Overview of physics at EIC:
From the perspective of an ordinary perturbative QCD theorist

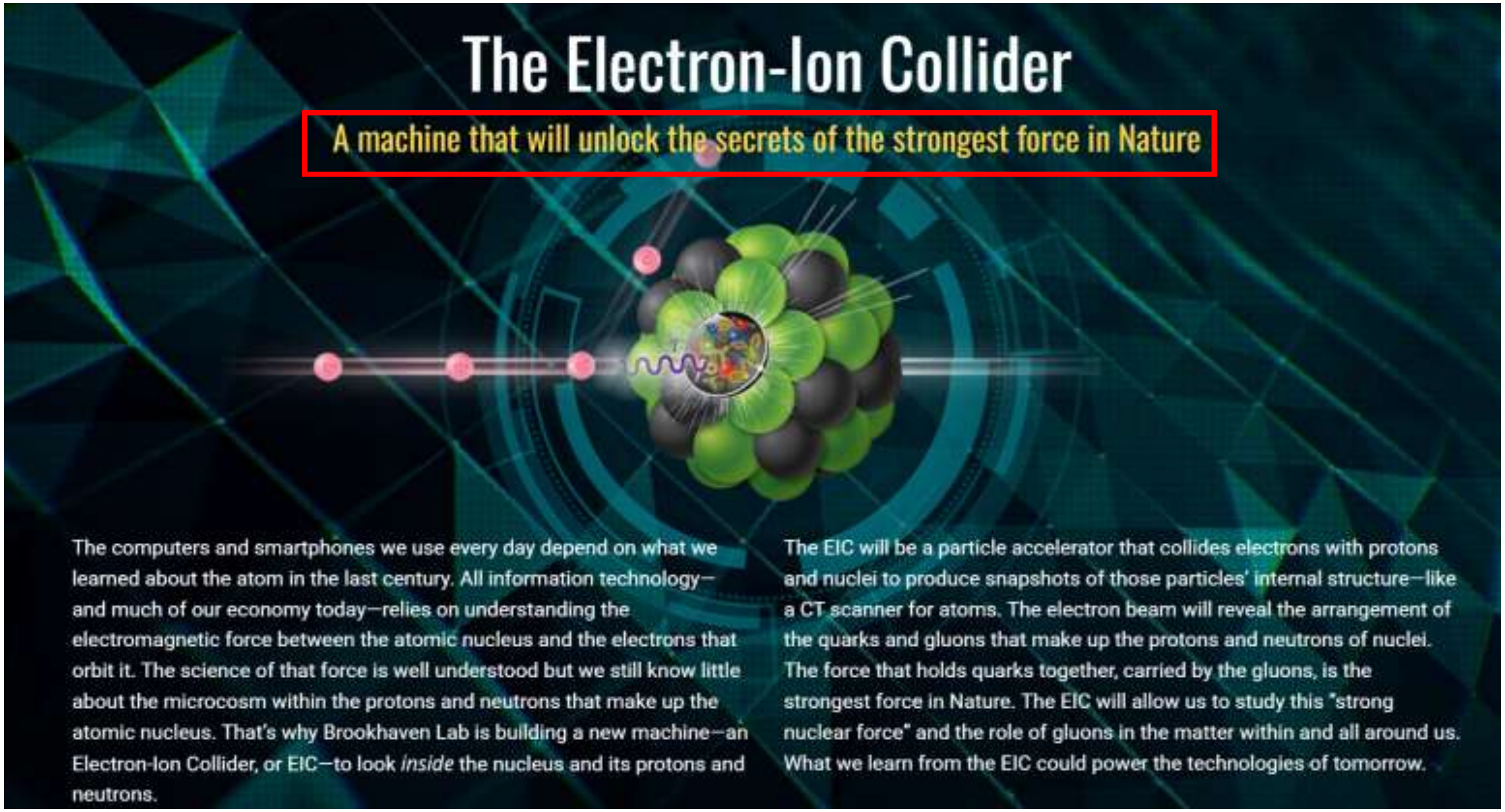
Shinsuke Yoshida

(South China Normal University)

QCD theory seminars, April 10, 2025

Introduction to EIC

Electron-Ion collider(EIC): next-generation collider experiment
plans to begin operation in 2032



https://www.bnl.gov/eic/

Introduction to EIC

Electron-Ion collider(EIC): next-generation collider experiment
plans to begin operation in 2032

The Electron-Ion Collider

A machine that will unlock the secrets of the strongest force in Nature

3D structure of protons and nuclei

The EIC will bring high-energy electrons into head-on collisions with high-energy protons or atomic nuclei to produce "freeze-frame" snapshots of those particles' inner structure, creating the first-ever tomographic 3D images of the "ocean" of gluons within. These images will tell scientists how gluons and quarks bind each other to form the particles within and around us.

More

Gluon saturation and the color glass condensate

Recent experiments and advances in theory suggest that protons, neutrons, and nuclei appear as dense "walls" of gluons at high energies, creating what may be among the strongest force fields in nature. Discovering and studying this form of matter, the "color glass condensate," will provide deeper insight into why matter in this subatomic realm is stable.

More

Solving the mystery of proton spin + mass

The EIC will be the world's first polarized electron-proton collider—meaning the "spins" of both colliding particles can be aligned in a controlled way. This will make it possible to experimentally solve the outstanding mystery of how the teeming quarks and gluons inside the proton combine their spins to generate the overall spin carried by the proton.

More

EIC school at Kyoto

"YITP International School on EIC Physics" March 2 – 13, 2026

@Panasonic Hall, Yukawa Institute for Theoretical Physics (YITP), Kyoto University

- ❖ Constantia Alexandrou (CyI and UCY, Cyprus: Member of the Council of PRACE)
  Hadron Structure from Lattice QCD

- ❖ Yoshitaka Hatta (BNL, USA: Group Leader of RBRC Theory Group)
  Generalized Parton Distributions (GPDs)

- ❖ Zhongbo Kang (UCLA, USA: Member of Advisory Board, BNL EIC Theory Institute)
  Transverse Momentum Dependent Distributions (TMDs)

- ❖ Anna Stasto (PSU, USA: Convener of EIC Theory WG)
  Small-x physics

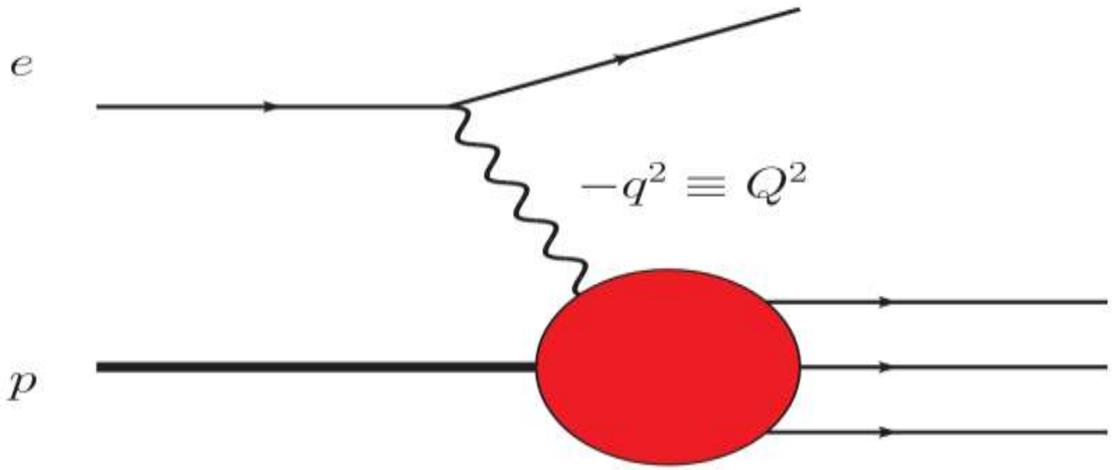
- ❖ Iain Stewart (MIT, USA: Member of Steering Committee, BNL EIC Theory Institute)
  Soft Collinear Effective Theory (SCET)

- ❖ Yong Zhao (ANL, USA)
  Large Momentum Effective Theory (LaMET)

1. Nucleon structure and *ep* collision

Electron-proton(ion) collision

Investigation of the nucleon structure was initiated by SLAC in the late 1960s



Virtual photon microscope

with "resolution" Q

Use a large Q^2 to probe internal structure

(Deep inelastic scattering(DIS))

Further investigation was inherited by later experiments (EMC, HERA, JLab,...)

Why we still need another experiment ?

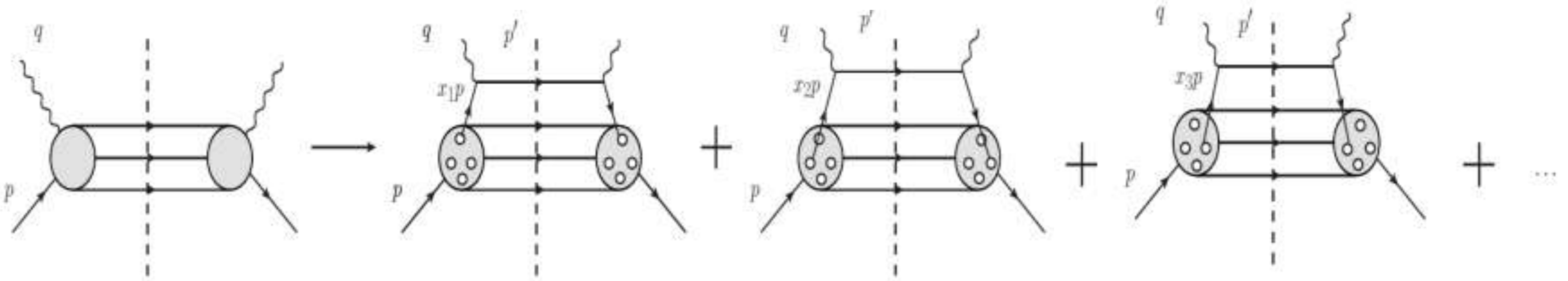
- Still a lot of mysteries lie in the nucleon structure

  · 3D structure and polarization · Gluon structure in small-x

  · Origins of the mass and the spin

Parton model

Parton model: scattering of the nucleon is viewed as incoherent scatterings of internal particles "partons"



