Nonlinear Chiral Kinetic Theory

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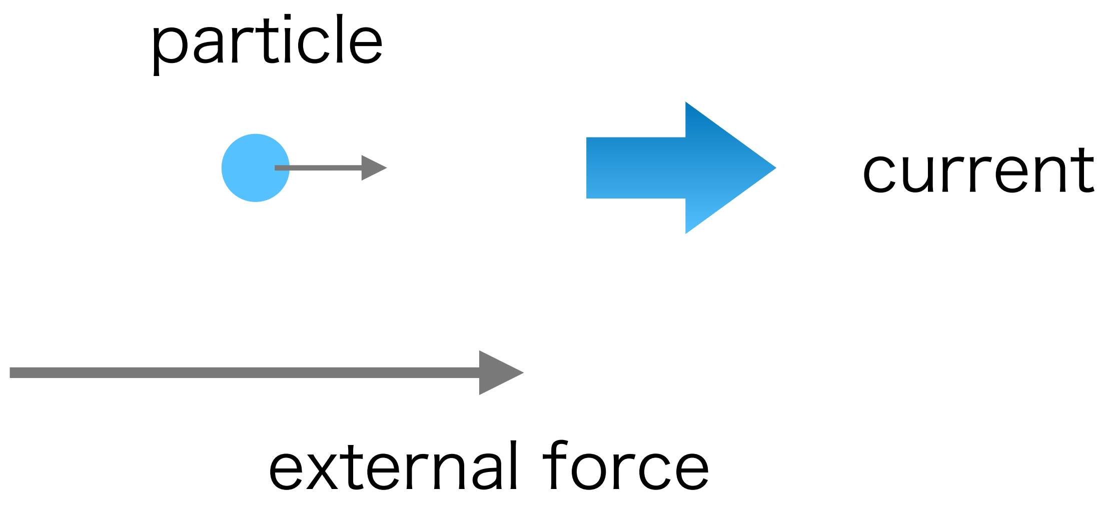
