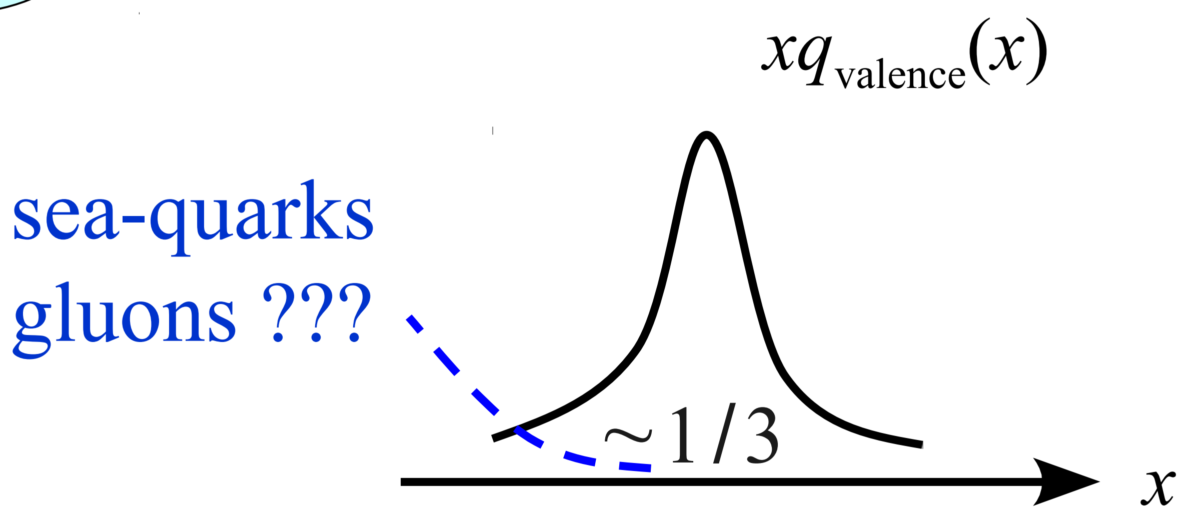
Parton Distribution Function

Valence and Sea Quarks and Gluons



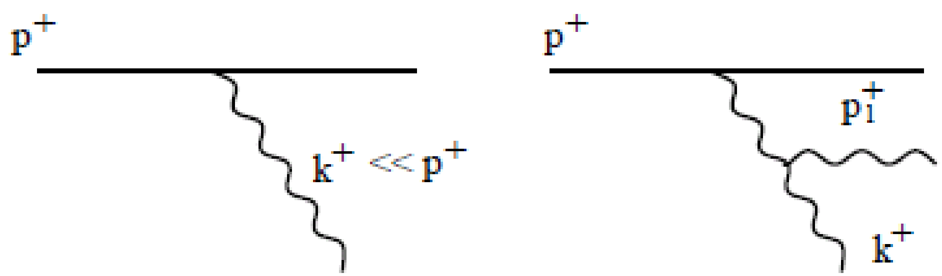
proton

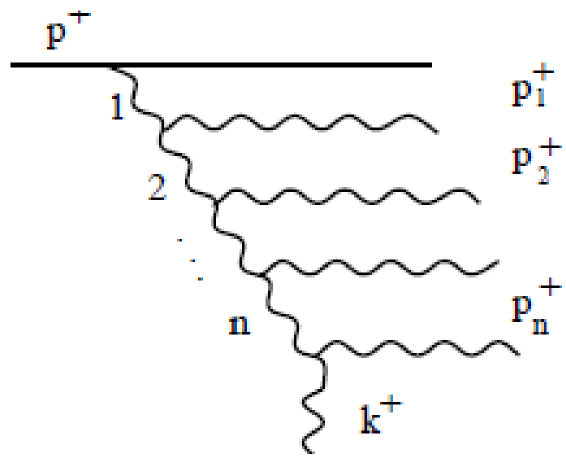
valence quark constituent



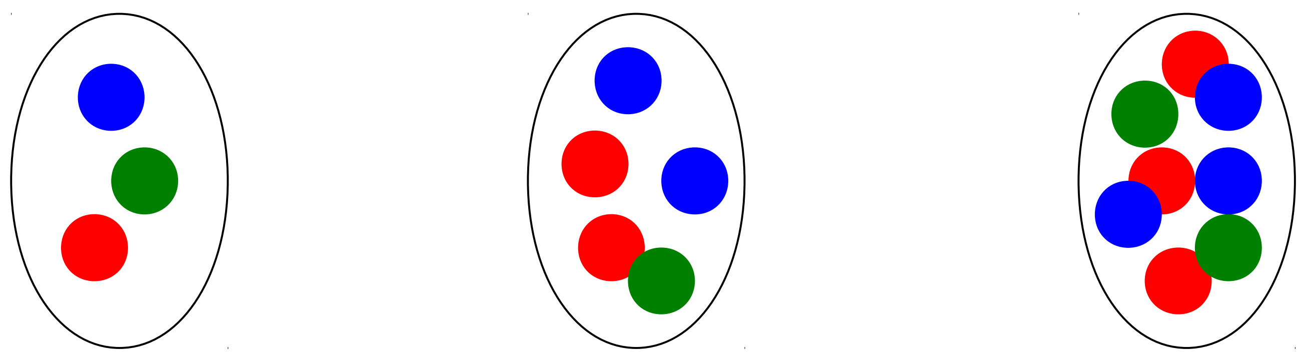
BFKL – Higher Energy with Fixed Q²

Gluon increases with (nearly) fixed transverse area





Graphical representation

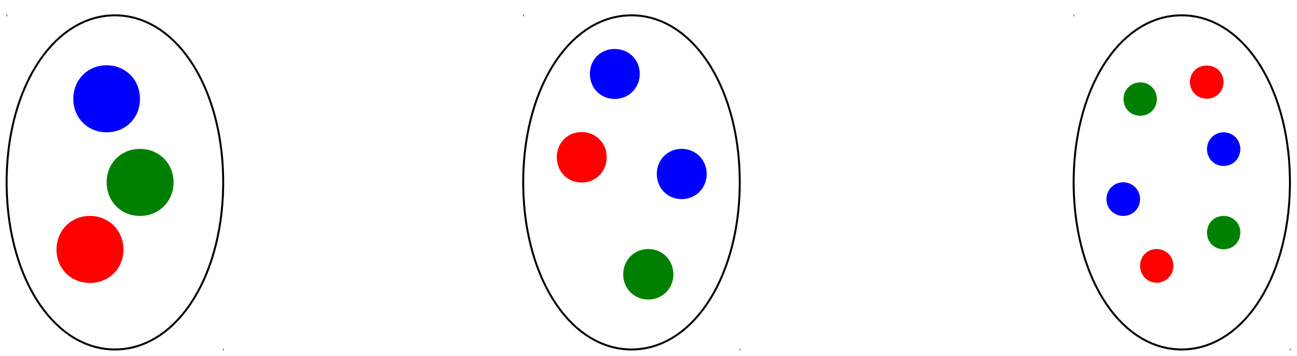


Higher Energy → Dense Gluon Matter

DGLAP – Larger Q^2 with Fixed Energy

Gluon slowly increases with decreasing area

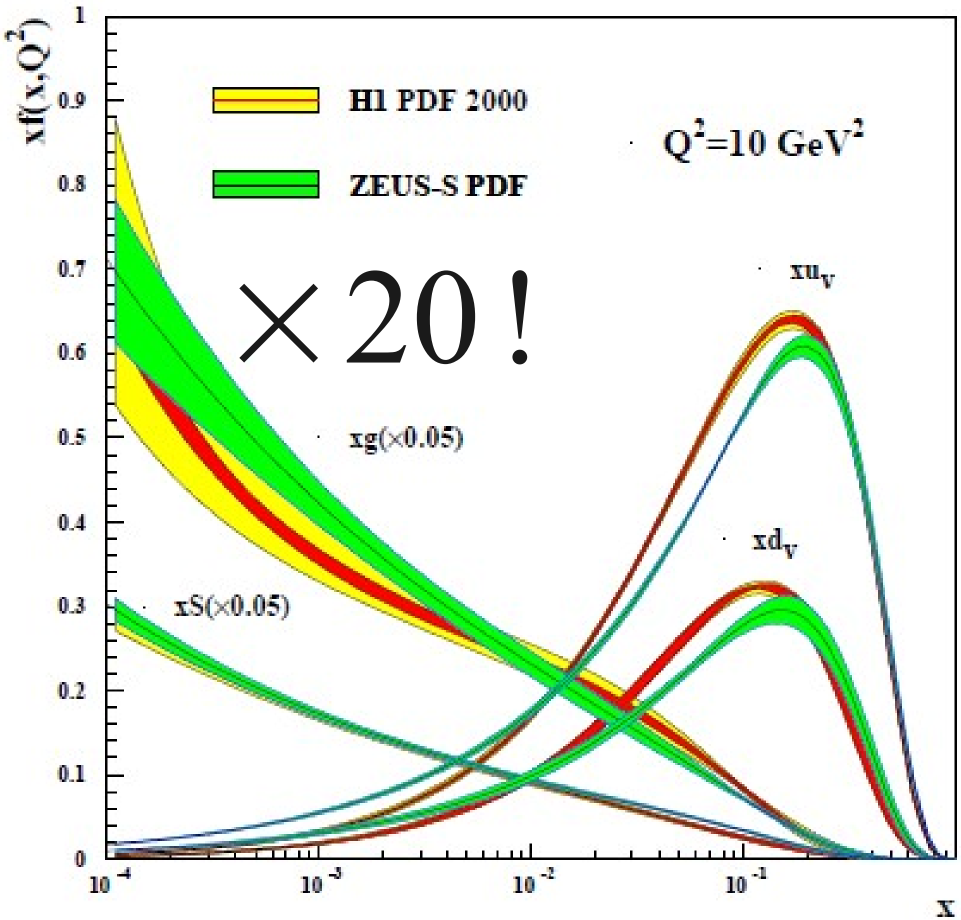
Graphical representation



large *Q* → Dilute Gluon Matter

Data from HERA

Quantum Evolution of PDFs at fixed Q^2



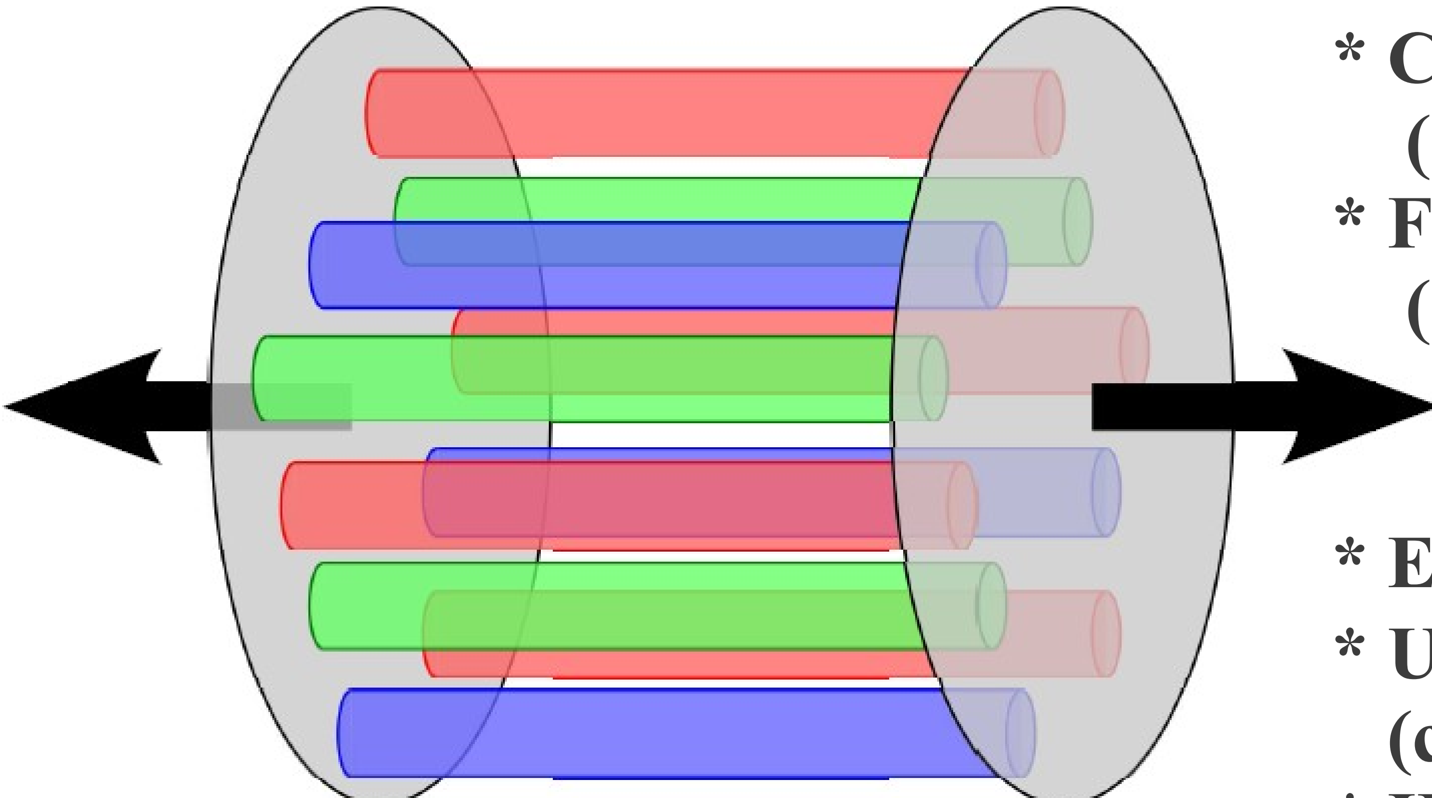
As the energy goes larger, **gluon** is becoming dominant.

High energy (large s and small t) processes are dominated by abundant **gluons**.

Coherent Gluon Fields

Glasma = (Color) Glass + Plasma

Intuitive Picture of Glasma



* Boost Invariance
* Coherent Fields
  (amp. ~ $1/g$)
* Flux Tube
  (size ~ $1/Q_s$)

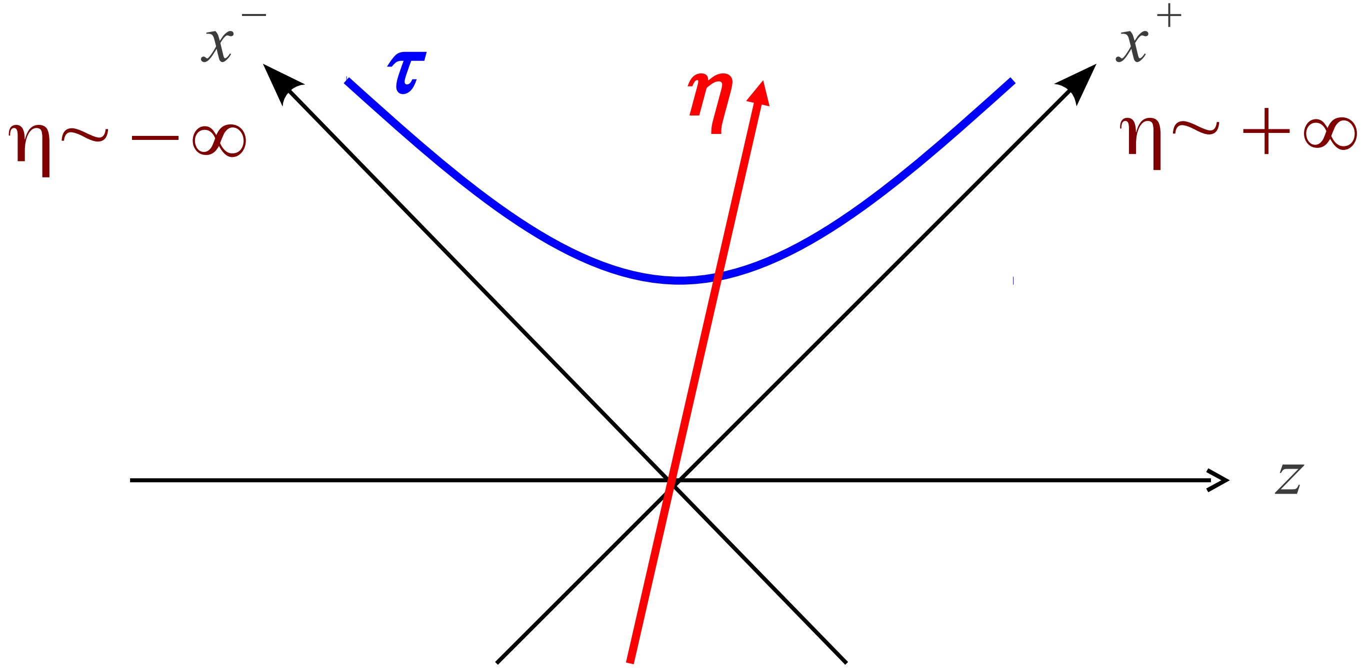
* Expanding
* Unstable in η
  (cascade to UV)
* Hydro Input?

Initial Dynamics of the HIC
should be described by
the classical eom of the Yang-Mills theory
in the expanding coordinates

