Real-time non-perturbative dynamics in Schwinger model: jet production, chiral magnetic wave, and more

Shuzhe Shi（施舒哲）

Tsinghua University

References:

2301.11991: Florio, Frenklakh, Ikeda, Kharzeev, Korepin, SS, Yu

2305.00996: Ikeda, Kharzeev, Meyer, SS

2305.05685: Ikeda, Kharzeev, SS

(池田一毅)

outline

- motivation: real time dynamics in QFT
- model set up
- jet production
    - jet production
    - vector and axial charge transport
    - phase structure
- summary and outlook

"first principle" microscopic theory: quantum field theory $L \leftrightarrow H$

perturbative calculation: scattering process, thermodynamics, transport

non-perturbative calculation(lattice QFT): thermodynamics, transport

real time dynamics of non-perturbative theory?

time evolution of a quantum (field) state:

$$|\psi(t)\rangle = \mathcal{T} e^{-i\int \hat{H}\, dt} |\psi(0)\rangle$$

Ideally, ***quantum simulation*** for ***full QCD in 3+1 D***, but …

Hamiltonian

1+1D Schwinger model with time-dependent external source

$$H(t) = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_z - g\gamma^1 A - m)\psi \right) \mathrm{d}x \, .$$

E: electric field

A: electric potential

$\psi, \bar{\psi}$: fermion field

$$L(t) = \int \left(- \frac{F^{\mu\nu} F_{\mu\nu}}{4} + \bar{\psi}(i\gamma^\mu \partial_\mu - g\gamma^\mu A_\mu - m)\psi \right) \mathrm{d}x \, .$$

Hamiltonian

1+1D Schwinger model with time-dependent external source

$$H(t) = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1\partial_z - g\gamma^1 A - m)\psi \right) \mathrm{d}x\,.$$

E: electric field

A: electric potential

$\psi, \bar{\psi}$: fermion field

static energy

coupling b/w electric and fermion fields

kinetic energy

electric field energy

Hamiltonian

1+1D Schwinger model with time-dependent external source

$$H(t) = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1\partial_z - g\gamma^1 A - m)\psi - j^1_{\text{ext}}(t)A \right) \mathrm{d}x \,.$$

$$L(t) = \int \left(-\frac{F^{\mu\nu}F_{\mu\nu}}{4} + \bar{\psi}(i\gamma^\mu\partial_\mu - g\gamma^\mu A_\mu - m)\psi - j^\mu_{\text{ext}}(t)A_\mu \right) \mathrm{d}x \,.$$

E: electric field

A: electric potential

$\psi, \bar{\psi}$: fermion field

coupling w/ external source (jets)

$$j^1_{\text{ext}}(x,t) = g\,[\delta(x-t) + \delta(x+t)]\,\theta(t)$$

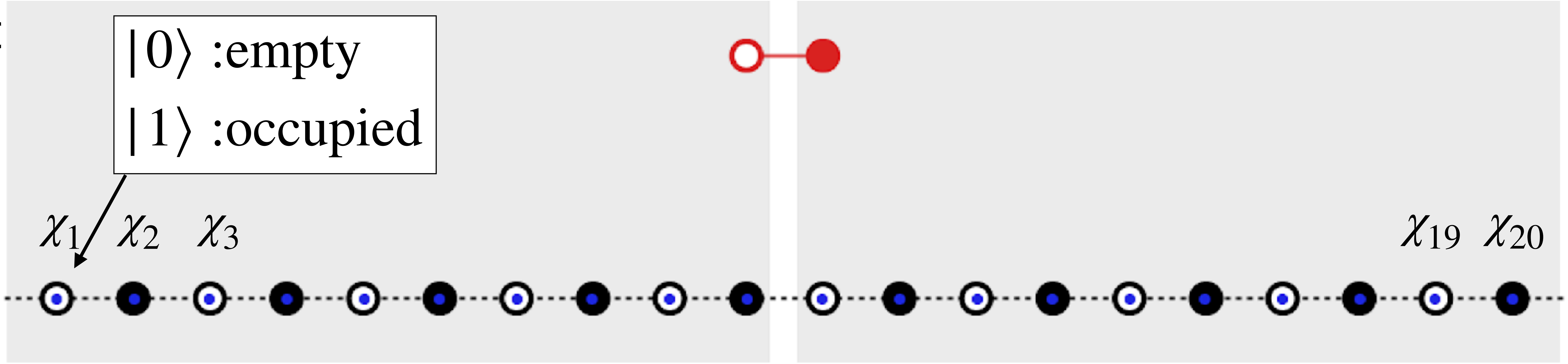
1+1D Schwinger model with time-dependent external source

$$H(t) = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_z - g\gamma^1 A - m)\psi - j^1_{\text{ext}}(t) A \right) \mathrm{d}x\,.$$

discretize and matrix(gate) representation:

Hamiltonian

1+1D Sc



discretize and matrix(gate) representation:

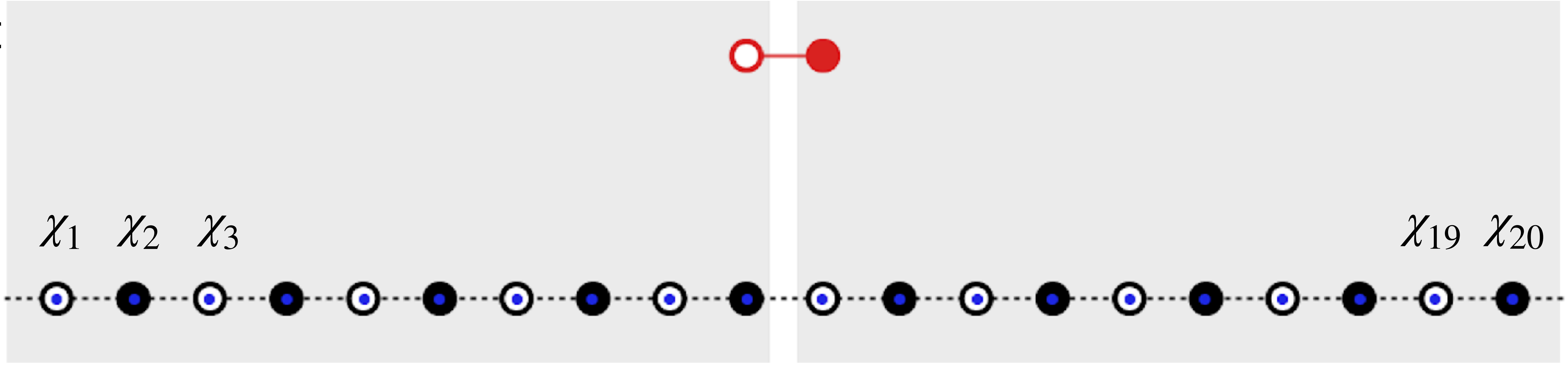
staggered fermion that satisfied anti-commutation: $\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b}\delta(x-y)$

$$\psi(x = a\ n) \quad \leftrightarrow \quad \frac{1}{\sqrt{a}}\begin{pmatrix}\chi_{2n} \\ \chi_{2n-1}\end{pmatrix}$$

Kogut-Susskind

Hamiltonian

1+1D Sc



Pauli matrices: $X,\ Y,\ Z$

discretize and matrix(gate) representation:

staggered fermion that satisfied anti-commutation: $\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b}\delta(x-y)$

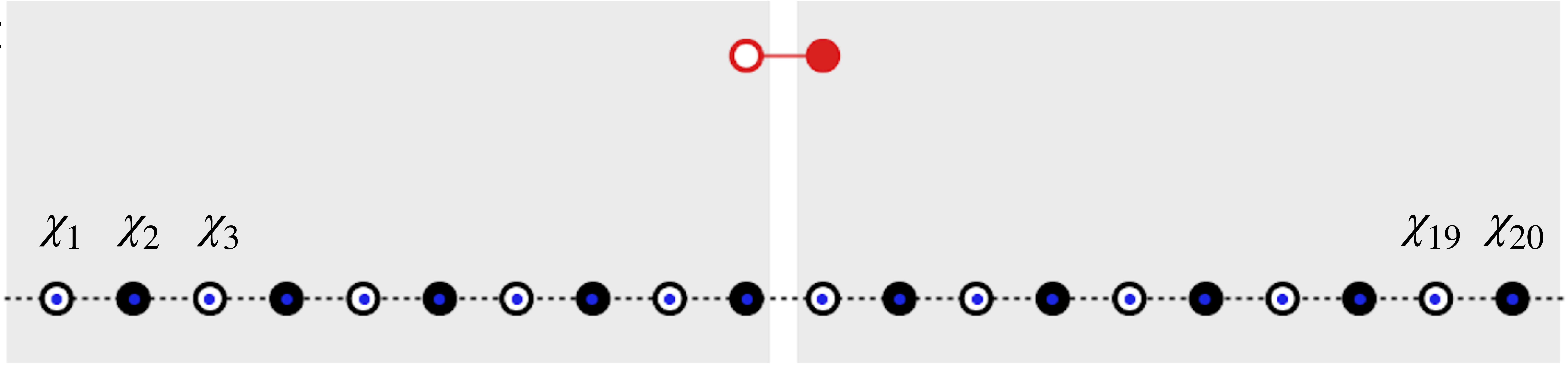
$$\psi(x = a\ n) \quad \leftrightarrow \quad \frac{1}{\sqrt{a}}\begin{pmatrix}\chi_{2n} \\ \chi_{2n-1}\end{pmatrix}$$

Kogut-Susskind

$$X_n \equiv \underset{1^{\mathrm{st}}}{I} \otimes \cdots \otimes \underset{(n-1)^{\mathrm{th}}}{I} \otimes \underset{n^{\mathrm{th}}}{X} \otimes \underset{(n+1)^{\mathrm{th}}}{I} \otimes \cdots \otimes I$$

Hamiltonian

1+1D Sc



Pauli matrices: X, Y, Z

discretize and matrix(gate) representation:

staggered fermion that satisfied anti-commutation: $\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b}\delta(x-y)$

$$\psi(x = a\ n) \quad \leftrightarrow \quad \frac{1}{\sqrt{a}}\begin{pmatrix} \chi_{2n} \\ \chi_{2n-1} \end{pmatrix}$$

Kogut-Susskind

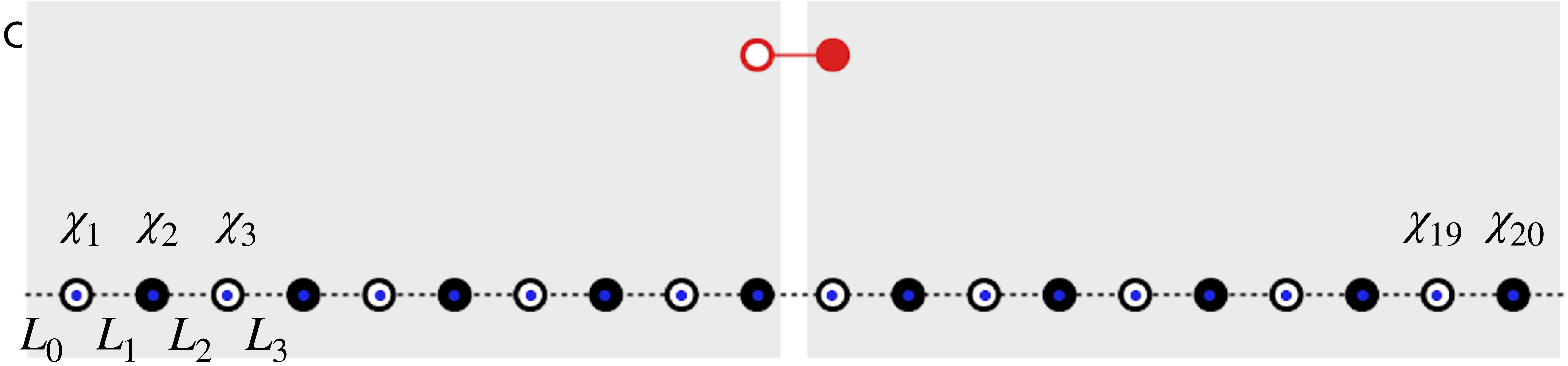
$$\chi_n = \frac{X_n - iY_n}{2}\prod_{m=1}^{n-1}(-iZ_m)$$

Jordan-Wigner

$$\{\chi_n^\dagger, \chi_m\} = \delta_{nm}\,, \quad \{\chi_n^\dagger, \chi_m^\dagger\} = \{\chi_n, \chi_m\} = 0\,.$$

Hamiltonian

1+1D Sc



Pauli matrices: X, Y, Z

discretize and matrix(gate) representation:

staggered fermion that satisfied anti-commutation: $\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b}\delta(x-y)$

gauge field fixed by Gauss' law: $\partial_1 E - g\,\bar{\psi}\gamma^0\psi = j^0_{\text{ext}}$

$E(x = an) \quad \leftrightarrow \quad L_n$

$$L_n - L_{n-1} - \frac{Z_n + (-1)^n}{2} = \frac{1}{g}\int_{(n-1/2)a}^{(n+1/2)a} \mathrm{d}x\, j^0_{\text{ext}}(x,t)\ ,$$

Hamiltonian

1+1D Schwinger model with time-dependent external source

$$H(t) = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_z - g\gamma^1 A - m)\psi - j^1_{\text{ext}}(t) A \right) \mathrm{d}x \,.$$

discretize and matrix(gate) representation:

Pauli matrices: X, Y, Z

staggered fermion that satisfied anti-commutation: $\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b}\delta(x-y)$

gauge field fixed by Gauss' law: $\partial_1 E - g\,\bar{\psi}\gamma^0\psi = j^0_{\text{ext}}$

$$H(t) = \frac{1}{4a}\sum_{n=1}^{N-1}(X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n + \frac{a\,g^2}{2}\sum_{n=1}^{N-1} L_n^2(t) \;.$$

why quantum computer?

dimension of state vector $= 2^N$ N : number of lattice sides

dimension of Hamiltonian $=$ ~~$2^N \times 2^N$~~ sparse $\sim 2N \times 2^N$

N	dimension	memory of Hamiltonian	# of qubit (N)
8	256	~ 131 kB	8
12	4,096	~ 3.1 MB	12
16	65,536	~ 67 MB	16
20	1,048,576	~ 1.3 GB	20
24	16,777,216	~ 26 GB	24
28	268,435,456	~ 481 GB	28

unrealistic in a "classical" computer,
but plausible in the state-of-art quantum computer?

why quantum computer?

dimension of state vector $= 2^N$

N : number of lattice sides

dimension of Hamiltonian $=$ ~~$2^N \times 2^N$~~ sparse $\sim 2N \times 2^N$

N	dimension	memory of Hamiltonian	# of qubit (N)
8	256	~ 131 kB	8
12	4,096	~ 3.1 MB	12
16	65,536	~ 67 MB	16
20	1,048,576	~ 1.3 GB	20
24	16,777,216	~ 26	
28	~~268,435,456~~	~~~ 481 GB~~	28

classical hardware in this work

performance not satisfying…

Hamiltonian

(repeating 4)

1+1D Schwinger model with time-dependent external source

$$H(t) = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_z - g\gamma^1 A - m)\psi - j^1_{\text{ext}}(t) A \right) \mathrm{d}x \,.$$

discretize and matrix(gate) representation:

Pauli matrices: X, Y, Z

staggered fermion that satisfied anti-commutation: $\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b}\delta(x-y)$

gauge field fixed by Gauss' law: $\partial_1 E - g\,\bar{\psi}\gamma^0\psi = j^0_{\text{ext}}$

$$H(t) = \frac{1}{4a}\sum_{n=1}^{N-1}(X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n + \frac{a\,g^2}{2}\sum_{n=1}^{N-1} L_n^2(t) \,.$$

time-dependent Schroedinger equation:

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -\, i\, H\,|\psi(t)\rangle$$

$$q_{n,t} \equiv \langle \psi^\dagger(a\,n)\psi(a\,n)\rangle_t = \frac{\langle Z_n\rangle_t + (-1)^n}{2a},$$

$$Q_t \equiv \int \langle \psi^\dagger(x)\psi(x)\rangle_t \,\mathrm{d}x = a\sum_{n=1}^{N} q_{n,t},$$

$$\nu_{n,t} \equiv \langle \bar\psi(a\,n)\psi(a\,n)\rangle_t = \frac{(-1)^n\langle Z_n\rangle_t}{2a},$$

$$\nu_t \equiv \int \langle \bar\psi(x)\psi(x)\rangle_t \,\mathrm{d}x = a\sum_{n=1}^{N} \nu_{n,t},$$

$$\Pi_{n,t} \equiv \langle E(a\,n)\rangle_t = g\,\langle L_n\rangle_t,$$

$$E_{\mathrm{ele},t} \equiv \frac{1}{2}\int \langle E^2(x)\rangle_t \,\mathrm{d}x = \frac{a\,g^2}{2}\sum_{n=1}^{N-1}\langle L_n^2\rangle_t\,.$$

I. jet production

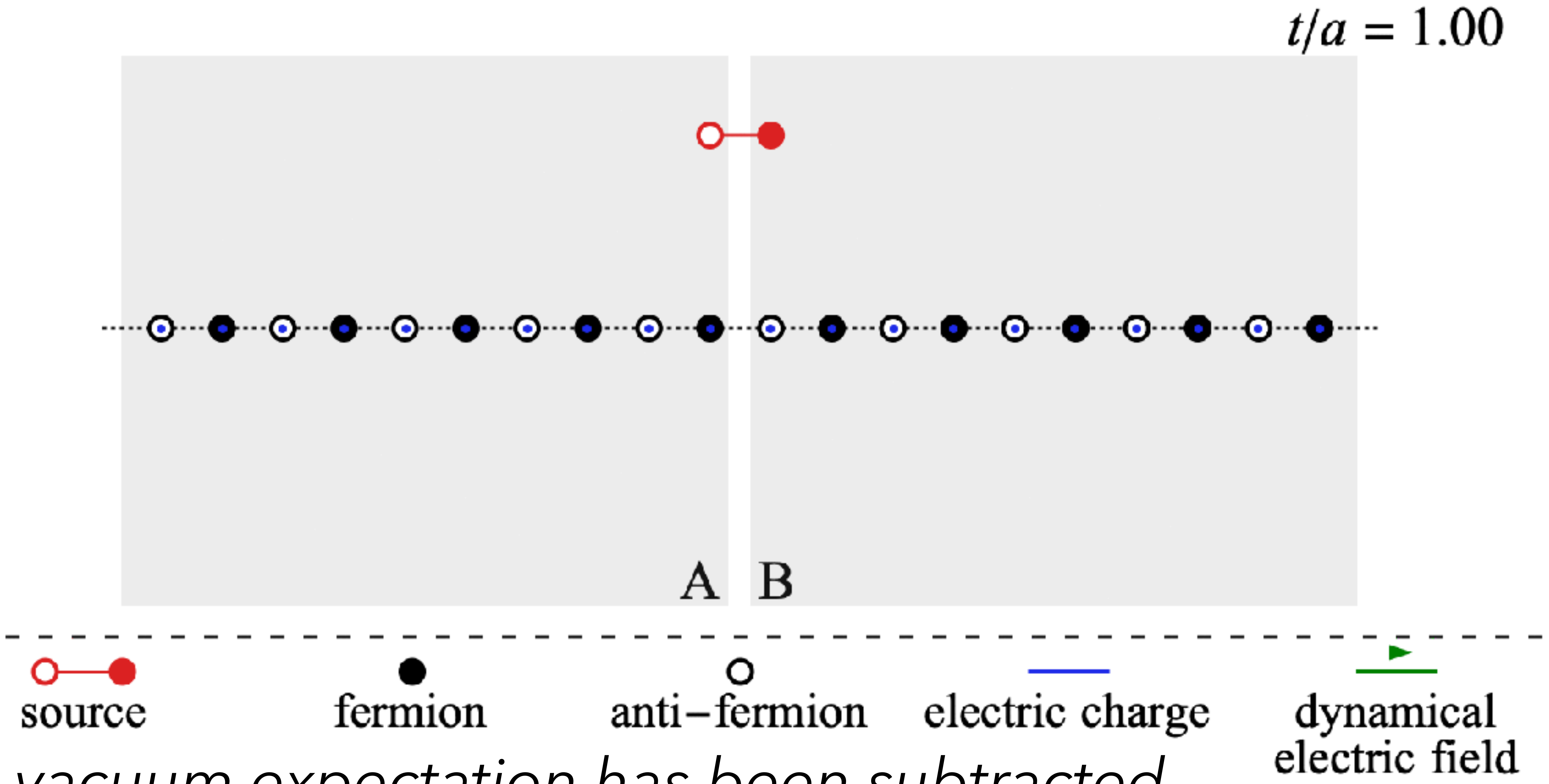
A. Florio, D. Frenklakh, K. Ikeda, D. Kharzeev, V. Korepin, *SS*, K. Yu

PhysRevLett.131.021902 (arXiv: 2305.05685)

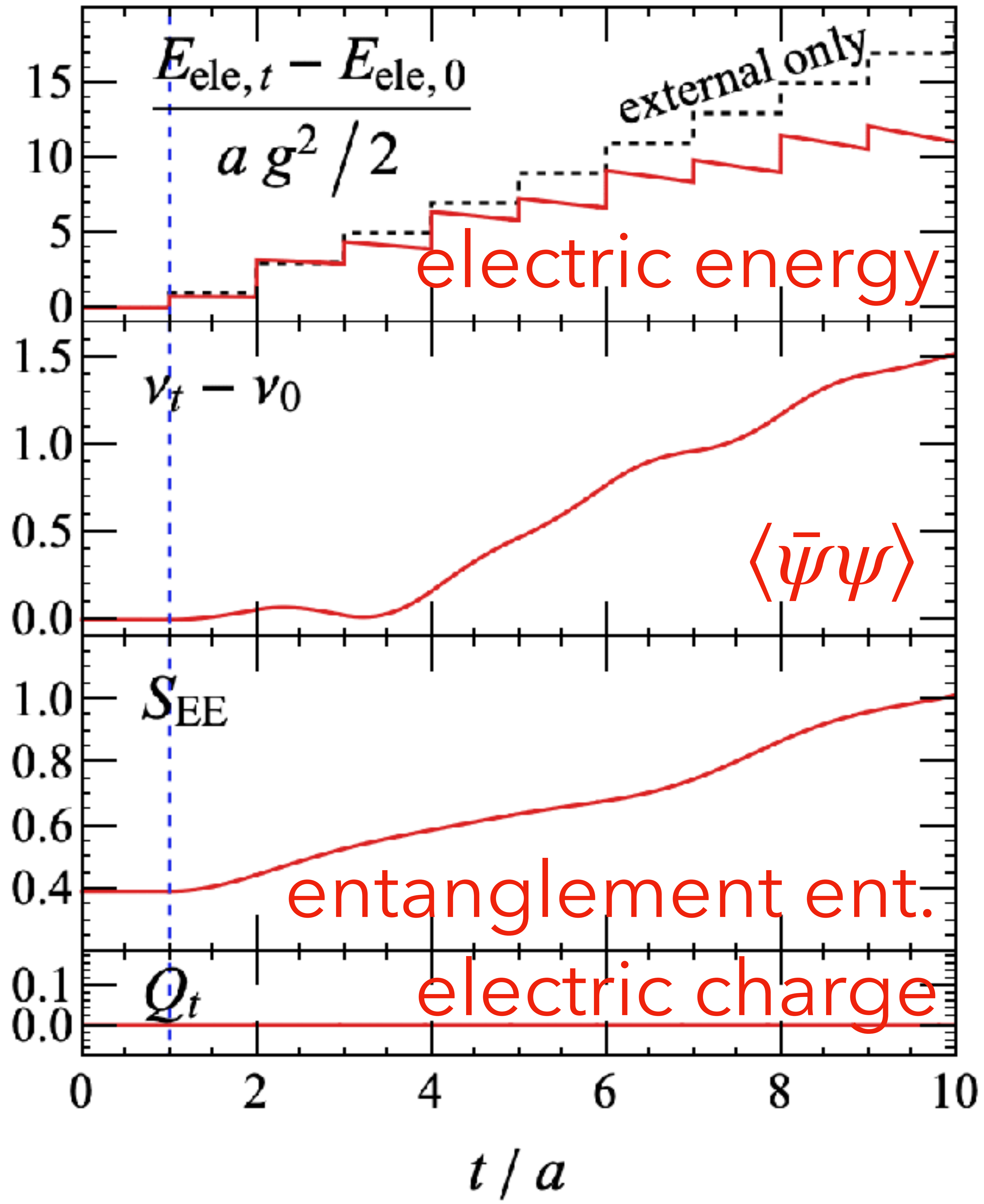
real-time evolution

initial state: vacuum $H(t=0)\,|\,\psi(t=0)\rangle = E_0\,|\,\psi(t=0)\rangle$

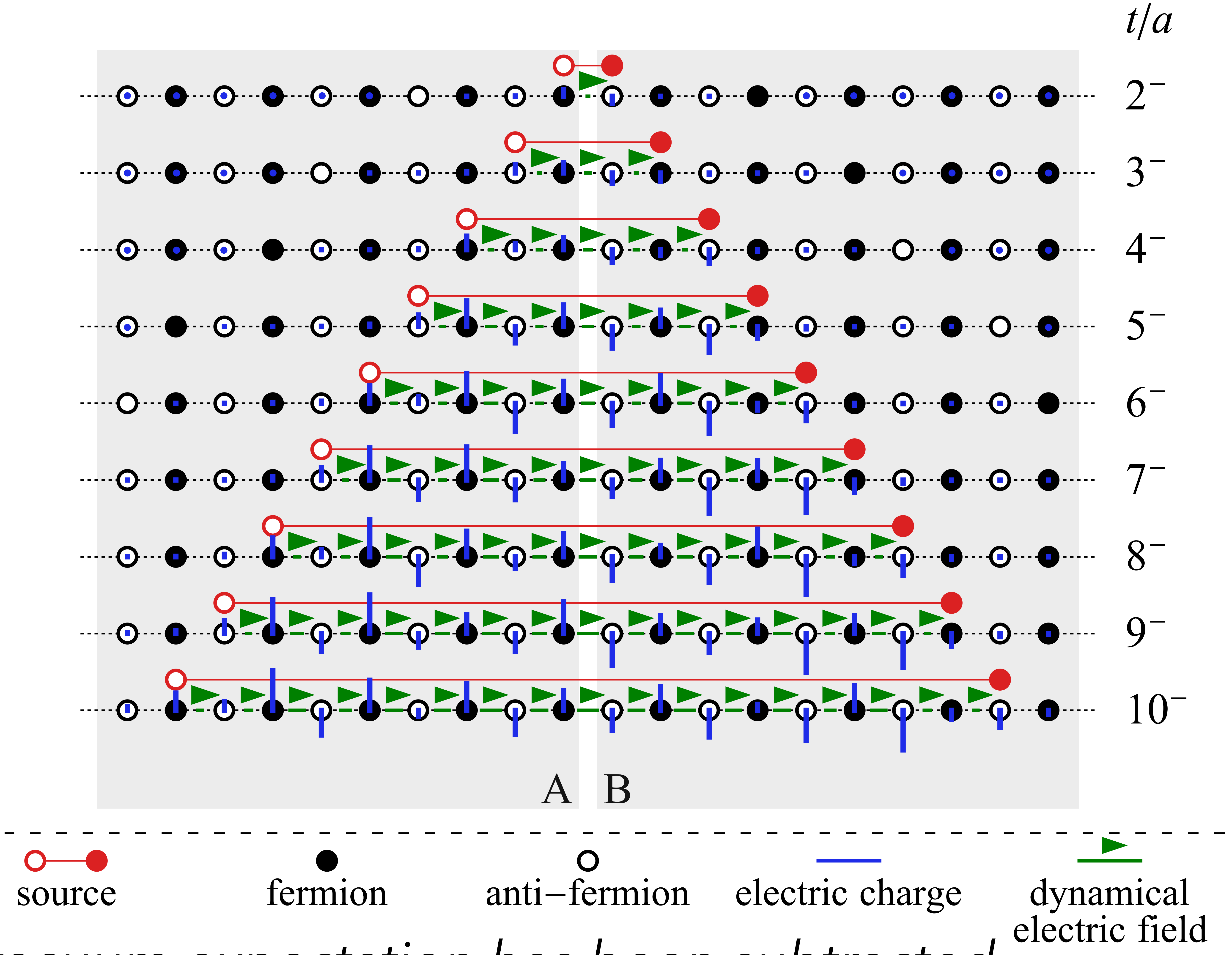
$$\frac{\partial}{\partial t}\,|\,\psi(t)\rangle = -\,i\,H(t)\,|\,\psi(t)\rangle$$