· Each parton behaves like a free particle and is scattered independently

· A scattered parton has parallel momentum to the parent proton with a fraction x

1-dimensional motion

The proton scattering is characterized by parton distribution function $f_q(x)$

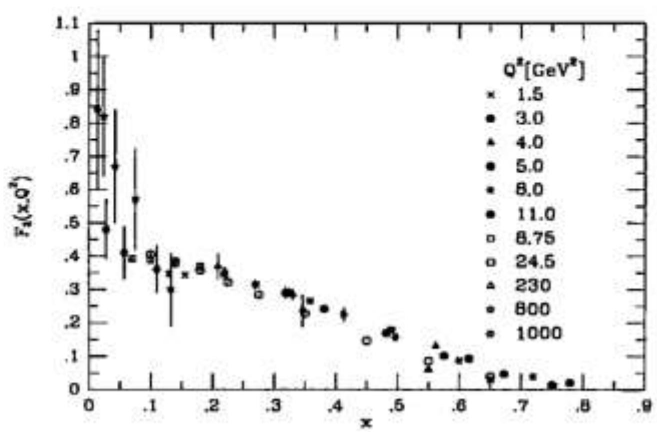


Fig. 4.2. The F_2 structure function from the SLAC-MIT, BCDMS, H1 and ZEUS collaborations.

$$F_1(x) = \frac{1}{2x}F_2(x) = \sum_{a=q,\bar{q}} \frac{e_a^2}{2} f_a(x)$$

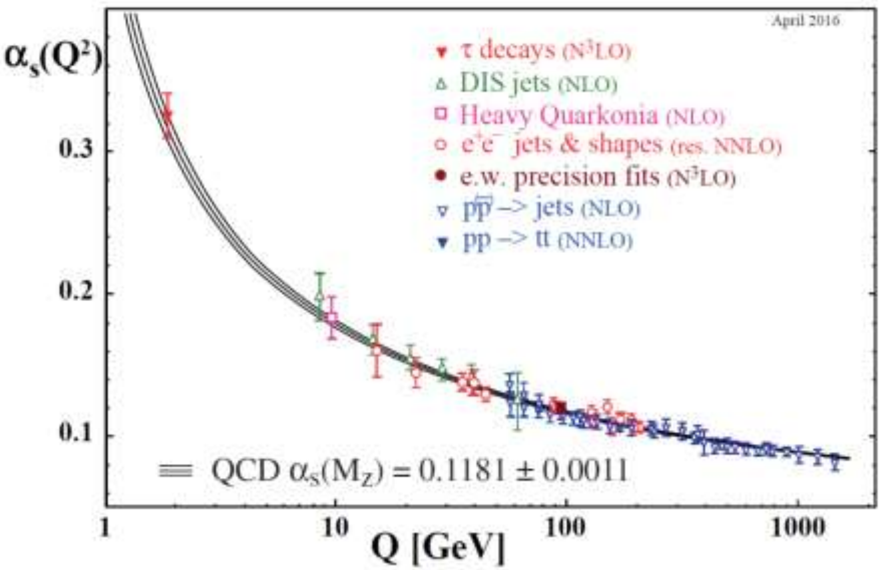
The structure functions depend on a single parameter (Bjorken scaling)

F_2 shows the scaling in the wide range of Q^2

$1\mathrm{GeV}^2 < Q^2 < 1000\mathrm{GeV}^2$

Nucleon structure and modified perturbation

QCD exhibits the asymptotic freedom



modified perturbation against UV divergence
(Renormalization)

$$\alpha_s \to \alpha_s(Q^2)$$

The parton model has been accepted as a basic picture for high-energy scatterings up to present

The asymptotic freedom opened the door to the understading of the nucleon structure by the first princlple calculation

Although a large Q^2 allows us a perturbative approach, it brings a problem at the same time

The virtual photon with large Q scatters a quark with small mass m_q

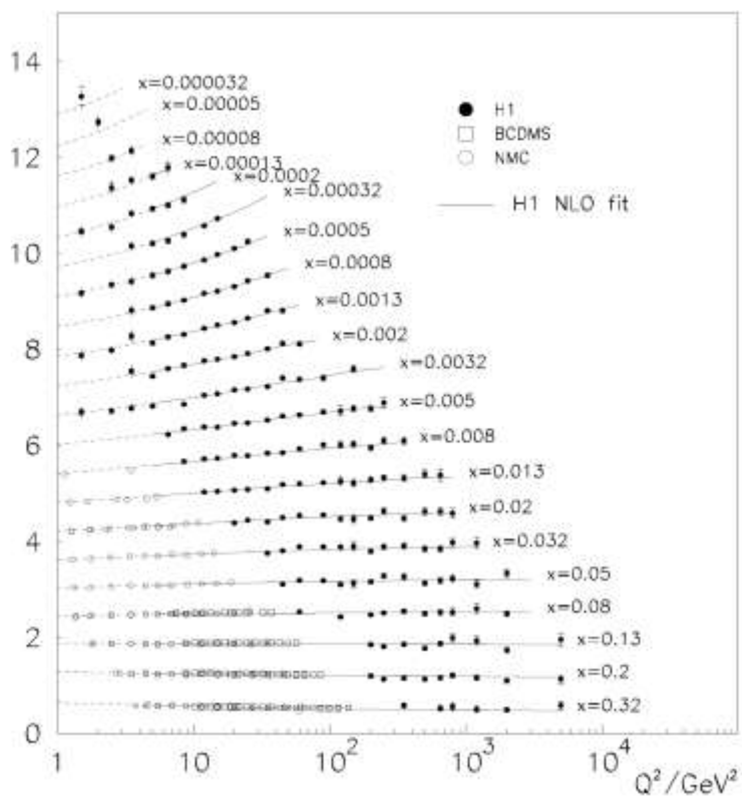
Large scale gap $m_q \ll Q$ could spoil the perturbation theory by the large log contribution $\ln \dfrac{Q}{m_q}$

mass singularity
(collinear divergence)

Construction of modified perturbation brings Q-dependence of parton distribution
(Collinear factorization)

$$f(x) \longrightarrow f(x, Q^2)$$

QCD improved parton model



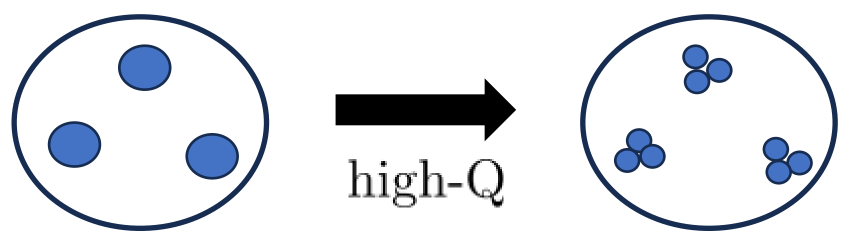
$$F_2(x) \to F_2(x, Q^2)$$

scaling violation

scaling violation is well controlled by DGLAP eq

parton distribution changes

with respect to resolution Q



2. Physics of TMD

3D structure of protons and nuclei

The EIC will bring high-energy electrons into head-on collisions with high-energy protons or atomic nuclei to produce "freeze-frame" snapshots of those particles' inner structure, creating the first-ever tomographic 3D images of the "ocean" of gluons within. These images will tell scientists how gluons and quarks bind each other to form the particles within and around us.

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Transverse-momentum-dependent distribution

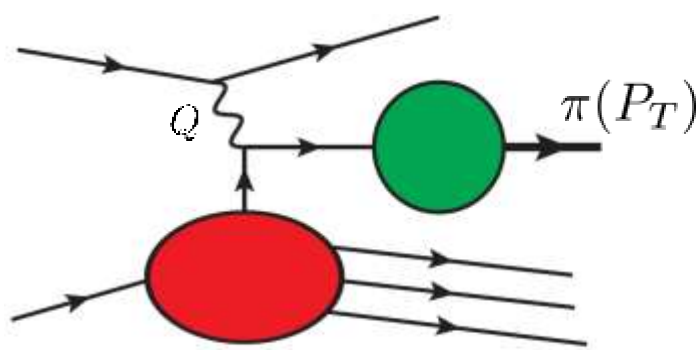
Understanding of 3D structure in the momentum space is one of the main goals of EIC

⟶ Determination of transverse-momentum-dependent(TMD) functions

processes with large virtuality Q and a small transverse momentum of observed particle P_T, $P_T \ll Q$

<u>semi-inclusive DIS(SIDIS)</u>

$$e^- + p \to e^- + \pi + X$$



A modified perturbation is needed to control large logs $\ln \dfrac{Q}{P_T}$

TMD factorization

J. C. Collins and D. E. Soper, Nucl. Phys. B193 (1981)

X. d. Ji, J. p. Ma and F. Yuan, Phys. Rev. D71 (2005)

TMD Handbook, arXiv:2304.03302

$$f(x) \longrightarrow f(x, k_T)$$

A modified perturbation can be constructed by introducing k_T-dependence in nonperturbative functions

The resummation of large logs can be achieved by the renormalization group equations of TMD functions

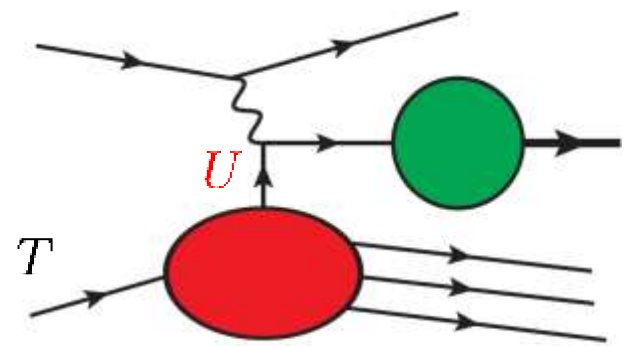
If this modified perturbative expansion exists,

k_T-dependent function brings a new polarization effect

D. W. Sivers, Phys. Rev. D41 (1990)

$$\frac{2}{M_N}(\vec{k}_T \times \vec{S}_\perp)_z f_{1T}^\perp(x, k_T)$$

unpolarized quark scattering inside the transversely polarized proton(Sivers effect)



not considered in the parton model

The single transverse-spin asymmetry(SSA) is an ideal probe: $A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$