Bjorken (Expanding) Coordinates

Proper Time and (space-time) Rapidity

$$\tau = \sqrt{t^2 - z^2}\,, \qquad \eta = \frac{1}{2}\ln\left[\frac{t+z}{t-z}\right]$$



Formulations

Time Evolution

$$E^i = \tau \partial_\tau A_i , \qquad E^\eta = \tau^{-1} \partial_\tau A_\eta$$

$$\partial_\tau E^i = \tau^{-1} D_\eta F_{\eta i} + \tau D_j F_{ji}$$

$$\partial_\tau E^\eta = \tau^{-1} D_j F_{j\eta}$$

Classical Equations of Motion in the Bjorken Coordinates

in the “temporal” gauge $A_\tau = 0$

Ensemble Average

$$\langle\langle \mathcal{O}[A] \rangle\rangle_{\rho_t , \rho_p} \sim \int D\rho_t \, D\rho_p W_x[\rho_t] W_{x'}[\rho_p] \, \mathcal{O}[\mathcal{A}[\rho_t , \rho_p]]$$

Stress Tensor (Energy, Pressures)

Quantum fluctuations partially included in the initial state

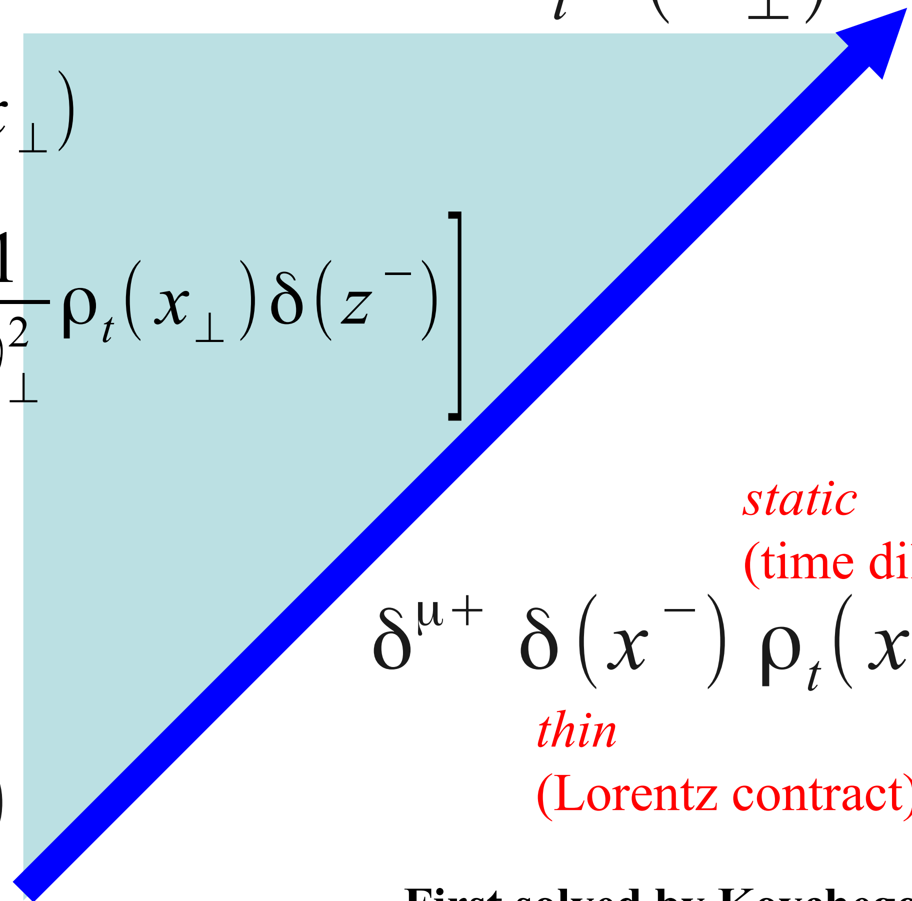
One Source Problem

One-source problem is solvable

$A^{+} = A^{-} = 0$ (gauge choice)

$$A_i = \alpha_i^{(1)} = -\frac{1}{ig} V(x_\perp)\,\partial_i V^\dagger(x_\perp)$$

$$V^{+}(x_\perp) = P\exp\left[-ig\int dz^{-}\,\frac{1}{\partial_\perp^2}\rho_t(x_\perp)\,\delta(z^{-})\right]$$



c.f. in EM

$\partial_\perp^2 \phi = -\rho$ (Poisson eq)

$\rightarrow \phi' = 0$ (Gauge trans)

$$\rightarrow A'_i = \frac{1}{ie} e^{ie\phi}\,\partial_i e^{-ie\phi} \quad (= -\partial_i \phi)$$

