Classification of color superconductivity revisited,

or the fate of bad diquarks in dense quark medium

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References: Y. Fujimoto, arXiv:2508.19222 [hep-ph]; 2508.19728 [hep-ph]

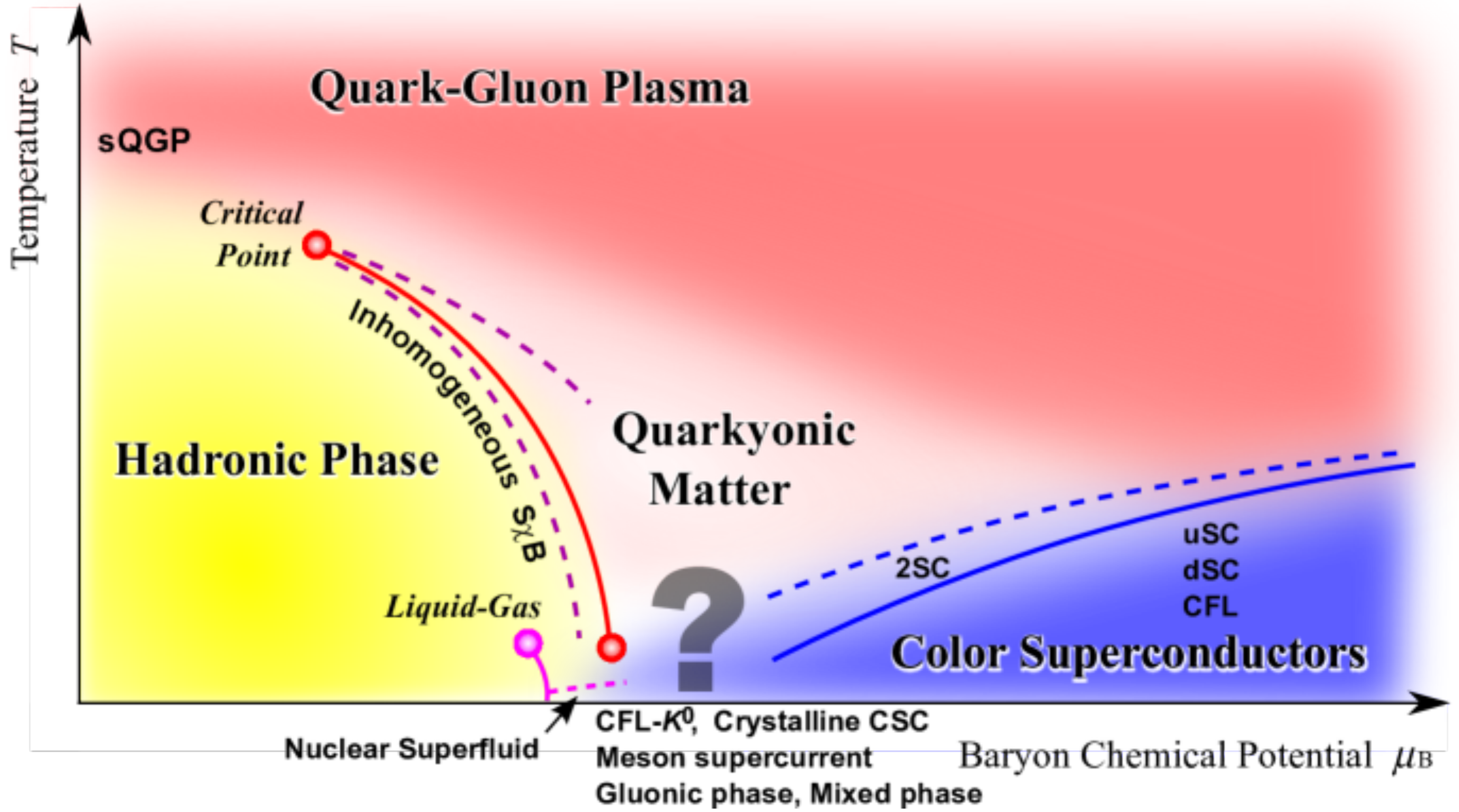
December 4, 2025 - QCD theory Seminars

Introduction

Why color superconductivity now?

1. Recent progress in QCD and QCD-like theories at finite μ
   → gap calculation for the precise comparison is necessary

2. Interest in finite-T & μ phase diagram w.r.t. neutron star mergers, etc.
   → Weak-coupling calculation can set the boundary condition



Fukushima,Hatsuda (2010)

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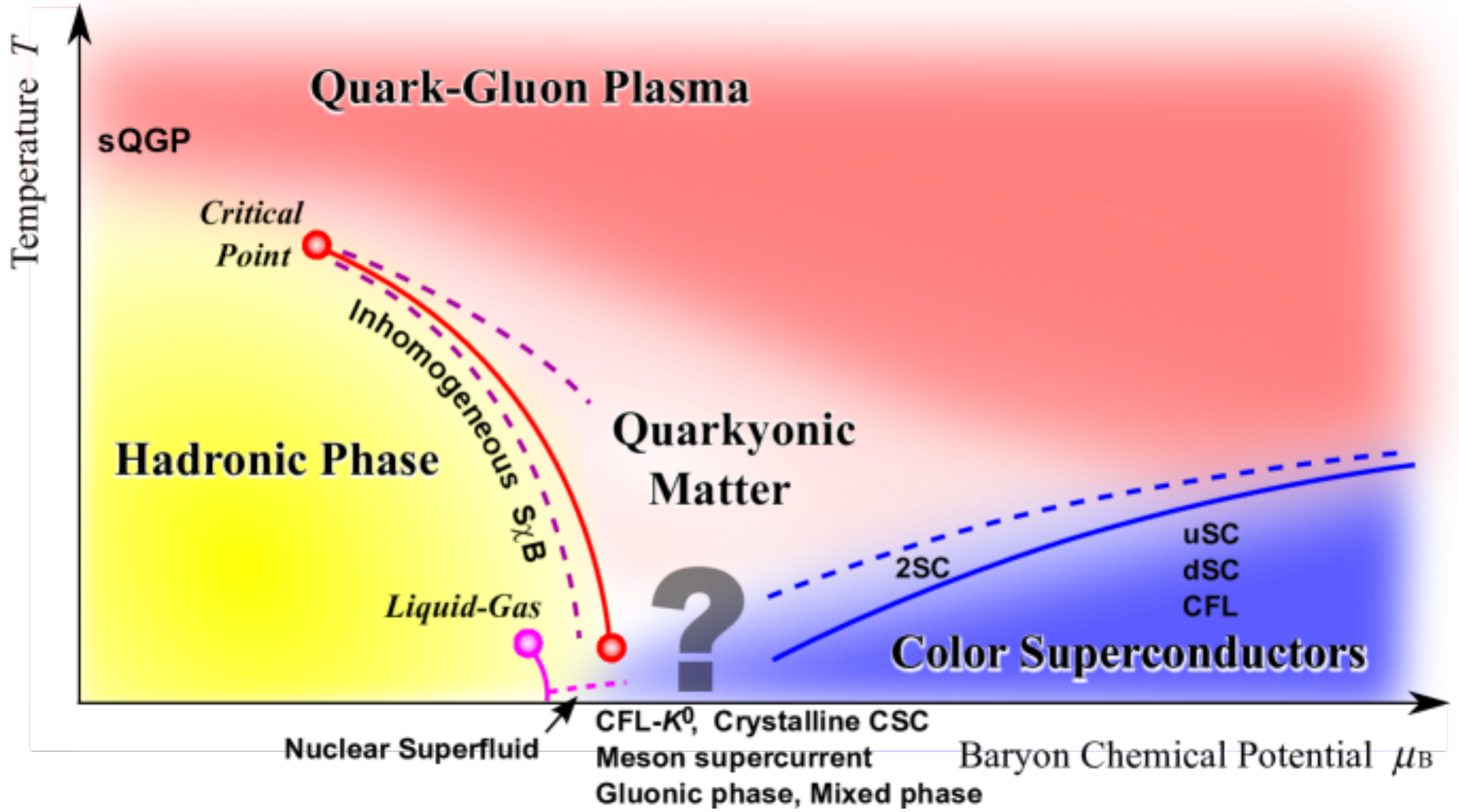


Fukushima,Hatsuda (2010)

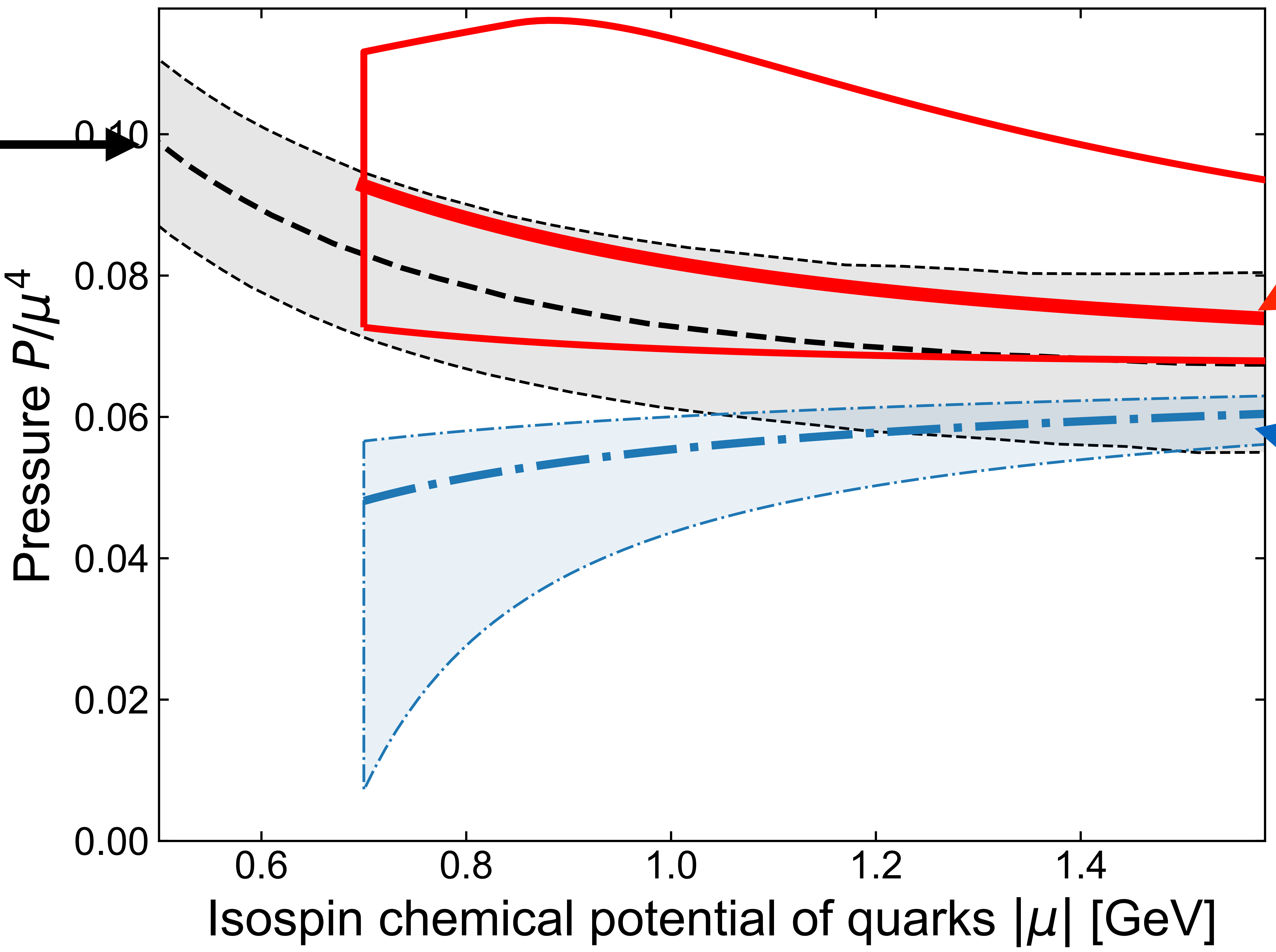
Recent lattice & weak-coupling QCD calculations

Fujimoto (2023)

QCD at finite isospin density:

Lattice QCD

Abbott et al. (2024)



$P_{\rm pQCD} + 6\frac{\mu^2\Delta^2}{4\pi^2}$

(w/ pairing gap at weak coupling)

$P_{\rm pQCD}$

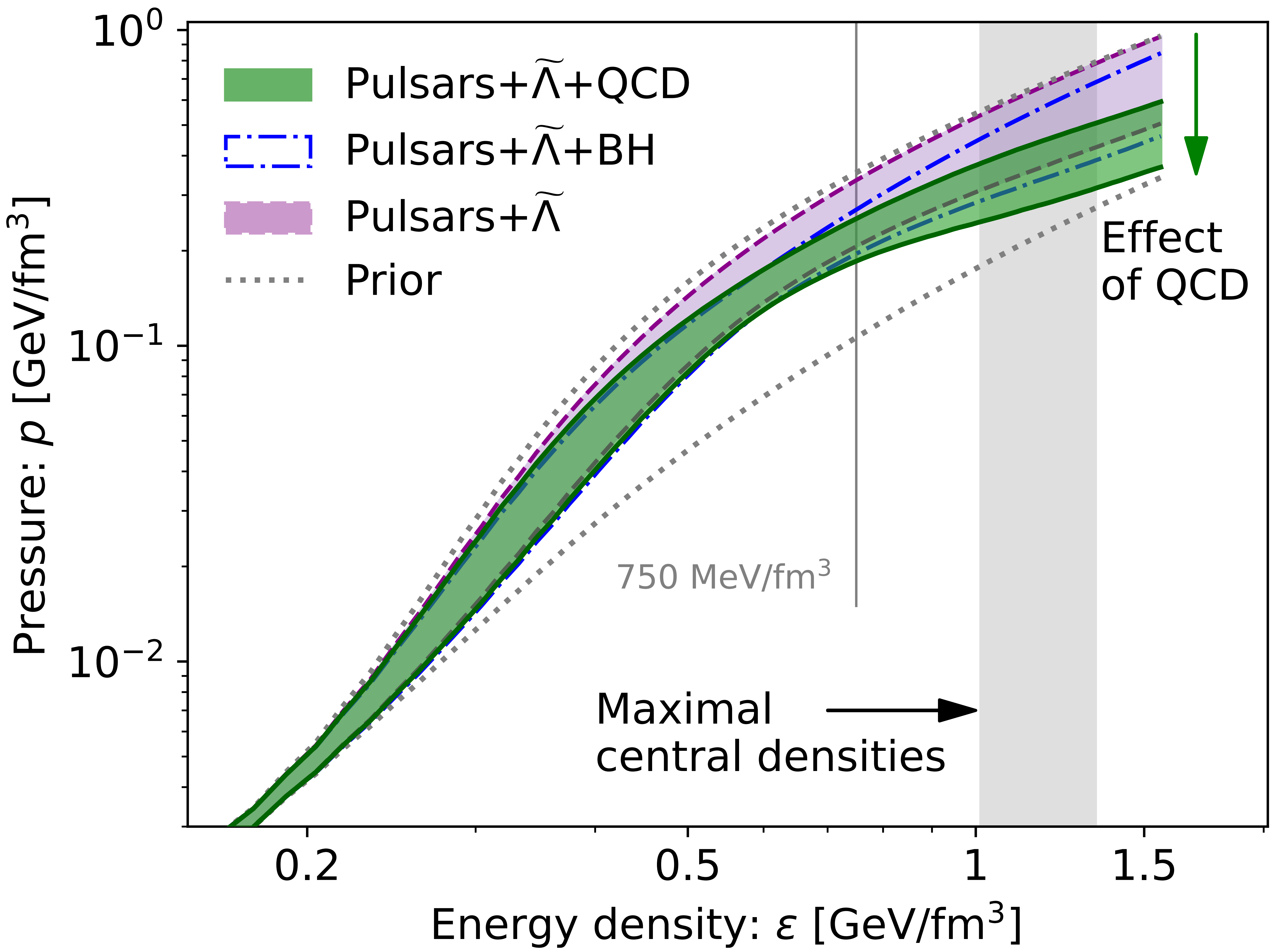
(w/o pairing gap)

pQCD band: scale variation

Unlike in the case of QCD at finite baryon density, the effect of the pairing gap is large

Role of weak-coupling QCD in constraining EoS

Gorda,Komoltsev,Kurkela (2022);
Komoltsev,Somasundaram, et al. (2023)



- QCD effect significantly softens the equation of state at high density
- The QCD result used here is the same weak-coupling expansion as in the previous slide
- Cross check between lattice & weak-coupling QCD is useful

