

# The phi meson in-medium polarization modes from theory and experiment

Philipp Gubler (JAEA)



H.J. Kim, P. Gubler and S.H. Lee, Phys. Lett. B **772**, 194 (2017).

H.J. Kim and P. Gubler, Phys. Lett. B **805**, 135412 (2020).

I.W. Park, H. Sako, K. Aoki, P. Gubler and S.H. Lee, Phys. Rev. D **107**, 074033 (2023).

Talk at QCD Theory Seminars,  
online,  
October 19, 2023

Work done in collaboration with  
HyungJoo Kim (Yonsei U.)  
InWoo Park (Yonsei U.)  
Hiroyuki Sako (JAEA)  
Kazuya Aoki (KEK)  
Su Hounng Lee (Yonsei U.)

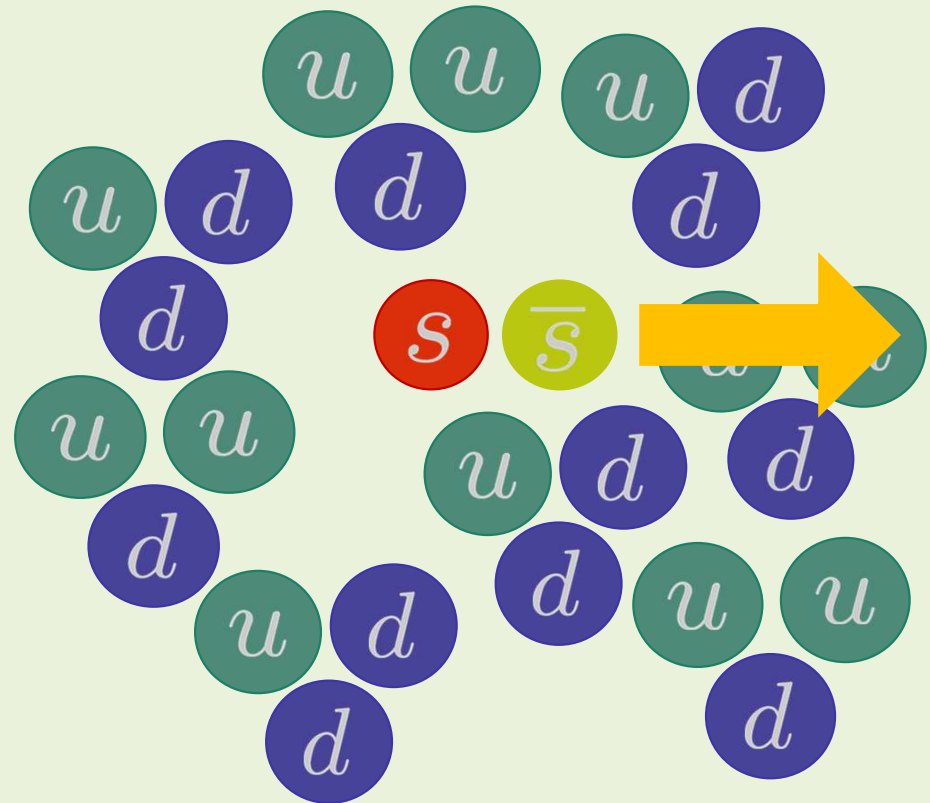
# Interest

$\phi$  meson



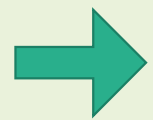
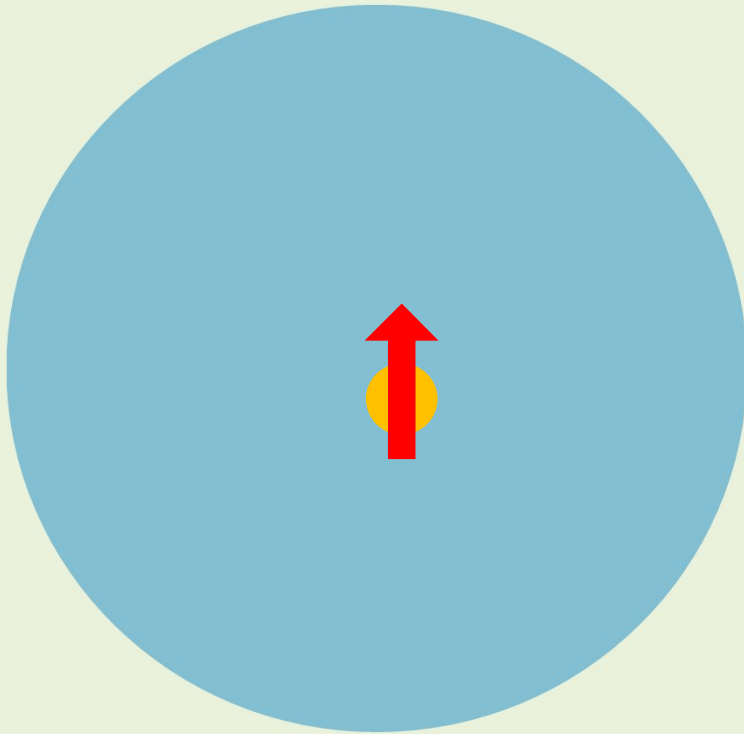
$$m_{\phi} = 1019 \text{ MeV}$$

$$\Gamma_{\phi} = 4.3 \text{ MeV}$$



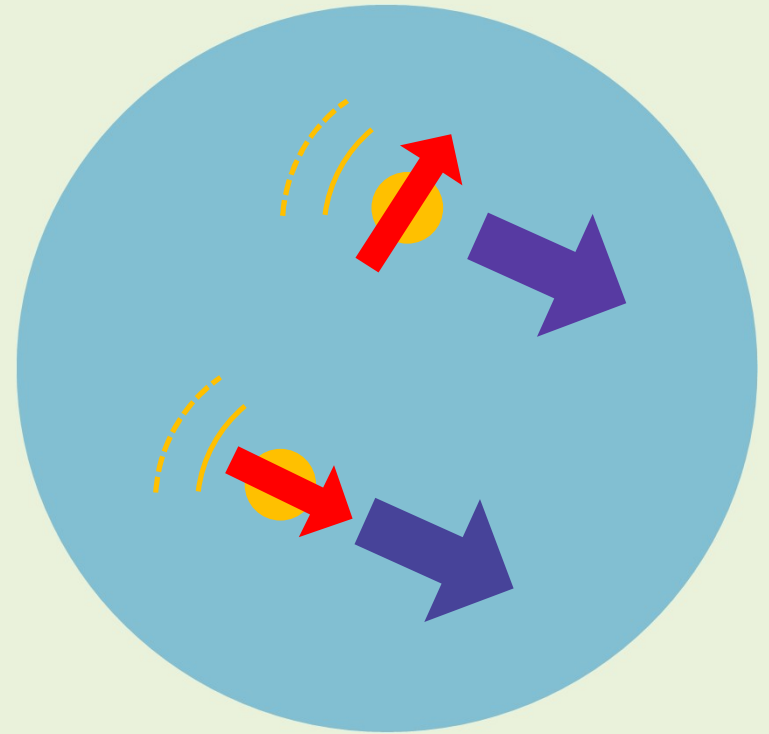
# Role of spin

meson at rest in nuclear matter



spin direction does not change physics  
(rotational symmetry)

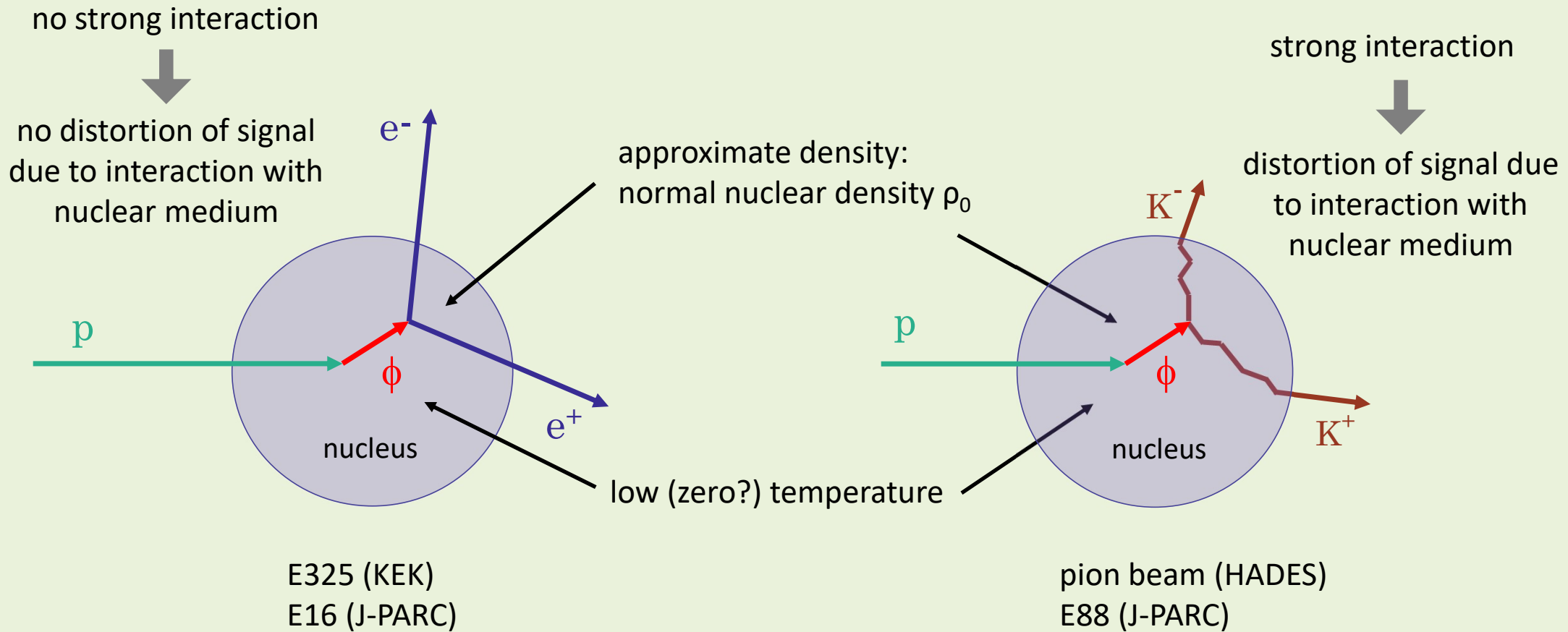
meson moving in nuclear matter



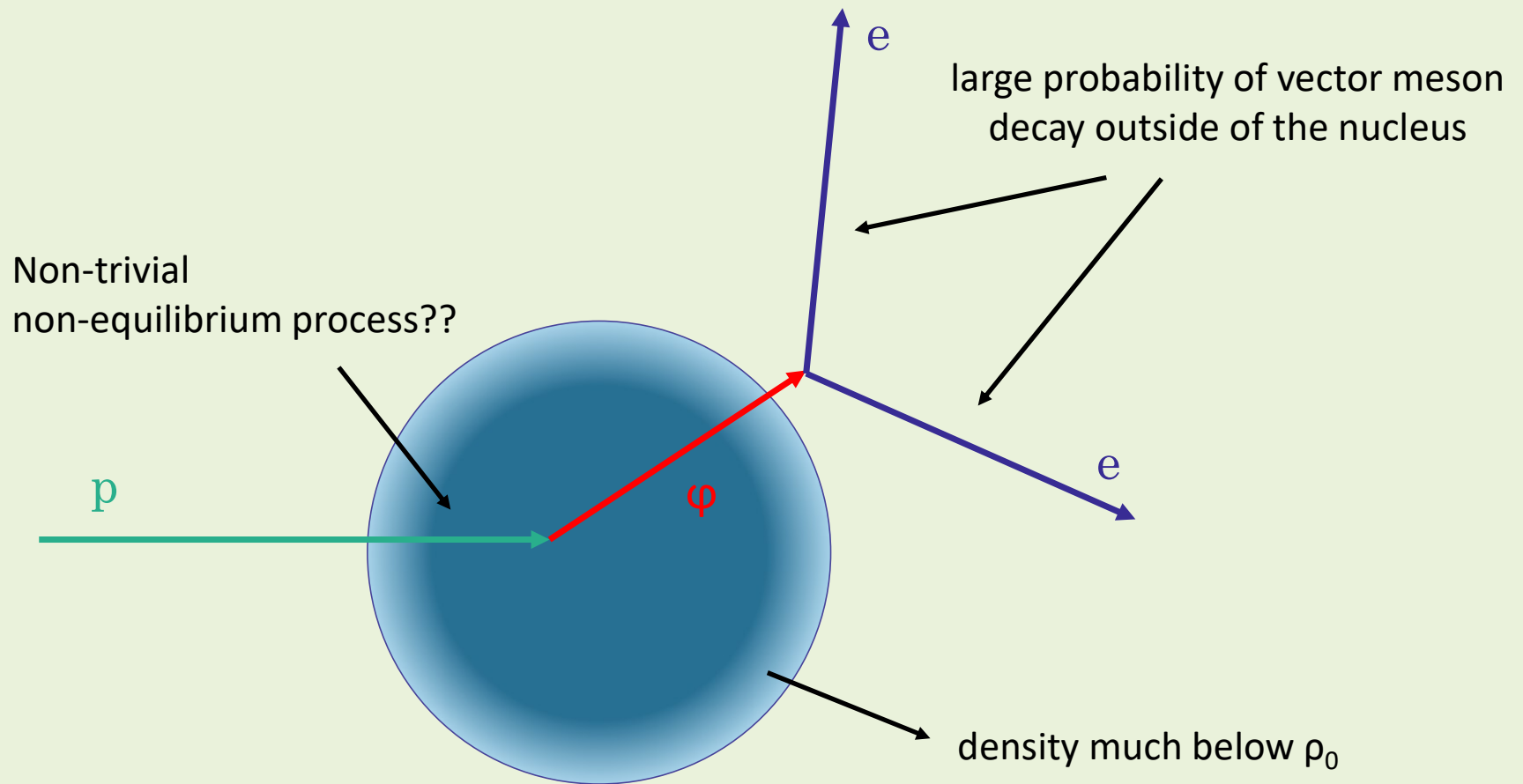
spin direction changes physics  
(broken rotational symmetry)

