Complex potential and its universality

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QCD Theory Seminar

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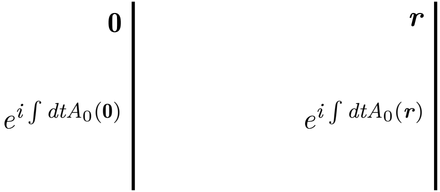
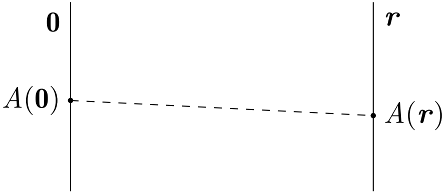
What is potential?

Potential in vacuum, e.g. QED

- Non-perturbative



- One photon exchange



Extract the potential from

- Energy shift $\Delta E(r)$
- Time dependence of the wave function $\propto e^{-iV(r)t}$

Long range behavior $V(r) \propto 1/r$

- Massless photons

Potential in the medium?

How to extend the vacuum definition?

- Free energy → real potential for thermodynamic description
- Real-time behavior → complex potential for dynamical description

$$\Psi(\boldsymbol{r},t) = \underbrace{\left\langle e^{-i\int_0^t dt'\,V(r;A(t'))} \right\rangle_T}_{\text{medium average for } A} \Psi(\boldsymbol{r},0) \xrightarrow[t\to\infty]{} \underbrace{e^{-iV(\boldsymbol{r})t}\Psi(\boldsymbol{r},0)}_{\text{oscillatory damping}}$$

Complex potential is a key quantity to the open system description

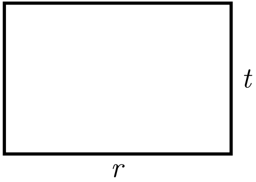
Contents

Complex potential between heavy quark pair

1. Definition using static heavy quark pair ($M = \infty$)

$$\Psi(\boldsymbol{r}, t) = \underbrace{\langle Q_c(\boldsymbol{0}, t)\, Q(\boldsymbol{r}, t)\, Q^\dagger(\boldsymbol{r}, 0)\, Q_c^\dagger(\boldsymbol{0}, 0)\rangle_T}_{\text{medium average of } e^{-iV(r;A_{\rm bkg})t}} \xrightarrow[t\to\infty]{} \underbrace{e^{-iV(\boldsymbol{r})t}\Psi(\boldsymbol{r}, 0)}_{\text{oscillatory damping}}$$

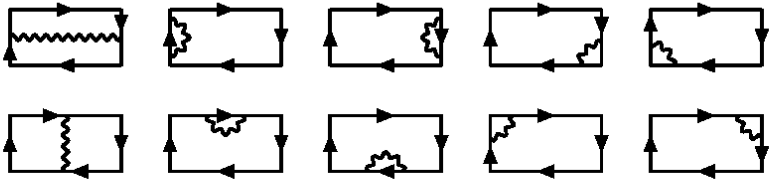
2. Time dependence of real-time thermal Wilson loop at late times



$$W(t, r) \sim e^{-iV(r)t}$$

Complex potential in perturbation theory

Leading order (HTL-resummed) perturbation at $r \sim 1/gT$ [Laine et al (07)]



$$V(r) = \underbrace{-\frac{C_F g^2}{4\pi}\left(m_D + \frac{e^{-m_\mathrm{D} r}}{r}\right)}_{\text{mass shift + screening}} \underbrace{-iC_F g^2 T \int \frac{d^3k}{(2\pi)^3} \frac{\pi m_\mathrm{D}^2 (1 - e^{i\boldsymbol{k}\cdot\boldsymbol{r}})}{k(k^2 + m_\mathrm{D}^2)^2}}_{\text{Landau damping} \sim \text{collisions}}$$

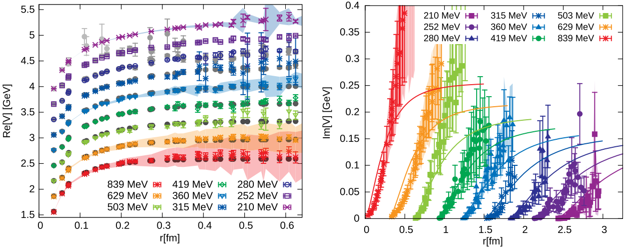
Next-to-leading order calculation [Carrington-Manuel-Soto (24)]