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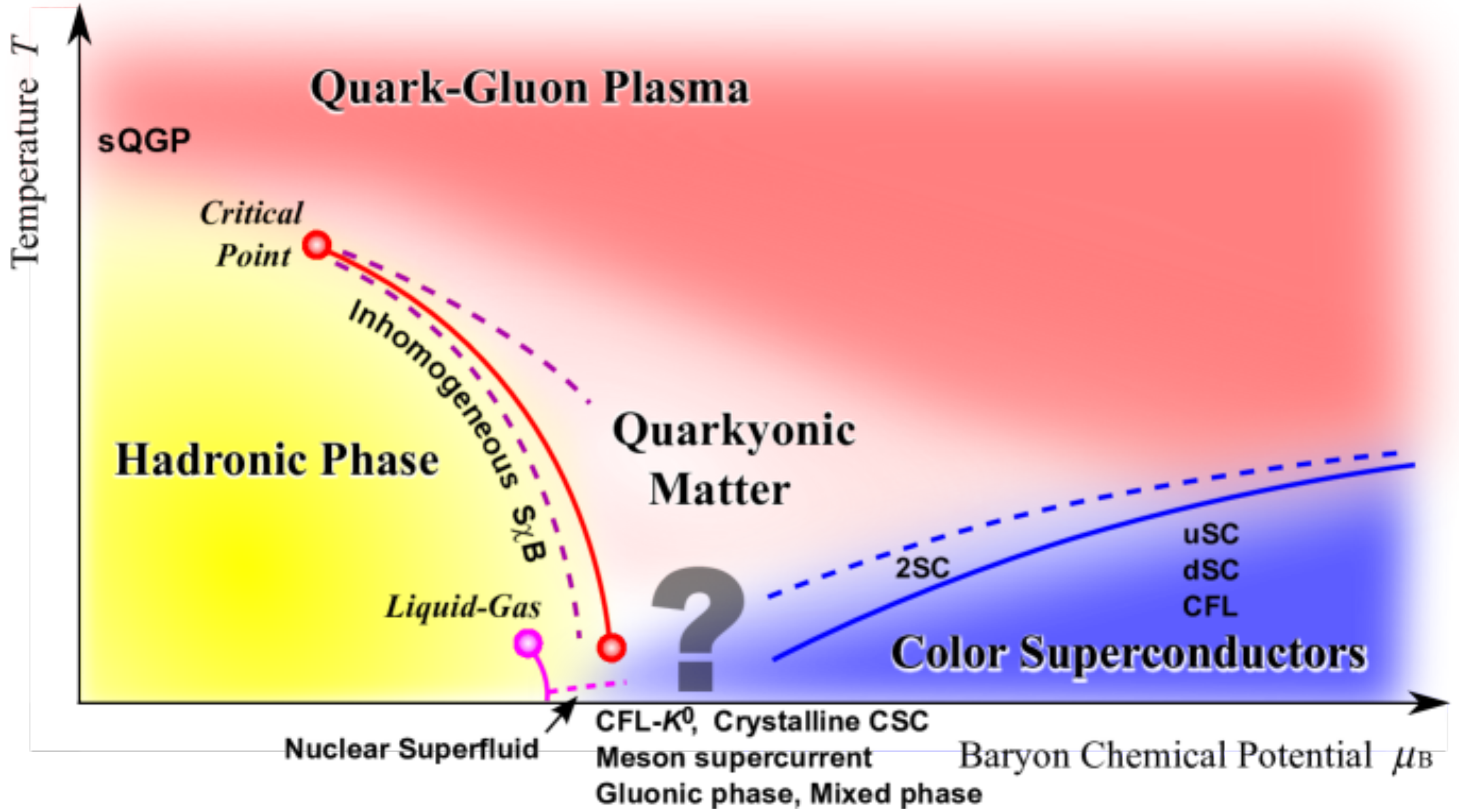
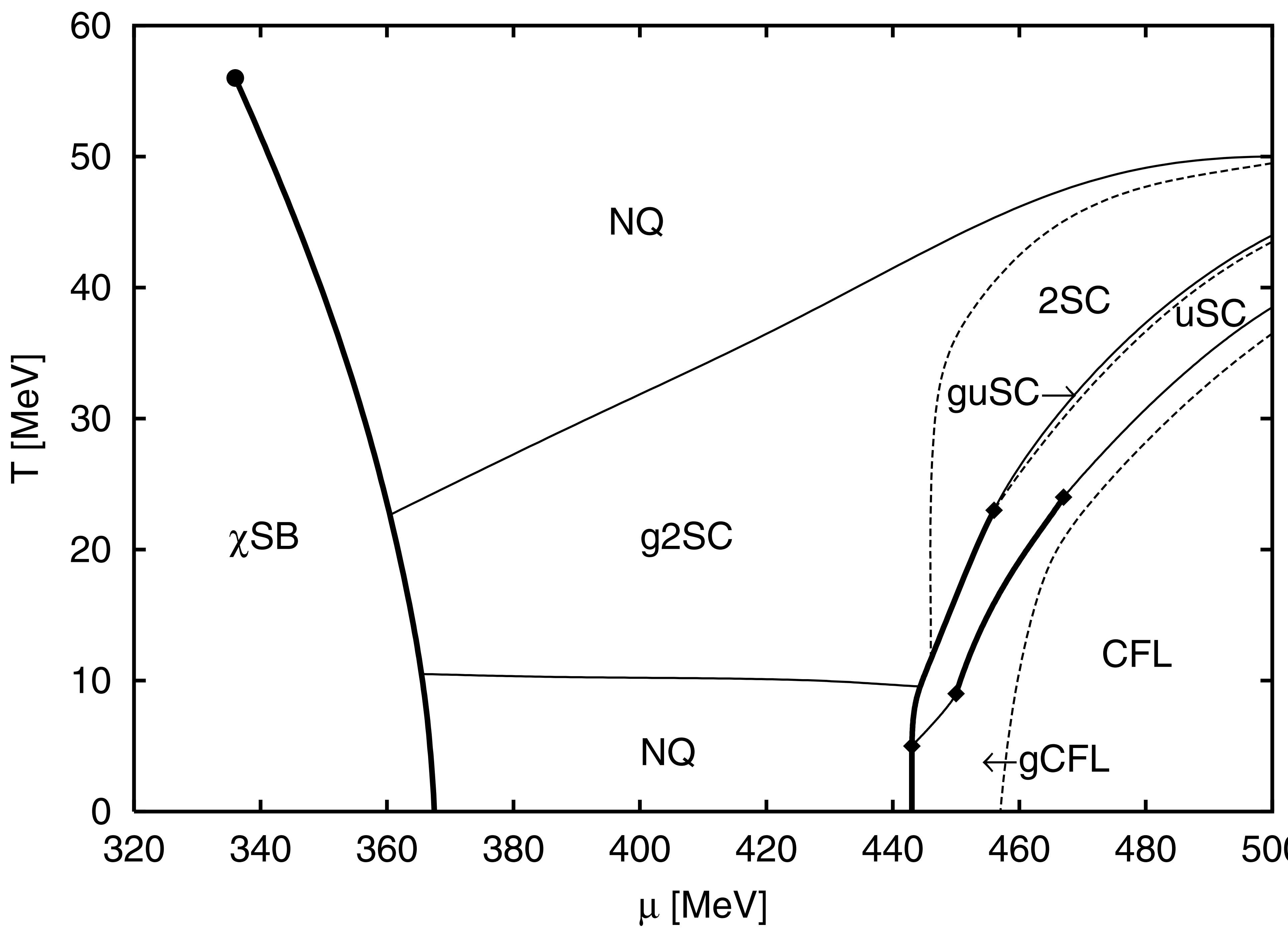


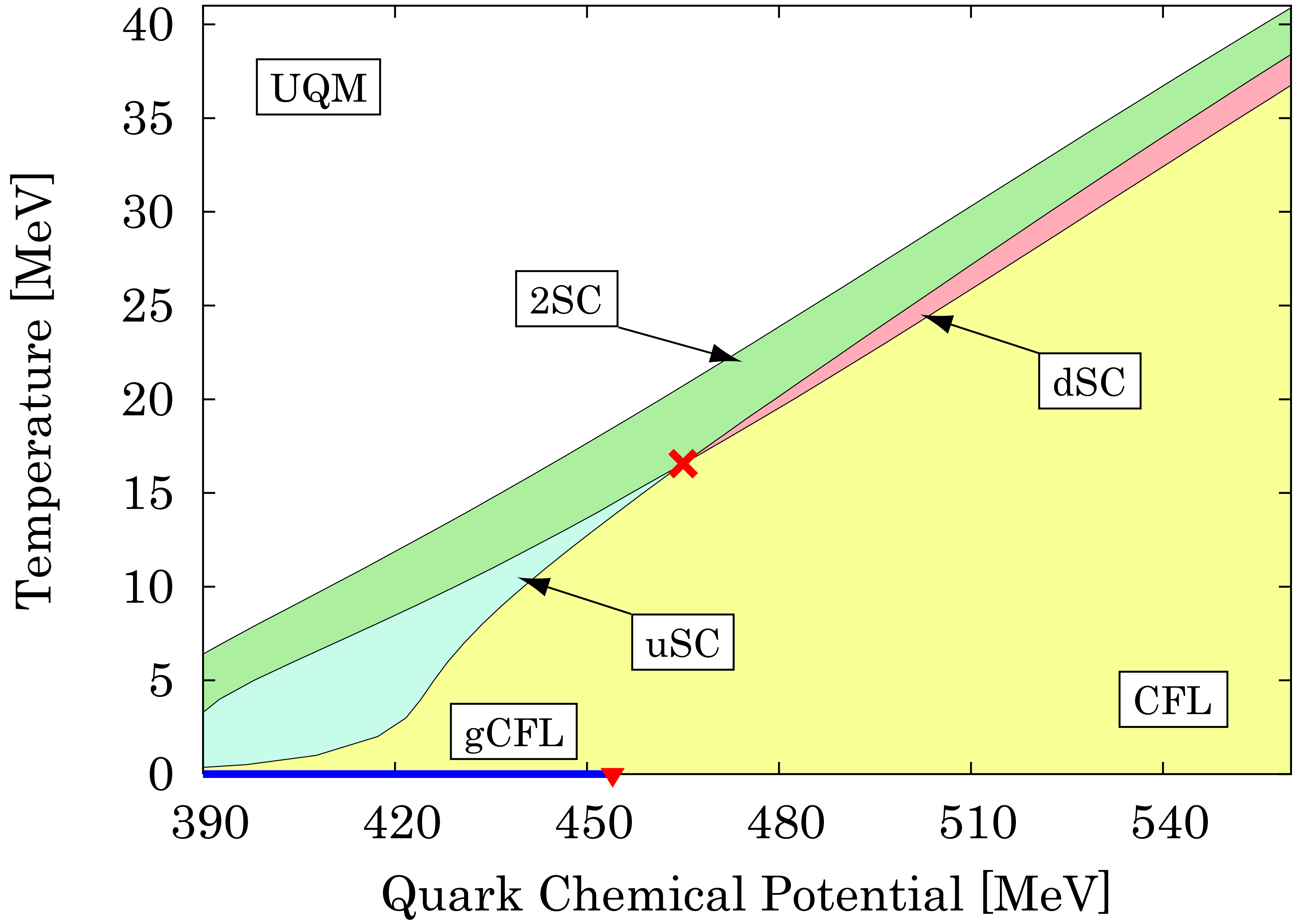
Fukushima,Hatsuda (2010)

Finite-T & μ phase diagram

E.g., a few NJL model analyses



Rüster et al. (2005)

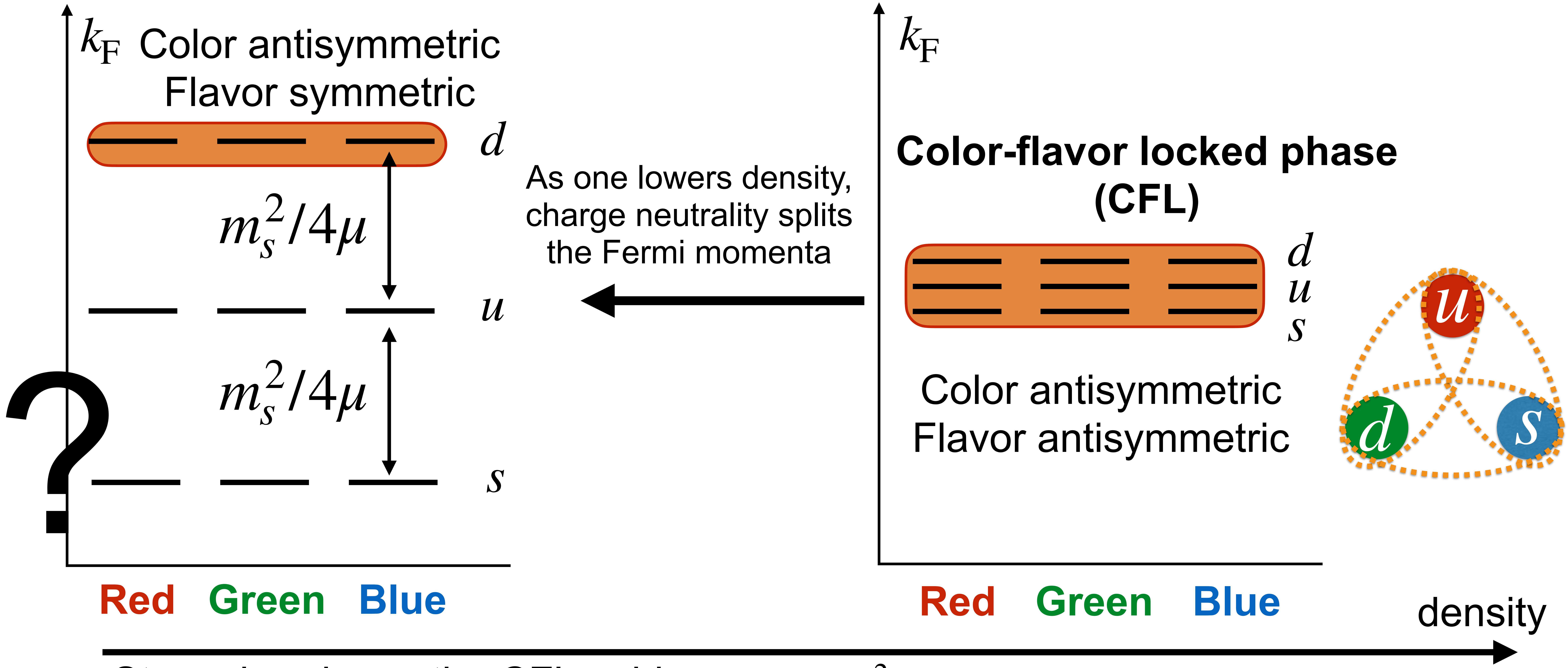


Fukushima (2005)

Still unsettled to date…

One can in principle set the boundary at $\mu \sim O(\mathrm{GeV})$ from weak-coupling QCD

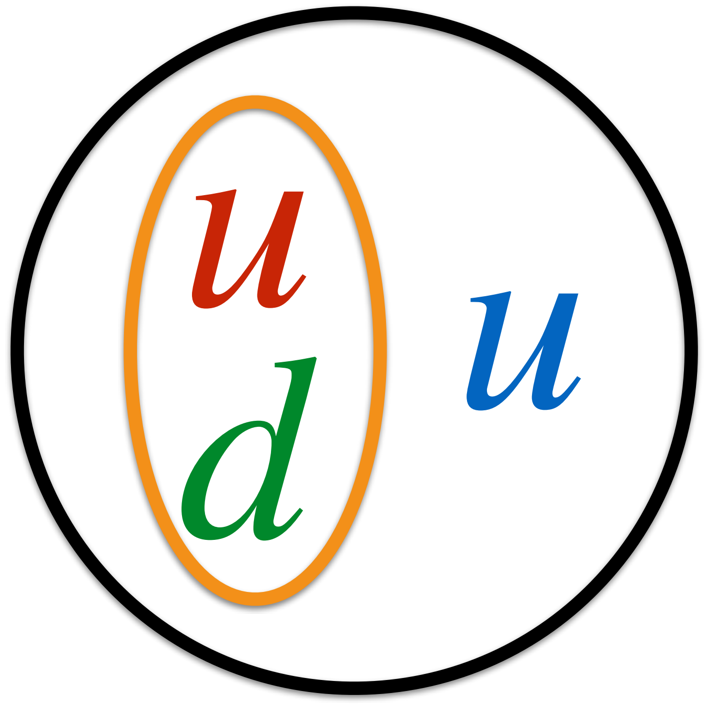
Complication: stress on the pairing



Stress breaks up the CFL pairing $\Delta \simeq m_s^2/(4\mu)$

→ less symmetric pairing in “bad” diquark channel

Digression: analogy to hadron physics



- Color superconductivity:
  caused by **diquark condensation**

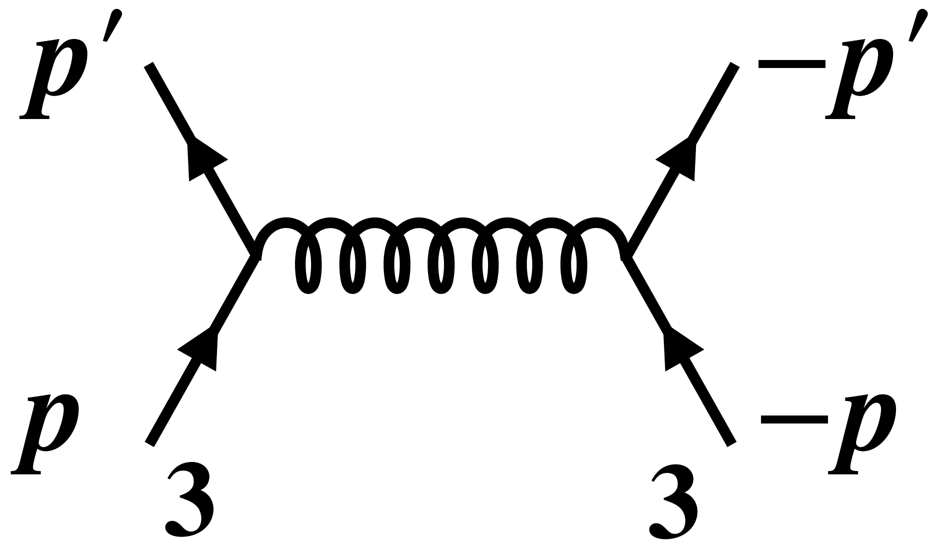
- Diquark as inspiration:
  quarks are tightly bound inside hadrons
  (nebulous concept like constituent quarks)

- Phenomenology: e.g. Jaffe (2004)
  - Baryon spectroscopy
  - $\Delta I = 1/2$ rule in weak non-leptonic decays
  - Structure functions in deep inelastic scattering, etc…

Digression: "good" and "bad" diquarks

Spin-spin interaction part of the Breit Hamiltonian from one-gluon exchange:

$$\hat{H} = -\alpha_s \sum_{i \neq j} M_{ij} \underset{\text{color}}{(\boldsymbol{t}_i \cdot \boldsymbol{t}_j)} \underset{\text{spin}}{(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)} \qquad M_{ij} \propto 1/m_i m_j$$



$$(t^a)_{ij}(t^a)_{kl} = \boxed{\underset{\text{color } \bar{\mathbf{3}}_A}{-\frac{N_c+1}{4N_c}(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})}} + \underset{\text{color } \mathbf{6}_S}{\frac{N_c-1}{4N_c}(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})}$$

$$(\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6})$$

- Spin-singlet (antisymmetric), flavor antisymmetric: $\hat{H}\,|\,\mathbf{0}\rangle = -\frac{3}{4}C\,|\,\mathbf{0}\rangle$ **good diquark [qq']**
- Spin-triplet (symmetric), flavor symmetric: $\hat{H}\,|\,\mathbf{1}\rangle = \frac{1}{4}C\,|\,\mathbf{1}\rangle$ **bad diquark (qq')**

we will see that bad diquark becomes important in the dense medium

Goal of this talk

- Revisit the weak-coupling calculation of the pairing gap

- Revisit the classification of the diquark condensate
  Bailin,Love (1984); Alford,Bowers,Cheyne,Cowan (2003); many other works …

- Dense medium is Lorentz non-invariant,
  and $J = L + S$ decomposition is unique
  → Classification by the term symbol ${}^{2S+1}L_J$ possible
  (similar to non-relativistic case)

- What is the ground state of the color superconductor
  for a given color & flavor representation?