First solved by Kovchegov

Formulations

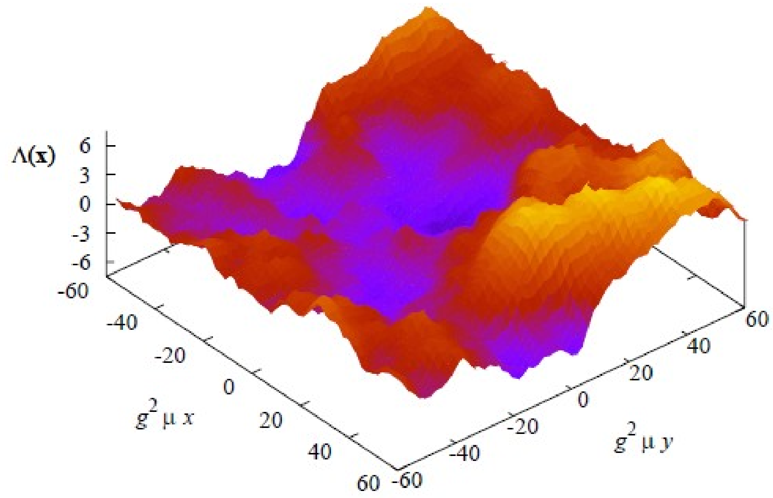
Initial Condition

MV Model

Color source distribution is Gaussian
(No spatial correlation in transverse direction)

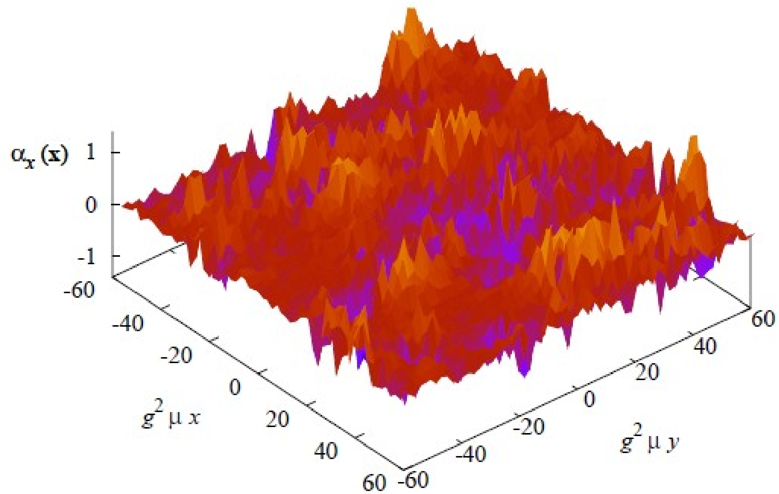
Solve the Poisson Eq

$$\partial_\perp^2 \Lambda^{(m)}(\boldsymbol{x}_\perp) = -\rho^{(m)}(\boldsymbol{x}_\perp)$$



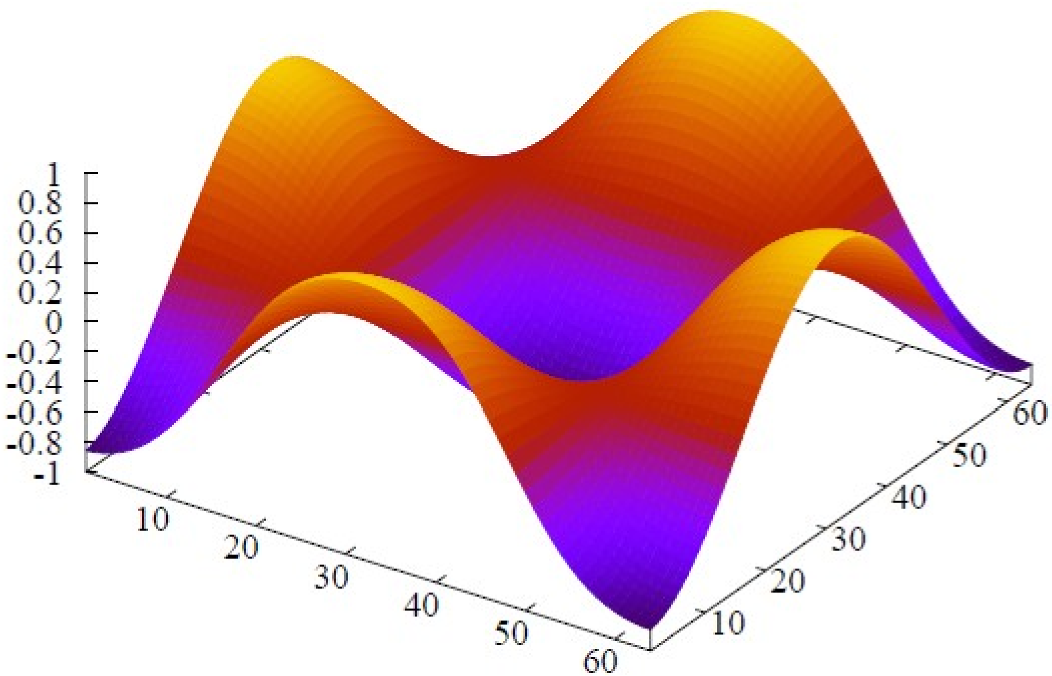
Gauge Configuration

$$e^{-ig\Lambda(\boldsymbol{x}_\perp)} e^{ig\Lambda(\boldsymbol{x}_\perp + \hat{i})} = \exp[-ig\alpha_i(\boldsymbol{x}_\perp)]$$



Example of Simulations

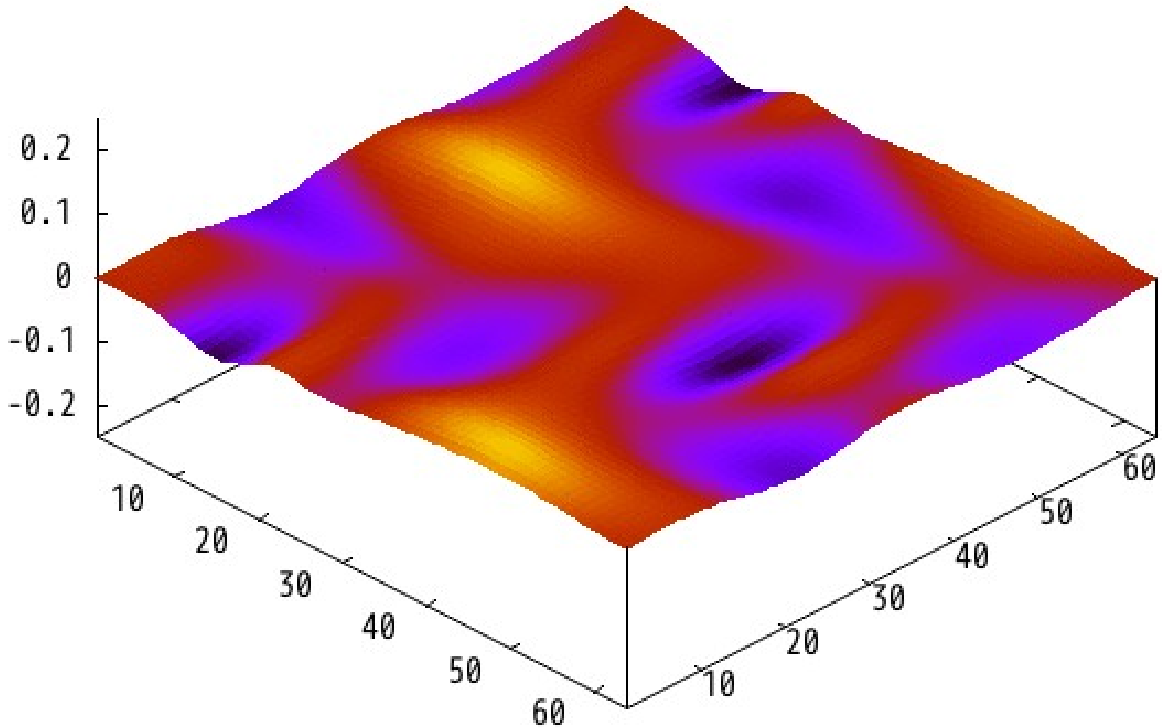
Demonstration with smooth source (one flux-tube)