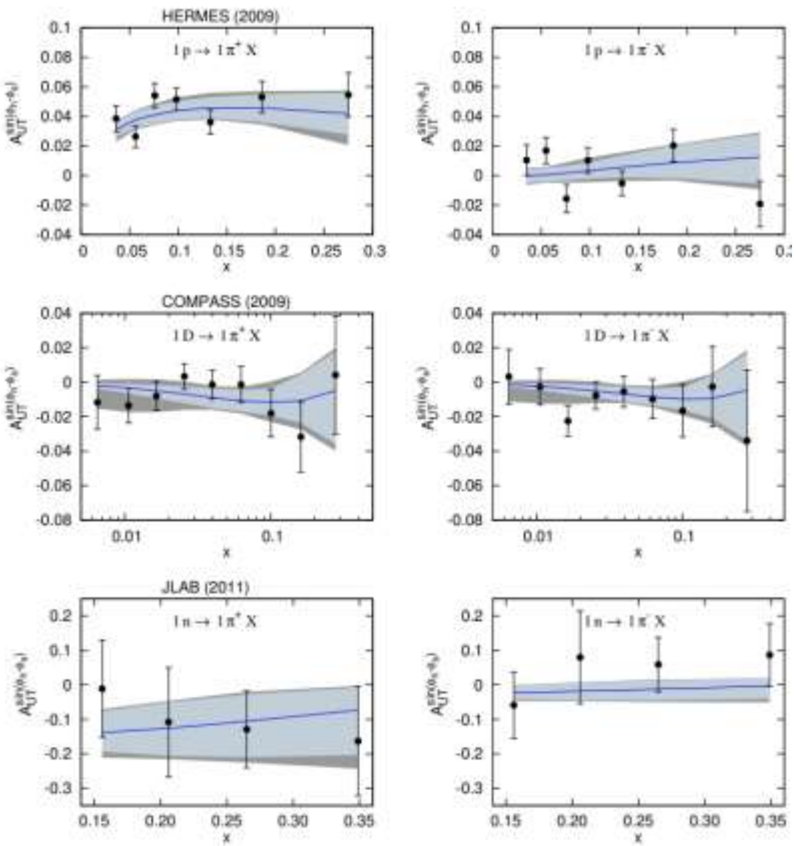
The parton model predicts a negligible SSA

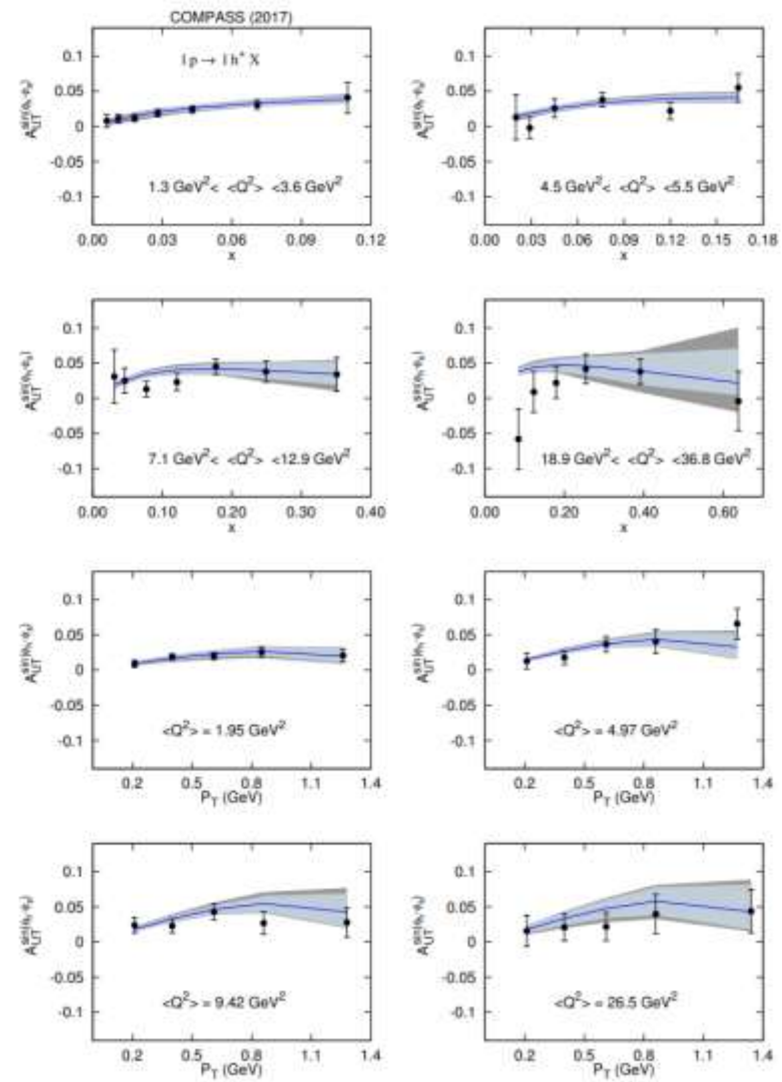
G. L. Kane, J. Pumplin and W. Repko, Phys. Rev. Lett. 41 (1978)

Successes of Sivers

M. Boglione, U. D'Alesio, C. Flore and J. O. Gonzalez-Hernandez, JHEP07 (2018)

SSA data in SIDIS has been reported in the past couple of decades





Reasonable description of the existing data

suggests the existence of Sivers function

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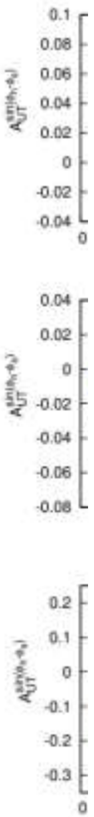
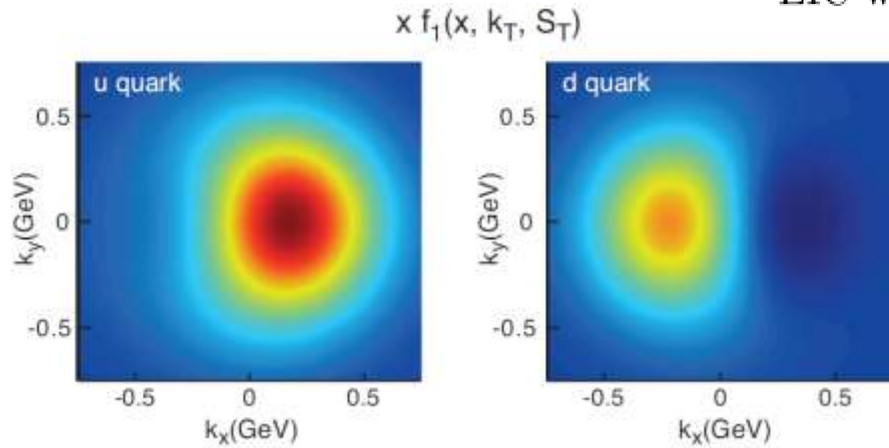


EIC white paper

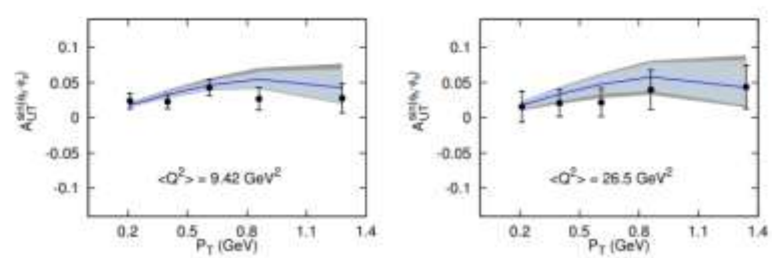


TMD framework has been intensively studied for decades

- gives the accessibility to novel nucleon structures beyond the parton model(3D structure, spin-mismatch scatterings)
- framework for a precision calculation as a resummed perturbation

Reasonable description of the existing data

suggests the existence of Sivers function



TMD evolution

TMD functions have two types of scale dependences

Lecture by
Iain Stewart

$$f(x, k_T) \longrightarrow f(x, k_T, \mu, \zeta)$$

renormalization

$$f_{1T}^{\perp}(x, b_T, \mu, \zeta) = \int d^2 k_T \, e^{-i\vec{k}_T\cdot\vec{b}_T} f_{1T}^{\perp}(x, k_T, \mu, \zeta)$$

Perturbative

$$\mu\frac{\partial}{\partial\mu} f(x, b_T, \mu, \zeta) = \gamma_q\left(\alpha_s(\mu), \frac{\zeta}{\mu^2}\right) f(x, b_T, \mu, \zeta) \qquad \zeta\frac{\partial}{\partial\zeta} f(x, b_T, \mu, \zeta) = \frac{\tilde{K}}{2} f(x, b_T, \mu, \zeta)$$

Collins-Soper Kernel receives a contribution from $\alpha_s(1/b_T)$

nonperturbative in large b_T

The perturbative parts have been calculated up to 3-loop, 4-loop

Soft collinear effective theory(SCET) provides a systematic way TMD Handbook, arXiv:2304.03302

Different modelings of the nonperturbative part

Scimemi and Vladimirov, JHEP06(2020) MAPTMD22, JHEP10(2022)

New challenge for lattice QCD theorists

Lattice Parton (LPC) Collaboration, JHEP 08 (2023)

A. Avkhadiev, P. E. Shanahan, M. L. Wagman and Y. Zhao,Phys. Rev. D108 (2023)

D. Bollweg, X. Gao, S. Mukherjee and Y. Zhao, Phys. Lett. B852 (2024)

Wide range of Q^2 from EIC will justify the validity of the TMD framework
as a resummed perturbation again the large logs

Non-universality of Sivers

$$\mathcal{F.T.}\langle PS_\perp|\bar{\psi}(0)\gamma^+\psi(x^-,x_T)|PS_\perp\rangle = \frac{2}{M_N}(\vec{k}_T\times\vec{S}_\perp)_z f_{1T}^\perp(x,k_T)+\cdots$$

$$\mathcal{F.T.}\langle PS_\perp|\bar{\psi}(0)\gamma^\mu\psi(x^-,x_T)|PS_\perp\rangle = \mathcal{F.T.}\langle P(-S_\perp)|\bar{\psi}(0)\gamma^\mu\psi(x^-,x_T)|P(-S_\perp)\rangle \Longrightarrow f_{1T}^\perp(x,k_T)=0$$

under PT

Does the Sivers function exist ? ⟵ Successes in the description of the SSA data

The gauge invariance requires the Wilson line $\mathcal{F.T.}\langle PS_\perp|\bar{\psi}(0)\gamma^\mu\mathcal{W}\psi(x^-,x_T)|PS_\perp\rangle$