Weak-coupling gap calculation: RG equation for superconductivity

Weak-coupling calculation

$$\ln\left(\frac{\Delta}{\mu}\right) = -\frac{\sqrt{3}\pi^2}{\sqrt{c}}\frac{1}{g} - 5\ln g + \ln\frac{256\pi^4}{e^{(\pi^2+4)/12c}} + o(g^0)$$

cf. Barrois (1978)

3) 1), 2)

- Color superconductivity: nonperturbative phenomenon but can be calculated at weak coupling

- There are three ways to read out the gap:
  1) Gap equation (Schwinger-Dyson eqn) Schafer,Wilczek (99); Pisarski,Rischke(99); Hong et al. (99); Wang,Rischke(01)
  2) Singularity in the fully renormalized two-particle vertex function Brown,Liu,Ren (99)
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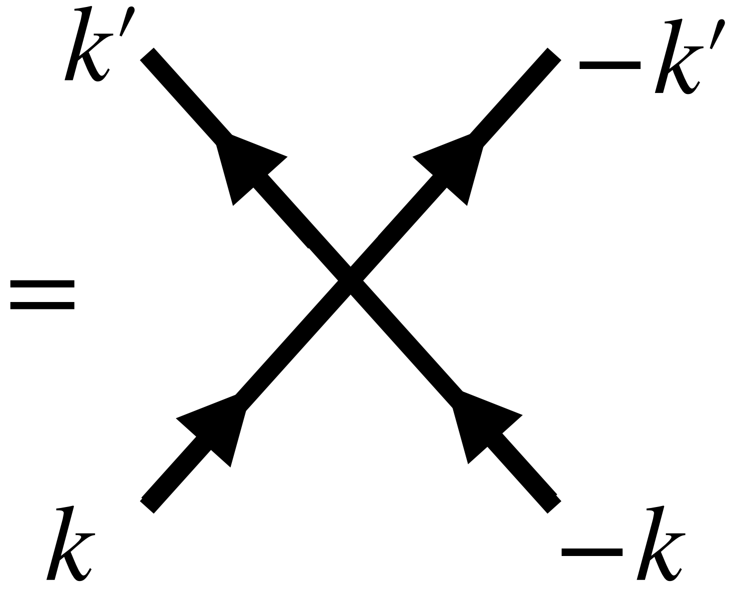
- 1) & 2) are known to give the same results.
  I showed that 3) can also give the same result as 1) & 2).

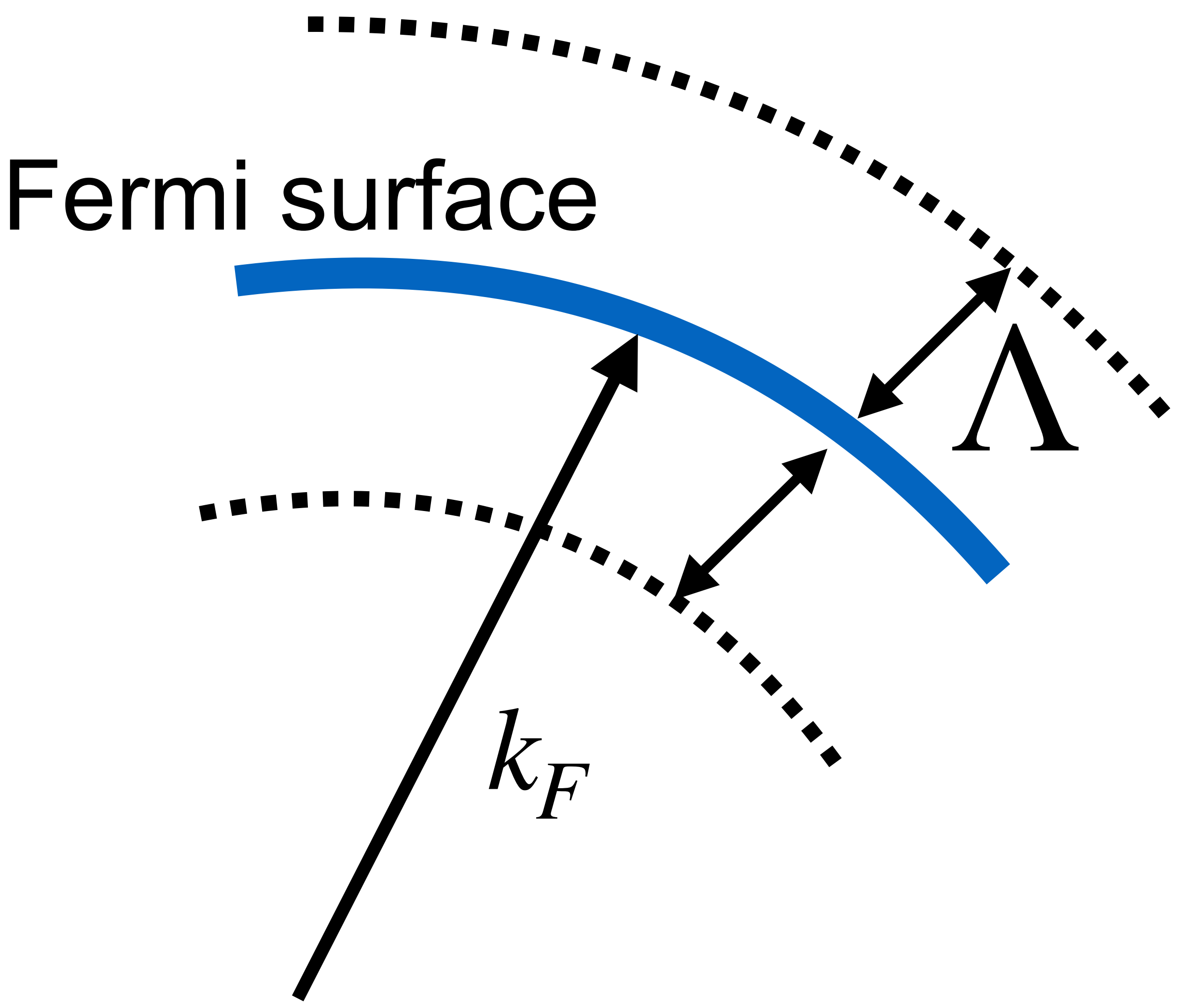
RG approach to BCS instability

Benfatto,Gallavotti (1990); Polchinski (1992); Shankar (1993)…

Consider an EFT with the UV cutoff $|l| < \Lambda$, ($l = k - k_F$)

$$S_{\text{int}} = \prod_{i=1}^{4} \int_{|l|<\Lambda} \frac{d^4 k_i}{(2\pi)^4} V(l_1, l_2, l_3, l_4)\bar{\psi}(l_4)\bar{\psi}(l_3)\psi(l_2)\psi(l_1)$$

$V =$ 



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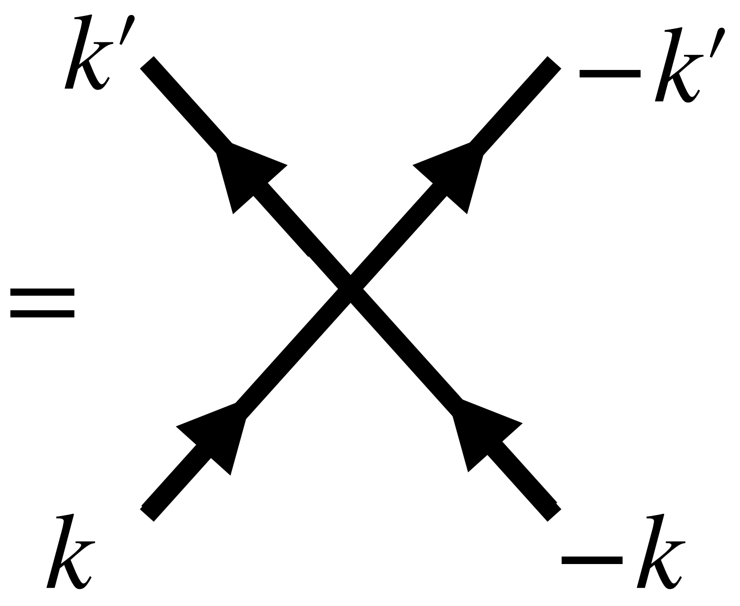
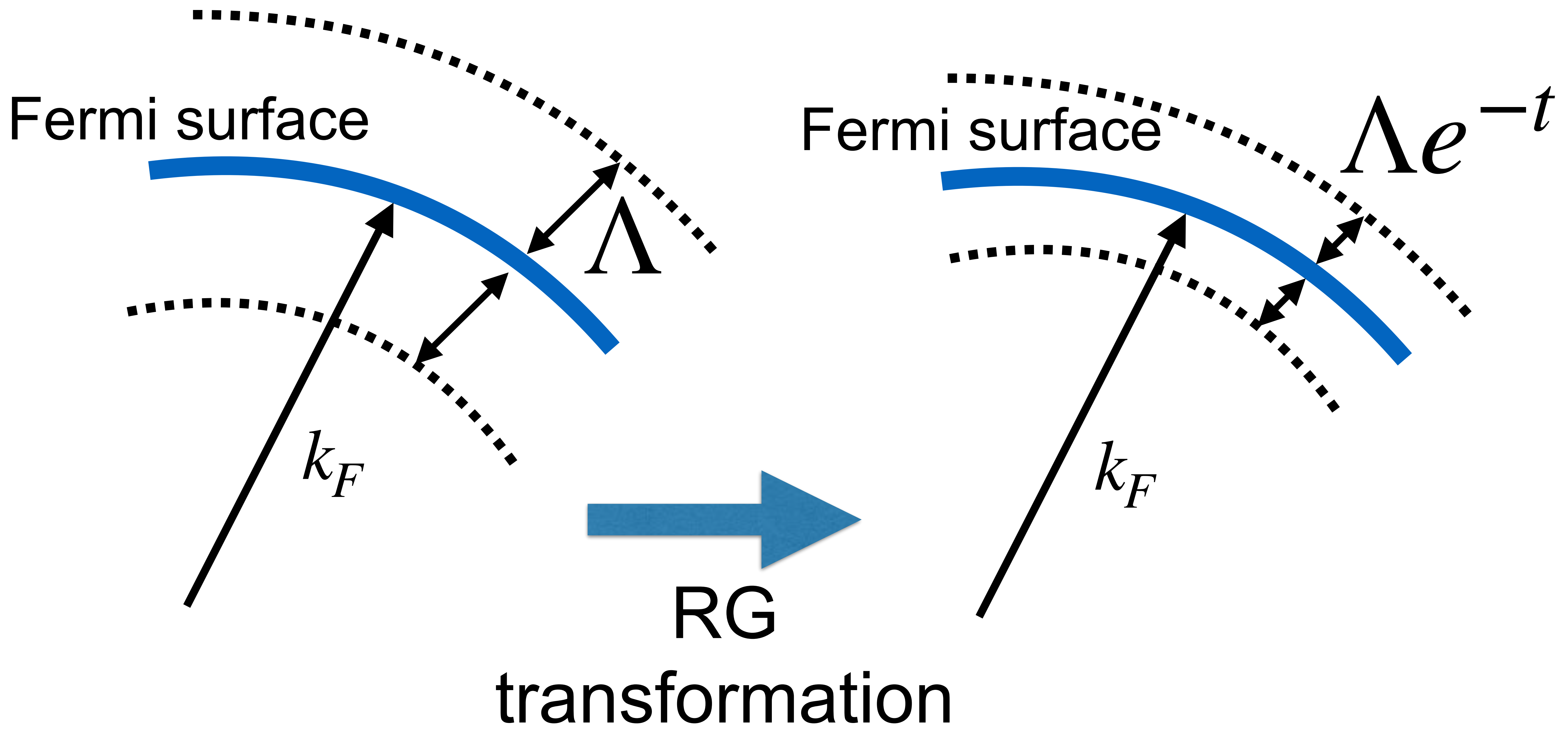
$V =$ 

RG transformation near the Fermi surface:



Slow mode: $\psi_< = \psi(l),\ 0 < |l| < \Lambda e^{-t}$

Fast mode: $\psi_> = \psi(l),\ \Lambda e^{-t} < |l| < \Lambda$

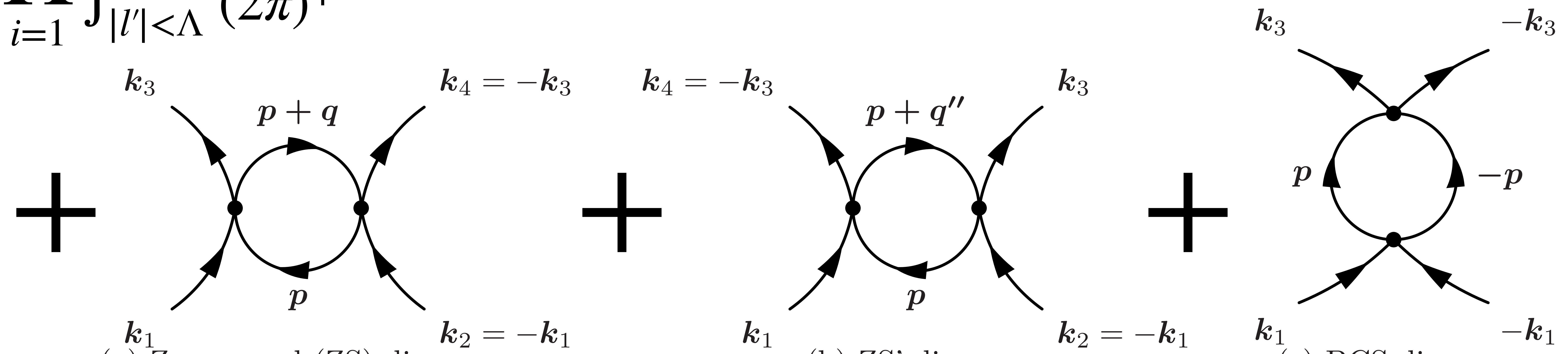
1. Integrate out the fast modes
   i.e., reduced the cutoff as $\Lambda \to \Lambda e^{-t}$
2. Introduce rescaled momenta:
   $l' = l\, e^{t}$ (l' goes up to Λ)
3. Rewrite in terms of rescaled field:
   $\psi'(l') = e^{-3t/2} \psi_<(l' e^{-t})$

RG approach to BCS instability

Benfatto,Gallavotti (1990); Polchinski (1992); Shankar (1993)…

Renormalized effective action (only in terms of the slow modes):

$$S'_{\mathrm{int}} = \prod_{i=1}^{4} \int_{|l'|<\Lambda} \frac{d^4 k'_i}{(2\pi)^4} V(l'_1 e^{-t}, l_2 e^{-t}, l_3 e^{-t}, l_4 e^{-t}) \bar{\psi}'(l'_4)\bar{\psi}'(l'_3)\psi'(l'_2)\psi'(l'_1)$$



(a) Zero sound (ZS) diagram (b) ZS' diagram (c) BCS diagram

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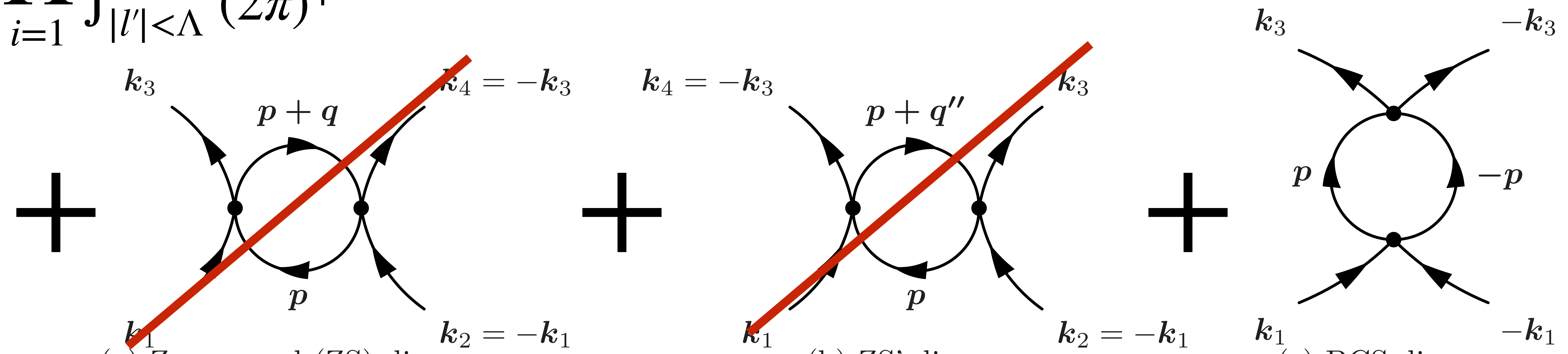