$$\mathrm{tr}\left[\tau^1 U_1^{(1)}(x, y)\right]$$

Example of Simulations

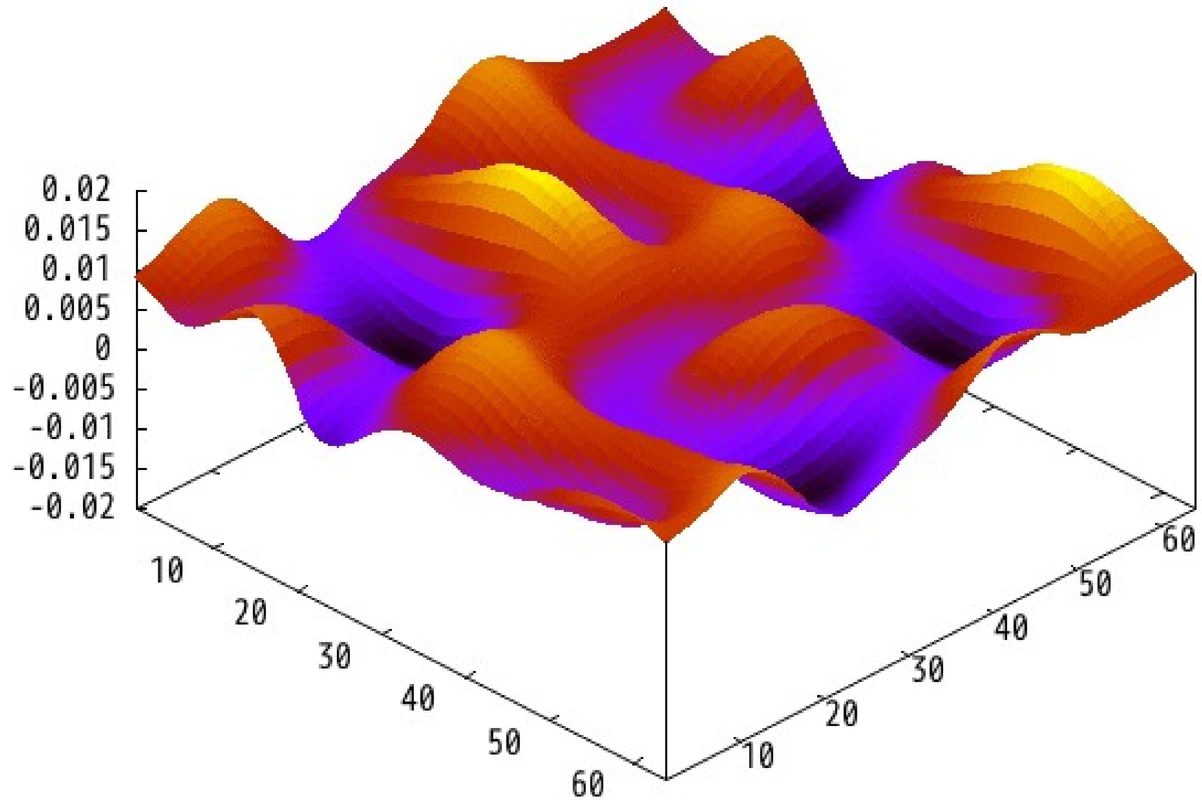
E_1



True electric field $\sim E_i / \tau$

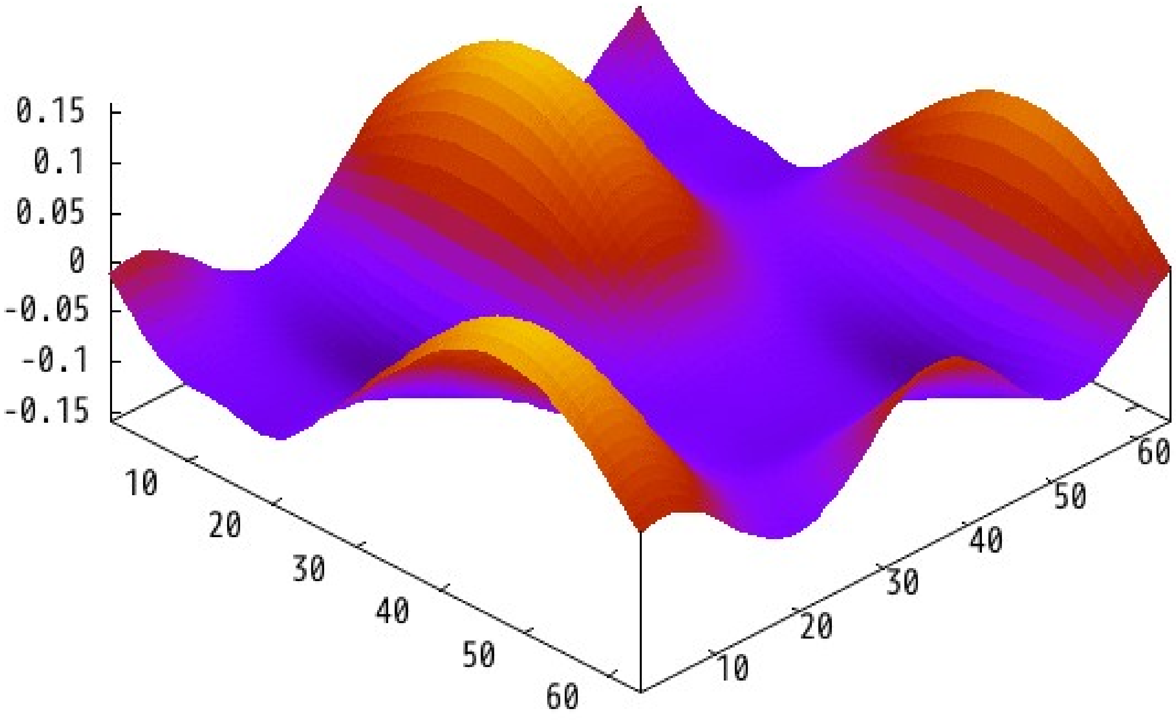
Example of Simulations

E_η



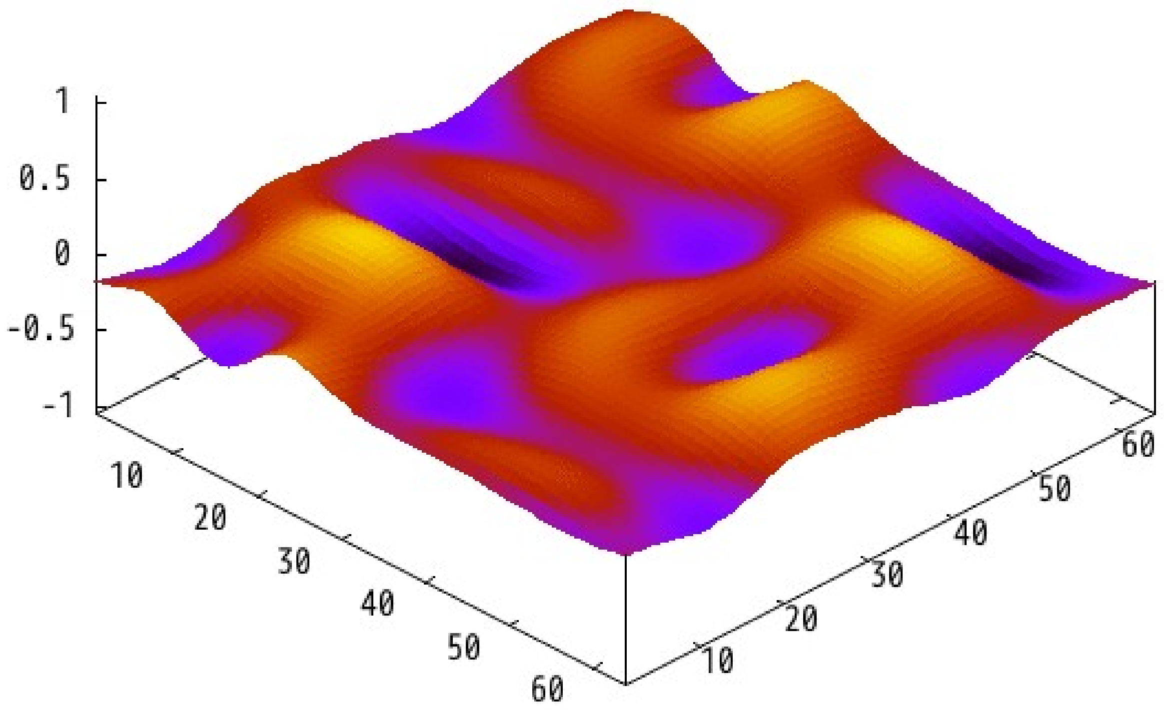
Example of Simulations

U_1



Example of Simulations

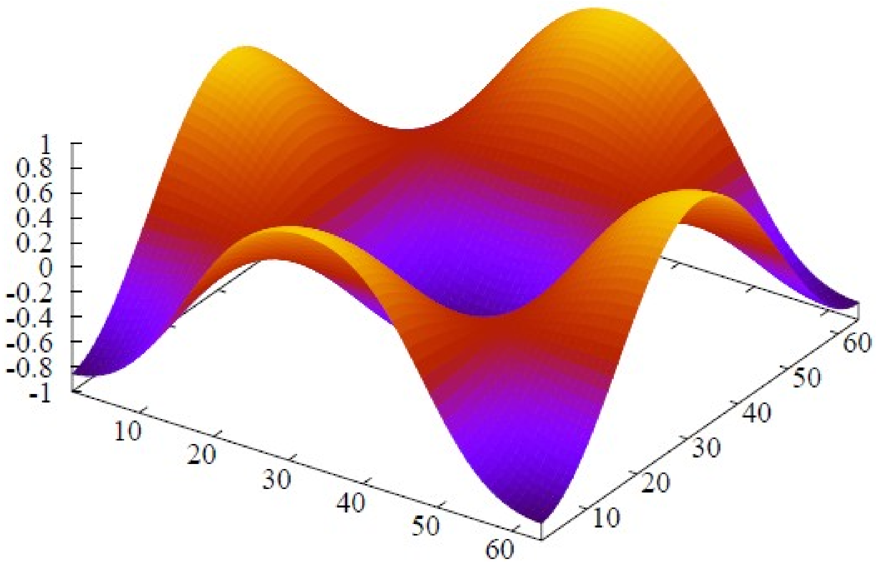
U_η

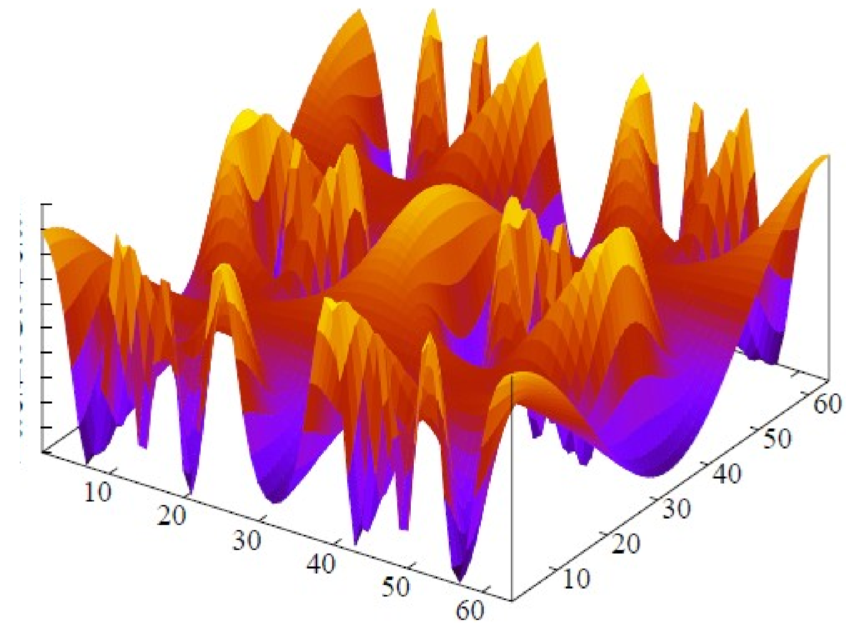


A_η , E_i fast modes (multiple time scales)

Large Amplitude Problem

Gauge fields as angle variables

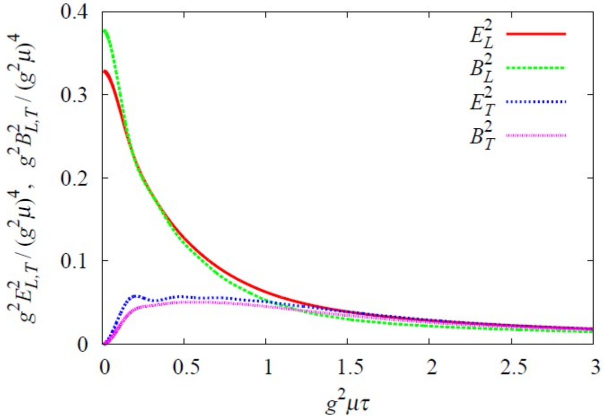


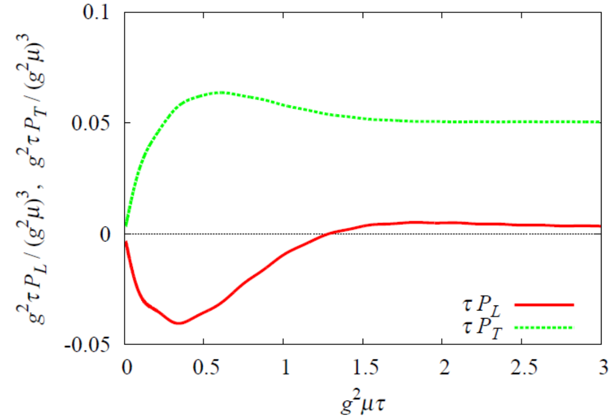


No problem if we use the continuum variables...
Why we stick to the link variables...?