Complex potential on the lattice

Analytic continuation of thermal Wilson loop to imaginary time

$$\underbrace{W(t=-i\tau, r)}_{\text{lattice}} = \int d\omega\, e^{-\omega\tau}\rho(\omega, r), \quad 0 \le \tau \le \beta$$

Bayesian reconstruction of $\rho(\omega, r)$ from $W(-i\tau, r) \to V_{\text{Re}}(r) + iV_{\text{Im}}(r)$



[Rothkopf-Burnier (15)]

- $V_{\text{Re}}(r)$ screening (2015), no screening (2024), $V_{\text{Im}}(r)$ increases with r

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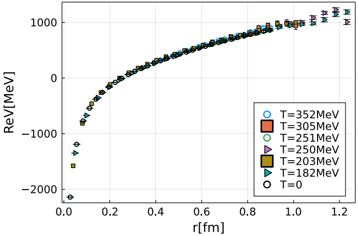

[HotQCD (24)]

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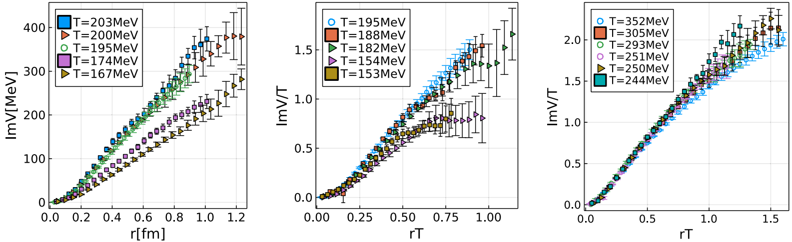

[HotQCD (24)]

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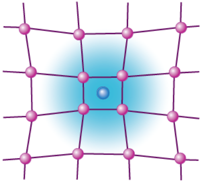
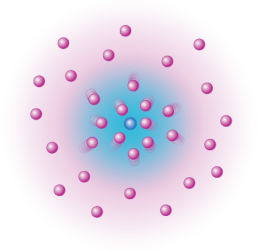
Polarons

- In metals, a conduction electron induces crystal polarization = Polaron [Landau-Pekar (48)]
  - Polaron mass $\gg$ electron mass, e.g. 432 times larger in NaCl
- Polaron broadly means (not necessarily heavy) impurity quasiparticles in cold atomic gas
  - Various mass ratios, e.g. Fermi polaron $^{133}\mathrm{Cs}$ in $^{6}\mathrm{Li}$ gas, Bose polaron $^{40}\mathrm{K}$ in $^{87}\mathrm{Rb}$ gas
  - Tunable coupling $\rightarrow$ attractive polarons, repulsive polarons





Heavy impurities can simulate heavy quark systems in the QGP

Polarons in a Fermi gas

Contact interaction $\propto \hat{n}\hat{n}_\Phi$

$$\hat{H} = \int d^3x \frac{1}{2m}|\boldsymbol{\nabla}\hat{\psi}_f|^2 - \mu|\hat{\psi}_f|^2 + g\underbrace{\hat{\psi}_f^\dagger\hat{\psi}_f\hat{\Phi}^\dagger\hat{\Phi}}_{=\hat{n}\hat{n}_\Phi}$$

We call complex "potential" after subtracting $2\times$single particle self energies

$$\bar{V}(\boldsymbol{r}) \equiv \lim_{t\to\infty}\frac{i}{\Psi(\boldsymbol{r},t)}\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t) \quad \to \quad V(\boldsymbol{r}) \equiv \bar{V}(\boldsymbol{r}) - \bar{V}(\infty)$$