(a) Zero sound (ZS) diagram (b) ZS' diagram (c) BCS diagram

RG equation:

$$\frac{dV_L(t)}{dt} = -V_L^2(t)$$

Partial wave expansion:

$$V(\theta) = \sum_l (2L+1) V_L P_L(\cos\theta)$$

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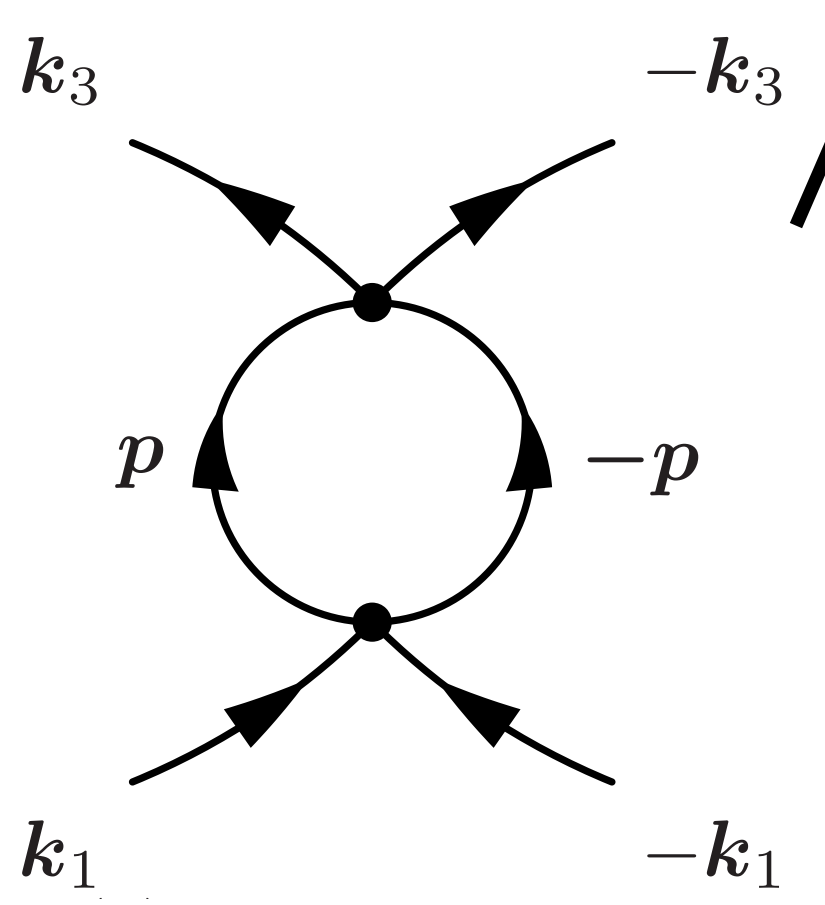

Solution:

$$V_L(t) = \frac{V_L(t=0)}{1 + V_L(t=0)\,t}$$

… singular at $t = -1/V_L(0)$ when $V_L(0) < 0$
(attractive interaction)

- Singularity → Break down of the Fermi liquid picture
  Manifestation of the BCS instability
- Pairing gap Δ = Energy scale Λ at the BCS instability
- From the scale parameter: $t = -\ln(\Lambda/\epsilon_F)$

$$\rightarrow \Delta = \epsilon_F \exp\left(-\frac{1}{|V_L(0)|}\right)$$

Gap in BCS approximation.
Ladder summation via RG eq

Modified RG equation

Benfatto,Gallavotti (1990); Polchinski (1992); Shankar (1993)…

Renormalized effective action (only in terms of the slow modes):

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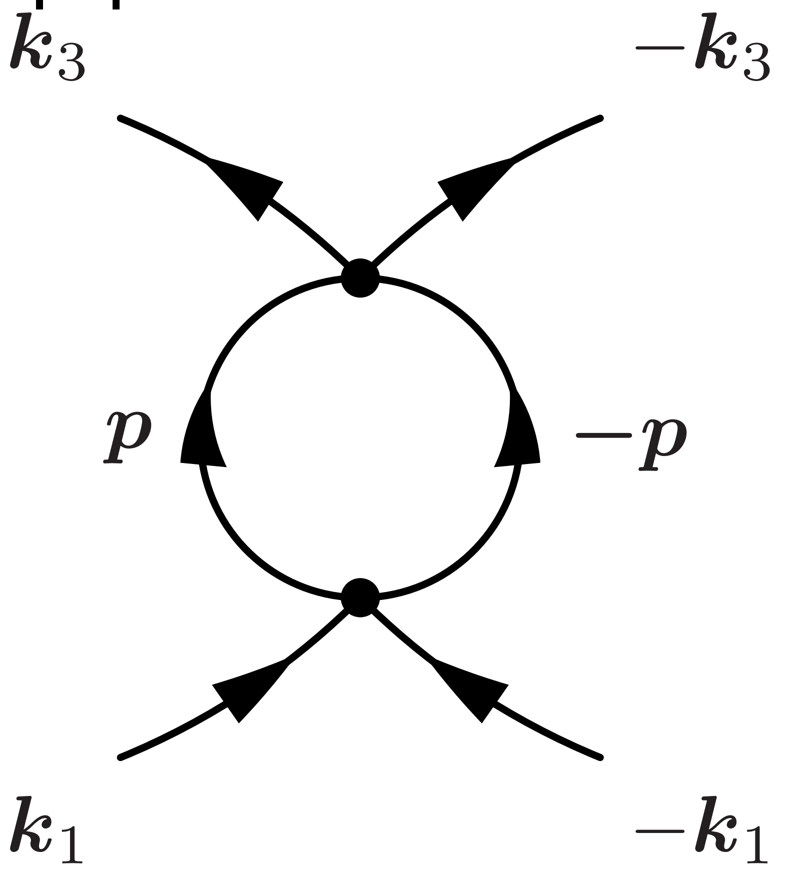
$+$



(c) BCS diagram

$$V_{L=0}(l_1, l_3) \simeq -\frac{g^2}{6\pi^2} \ln\left(\frac{\mu}{g^2 |l_1 - l_3|}\right)$$

Tree-level amplitude is sensitive to d.o.f near the Fermi surface
→ renormalization at the tree-level

RG equation:

Son (1998); Hsu,Schwetz (1999); Fujimoto (2025)

$$\frac{dV_L(t)}{dt} = -V_L^2(t) - \frac{g^2}{6\pi^2}$$

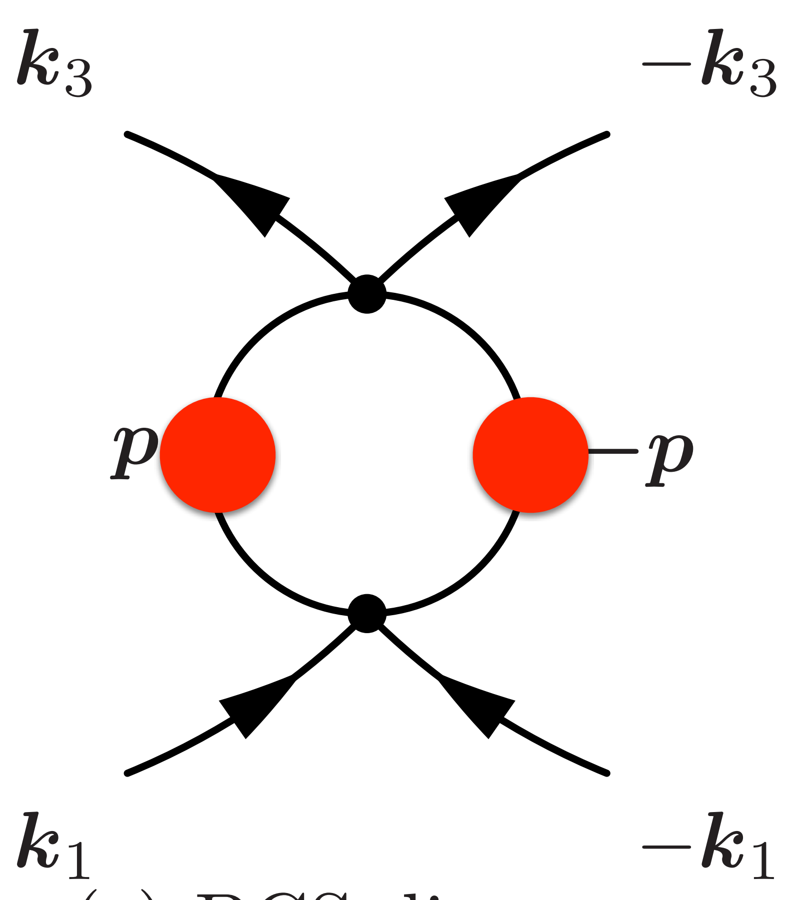
$$\Delta \propto \exp\left(-\frac{1}{g}\right)$$

NB: w/o the const. term:

$$\Delta \propto \exp\left(-\frac{1}{g^2}\right)$$

Further modification to RG equation

One has to use the resummed propagator for fermions…



(c) BCS diagram

→ wave function renormalization

RG equation:

$$\frac{dV_L(t)}{dt} = -Z(t)V_L^2(t) - \frac{g^2}{6\pi^2} \qquad Z(t) = \left(1 + \frac{g^2}{9\pi^2}t\right)^{-1}$$

$$\Rightarrow \quad \ln\left(\frac{\Delta}{\mu}\right) = -\frac{\sqrt{3}\pi^2}{\sqrt{c}}\frac{1}{g} - 5\ln g + \ln\frac{256\pi^4}{e^{(\pi^2+4)/12c}} + o(g^0)$$

Intermediate summary:

$$\ln\left(\frac{\Delta}{\mu}\right) = -\frac{\sqrt{3}\pi^2}{\sqrt{c}}\frac{1}{g} - 5\ln g + \ln\frac{256\pi^4}{e^{(\pi^2+4)/12c}} + o(g^0)$$

1), 2), **3)**

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- Advantage of RG method 3): one only has to calculate a **tree-level** QCD amplitude whereas the other methods require **one-loop** calculation
  → portability toward the higher-order computation?

Helicity amplitude & classification based on it

Problem: pairing below the CFL density

k_F Color antisymmetric
Flavor symmetric

d

$m_s^2/4\mu$

u

s

Red Green Blue

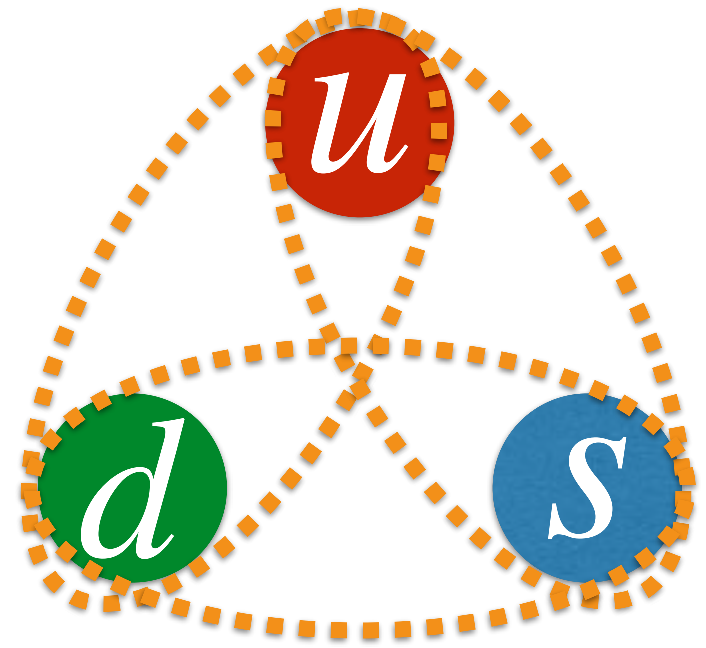
As one lowers density, charge neutrality splits the Fermi momenta

k_F

Color-flavor locked phase (CFL)

d
u
s

Color antisymmetric
Flavor antisymmetric



Red Green Blue

density

What is the favored pairing below the CFL density??

Stress breaks up the CFL pairing when $\Delta \simeq m_s^2/(4\mu)$

Preceding analysis based on NJL model

Alford,Bowers,Cheyne,Cowan (2002); Alford,Cowan (2005)

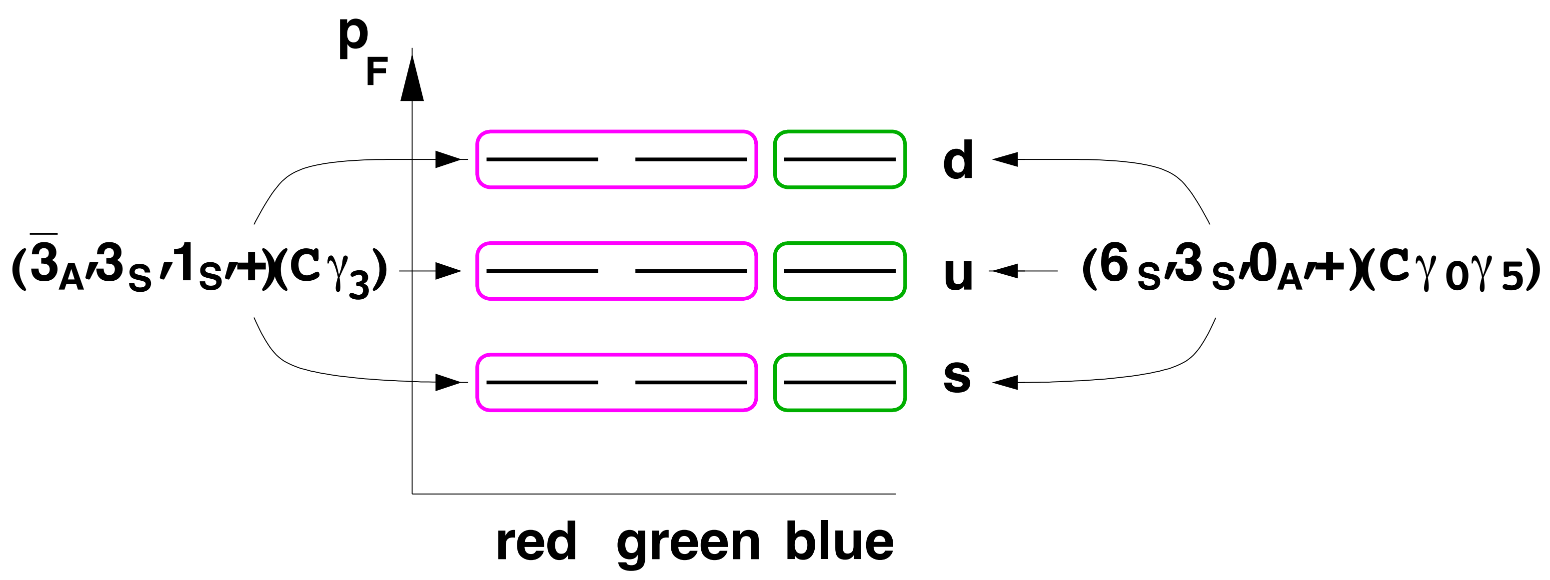


FIG. 1: Pictorial representation of simple single-flavour pairing in neutral quark matter. This will be referred to as the $(1SC)^3$ phase in the text. The requirement of electric neutrality and a nonzero strange quark mass forces the Fermi momenta of the three flavours apart. Two colours of each flavour form $(\bar{\mathbf{3}}_A, \mathbf{3}_S, 1, +)(C\gamma_3)$ Cooper pairs (1SCu, 1SCd and 1SCs). The third colour of each flavour forms $(\mathbf{6}_S, \mathbf{3}_S, 0, +)(C\gamma_0\gamma_5)$ pairs.