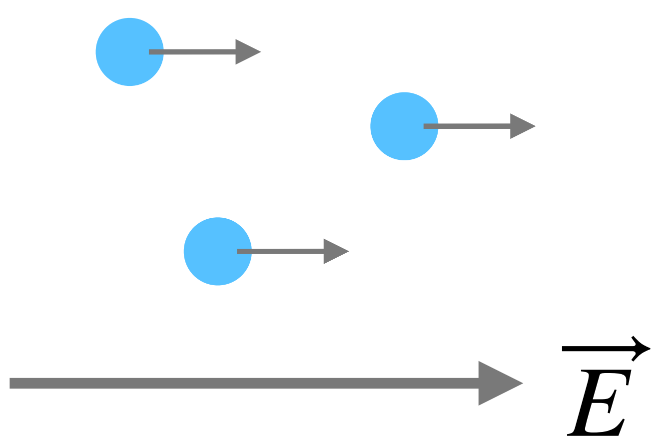
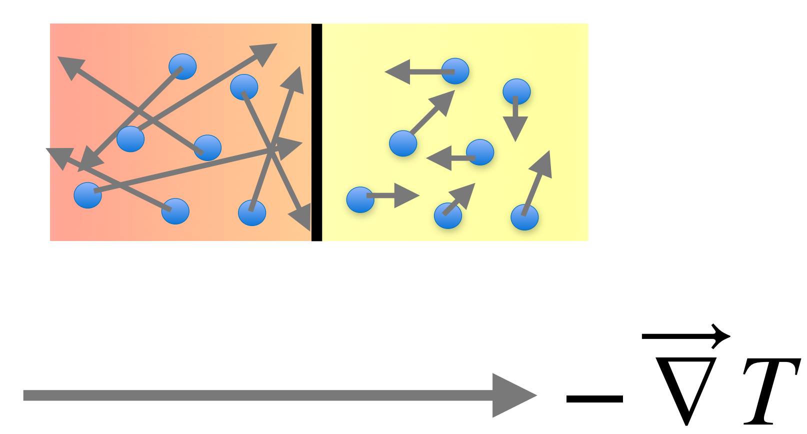
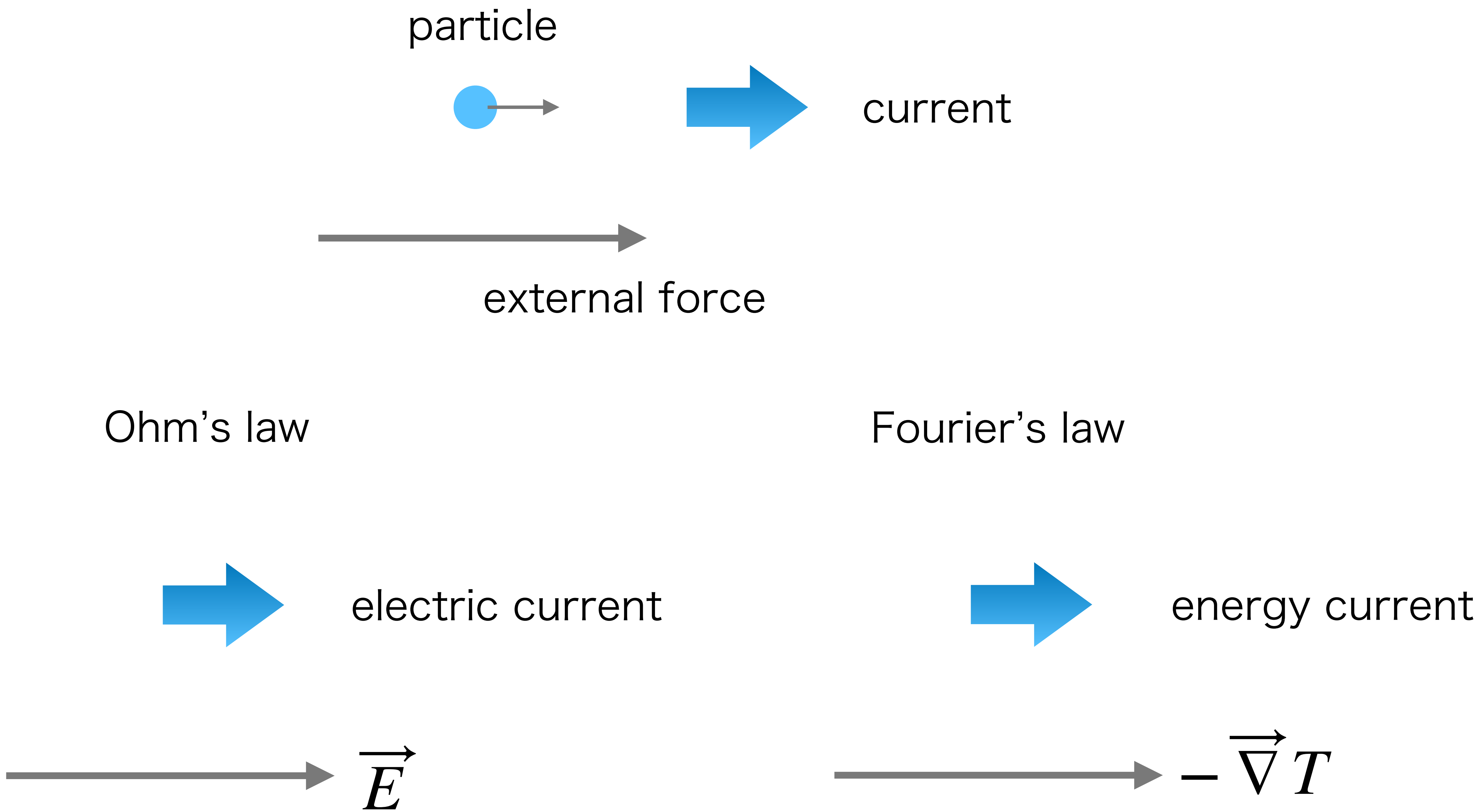
Transport Phenomena