Perturbative formula by retarded Green's function

$$V_{\mathrm{Re}}(\boldsymbol{r}) = -g^2 \lim_{\omega\to 0} G^R(\boldsymbol{r},\omega), \quad V_{\mathrm{Im}}(\boldsymbol{r}) = -g^2 \lim_{\omega\to 0}\frac{2T}{\omega}\mathrm{Im}\,G^R(\boldsymbol{r},\omega)$$

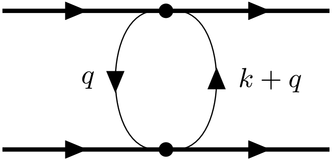
$$G^R(\boldsymbol{r},\omega) \equiv i\int_0^\infty dt e^{i\omega t}\langle[\hat{n}(\boldsymbol{r},t),\hat{n}(\boldsymbol{0},0)]\rangle$$

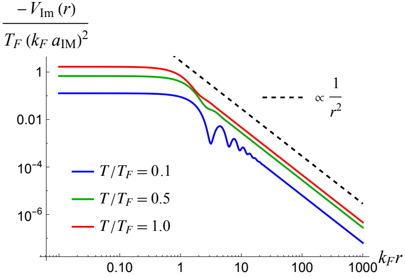
Complex potential in a Fermi gas

Complex potential [Sighinolfi et al (22)]

$$V_{\rm Re}(\boldsymbol{r}) = -g^2 \int_{\boldsymbol{k},\boldsymbol{q}} e^{i(\boldsymbol{k}-\boldsymbol{q})\cdot\boldsymbol{r}} \frac{n_F(\xi_{\boldsymbol{k}}) - n_F(\xi_{\boldsymbol{q}})}{-\xi_{\boldsymbol{k}} + \xi_{\boldsymbol{q}}} \quad \rightarrow \quad \text{virtual process, RKKY interaction}$$

$$V_{\rm Im}(\boldsymbol{r}) = -2g^2\, T \int_{\boldsymbol{k},\boldsymbol{q}} e^{i(\boldsymbol{k}-\boldsymbol{q})\cdot\boldsymbol{r}} 2\pi\delta(\xi_{\boldsymbol{k}} - \xi_{\boldsymbol{q}}) \frac{1}{T} n_F(\xi_{\boldsymbol{k}})\left[1 - n_F(\xi_{\boldsymbol{k}})\right] \quad \rightarrow \quad \text{real process}$$





$V_{\rm Im} \propto r^{-2}$ at long distance

Polarons in a superfluid

Contact interaction $\propto \hat{n}\hat{n}_\Phi$

$$\hat{H}_{\text{eff}} = \int d^3x \frac{\hat{n}}{2m}(\boldsymbol{\nabla}\hat{\varphi})^2 + \epsilon(\hat{n}) + g\hat{n}\hat{\Phi}^\dagger\hat{\Phi}$$
$$\simeq \int d^3x \frac{\bar{n}}{2m}(\boldsymbol{\nabla}\hat{\varphi})^2 + \frac{1}{2\chi}(\delta\hat{n})^2 + \frac{1}{2m}\delta\hat{n}(\boldsymbol{\nabla}\hat{\varphi})^2 + g\bar{n}\hat{\Phi}^\dagger\hat{\Phi} + g\delta\hat{n}\hat{\Phi}^\dagger\hat{\Phi}$$

Maybe, effective Lagrangian is easier to understand the physics

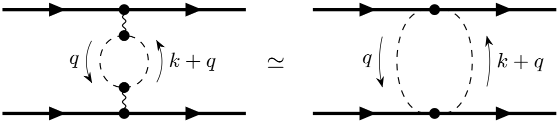
$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{ph}}(\varphi) + \mathcal{L}_{\text{pol}}(\Phi) + \underbrace{g\left[\sqrt{\chi}\partial_t\varphi + \frac{1}{2m}(\boldsymbol{\nabla}\varphi)^2\right]\Phi^\dagger\Phi}_{\text{contact interaction } nn_\Phi},$$