# The $\phi$ meson in pA collisions

Experiments to be discussed in this talk



# Complications of the experimental measurement



# Baseline: $\phi$ meson at rest in nuclear matter

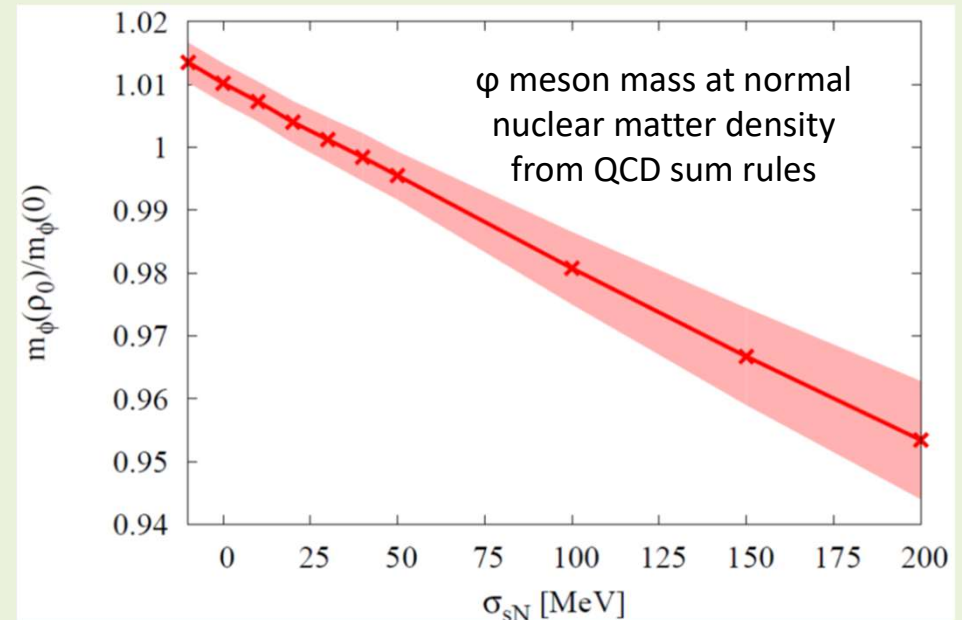
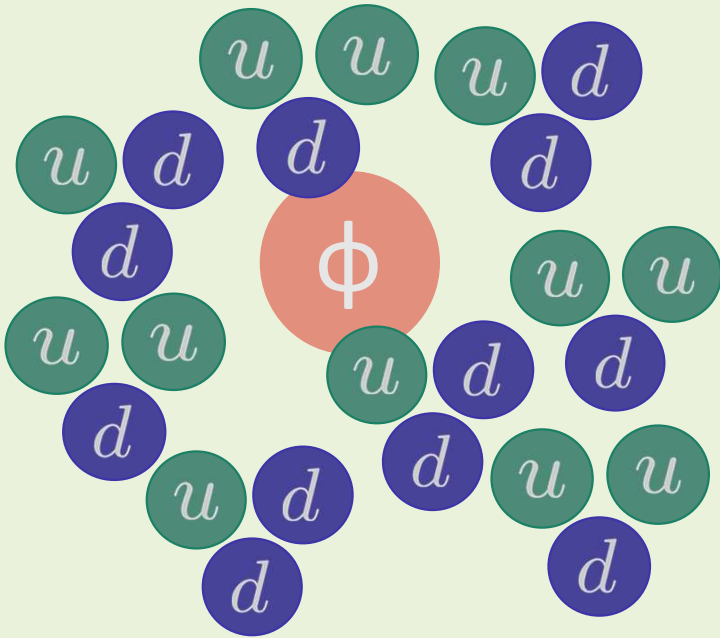
The  $\phi$  meson mass in nuclear matter probes the strange quark condensate at finite density

$$|\langle \bar{s}s \rangle_\rho| \quad \rightarrow$$

$$\rightarrow m_\phi \quad \rightarrow ?$$

T. Hatsuda and S.H. Lee, Phys. Rev. C **46**, R34 (1992).

P. Gubler and K. Ohtani, Phys. Rev. D **90**, 094002 (2014).



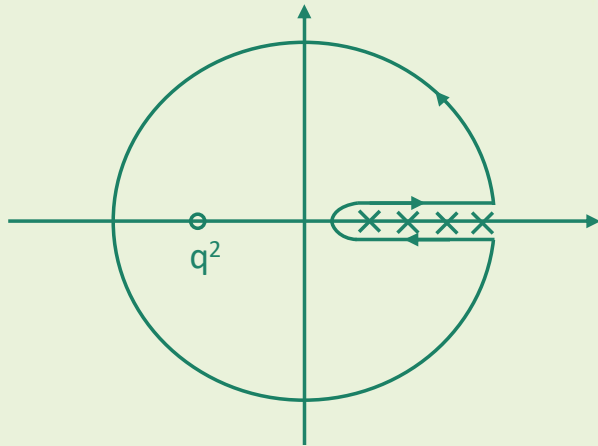
$$|\langle \bar{s}s \rangle_\rho| = |\langle \bar{s}s \rangle_0| - \frac{\rho}{m_s} \sigma_{sN} + \dots$$

# QCD sum rules

Makes use of the analytic properties of the correlation function:

$$\Pi^{\mu\nu}(q^2) = i \int d^4x e^{iqx} \langle T[j^\mu(x) j^\nu(0)] \rangle_\rho$$

$j^\mu(x) = \bar{s}(x)\gamma^\mu s(x)$



$$\rightarrow \Pi^{\mu\nu}(q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} \Pi^{\mu\nu}(s)}{s - q^2 - i\epsilon}$$

spectral function

$$\begin{aligned} &\langle \bar{s}s \rangle_\rho, \\ &\langle G_{\mu\nu}^a G^{a\mu\nu} \rangle_\rho, \\ &\langle \bar{s}\sigma_{\mu\nu} \frac{\lambda^a}{2} G^{a\mu\nu} s \rangle_\rho, \\ &\langle \bar{s}s\bar{s}s \rangle_\rho, \end{aligned}$$

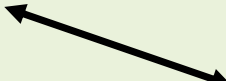
scalar condensates:  
**same for longitudinal and transverse modes**

$$\begin{aligned} &\langle ST\bar{s}\gamma^\alpha iD^\beta s \rangle_\rho, \\ &\langle STG_\mu^{a\alpha} G^{a\mu\beta} \rangle_\rho, \\ &\langle ST\bar{s}\gamma^\alpha iD^\beta iD^\gamma iD^\delta s \rangle_\rho \end{aligned}$$

non-scalar condensates:  
**cause difference between longitudinal and transverse modes**

# Structure of QCD sum rules for the $\varphi$ meson channel

(after application of the Borel transform)

$$\chi(x) = \bar{s}(x)\gamma_\mu s(x)$$


$$\frac{1}{M^2} \int_0^\infty ds e^{-\frac{s}{M^2}} \rho(s) = c_0(\rho) + \frac{c_2(\rho)}{M^2} + \frac{c_4(\rho)}{M^4} + \frac{c_6(\rho)}{M^6} + \dots$$

**In Vacuum**

$$\text{Dim. 0: } c_0(0) = 1 + \frac{\alpha_s}{\pi}$$

$$\text{Dim. 2: } c_2(0) = -6m_s^2$$

$$\text{Dim. 4: } c_4(0) = \frac{\pi^2}{3} \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle + 8\pi^2 m_s \langle 0 | \bar{s}s | 0 \rangle$$

$$\text{Dim. 6: } c_6(0) = -\frac{448}{81} \kappa \pi^3 \alpha_s \langle 0 | \bar{s}s | 0 \rangle^2$$



## Structure of QCD sum rules for the $\varphi$ meson

$$\frac{1}{M^2} \int_0^\infty ds e^{-\frac{s}{M^2}} \rho(s) = c_0(\rho) + \frac{c_2(\rho)}{M^2} + \frac{c_4(\rho)}{M^4} + \frac{c_6(\rho)}{M^6} + \dots$$

**At finite density**

(within the linear density approximation)

Dim. 0:  $c_0(\rho) = c_0(0)$

$$\langle \bar{s}s \rangle_\rho = \langle 0 | \bar{s}s | 0 \rangle + \langle N | \bar{s}s | N \rangle \rho + \dots$$

Dim. 2:  $c_2(\rho) = c_2(0)$

Dim. 4:  $c_4(\rho) = c_4(0) + \rho \left[ -\frac{2}{27} M_N + \frac{56}{27} m_s \langle N | \bar{s}s | N \rangle \right. \\ \left. + \frac{4}{27} m_q \langle N | \bar{q}q | N \rangle + A_2^s M_N - \frac{7}{12} \frac{\alpha_s}{\pi} A_2^g M_N \right]$

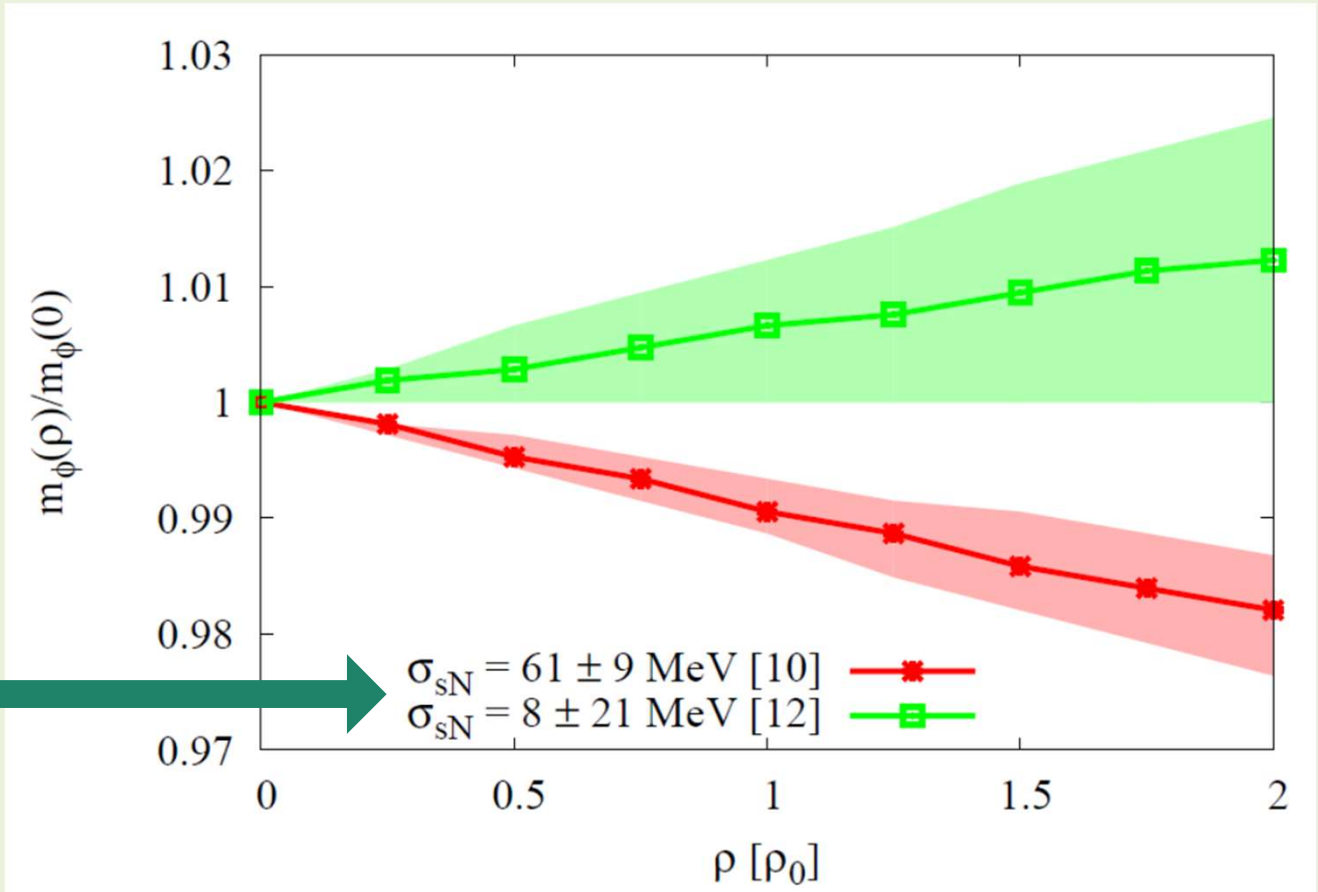
Dim. 6:  $c_6(\rho) = c_6(0) + \rho \left[ -\frac{896}{81} \kappa_N \pi^3 \alpha_s \langle \bar{s}s \rangle \langle N | \bar{s}s | N \rangle - \frac{5}{6} A_4^s M_N^3 \right]$

# Results for the $\phi$ meson mass at rest

Most important parameter, that determines the behavior of the  $\phi$  meson mass at finite density:

Strangeness content of the nucleon

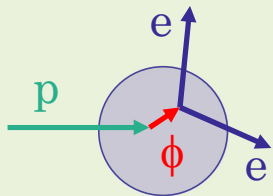
$$\sigma_{sN} = m_s \langle N | \bar{s}s | N \rangle$$



P. Gubler and K. Ohtani, Phys. Rev. D **90**, 094002 (2014).

# Experimental results

(E325, KEK)



12 GeV  
pA-reaction

slow  $\phi$ s

Pole mass:

$$\frac{m_\phi(\rho)}{m_\phi(0)} = 1 - k_1 \frac{\rho}{\rho_0}$$

$0.034 \pm 0.007$

intermediate  
 $\phi$ s

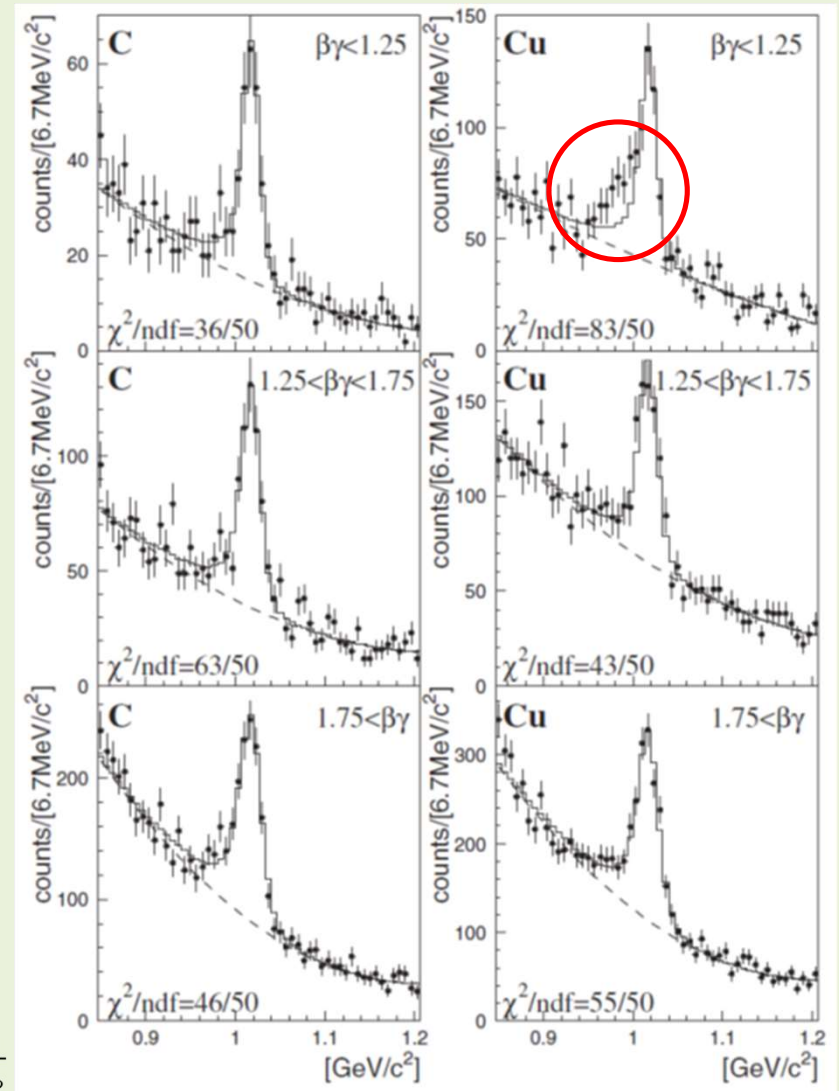
Pole width:

$$\frac{\Gamma_\phi(\rho)}{\Gamma_\phi(0)} = 1 + k_2 \frac{\rho}{\rho_0}$$

$2.6 \pm 1.5$

fast  $\phi$ s

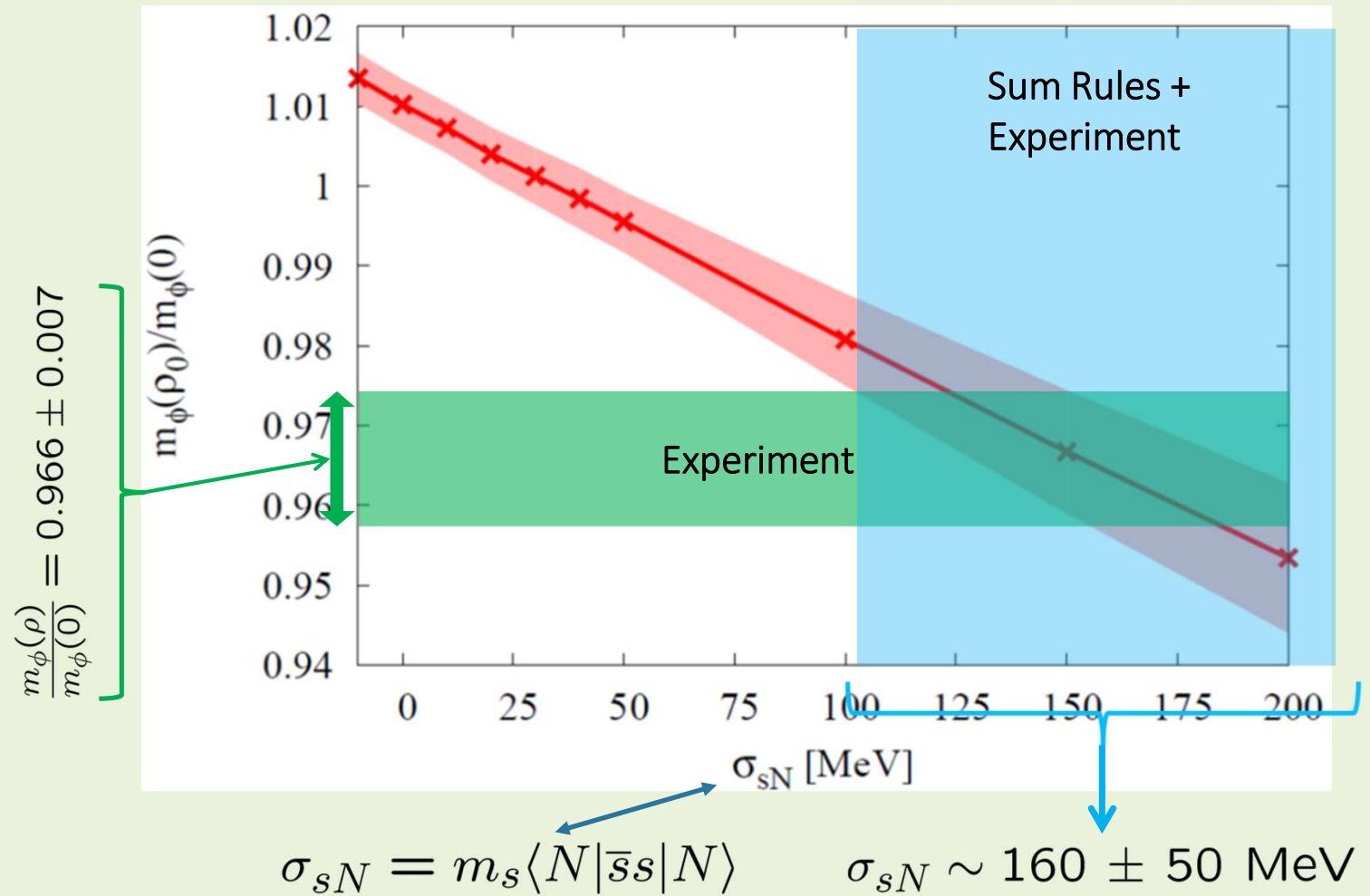
$$\beta\gamma = \frac{|\vec{p}|}{m_\phi}$$



R. Muto et al. (E325 Collaboration), Phys. Rev. Lett. **98**, 042501 (2007).

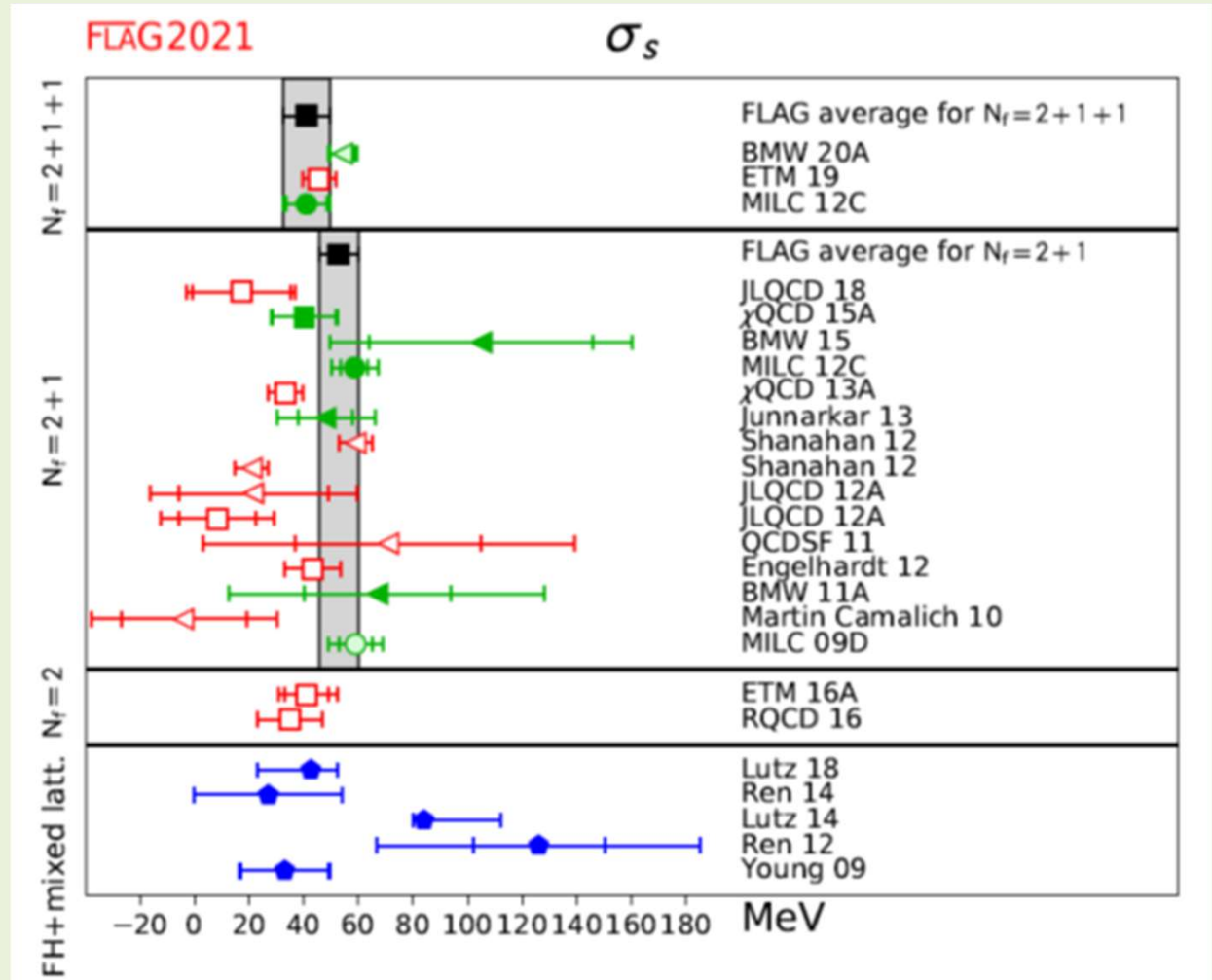
# Comparison between theory and experiment

R. Muto et al.  
(KEK, E325 Collaboration),  
Phys. Rev. Lett. **98**,  
042501 (2007).



# What does lattice QCD say about the strange sigma term?

$$\sigma_{sN} = m_s \langle N | \bar{s}s | N \rangle$$



<http://flag.unibe.ch/2021/>

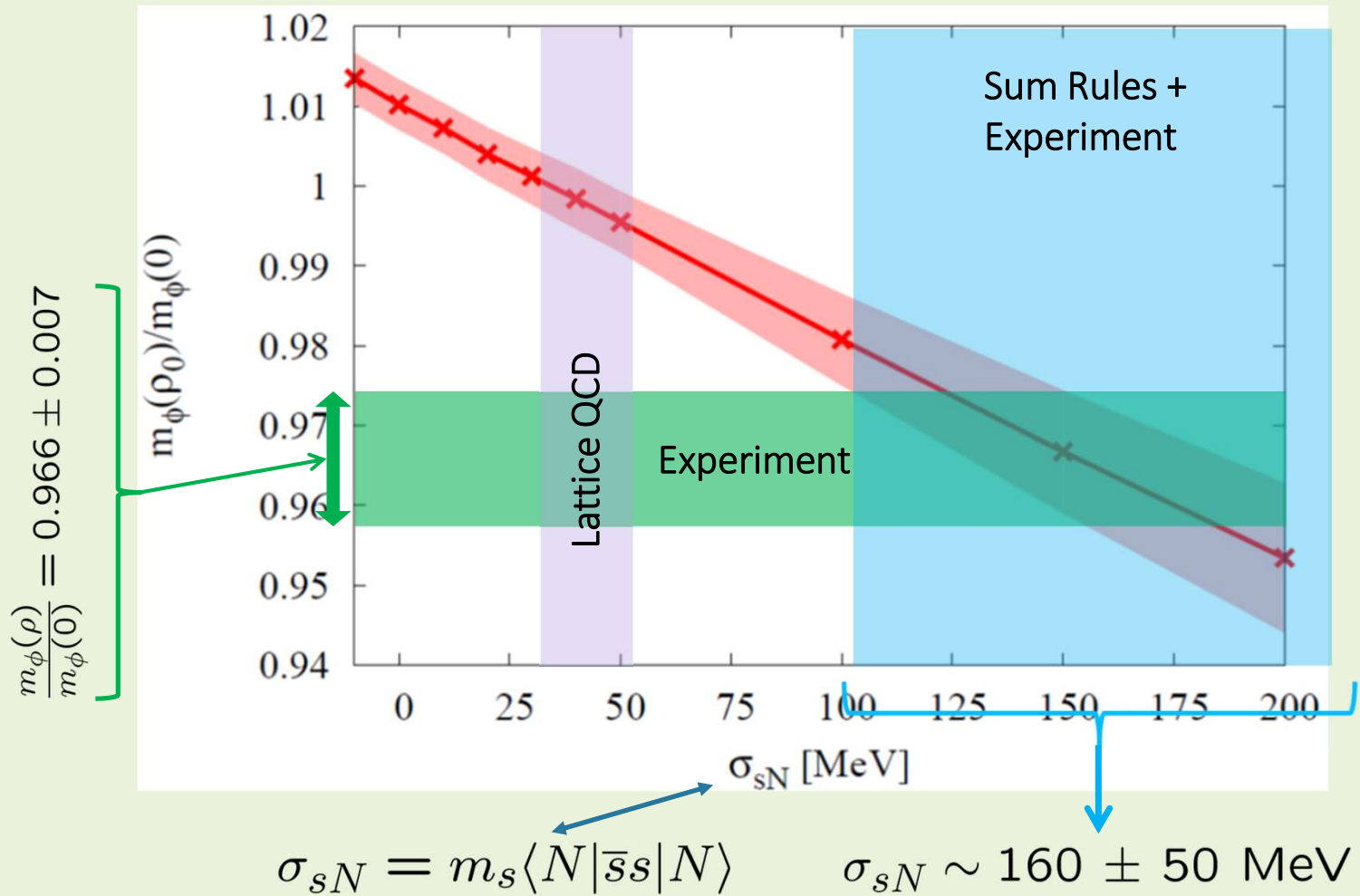
# Comparison between theory and experiment

Not consistent?

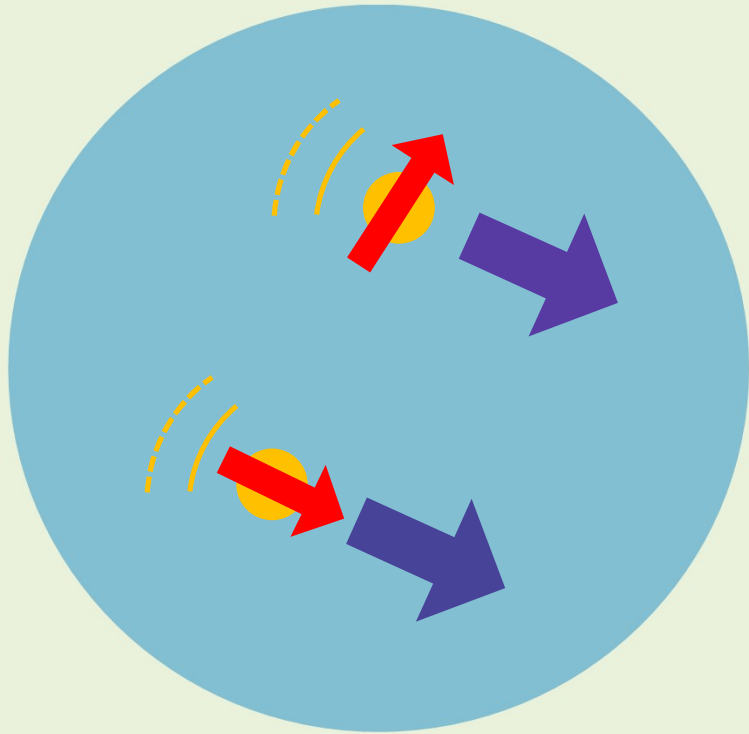
R. Muto et al.  
(KEK, E325 Collaboration),  
Phys. Rev. Lett. **98**,  
042501 (2007).



Measurement will be repeated at the J-PARC E16 experiment (with 100 times increased statistics!)