Some Known Facts about Glasma Simulation

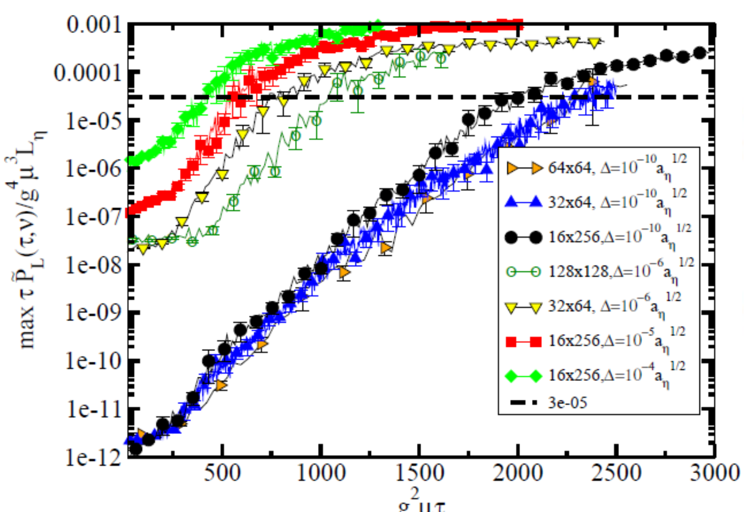
Free streaming after $1/Q_s$

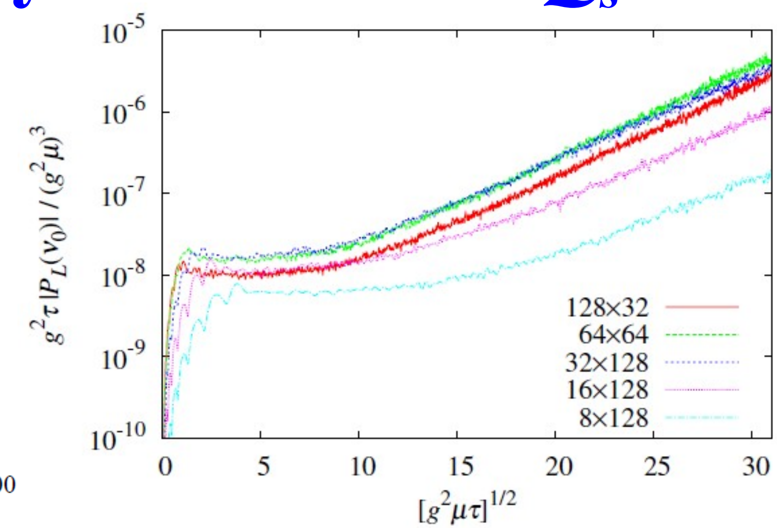




$$P_L \simeq 0$$

Glasma instability at $100\sim1000/Q_s$





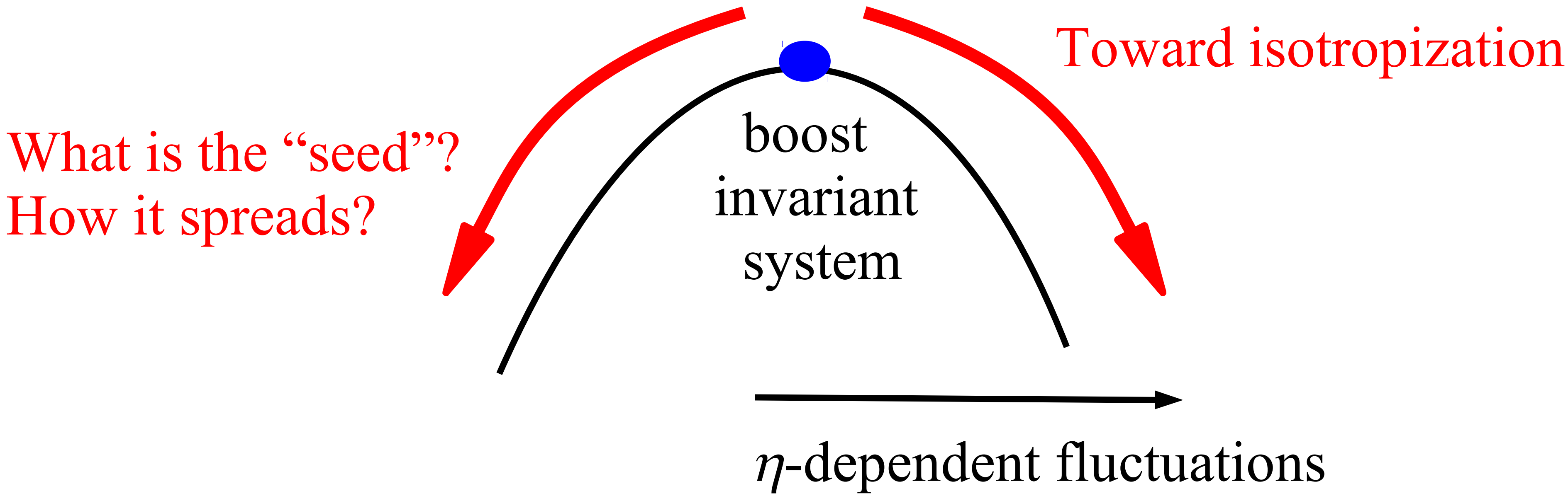
Instability ends when non-Abelianized
Then spectrum shows asymptotic scaling.

Romatschke-Venugopalan

Fukushima-Gelis

Boost Invariance Violation

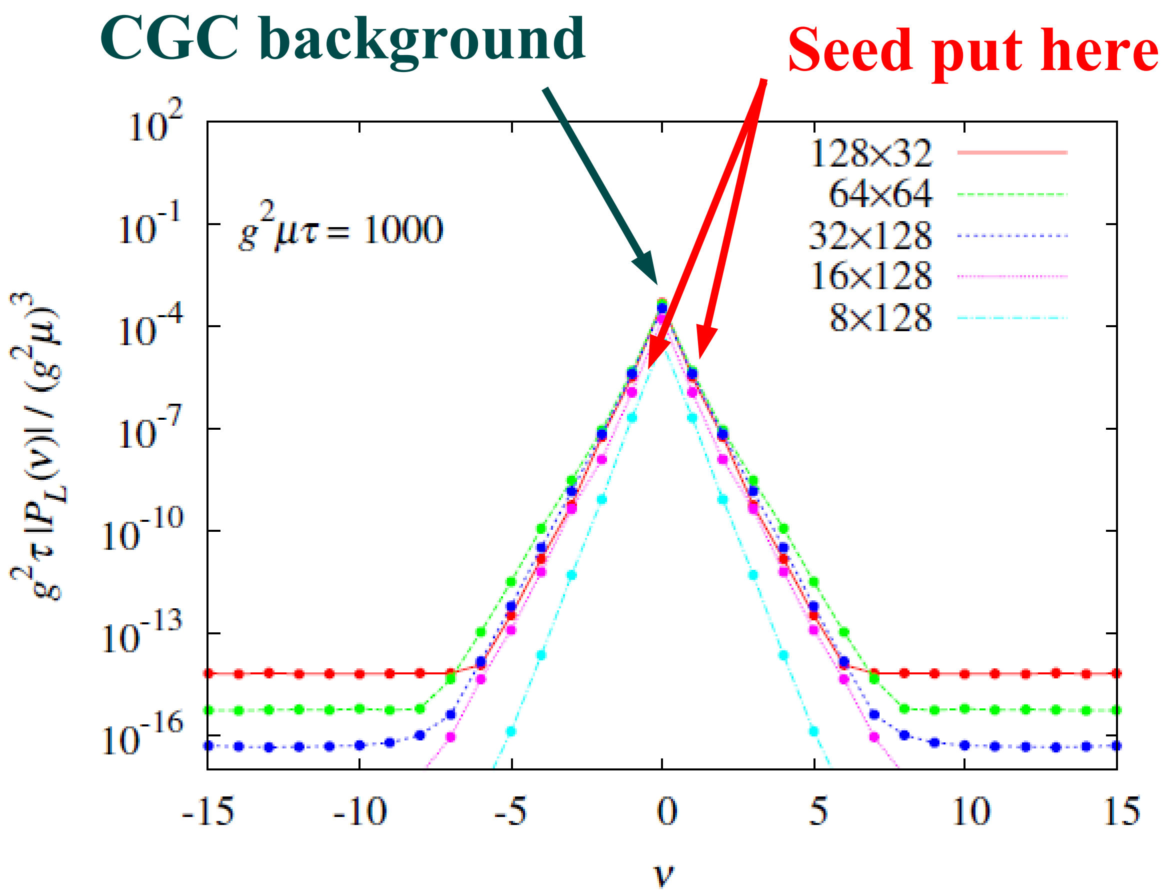
Boost-invariant Glasma sits on the top of the potential maximum (seemingly stable without any perturbation)



Isotropization does not necessarily mean thermalization.
If thermalized, the system must be isotropic.

