The inevitable quark three-body Force and its implications for exotic states

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- Introduction
- Why are Exotics interesting
- Quark model: failure with two-body
- Quark model: resolution using three-body
- Implications for Exotics

Acknowledgments:

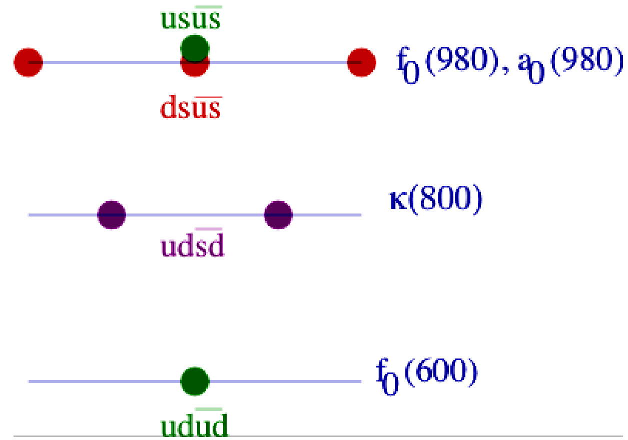
Previous works+

S. Noh, A. Park, H. Yun, S. Cho, SHL: PLB 862(2025)139278

Jongheon Baek, A. Park, S. Noh, H. Yun, K. Han, SHL, in preparation

Recent findings of Exotics: But an old topic

☞ Tetraquark:

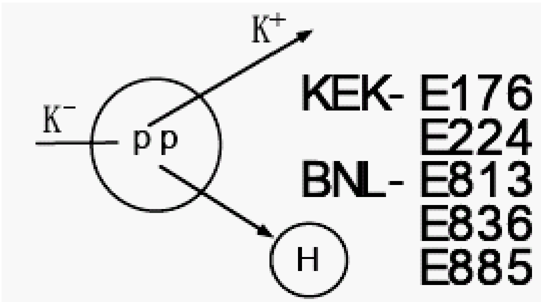


- scalar tetraquark (Jaffe 76)
  → Still controversial

But ALICE(Junlee Kim) analysis suggests

f_0 is most likely a $(\bar{q}q)$ without $(\bar{s}s)$

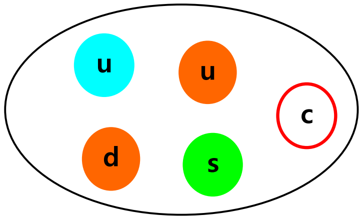
☞ Dibaryon



- H (ududss) dibaryon (Jaffe 77):
  → experimentally not found
- May be $\Lambda\bar{\Lambda}$

☞ Pentaquark

- $P_{\bar{c}s}$ (Gignoux, Silvestre-Brac, Richard 87)
- $P_{\bar{c}s}$ (udus$\bar{c}$) (Lipkin 87)
  → Fermilab E791 : not found $P^0_{\bar{c}s} \to K^{*0} K^- p$

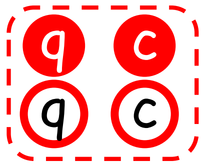
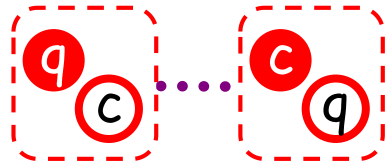
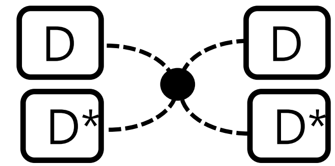


- Θ^+ (Diakonov, Petrov, Polyaov 97)
  → LEPS 2003 but not confirmed

Few examples of recent findings that could be probed in HIC

	Tetraquark	Mass	Quark content	2-body Threshold	Observed mode	Exp
Bound Near Threshold	$\chi_{c1}(3872)$ $X(3872)$	3871.65	$[c\bar{c}q\bar{q}]$	$\bar{D}^0 D^{*0}$(3871.69) $D^- D^{*+}$(3879.92)	$J/\psi\pi^-\pi^+$ …..	Belle ..
Bound Near Threshold	$T_{cc}(3875)$	3875	$[c\bar{u}c\bar{d}]$	$D^0 D^{*+}$(3875.26) $D^+ D^{*0}$(3876.51)	$D^0 D^0 \pi^+$	LHCb
Above Threshold	$T^{\theta}_{\psi s1}$(4000) $Z_{cs}(3872)$	4003+i(131)	$[c\bar{c}u\bar{s}]$	$\bar{D}^0 D_s^{*+}$ (3977) $J/\psi K^+$(3590.58)	$J/\psi K^+$	LHCb (BES?)
Above Threshold	$X(5568)$	5568+i(21.9)	$[b\bar{d}u\bar{s}]$	$B^0 K^+$(5773) $B_s^0 \pi^\pm$ (5506.49)	$B_s^0 \pi^\pm$	D0
Above Threshold	$T^{a}_{c\bar{s}0}$(2900)	2908+i(136)	$[c\bar{s}u\bar{d}]$	2251.77	$D_s^+ \pi^+$	LHCb
Above Threshold	X(6600) X(6900)		$[c\bar{c}c\bar{c}]$	6193.8 MeV	$J/\psi J/\psi$	CMS LHCb

Types of Exotic particles

	Compact multiquark	Molecule	Resonance
Picture			
Size Threshold width	$\langle r \rangle <$ 0.6 fm Near threshold or other small	$\langle r \rangle >$ 2 fm Near threshold small	$\langle r \rangle \sim$ 1 fm Above threshold or other large
Typical mode l used	Quark Model	Meson exchange models	Unitary approach Quark model
	Effective field theory: constants QCD sum rules: uncertainty		

- In some cases, two pictures seem possible. Compact and Molecular
- Yet, there are common features to exotics not seen in usual hadrons

Why are exotics interesting?

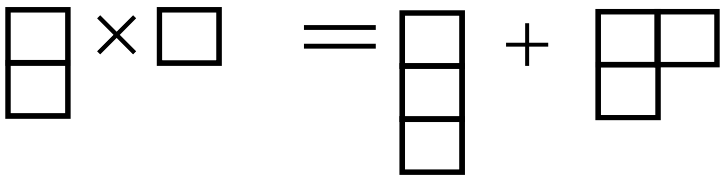
- A New color configuration

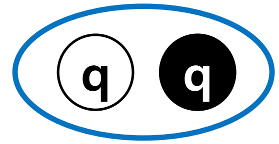
- Gateway to multi-quark states and high-density

So far only $(q\bar{q})_{C=1}$, $(qq)_{C=\bar{3}}$ color states are seen

- Color state of Meson: $\bar{3}\times 3 = 1 + 8$

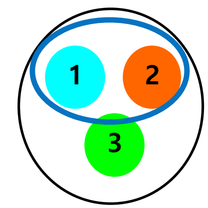
$(\bar{q})_{C=\bar{3}} \times (q)_{C=3}$ $\quad$ $(q\bar{q})_{C=1}$ $\quad$ $(q\bar{q})_{C=8}$





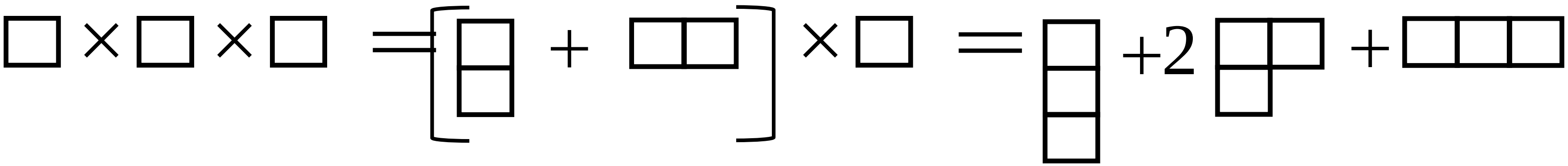
$(q\bar{q})_{C=1}$

- Color state of Baryon: $3\times 3\times 3 = (\bar{3}+6)\times 3 = 1 + 2\cdot 8 + 10$



$(qq)_{C=\bar{3}}$

$(q)_{C=3} \times (q)_{C=3} \times (q)_{C=3}$ $\quad$ $(qq)_{C=\bar{3}}$ $\quad$ $(qq)_{C=6}$ $\quad$ $(qq)_{C=\bar{3}}$



- But, Color states $(q\bar{q})_{C=8}$ and $(qq)_{C=6}$ are attractive in the Spin=1 channel

A new color configuration of SU(3)

☞ Usual ground state hadron $(q\bar{q})_{C=1}$ $(qq)_{C=\bar{3}}$ or $(\overline{qq})_{C=3}$

☞ But Exotics contain additional color configurations with higher degeneracy
For example: Tetraquark state

$$3\times 3\times \bar{3}\times \bar{3} = (\bar{3}+6)\times(3+\bar{6}) = 3\times\bar{3}+6\times\bar{6}+\ldots$$

$$(qq)_{C=\bar{3}}\otimes(\overline{qq})_{C=3} \quad \text{and} \quad (qq)_{C=6}\otimes(\overline{qq})_{C=\bar{6}}$$

degeneracy: 3×3 and 6×6

$$3\times\bar{3}\times 3\times\bar{3} = (1+8)\times(1+8) = 1\times 1+8\times 8+\ldots$$

$$(q\bar{q})_{C=1}\otimes(q\bar{q})_{C=1} \quad \text{and} \quad (q\bar{q})_{C=8}\otimes(q\bar{q})_{C=8}$$

degeneracy: 1×1 and 8×8

Also, Wisdom from Nuclear Physics

❑ Need of multi-nucleon force (Lessons from G.E. Brown, S.N. Yang ...)