# $\phi$ meson **moving** in nuclear matter



$\phi$  meson properties depend on the spin polarization (longitudinal or transverse)



Broken  
Lorentz symmetry

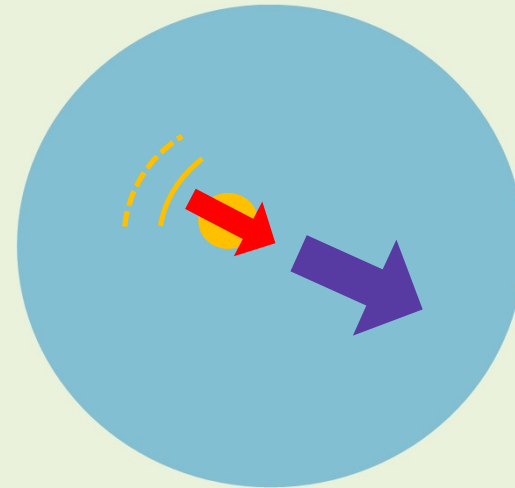
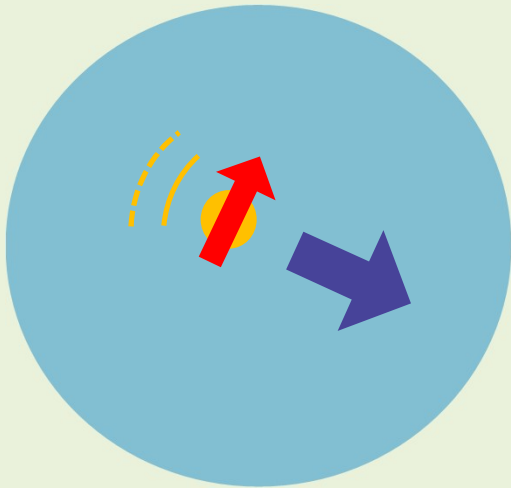
- ★ Potential effect on mass shift measurement?
- ★ Splitting between different polarization modes?

The non-zero momentum case:  
Disentangling longitudinal and transverse components

$$\Pi^{\mu\nu}(\omega^2, \vec{q}^2)$$

$$\Pi_L(\omega^2, \vec{q}^2) = \frac{1}{\vec{q}^2} \Pi_{00}$$

$$\Pi_T(\omega^2, \vec{q}^2) = -\frac{1}{2} \left( \frac{1}{\vec{q}^2} \Pi_{00} + \frac{1}{q^2} \Pi_{\mu}^{\mu} \right)$$

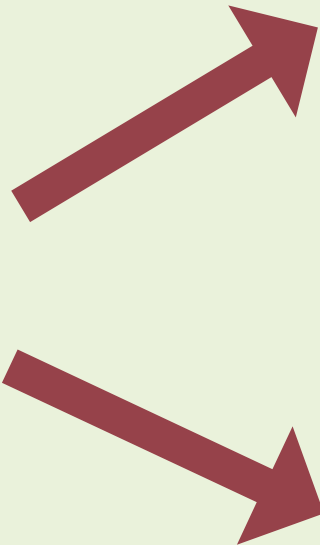




# The $\phi$ meson with non-zero momentum

$$\frac{1}{\omega^2 - m_\phi^2(0)}$$

zero momentum



$$\frac{1}{\omega^2 - \vec{q}^2 - m_{\phi,L}^2(\vec{q}^2)}$$

longitudinal  
part

$$\frac{1}{\omega^2 - \vec{q}^2 - m_{\phi,T}^2(\vec{q}^2)}$$

transverse  
part

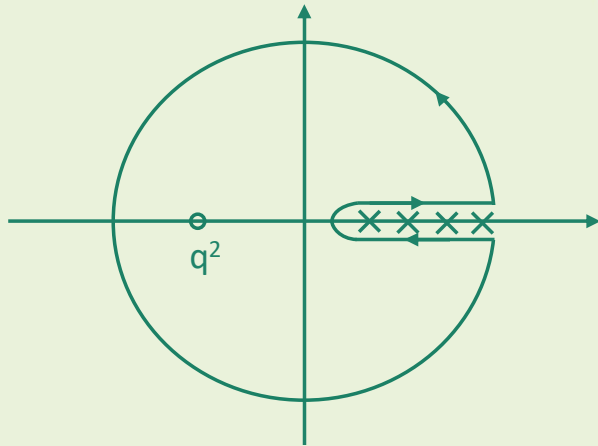
non-zero momentum  $\vec{q}$

# QCD sum rules

Makes use of the analytic properties of the correlation function:

$$\Pi^{\mu\nu}(q^2) = i \int d^4x e^{iqx} \langle T[j^\mu(x) j^\nu(0)] \rangle_\rho$$

$j^\mu(x) = \bar{s}(x)\gamma^\mu s(x)$



$$\rightarrow \Pi^{\mu\nu}(q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} \Pi^{\mu\nu}(s)}{s - q^2 - i\epsilon}$$

spectral function

$$\begin{aligned} &\langle \bar{s}s \rangle_\rho, \\ &\langle G_{\mu\nu}^a G^{a\mu\nu} \rangle_\rho, \\ &\langle \bar{s}\sigma_{\mu\nu} \frac{\lambda^a}{2} G^{a\mu\nu} s \rangle_\rho, \\ &\langle \bar{s}s\bar{s}s \rangle_\rho, \end{aligned}$$

scalar condensates:  
**same for longitudinal and transverse modes**

$$\begin{aligned} &\langle ST\bar{s}\gamma^\alpha iD^\beta s \rangle_\rho, \\ &\langle STG_\mu^{a\alpha} G^{a\mu\beta} \rangle_\rho, \\ &\langle ST\bar{s}\gamma^\alpha iD^\beta iD^\gamma iD^\delta s \rangle_\rho \end{aligned}$$

non-scalar condensates:  
**cause difference between longitudinal and transverse modes**

# Condensates that appear in the vector channel

## Quark condensates

$$\langle \bar{q}jq \rangle \equiv \langle g \bar{q} \gamma_\mu (D_\nu G_{\mu\nu}) q \rangle,$$

$$\langle j_5 j_5 \rangle \equiv \langle g^2 \bar{q} t^a \gamma_5 \gamma_\mu q \bar{q} t^a \gamma_5 \gamma_\mu q \rangle,$$

$$A_{\alpha\beta} \equiv \langle g \bar{q} (D_\mu G_{\alpha\mu}) \gamma_\beta q |_{ST} \rangle,$$

$$B_{\alpha\beta} \equiv \langle g \bar{q} \{iD_\alpha, \tilde{G}_{\beta\mu}\} \gamma_5 \gamma_\mu q |_{ST} \rangle,$$

$$C_{\alpha\beta} \equiv \langle m \bar{q} D_\alpha D_\beta q |_{ST} \rangle,$$

$$F_{\alpha\beta} \equiv \langle \bar{q} \gamma_\alpha i D_\beta q |_{ST} \rangle,$$

$$H_{\alpha\beta} \equiv \langle g^2 \bar{q} t^a \gamma_5 \gamma_\alpha q \bar{q} t^a \gamma_5 \gamma_\beta q \rangle,$$

$$K_{\alpha\beta\gamma\delta} \equiv \langle \bar{q} \gamma_\alpha D_\beta D_\gamma D_\delta q |_{ST} \rangle$$

scalar

non-scalar

## Gluon condensates

$$\langle G^2 \rangle \equiv \langle g^2 G_{\mu\nu}^a G_{\mu\nu}^a \rangle,$$

$$\langle G^3 \rangle \equiv \langle g^3 f^{abc} G_{\mu\nu}^a G_{\nu\lambda}^b G_{\lambda\mu}^c \rangle,$$

$$\langle j^2 \rangle \equiv \langle g^2 (D_\mu G_{\alpha\mu}^a) (D_\nu G_{\alpha\nu}^a) \rangle,$$

$$G_{2\alpha\beta} \equiv \langle g^2 G_{\alpha\mu}^a G_{\beta\mu}^a |_{ST} \rangle,$$

$$X_{\alpha\beta} \equiv \langle g^2 G_{\mu\nu}^a D_\beta D_\alpha G_{\mu\nu}^a |_{ST} \rangle,$$

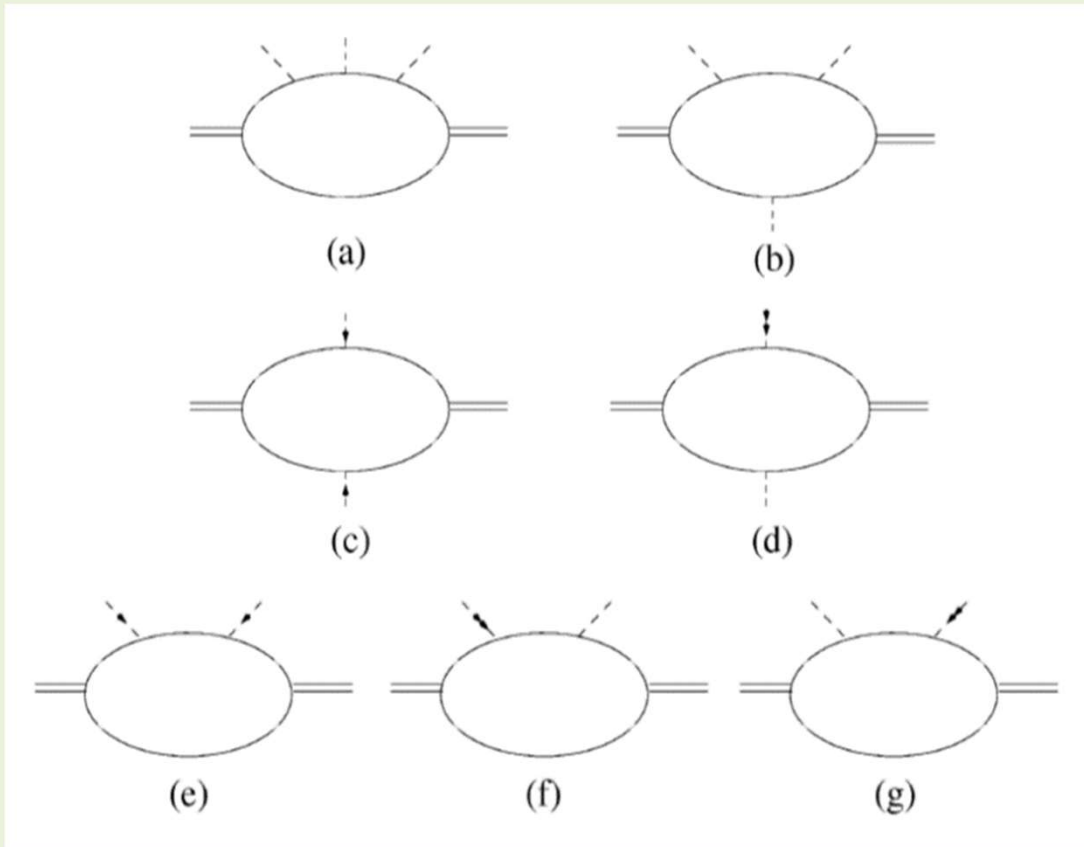
$$Y_{\alpha\beta} \equiv \langle g^2 G_{\alpha\mu}^a D_\mu D_\nu G_{\beta\nu}^a |_{ST} \rangle,$$

$$Z_{\alpha\beta} \equiv \langle g^2 G_{\alpha\mu}^a D_\beta D_\nu G_{\mu\nu}^a |_{ST} \rangle,$$

$$G_{4\alpha\beta\gamma\delta} \equiv \langle g^2 G_{\alpha\mu}^a D_\delta D_\gamma G_{\beta\mu}^a |_{ST} \rangle$$

Wilson coefficients were not yet available until recently

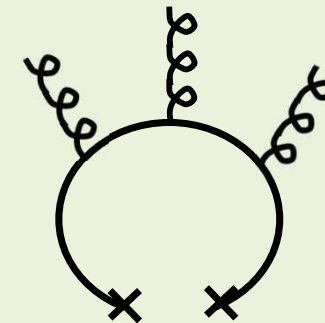
# OPE calculation



Mass singularities in  
chiral limit!

$$\frac{1}{m^2}, \log\left(\frac{\mu^2}{m^2}\right), \dots$$

Subtract corresponding quark  
condensate contribution

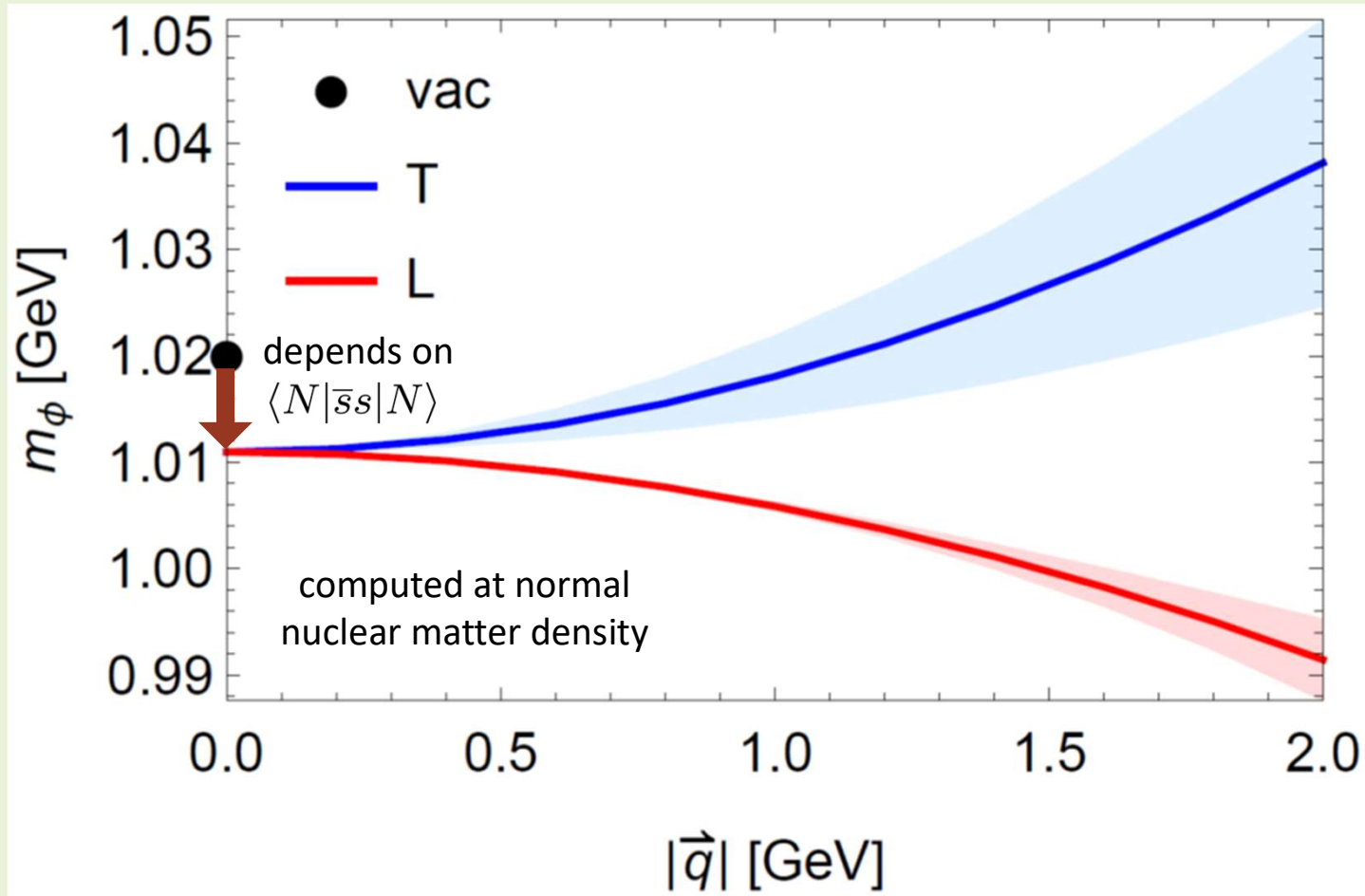


$$\langle \bar{q} \Gamma(D, G) q \rangle$$

S. Kim and S.H. Lee, Nucl. Phys. **679**, 517 (2001).

H.J. Kim, P. Gubler and S.H. Lee, Phys. Lett. B **772**, 194 (2017).