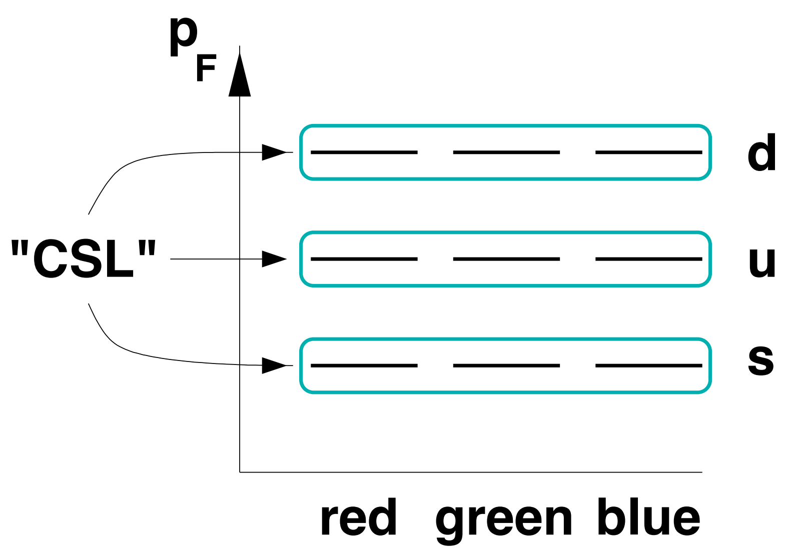
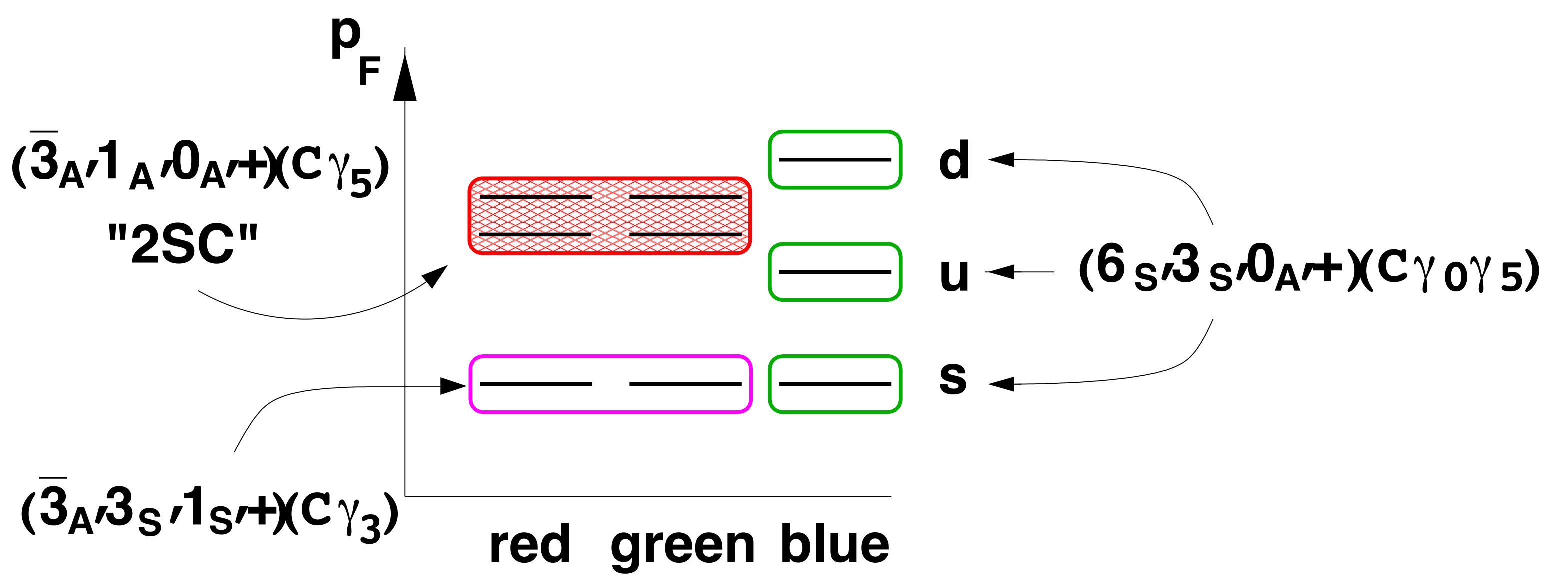
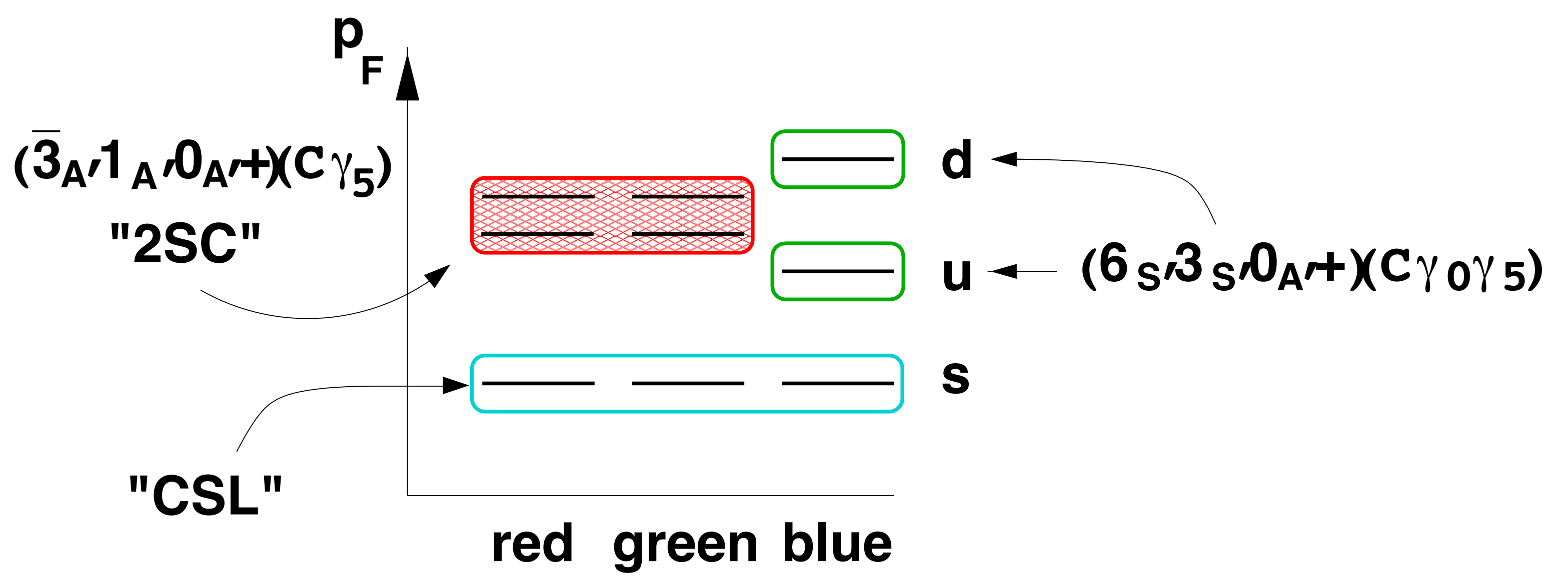


FIG. 3: Pictorial representation of CSL pairing in neutral quark matter. This will be referred to as the $(CSL)^3$ phase in the text. It is composed of up, down and strange quark CSL condensates, labelled CSLu, CSLd and CSLs respectively. The requirement of electric neutrality and a nonzero strange quark mass forces the Fermi momenta of the three flavours apart. The red, green and blue colours of each flavour pair in a colour-antisymmetric channel.





Based on comparison of free energy:

$$\Omega_{\text{paired}} = \Omega_{\text{unpaired}} - C\mu^2\Delta^2$$

→ Larger Δ is favored

→ Larger attraction leads to larger Δ

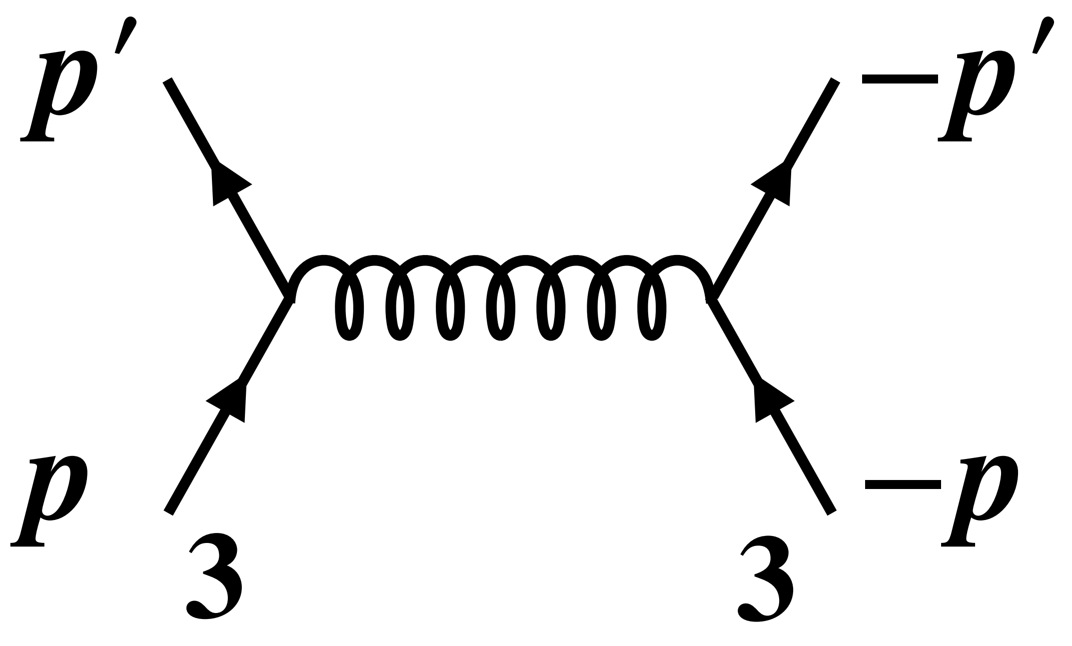
→ Classification of the (attractive) interaction between quarks

One-gluon exchange attraction

- **Cooper instability:**

Fermi surface
Attractive interaction $\Big\}$ Cooper pair (diquark condensation) → **Superconductivity**

- **One-gluon exchange (OGE)** amplitude for quarks



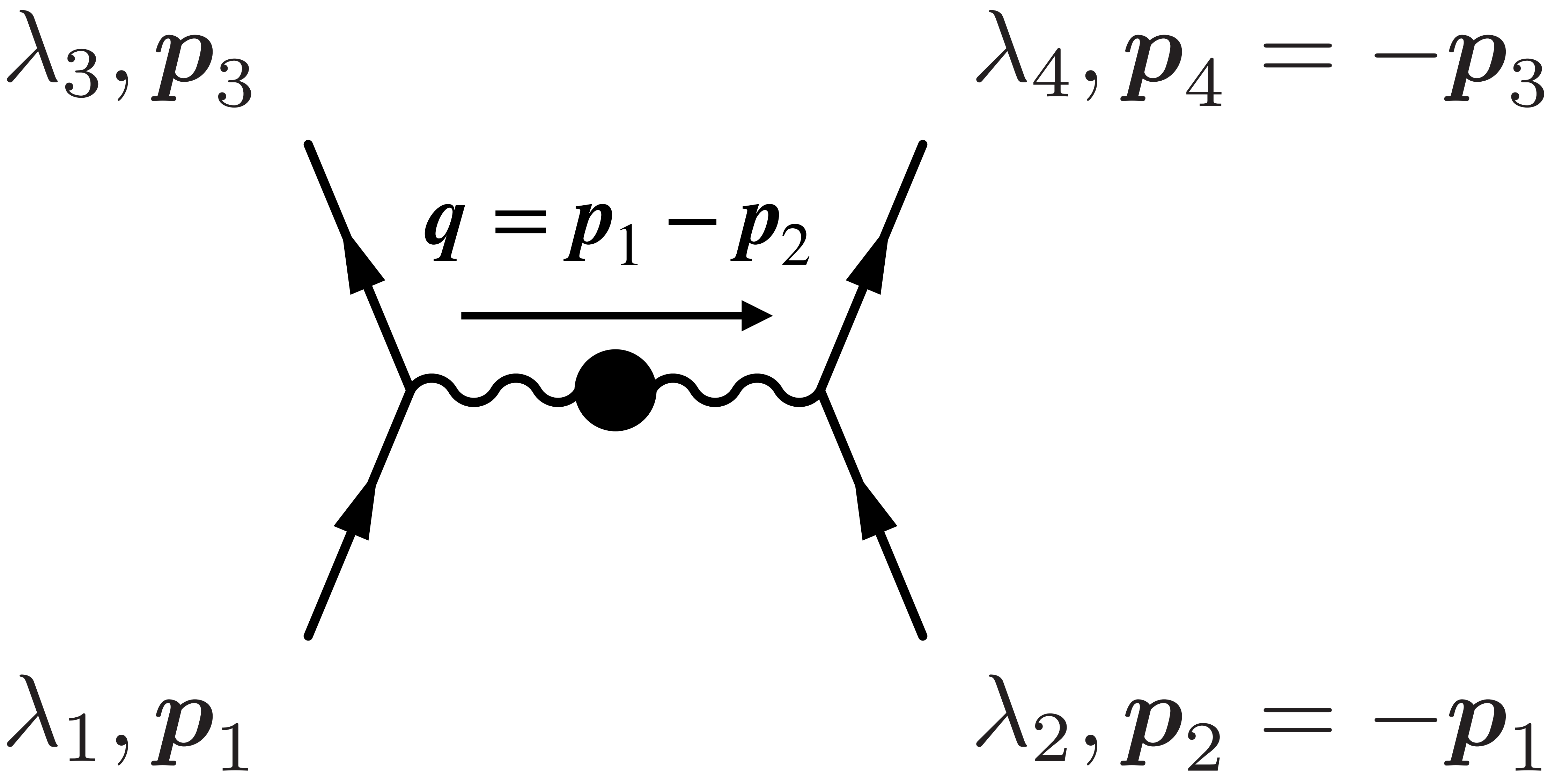
$$(t^a)_{ij}(t^a)_{kl} = \boxed{-\frac{N_c+1}{4N_c}\underbrace{(\delta_{ij}\delta_{kl}-\delta_{il}\delta_{kj})}_{\text{color }\bar{\mathbf{3}}_A}} + \frac{N_c-1}{4N_c}\underbrace{(\delta_{ij}\delta_{kl}+\delta_{il}\delta_{kj})}_{\text{color }\mathbf{6}_S}$$

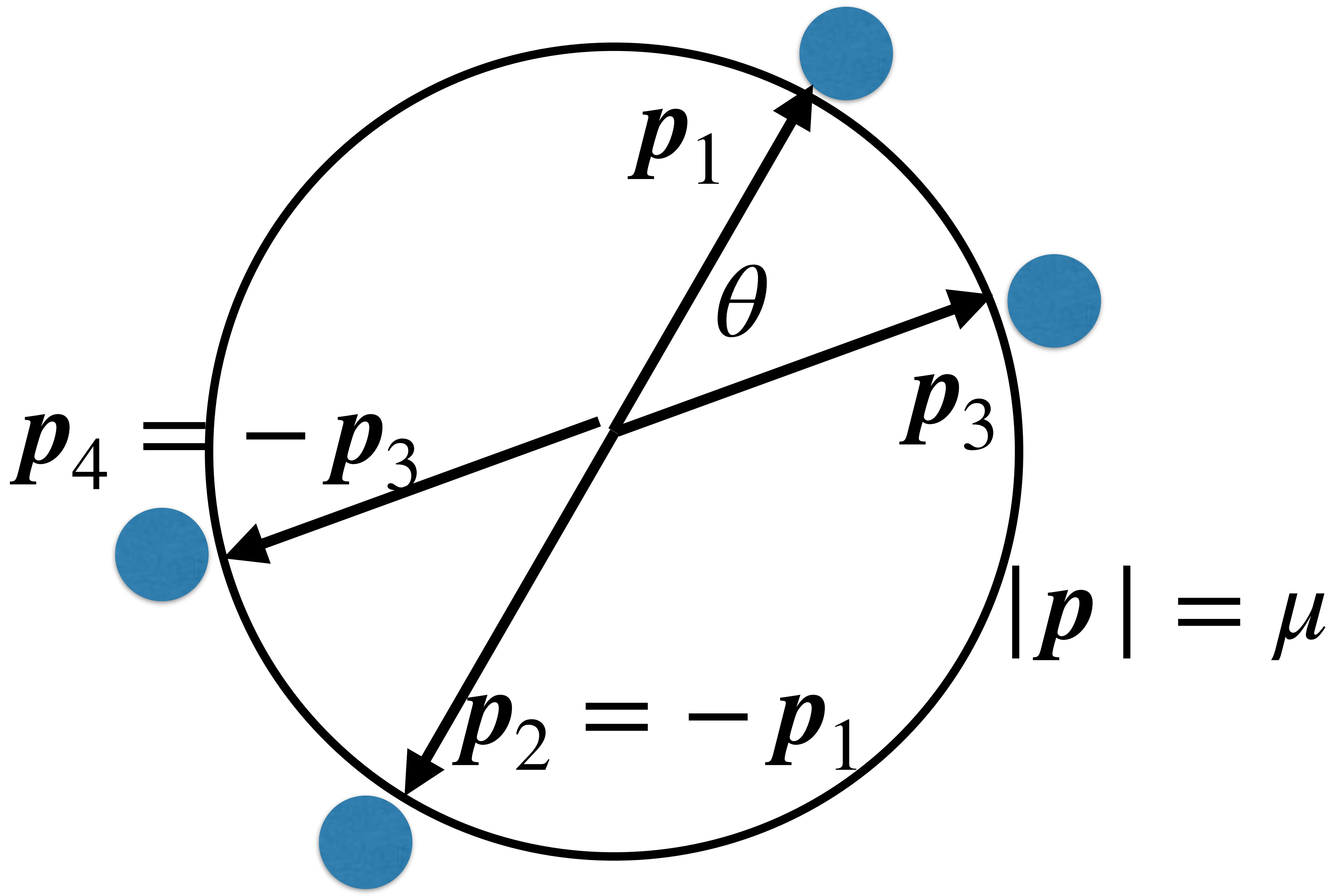
$$(\mathbf{3}\otimes\mathbf{3}=\bar{\mathbf{3}}\oplus\mathbf{6})$$