1. Nucleon
2. Deuteron: pion-exchange + D-wave mixing
3. Triton, He3: Need extra three-body force
4. Also, 4-body force for N>3 system

❑ Not much work on quark 3-body force

1. Meson, quark-antiquark
2. Baryon: 3-quark
3. Tetraquark: 2 quark+2 antiquark
4. Penta-quark: Also, 4-body force for N>3 system

Quark Model

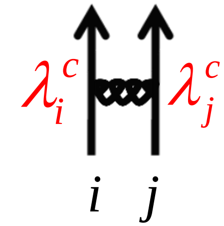
- Two-body quark force: color-color and color-spin interaction

- Failure

Quark Model: two-body interaction

❑ Color-Color interaction:

$$H = \sum_{i=1}^{n}\left(m_i + \frac{p_i^2}{2m_i}\right) - \sum_{i<j}^{n}\left(\lambda_i^c\lambda_j^c\right)V_{ij}^C\left(r_{ij}\right) - \sum_{i<j}^{n}\frac{\left(\lambda_i^c\lambda_j^c\right)\left(\sigma_i\sigma_j\right)}{m_i m_j}V_{ij}^{SS}\left(r_{ij}\right)$$



$$V_{ij}^C\left(r_{ij}\right) = \left(-\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0} - D\right)$$

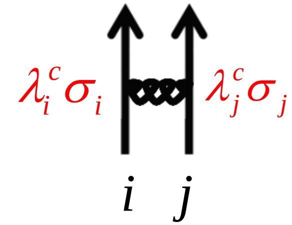
☞ Color-Color factors: important only within a color singlet configuration

$$\sum_{i<j}^{N}\left(\lambda_i^c\lambda_j^c\right) = \frac{1}{2}\left[\left(\lambda_1^c + \ldots + \lambda_N^c\right)^2 - \lambda_1^2 - \ldots\lambda_N^2\right] = 0 - \frac{8}{3}\left(N_{B_1} + N_{B_2}\right) = \sum_{i<j}^{N_{B_1}}\left(\lambda_i^c\lambda_j^c\right) + \sum_{i<j}^{N_{B_2}}\left(\lambda_i^c\lambda_j^c\right)$$

$$N = N_{B_1} + N_{B_2}$$

❑ Color-Spin:

$$H=\sum_{i=1}^{n}\left(m_i+\frac{p_i^2}{2m_i}\right)-\sum_{i<j}^{n}\left(\lambda_i^c\lambda_j^c\right)V_{ij}^{C}\left(r_{ij}\right)-\sum_{i<j}^{n}\frac{\left(\lambda_i^c\lambda_j^c\right)\left(\sigma_i\sigma_j\right)}{m_i m_j}V_{ij}^{CS}\left(r_{ij}\right)$$



$$V_{ij}^{CS}=\frac{\hbar^2\kappa'_{ij}}{m_i m_j c^2 r_{0ij}}\frac{\exp\{-(r_{ij}/r_{0ij})^2\}}{r_{ij}}F_i^c F_j^c\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j,\qquad r_{0ij}=\left(\alpha+\frac{\beta\, m_i\, m_j}{m_i+m_j}\right)^{-1},\quad \kappa'_{ij}=\kappa_0\left(1+\frac{\gamma\, m_i\, m_j}{m_i+m_j}\right)$$

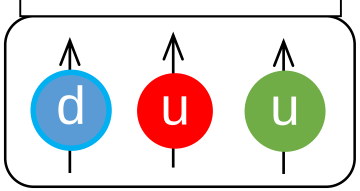
☞ Color-spin factors

	Q-Q				$Q\text{-}\bar{Q}$			
Color	$\bar{3}$	6	$\bar{3}$	6	1	8	1	8
Flavor	A	A	S	S				
Spin	A(0)	S(1)	S(1)	A(0)	0	0	1	1
$K=-\left(\lambda_i^c\lambda_j^c\right)\left(\sigma_i^s\sigma_j^s\right)$	-8	-4/3	8/3	4	-16	2	16/3	-2/3

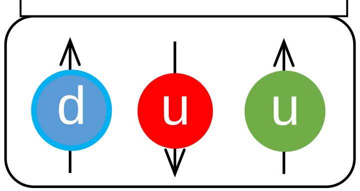
$K<0$ attraction; $K>0$ repulsion

$$M_\Delta-M_P\approx[+8-(-8)]\times\left[\int\frac{dx_{12}}{m_1m_2}V_{12}^{SS}\left(x_{12}\right)\left|\psi\left(x_{12}\right)\right|^2\right]\approx 290\text{ MeV}\qquad K=1\ \rightarrow\ 18\text{ MeV}$$

Delta $K=+8$



Proton $K=-8$



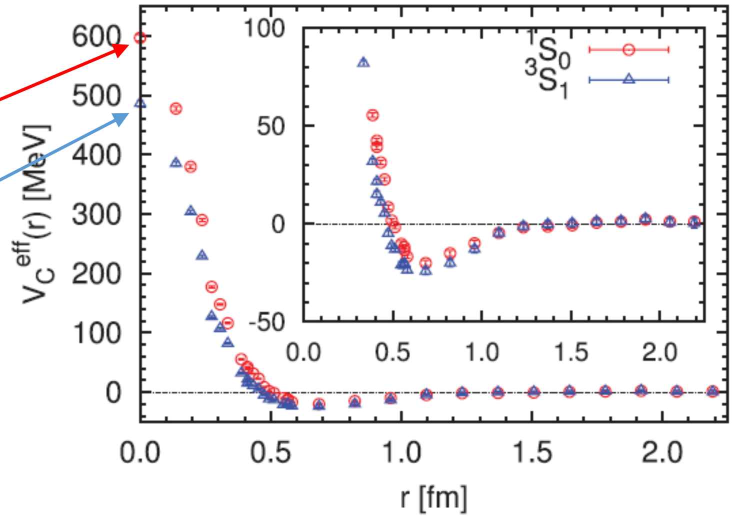
Quark Model vs Lattice comparison : A.Park, Lee, Inoue, Hatsuda, EPJA 56(2020)3,93

☞ NN force in SU(2) spin 1 vs spin 0 channel: comparison to lattice

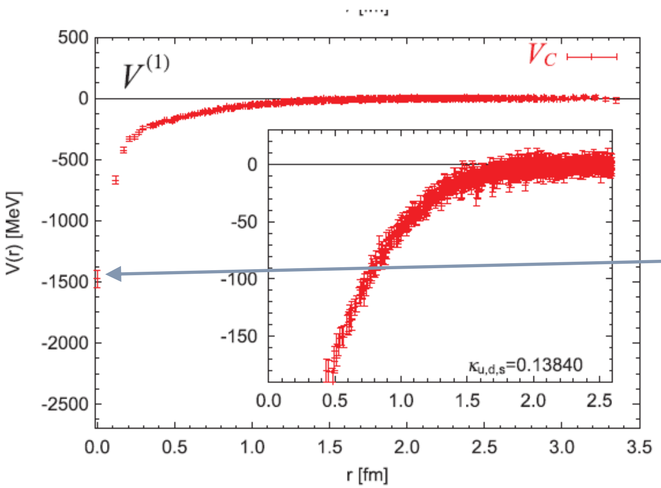
$$K_{2-N} = K_{6-quark} - \left(K_{1N} + K_{1N} \right)$$

$$\frac{K_{2-N}^{S=0}}{K_{2-N}^{S=1}} = 1.29$$ → comparison

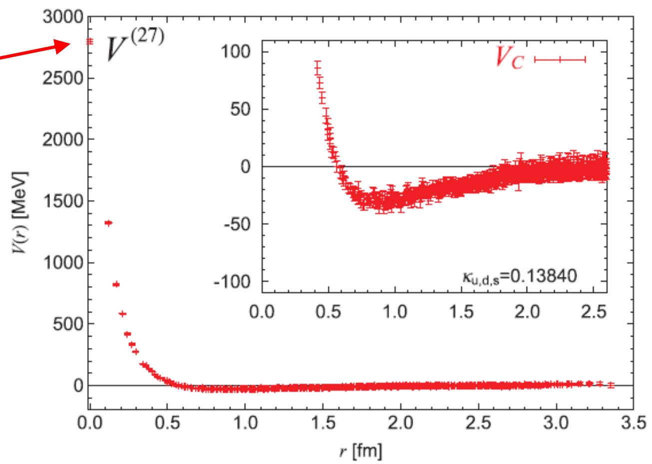
QCD HAL collaboration



☞ H dibaryon channel: Flavor 1 vs Flavor 27



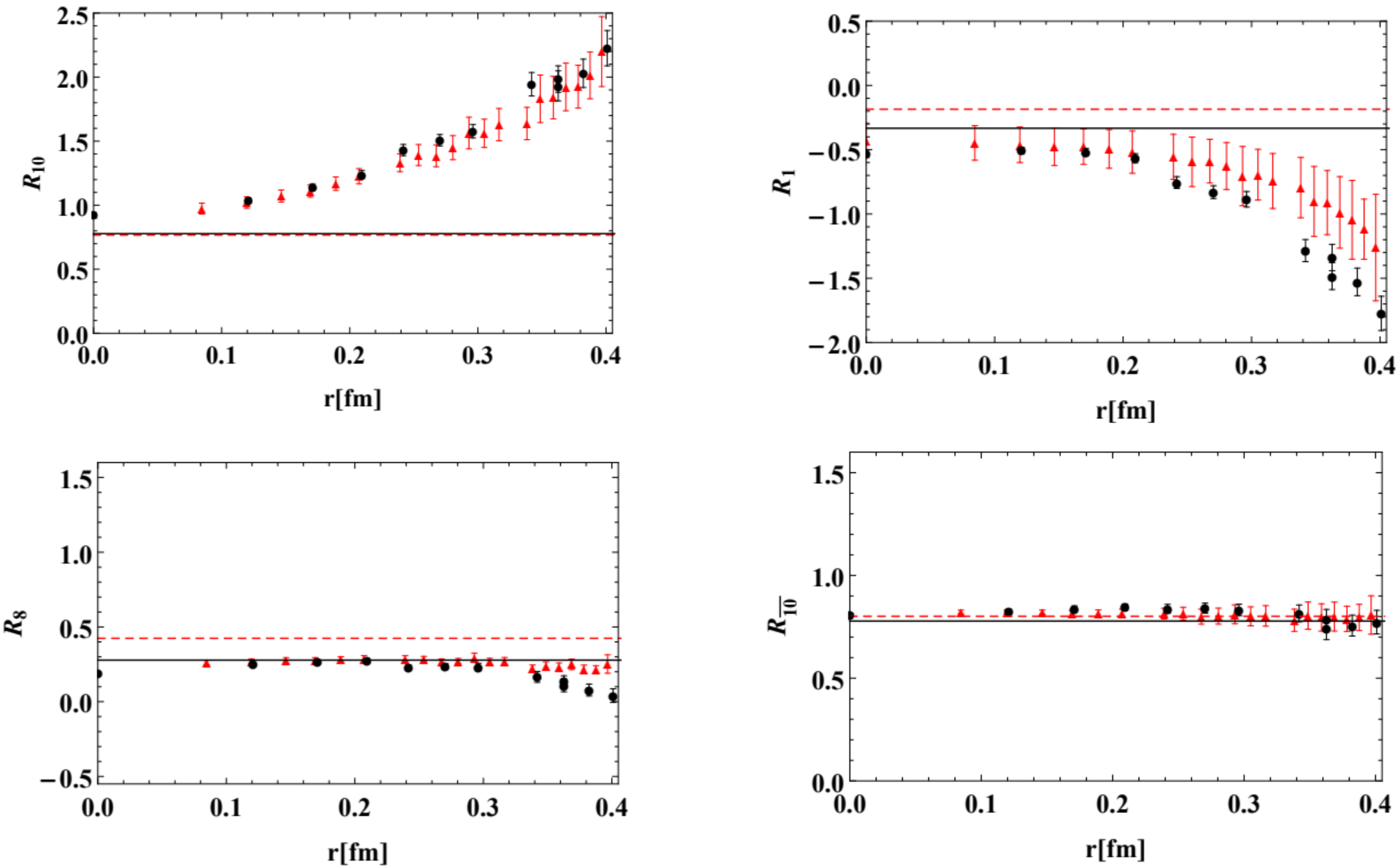
$$\frac{K_{2-N}^{F=27}}{K_{2-N}^{F=1}} = -3$$