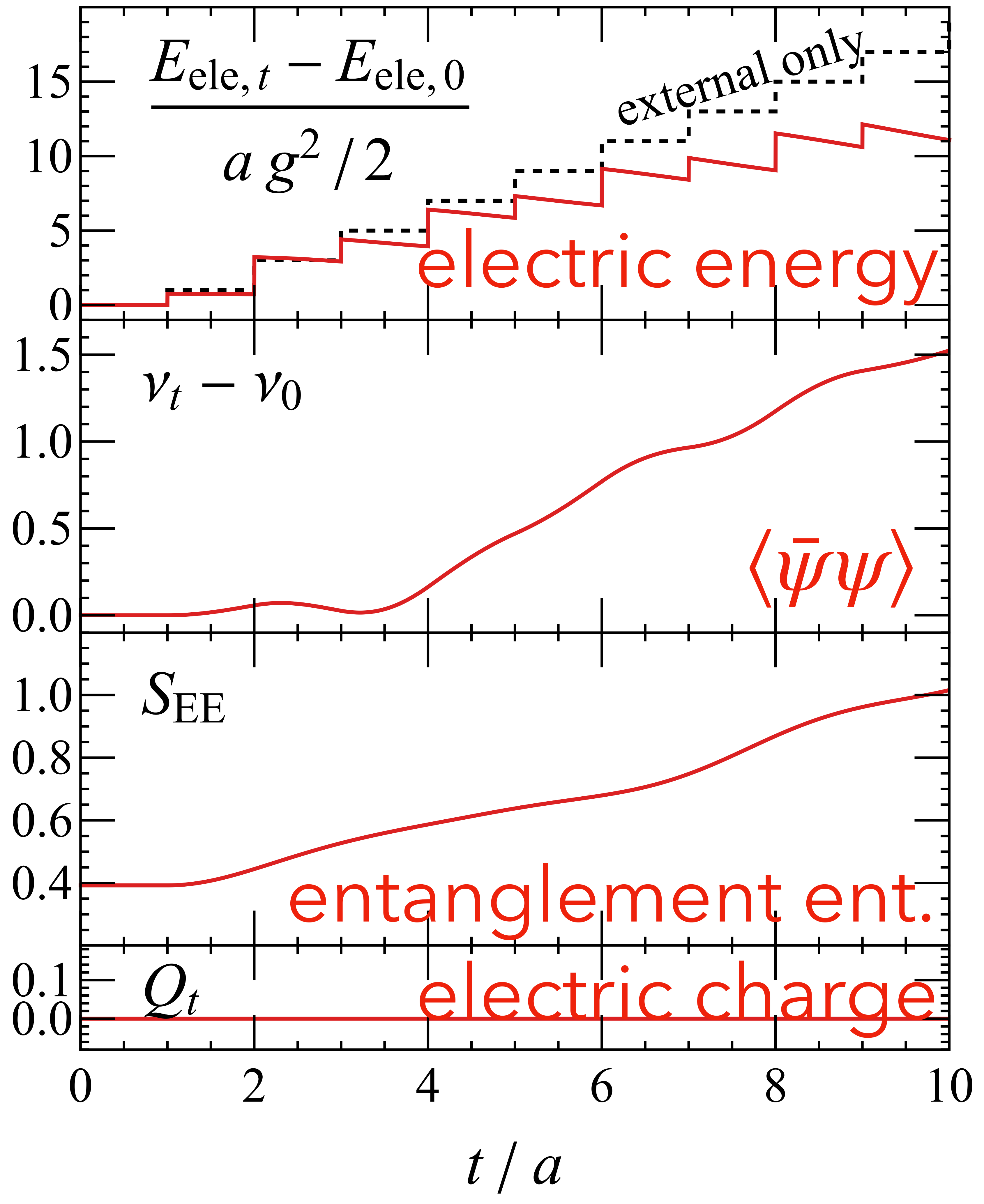
vacuum expectation has been subtracted



vacuum modification



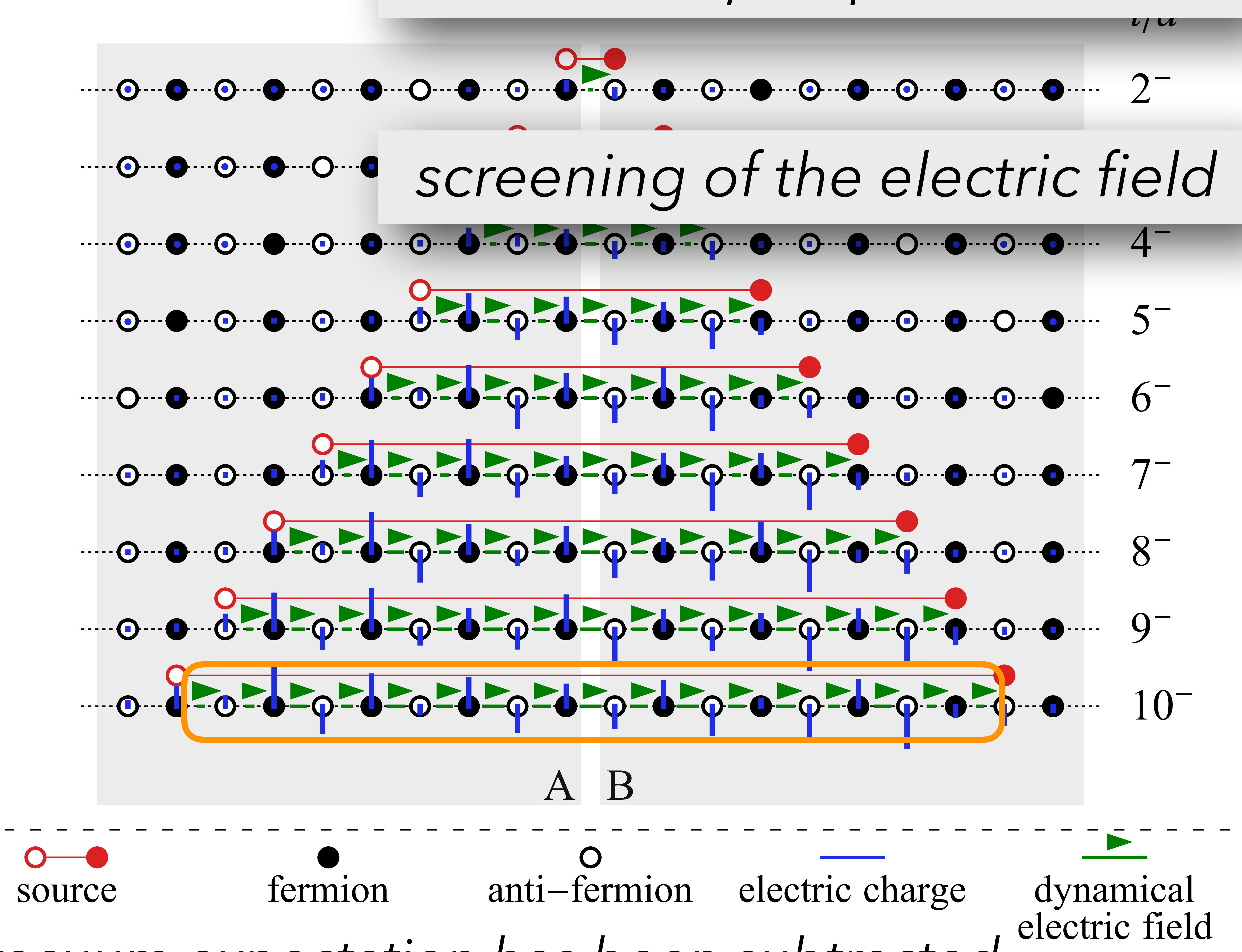
vacuum expectation has been subtracted



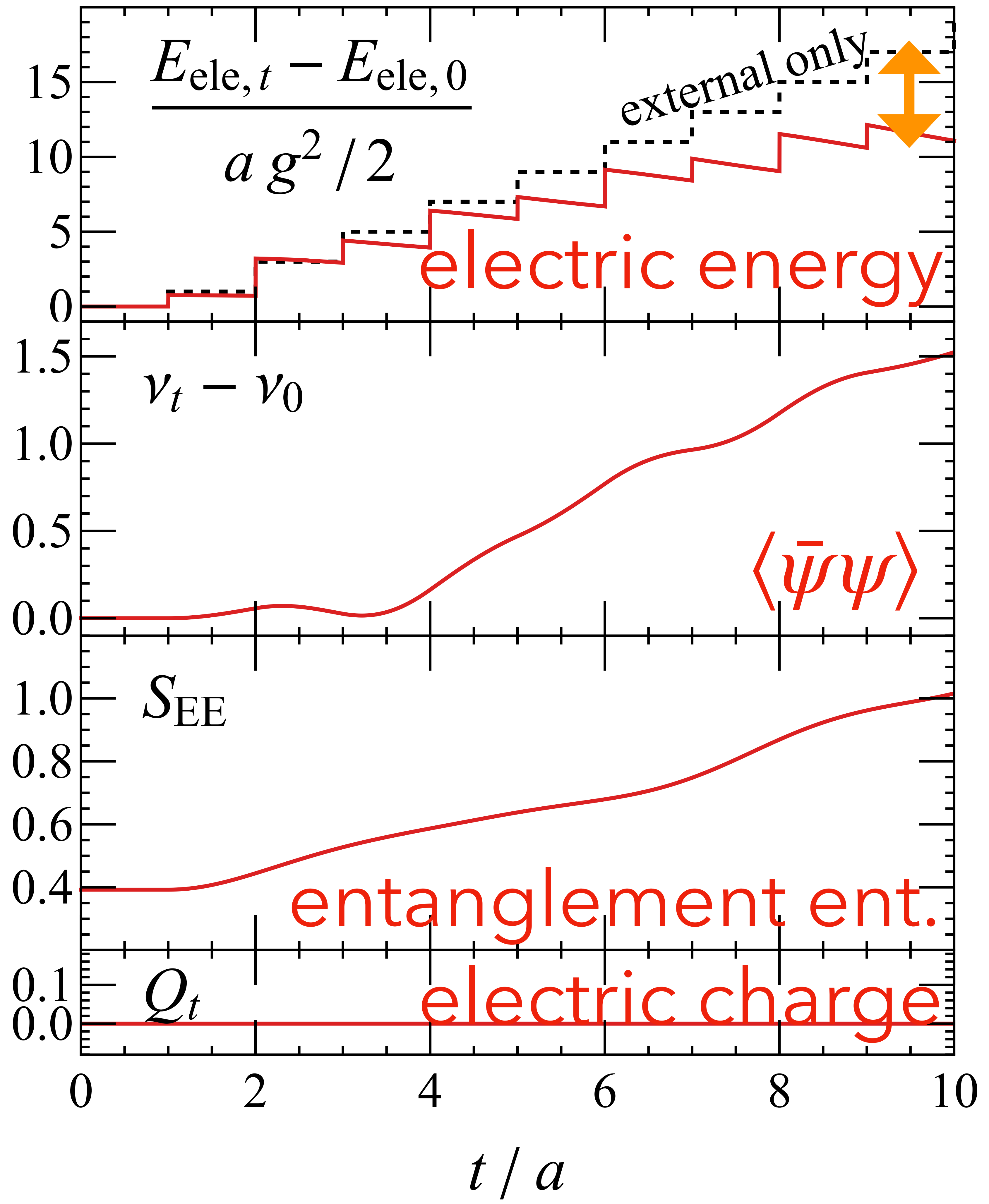
vacuum modification

effects of pair production:

screening of the electric field



vacuum expectation has been subtracted

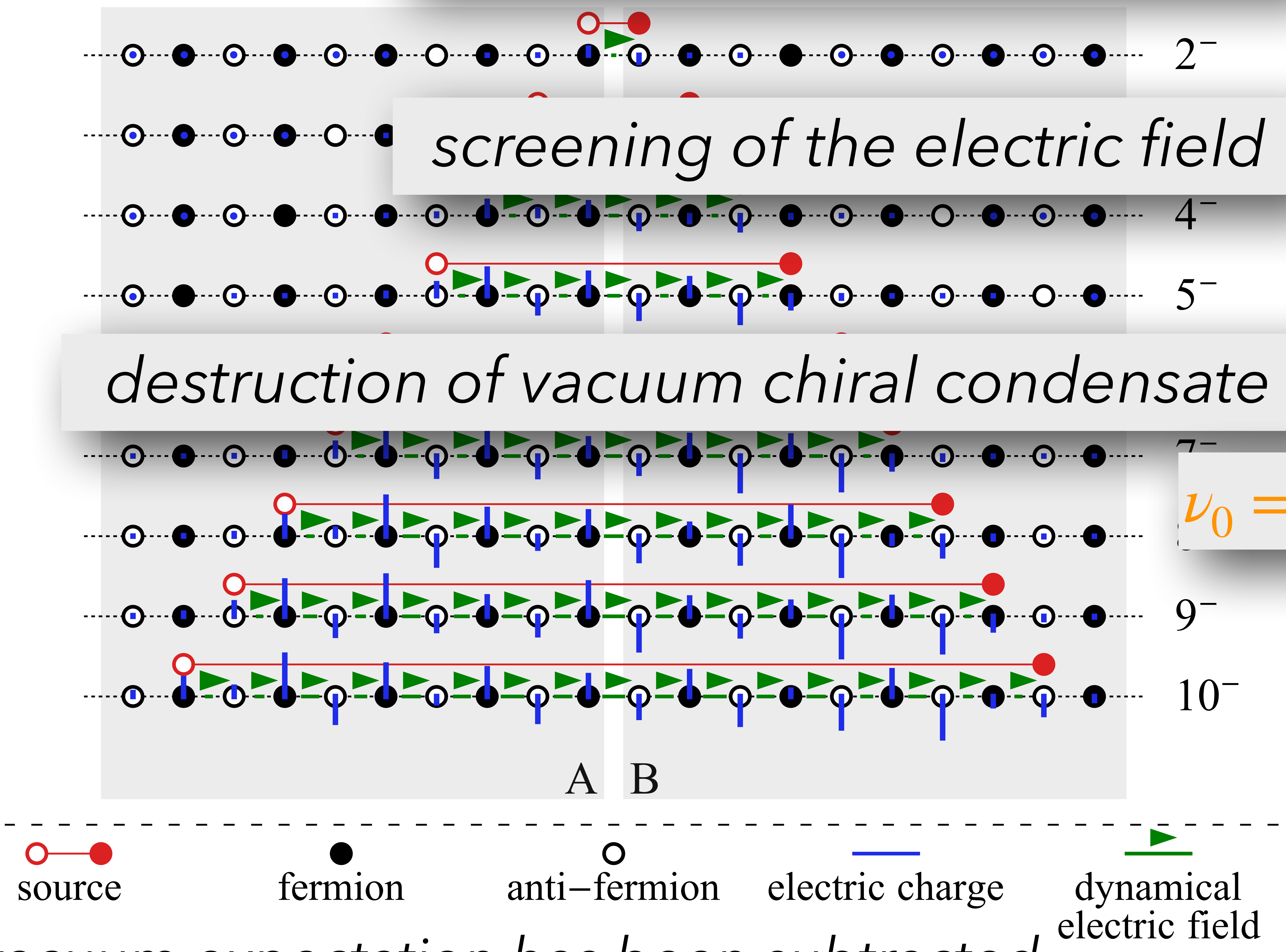


vacuum modification

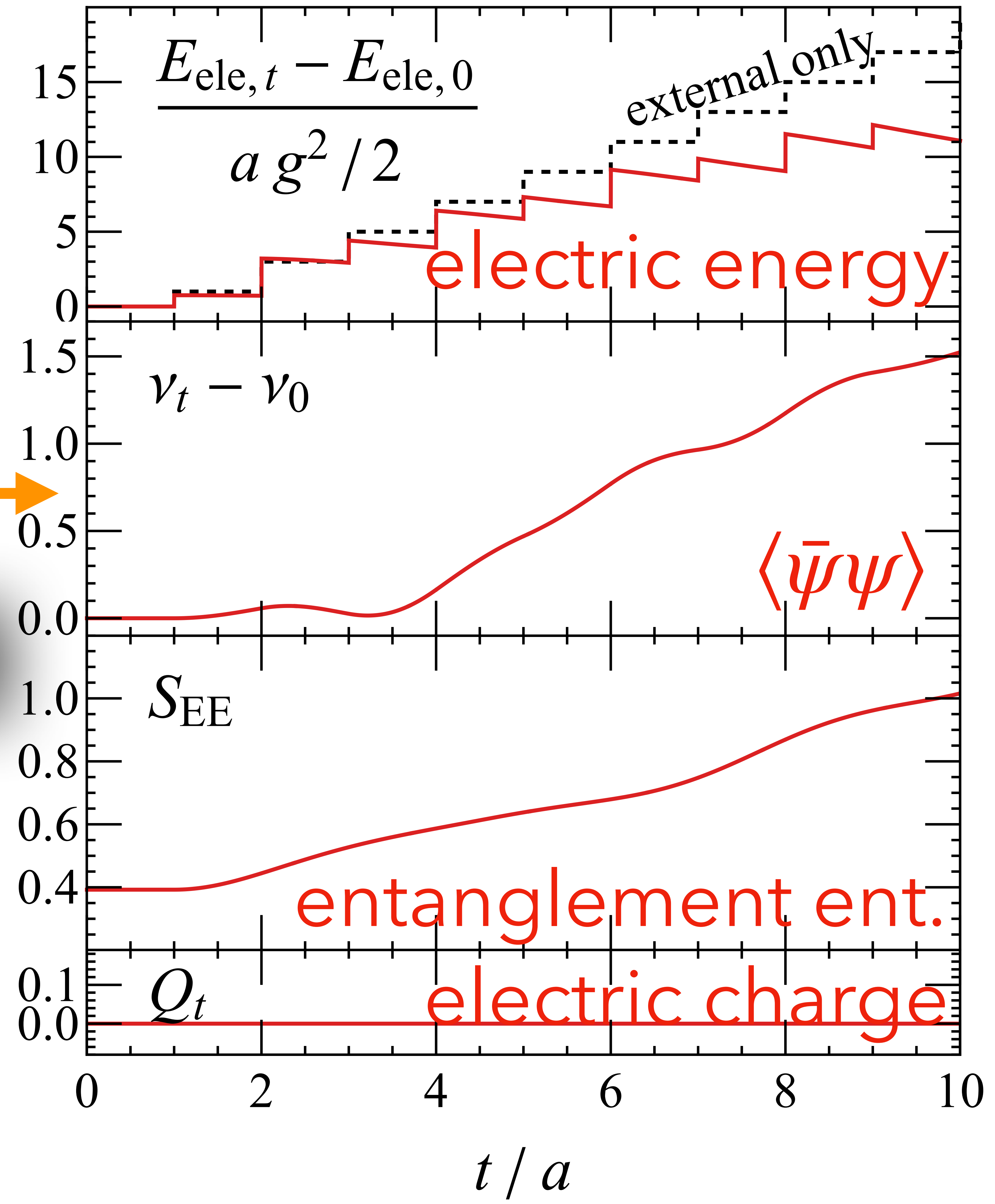
effects of pair production:

screening of the electric field

destruction of vacuum chiral condensate



$\nu_0 = -5.16$



vacuum expectation has been subtracted

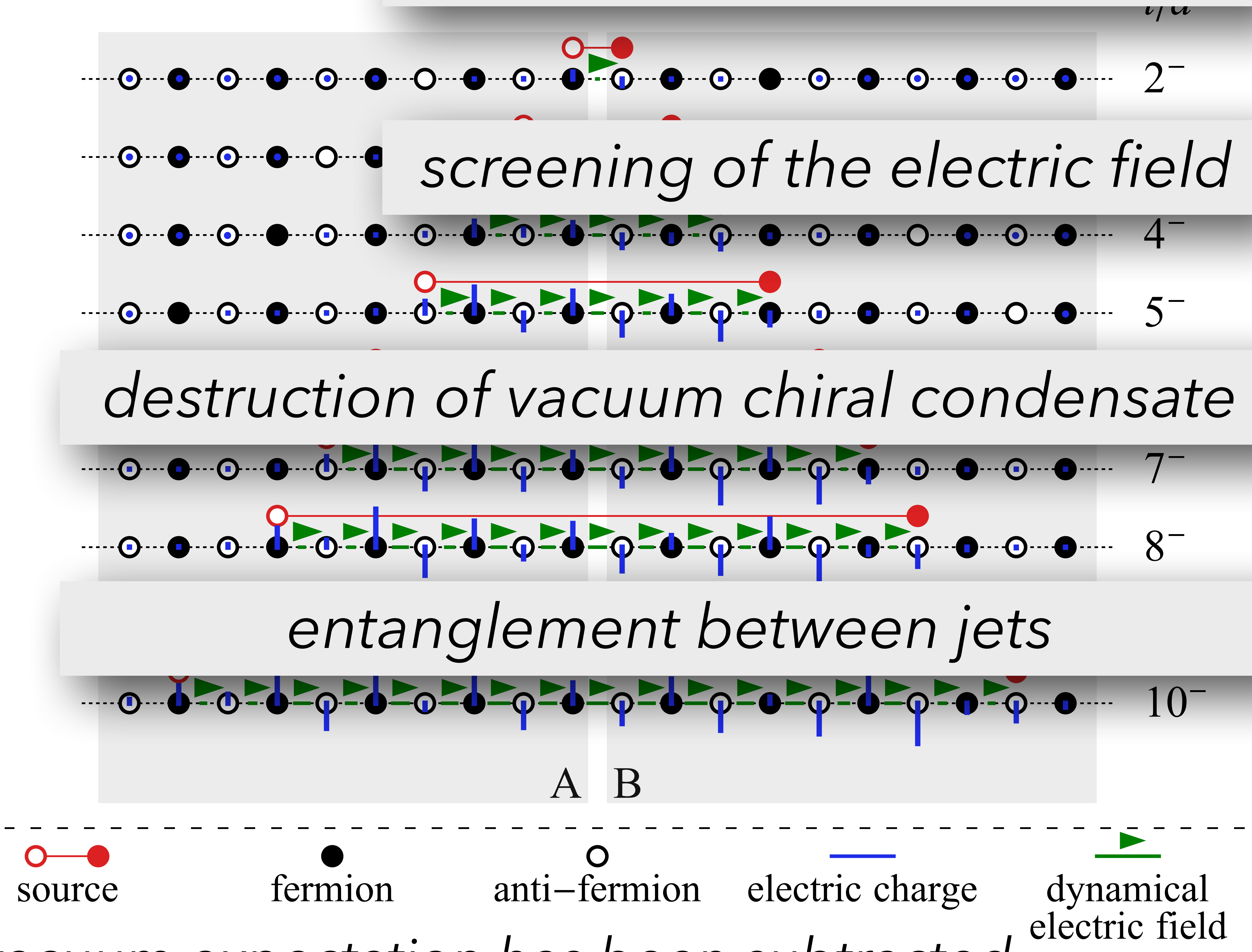
vacuum modification

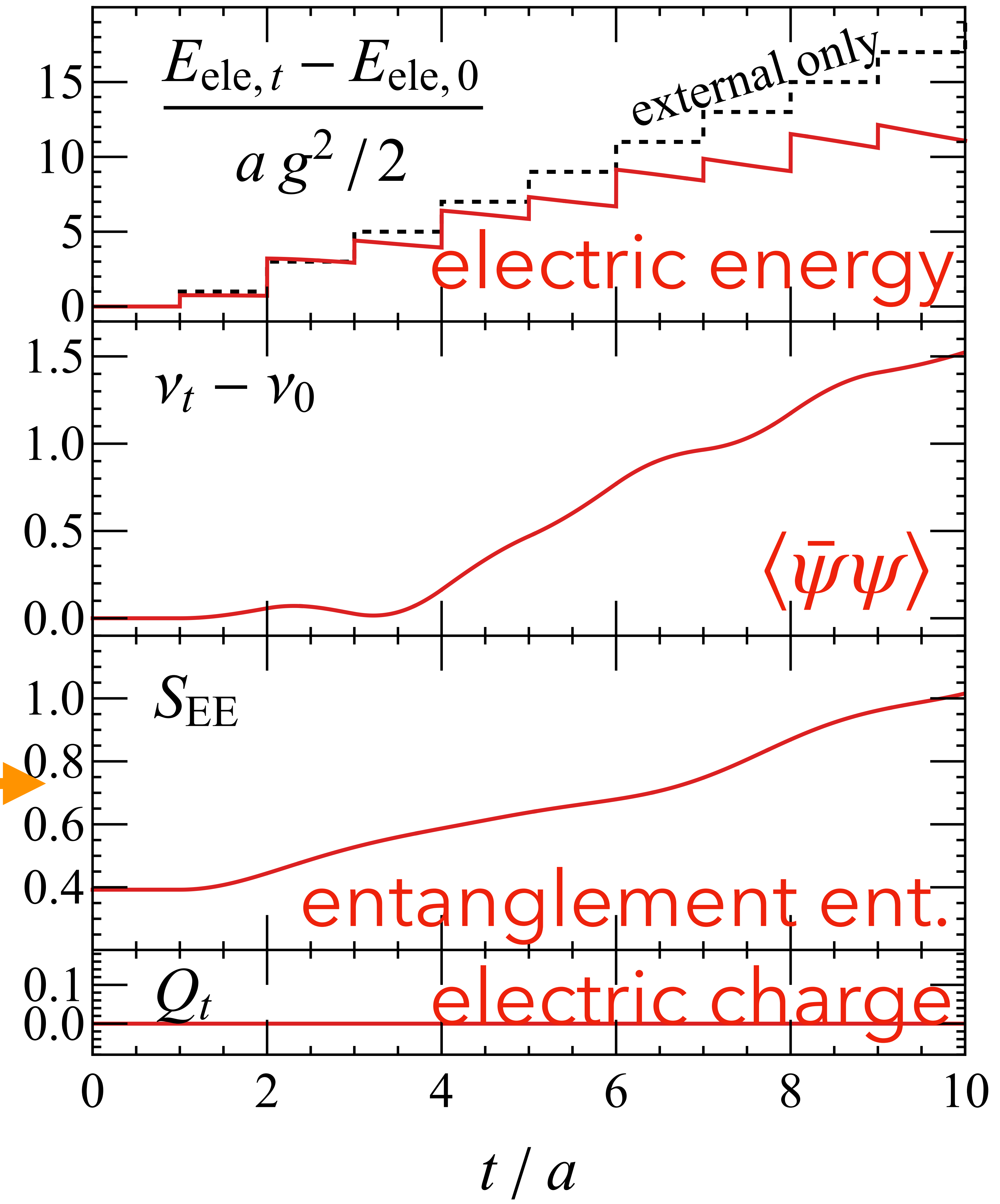
effects of pair production:

screening of the electric field

destruction of vacuum chiral condensate

entanglement between jets



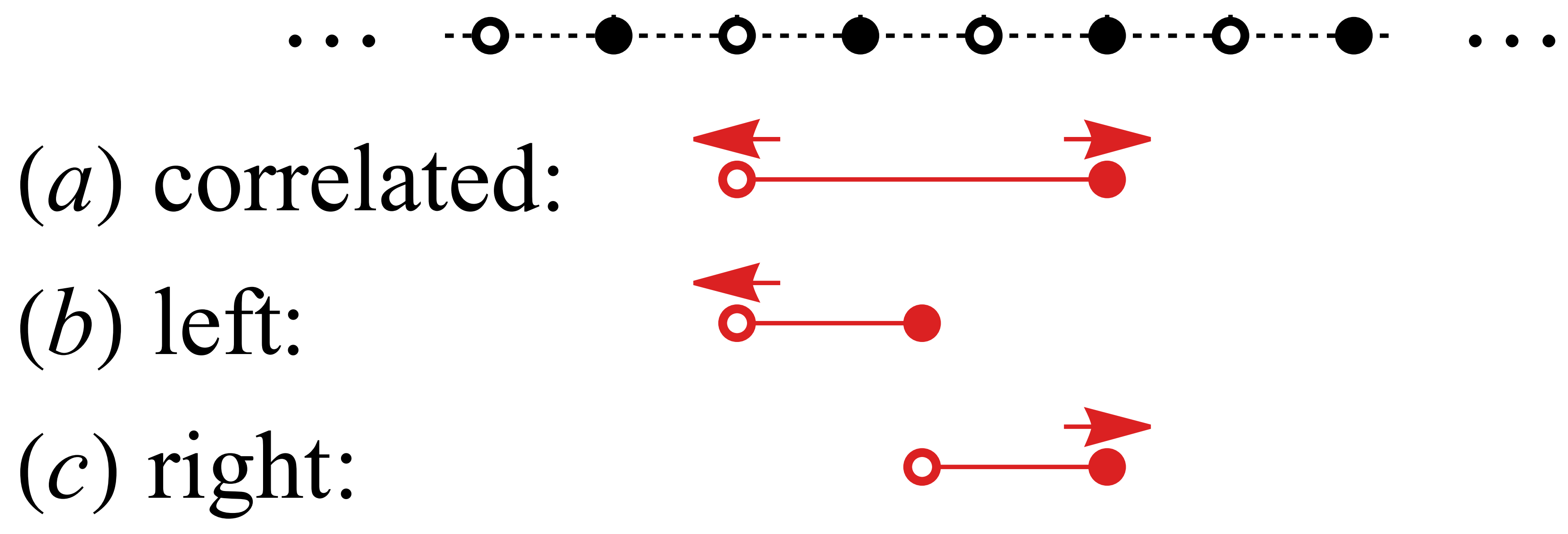


vacuum expectation has been subtracted

Background estimation: uncorrelated sources

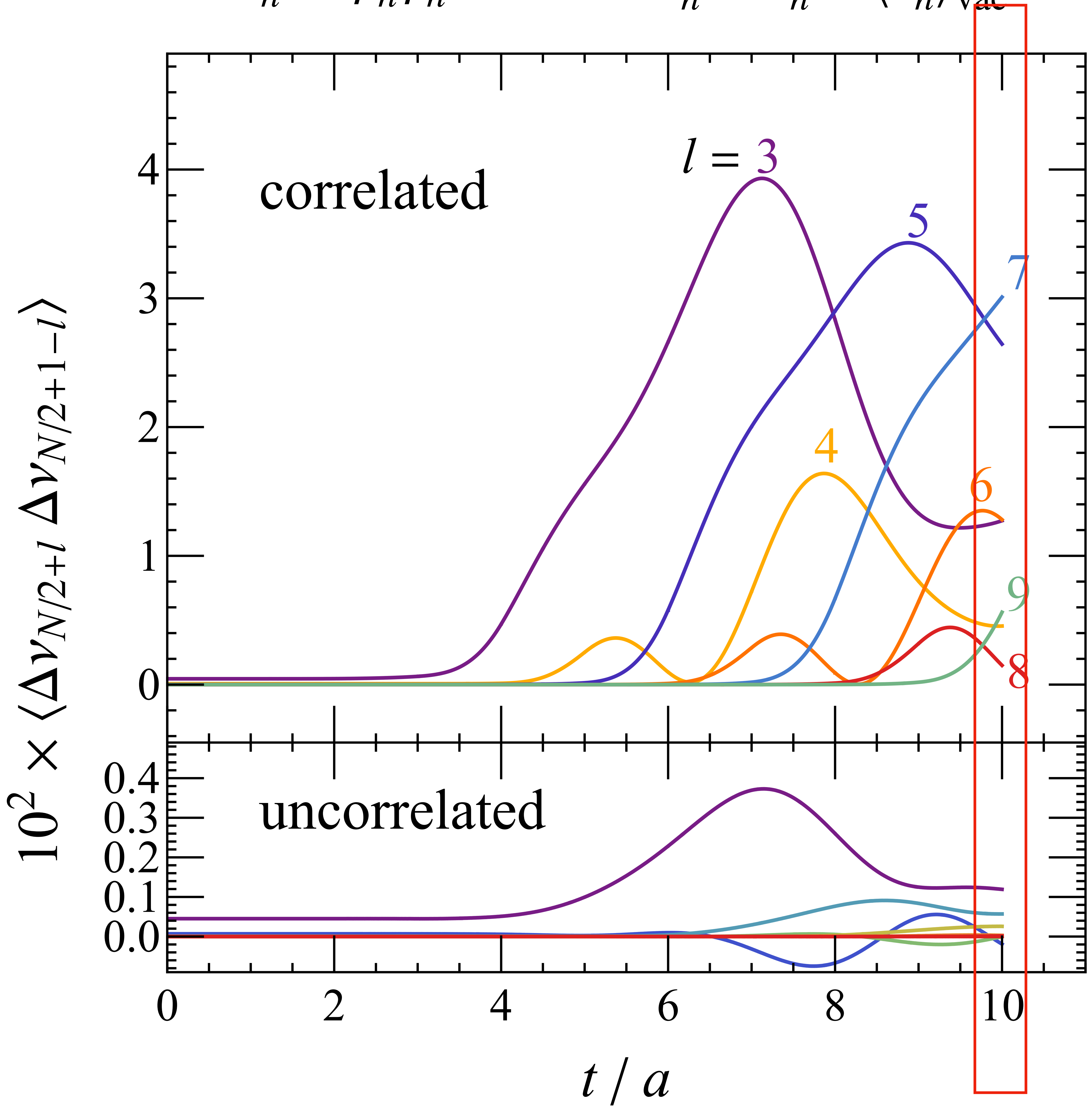
$$|\psi_{\text{uncorr}}\rangle = \frac{1}{\sqrt{2}}|\psi_{\text{left}}\rangle + \frac{e^{i\varphi}}{\sqrt{2}}|\psi_{\text{right}}\rangle$$