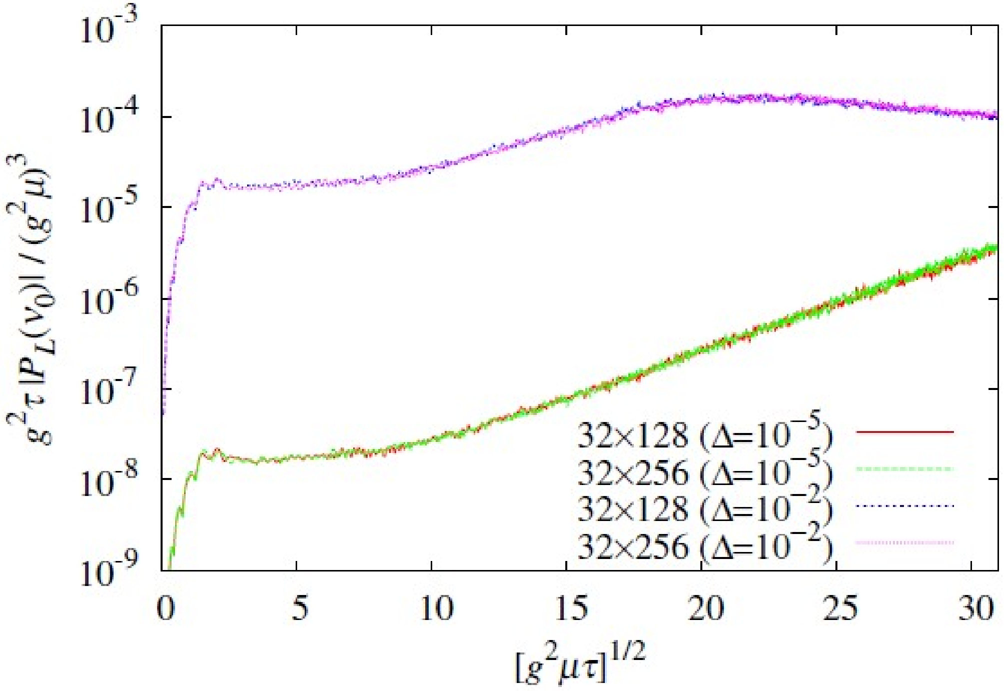
Mode Amplitudes

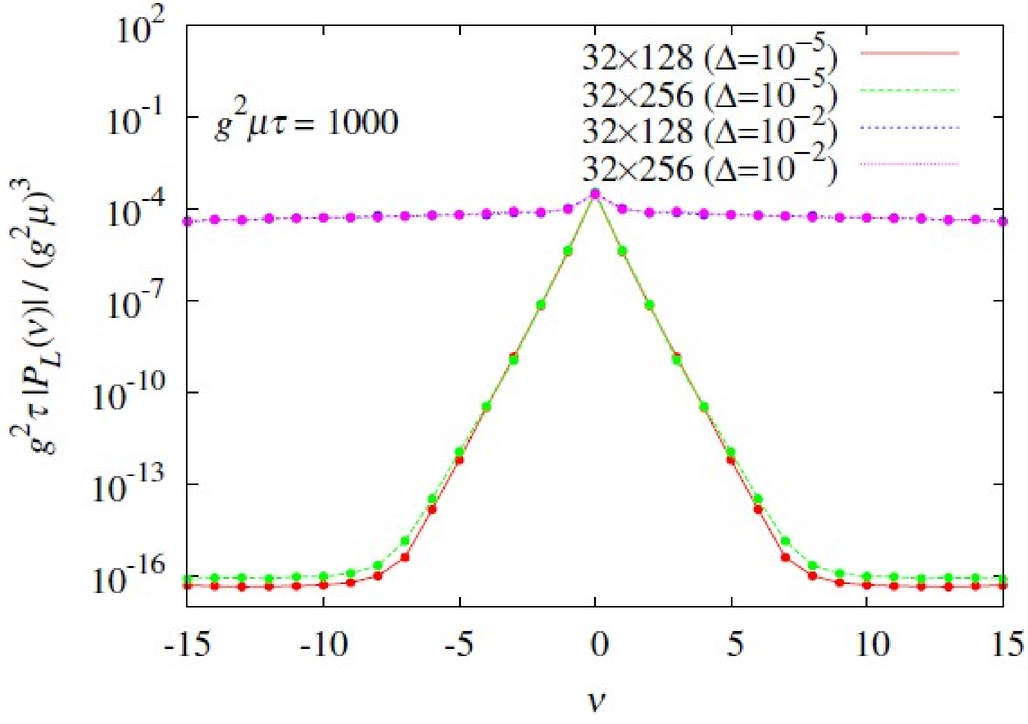
Instability spreads from lower to higher wave-number modes (conjugate to rapidity)



Longitudinal Size Dependence

Completely free from artifacts

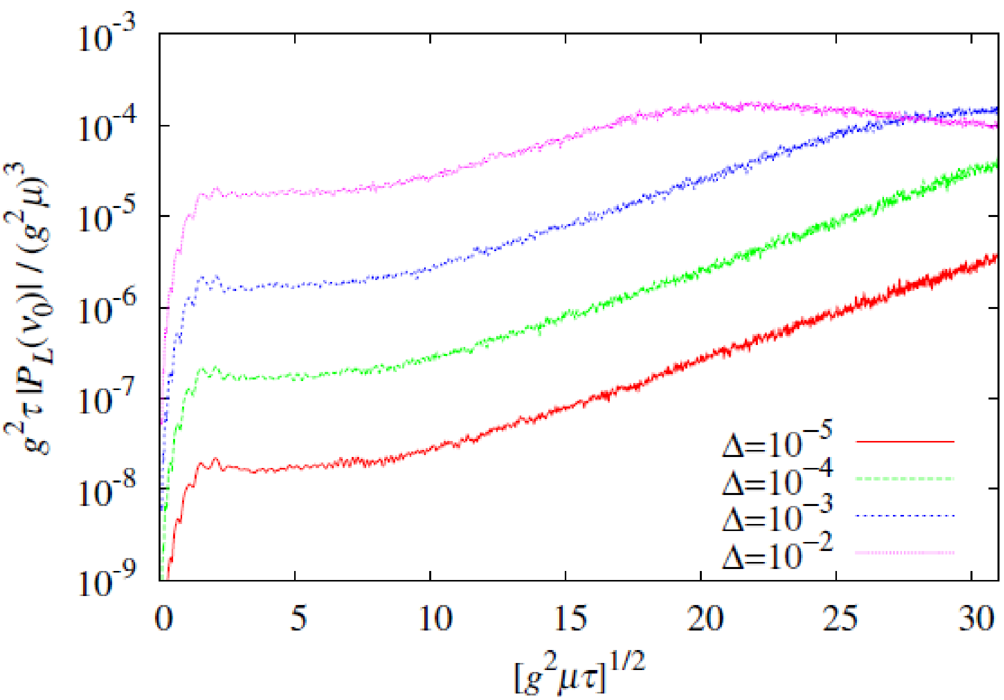


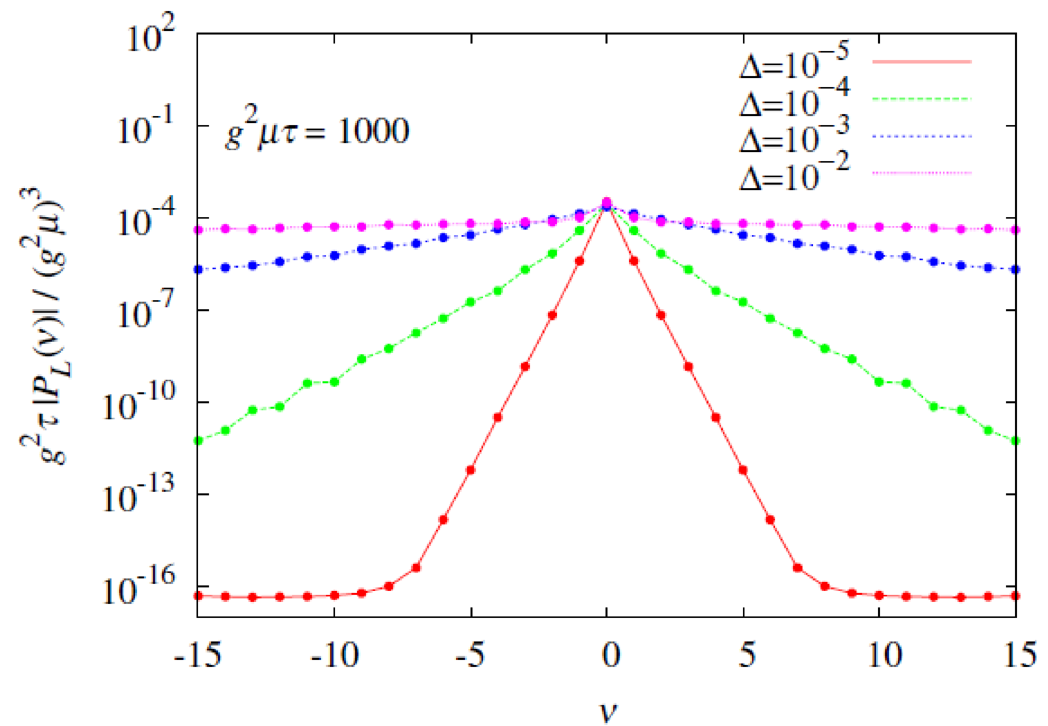


No dependence on the longitudinal size as it should not

Seed Magnitude Dependence

η-fluctuations are in the linear regime



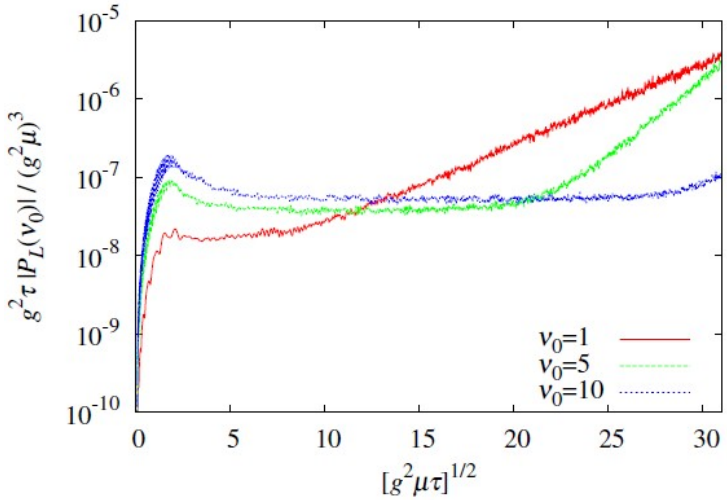


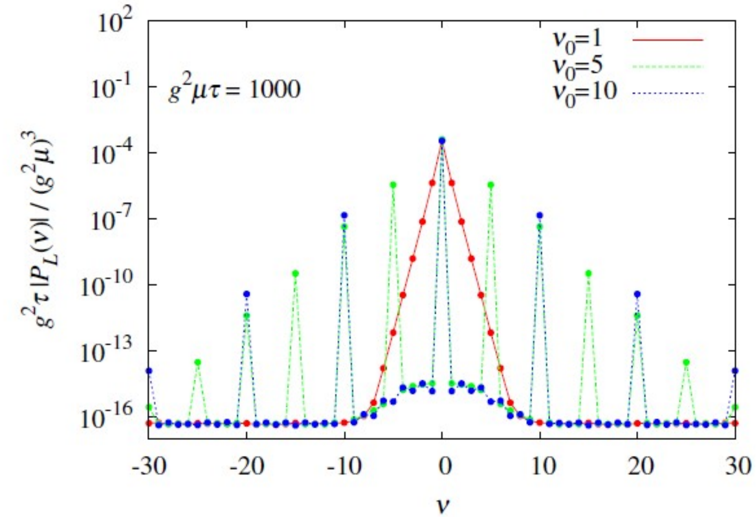
Simply proportional to the seed magnitude

How to fix it in principle?