(HAL QCD Collaboration)

☞ Full quark model calculation vs Lattice in SU(3) A.Park, Lee, Inoue, Hatsuda, EPJA 56(2020)3,93

$$\mathcal{R}_\ell^{\rm CQM} = \frac{V_{\rm CQM}(F_\ell)}{V_{\rm CQM}(F_{27})}. \qquad \mathcal{R}_\ell^{\rm LQCD} = \frac{V_{\rm LQCD}(F_\ell)}{V_{\rm LQCD}(F_{27})}.$$



Use this quark model to Fit the meson spectrum

- S. Noh et al. PLB862 (2025) 139278: Single Gaussian wave function

$$\sigma = \left(\frac{1}{N-1}\sum_{i=1}^{N}\left(M_i^{Thr} - M_i^{Exp}\right)^2\right)^{1/2} = 5.86 \text{ MeV}$$

Particle	Experimental Value (MeV)	Mass (MeV)	Variational Parameter (fm^{-2})
η_c	2983.6	2996.9	$a = 13.1$
J/ψ	3096.9	3089.6	$a = 11.1$
D	1864.8	1864.1	$a = 4.5$
D^*	2010.3	2010.7	$a = 3.7$
π	139.57	139.39	$a = 4.6$
ρ	775.11	775.49	$a = 2.2$
K	493.68	494.62	$a = 4.6$
K^*	891.66	888.82	$a = 2.8$

- Fit using Exact wave function: Jongheon Baek et al. in preparation

$$\sigma_{\text{Meson}} = 1.36 \text{ MeV}$$

Parameters

$\kappa = 85.2189$ MeV fm $a_0 = 0.0298683$ (fm/MeV)$^{1/2}$

$D = 1082.19$ MeV $\alpha = 0.911659$ fm^{-1}

$m_{ud} = 316.551$ MeV $\beta = 0.000906491$ MeV^{-1}

$m_s = 595.328$ MeV $\gamma = 0.00120125$ (MeV fm)$^{-1}$

$m_c = 1882.96$ MeV $\kappa_0 = 207.473$ MeV

Meson ($\mathcal{J}^{P(C)}$)	Experimental Value (MeV)	1 Gaussian ($\sigma = 5.86$)	30 Gaussians ($\sigma = 1.36$)
$\eta_c(0^{-+})$	2983.6	2996.9(+13.3)	2984.0(+0.4)
$J/\Psi(1^{--})$	3096.9	3089.6(−7.3)	3095.8(−1.1)
$D(0^-)$	1864.8	1864.1(−0.7)	1863.5(−1.3)
$D^*(1^-)$	2010.3	2010.7(+0.4)	2013.3(+3.0)
$\pi(0^{-+})$	139.57	139.39(−0.18)	139.58(+0.01)
$\rho(1^{--})$	775.11	775.49(+0.38)	774.17(−0.94)
$K(0^-)$	493.68	494.62(+0.94)	493.77(+0.09)
$K^*(1^-)$	891.66	888.82(−2.84)	891.45(−0.21)

When using parameters from meson fit to calculate Baryons

- S. Noh et al. PLB862 (2025) 139278
  Single Gaussian wave function

$$\sigma_{\text{Meson}} = 5.86\ \text{MeV} \rightarrow \sigma_{\text{Baryon}} = 59.26\ \text{MeV}$$

Particle	Experimental Value (MeV)	Mass (MeV)	Variational Parameters (fm^{-2})
Λ_c	2286.5	(2281.6)	$a_1 = 2.9, a_2 = 3.7$
Σ_c	2452.9	(2480.9)	$a_1 = 2.1, a_2 = 3.8$
Λ	1115.7	(1134.1)	$a_1 = 2.8, a_2 = 2.7$
Σ	1192.6	(1231.6)	$a_1 = 2.1, a_2 = 3.1$
Ξ_{cc}	3621.2	(3606.3)	$a_1 = 7.6, a_2 = 3.1$
Σ_c^*	2518.5	(2567.7)	$a_1 = 2.0, a_2 = 3.4$
Σ^*	1383.7	(1455.2)	$a_1 = 1.9, a_2 = 2.4$
p	938.27	(1005.3)	$a_1 = 2.4, a_2 = 2.4$
Δ	1232	(1346.8)	$a_1 = 1.8, a_2 = 1.8$

- Exact wave function: Jongheon Baek et al. in preparation

$$\sigma_{\text{Meson}} = 1.36\ \text{MeV} \rightarrow \sigma_{\text{Baryon}} = 46.28\ \text{MeV}$$

Parameters from meson fit

$\kappa = 85.2189$ MeV fm, $a_0 = 0.0298683\ (\text{fm/MeV})^{1/2}$

$D = 1082.19$ MeV, $\alpha = 0.911659\ \text{fm}^{-1}$

$m_{ud} = 316.551$ MeV, $\beta = 0.000906491\ \text{MeV}^{-1}$

$m_s = 595.328$ MeV, $\gamma = 0.00120125\ (\text{MeV fm})^{-1}$

$m_c = 1882.96$ MeV, $\kappa_0 = 207.473$ MeV

Baryon ($M_{\text{exp}}, \mathcal{J}^P$)	M_S (MeV) ($\sigma = 72.13$)	$M_{S,P,D}^{\text{GEM}}$ (MeV) ($\sigma = 46.28$)	ΔM (MeV)
$p(938.27, 1/2^+)$	1036.17(+97.90)	997.56(+59.29)	-38.61
$\Delta(1232, 3/2^+)$	1348.35(+116.35)	1313.42(+81.42)	-37.93
$\Sigma(1197.45, 1/2^+)$	1255.90(+58.45)	1231.40(+33.95)	-24.50
$\Lambda(1115.68, 1/2^+)$	1168.64(+52.96)	1124.22(+8.54)	-44.42
$\Sigma_c(2453.97, 1/2^+)$	2495.92(+41.95)	2476.24(+22.27)	-19.68
$\Lambda_c(2286.46, 1/2^+)$	2315.78(+29.32)	2294.00(+7.54)	-21.78
$\Xi_{cc}(3621.46, ?^?)$[a]	3630.22(+8.76)	3619.29(−2.17)	-10.93
$\Sigma^*(1383.7, 3/2^+)$	1462.16(+78.46)	1433.87(+50.17)	-28.28
$\Sigma_c^*(2518.48, 3/2^+)$	2587.03(+68.55)	2570.33(+51.85)	-16.70

Failure when going from meson to baryon Example 1

- Different quark mass for meson and baryon

- M. Karliner, J.L. Rosner, PRD90 (2014)094007

TABLE I. Quark model description of ground-state baryons containing u, d, s. Here we take $m_u^b = m_d^b \equiv m_q^b = 363$ MeV, $m_s^b = 538$ MeV, and hyperfine interaction term $a/(m_q^b)^2 = 50$ MeV.

State (mass in MeV)	Spin	Expression for mass [24]	Predicted mass (MeV)
$N(939)$	1/2	$3m_q^b - 3a/(m_q^b)^2$	939
$\Delta(1232)$	3/2	$3m_q^b + 3a/(m_q^b)^2$	1239
$\Lambda(1116)$	1/2	$2m_q^b + m_s^b - 3a/(m_q^b)^2$	1114
$\Sigma(1193)$	1/2	$2m_q^b + m_s^b + a/(m_q^b)^2 - 4a/m_q^b m_s^b$	1179
$\Sigma(1385)$	3/2	$2m_q^b + m_s^b + a/(m_q^b)^2 + 2a/m_q^b m_s^b$	1381
$\Xi(1318)$	1/2	$2m_s^b + m_q^b + a/(m_s^b)^2 - 4a/m_q^b m_s^b$	1327
$\Xi(1530)$	3/2	$2m_s^b + m_q^b + a/(m_s^b)^2 + 2a/m_q^b m_s^b$	1529
$\Omega(1672)$	3/2	$3m_s^b + 3a/(m_s^b)^2$	1682

TABLE II. Quark model description of ground-state mesons containing u, d, s. Here we take $m_u^m = m_d^m \equiv m_q^m = 310$ MeV, $m_s^m = 483$ MeV, $b/(m_q^m)^2 = 80$ MeV.