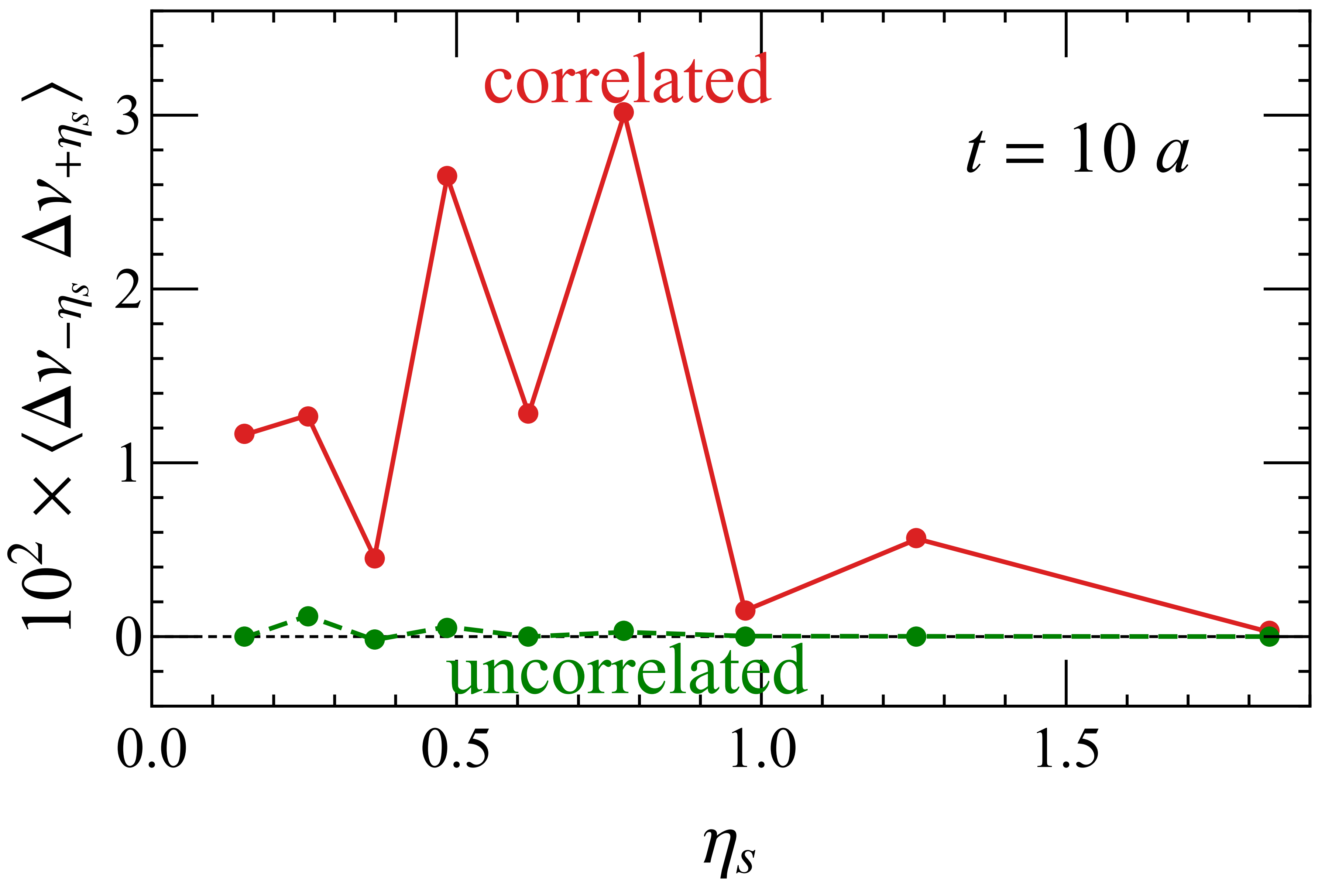
$$\begin{aligned}
&\langle\langle\psi_{\text{uncorr}}|O|\psi_{\text{uncorr}}\rangle\rangle \\
&\equiv \int \langle\psi_{\text{uncorr}}|O|\psi_{\text{uncorr}}\rangle \frac{\mathrm{d}\varphi}{2\pi} \\
&= \frac{\langle\psi_{\text{left}}|O|\psi_{\text{left}}\rangle}{2} + \frac{\langle\psi_{\text{right}}|O|\psi_{\text{right}}\rangle}{2}
\end{aligned}$$

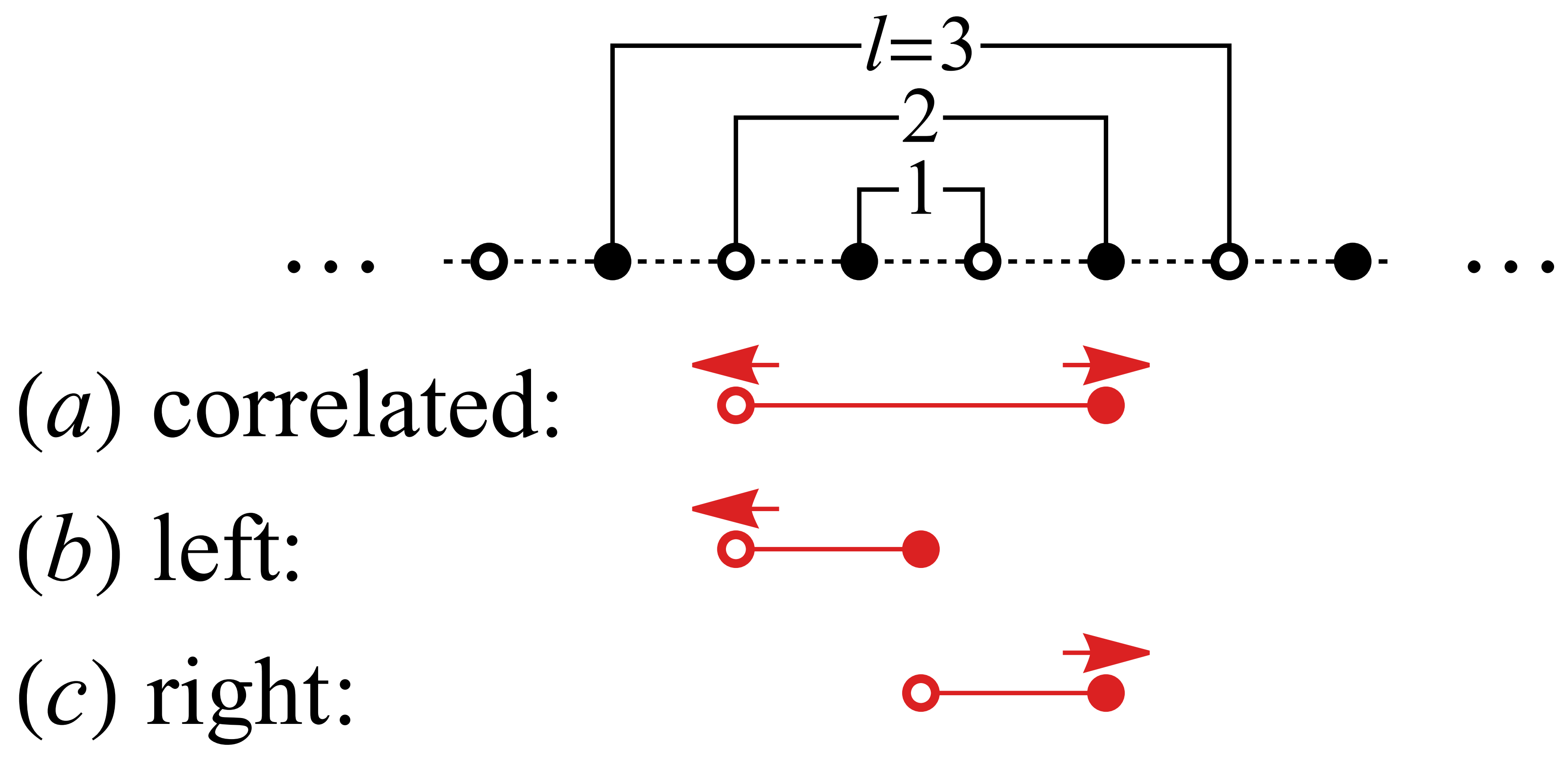


observables of entanglement

$\nu_n \equiv \bar{\psi}_n \psi_n \qquad \Delta\nu_n \equiv \nu_n - \langle \nu_n \rangle_{\text{vac}}$







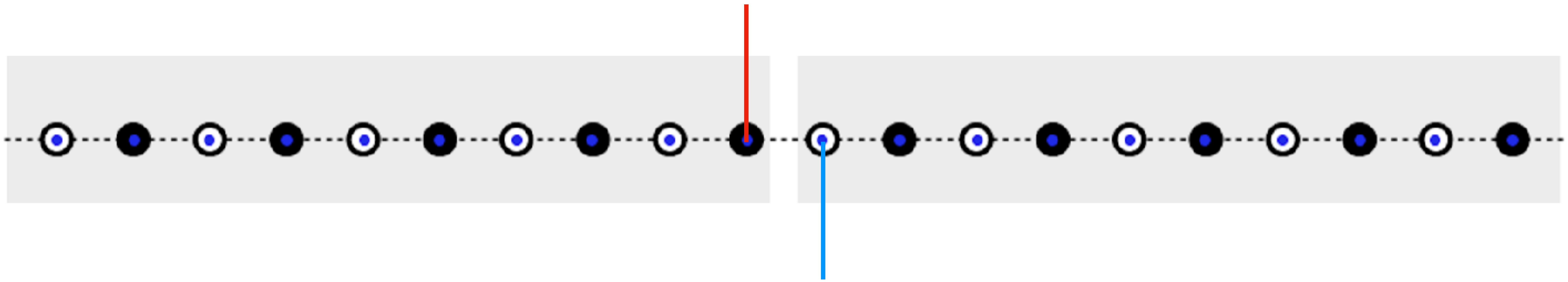
II. Vector & Axial Charge Transport

K. Ikeda, D. Kharzeev, S. Shi,
Phys. Rev. D in press(arXiv: 2305.05685)

axial and vector transport in Schwinger model

$$\partial_t Q + \partial_z J = 0$$

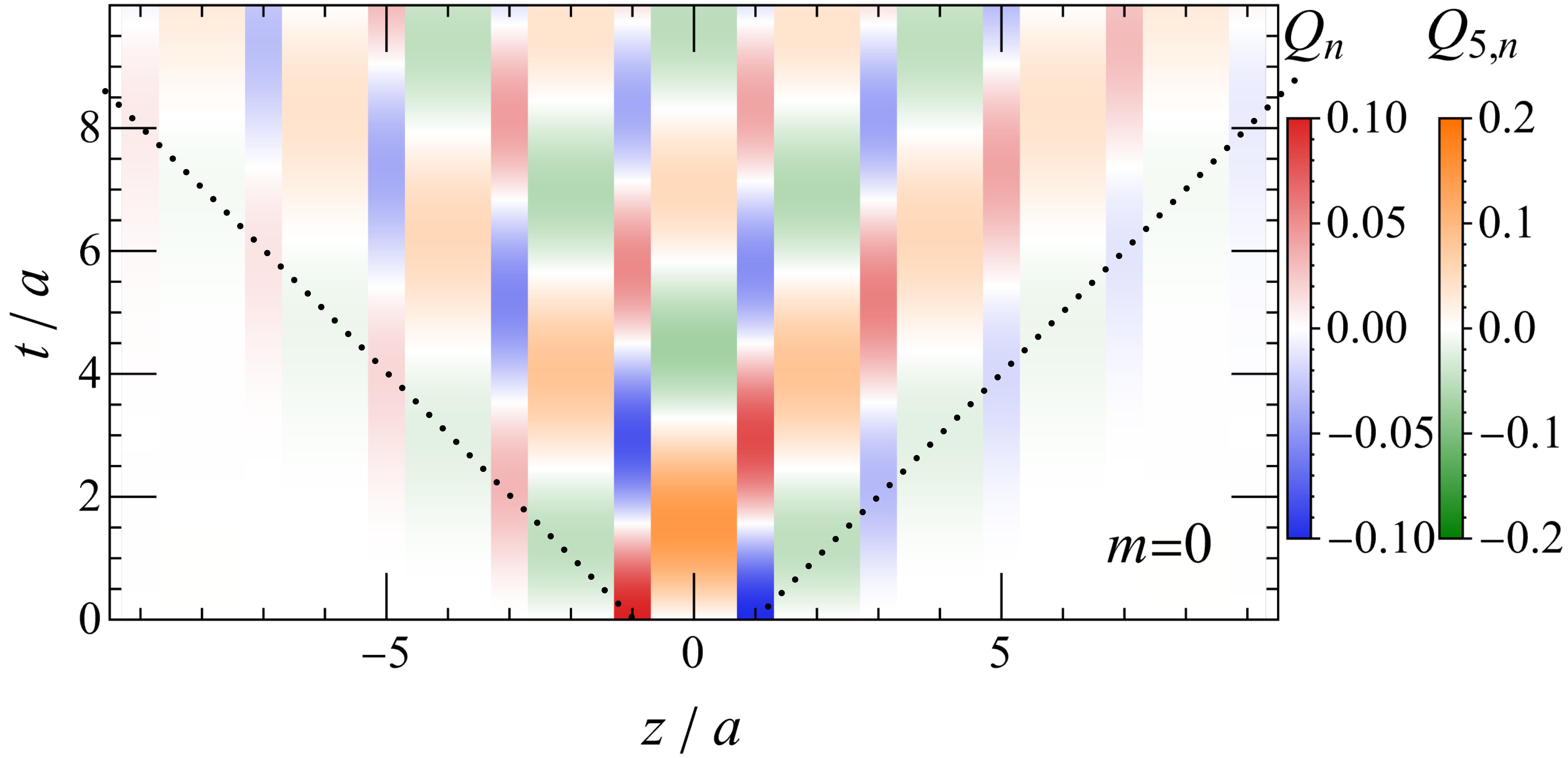
in Schwinger model: $$Q = J_5, \quad J = -Q_5$$