3D Wilson line changes its path under PT

PT connects two different functions

$$f_{1T}^{\perp\text{SIDIS}}(x,k_T) = -f_{1T}^{\perp\text{DY}}(x,k_T)$$

Important test of the TMD framework

Experimental data so far seems to support the sign change within large error bars

Sivers effect within the TMD framework has reached a high level of precision
NLO hard cross section + TMD evolution
M. G. Echevarria, Z. B. Kang and J. Terry, JHEP01 (2021)

Lecture by
Zhongbo Kang

Role of TMD in Physics

- small transverse momentum + large scale could often happen in high energy reactions

  cf. q_T-spectrum of a heavy particle production (Z, W, Higgs...): $q_T \ll M$

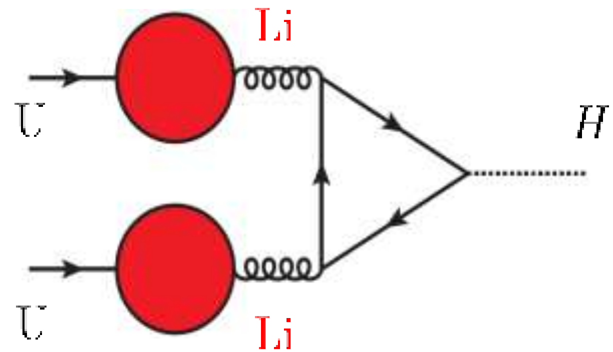
  G. Aad *et al.* [ATLAS], Phys. Lett. B705 (2011)

Novel polarization could happen

linearly polarized gluon TMD

$$\left(\frac{p_T^\mu p_T^\nu}{M_N^2} + g_\perp^{\mu\nu}\frac{p_T^2}{2M_N^2}\right) h_1^{\perp g}(x, p_T)$$

linearly polarized gluon inside the unpolarized proton



- TMD polarizations can be used as a tool to pin down a new physics

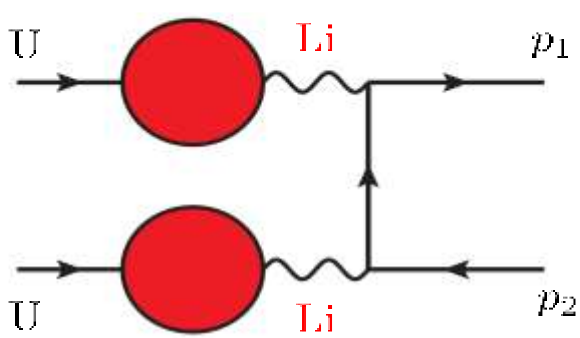
  Contribution from the above linearly polarized gluon changes its sign between the scalar and the pseudoscalar Higgs productions

  D. Boer *et al.*, Phys. Rev. Lett. 108 (2012)

  D. Gutierrez-Reyes, S. Leal-Gomez, I. Scimemi and A. Vladimirov, JHEP11 (2019)

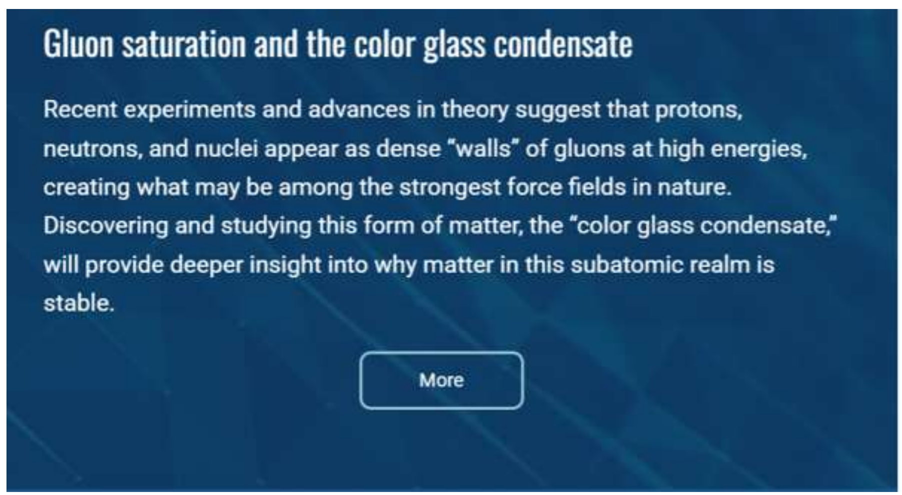
  Linearly polarized photon may be used as a signal of a strong magnetic field

  C. Li, J. Zhou and Y. J. Zhou, Phys. Lett. B795 (2019)

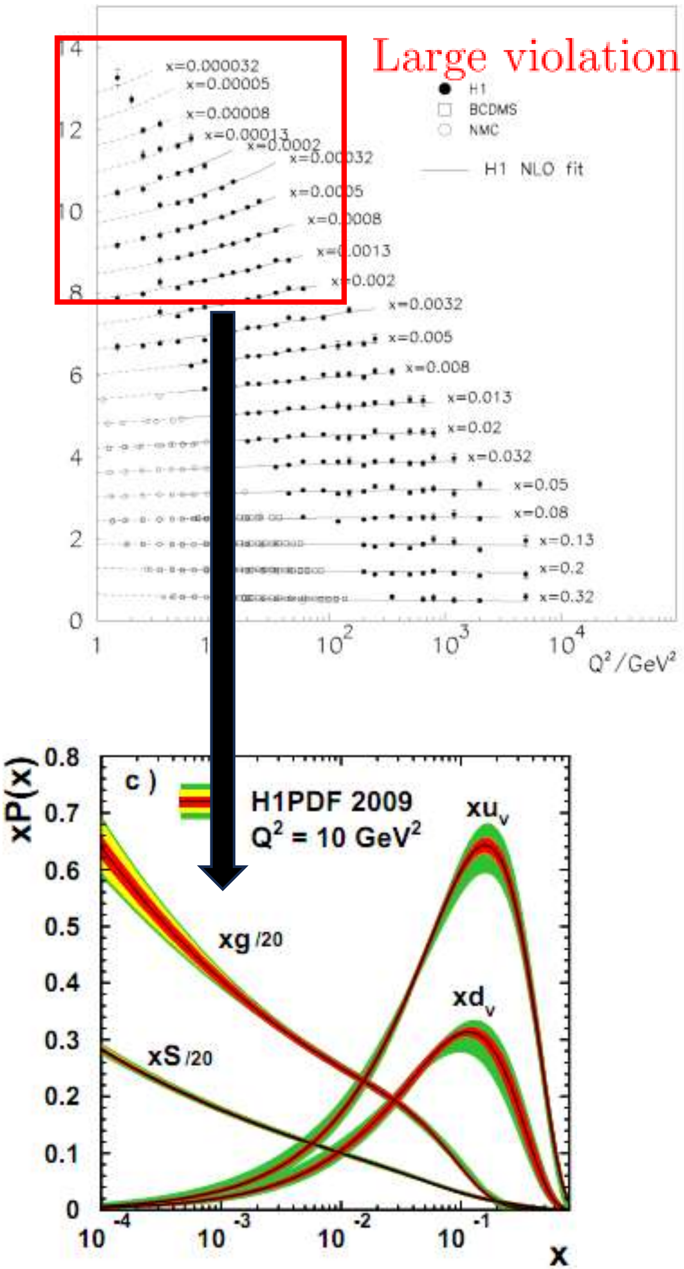


$$\left(2\frac{p_T^\mu p_T^\nu}{p_T^2} + g_\perp^{\mu\nu}\right) h_1^{\perp\gamma}(x, p_T)$$

3. Small-x physics



Gluon density in small-x



Large violation in small-x

$$f(x) \longrightarrow f(x, Q^2)$$

DGLAP equation

$$\frac{\partial}{\partial \ln Q^2} f_q(x,Q) = \frac{\alpha_s(Q)}{2\pi}\int_x^1 \frac{dy}{y} P_{qq}(\frac{x}{y}) f_q(y,Q) + \frac{\alpha_s(Q)}{2\pi}\int_x^1 \frac{dy}{y} P_{qg}(\frac{x}{y}) \underline{f_g(y,Q)}$$

gluon density

Gluon density can be observed
as the subleading contribution in DIS

Bjorken x: $\quad x_B = \dfrac{Q^2}{2p\cdot q} = \dfrac{Q^2}{s+Q^2} \simeq \dfrac{Q^2}{s}$

It is small in Regge-Gribov limit: fixed Q^2, $s \to \infty$

$\longrightarrow$ large log enhancement from $\ln(1/x)$

Can we construct a modified perturbation
with small-x resummed gluon density ?

BFKL equation

Balitsky-Fadin-Kuraev-Lipatov

Summation of large log $\ln(1/x)$ can be performed by BFKL equation

$$(\text{Gluon dentity}) \sim \left(\frac{1}{x}\right)^{\alpha_p - 1} \qquad \alpha_p - 1 \simeq 0.79$$

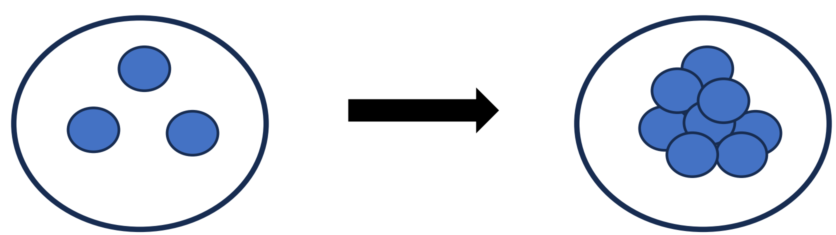
The total cross section in pp collision: σ_{tot}

BFKL shows $\sigma_{\text{tot}} = s^{\alpha_p - 1}$ but data shows $\sigma_{\text{tot}} = s^{0.1}$

unitary bound says it should be up to logarithmic dependence $\ln^2 s$ in $s \to \infty$

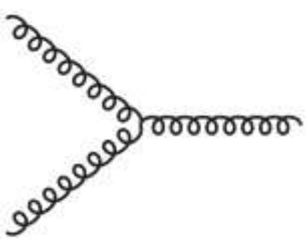
What are we overlooking ?