State (mass in MeV)	Spin	Expression for mass [24]	Predicted mass (MeV)
$\pi(138)$	0	$2m_q^m - 6b/(m_q^m)^2$	140
$\rho(775)$, $\omega(782)$	1	$2m_q^m + 2b/(m_q^m)^2$	780
$K(496)$	0	$m_q^m + m_s^m - 6b/(m_q^m m_s^m)$	485
$K^*(894)$	1	$m_q^m + m_s^m + 2b/(m_q^m m_s^m)$	896
$\phi(1019)$	1	$2m_s^m + 2b/(m_s^m)^2$	1032

Failure when going from meson to baryon Example 2

- Not quite consistent between meson and baryon

- SHLee, S. Yasui, W.Liu, Che-Ming Ko, EPJ C54 (2008) 259

Table 1. Baryon mass differences. The first column is a fit to the approximate difference between experimental Δ and N masses. Units are in MeV

Diff.	$\Delta - N$	$\Sigma - \Lambda$	$\Sigma_c - \Lambda_c$	$\Sigma_b - \Lambda_b$
Form.	$\frac{3C_B}{2m_u^2}$	$\frac{C_B}{m_u^2}(1-\frac{m_u}{m_s})$	$\frac{C_B}{m_u^2}(1-\frac{m_u}{m_c})$	$\frac{C_B}{m_u^2}(1-\frac{m_u}{m_b})$
Fit	290	77	154	180
Exp.	290	75	170	192

$$\left[\int \frac{dx_{12}}{m_1 m_2} V_{12}^{SS}(x_{12}) |\psi(x_{12})|^2\right] \approx \text{same for all quarks}$$

Table 2. Meson mass differences. The first column is a fit to the approximate difference between experimental ρ and π masses. Units are in MeV

Diff.	$\rho - \pi$	$K^* - K$	$D^* - D$	$B^* - B$
Form.	$\frac{C_M}{m_u^2}$	$\frac{C_M}{m_u m_s}$	$\frac{C_M}{m_u m_c}$	$\frac{C_M}{m_u m_b}$
Fit	635	381	127	41
Exp.	635	397	137	46

$$\frac{C_M}{C_B} = 3.28$$

- But quark model with only 2-body interaction leads to $\frac{C_M}{C_B} = 2$

Quark Model

- Nuclear Physics lesson: from Deuteron to Triton need three-body force

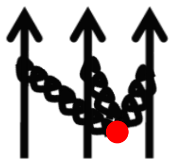
- Three-body quark force: Lessons from Nuclear Physics

- Resolution of the Failure

Quark-Three-body force Early works

- V. Dmitrasinovic, PLB499(2001)135: S. Pepin, Fl Stancu, PRD65 (2002)

$$C_{123}=\begin{cases} d^{abc}F_1^aF_2^bF_3^c \\ if^{abc}F_1^aF_2^bF_3^c \end{cases}, \quad \text{where} \quad F_i^a=\frac{\lambda_i^a}{2} \text{ for quark } i$$



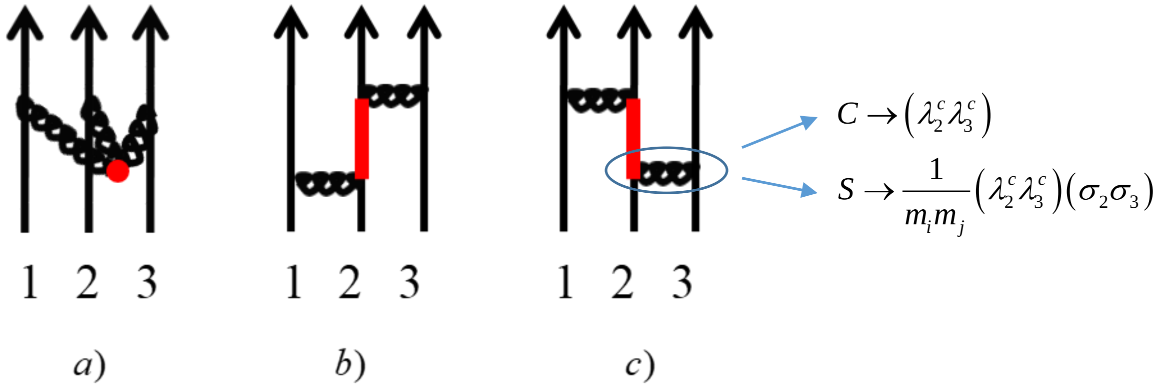
$$\left[F^a,F^b\right]=if^{abc}F^c \quad \text{and} \quad \left\{F^a,F^b\right\}=\frac{1}{3}\delta^{ab}+d^{abc}F^c$$

- For a Baryon, $C_{123}=\begin{cases} d^{abc}F_1^aF_2^bF_3^c=\frac{10}{9} \\ if^{abc}F_1^aF_2^bF_3^c=0 \end{cases}$, does not improve Δ-N mass splitting

- When anti-quark are involved, $\tilde{C}_{123}=\begin{cases} -d^{abc}F_1^aF_2^b\bar{F}_3^c \\ d^{abc}F_1^a\bar{F}_2^b\bar{F}_3^c \\ -d^{abc}\bar{F}_1^a\bar{F}_2^b\bar{F}_3^c \end{cases}$, '-' put in by hand

$$\left[\bar{F}^a,\bar{F}^b\right]=if^{abc}\bar{F}^c \quad \text{and} \quad \left\{\bar{F}^a,\bar{F}^b\right\}=\frac{1}{3}\delta^{ab}-d^{abc}\bar{F}^c$$

- Origin could be similar to Nuclear-Three-body force

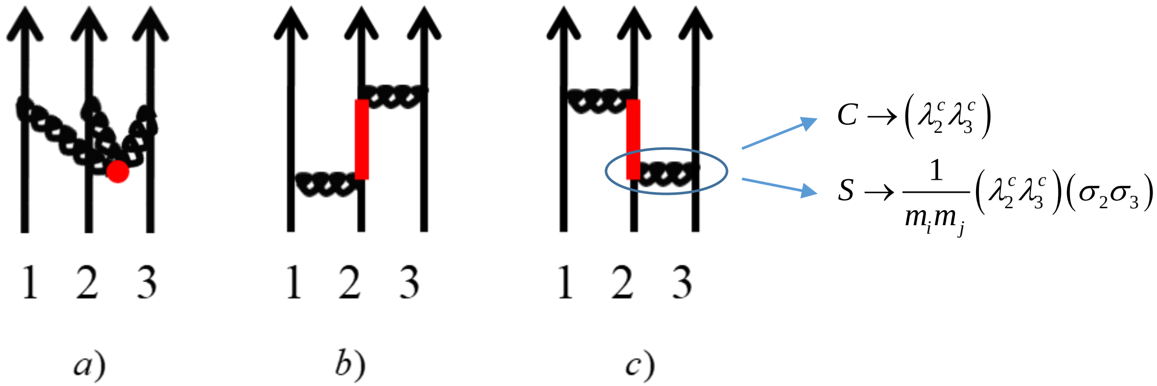


$$L_{123}^{C-C} = \frac{4}{3}\left(\frac{\lambda_2^c\lambda_3^c}{m_1} + \frac{\lambda_1^c\lambda_3^c}{m_2} + \frac{\lambda_1^c\lambda_2^c}{m_3}\right) + 2d^{abc}(\lambda_1^a\lambda_2^b\lambda_3^c)\left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3}\right)$$

$$L_{123}^{S-S} = \frac{1}{m_1m_2m_3}\left[\frac{4}{3}\left(\frac{(\sigma_2\cdot\sigma_3)(\lambda_2^c\lambda_3^c)}{m_1^2} + \frac{(\sigma_1\cdot\sigma_3)(\lambda_1^c\lambda_3^c)}{m_2^2} + \frac{(\sigma_1\cdot\sigma_2)(\lambda_1^c\lambda_2^c)}{m_3^2}\right)\right.$$
$$\left. + 2d^{abc}(\lambda_1^a\lambda_2^b\lambda_3^c)\left(\frac{\sigma_2\cdot\sigma_3}{m_1^2} + \frac{\sigma_1\cdot\sigma_3}{m_2^2} + \frac{\sigma_1\cdot\sigma_2}{m_3^2}\right) - 2\epsilon_{ijk}\sigma_1^i\sigma_2^j\sigma_3^k f^{abc}\lambda_1^a\lambda_2^b\lambda_3^c\left(\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{1}{m_3^2}\right)\right].$$

$$L_{123}^{C-S} = \frac{4}{3}\left[\frac{(\lambda_1^c\lambda_3^c)}{m_2}\left(\frac{\sigma_2\cdot\sigma_3}{m_2m_3} + \frac{\sigma_1\cdot\sigma_2}{m_2m_1}\right) + \frac{(\lambda_1^c\lambda_2^c)}{m_3}\left(\frac{\sigma_3\cdot\sigma_2}{m_2m_3} + \frac{\sigma_1\cdot\sigma_3}{m_3m_1}\right) + \frac{(\lambda_2^c\lambda_3^c)}{m_1}\left(\frac{\sigma_1\cdot\sigma_3}{m_1m_3} + \frac{\sigma_1\cdot\sigma_2}{m_2m_1}\right)\right]$$
$$+ 2d_{abc}(\lambda_1^a\lambda_2^b\lambda_3^b)\left[\frac{1}{m_2}\left(\frac{\sigma_2\cdot\sigma_3}{m_2m_3} + \frac{\sigma_1\cdot\sigma_2}{m_2m_1}\right) + \frac{1}{m_3}\left(\frac{\sigma_3\cdot\sigma_2}{m_2m_3} + \frac{\sigma_1\cdot\sigma_3}{m_3m_1}\right) + \frac{1}{m_1}\left(\frac{\sigma_1\cdot\sigma_3}{m_1m_3} + \frac{\sigma_1\cdot\sigma_2}{m_2m_1}\right)\right].$$

- Natural explanation for “-” sign



- For example $L_{123}^{C-C} = \frac{4}{3}\left(\frac{\lambda_2^c\lambda_3^c}{m_1} + \frac{\lambda_1^c\lambda_3^c}{m_2} + \frac{\lambda_1^c\lambda_2^c}{m_3}\right) + 2d^{abc}(\lambda_1^a\lambda_2^b\lambda_3^c)\left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3}\right),$

  the second terms comes from

$$\left[(\lambda_1^a\lambda_2^a)\frac{1}{m_2}(\lambda_2^b\lambda_3^b) + (\lambda_3^b\lambda_2^b)\frac{1}{m_2}(\lambda_2^a\lambda_1^a)\right] = \frac{1}{m_2}\left[\lambda_1^a\left(\lambda_2^a\lambda_2^b + \lambda_2^b\lambda_2^a\right)\lambda_3^b\right] \qquad \{\lambda^a, \lambda^b\} = \frac{4}{3}\delta^{ab} + 2d^{abc}\lambda^c$$