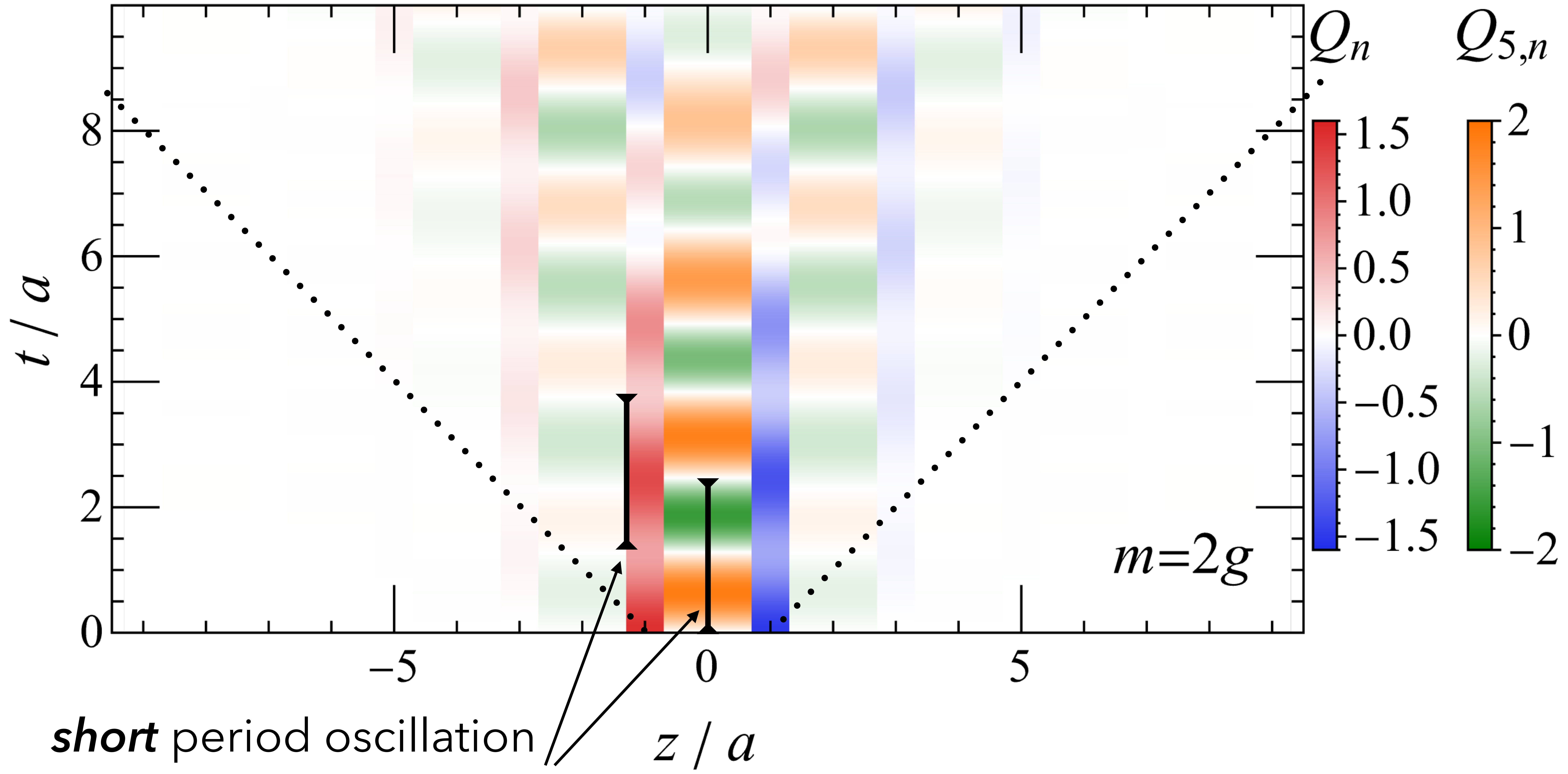


$$Q_n \equiv \langle \bar{\psi}(a\,n)\gamma^0\psi(a\,n)\rangle = \frac{\langle Z_n \rangle + (-1)^n}{2a},$$

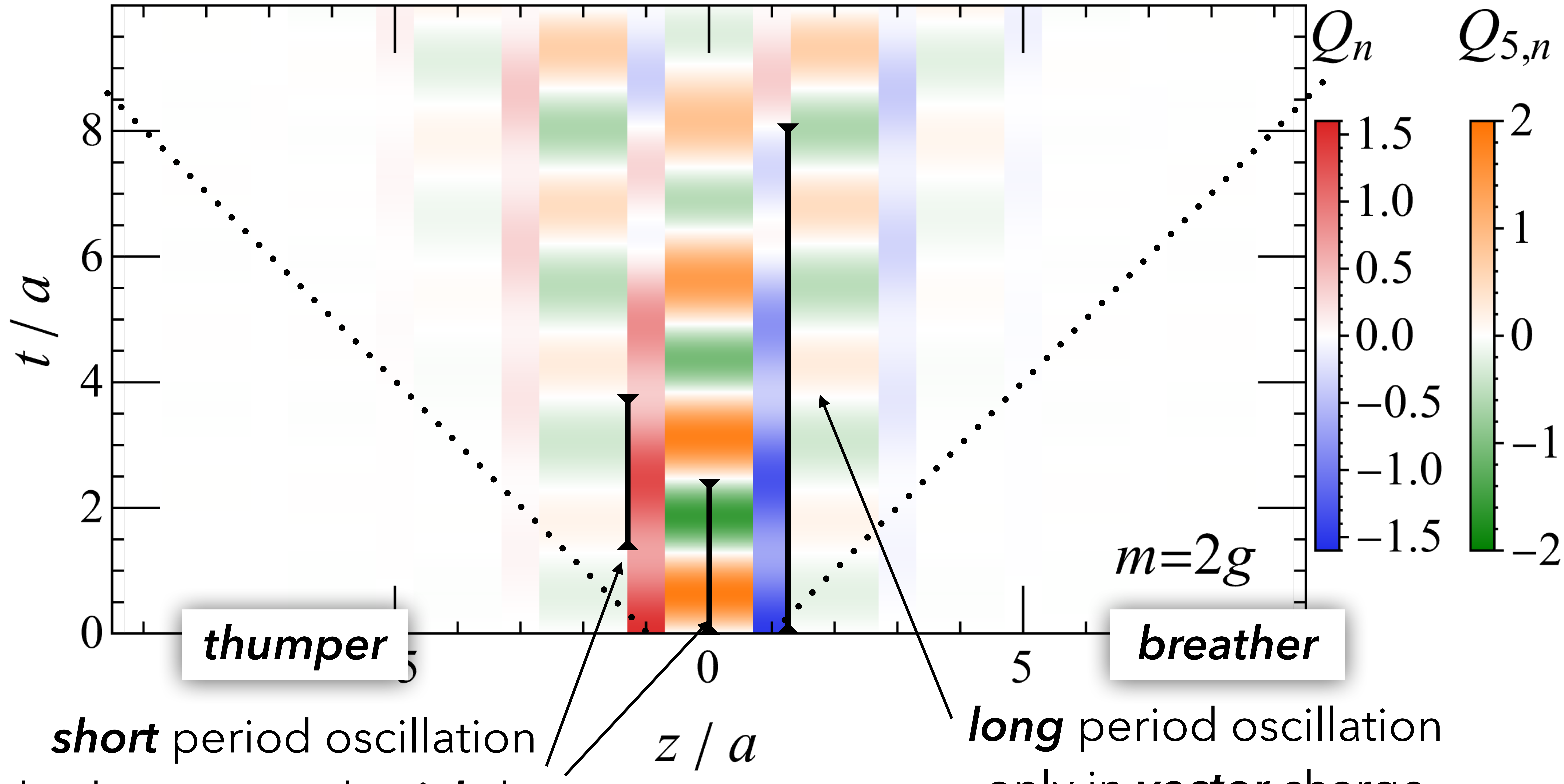
$$Q_{5,n} \equiv \langle \bar{\psi}(a\,n)\gamma^5\gamma^0\psi(a\,n)\rangle = \frac{\langle X_n Y_{n+1} - Y_n X_{n+1}\rangle}{4a}$$

massless case





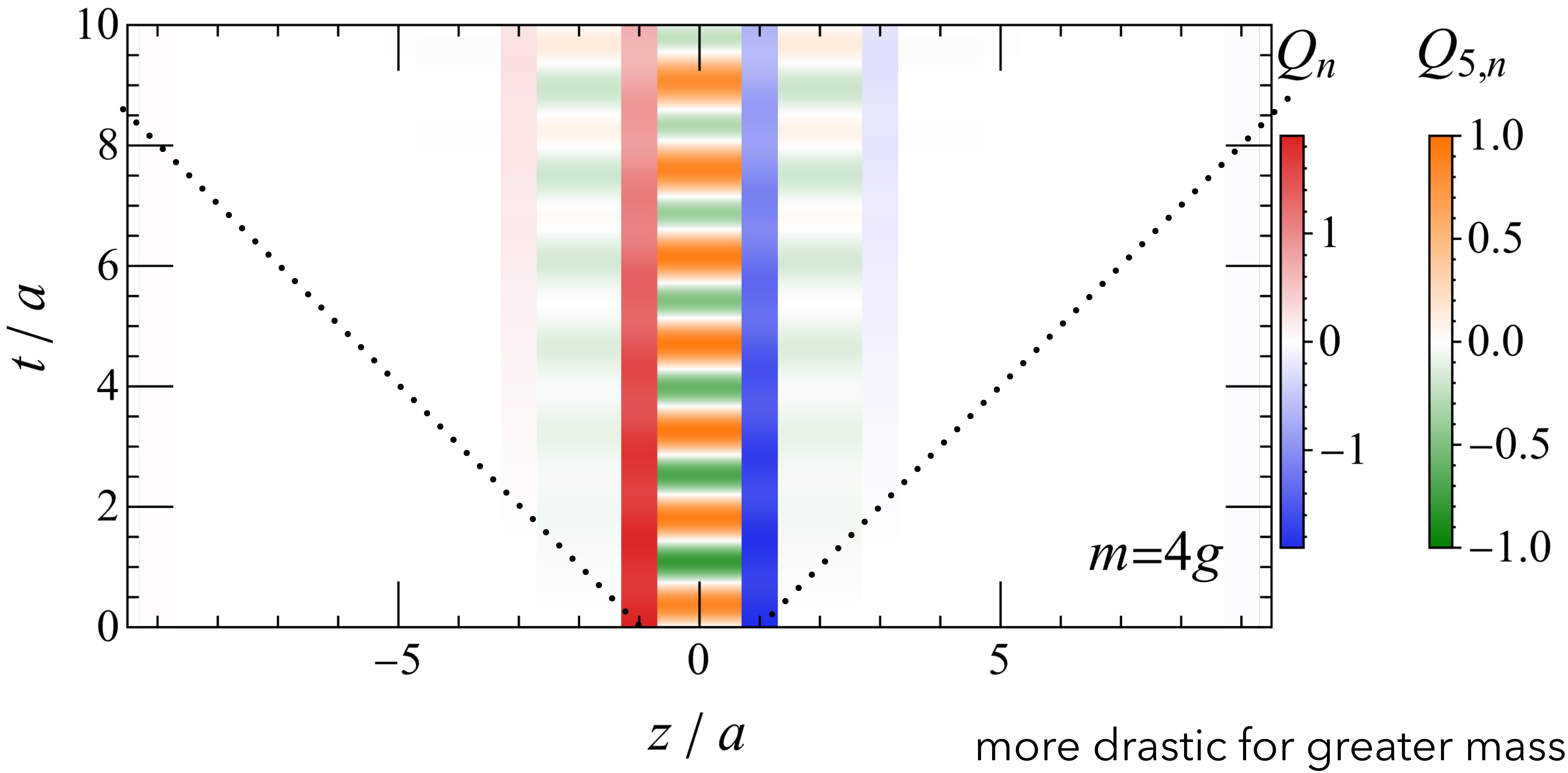
short period oscillation
in both ***vector*** and ***axial*** charge



short period oscillation
in both ***vector*** and ***axial*** charge

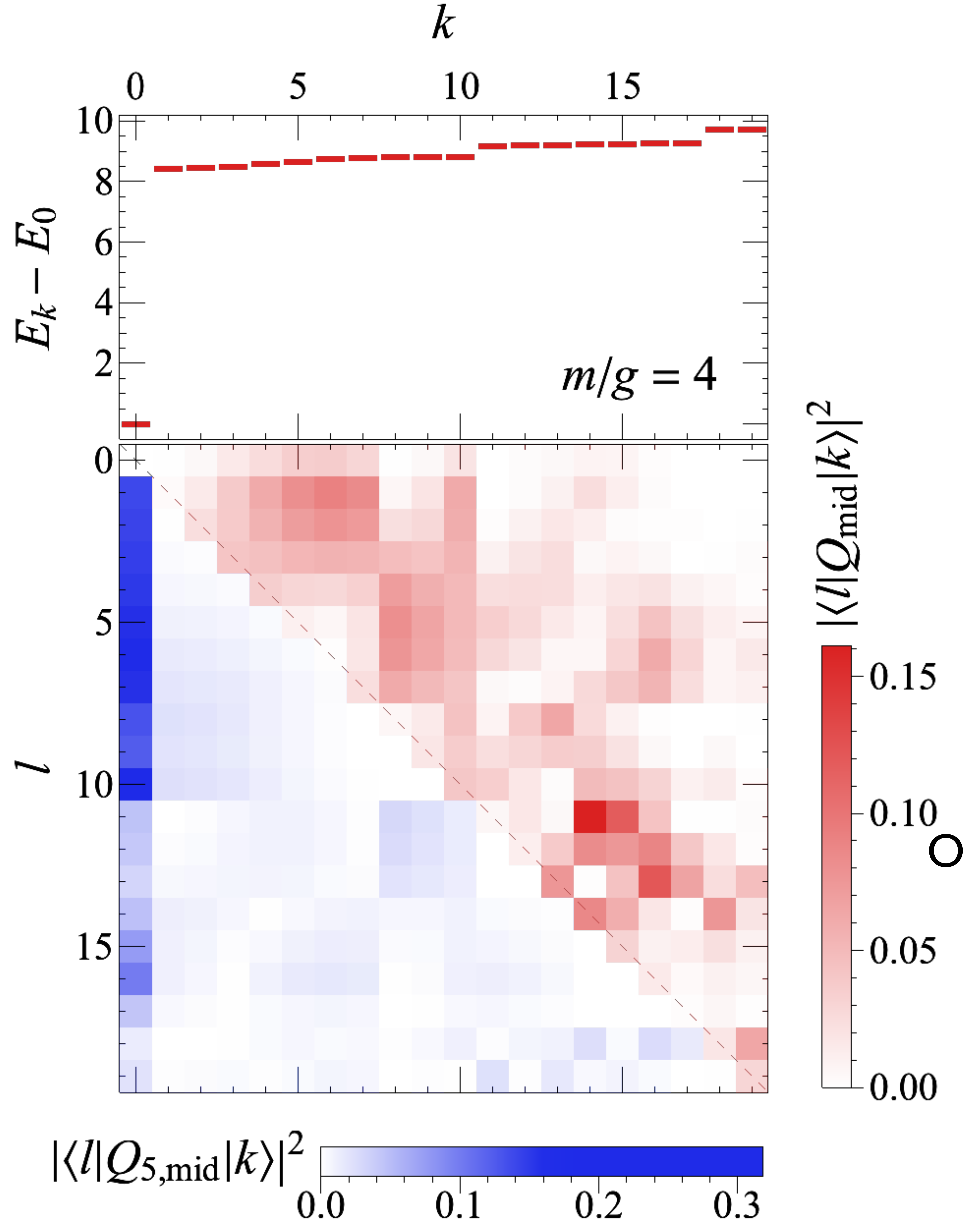
long period oscillation
only in ***vector*** charge