Regge-Gribov limit: fixed Q^2, $s \to \infty$ density increases with a fixed resolution



start overlapping

QCD exhibits the gluons' self-interaction



possibility of recombination

↓

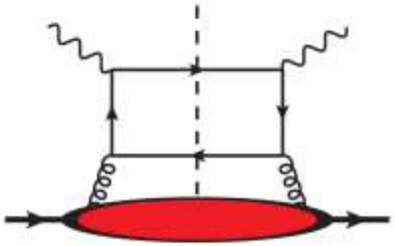
BK equation

If this piture is correct there should be the point of saturation density

verification in experiment

Color-glass-condensate EFT

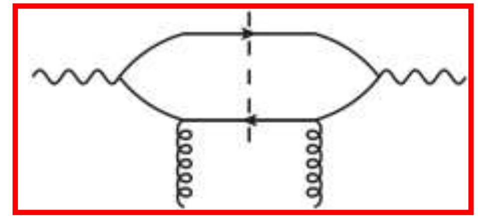
· conventional parton picture $\gamma^* p \to q\bar{q}$(gluon scattering channel)



$=$

probabilistic density of gluon

$\otimes$



Cross section of gluon scattering

· color-glass-condensate(CGC) EFT

$$\langle\langle O \rangle\rangle_Y = \int [D\rho] W_Y[\rho] \langle O \rangle [A[\rho]]$$

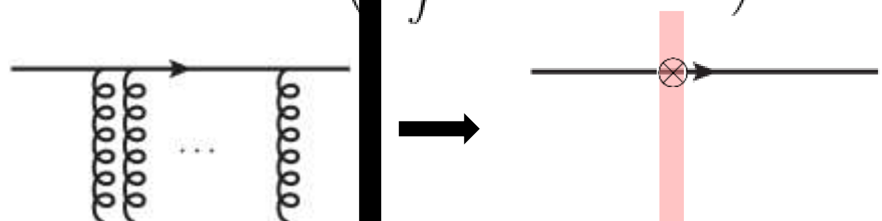
$$Y = \log(1/x)$$

$$\frac{\partial}{\partial Y}\langle\langle O \rangle\rangle_Y = \langle\langle H_{\mathrm{JIMWLK}} O \rangle\rangle_Y$$

JIMWLK eq

path integral by the small-x effective action
in terms of the Wilson line

$$V(x_\perp) = \mathrm{P}\exp\left(ig \int dx^- A^+(x^-, x_\perp)\right)$$

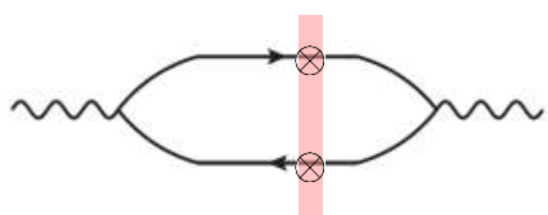


Y-evolution is consistent with BFKL in dilute

J. Jalilian-Marian *et al.*, Nucl. Phys. B504 (1997)

with BK in dense

E. Ferreiro, E. Iancu, A. Leonidov and L. McLerran,
Nucl. Phys. A703 (2002)



$\sim \mathrm{Tr}[V(x_\perp) V^\dagger(y_\perp)]$

Era of NLO studies

NLO results for eA has been published one after another in recent years

NLO JIMWLK kernel H_{JIMWLK} A. Kovner, M. Lublinsky and Y. Mulian, Phys. Rev. D89 (2014)

Inclusive DIS arXiv: 1009.4729, 1207.3844, 1112.4501, 1606.00777, 1708.06557, 1711.08207, 2103.14549, 2112.03158, 2204.02486, 2211.03504

Inclusive dijet
arXiv: 2108.06347, 2204.11650, 2208.13872, 2304.03304

Exclusive light vector meson
arXiv: 1612.08026, 2203.16911

Exclusive heavy vector meson
arXiv:2104.02349, 2204.14031

Single inclusive and dihadron
arXiv:2207.03606, 2210.03208, 2211.04837

Diffractive dihadron
arXiv: 2211.05774

Diffractive DIS
arXiv: 2401.17251

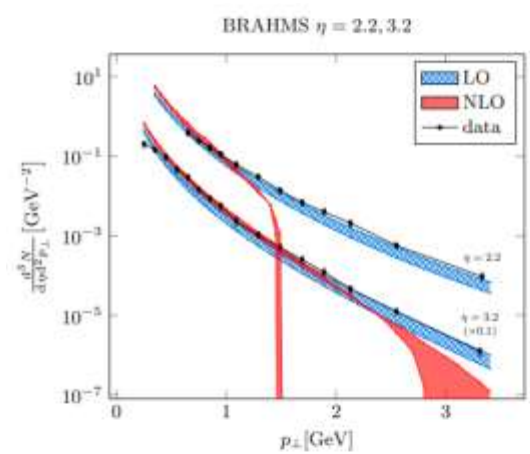
Diffractive dijet
arXiv: 1606.00419, 2112.06353, 2207.06268

Inclusive photon+dijet
arXiv: 1911.04530

Inclusive dihadron
arXiv: 2405.19404

A. M. Stasto, B. W. Xiao and D. Zaslavsky, Phys. Rev. Lett. 112 (2014)

First numerical study at NLO

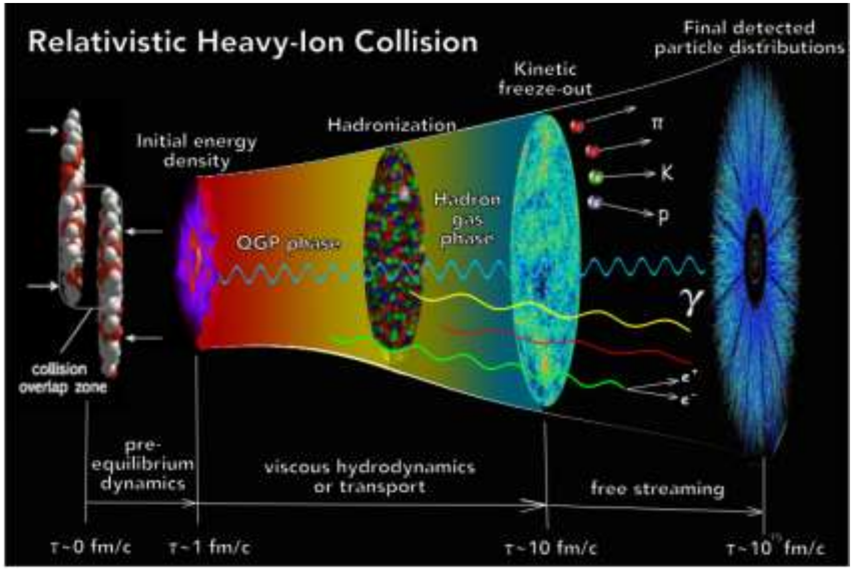


Lecture by Anna Stasto

High statistical data in various processes at EIC is expected to shows a signal of CGC

Role of small-x in physics

Initial condition of Heavy-Ion Collisions

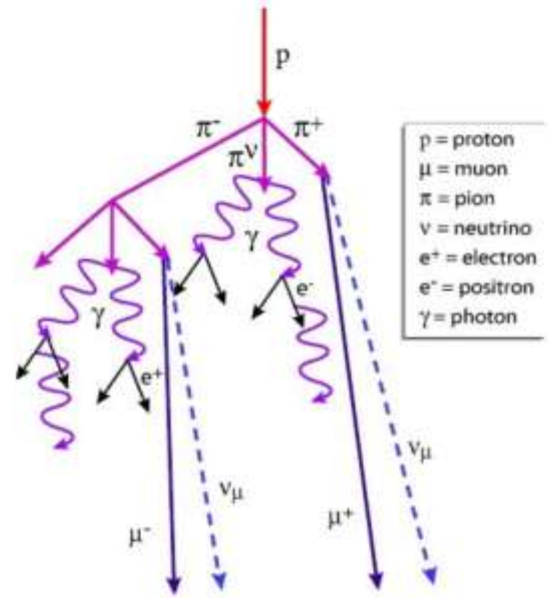


P. Achenbach *et al.*, Nucl. Phys. A1047 (2024)

Quark-Gluon-Plasma is produced by two energetic heavy ions collision in lab

The precise understanding of the initial conditions is essential for understanding the properties of the quark-gluon plasma

Origin of cosmic rays



Proton's energy could reach 10^{20}GeV which is much larger than the energy of the proton beam at LHC

Produced cosmic ray could reflect nontrivial dynamics after the collision

4. Mass and spin of the proton

Solving the mystery of proton spin + mass

The EIC will be the world's first polarized electron-proton collider—meaning the "spins" of both colliding particles can be aligned in a controlled way. This will make it possible to experimentally solve the outstanding mystery of how the teeming quarks and gluons inside the proton combine their spins to generate the overall spin carried by the proton.

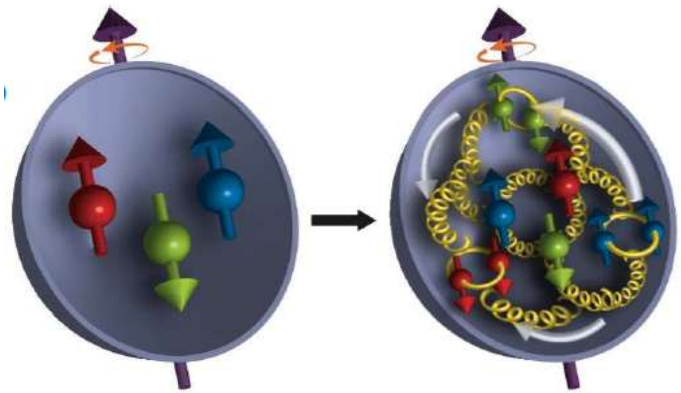
More

Origin of mass and spin of the proton

High energy reactions show the uud quark mass is just 1% of the proton mass $938\text{MeV}/c^2$

What is the origin of the mass ?