Attractive
→ Color superconductivity

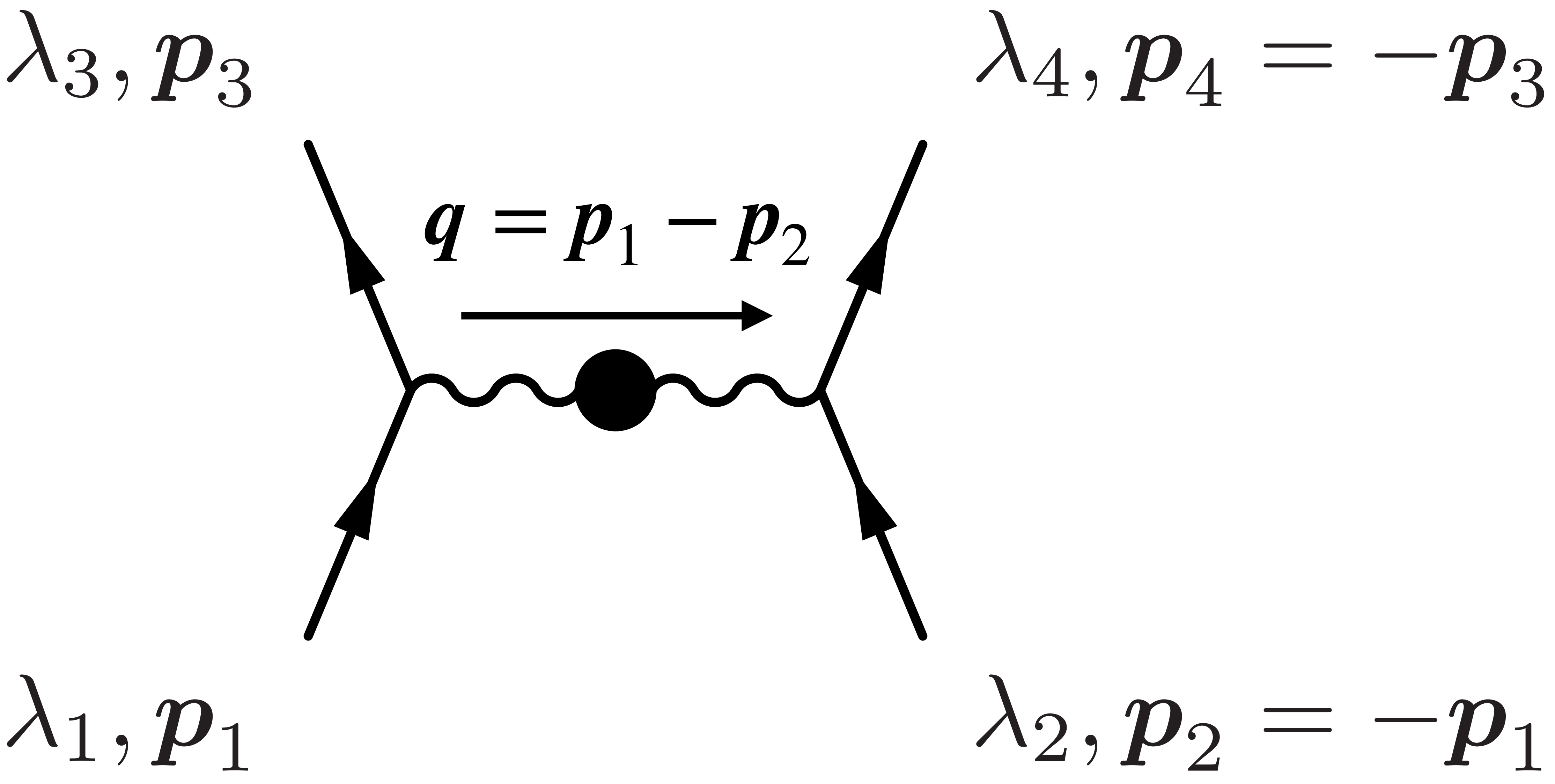
There are other quantum numbers, such as **spin (helicity) and flavor**
→ enriches (complicates?) the pairing pattern

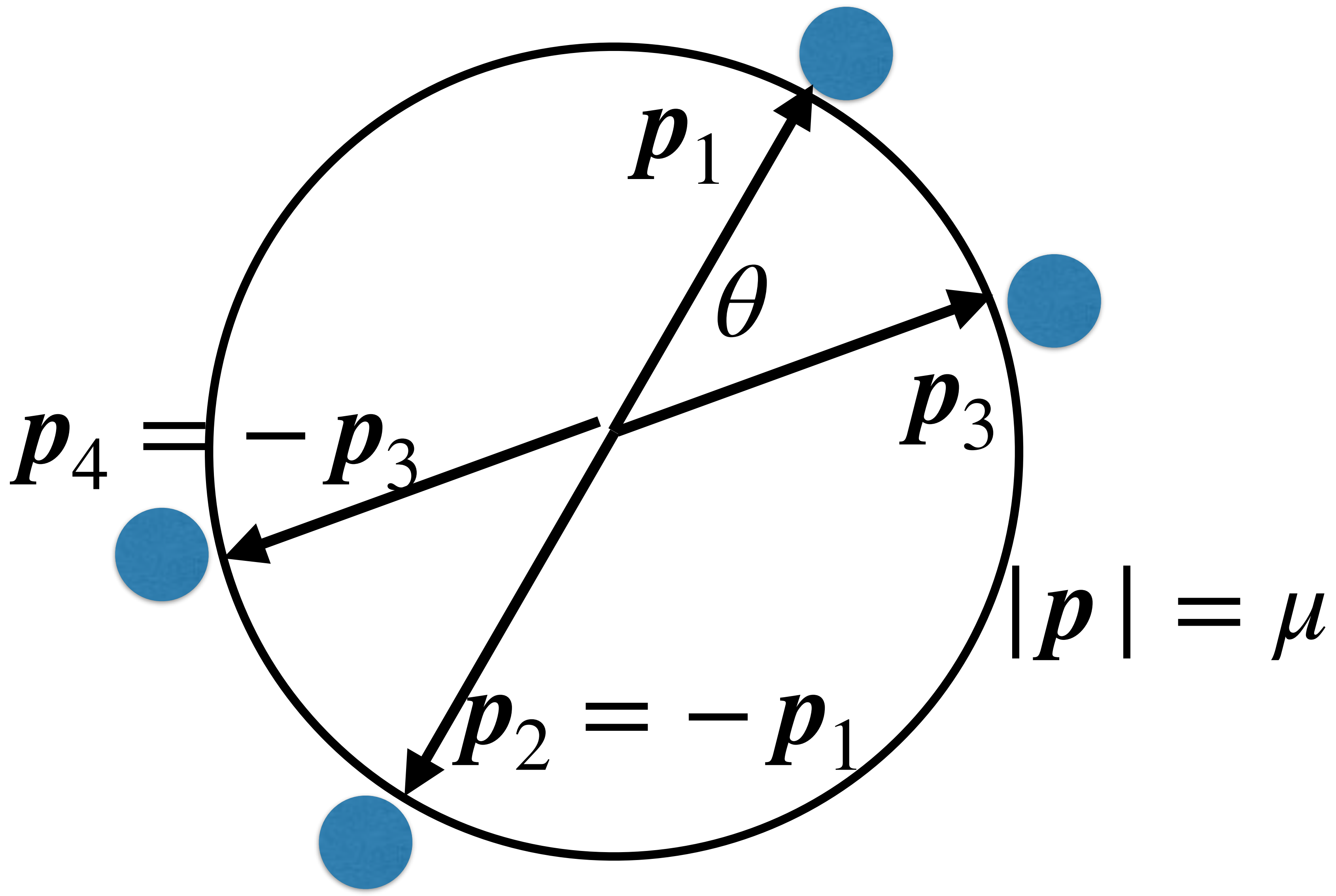
One-gluon exchange (OGE) amplitude





One-gluon exchange (OGE) amplitude





$$D^{\mathrm{E/M}} = -\frac{1}{q_0^2 - \boldsymbol{q}^2 - \Pi^{\mathrm{E/M}}}$$

$$\Pi^E \simeq m_{\mathrm{D}}^2, \quad \Pi^M \simeq -i\frac{\pi}{4} m_{\mathrm{D}}^2 \frac{q_0}{|\boldsymbol{q}|}$$

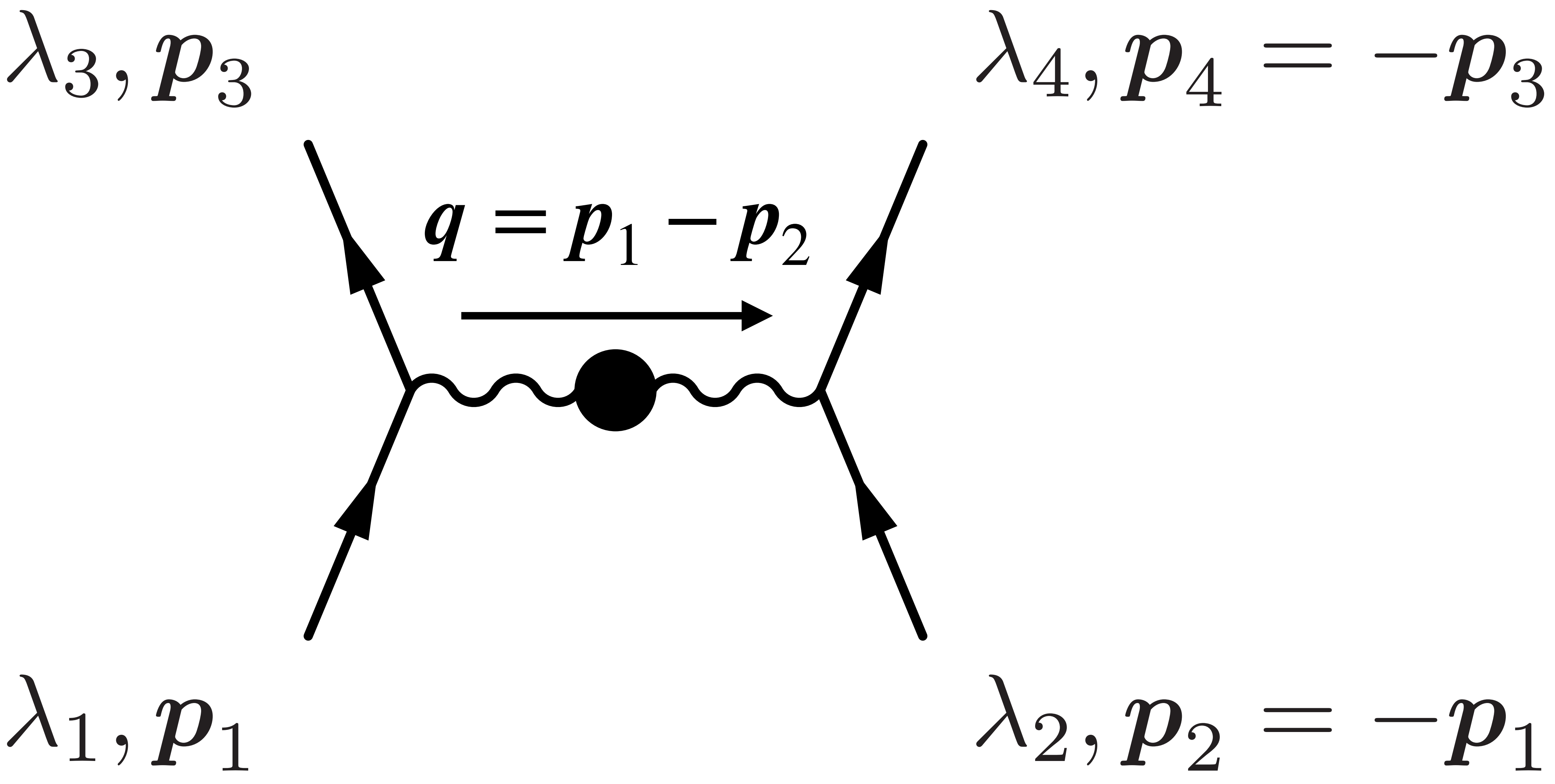
Debye screening

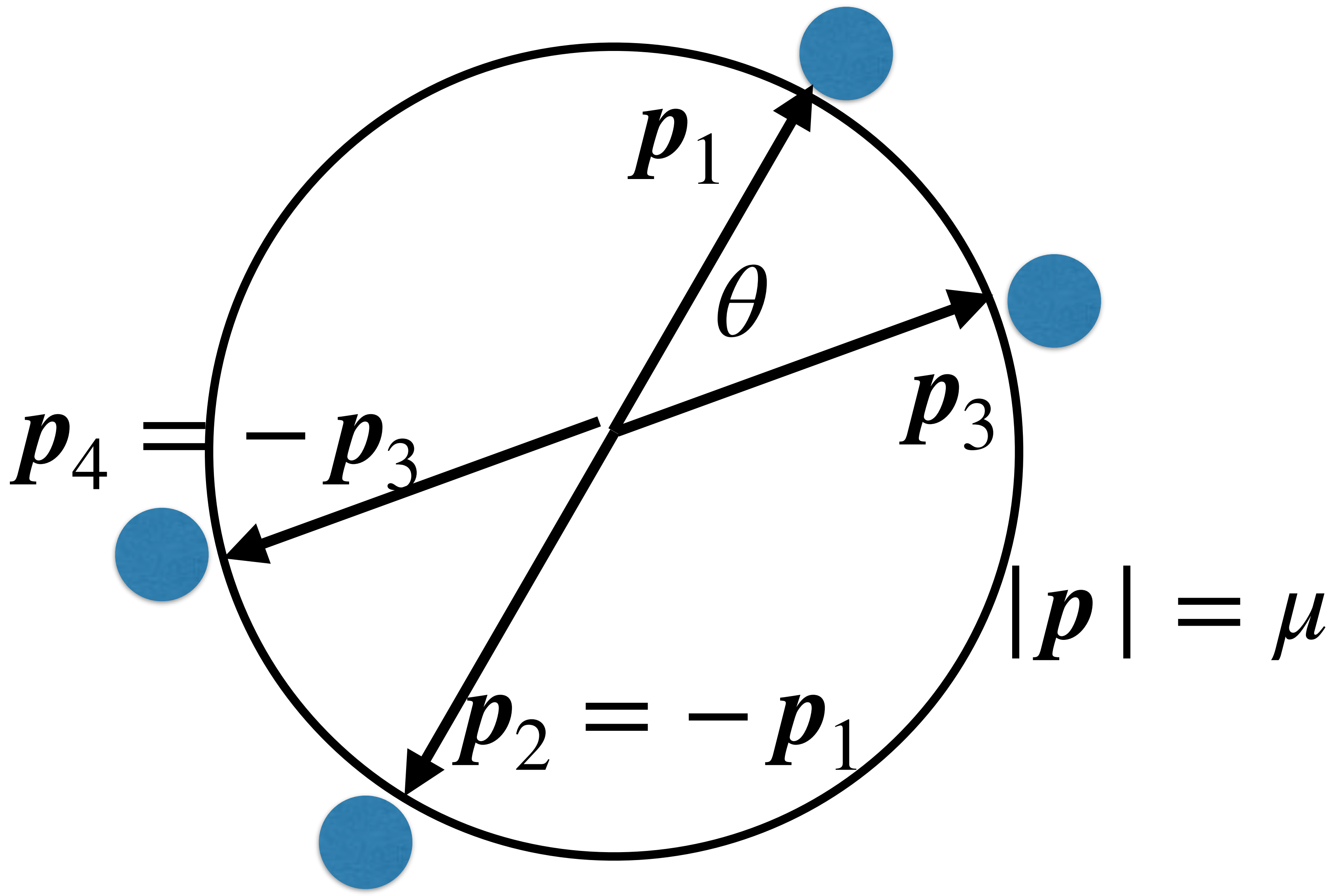
$m_{\mathrm{D}} \simeq g\mu$

Landau damping

→ dynamical screening

One-gluon exchange (OGE) amplitude





$$D^{\mathrm{E/M}} = -\frac{1}{q_0^2 - \boldsymbol{q}^2 - \Pi^{\mathrm{E/M}}}$$

$$\Pi^E \simeq m_{\mathrm{D}}^2, \quad \Pi^M \simeq -i\frac{\pi}{4} m_{\mathrm{D}}^2 \frac{q_0}{|\boldsymbol{q}|}$$

Debye screening

$m_{\mathrm{D}} \simeq g\mu$

Landau damping

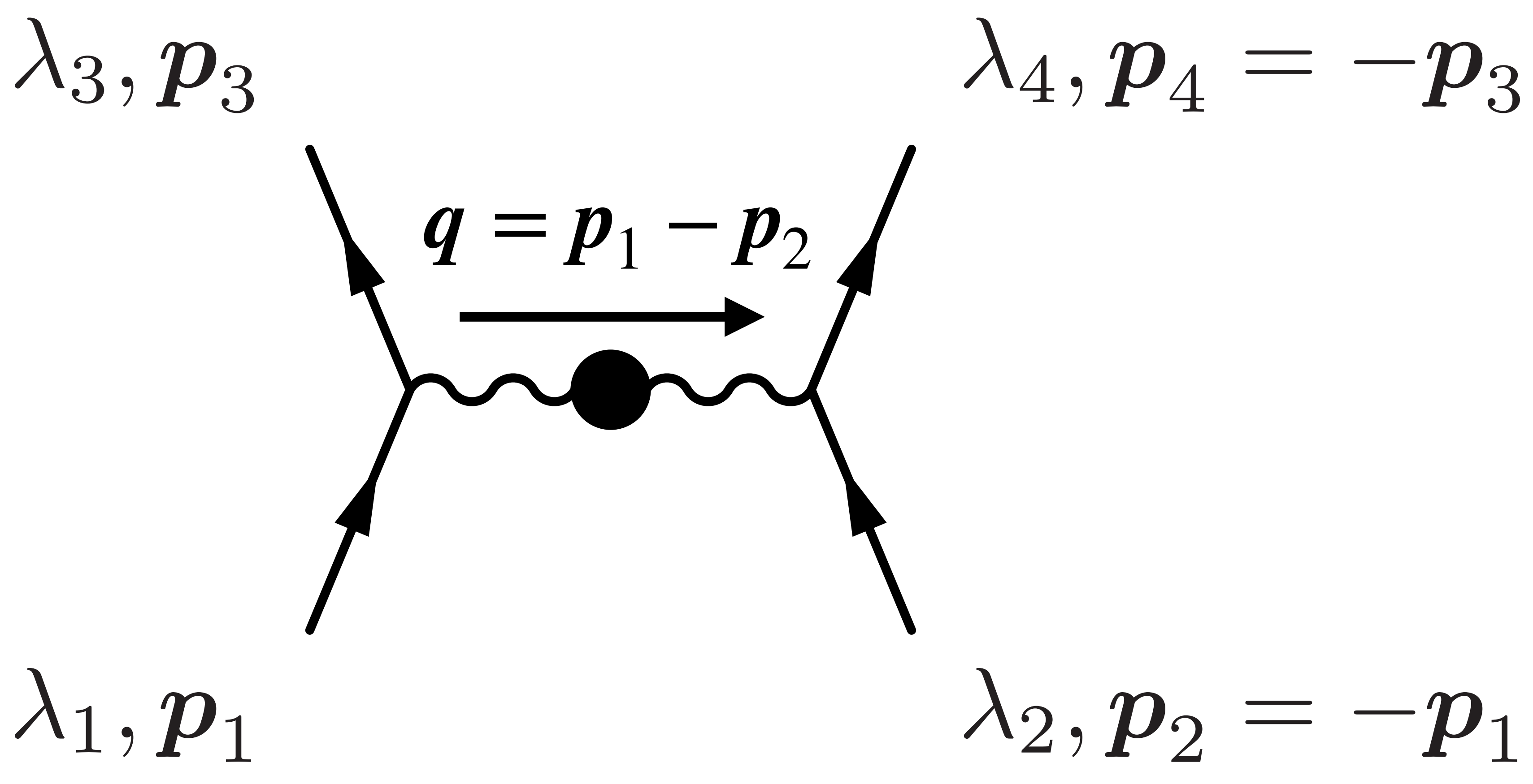
$\rightarrow$ dynamical screening

Scattering amplitude $\mathscr{M}_{\lambda_1\lambda_2;\lambda_3\lambda_4}$

$$\mathscr{M}_{++;++} = (\boldsymbol{t}_1 \cdot \boldsymbol{t}_2) g^2 \left[D^{\mathrm{E}} \cos^2\frac{\theta}{2} + D^{\mathrm{M}} \left(\cos^2\frac{\theta}{2} + 2\sin^2\frac{\theta}{2} \right) \right]$$

$$\mathscr{M}_{+-;+-} = (\boldsymbol{t}_1 \cdot \boldsymbol{t}_2) g^2 \left[D^{\mathrm{E}} \cos^2\frac{\theta}{2} + D^{\mathrm{M}} \cos^2\frac{\theta}{2} \right]$$