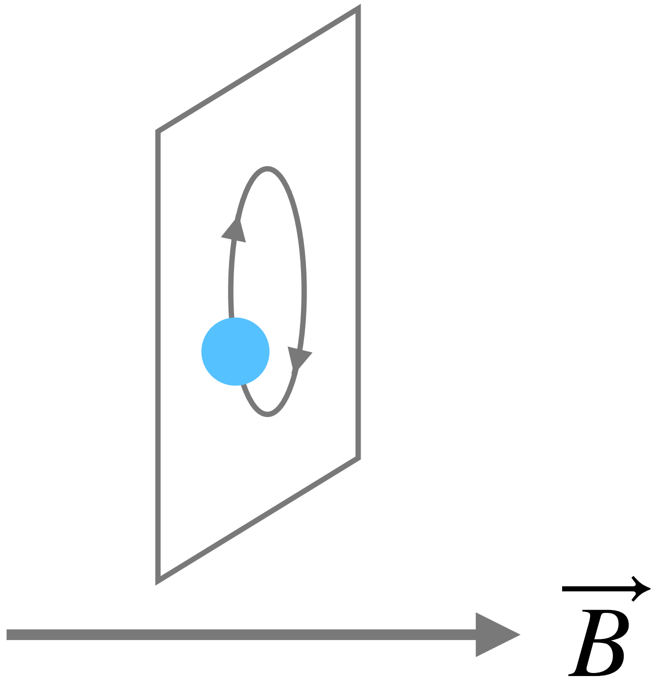


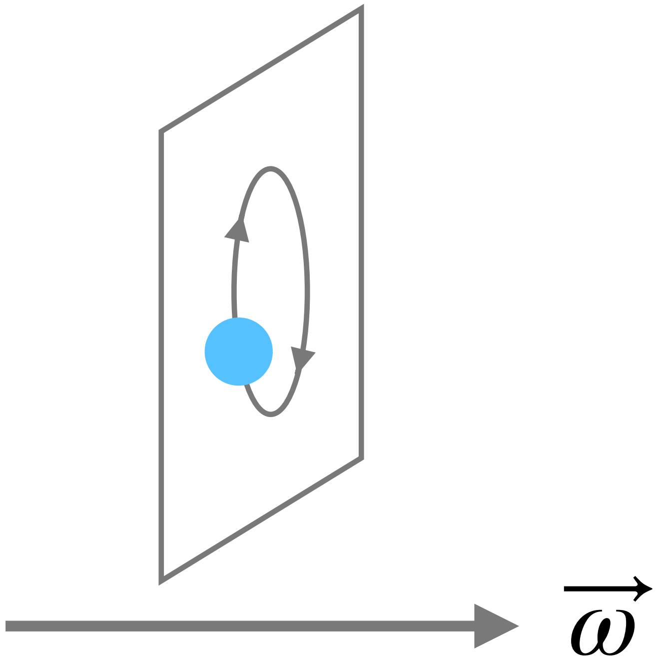
Transport Phenomena



Transport Phenomena

- classical

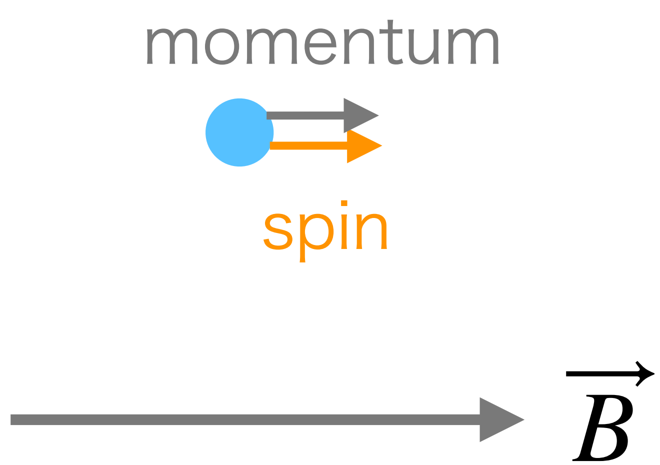




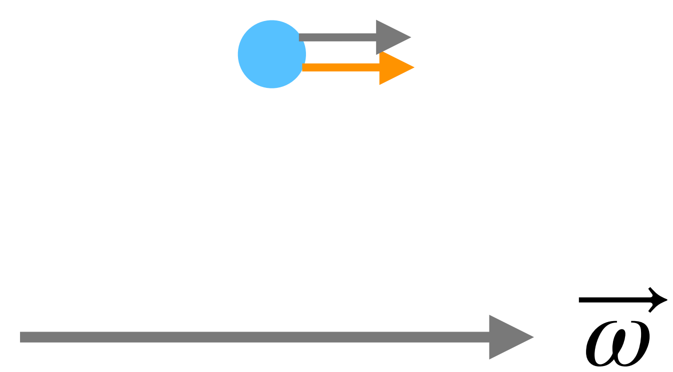
Transport Phenomena

- quantum and massless

magnetization coupling

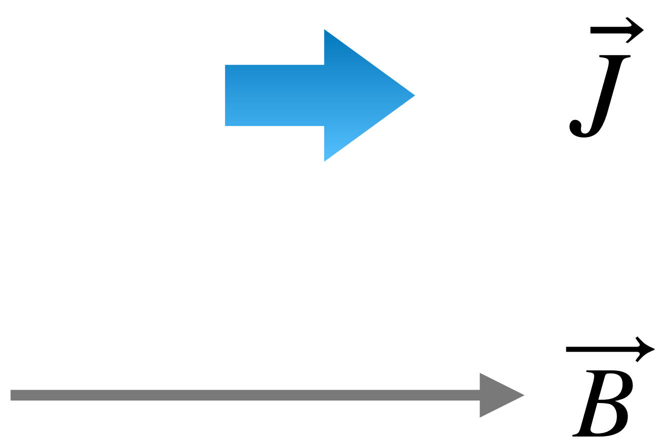


spin-vorticity coupling



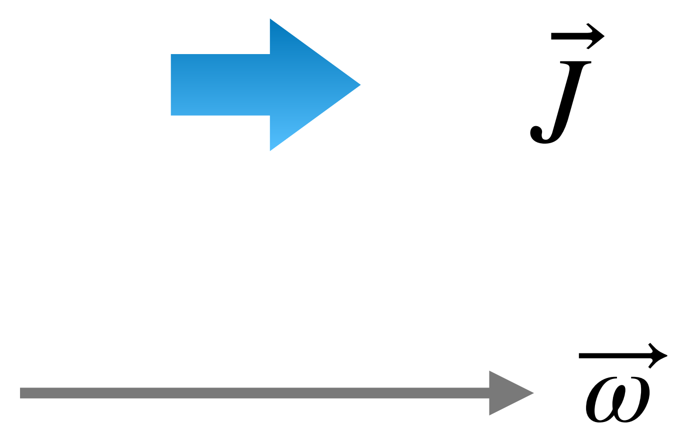
Transport Phenomena

- quantum and massless



chiral magnetic effect

Vilenkin (1980)
Fukushima, Kharzeev, Warringa (2008)