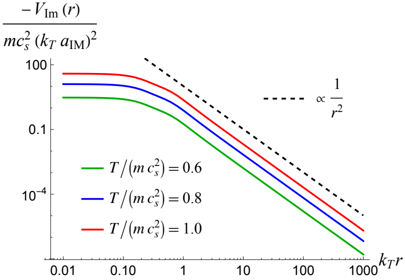
Complex potential in a superfluid

Complex potential: imaginary part [Akamatsu-Endo-Fujii-Hongo (24)], real part [Fujii-Hongo-Enss (22)]

$$V_{\mathrm{Re}}(\boldsymbol{r}) = -\frac{g^2}{m^2}\int_{\boldsymbol{k},\boldsymbol{q}} e^{i(\boldsymbol{k}-\boldsymbol{q})\cdot\boldsymbol{x}}\frac{(\boldsymbol{k}\cdot\boldsymbol{q})^2}{4c_s^2 kq}\left[\frac{1+n(c_s k)+n(c_s q)}{c_s(k+q)} - \frac{n(c_s k)-n(c_s q)}{c_s(k-q)}\right]$$

$$V_{\mathrm{Im}}(\boldsymbol{r}) = -\frac{2\pi g^2}{m^2}\int_{\boldsymbol{k},\boldsymbol{q}} e^{i(\boldsymbol{k}-\boldsymbol{q})\cdot\boldsymbol{r}}\frac{(\boldsymbol{q}\cdot\boldsymbol{k})^2}{4E_{\boldsymbol{k}}^2}\delta(E_{\boldsymbol{k}}-E_{\boldsymbol{q}})[1+n_B(E_{\boldsymbol{k}})]n_B(E_{\boldsymbol{k}})$$





Two phonon exchange gives $V_{\mathrm{Re}} \propto r^{-6}$ and $V_{\mathrm{Im}} \propto r^{-2}$ at long distance

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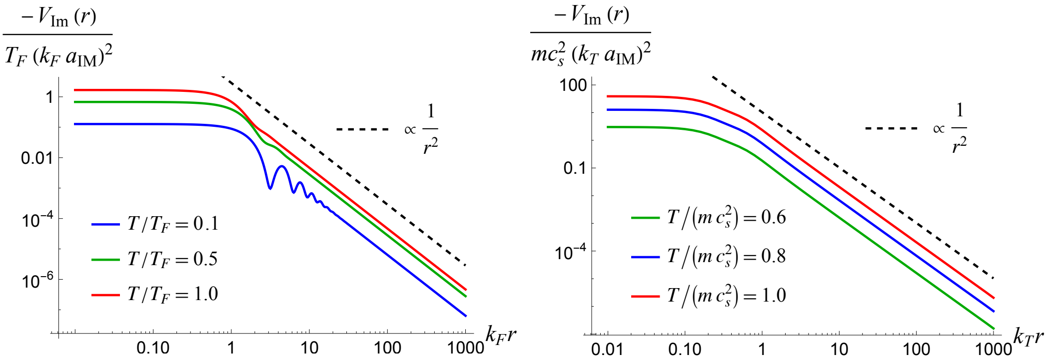
Universal imaginary potential

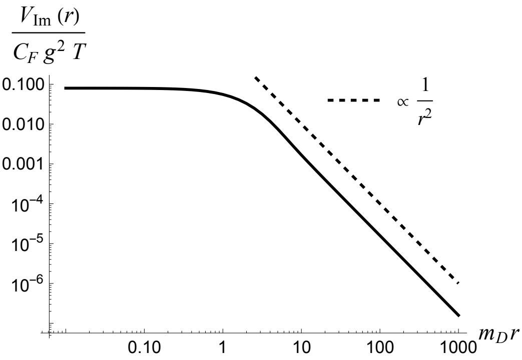
Polarons in a Fermi gas and in a superfluid



Due to gapless excitation of a particle-hole pair? Due to massless nature of phonons?