even more massive



more drastic for greater mass

origin of fast and slow oscillating modes



$$H|k\rangle = E_k|k\rangle$$

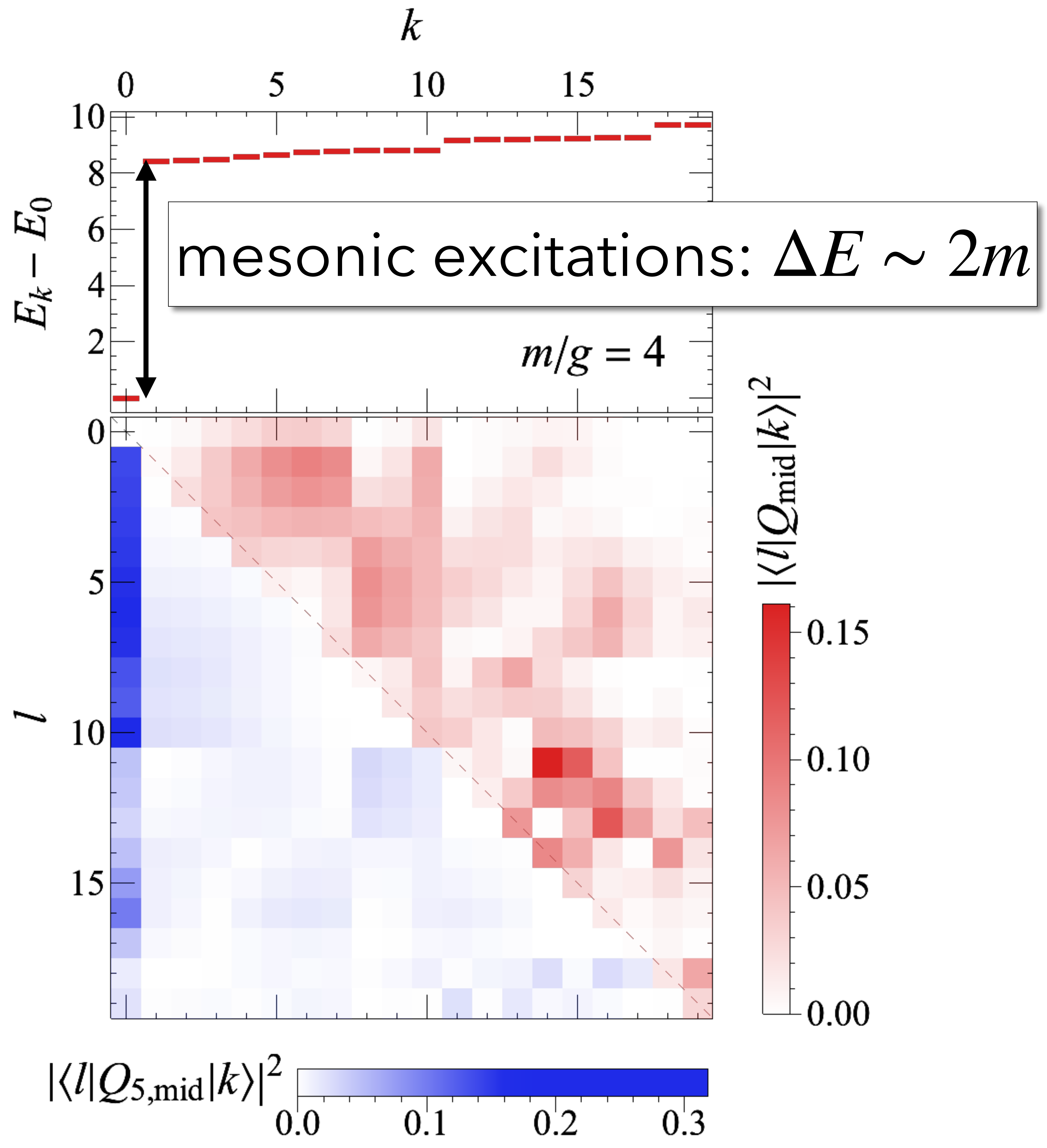
$$|\Psi(t=0)\rangle = \sum_k c_k|k\rangle$$

$$O(t) \equiv \langle\Psi(t)|O|\Psi(t)\rangle = \sum_{k,l} c_k c_l^* e^{i(E_l-E_k)t}\langle l|O|k\rangle$$

oscillation *frequency* ← energy difference

oscillation *strength* ← matrix element

origin of fast and slow oscillating modes

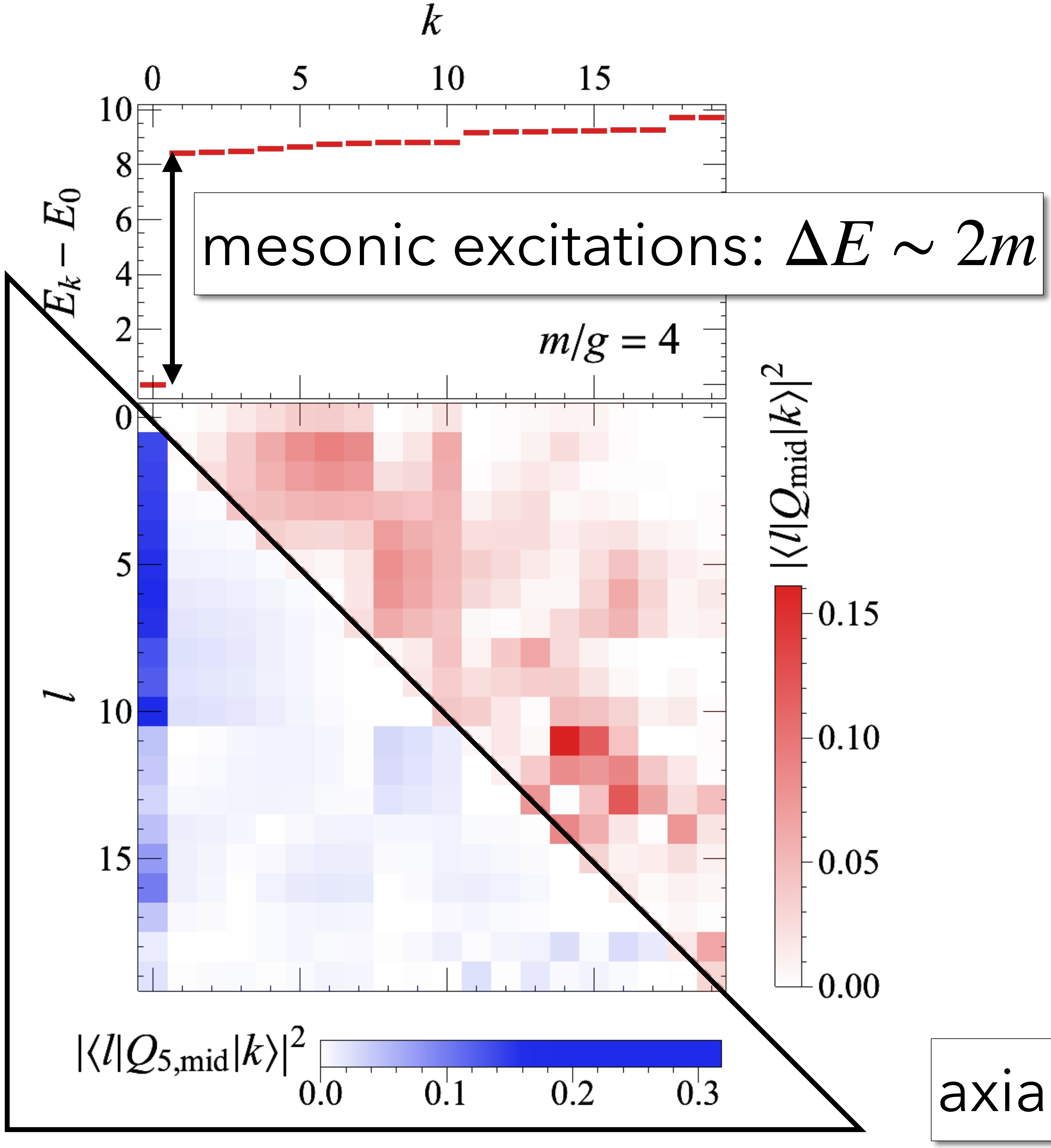


$$H\,|\,k\rangle = E_k\,|\,k\rangle$$

$$|\,\Psi(t=0)\rangle = \sum_k c_k\,|\,k\rangle$$

$$O(t) \equiv \langle \Psi(t)\,|\,O\,|\,\Psi(t)\rangle = \sum_{k,l} c_k c_l^* \underline{e^{i(E_l - E_k)t}}\,\underline{\langle l\,|\,O\,|\,k\rangle}$$

origin of fast and slow oscillating modes



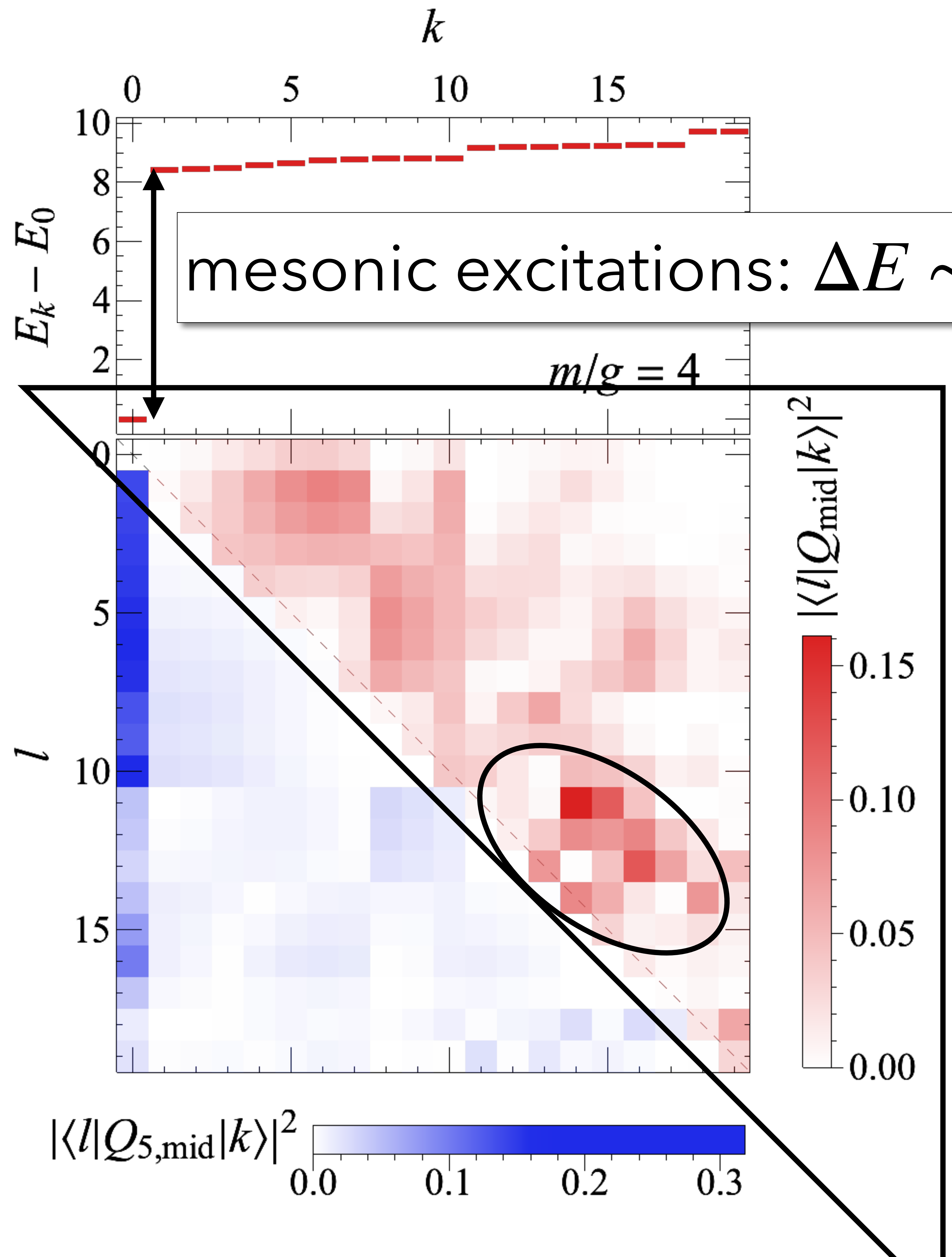
$$H|k\rangle = E_k|k\rangle$$

$$|\Psi(t=0)\rangle = \sum_k c_k|k\rangle$$

$$O(t) \equiv \langle\Psi(t)|O|\Psi(t)\rangle = \sum_{k,l} c_k c_l^* \underline{e^{i(E_l - E_k)t}} \underline{\langle l|O|k\rangle}$$

axial charge: ground state ↔ excitation

origin of fast and slow oscillating modes



$$H|k\rangle = E_k|k\rangle$$

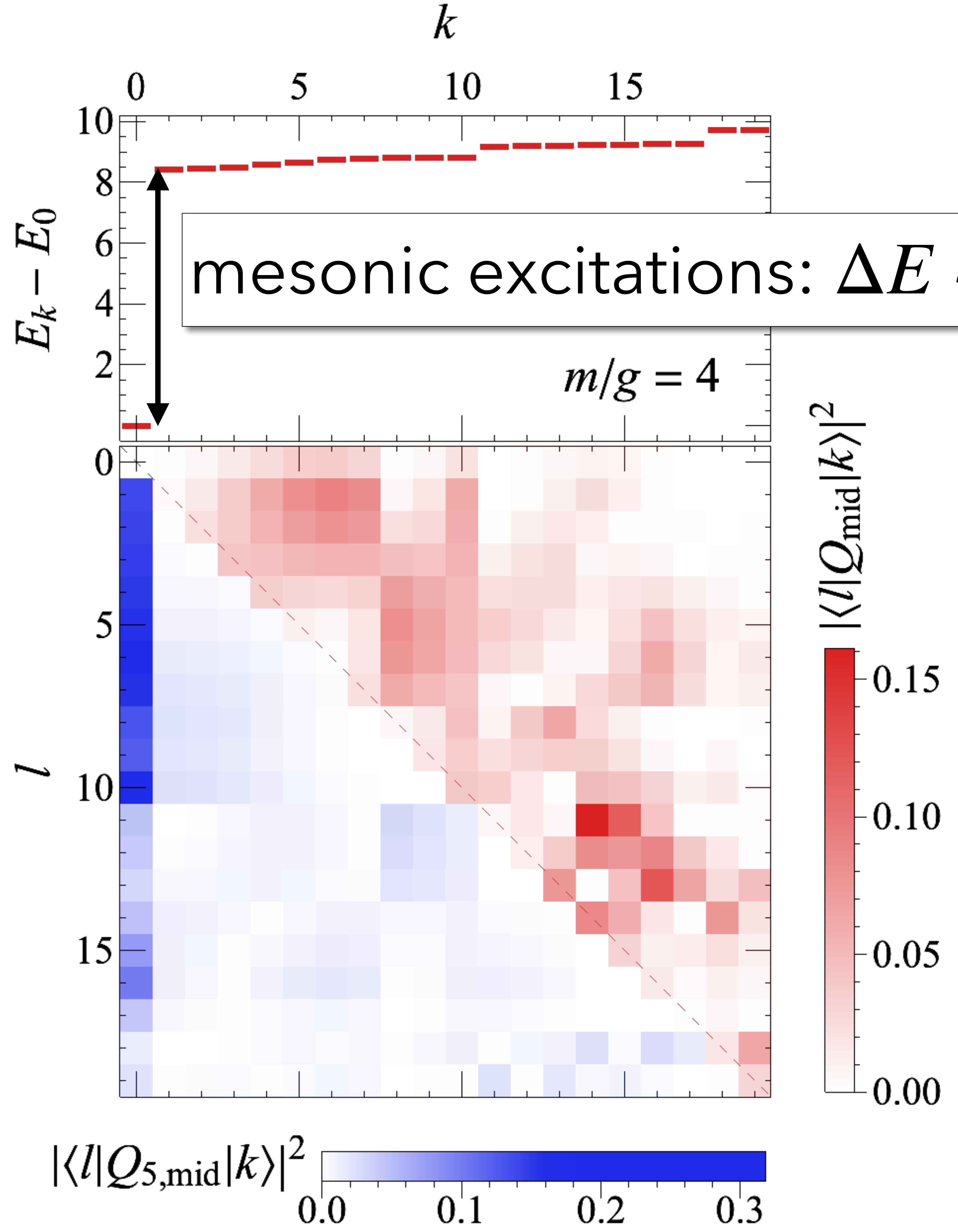
$$|\Psi(t=0)\rangle = \sum_k c_k|k\rangle$$

$$O(t) \equiv \langle\Psi(t)|O|\Psi(t)\rangle = \sum_{k,l} c_k c_l^* e^{i(E_l - E_k)t}\langle l|O|k\rangle$$