chiral vortical effect

Vilenkin (1979), Son, Surowka (2009)
Landsteiner, Megias, Pena-Benitez (2011)

Chiral Transport Phenomena

chiral magnetic effect

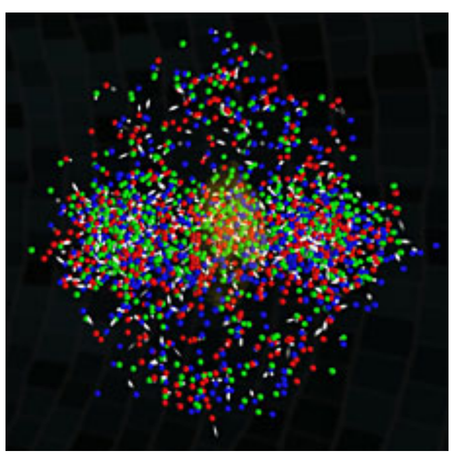
$$\vec{J} = \sigma_{\mathrm{CME}} \vec{B}$$

chiral vortical effect

$$\vec{J} = \sigma_{\mathrm{CVE}} \vec{\omega}$$

from quantum anomaly in Dirac theory

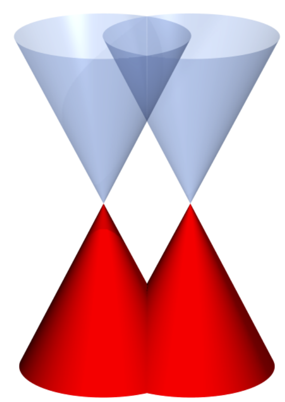
hep



quark-gluon plasma

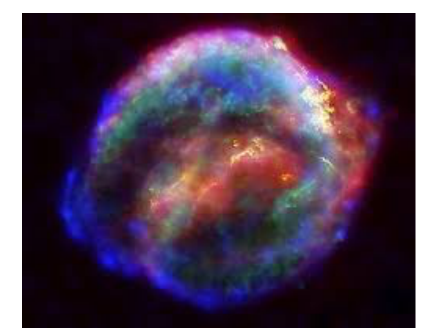
cond-mat

Vazifeh, Franz (2013)