- When 2 is an antiquark

$$\left[(\lambda_1^a\bar{\lambda}_2^a)\frac{1}{m_2}(\bar{\lambda}_2^b\lambda_3^b) + (\lambda_3^b\bar{\lambda}_2^b)\frac{1}{m_2}(\bar{\lambda}_2^a\lambda_1^a)\right] = \frac{1}{m_2}\left[\lambda_1^a\left(\bar{\lambda}_2^a\bar{\lambda}_2^b + \bar{\lambda}_2^b\bar{\lambda}_2^a\right)\lambda_3^b\right] \qquad \{\bar{\lambda}^a, \bar{\lambda}^b\} = \frac{4}{3}\delta^{ab} - 2d^{abc}\bar{\lambda}^c$$

- Improves Δ-N splitting

$$\mathrm{H}_{\mathrm{Total}} = \mathrm{H}_{\text{2-body}} + A \cdot L^{C-C} + B \cdot L^{S-S} + C \cdot L^{C-S}$$

$$\mathrm{H}_{\text{3-body}} = \begin{cases} A\dfrac{128}{3m_q} - \left(B\dfrac{128}{3m_q^5} + C\dfrac{256}{3m_q^3} \right) & \text{For N} \\ A\dfrac{128}{3m_q} + \left(B\dfrac{128}{3m_q^5} + C\dfrac{256}{3m_q^3} \right) & \text{For } \Delta \end{cases}$$

- Quark-three-body force leads to a better simultaneous fit for mesons and baryons

  (Simple Gaussian wave function for baryons with constant factors)

$$\mathrm{H}_{\mathrm{Total}} = \mathrm{H}_{\text{2-body}} + A \cdot L^{C-C} + B \cdot L^{S-S} + C \cdot L^{C-S}$$

$$A = -367.522\,\mathrm{MeV}^2,\ B = -2.85156 \times 10^{11}\,\mathrm{MeV}^6,\ C = -7.68351 \times 10^6\,\mathrm{MeV}^4.$$

S. Noh et al. PLB862 (2025) 139278

Meson

Particle	Experimental Value (MeV)	Mass (MeV)	Variational Parameter (fm^{-2})
η_c	2983.6	2996.9	a = 13.1
J/ψ	3096.9	3089.6	a = 11.1
D	1864.8	1864.1	a = 4.5
D^*	2010.3	2010.7	a = 3.7
π	139.57	139.39	a = 4.6
ρ	775.11	775.49	a = 2.2
K	493.68	494.62	a = 4.6
K^*	891.66	888.82	a = 2.8

$$\sigma_{\mathrm{Meson}} = 5.86\ \mathrm{MeV}$$

Baryon

Particle	Experimental Value (MeV)	Mass (MeV)	Variational Parameters (fm^{-2})
Λ_c	2286.5	2266.7 (2281.6)	a_1 = 2.9, a_2 = 3.7
Σ_c	2452.9	2441.6 (2480.9)	a_1 = 2.1, a_2 = 3.8
Λ	1115.7	1113.6 (1134.1)	a_1 = 2.8, a_2 = 2.7
Σ	1192.6	1196.5 (1231.6)	a_1 = 2.1, a_2 = 3.1
Ξ_{cc}	3621.2	3586.8 (3606.3)	a_1 = 7.6, a_2 = 3.1
Σ_c^*	2518.5	2522.9 (2567.7)	a_1 = 2.0, a_2 = 3.4
Σ^*	1383.7	1398.9 (1455.2)	a_1 = 1.9, a_2 = 2.4
p	938.27	980.47 (1005.3)	a_1 = 2.4, a_2 = 2.4
Δ	1232	1272.1 (1346.8)	a_1 = 1.8, a_2 = 1.8

$$\sigma_{\mathrm{Baryon}} = 59.26\ \mathrm{MeV} \rightarrow \quad \sigma_{\mathrm{Baryon}}\left(\text{with three-body}\right) = 25.90\ \mathrm{MeV}$$

❑ With exact wave function and minimal r-dependence (Jongheon Baek)

$$L_{f,\text{Yuk}}^{\text{3-quark}} = \sum_{i<j<k} \left(A L_{ijk}^{C-C} + B L_{ijk}^{S-S} + C L_{ijk}^{C-S} \right) \exp\left[-\frac{c}{\hbar} (m_{ij} r_{ij} + m_{jk} r_{jk} + m_{ki} r_{ki}) \right]$$

Meson

Meson ($\mathcal{J}^{P(C)}$)	Experimental Value (MeV)	1 Gaussian ($\sigma = 5.86$)	30 Gaussians ($\sigma = 1.36$)
$\eta_c(0^{-+})$	2983.6	2996.9(+13.3)	2984.0(+0.4)
$J/\Psi(1^{--})$	3096.9	3089.6(−7.3)	3095.8(−1.1)
$D(0^-)$	1864.8	1864.1(−0.7)	1863.5(−1.3)
$D^*(1^-)$	2010.3	2010.7(+0.4)	2013.3(+3.0)
$\pi(0^{-+})$	139.57	139.39(−0.18)	139.58(+0.01)
$\rho(1^{--})$	775.11	775.49(+0.38)	774.17(−0.94)
$K(0^-)$	493.68	494.62(+0.94)	493.77(+0.09)
$K^*(1^-)$	891.66	888.82(−2.84)	891.45(−0.21)

$A = -23000.3\ \text{MeV}^2, \quad B = 5.94444 \times 10^{13}\ \text{MeV}^6, \quad C = -8.78859 \times 10^8\ \text{MeV}^4.$

Baryon

Baryon ($M_{\text{exp}}, \mathcal{J}^P$)	M_S (MeV) ($\sigma = 72.13$)	$M_{S,P,D}^{\text{GEM}}$ (MeV) ($\sigma = 46.28$)	ΔM (MeV)
$p(938.27, 1/2^+)$	1036.17(+97.90)	997.56(+59.29)	-38.61
$\Delta(1232, 3/2^+)$	1348.35(+116.35)	1313.42(+81.42)	-37.93
$\Sigma(1197.45, 1/2^+)$	1255.90(+58.45)	1231.40(+33.95)	-24.50
$\Lambda(1115.68, 1/2^+)$	1168.64(+52.96)	1124.22(+8.54)	-44.42
$\Sigma_c(2453.97, 1/2^+)$	2495.92(+41.95)	2476.24(+22.27)	-19.68
$\Lambda_c(2286.46, 1/2^+)$	2315.78(+29.32)	2294.00(+7.54)	-21.78
$\Xi_{cc}(3621.46, ?^?)$[a]	3630.22(+8.76)	3619.29(−2.17)	-10.93
$\Sigma^*(1383.7, 3/2^+)$	1462.16(+78.46)	1433.87(+50.17)	-28.28
$\Sigma_c^*(2518.48, 3/2^+)$	2587.03(+68.55)	2570.33(+51.85)	-16.70

With r-dependence

Mass ($M_{\text{exp}}, \mathcal{J}^P$)	with $L^{\text{3-quark}}$ ($\sigma = 12.71$)	with $L_{f,\text{Yuk}}^{\text{3-quark}}$ ($\sigma = 3.67$)	ΔL (MeV)
$p(938.27, 1/2^+)$	958.00(+19.73)	940.31(+2.04)	−17.69
$\Delta(1232, 3/2^+)$	1243.64(+11.64)	1231.28(−0.72)	−12.36
$\Sigma(1197.45, 1/2^+)$	1187.48(−9.97)	1192.9(−4.55)	+5.42
$\Lambda(1115.68, 1/2^+)$	1111.78(−3.90)	1110.94(−4.74)	−0.84
$\Sigma_c(2453.97, 1/2^+)$	2441.86(−12.11)	2455.27(+1.30)	+13.41
$\Lambda_c(2286.46, 1/2^+)$	2290.70(+4.23)	2292.53(+6.07)	+1.84
$\Xi_{cc}(3621.46, ?^?)$[b]	3599.99(−21.47)	3618.58(−2.88)	+18.59
$\Sigma^*(1383.7, 3/2^+)$	1378.37(−5.33)	1383.11(−0.59)	+4.74
$\Sigma_c^*(2518.48, 3/2^+)$	2518.59(+0.11)	2522.11(+3.63)	+3.52

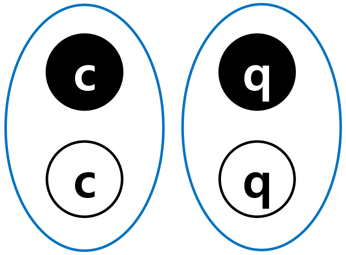
$$\sigma_{\text{Meson}} = 1.36\ \text{MeV} \rightarrow \sigma_{\text{Baryon}}(\text{no three-body}) = 46.28\ \text{MeV} \rightarrow \sigma_{\text{Baryon}}(\text{with three-body}) = 3.67\ \text{MeV}$$

→ With three-body force, the Baryon spectrum is well reproduced with the parameters from the meson fit

Implications for Exotics

X(3872)

$I^G\left(J^{PC}\right)=0^+\left(1^{++}\right)$



☞ Color-spin $\left(\text{C=color, S=spin}\right)$

$$K_{X(3872)}-K_D-K_{D^*}=\begin{pmatrix} \frac{16}{3}\frac{1}{m_c^2}+\frac{16}{3}\frac{1}{m_q^2}+\frac{32}{3}\frac{1}{m_c m_q} & 0 \\ 0 & -\frac{2}{3}\frac{1}{m_c^2}-\frac{2}{3}\frac{1}{m_q^2}-\frac{4}{3}\frac{1}{m_c m_q} \end{pmatrix}$$

Upper-left term: $\sim +140$ MeV, for $\left(c\bar{c}\right)^{C=1}_{S=1}\otimes\left(q\bar{q}\right)^{C=1}_{S=1}$

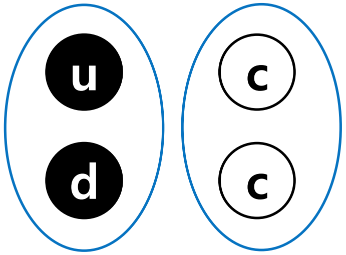
Lower-right term: ~ -20 MeV, for $\left(c\bar{c}\right)^{C=8}_{S=1}\otimes\left(q\bar{q}\right)^{C=8}_{S=1}$