What are the roles of quarks and gluons ?



EMC experiment observed small quark spin

$$\frac{1}{2}\Delta\Sigma = \frac{1}{2}(\Delta u + \Delta d + \Delta s)$$
$$= \frac{1}{2}(0.12 \pm 0.17 + (-0.19 \pm 0.06))$$

"Spin crisis"

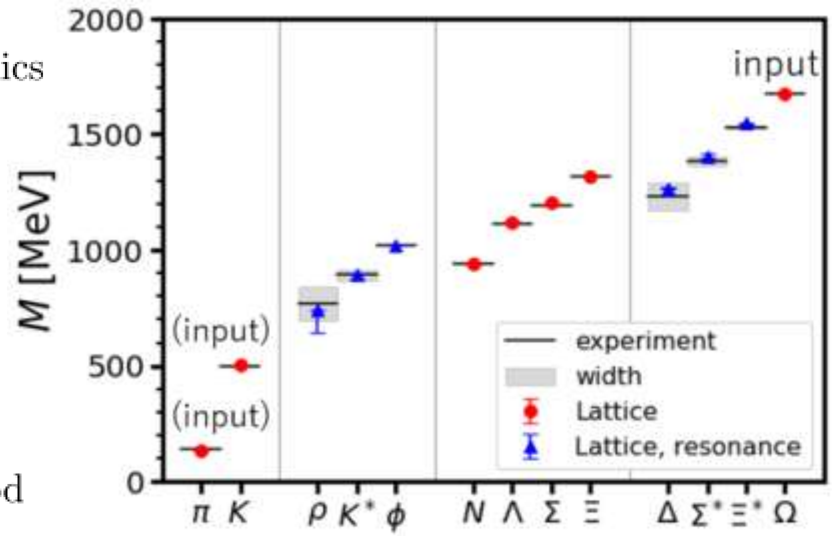
How much are the other contributions ?

(gluon spin, OAMs)

How can we access them ?

Proton mass decomposition

T. Aoyama *et al.* [HAL QCD], Phys. Rev. D110 (2024)



No doubt that the nucleon mass arises from QCD dynamics

Is more microscopic understanding possible ?

$$\langle p|\underline{T^{\mu\nu}}|p\rangle = 2P^\mu P^\nu$$

QCD energy momentum tensor(EMT)

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$$

roles of quarks and gluons could be separately understood

There are two decompositions

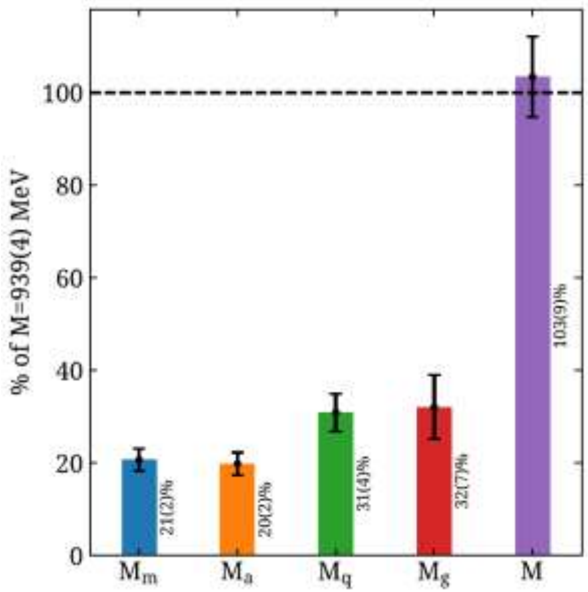
1. rest energy $\quad 2M^2 = \langle p|T^{00}|p\rangle\Big|_{\vec{p}=0} \quad M = M_m + M_q + M_g + \frac{1}{4}M_a$
2. invariant mass $\quad 2M^2 = \langle p|T^\mu_\mu|p\rangle \quad M = M_m + M_a$

$$M_m = \sum_q \sigma_q \qquad M_q = \frac{3}{4}\left(M\sum_q\langle x\rangle_q - M_m\right) \qquad M_g = \frac{3}{4}M\sum\langle x\rangle_g$$

two total momentum fractions $\langle x\rangle_q$ and $\langle x\rangle_g$
and the sigma terms σ_q

C. Alexandrou et al., PRD 101, 094513 (2020)
PRD 102, 054517 (2020)

→ calculable on lattice



Talk by C. Alexandrou

at 3rd Proton Mass Workshop

The charm quark sigma term σ_c gives a sizable contribution

Lecture by

Constantia Alexandrou

<u>Is there anything pQCD theorists can do ?</u>

Gravitational form factors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{\{\mu}P^{\nu\}} + B_{q,g}(t)\frac{P^{\{\mu}i\sigma^{\nu\}\alpha}\Delta_\alpha}{2M} + D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)Mg^{\mu\nu}\Big]u(p)$$

$$P = \frac{p+p'}{2} \qquad \Delta = p' - p \qquad t = \Delta^2$$

$$M = \frac{1}{2M}\langle p|T^{00}|p\rangle\Big|_{\vec{p}=0} = M\left(A_q(0) + A_g(0) + \bar{C}_q(0) + \bar{C}_g(0)\right)$$

$$M = \frac{1}{2M}\langle p|T^{\mu}_{\ \mu}|p\rangle = M\left(A_q(0) + A_g(0) + 4[\bar{C}_q(0) + \bar{C}_g(0)]\right)$$

$$A_q(0) + A_g(0) = 1$$

total momentum fraction

$$\bar{C}_q(0) + \bar{C}_g(0) = 0$$

conservation law$(\Delta_\mu T^{\mu\nu}_{q+g} = 0)$

just have to evaluate $A_q(0)$ and $\bar{C}_q(0)$

$A_q(0) = \int_0^1 dx\, xq(x)$ ⟵ moment of the parton distribution function

global QCD analysis at NNLO (CT18) $\quad A_q(0, \mu = 1.3\mathrm{GeV}) = 0.613$

Perturbative matching

2-loop Y. Hatta, A. Rajan and K. Tanaka, JHEP12 (2018)

3-loop K. Tanaka, JHEP03 (2023)

$$
\begin{aligned}
\bar{C}_q(\mu) = -\bar{C}_g(\mu) = &-\frac{1}{4}\left(\frac{n_f}{4C_F+n_f}+\frac{2n_f}{3\beta_0}\right)+\frac{1}{4}\left(\frac{2n_f}{3\beta_0}+1\right)\frac{\langle N(p)|\, m\bar{\psi}\psi\, |N(p)\rangle}{2M^2} \\
&-\frac{4C_F A_q(\mu_0)+n_f\left(A_q(\mu_0)-1\right)}{4(4C_F+n_f)}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F+2n_f}{3\beta_0}} \\
&+\frac{\alpha_s(\mu)}{4\pi}\left(-\frac{n_f(34C_A+49C_F)}{108\beta_0}+\frac{\beta_1 n_f}{6\beta_0^2}\right. \\
&\left.+\left[\frac{n_f(34C_A+157C_F)}{108\beta_0}+\frac{C_F}{3}-\frac{\beta_1 n_f}{6\beta_0^2}\right]\frac{\langle N(p)|\, m\bar{\psi}\psi\, |N(p)\rangle}{2M^2}\right)-\frac{1}{4}A_q^{\mathrm{NLO}}(\mu) \\
&+\left(\frac{\alpha_s(\mu)}{4\pi}\right)^2\left(\frac{n_f^2}{\beta_0}\left[\frac{697C_A}{1458}+\frac{169C_F}{2916}\right]+n_f\left[\frac{17\beta_1 C_A}{54\beta_0^2}+\frac{\beta_2}{6\beta_0^2}+\frac{49\beta_1 C_F}{108\beta_0^2}\right.\right. \\
&\left.+\frac{1}{\beta_0}\left\{\left(\frac{401}{648}-\frac{26\zeta(3)}{9}\right)C_A C_F+\left(2\zeta(3)-\frac{67}{27}\right)C_A^2+\left(\frac{8\zeta(3)}{9}-\frac{2407}{2916}\right)C_F^2\right\}-\frac{\beta_1^2}{6\beta_0^3}\right] \\
&+\left[-\frac{n_f^2}{\beta_0}\left(\frac{697C_A}{1458}+\frac{1789C_F}{2916}\right)+n_f\left(-\frac{17\beta_1 C_A}{54\beta_0^2}-\frac{\beta_2}{6\beta_0^2}-\frac{157\beta_1 C_F}{108\beta_0^2}+\frac{\beta_1^2}{6\beta_0^3}-\frac{17C_F}{27}\right)\right. \\
&+\frac{n_f}{\beta_0}\left\{\left(\frac{26\zeta(3)}{9}+\frac{4315}{648}\right)C_A C_F+\left(\frac{67}{27}-2\zeta(3)\right)C_A^2+\left(\frac{11803}{2916}-\frac{8\zeta(3)}{9}\right)C_F^2\right\} \\
&\left.\left.+\frac{61C_A C_F}{108}-\frac{C_F^2}{27}\right]\frac{\langle N(p)|\, m\bar{\psi}\psi\, |N(p)\rangle}{2M^2}\right)-\frac{1}{4}A_q^{\mathrm{NNLO}}(\mu)\,,
\end{aligned}
$$

two nonperturbative inputs

$A_q(0)$ and σ_q

PDF

cannot determined in a hard process

Analysis by effective theory

Hoferichter, Elvira, Kubis, Meißner, PRL115, 092301

Lattice

Alexandrou, *et al.*, PRD102, 054517

$$\bar{C}_q(0,\mu=1\mathrm{GeV}) = -0.18$$

K. Tanaka, JHEP03 (2023)

$$M = \frac{1}{2M}\langle p|T^{00}|p\rangle\Big|_{\vec{p}=0} = M\Big(\underline{A_q(0)} + A_g(0) + \underline{\bar{C}_q(0)} + \bar{C}_g(0)\Big)$$