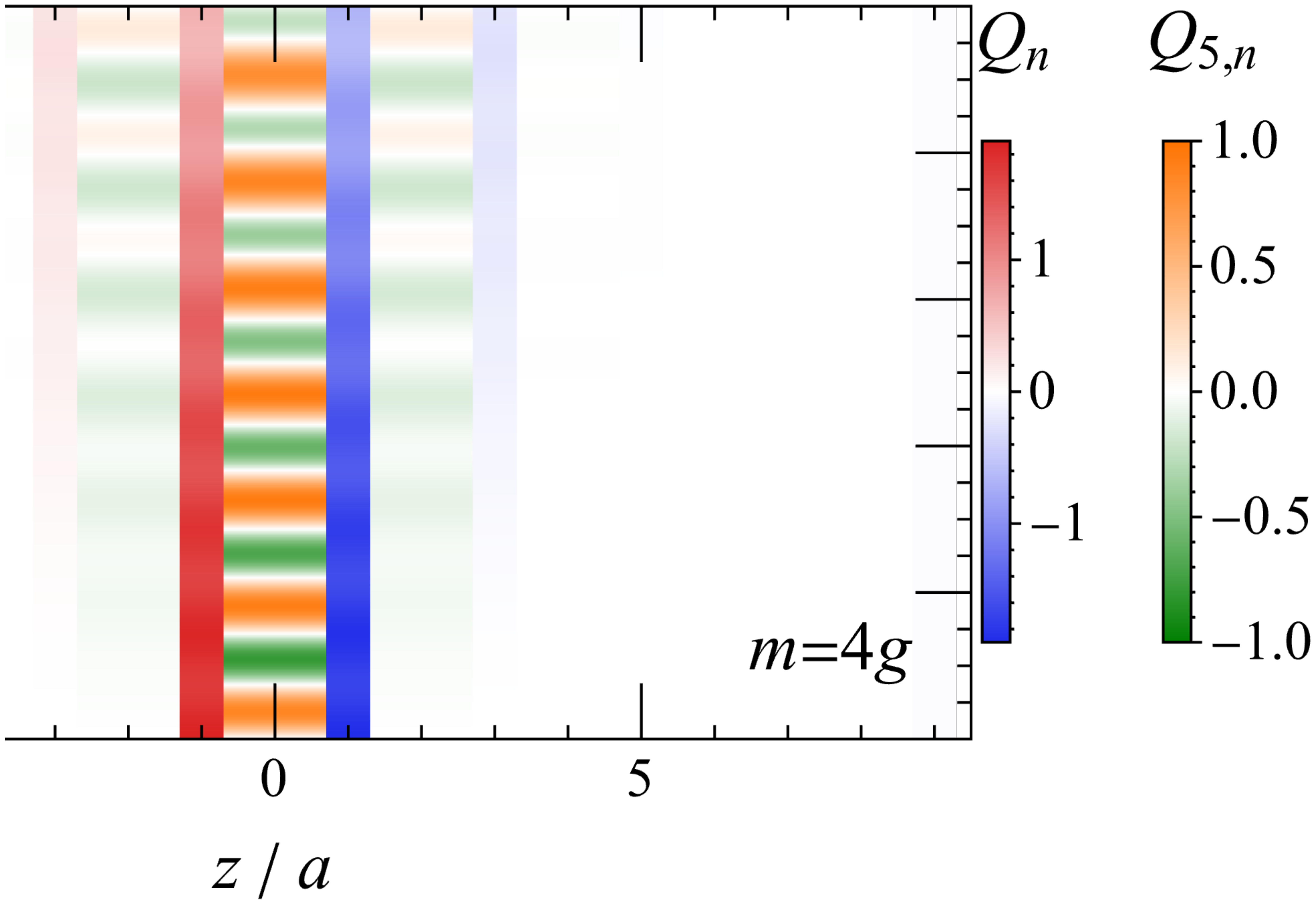
vector charge: excitation ↔ excitation

axial charge: ground state ↔ excitation

origin of fast and slow oscillating modes



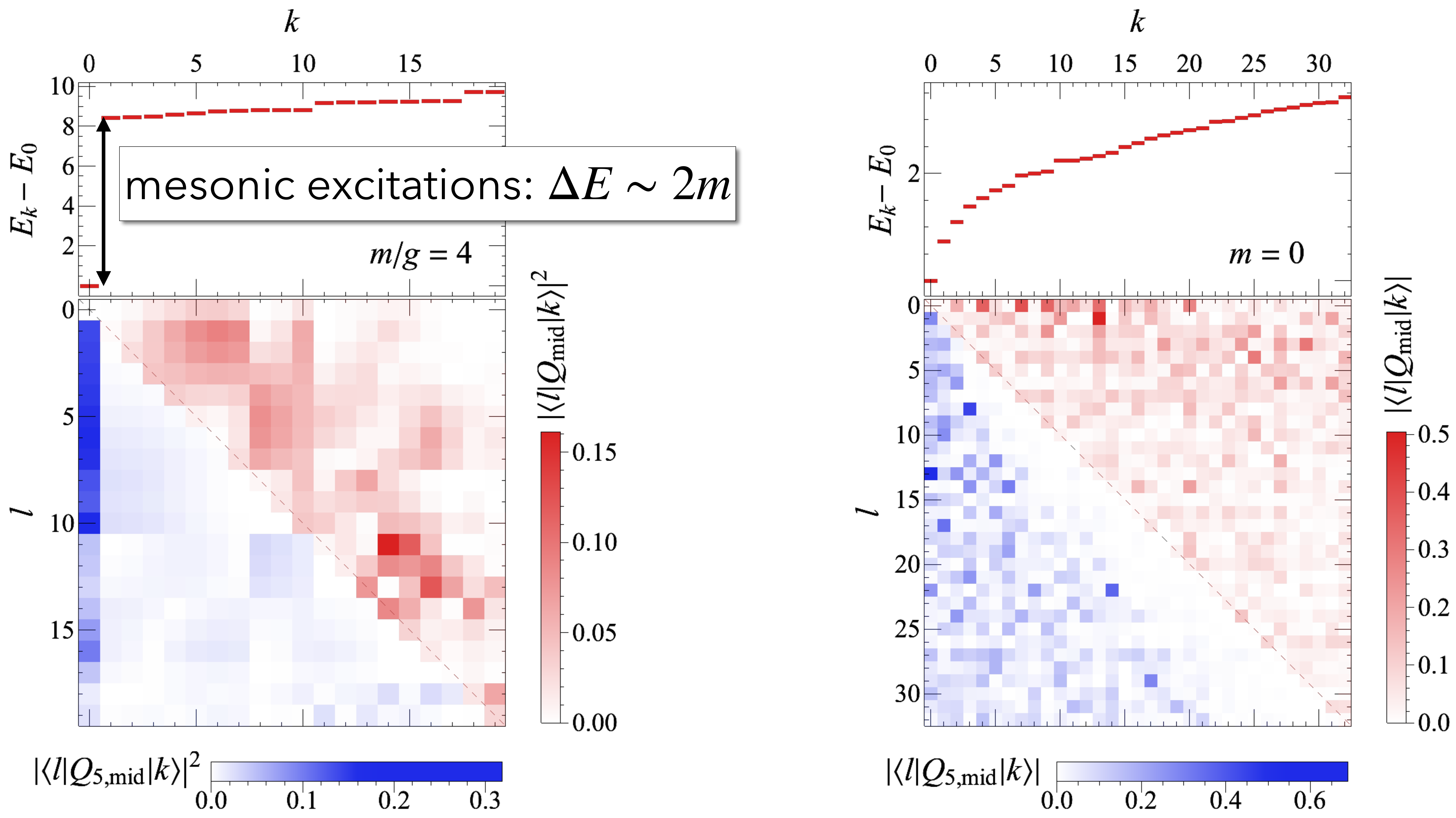
mesonic excitations: $\Delta E \sim 2m$



vector charge: excitation ↔ excitation

axial charge: ground state ↔ excitation

real-time evolution



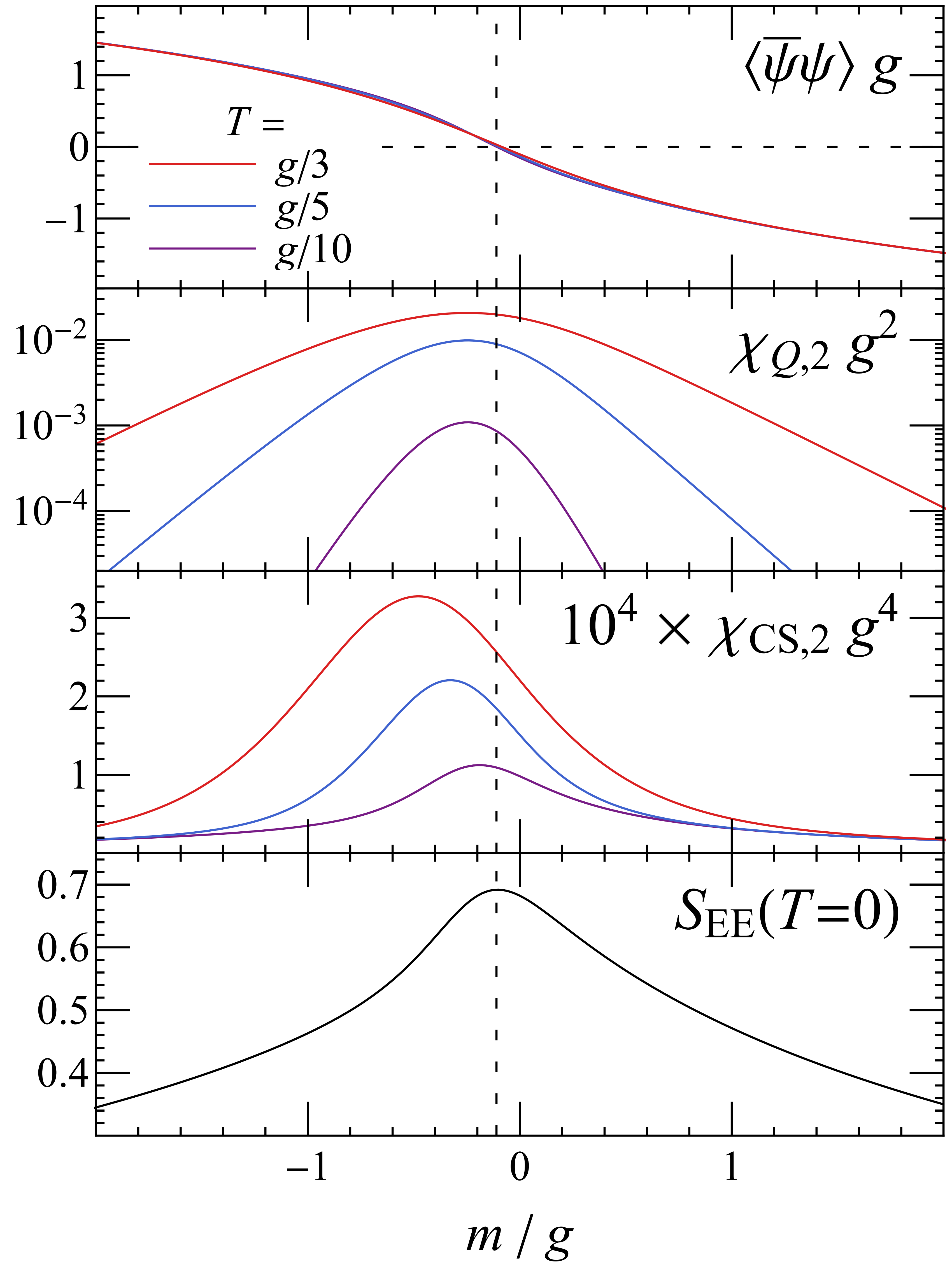
III. Phase Structure

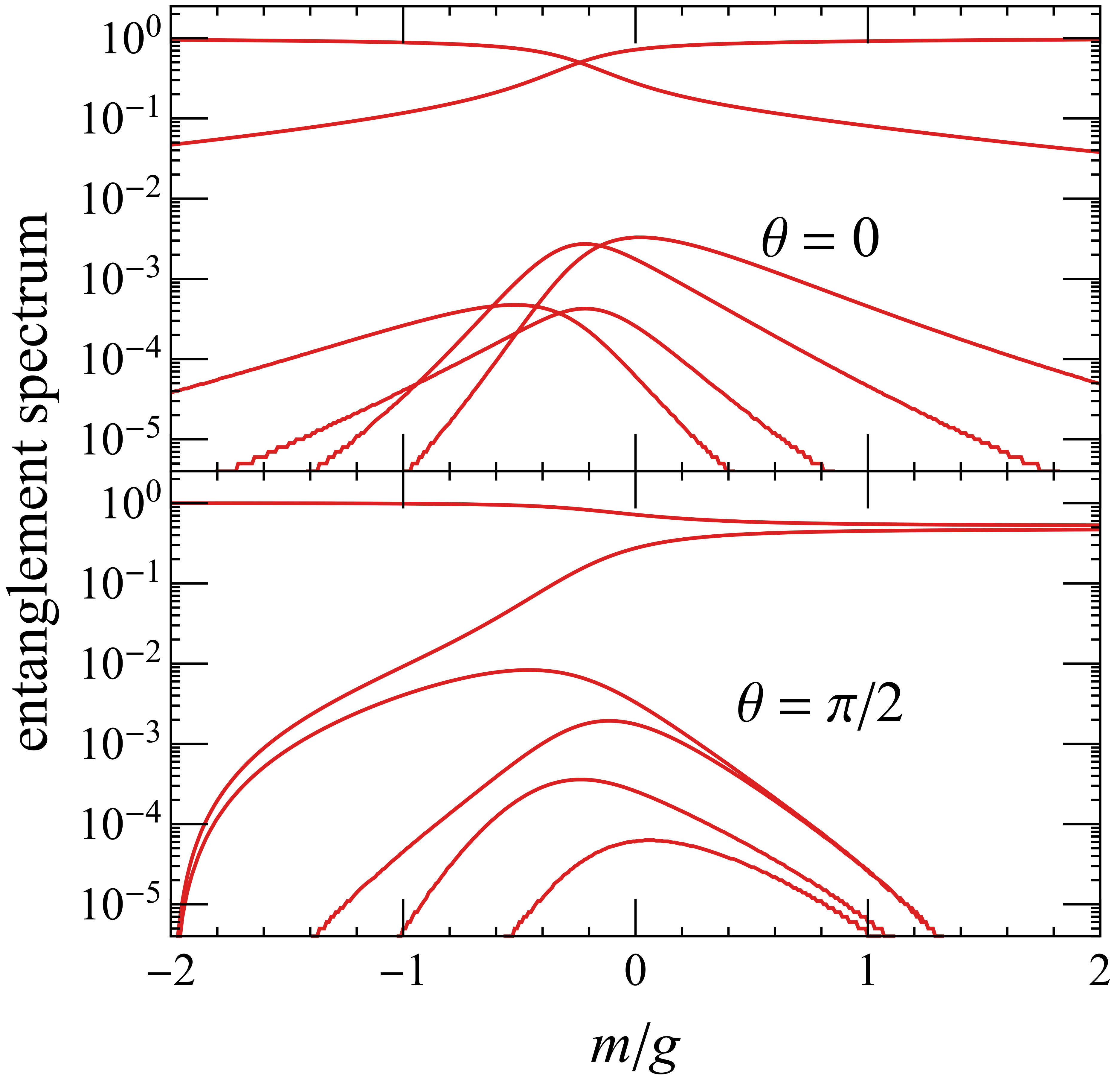
K. Ikeda, D. Kharzeev, R. Meyer, S. Shi, arXiv: 2305.00996

- finite temperature, finite chemical potential:

$$\langle O \rangle_{\rm th} \equiv {\rm Tr}(\rho_{\rm th}\, O) \qquad \rho_{\rm th} \equiv \frac{e^{-(H-\mu Q)/T}}{{\rm Tr}(e^{-(H-\mu Q)/T})}$$

different signals of the critical point





summary

- real-time dynamics in Schwinger model
    - jet production: spread out of light cone, creation of fermion-antifermion pairs
    - charge transport: thumper and breather modes
    - critical point detected by different signals
- Need ***quantum computers*** to approach the continuum limit.

Students and Postdocs are welcome!

(so are questions/comments)