A counterexample

Quarkonium in QGP [Laine+ (07), Beraudo+ (08), Brambilla+ (08)], $V_{\rm Im} \propto +1/r^2$ at large distance

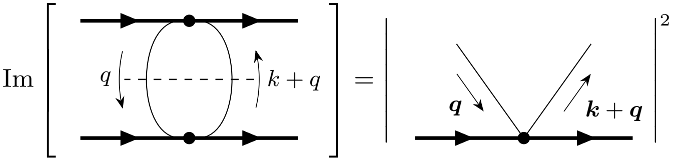


Gluons are massive due to screening → no massless excitations

Physics behind the universal imaginary potential at long distances

[Akamatsu-Endo-Fujii-Hongo (24)]

Common properties: 2-body collisions



$$\widetilde{V}_{\rm Im}(\boldsymbol{k}) \propto -\int_{\boldsymbol{q}} |\mathcal{M}_{\boldsymbol{k}+\boldsymbol{q},\boldsymbol{q}}|^2 \underbrace{\delta(E_{\boldsymbol{k}+\boldsymbol{q}} - E_{\boldsymbol{q}})}_{\text{instantaneous pot.}} n(E_{\boldsymbol{q}}) \big[1 \pm n(E_{\boldsymbol{k}+\boldsymbol{q}})\big],$$

Long distance limit $(k \to 0)$

- Delta function: $\delta(E_{\boldsymbol{k}+\boldsymbol{q}} - E_{\boldsymbol{q}}) = \delta(\cos\theta_{\boldsymbol{q}})/v_q k$
- The other parts approach constant $\neq 0$
- In total, $\tilde{V}_{\rm Im}(\boldsymbol{k}) \propto 1/k \;\to\; V_{\rm Im}(\boldsymbol{r}) \propto 1/r^2$

Universal imaginary potential $1/r^2$ in the collisional regime

Contents

Proposal for observation in cold atomic experiment?

Radio-frequency interferometry signal

$$I(\boldsymbol{r},t) \simeq 1 + e^{\bar{V}_{\mathrm{Im}}(\boldsymbol{r})t}\cos(\bar{V}_{\mathrm{Re}}(\boldsymbol{r})t)$$

Spectral width of bipolaron at low temperature

$$\frac{1}{2}\Gamma \simeq -\int d^3r \underbrace{|\Psi_b(\boldsymbol{r})|^2}_{\text{bound state}} \bar{V}_{\mathrm{Im}}(\boldsymbol{r})$$

Medium density fluctuation induced by a single impurity $n_\Phi(\boldsymbol{r},t) = g\theta(t)\delta(\boldsymbol{r})$

$$\delta n(\boldsymbol{r},t) \simeq -g\int_{t'>0} G_R(\boldsymbol{r},t-t') \simeq \frac{1}{g}\left[1 - e^{-2TV_{\mathrm{Re}}(\boldsymbol{r})t/V_{\mathrm{Im}}(\boldsymbol{r})}\right] V_{\mathrm{Re}}(\boldsymbol{r})$$

Does the universal behavior r^{-2} persist at the longest distances?

Contact interaction $\propto n n_\Phi$

1. n is a conserved density $\to$ hydrodynamics governs the longest distances

$$\tilde{V}_{\rm Im}(\boldsymbol{k}) \propto \lim_{\omega\to 0} \frac{1}{\omega} {\rm Im}\, G^R(\boldsymbol{k},\omega)$$

2. Hydrodynamic peaks [Landau-Placzek (34)]

$$\frac{1}{\omega} {\rm Im}\, G^R(\boldsymbol{k},\omega) \propto \underbrace{\frac{c_v}{c_p} \frac{c^2 \Gamma k^4}{(\omega^2 - c^2 k^2)^2 + (\omega \Gamma k^2)^2}}_{\text{Brillouin peaks}} + \underbrace{\left(1 - \frac{c_v}{c_p}\right) \frac{D_T k^2}{\omega^2 + (D_T k^2)^2}}_{\text{Rayleigh peak}} + \cdots$$

$$\tilde{V}_{\rm Im}(\boldsymbol{k}) \propto \frac{1}{D_T k^2} \quad \text{(Rayleigh peak)} \quad \to \quad V_{\rm Im}(\boldsymbol{r}) \propto \frac{1}{D_T r}$$