☞ Color-color interaction of $\left(c\bar{c}\right)^{C=8}_{S=1}\otimes\left(q\bar{q}\right)^{C=8}_{S=1}$ is repulsive

To overcome additional kinetic term attraction has to be >100 MeV

Full quark model calculation → Fall apart to two mesons
(W. Park, SHL, NPA925 (2014) 161)

Tcc(3875) $\quad I^G\left(J^P\right)=0^+\left(1^+\right)$



☞ Color-spin

$$K_{T_{cc}(3875)}-K_D-K_{D^*}=\begin{pmatrix} -8\frac{1}{m_q^2}+\frac{8}{3}\frac{1}{m_c^2}+\frac{32}{3}\frac{1}{m_cm_q} & -8\sqrt{2}\frac{1}{m_cm_q} \\ -8\sqrt{2}\frac{1}{m_cm_q} & -\frac{4}{3}\frac{1}{m_q^2}+4\frac{1}{m_c^2}+\frac{32}{3}\frac{1}{m_cm_q} \end{pmatrix} \begin{matrix} (ud)_{S=0}^{C=\bar{3}}\otimes(\overline{cc})_{S=1}^{C=3} \\ (ud)_{S=1}^{C=6}\otimes(\overline{cc})_{S=0}^{C=\bar{6}} \end{matrix}$$

Top-left element: □ −100 MeV

Bottom-right element: □ +17 MeV

☞ Color-color interaction of $(ud)_{S=0}^{C=\bar{3}}\otimes(\overline{cc})_{S=1}^{C=3}$ is attractive

Full quark model calculation → Could be compact

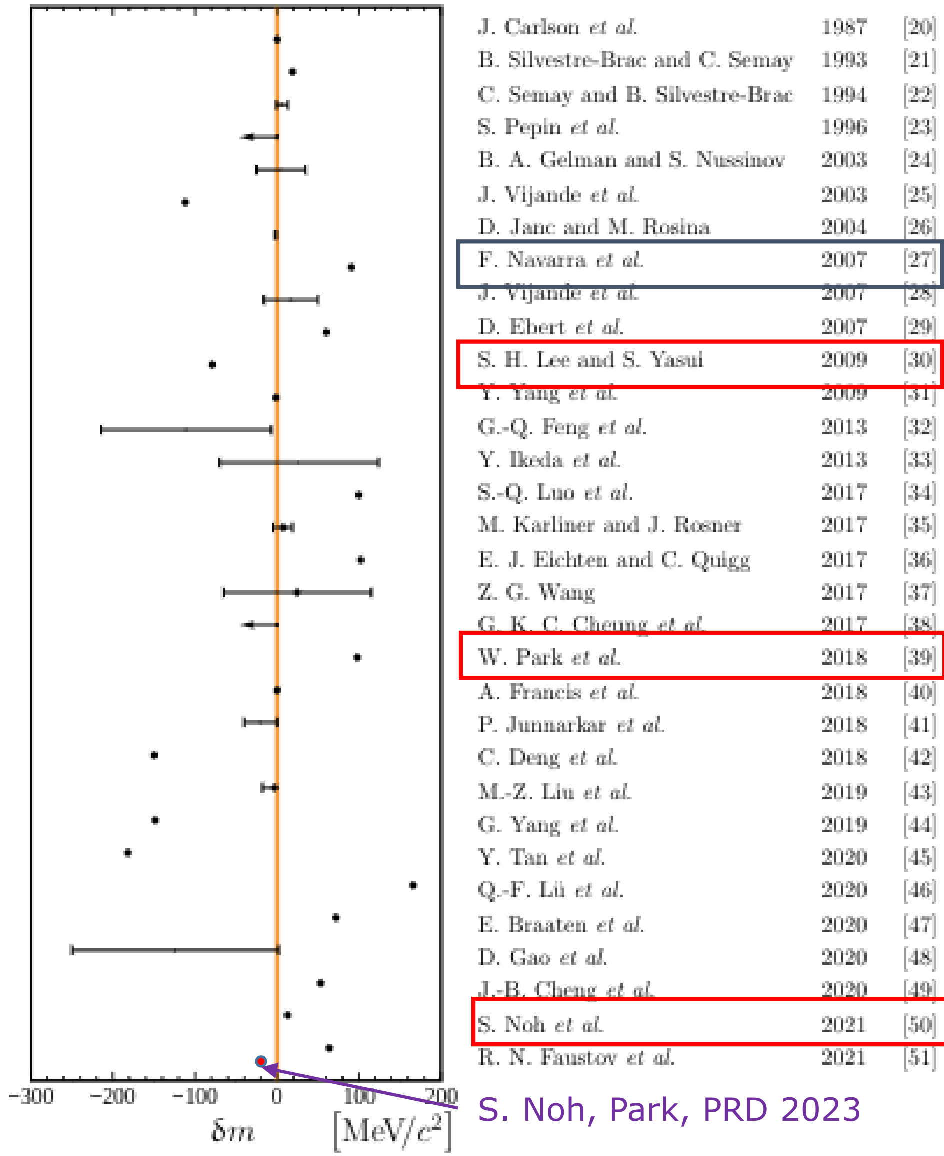
Full Quark Model calculation suggests: ex S.Noh, W.Park, Lee, PRD10(2021)114009

-2021- Tcc(3875) LHCb coll.

☞ There is a strong short range attraction for Tcc → Could be compact, but depends sensitively on parameters:

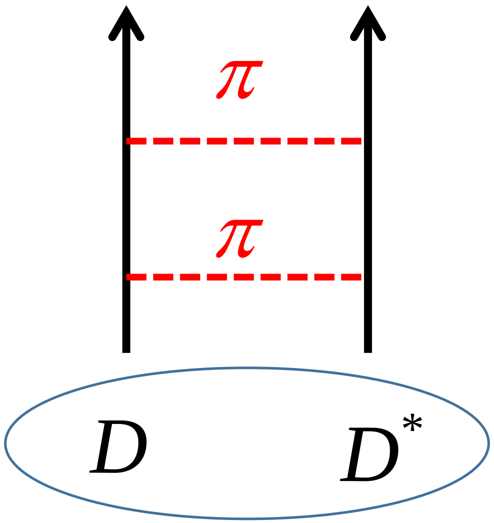
☞ The short range attraction for X(3872) is very weak

→ Can not be compact



Can X(3872) be D-$\bar{D}^*$ and Tcc be D-D^* Molecules?

Perspectives from the p-exchange



$$M\left(J_M, I_M\right)$$

Especially important when

$J_M \neq 0$ Mixing with D-wave

and

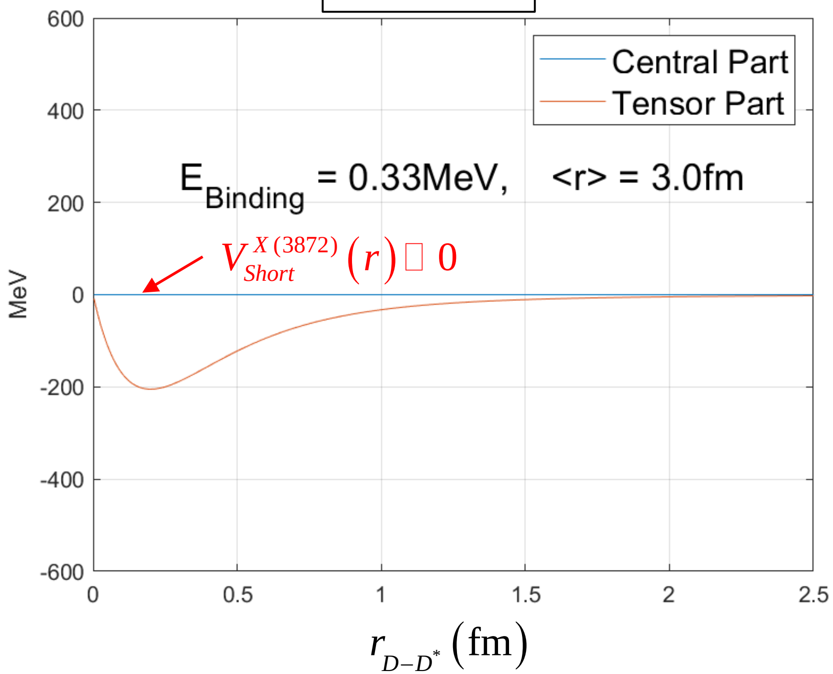
$I_M < \left(I_D + I_{D^*}\right)$ Mixing is strong

D-wave mixing through p-exchange Tcc: N.A. Tornqvist (94) + Short range attraction (D. Park, et al)

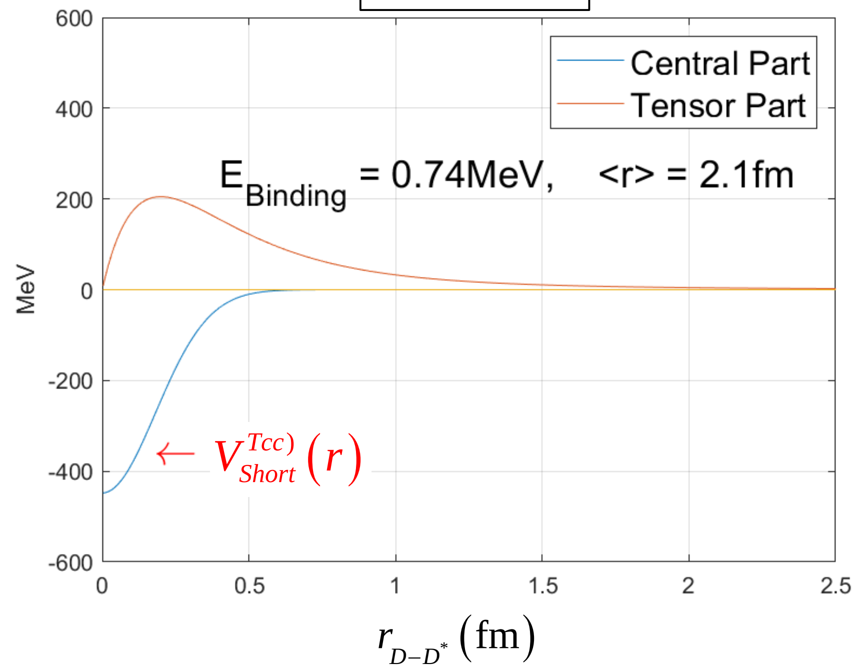
☞ $$V(r)^{-:X(3872):\ D-\bar{D}^*}_{+:Tcc:\ D-\bar{D}^*} = V_{Short}(r)\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} \mp 3V_0\left[\begin{pmatrix}0 & -\sqrt{2}\\ -\sqrt{2} & 1\end{pmatrix}T_\pi(r)\right]$$