Weyl semimetal

astro



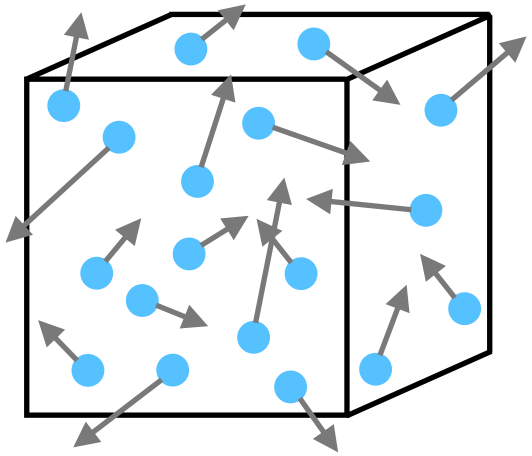
neutrino

for matter in early universe
(2010)

→

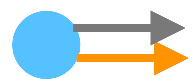
for any chiral matter
(present)

Chiral Kinetic Theory



Boltzmann eq.

+



chirality

=

Chiral Kinetic Theory(CKT)

Stephanov, Yin (2012)
Son, Yamamoto (2012)
Chen et al. (2013)

- Lorentz covariance
- collision
- mass correction
- spin polarization

- strong magnetic field
- gravitational field
- other derivations

Hidaka, Pu, Wang, Yang (2022)

Limitation 1 : magneto-vortical transport

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\hbar\,\partial_\mu - eA_\mu)\psi$$

$$\longrightarrow \quad O(\partial) \sim O(\hbar)$$

$O(1)$

$J^0 \sim \mu^3$

Fermi sphere

Boltzmann

$O(\hbar)$

$\vec{J} \sim \mu\vec{B}$ $\quad$ $\vec{J} \sim \mu^2\vec{\omega}$

CME $\quad$ CVE

linear CKT

$O(\hbar^2)$

Hattori, Yin (2016)

$J^0 \sim \vec{B}\cdot\vec{\omega}$

nonlinear CKT

$B \sim \partial A \qquad \omega \sim \partial u$

Limitation 1 : magneto-vortical transport

$$O(\hbar) \qquad\qquad O(\hbar^2)$$

$$\vec{J} \sim \mu\vec{B} \quad \vec{J} \sim \mu^2\vec{\omega} \qquad\qquad J^0 \sim \vec{B}\cdot\vec{\omega}$$

anomaly, nondissipative   anomaly? nondissipative?

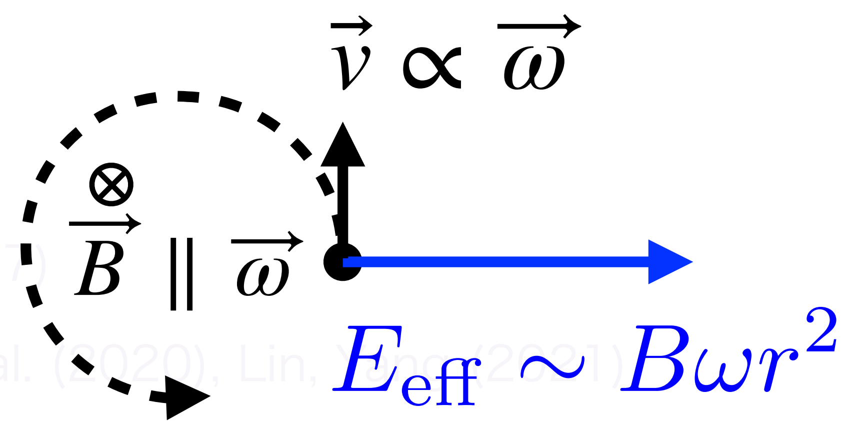
- linear response Hattori, Yin (2016)
- rotating fermions Ebihara, Fukushima, KM (2017)
- near-equilibrium Wigner function Yang et al. (2020), Lin, Yang (2021)

Limitation 1 : magneto-vortical transport

$O(\hbar)$	$O(\hbar^2)$
$\vec{J} \sim \mu \vec{B} \quad \vec{J} \sim \mu^2 \vec{\omega}$	$J^0 \sim \vec{B} \cdot \vec{\omega}$
anomaly, nondissipative	anomaly? nondissipative?