0.6 0.4 -0.18 0.18

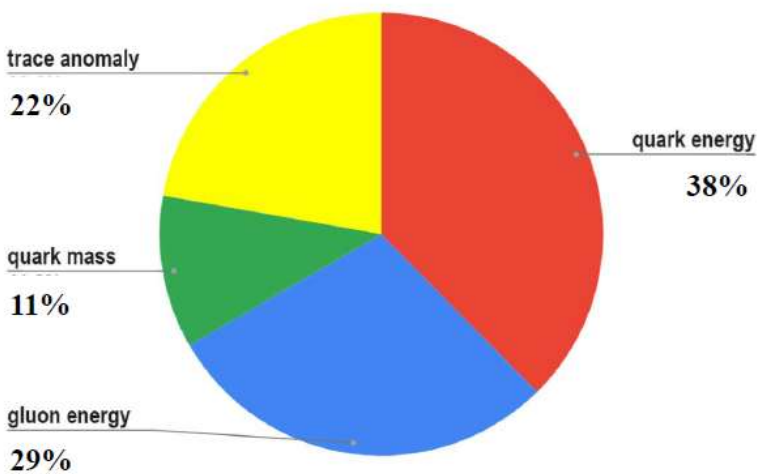
$$M = \frac{1}{2M}\langle p|T^{\mu}{}_{\mu}|p\rangle = M\Big(\underline{A_q(0)} + A_g(0) + 4[\underline{\bar{C}_q(0)} + \bar{C}_g(0)]\Big)$$

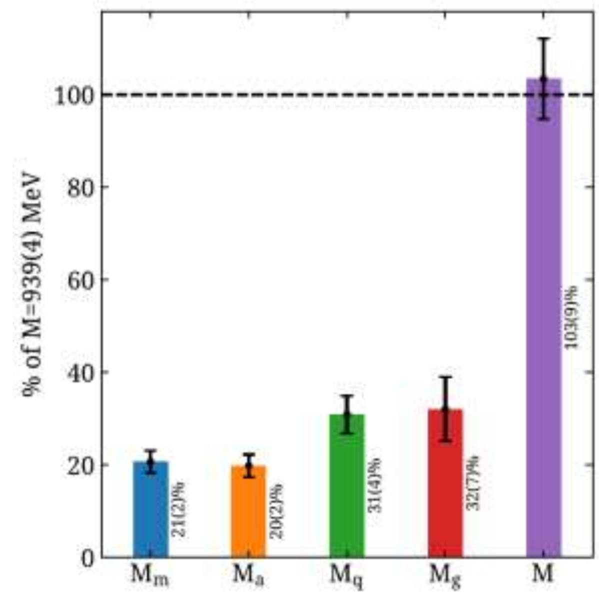
0.6 0.4 -0.18 0.18

<u>Mass decomposition in perturbative approach</u>

talk by K. Tanaka

@Hadron2025





How can EIC contribute to this problem ?

Near-threshold quarkonium production, Sullivan process, $\cdots$

Need more idea

Proton spin decomposition

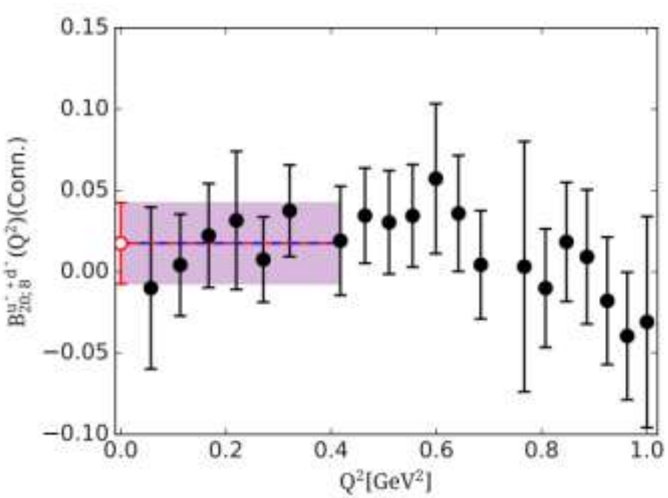
Ji's sum rule

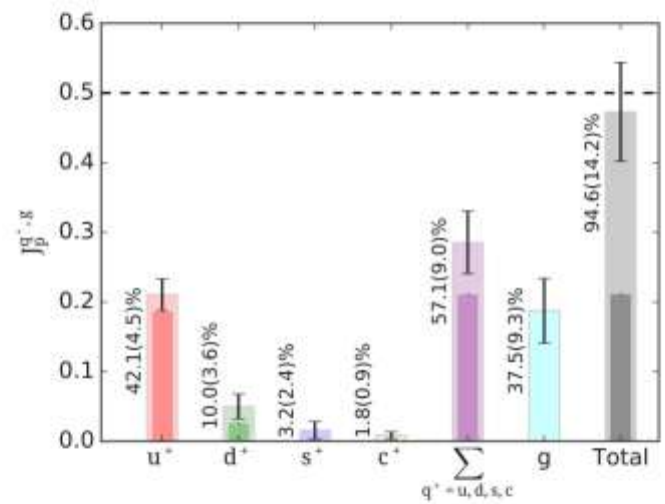
$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Big(A_{q+g}(0) + B_{q+g}(0)\Big)$$

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{\{\mu}P^{\nu\}} + B_{q,g}(t)\frac{P^{\{\mu}i\sigma^{\nu\}\alpha}\Delta_\alpha}{2M} + D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)Mg^{\mu\nu}\Big]u(p)$$

$\Delta_\alpha \to 0$ in $p' \to p$

lattice result C. Alexandrou *et al.*, Phys. Rev. D101 (2020)





$$\frac{1}{2}\Big(A_{q,g}(0)+B_{q,g}(0)\Big)=\frac{1}{2}\int dx\, x\Big(H_{q,g}(x,0,0)+E_{q,g}(x,0,0)\Big)$$

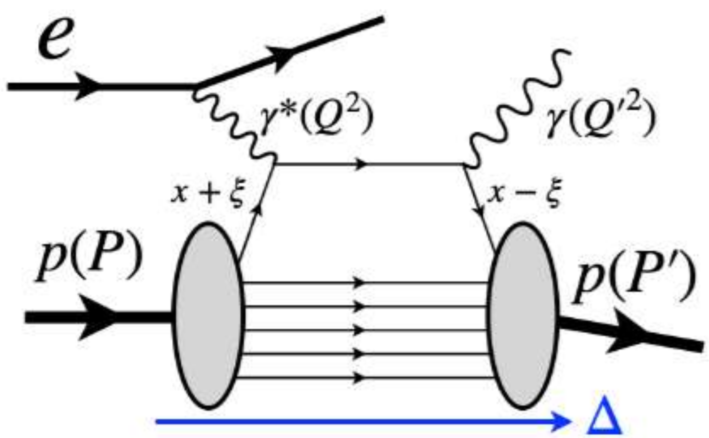
generalized parton distribution(GPD) functions

$q(x)$:parton distribution function

extracted from the off-foward matrix, difficult

$e_q \int dx\, E_q(x,\xi,t) = F_2(t)$ electromagnetic form factors

possible to access x-dependence ?



Deeply Virtual Compton Scattering(DVCS)

Exclusive reaction

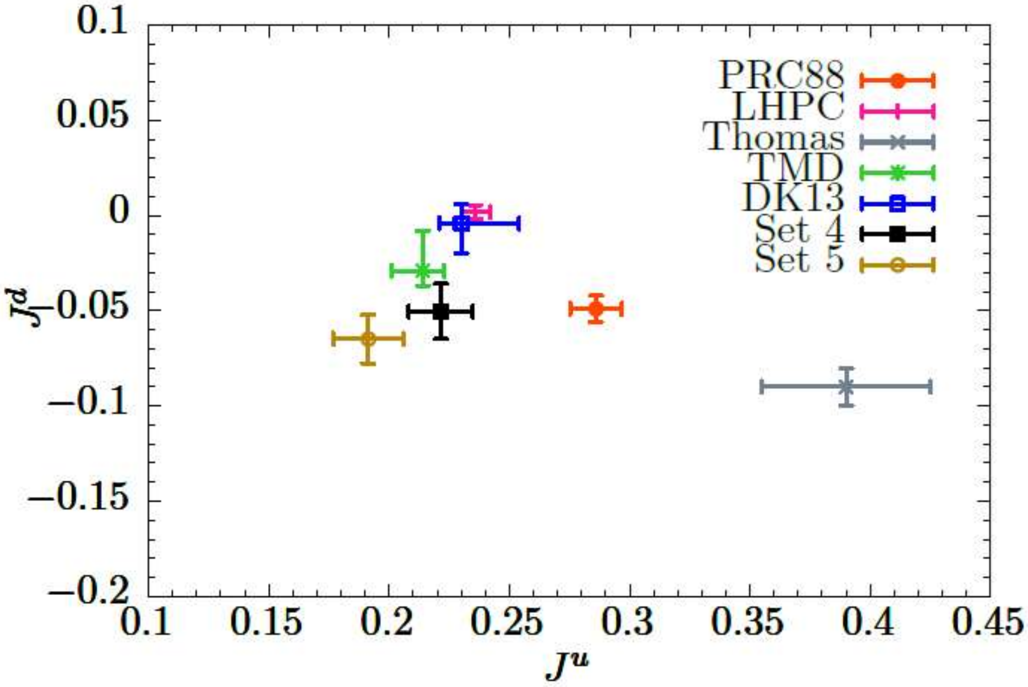
GPDs appear at the amplitude level

Many different cross sections with respect to the polarizations and the sign of beam charge

The cross sections receive contributions from Compton form factors(CFFs)

$$\mathcal{F}(\xi,t)=e_q\int dx\Big[\frac{1}{\xi-x-i\epsilon}+\frac{1}{\xi+x+i\epsilon}\Big]F_q(x,\xi,t)+O(\alpha_s) \qquad F_q=\{H_q,E_q,\cdots\}$$