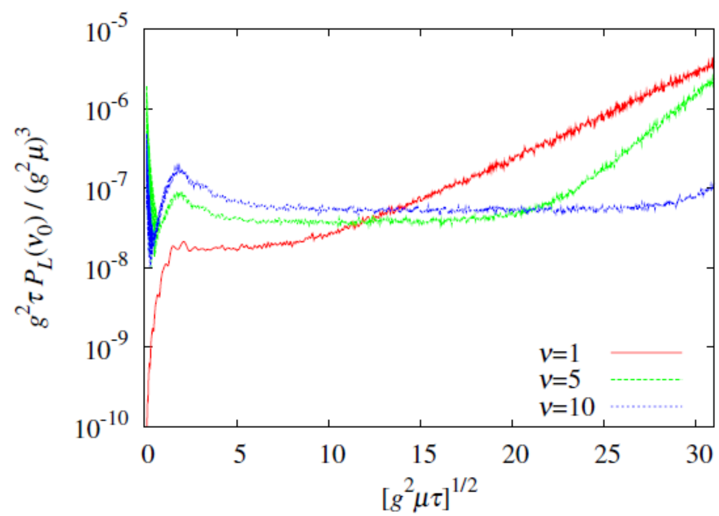
Fluctuations with Multiple Wave-Numbers

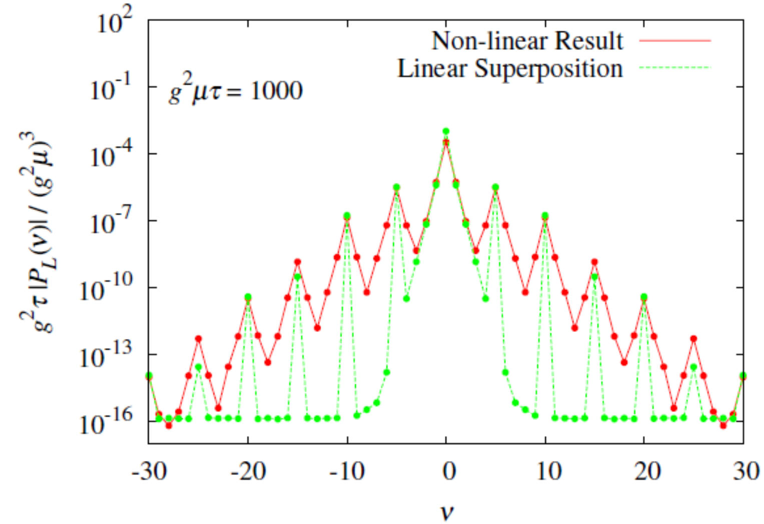
Individual simulations





Simultaneous simulations



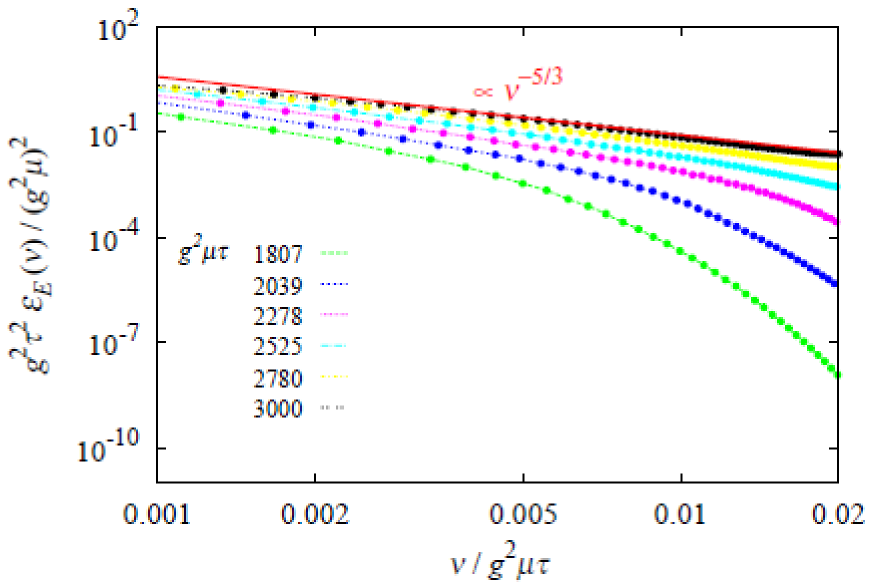


Energy Cascade Behavior?

$$\varepsilon_E = \int \frac{d\nu}{2\pi} \varepsilon_E(\nu)$$

This is not gauge inv.

$$\varepsilon_E(\nu) \equiv \left\langle \mathrm{tr}\left[E^{\eta a}(-\nu) E^{\eta a}(\nu) + \tau^{-2}\left(E^{ia}(-\nu) E^{ia}(\nu) \right) \right] \right\rangle$$



$$[\nu / c\tau] = l^{-1}$$

$$[V_\perp (c\tau)^2 \varepsilon_E(\nu)] = l^3 t^{-2}$$

$$[\psi] = l^2 t^{-3}$$

$$\tau^2 \varepsilon_E(\nu) \propto (\nu/\tau)^{-5/3}$$

Safe from gauge ambiguity $A_\tau = 0$ (gauge) $A_\eta \propto \tau^2 = 0$ at $\tau = 0$

Kolmogorov's Picture of a Turbulence

Dimensional Analysis

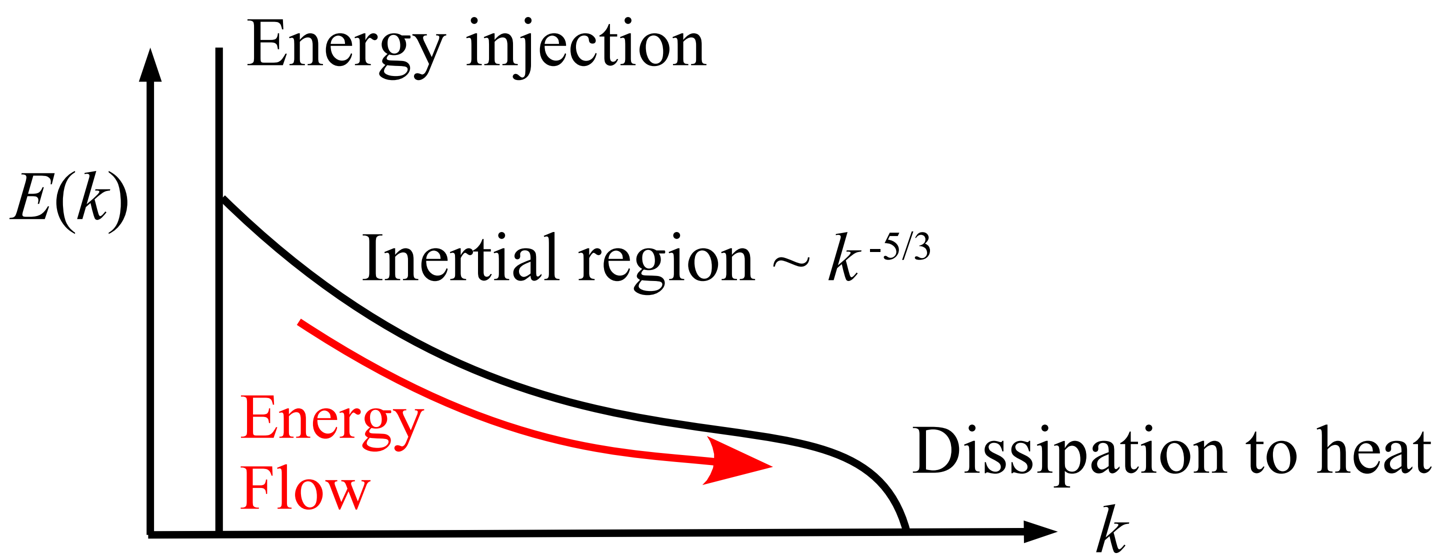
c.f. $[E]=l^2 t^{-2}$

wave-number: $[k]=l^{-1}$

Fourier component of the energy: $[E(k)]=l^3 t^{-2}$

energy flow rate: $[\psi]=l^2 t^{-3}$

$$E(k)\propto k^{\alpha}\psi^{\beta} \quad \Rightarrow \quad \alpha=-5/3,\ \beta=2/3$$



c.f. bottom-up thermalization

c.f. Kolmogorov's Hypothesis

Scaling functions of ν (viscosity) and ψ (energy flow rate) in a homogeneous isotropic turbulence

Kolmogorov Length Scale ~ $\eta = \nu^{3/4} \psi^{-1/4}$

Kolmogorov Time Scale ~ $\sigma = \nu^{1/2} \psi^{-1/2}$

Only dependence on ψ (in an inertial region up to the Kolmogorov length scale)

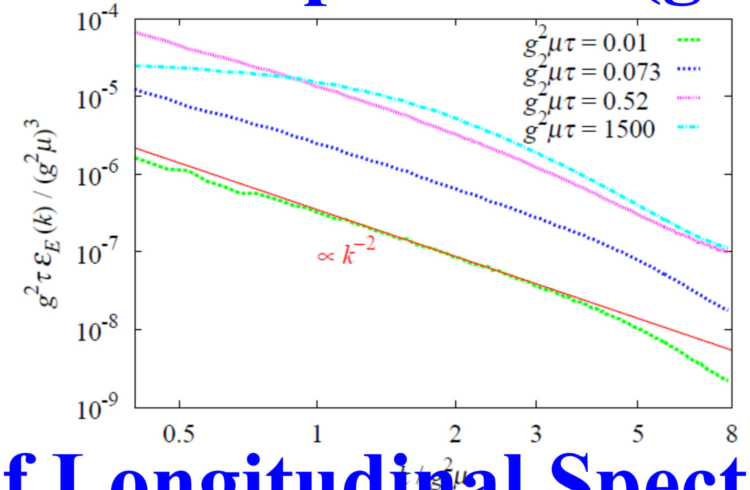
Velocity p-point Function

$$S_2(r) = C \psi^{2/3} r^{2/3}$$

$$S_p(r) = C_p \psi^{p/3} r^{p/3}$$

Evolution of Energy Spectra

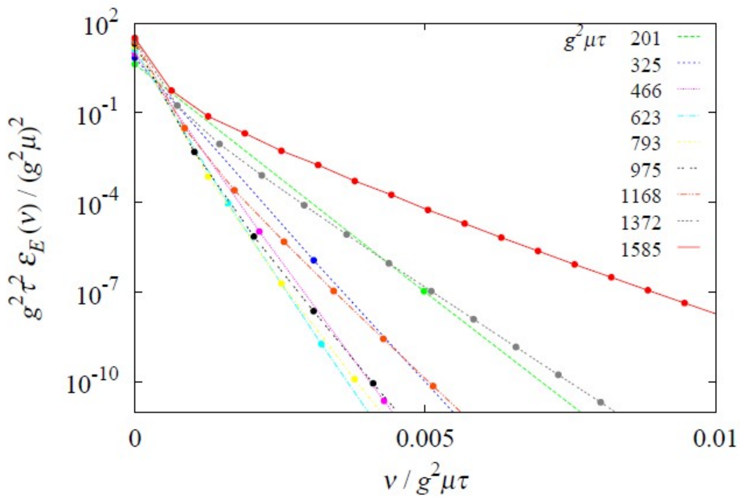
Initial Transverse Spectrum (gauge non-inv.)