# Results for the $\phi$ meson mass with non-zero momentum



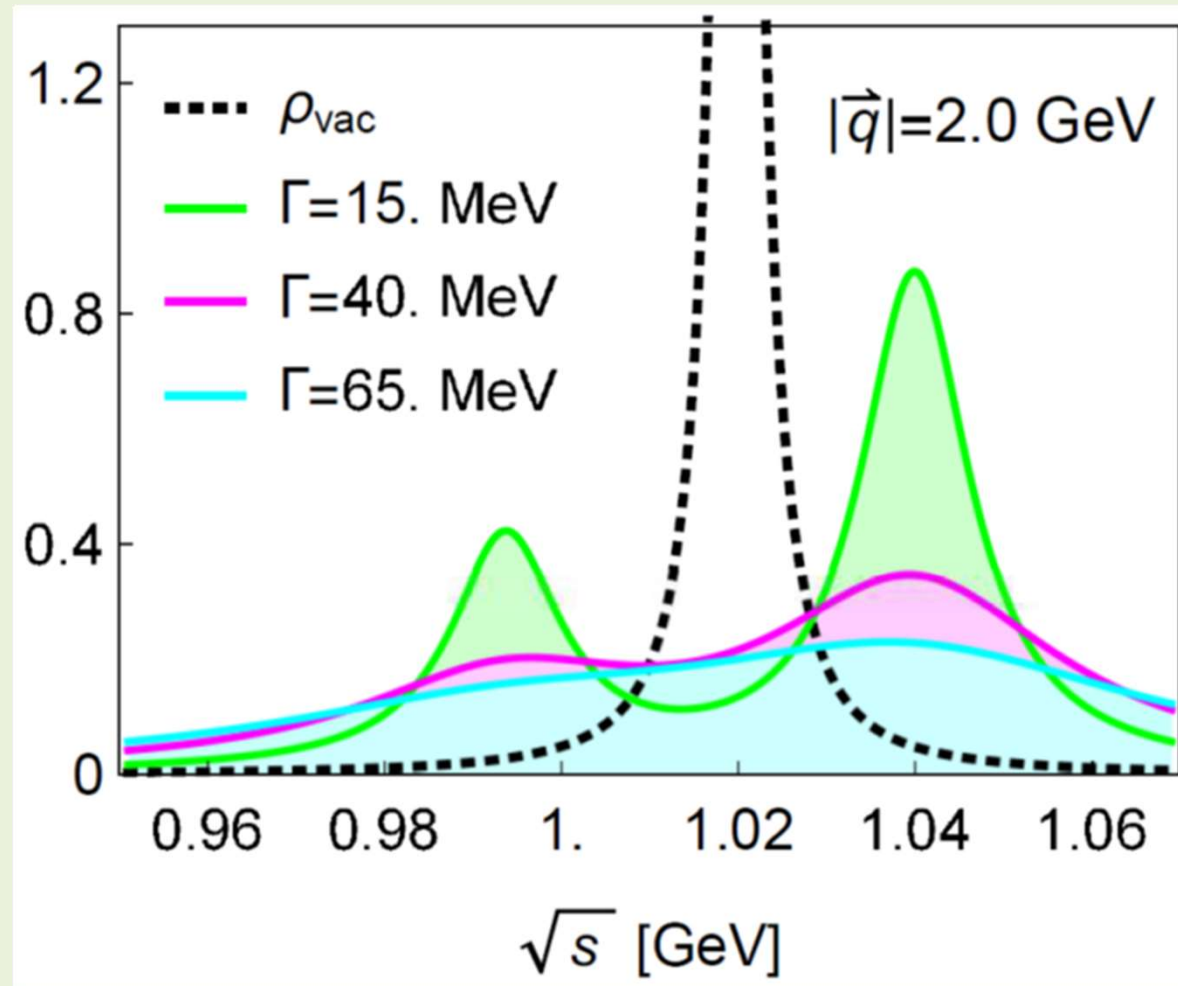
caused by

$$\langle N | \mathcal{S} \mathcal{T} \bar{s} \gamma^\alpha i D^\beta s | N \rangle + \langle N | \mathcal{S} \mathcal{T} G_\mu^{a\alpha} G^{a\mu\beta} | N \rangle$$

caused by

$$\langle N | \mathcal{S} \mathcal{T} G_\mu^{a\alpha} G^{a\mu\beta} | N \rangle$$

# The angle-averaged di-lepton spectrum

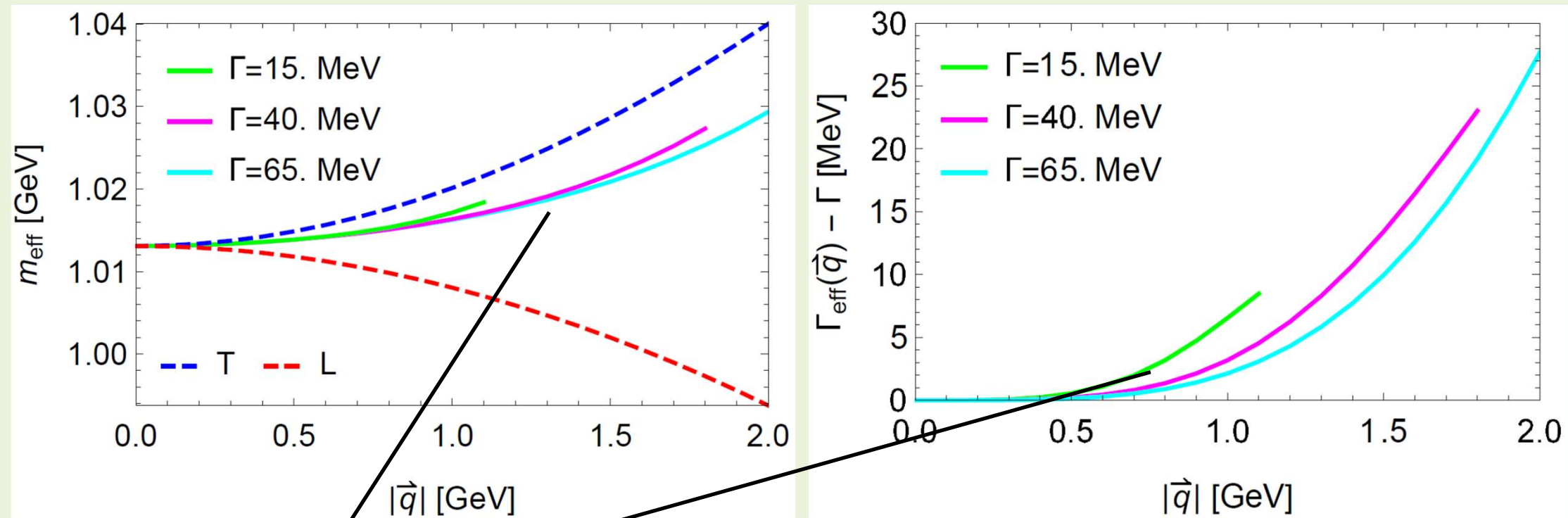


A double peak?

Computed at  
normal nuclear  
matter density

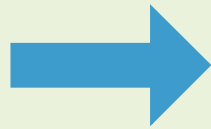
# The angle-averaged di-lepton spectrum

Even without a double peak, momentum effects can be observed

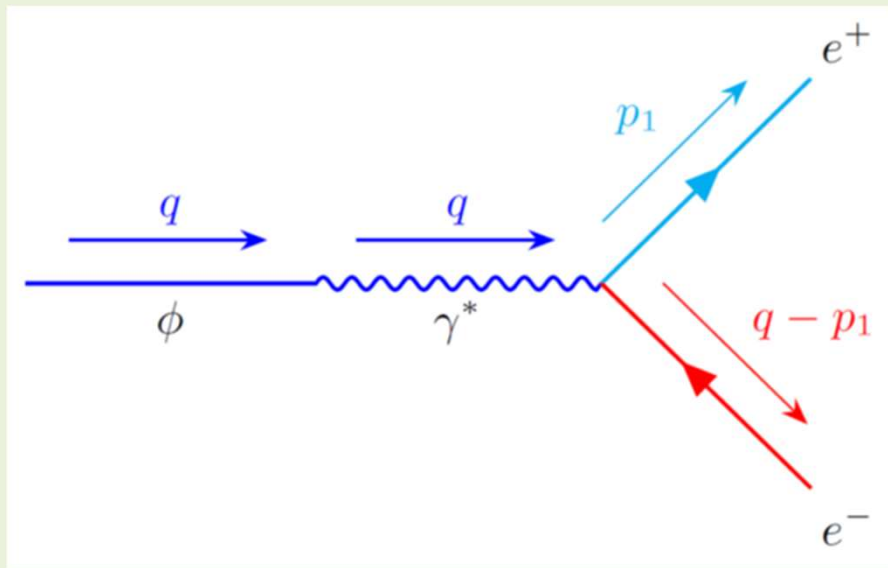


Results of one-peak fits

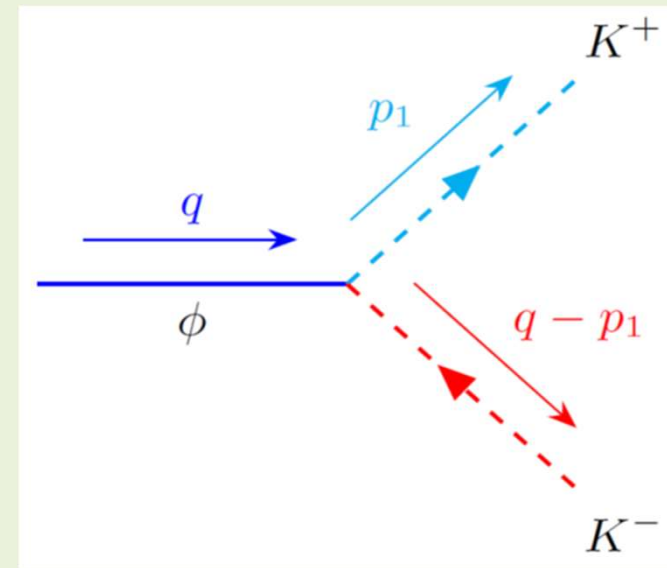
# Can the two polarizations be disentangled?



Look at the angular distributions of various decay channels



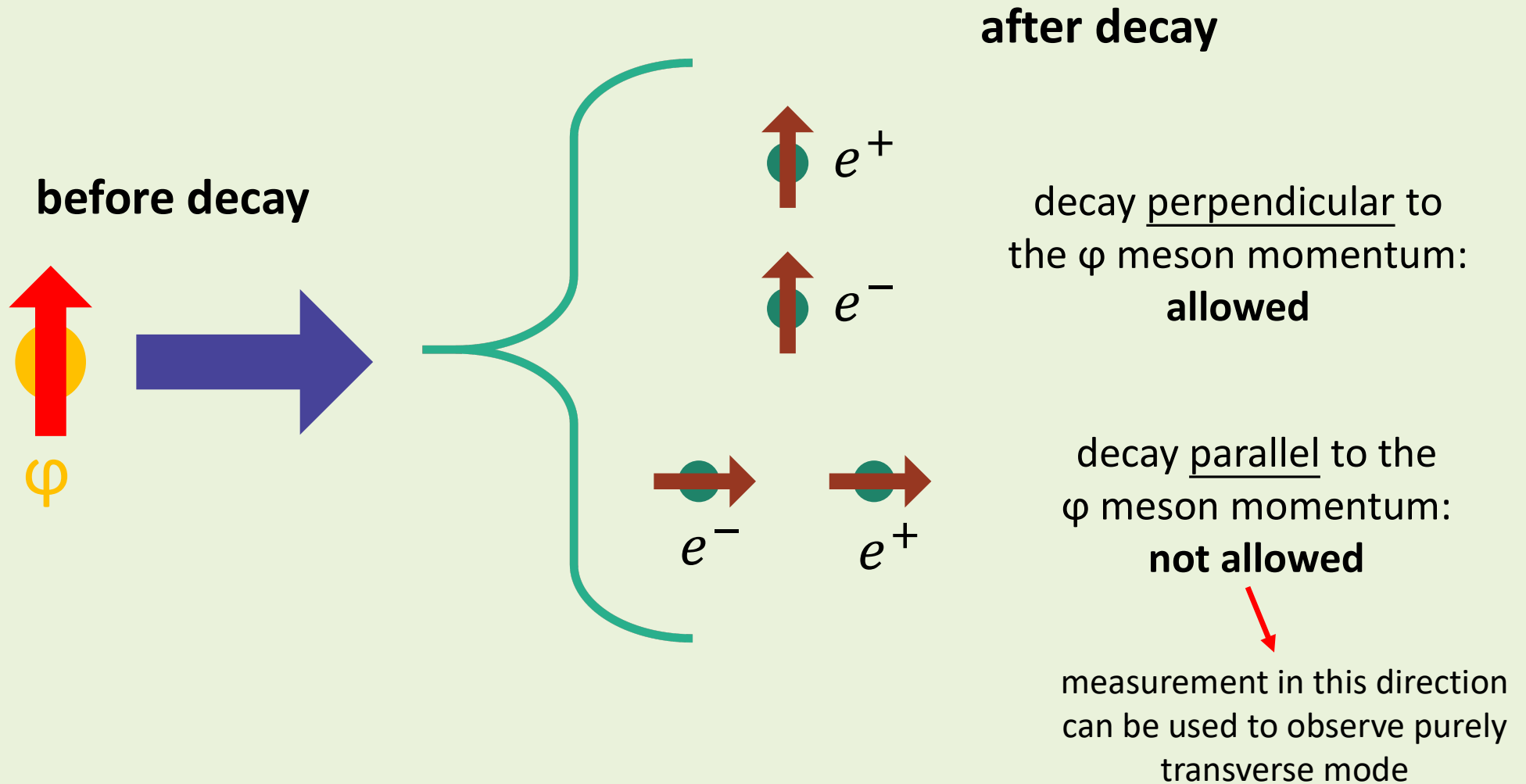
To be measured soon at the J-PARC E16 experiment