H. Hashamipour, M. Goharipour, K. Azizi and S. V. Goloskokov, Phys. Rev. D105 (2022)



Lattice: LHPC

J. D. Bratt *et al.* [LHPC], Phys. Rev. D82 (2010)

Model: Thomas, TMD

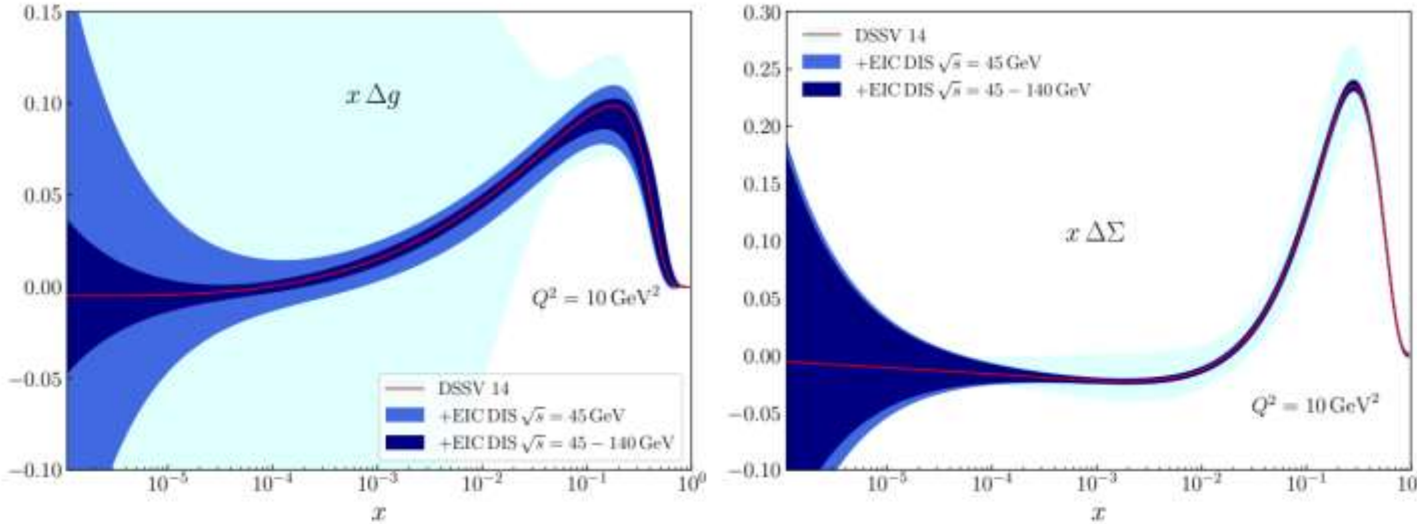
GPD: the others

Total quark spin and gluon spin has been well determined by experiment

$\Delta\Sigma \simeq 30\%$ $\quad\quad$ Δg: small positive, but large uncertainty in small-x

EIC has potential to reduce much more uncertainties

EIC Yellow Report



Can we combine them to the spin decomposition ?

<u>Jaffe-Manohar sum rule</u>

R. L. Jaffe and A. Manohar, Nucl. Phys. B337 (1990)

X.S. Chen et al, Phys. Rev. Lett. 100 (2008)

$$1/2 = \Delta\Sigma + \Delta g + L^q + L^g$$

OAMs in JM sum rule are twist-3

Y. Hatta and SY, JHEP 10 (2012) 080

$$L^q(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'}(H_q(x') + E_q(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\tilde{H}_q(x')$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \Phi_F(x_1, x_2)\mathcal{P}\frac{3x_1 - x_2}{x_1^2(x_1 - x_2)^2} - x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{\Phi}_F(x_1, x_2)\mathcal{P}\frac{1}{x_1^2(x_1 - x_2)}$$

$$L^g(x) = \frac{x}{2}\int_x^{\epsilon(x)} \frac{dx'}{x'^2}(H_g(x') + E_g(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\Delta G(x')$$
$$+2x\int_x^{\epsilon(x)} \frac{dx'}{x'^3}\int dX\,\Phi_F(X, x') + 2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{M}_F(x_1, x_2)\mathcal{P}\frac{1}{x_1^3(x_1 - x_2)}$$
$$+2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 M_F(x_1, x_2)\mathcal{P}\frac{2x_1 - x_2}{x_1^3(x_1 - x_2)^2}$$

Experimental determination is highly challenging

Another possible way

C. Lorce and B. Pasquini, Phys. Rev. D84, 014015 (2011)
C. Lorce, B. Pasquini, X. Xiong and F. Yuan, Phys. Rev. D85 (2012)
Y. Hatta, Phys. Lett. B708 (2012)

$$L_q^z(x) = \int dx \int d^2 b_T \int d^2 k_T \, (\vec{b}_T \times \vec{k}_T)_z f_q(x, \vec{k}_T, \vec{b}_T)$$

Wigner distribution

position momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

$$= -\int dx \int d^2 k_T \, \frac{\vec{k}_T^2}{M_N^2} F_{1,4}^q(x, 0, \vec{k}_T^2, 0, 0)$$

S. Meissner, A. Metz and M. Schlegel, JHEP08 (2009)

Generalized TMD(GTMD)

Direct measurement of GTMDs has been discussed

$$\frac{d\sigma_T}{dt dQ^2 dx_B d\phi} = \frac{(N_c^2-1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)} [1+(1-y)^2]$$
$$\times \Big\{ \Big[|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 + 2\frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2} + \mathcal{G}_{1,2}|^2 \Big]$$
$$+ \cos(2\phi) a \left[-|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 \right]$$
$$+ \lambda \sin(2\phi)\, 2a \, \mathrm{Re} \left[(i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*) \right] \Big\}$$

$$e + p \to e + p + \pi^0$$

S. Bhattacharya, D. Zheng and J. Zhou, Phys. Rev. Lett. 133 (2024)

$$d\sigma_{\mathrm{DSA}} = \frac{1}{4}(d\sigma^{++} - d\sigma^{+-} - d\sigma^{-+} + d\sigma^{--})$$

$$e + p \to e + p + \mathrm{di-jet}$$

$$ih_p \frac{1+\xi}{2M^2} \epsilon^{ij} k_\perp^i \Delta_\perp^j \left[2 \left(H_g - \frac{\xi^2}{1-\xi^2} E_g \right) F_{1,4}^g - E_g F_{1,2}^g \right]$$

S. Bhattacharya, R. Boussarie and Y. Hatta, Phys. Rev. D111 (2025)

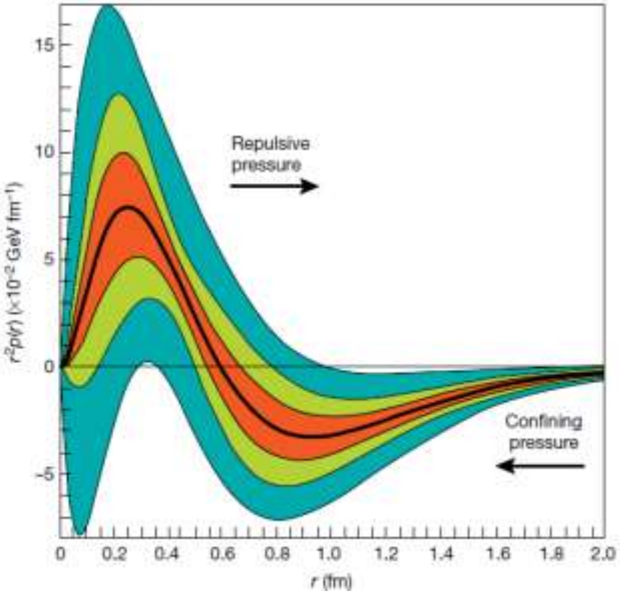
There is still much room for discussion on how to measure OAM mass, spin — spin

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{\{\mu}P^{\nu\}} + B_{q,g}(t)\frac{P^{\{\mu}i\sigma^{\nu\}\alpha}\Delta_\alpha}{2M}$$
$$+D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)Mg^{\mu\nu}\Big]u(p)$$

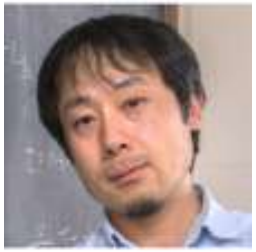
mass

pressure inside the proton

$$p_{q,g}(r) = \frac{1}{6Mr^2}\frac{d}{dr}r^2\frac{d}{dr}D_{q,g}(r) - M\bar{C}_{q,g}(r)$$



still more interesting issues



Lecture by Yoshitaka Hatta

V. D. Burkert, L. Elouadrhiri and F. X. Girod, Nature 557 (2018)

Nonperturbative approach

Lattice computation has played the central role in the mass and the spin decompositions

How about for the distribution functions ?

Parton distribution function

$$f_q(x,Q^2)=\int\frac{d\xi^-}{4\pi}e^{ixP^+\xi^-}\langle P|\bar{\psi}(\xi^-)\gamma^+\exp\left(-ig\int_0^{\xi^-}d\eta^-A^+(\eta^-)\right)\psi(0)|P\rangle$$

$$\xi^-=\frac{1}{\sqrt{2}}(t-z)$$

Lattice calcultion is difficult even for the simplest PDF

Quasi distribution method

$$O_{\text{lattice}}=C\otimes f_q$$

Perturbative coefficient PDF

-PDF

X. Xiong, X. Ji, J. H. Zhang and Y. Zhao, Phys. Rev. D90 (2014)

-TMD

X. Ji, L. C. Jin, F. Yuan, J. H. Zhang and Y. Zhao, Phys. Rev. D99 (2019)

-GPD

X. Ji, A. Schäfer, X. Xiong and J. H. Zhang, Phys. Rev. D92 (2015)

Lecture by Yong Zhao

Ken Wilson Award in 2022

Applications of quantum computing

- Parton distribution function

H. Lamm, S. Lawrence, and Y. Yamauchi, Phys. Rev. Res. 2 (2020)

N. Mueller, A. Tarasov, and R. Venugopalan, Phys. Rev. D102 (2020)

M. G. Echevarria et al., Phys. Rev. D104 (2021)

W. Qian, R. Basili, S. Pal, G. Luecke, and J. P. Vary, Phys. Rev. Res. 4 (2022)

T. Li et al., Phys. Rev. D105, L111502 (2022)

Z. B. Kang, N. Moran, P. Nguyen and W. Qian, arXiv:2501.09738

extension to TMD and GPD is possible ?

- Parton fragmentation function

never possible to calculate on lattice

T. Li, H. Xing, D. B. Zhang, arXiv:2406.05683

Application of the quantum computing to the structure functions
still has much room for discussion

Summary

I briefly reviewed the current status of the subjects related to the main objectives of EIC experiment

The objectives of the EIC are diverse and broad. It is difficult to cover all areas with a small team.

We warmly welcome young people and those from other fields!
Especially Japanese participants!