Helicity amplitude



Jacob,Wick (1959); Bailin,Love (1984)

Helicity amplitude

$$\mathscr{M}_{\lambda_1,\lambda_2;\lambda_3\lambda_4}(\theta) = \sum_J (2J+1)\mathscr{H}^J_{\lambda_1\lambda_2} d^J_{\lambda\lambda'}(\theta)$$

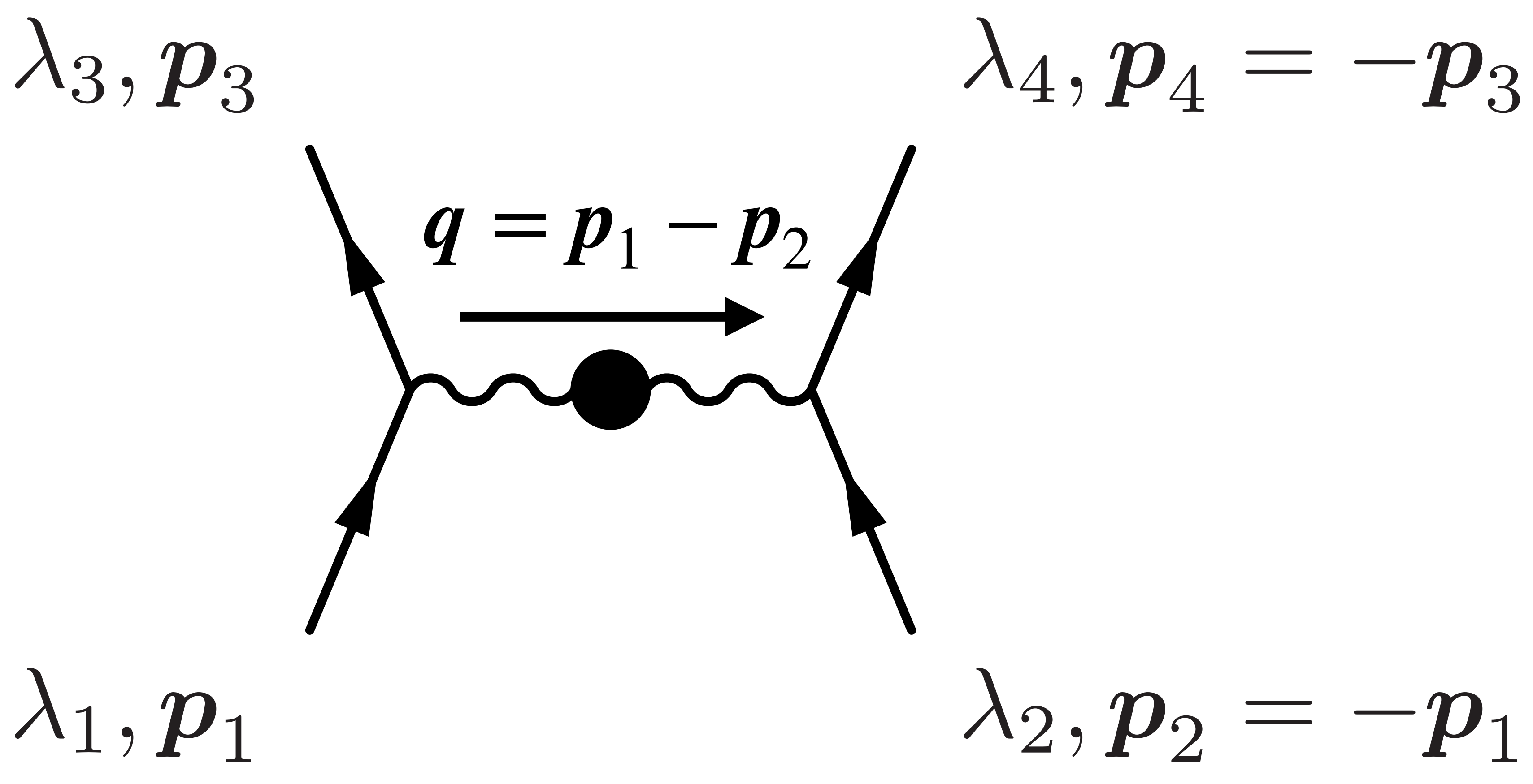
Wigner d-matrix

$(\lambda = \lambda_1 - \lambda_2, \quad \lambda' = \lambda_3 - \lambda_4)$

When $\lambda = \lambda' = 0$, $d^J_{00}(\theta) = P_J(\cos\theta)$

Helicity amplitude

Jacob,Wick (1959); Bailin,Love (1984)



Helicity amplitude

$$\mathscr{M}_{\lambda_1,\lambda_2;\lambda_3\lambda_4}(\theta) = \sum_J (2J+1) \mathscr{H}^J_{\lambda_1\lambda_2} d^J_{\lambda\lambda'}(\theta)$$

Wigner d-matrix

$(\lambda = \lambda_1 - \lambda_2, \quad \lambda' = \lambda_3 - \lambda_4)$

Decomposition in terms of canonical *LS* states:

$$|J;\lambda_1\lambda_2\rangle = \sum_{LS} \sqrt{\frac{2L+1}{2J+1}} C^{J\lambda}_{L0S\lambda} C^{S\lambda}_{\frac{1}{2}\lambda_1\frac{1}{2}(-\lambda_2)} |J;LS\rangle$$

$$\mathscr{H}^J_{++} = \frac{1}{2}\mathscr{H}^{S=0,L=J}_{++} + \frac{J}{2(2J+1)}\mathscr{H}^{S=1,L=J-1}_{++} + \frac{J+1}{2(2J+1)}\mathscr{H}^{S=1,L=J+1}_{++}$$

RG equations for these amplitude evolve independently

Hsu,Schwetz (99);
Fujimoto (25)

Decoupling of the RG equations

RG equations for a helicity amplitude with different (S, L) evolve independently:

$$\frac{d\mathscr{H}_{++}^{S=0,L=J}}{dt} = -\frac{NZ(t)}{2}\left(\mathscr{H}_{++}^{S=0,L=J}\right)^2 - \frac{g^2}{3\mu^2}$$

$$\frac{d\mathscr{H}_{++}^{S=1,L=J\pm1}}{dt} = -\frac{NZ(t)}{6}\left(\mathscr{H}_{++}^{S=1,L=J\pm1}\right)^2 + \frac{g^2}{9\mu^2}$$

$$N = \mu^2/2\pi^2$$

$$Z(t) = \left(1 + \frac{g^2}{9\pi^2}t\right)^{-1}$$

The decoupling is guaranteed by the orthogonality of the Legendre polynomial:

$$\mathscr{H}_{++}^{S=0,L=J} = -(2g^2/3)(D_L^{\mathrm{E}} + 3D_L^{\mathrm{M}})$$

$$\mathscr{H}_{++}^{S=1,L=J\pm1} = -(2g^2/3)(D_L^{\mathrm{E}} - D_L^{\mathrm{M}})$$

NB: this is for color $\bar{\mathbf{3}}$ channel

Angular momentum decomposition

- **In vacuum, or conventionally,:**
  - J is an only good quantum number
  - S & L are not conserved separately due to Lorentz transformation

- **At finite density:**
  - the special rest frame of the dense medium
    → Lorentz invariance explicitly broken
  - rotational symmetry alone remains good symmetry
  - RG equations for different spin & orbital angular momentum channel decouple

- So, the pairing problem decouples between independent states labeled by:
  (color, flavor, incoming helicity, ${}^{2S+1}L_J$)

Classification

Constraint: total diquark wave function has to be antisymmetric

-1 = (color) x (flavor) x (spin) x $(-1)^L$

Table of the most attractive channels with largest pairing gap for a given color, flavor reps.:

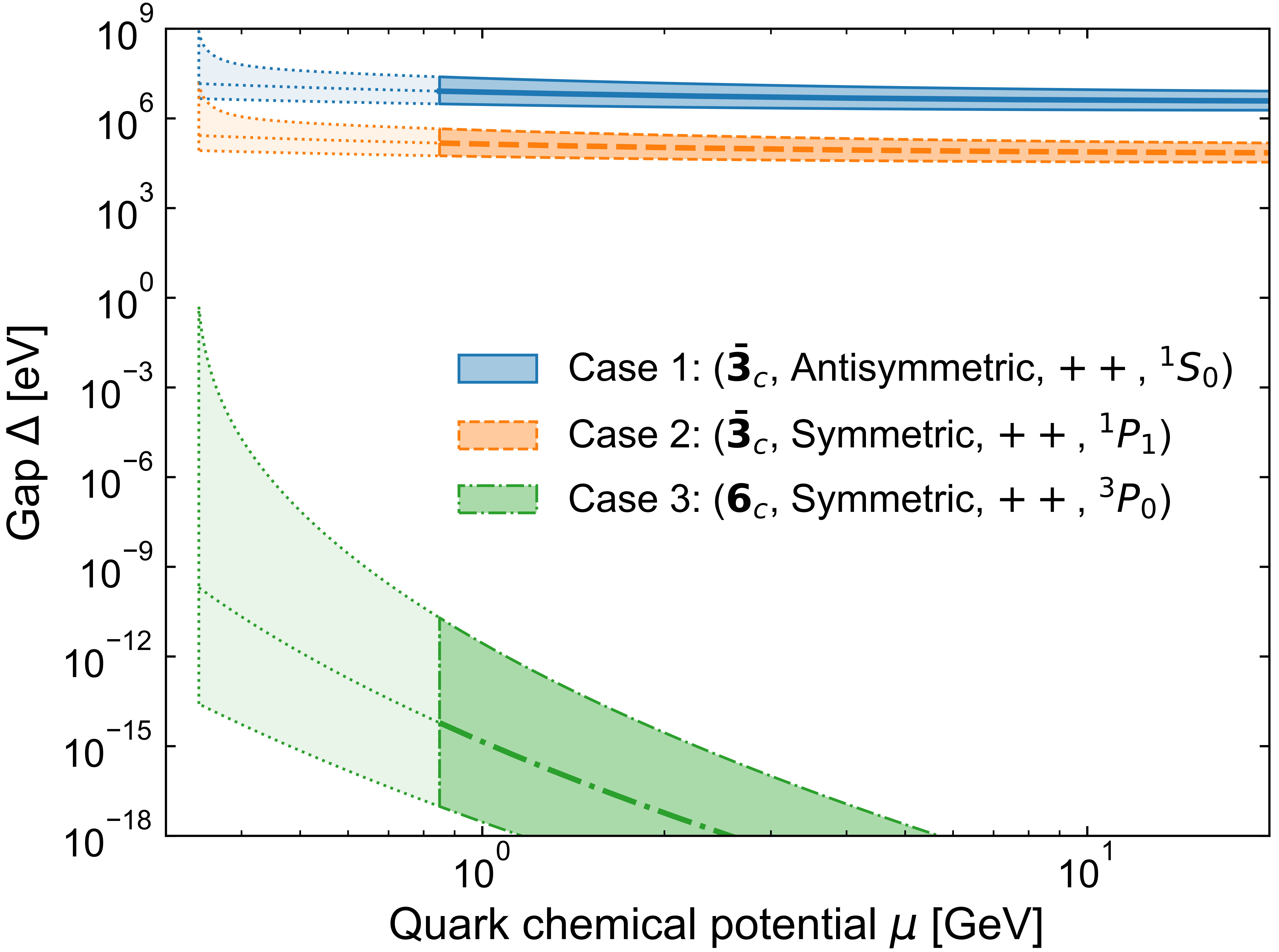
Color	Flavor	Helicity	$^{2S+1}L_J$
$\bar{\mathbf{3}}$	Antisymmetric	$++$	$^1S_0$
$\bar{\mathbf{3}}$	Symmetric	$++$	$^1P_1$
$\mathbf{6}$	Symmetric	$++$	$^3P_0$
$\mathbf{6}$	Antisymmetric	$++$	$^3S_1$

"good" diquark
CFL

"bad" diquark
Single-flavor pairing

very weak pairing (**repulsive** in vacuum)

Gap as a function of chemical potential

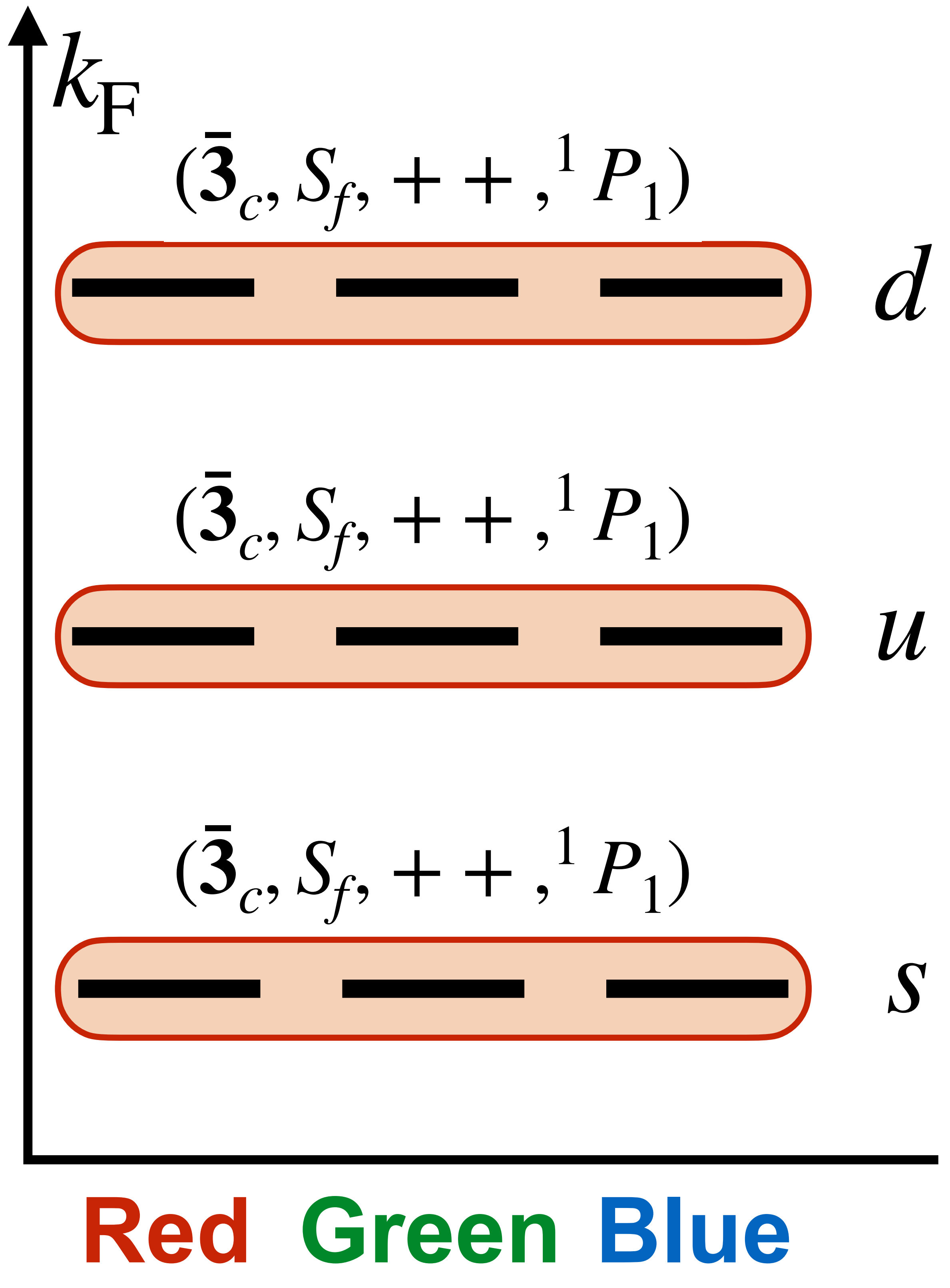


Pairing in weak coupling regime

Schafer (2000); Schmitt (2005); Fujimoto (2025)