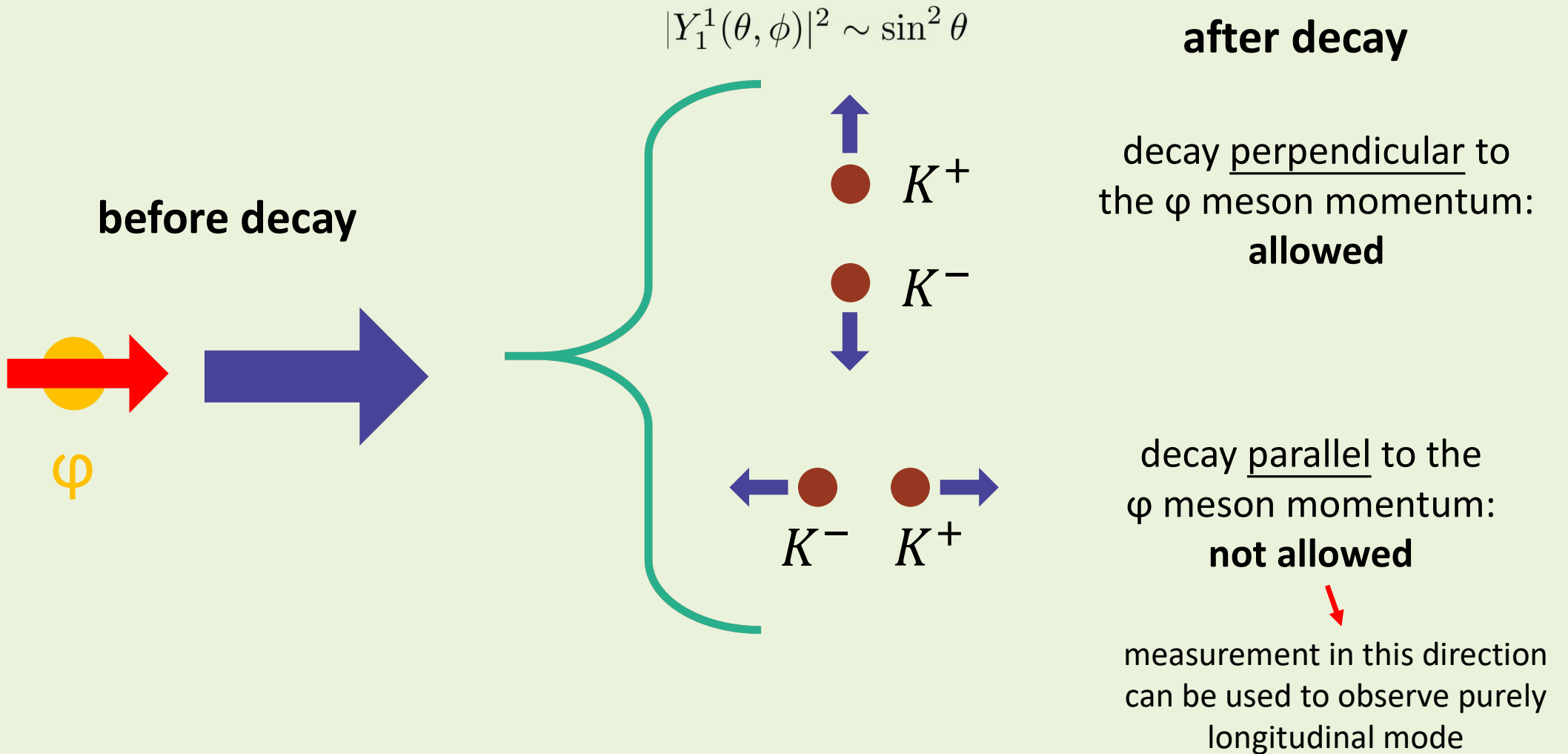
New E88 experiment at J-PARC (in a few years)



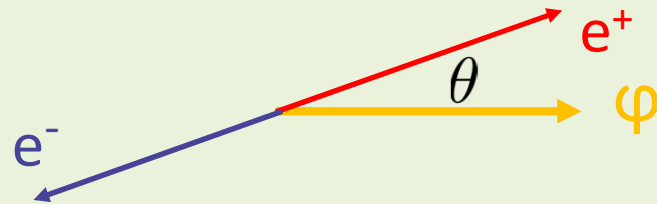
# A simple example of dilepton decay of a longitudinally polarized $\phi$



# A simple example of $K^+K^-$ decay of a transversely polarized $\varphi$

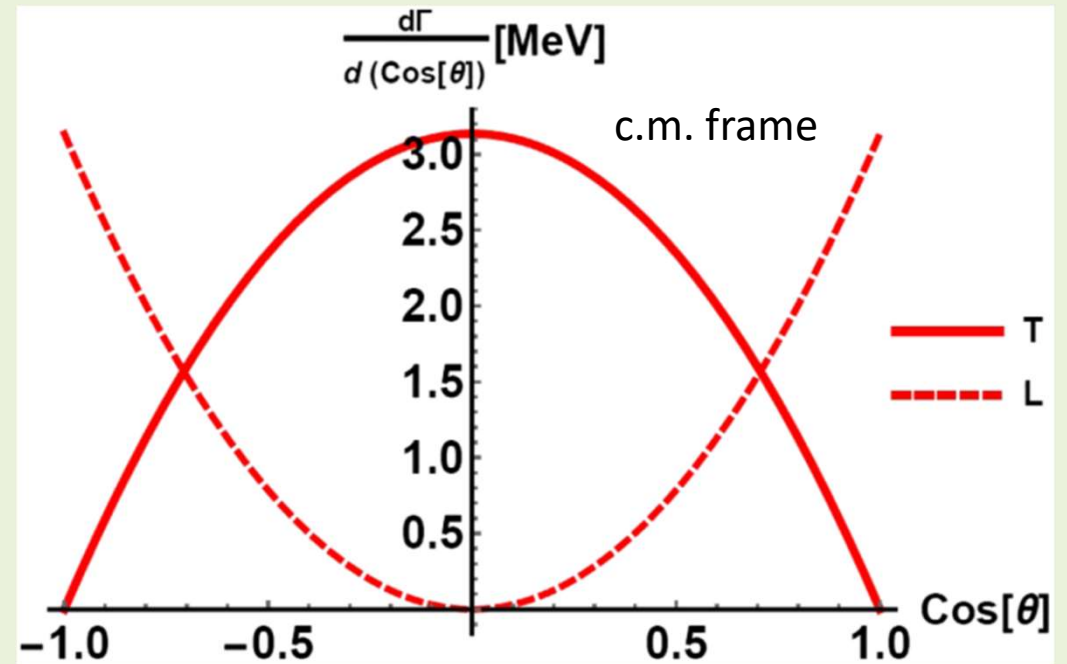
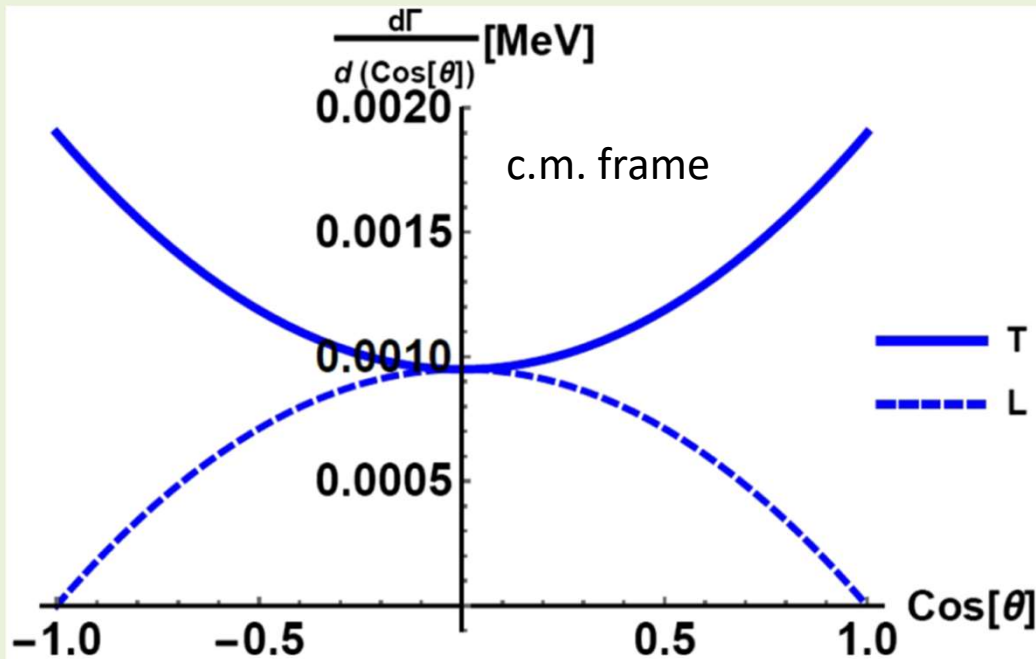


# Summary of $\varphi$ meson dilepton and $K^+K^-$ decays



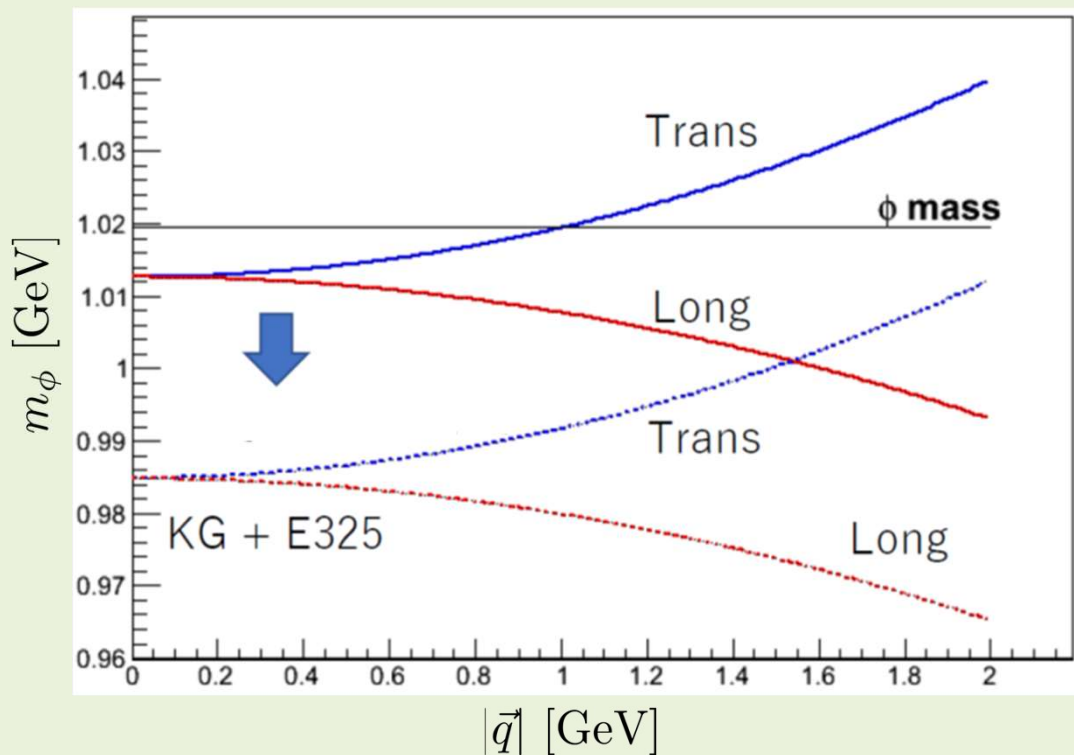
Dilepton decay angular distribution

$K^+K^-$  decay angular distribution



# Is it possible to disentangle the two polarization modes?

Model setup:



$$\frac{m_{\phi}^{L/T}(\vec{q})}{m_{\phi}^{\text{vac}}} = 1 + \left[ a + b^{L/T} \vec{q}^2 \right] \frac{\rho_N}{\rho_0}$$

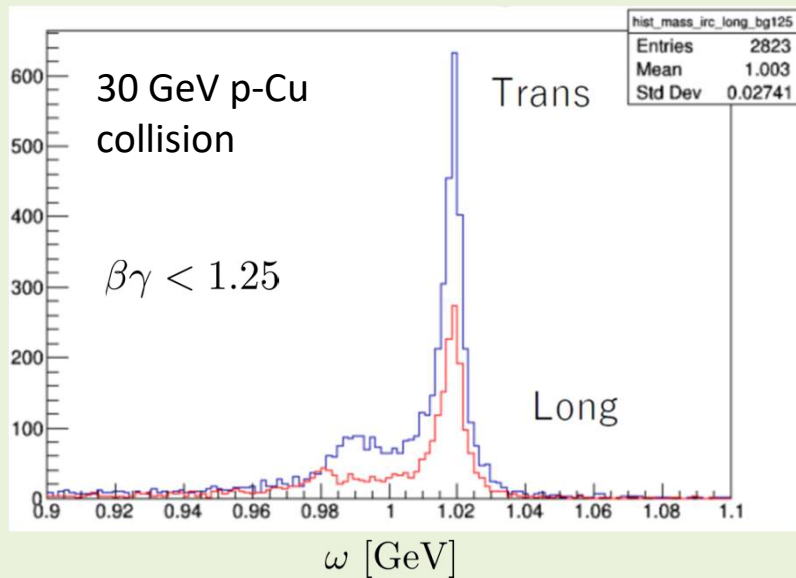
adjusted to the mass shift  
observed by the E325 experiment:  
 $a = -0.034$   
[R. Muto et al.,  
Phys. Rev. Lett. **98**, 042501 (2007).]

same as in:  
H.J. Kim and P. Gubler,  
Phys. Lett. B **805**, 135412 (2020).