Central Part $= V_{Short}(r)$; Tensor Part $= \pm T_\pi(r)$

X(3872)



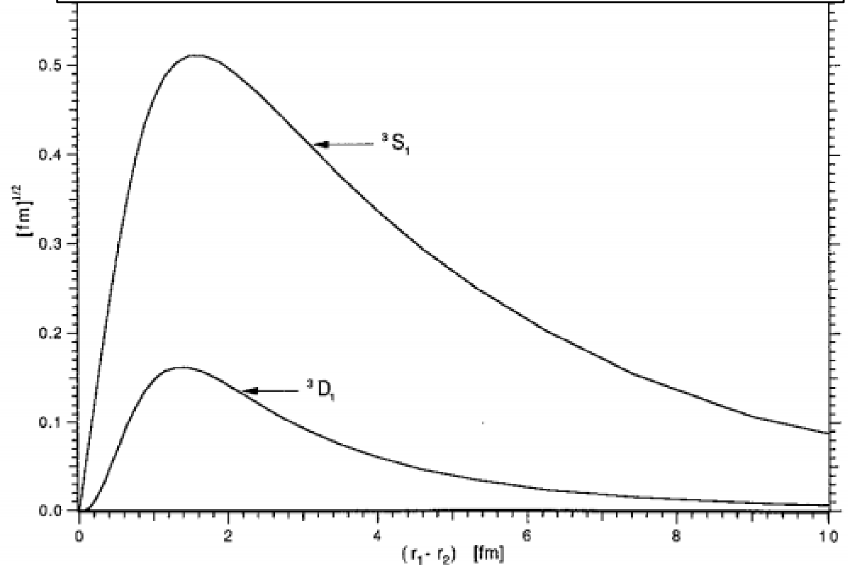
$T_{cc}(3875)$



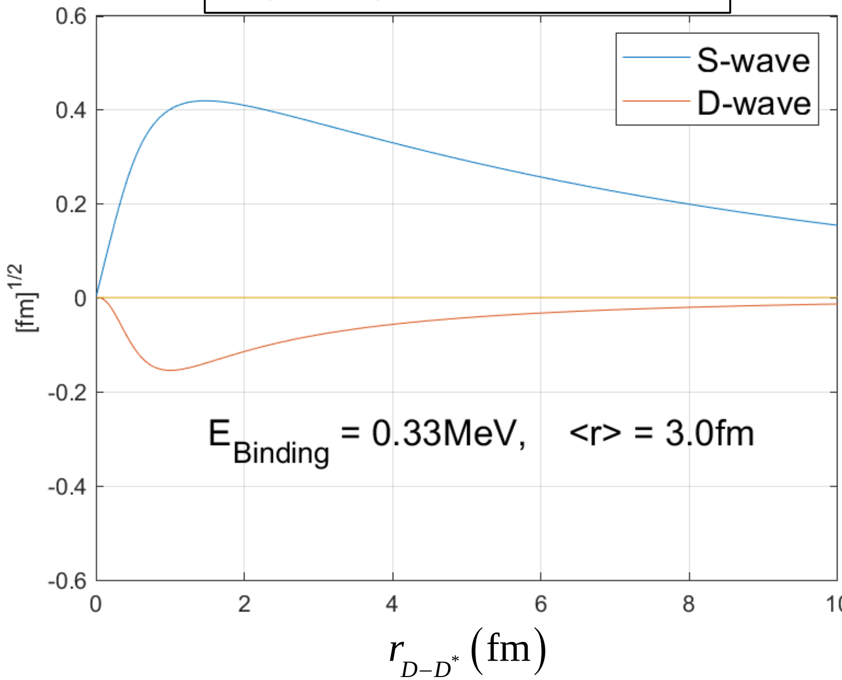
- Wave functions:
  Similar to that of Deuteron

- Both X(3872) : D and Tcc(3875)
could be a large molecular configuration

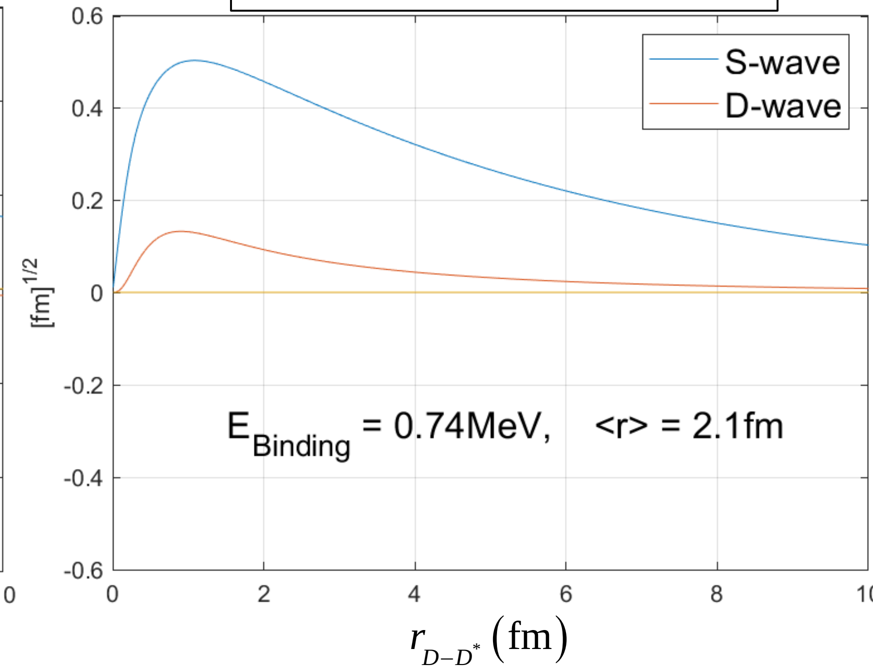
Deuteron – wave function (Törnqvist94)



$X(3872)$ – wave function



$T_{cc}(3875)$ – wave function



- Effect of 3-quark-interaction on Tetraquarks: Repulsive

$$\mathrm{H}_{\mathrm{Total}} = \mathrm{H}_{\text{2-body}} + A \cdot L^{C-C} + B \cdot L^{S-S} + C \cdot L^{C-S}$$

Particle	Measured mass (MeV)	$\sum_{i<j<k} L_{ijk}^{C-C}$	$\sum_{i<j<k} L_{ijk}^{S-S}$	$\sum_{i<j<k} L_{ijk}^{C-S}$
T_{cc}	3875	-4.84236	0.0319013	20.9444
$X(3872)$	3872	19.3694	0.0427164	-1.36541

Hence both Tcc and X can not be compact multiquark states

Summary

- Exotics are gateways to multiquark configurations and dense matter

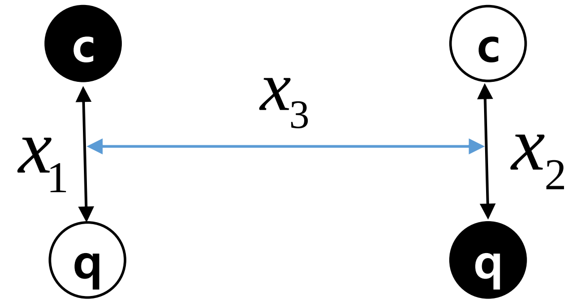
- To fit meson and baryon masses in the quark model, one needs quark three-body forces: Nuclear Physics motivated three-body forces improve the fit to the baryon spectrum

- The quark three-body force provides repulsion for X(3872) and Tcc: Hence, these states are most likely molecular or hybrid states

- In principle, one might need quark four-body force: Hence, we need experiments to determine the structure of exotics: Fortunately, are suppressed in 1/m

Quark Spatial wave function of X(3872) : $D\bar{D}^*$ molecule

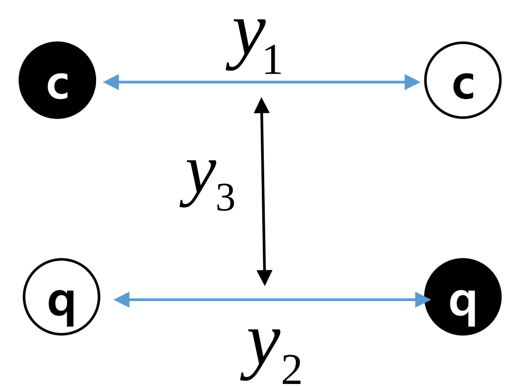
☞ S-wave in $(q\bar{c}),(c\bar{q})$ basis $\quad D-\bar{D}^*$



$$\psi_1^{Spatial} \propto \exp\left[-a_1 x_1^2 - a_2 x_2^2 - a_3 x_3^2\right]$$

$$R_{D \text{ or } D^*} \approx 0.55 \text{ fm}, \quad R_{D-D^*} \approx 4 \text{ fm}$$

☞ Transformation into $(c\bar{c}),(q\bar{q})$ basis

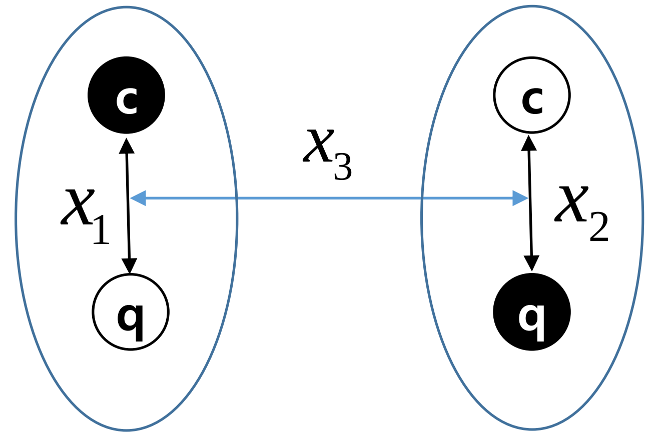


$$\psi_1^{Spatial} \propto \exp\left[-b_1 y_1^2 - b_{12} y_1 \cdot y_2 - b_2 y_2^2 - b_3 y_3^2\right]$$

$$R_{(c\bar{c})} \approx 4.01 \text{ fm}, \quad R_{(q\bar{q})} \approx 4.06 \text{ fm}, \quad R_{(c\bar{c})-(q\bar{q})} \approx 0.394 \text{ fm}$$