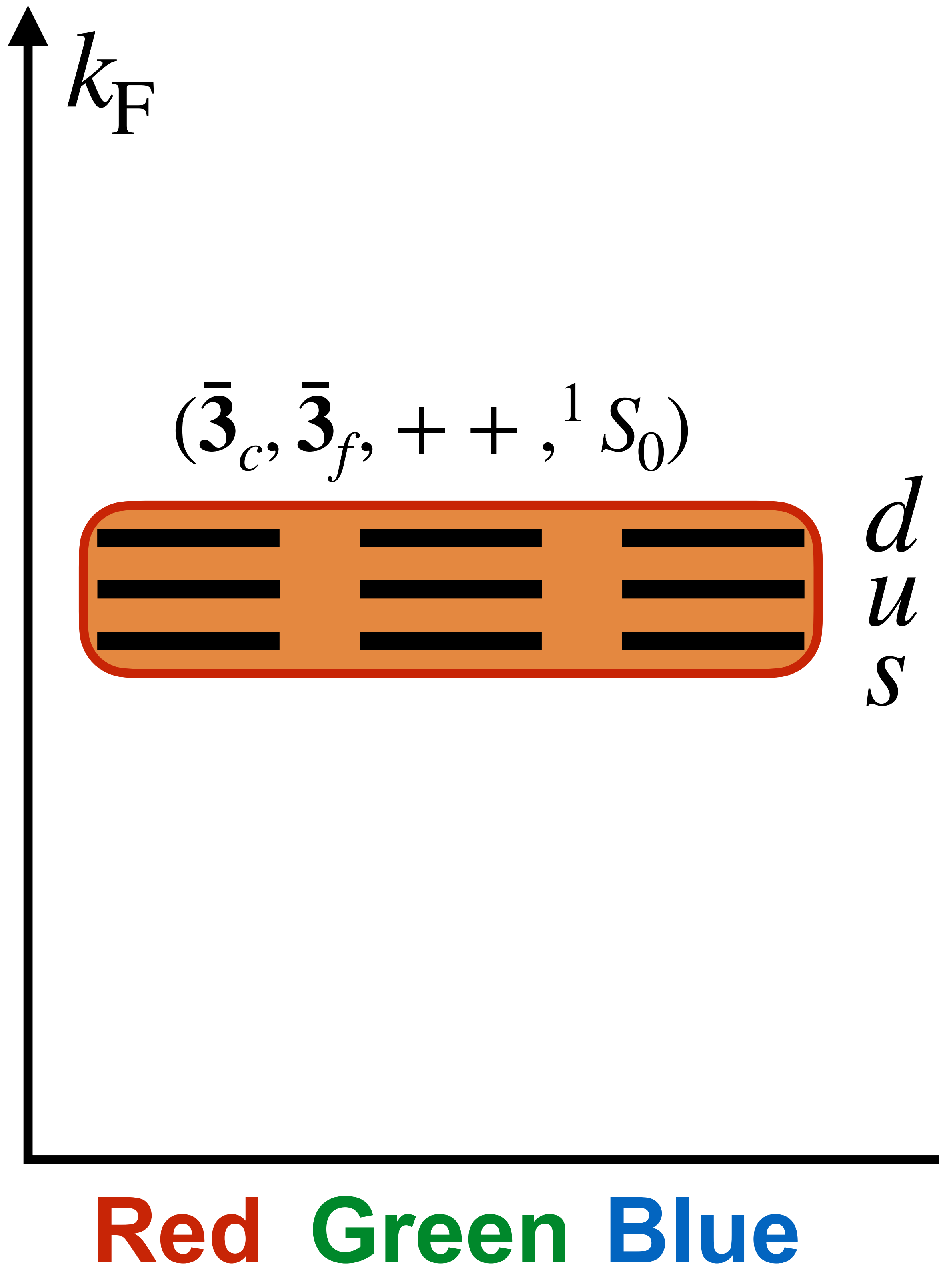
Schematic figure of the Fermi momenta:

Color-spin locked (CSL)



Lower μ_B
$\rightarrow$ strange quark mass tries to pull k_F of each quark apart

CFL



QCD phase diagram in the weak-coupling regime

Fujimoto, work in progress (2025)

Summary

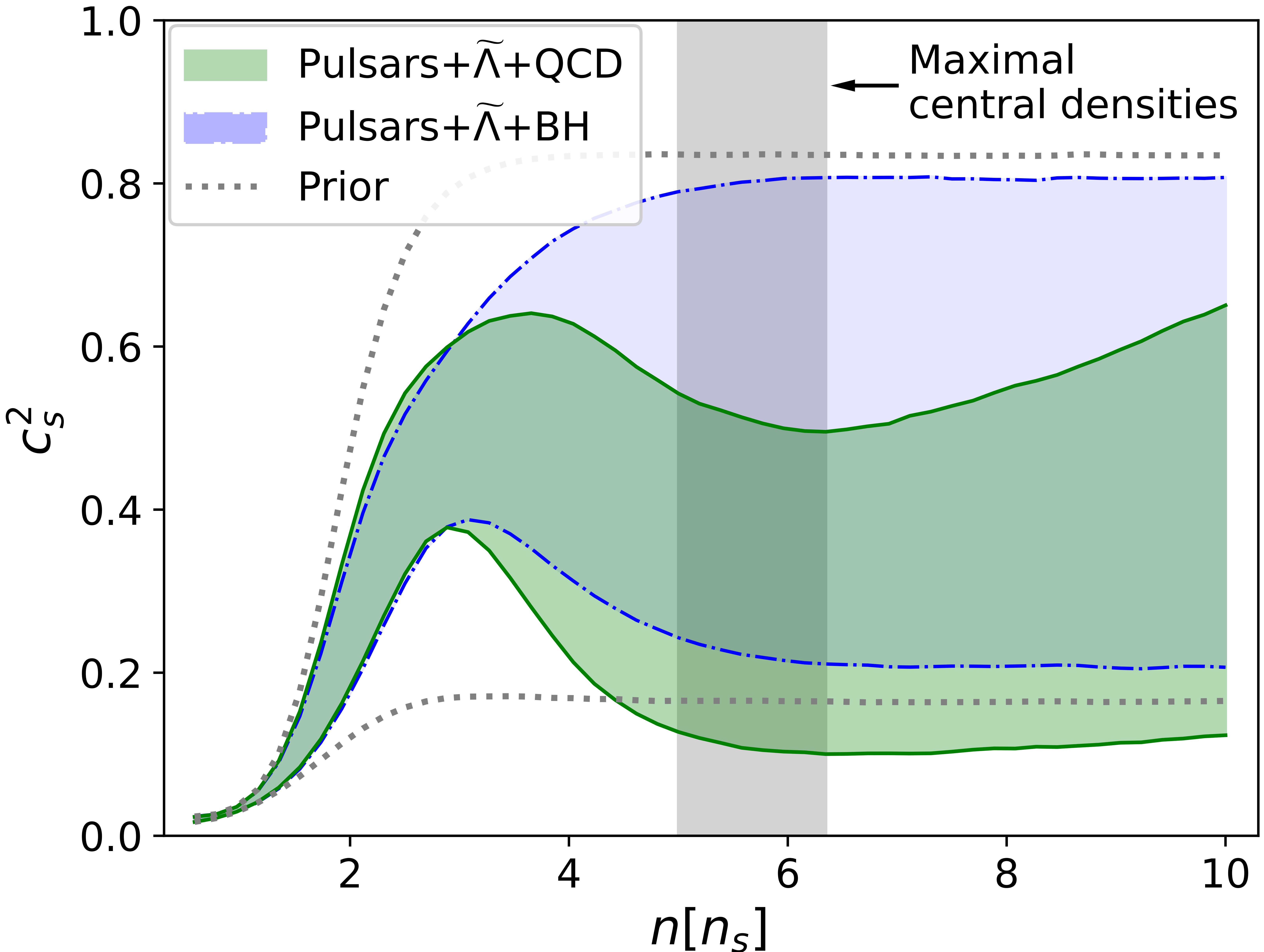
- Classification of color superconductivity by the color, flavor, helicity (chirality), term symbol ${}^{2S+1}L_J$ as in non-relativistic case

- Single-flavor pairing inevitable at lower density
  $\rightarrow$ color-spin locked phase?

- Determination of the superconductivity gap & the phase diagram in the weak coupling regime underway

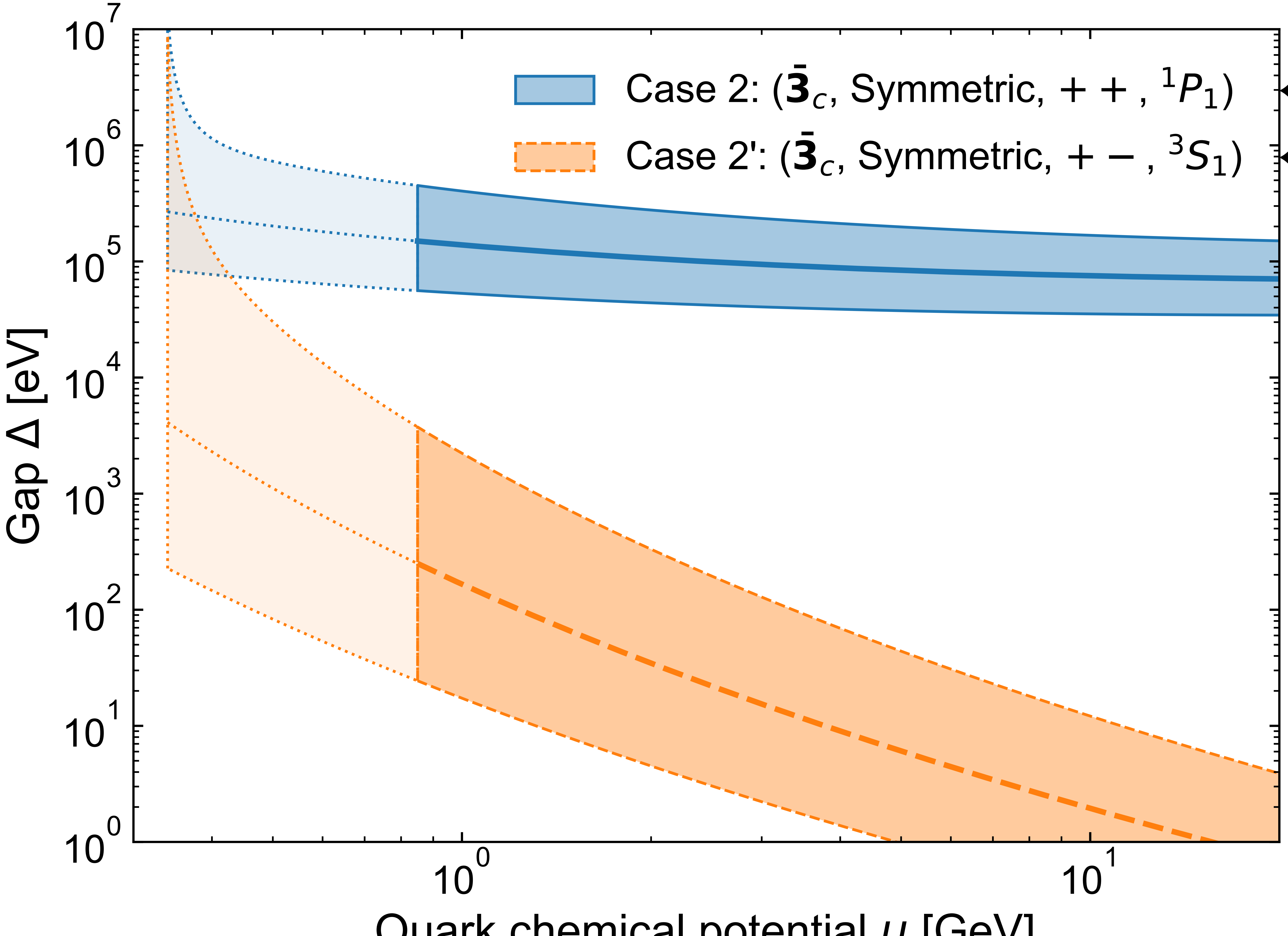
Bonus material

QCD constraint: speed of sound



Comparison of the gap

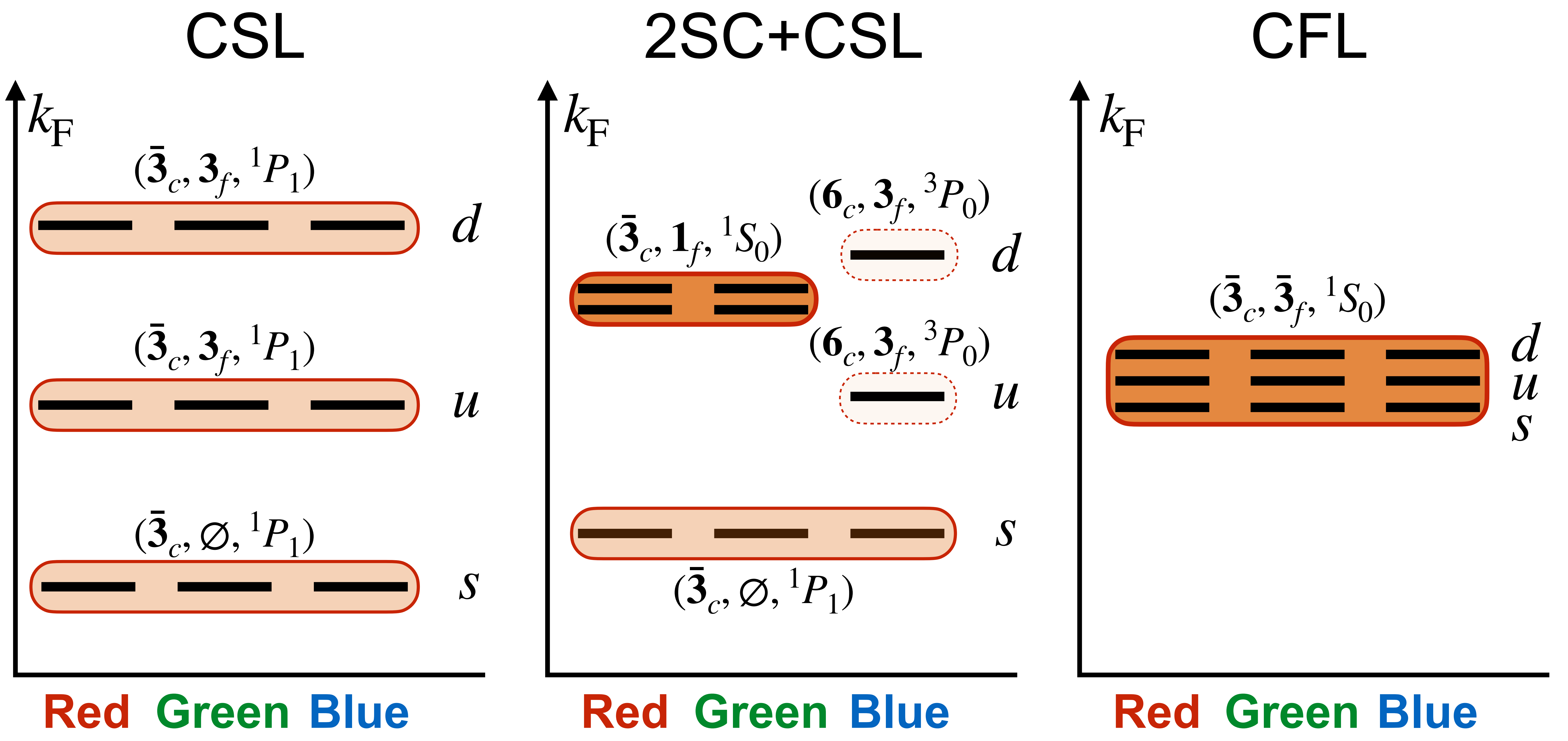
When (Color, flavor) = $(\bar{\mathbf{3}}_c, S_f)$, two possibilities: $(\text{helicity}, {}^{2S+1}L_J) = \begin{cases} (++, {}^1P_1) \\ (+-, {}^3S_1) \end{cases}$



← longitudinal
← transverse

Why CSL?

More pairing (in all the colors) is of course energetically favorable



$$\langle \psi_\alpha^\top C\gamma^5 \nabla^i \psi_\beta \rangle \propto \epsilon_{\alpha\beta\gamma}\phi^{\gamma i}$$

$$\phi^{\gamma i} = \delta^{\gamma i}\Delta_{{}^1P_1}$$

i: space

color cannot be gauge-rotated

$$\langle \psi_\alpha^\top C\gamma^5 \psi_\beta \rangle \propto \epsilon_{\alpha\beta 3}\Delta_{{}^1S_0}$$

α, β: color

color can always be gauge-rotated to 3rd direction

Helicity amplitudes in different channels

Color	Flavor	Helicity	${}^{2S+1}L_J$
$\bar{\mathbf{3}}$	Antisymmetric	$++$	${}^1S_0$
$\bar{\mathbf{3}}$	Symmetric	$++$	${}^1P_1$
$\mathbf{6}$	Symmetric	$++$	${}^3P_0$
$\mathbf{6}$	Antisymmetric	$++$	${}^3S_1$

$$\mathcal{H}^{{}^1S_0}_{++} = -\frac{2g^2}{3}\left(D^{\mathrm{E}}_0 + 3D^{\mathrm{M}}_0\right) .$$

$$\mathcal{H}^{{}^1P_1}_{++} = -\frac{2g^2}{3}\left(D^{\mathrm{E}}_1 + 3D^{\mathrm{M}}_1\right)$$

$$\mathcal{H}^{{}^3P_0}_{++} = -\frac{g^2}{3}\left(D^{\mathrm{M}}_1 - D^{\mathrm{E}}_1\right)$$