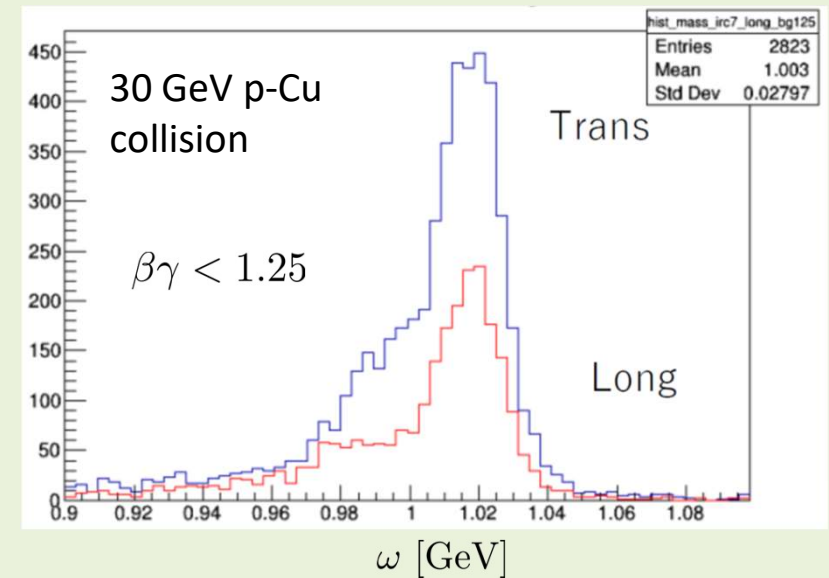
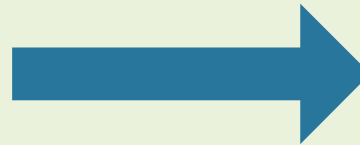
Analysis done by Kazuya Aoki (KEK)

# A simple Monte Carlo simulations under realistic experimental conditions

- ★ Momentum distribution from JAM (nuclear cascade code)
- ★ Breit-Wigner spectral function with density dependent mass
- ★ Radiative corrections (photon emission) of final state dileptons included
- ★ Target density: assumed to have a Woods-Saxon shape
- ★  $\phi$  production: assumed to be proportional to density

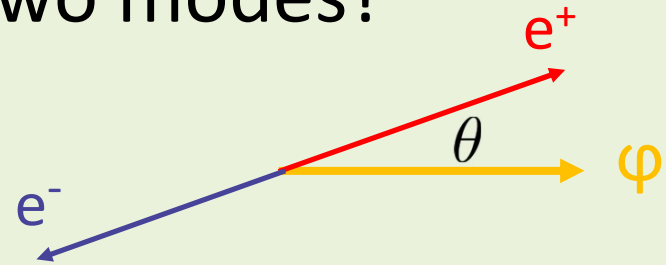


smeared by the  
expected mass  
resolution (7 MeV)

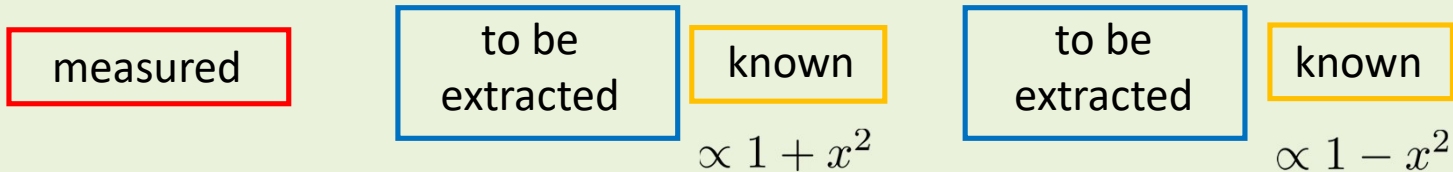


# How to disentangle the two modes?

$$x = \cos(\theta)$$



$$G(\omega, x) = g_T(\omega) f_T(x) + g_L(\omega) f_L(x)$$



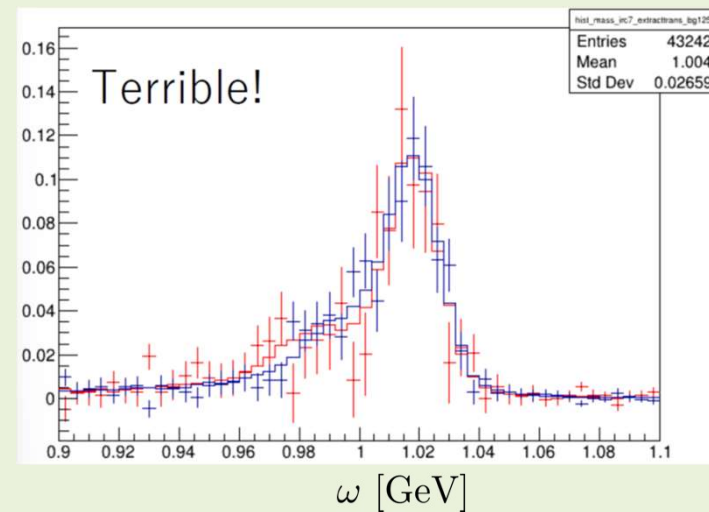
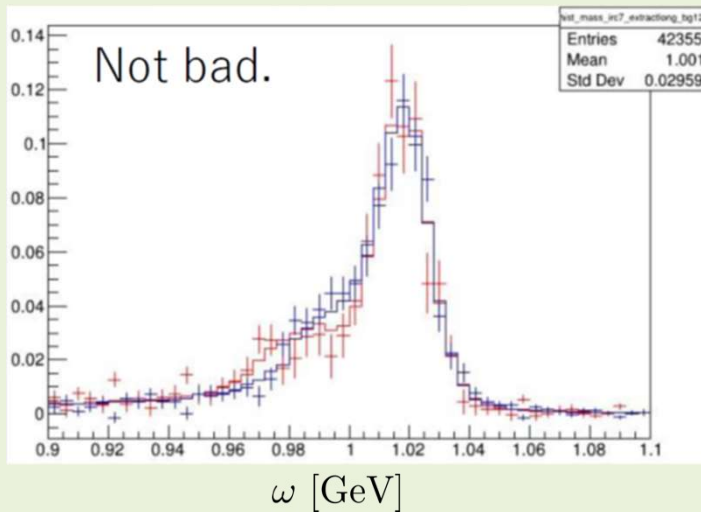
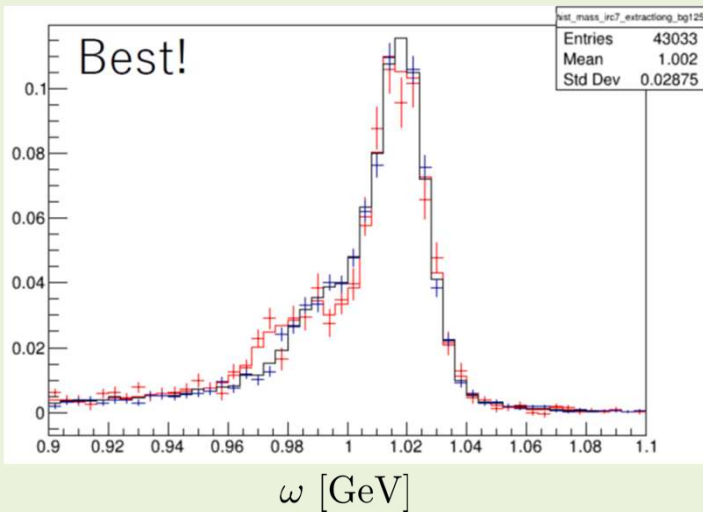
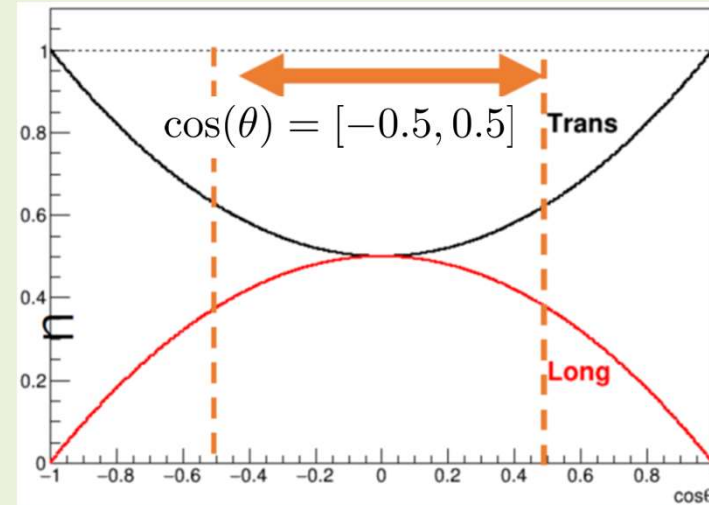
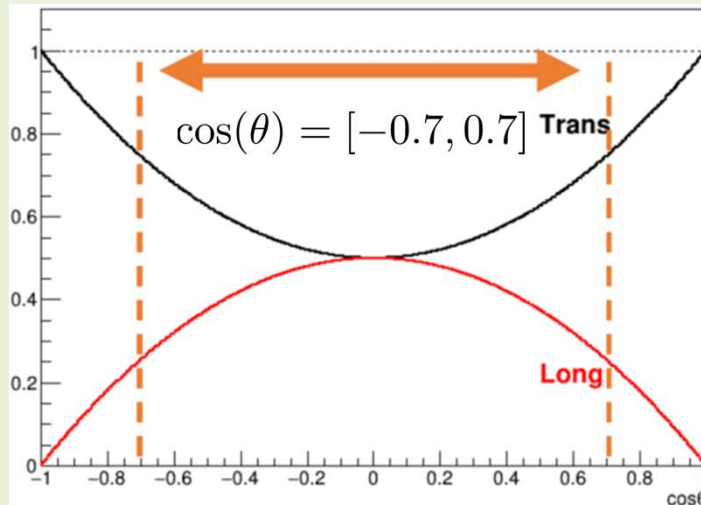
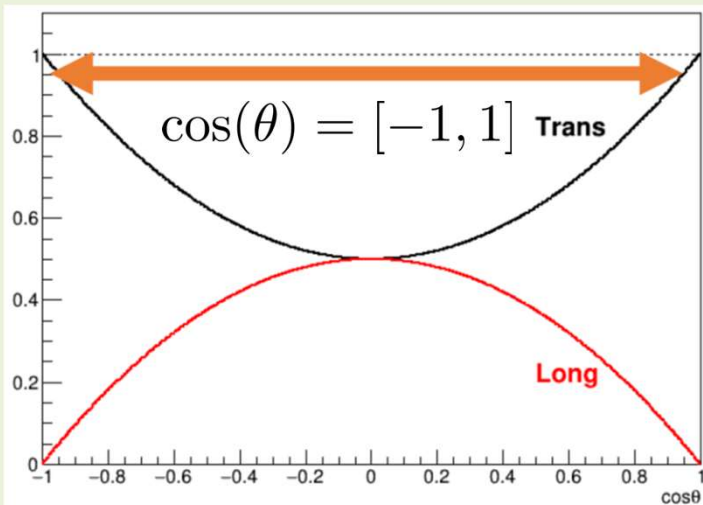
Find functions  $h_{T/L}(x)$  such that:

depend on acceptance of the experiment

$$\int_a^b dx h_T(x) f_L(x) = 0$$

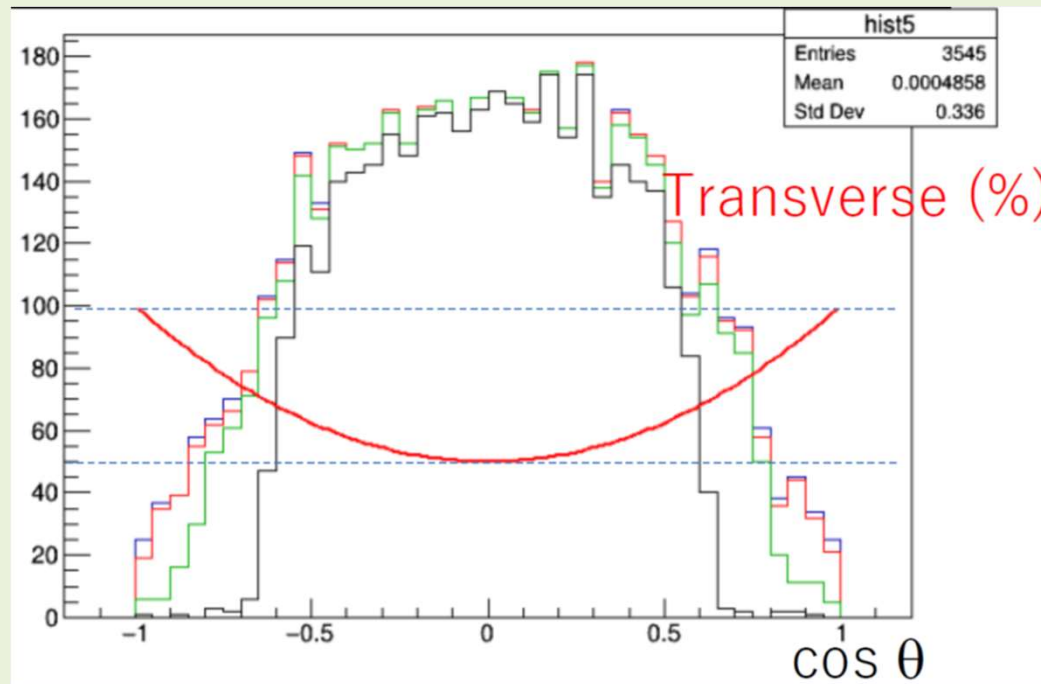
$$\int_a^b dx h_L(x) f_T(x) = 0$$

# Results depend on the angular acceptance!



# What will the angular acceptance look like at the J-PARC E16 experiment

Acceptance corr. +  $\beta\gamma < 1.25$  + low momentum cuts



Different low momentum cuts

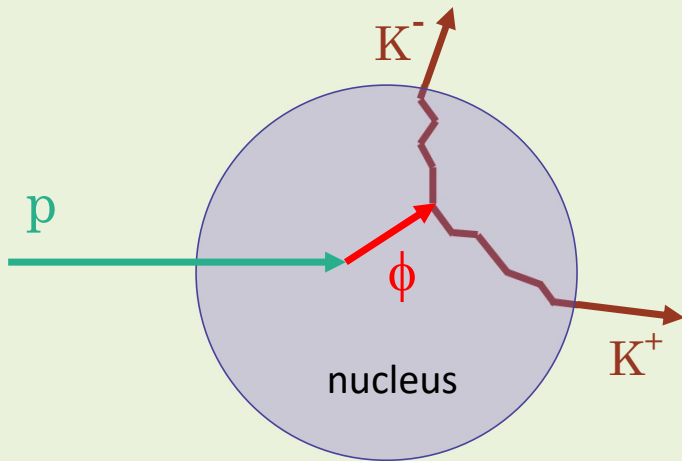


$\cos(\theta) = [-0.7, 0.7]$  seems realistic

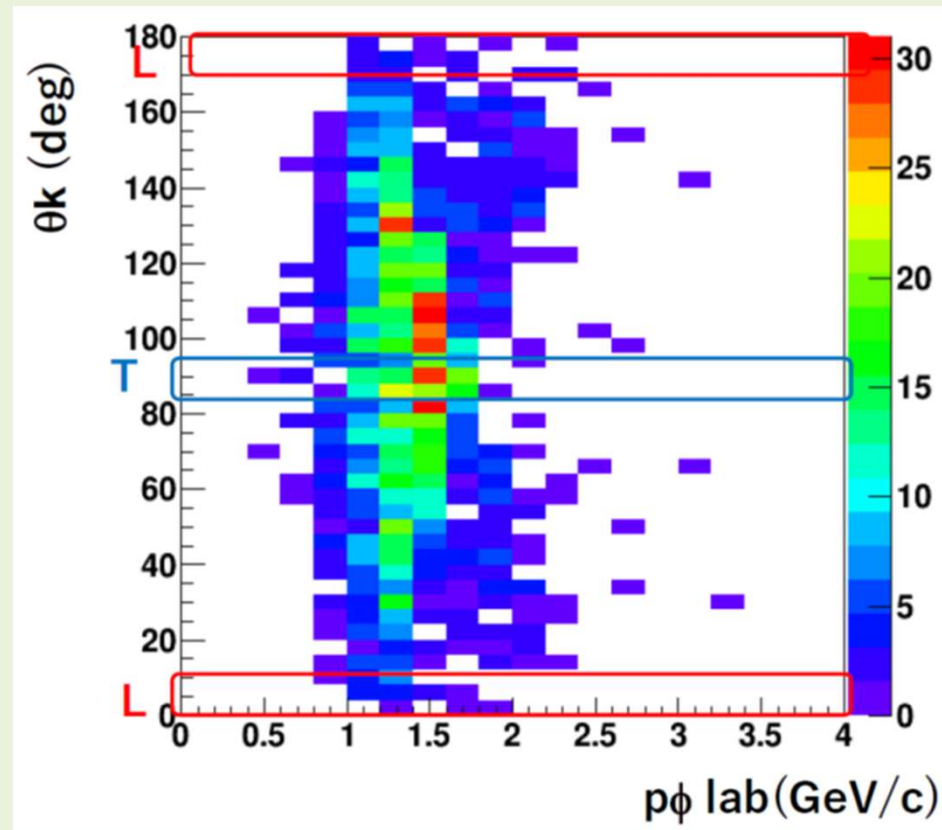


# The situation could be better for J-PARC E88 ( $K^+K^-$ decay)

experimental acceptance of  $K^+K^-$  decay for E88



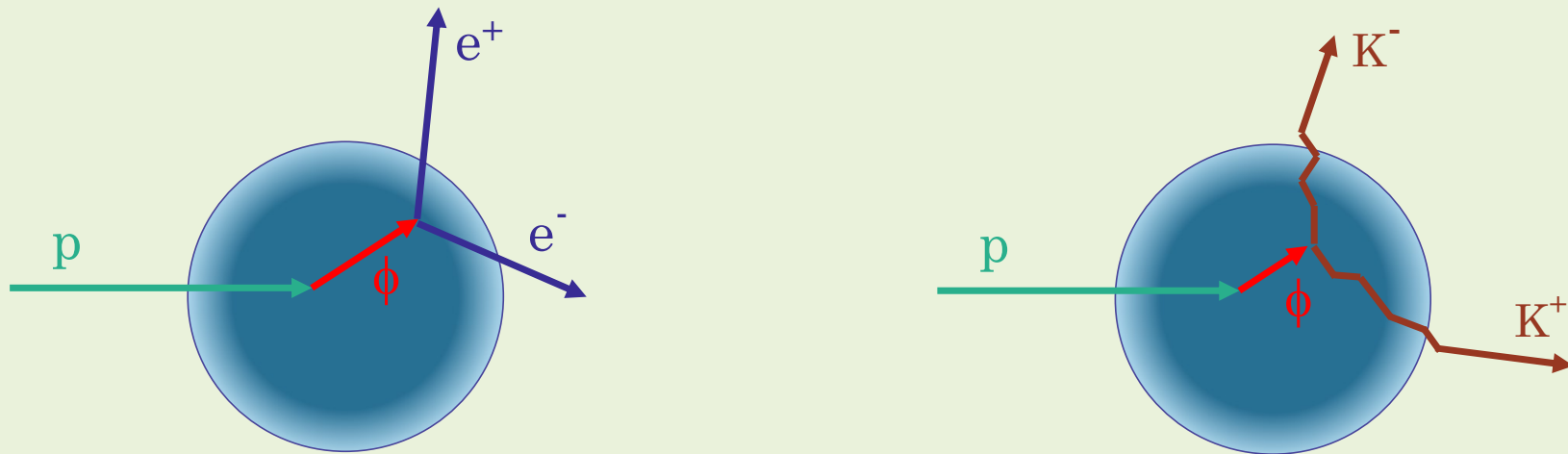
Remaining issue:  
How large are the final  
state interactions?



Analysis done by H. Sako (JAEA)

# Further tasks for theory

Have a good understanding of the production mechanisms of the  $\phi$  mesons in nuclei from pA reactions.



- ★ Where (and at what densities) is the  $\phi$  meson produced and where does it decay?
- ★ How do the final state interactions of the decay particles influence the decay spectrum (especially for  $K^+K^-$ )?



Realistic transport simulations using a transport approach  
(calculations using the PHSD code are ongoing)

# Summary and conclusions

- ★ Dispersion relations of hadrons can be non-trivially modified in nuclear matter.
- ★ For the  $\phi$  meson, the longitudinal and transverse modes are shifted in opposite directions with increasing momentum.



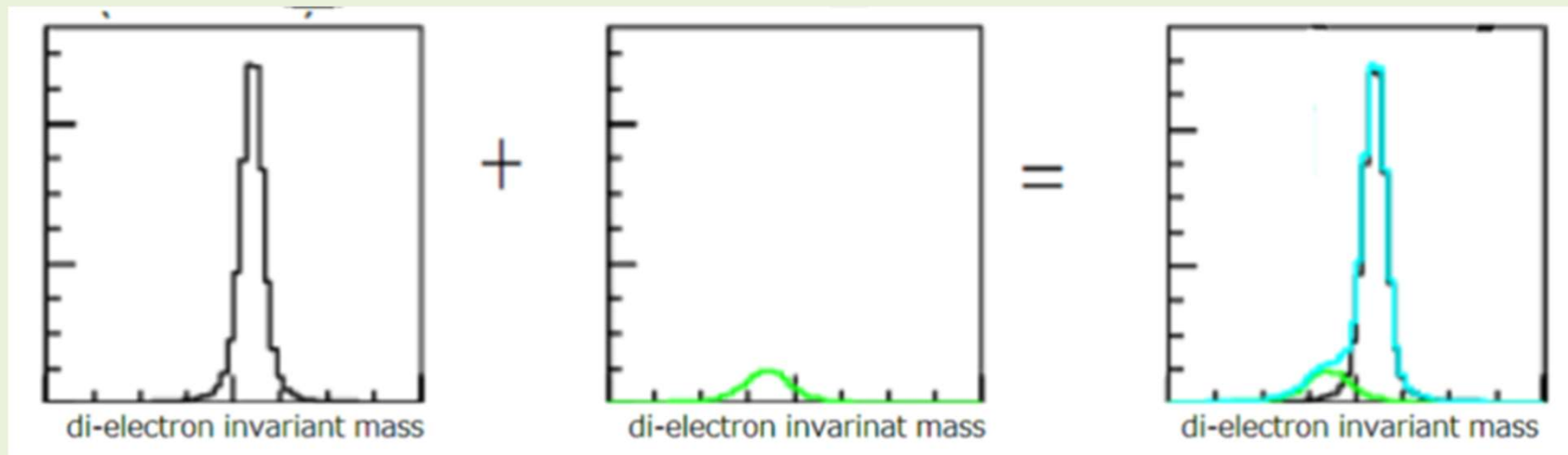
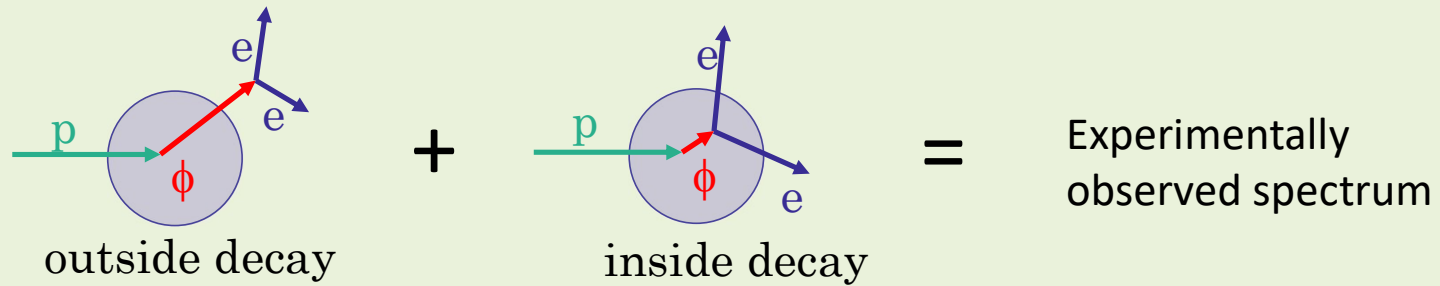
May be observed as a small **positive mass shift + width increase** at the E16 experiment at J-PARC



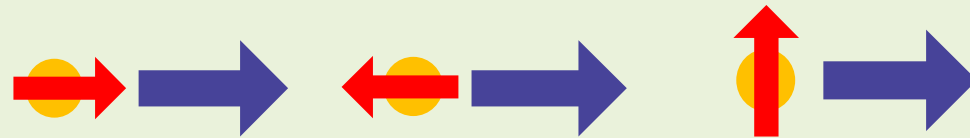
Making use of the angular dependences of the dilepton and  $K^+K^-$  decay channels, it is possible to **disentangle the longitudinal and transverse polarization modes**

Backup slides

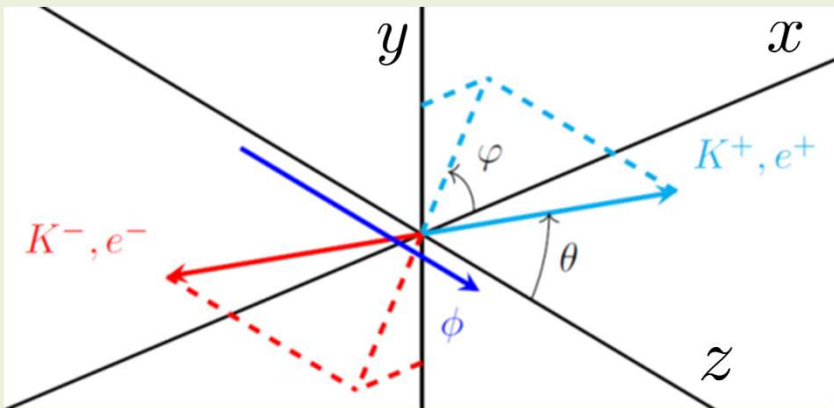
# Experimental di-lepton spectrum



# Full angular distribution of dilepton decay



Initial polarization:  $|V\rangle = a_{+1}|+1\rangle + a_{-1}|-1\rangle + a_0|0\rangle$



Transverse polarization

Longitudinal polarization

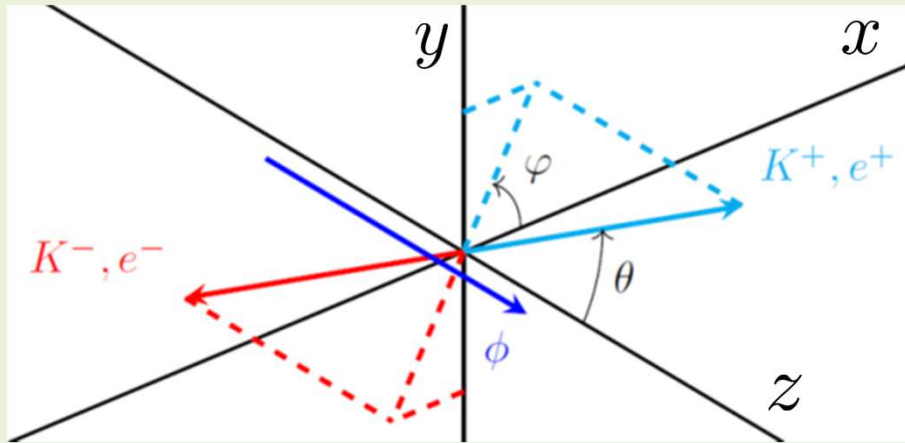
$\theta$ : polar angle

$\phi$ : azimuthal angle

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\Omega} = \frac{3}{16\pi} \left[ (|a_{+1}|^2 + |a_{-1}|^2)(1 + \cos^2 \theta) + 2|a_0|^2(1 - \cos^2 \theta) + 2\text{Re}(a_{+1}a_{-1}^*) \sin^2 \theta \cos 2\phi + \dots \right]$$

other  $\phi$ -dependent terms

## Full angular distribution of dilepton decay



$\theta$ : polar angle  
 $\phi$ : azimuthal angle

With

$$|a_{+1}|^2 + |a_{-1}|^2 + |a_0|^2 = 1, \quad |a_0|^2 = \rho_{00}$$

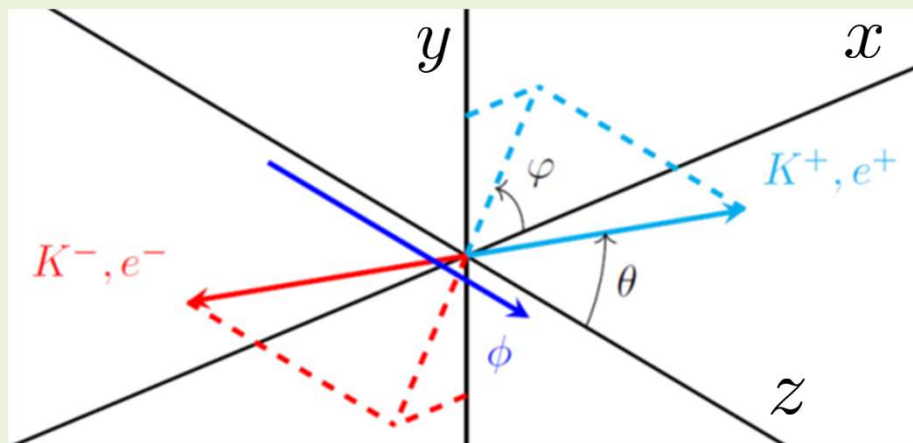
00-component of spin-density matrix

$$\rightarrow \frac{1}{\Gamma} \frac{d\Gamma}{d\Omega} = \frac{3}{16\pi} \left[ 1 + \cos^2 \theta + \rho_{00} (1 - 3 \cos^2 \theta) + \dots \right]$$

$$\rightarrow \rho_{00} = \frac{1}{3} \quad \text{Unpolarized case: vanishing } \theta\text{-dependence}$$

$\phi$ -dependent terms

## Full angular distribution of $K^+K^-$ decay



$\theta$ : polar angle  
 $\phi$ : azimuthal angle

Transverse modes

Longitudinal mode

$$\begin{aligned} \frac{1}{\Gamma} \frac{d\Gamma}{d\Omega} &= \frac{3}{16\pi} \left[ \underbrace{(|a_{+1}|^2 + |a_{-1}|^2)}_{\text{Transverse modes}} \sin^2 \theta + \overset{\text{Longitudinal mode}}{2|a_0|^2} \cos^2 \theta \right. \\ &\quad \left. - 2\text{Re}(a_{+1}a_{-1}^*) \sin^2 \theta \cos 2\phi + \dots \right] \\ &= \frac{3}{16\pi} \left[ 1 - \cos^2 \theta - \rho_{00}(1 - 3 \cos^2 \theta) + \dots \right] \end{aligned}$$

$\phi$ -dependent terms