3. Near the critical point (model F)

$$\frac{c_v}{c_p} \propto \frac{1}{c^2 \chi_T} \to 0 \quad \text{(Landau Placzek ratio)}, \quad D_T \to \infty \quad \text{(critical slowing down)}$$

Spectrum by light scattering

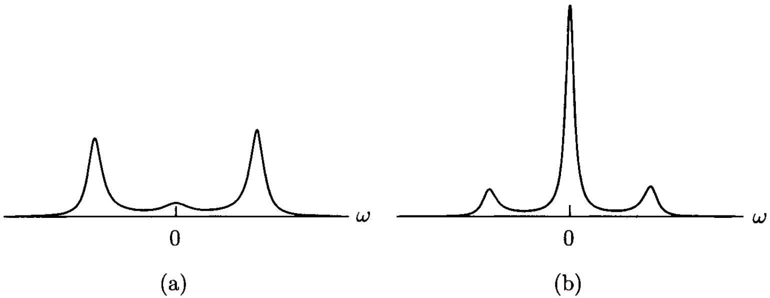
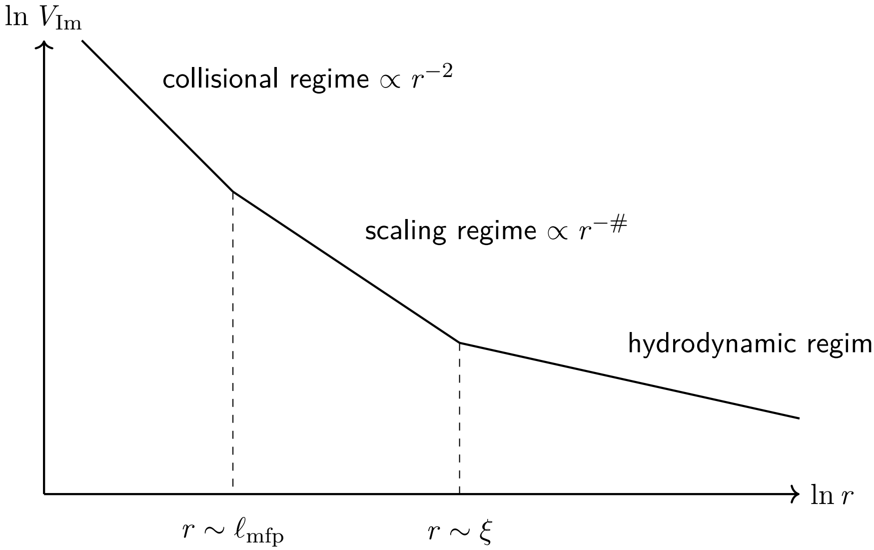


Fig. 8.4.2. Normalized light scattering intensity $nS_{nn}(\mathbf{q},\omega)/(\partial n/\partial p)_T = [2\hbar/(1-e^{-\beta\hbar\omega})]n\chi''_{nn}(\mathbf{q}\omega)/(\partial n/\partial p)_T$ as a function of ω at fixed $\mathbf{q}$. (a) Far from the critical point, where the Brillouin peaks dominate. (b) Closer to the critical point, where the Rayleigh peak dominates.

[Figure taken from Chaikin-Lubensky]

Three regimes near the critical point [Lau-Akamatsu-Endo-Fujii-Hongo, in prep.]



Summary

Universal power law regimes for $V_{\mathrm{Im}}(r) \propto r^{-2}$ is due to collisions

- Collisional regime: $V_{\mathrm{Im}}(r) \propto r^{-2}$ at $r \ll \ell_{\mathrm{mfp}}$
- Scaling regime (for $\propto n n_{\Phi}$ interaction): $V_{\mathrm{Im}}(r) \propto r^{-\#}$ at $\ell_{\mathrm{mfp}} \ll r \ll \xi$
- Hydrodynamic regime (for $\propto n n_{\Phi}$ interaction): $V_{\mathrm{Im}}(r) \propto r^{-1}$ at $\xi \ll r$

Proposals for experimental observations

- Radio-frequency interferometry signal
- Spectral width of bipolaron at low temperature
- Medium density fluctuation induced by a single impurity