Quark Color-spin wave function of X(3872): $D\bar{D}^*$ molecule

$I^G(J^{PC}) = 0^+(1^{++})$

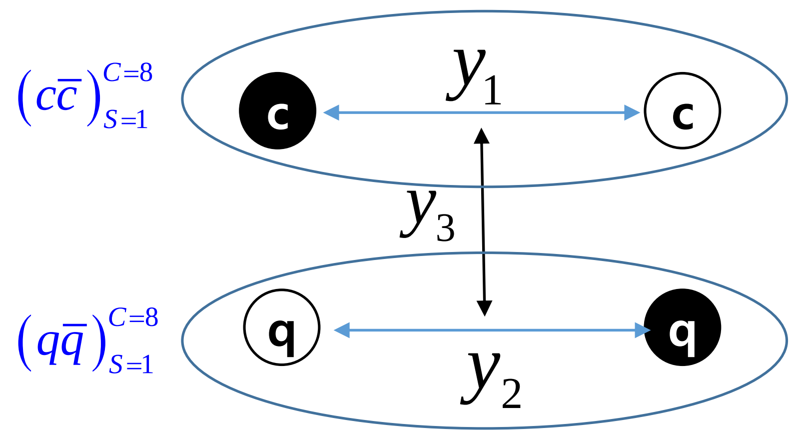


☞ In $(q\bar{c}),(c\bar{q})$ basis

$$|1'\rangle = (q\bar{c})^{C=1}_{S=0} \otimes (c\bar{q})^{C=1}_{S=1} \quad \rightarrow D - \bar{D}^*$$

$$|2'\rangle = (q\bar{c})^{C=8}_{S=0} \otimes (c\bar{q})^{C=8}_{S=1}$$

☞ Transformation into $(c\bar{c}),(q\bar{q})$ basis



$$|1\rangle = (c\bar{c})^{C=8}_{S=1} \otimes (q\bar{q})^{C=8}_{S=1}$$

$$|2\rangle = (c\bar{c})^{C=1}_{S=1} \otimes (q\bar{q})^{C=1}_{S=1}$$

➡ $$|1'\rangle = \frac{2\sqrt{2}}{3}|1\rangle + \frac{1}{3}|2\rangle$$

$$|2'\rangle = -\frac{1}{3}|1\rangle + \frac{2\sqrt{2}}{3}|2\rangle$$

→ $D\bar{D}^*$ is mostly composed of $(c\bar{c})^{C=8}_{S=1} \otimes (q\bar{q})^{C=8}_{S=1}$

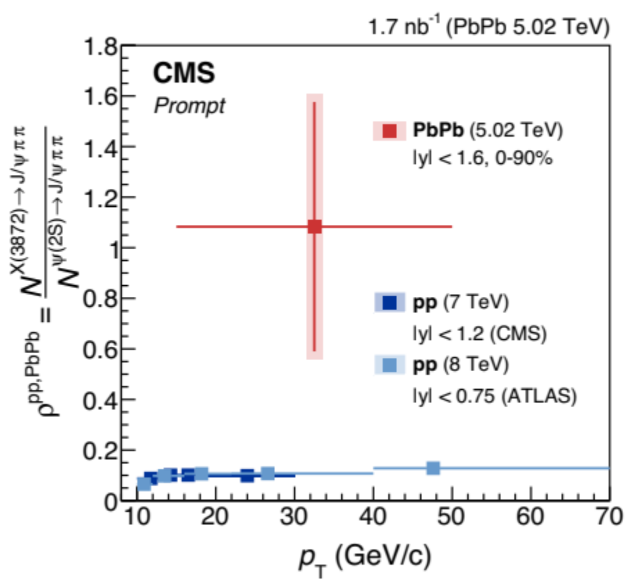
Possible explanation of abundant production of X(3872)

☞ $D\bar{D}^*$ is mostly composed of $(c\bar{c})^{C=8}_{S=1} \otimes (q\bar{q})^{C=8}_{S=1}$

☞ $R_{(c\bar{c})}$ □ 4.01 fm , $R_{(q\bar{q})}$ □ 4.06 fm, $R_{(c\bar{c})-(q\bar{q})}$ □ 0.394 fm

☞ ψ(2S) production at high Pt is dominated by $(c\bar{c})^{C=8}_{S=1}$ but has to be multiplied by a small overlap into color singlets: NRQCD

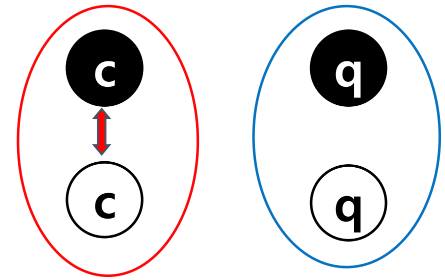
☞ X(3872) production at high Pt might be a direct recombination of $(c\bar{c})^{C=8}_{S=1}$ with $(q\bar{q})^{C=8}_{S=1}$ in QGP



X(3872) : W. Park, SHLee, NPA924(2014) 161

$$\text{X(3872)} \begin{cases} (c\bar{c}) \rightarrow (C=8, S=1) \\ (q\bar{q}) \rightarrow (C=8.S=1) \end{cases}$$

$$H_{cc} = \lambda_c^a \left(\lambda_c^a \frac{g}{r_{cc}} \right) \; ?$$



Color-Color (X(3872))

$$\lambda_c^a \left(\lambda_c^a\right) = \frac{1}{2}\left[\left(\lambda_c^a + \lambda_c^a\right)^2 - \lambda_c^2 - \left(\lambda_c^a\right)^2\right]$$

$$\frac{1}{4}\lambda^2 = C = \frac{1}{3}\left(p^2 + q^2 + pq + 3(p+q)\right) \qquad C(p=1, q=1) = 3, \quad C_f(p=1, q=0) = \frac{4}{3}$$

If cc is in $(C=8, S=1)$

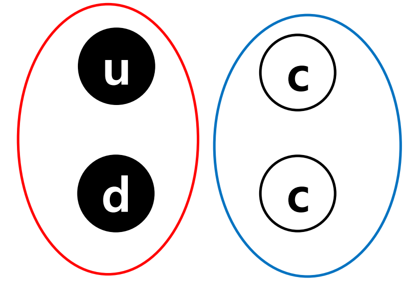
$$\lambda_c^a\left(\lambda_c^a\right) = \frac{4}{2}\left[3 - 2\frac{4}{3}\right] = \frac{2}{3} > 0$$

No additional attraction from color-color interaction

→ X(3872) can not be compact multiquark state

Tcc(3875) : W. Park, SHLee, NPA924(2014) 161

$$\text{Tcc(3875)} \begin{cases} (ud) \to (C = \bar{3}, S = 0) \\ (\overline{cc}) \to (C = 3. S = 1) \end{cases} \qquad H_{cc} = \lambda_c^a \left(\lambda_c^a \frac{g}{r_{cc}} \right) \ ?$$



Color-Color (Tcc)

$$\lambda_c^a \left(\lambda_c^a \right) = \frac{1}{2}\left[\left(\lambda_c^a + \lambda_c^a \right)^2 - \lambda_c^2 - \left(\lambda_c^a \right)^2 \right]$$

$$\frac{1}{4}\lambda^2 = C = \frac{1}{3}\left(p^2 + q^2 + pq + 3(p+q) \right) \qquad C(p=0, q=1) = \frac{4}{3}, \quad C(p=1, q=0) = \frac{4}{3}$$

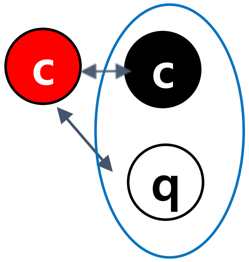
If $\overline{cc}$ is in $(C = 3, S = 1)$ $\qquad \lambda_c^a \left(\lambda_c^a \right) = \frac{4}{2}\left[\frac{4}{3} - 2\frac{4}{3} \right] = -\frac{8}{3} < 0$

Hence there is additional attraction

→ Tcc(3875) could be a compact multiquark state

Why Heavy quarks are needed for multiquark configuration

☞ Color-color interaction becomes stronger (Karliner Rosner)



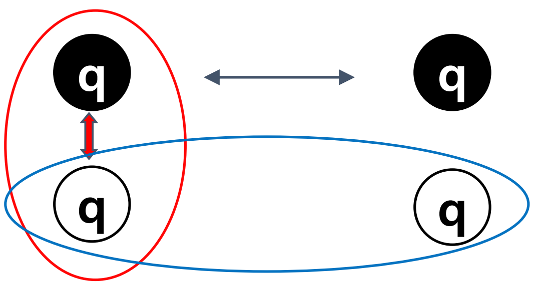
$$H_{cc} = \ldots + \lambda_i^c \lambda_j^c \left(\frac{g^2}{r_{ij}} \right) + \ldots. \qquad r \approx \frac{1}{mg^2}, \qquad E_C \approx -mg^4$$

☞ Color-spin interaction becomes weaker with heavy quarks

When all light quarks

Fall apart into two mesons

$$-16 \frac{1}{m_q m_q}$$

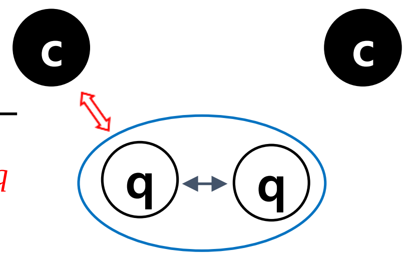


$$-8 \frac{1}{m_q m_q}$$

When heavy quarks,

could be compact (Tcc)

$$-16 \frac{1}{m_c m_q}$$



$$-8 \frac{1}{m_q m_q}$$