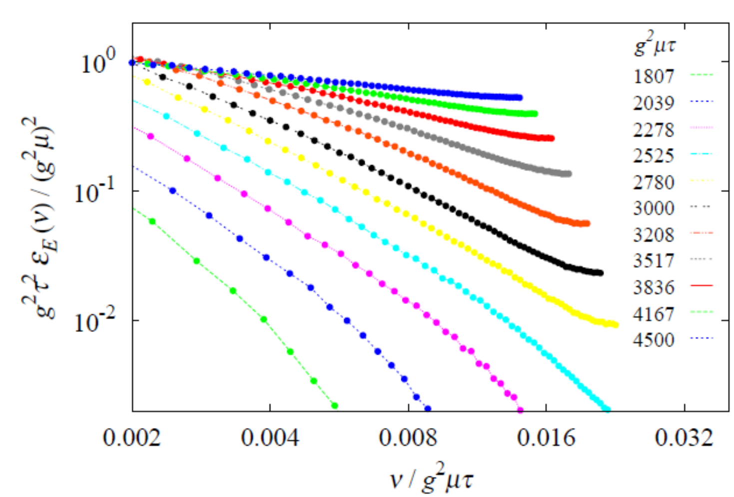


No Turbulence

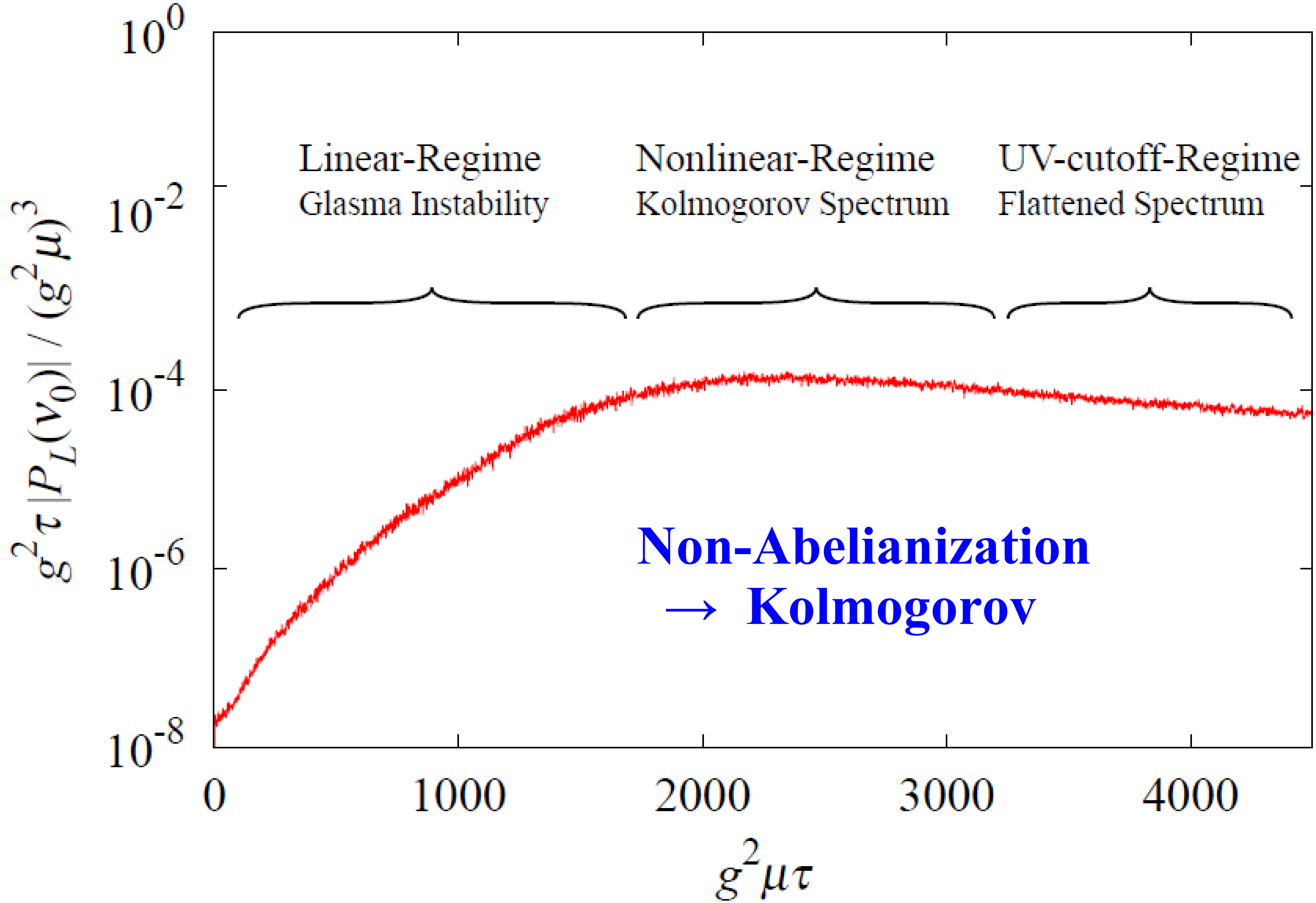
Evolution of Longitudinal Spectrum

Kolmogorov?





Three Regimes in the Evolution

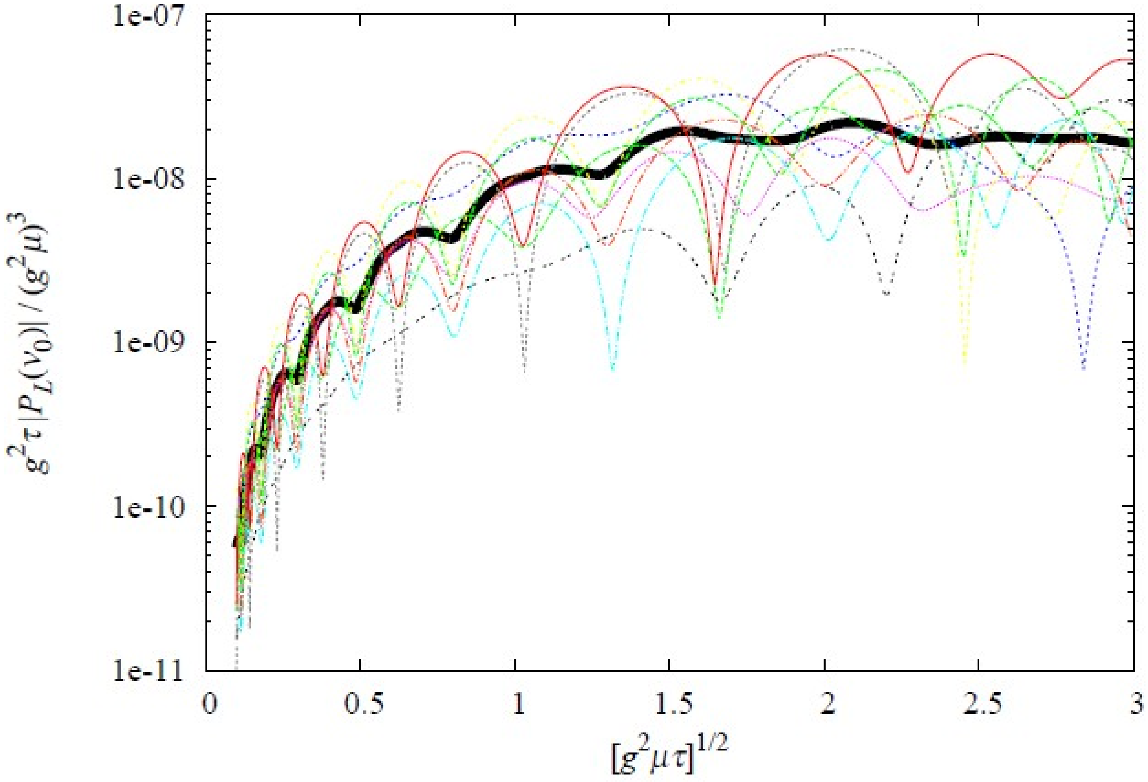


Future Extensions

- Evolution at a very initial stage of the collision (Glasma instability is still too slow...)
- How to demonstrate the scenario of the Bose-Einstein condensation in the Glasma simulation?
- Effects from the strong fields in the CGC initial condition – topology and particle production

Initial Rise

Still weak but seen till ~1/*Qs*

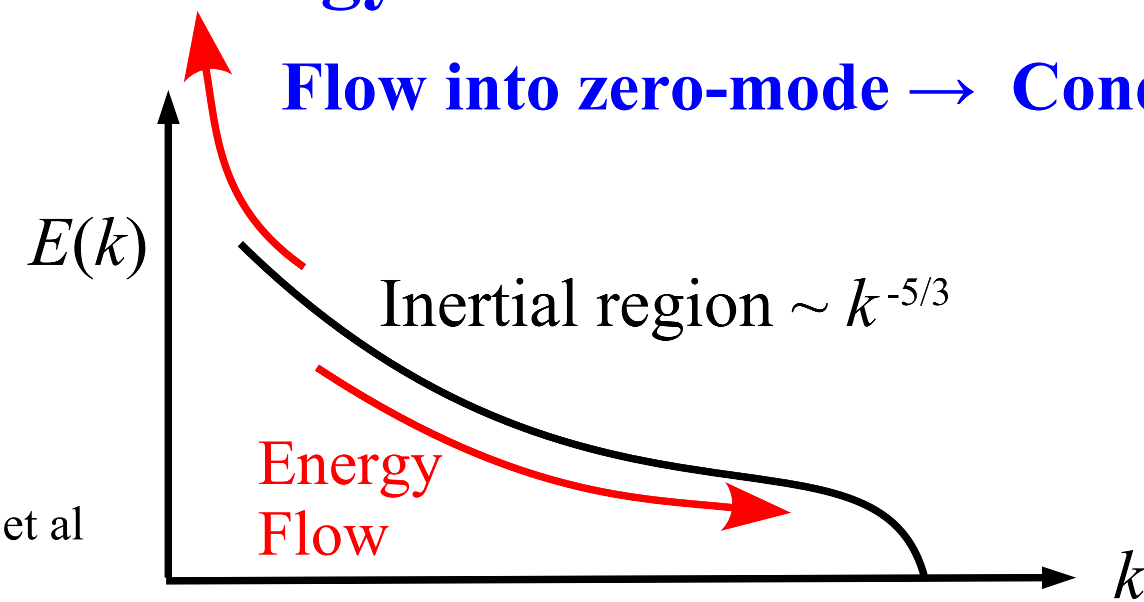


Implication? Wrong choice of initial configuration? Parametric Resonance? Chaos? (Kunihiro,Muller,Yamamoto,et al)

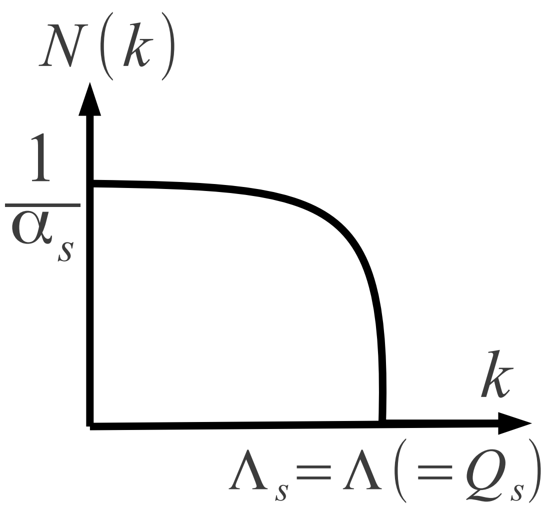
Bose-Einstein Condensation

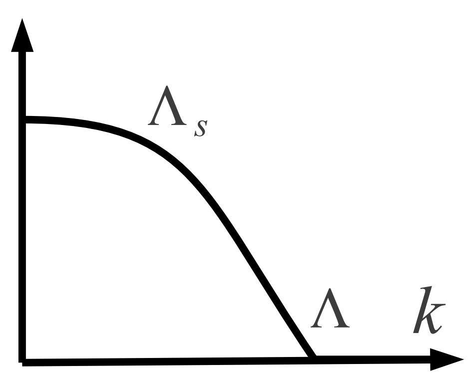
Backward Energy Flow

Flow into zero-mode → Condensate!



Blaizot,McLerran,et al
Berges et al





What happens in the expanding Glasma?

No conclusion, but lots of questions

- No one really understands what is going on in the heavy-ion collision at present...
- Really thermalization? If hydro works, it is sufficient, isn't it? Why thermalization needed?
- Glasma instability is too slow. What is missing??? Some people make huge efforts to formulate initial quantum fluctuations... no hope to cure this...
- Kolmogorov spectrum relevant to reality? Only academic thing in the simulation, or...?
- Bose-Einstein scenario is not so exotic as one might feel at the first glance, but...?