- linear response Hattori, Yin (2016)
- near-equilibrium Wigner function Yang et al. (2020)

Thermal Field Theory, but⋯



Equilibrated?

Check it from nonlinear CKT

Limitation 2 : trace anomaly

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\hbar\,\partial_\mu - eA_\mu)\psi$$

$$\longrightarrow \quad O(\partial) \sim O(\hbar)$$

$O(1)$

$$\partial_\mu J^\mu = 0$$

$$T^\mu{}_\mu = 0$$

Boltzmann

$O(\hbar)$

$$\partial_\mu J^\mu \sim \vec{E}\cdot\vec{B}$$

chiral anomaly

linear CKT

$$E, B \sim \partial A$$

$O(\hbar^2)$

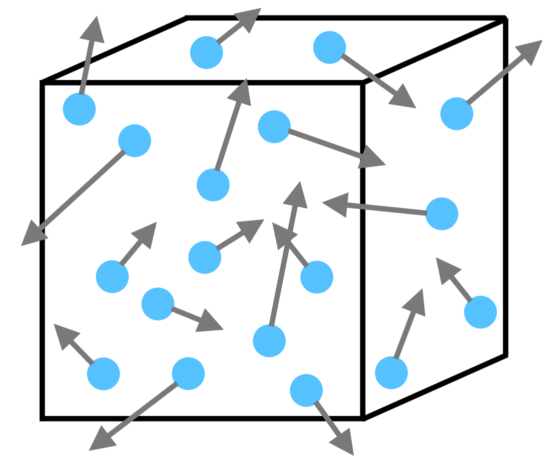
$$T^\mu{}_\mu \sim \beta(e) F_{\mu\nu}^2$$

trace anomaly

nonlinear CKT

Limitation 2 : trace anomaly

$$\mathcal{L}_{\mathrm{QED}} = -\frac{1}{4}F_{\mu\nu}^2 + \bar{\psi}\,\mathrm{i}\not{D}\psi \quad \longleftrightarrow$$



nonlinear CKT

$$O(\hbar)$$

$$\partial_\mu J^\mu \sim \vec{E}\cdot\vec{B} \quad \longleftrightarrow \quad \text{Berry monopole}$$

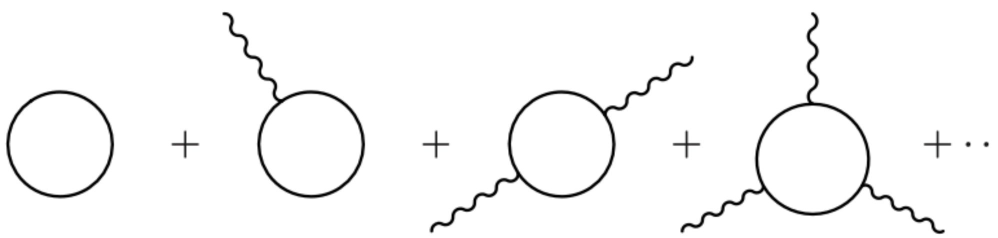
$$O(\hbar^2)$$

$$T^\mu{}_\mu \sim \beta(e)F_{\mu\nu}^2 \quad \longleftrightarrow \quad ???$$

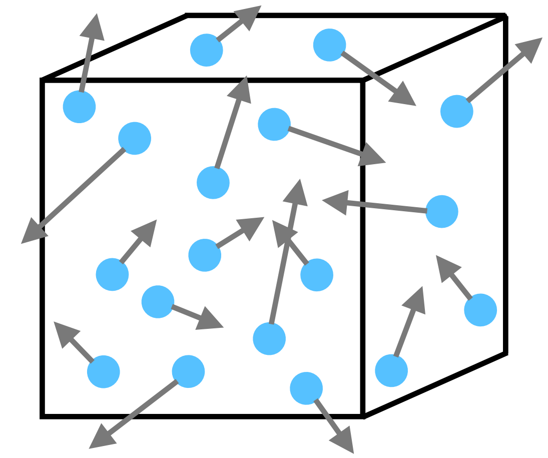
Limitation 2 : trace anomaly

$$\mathcal{L}_{\rm EH} = -\frac{1}{4}F_{\mu\nu}^2 + \mathcal{L}_{\rm fer}$$

$\shortparallel$



Euler-Heisenberg theory



nonlinear CKT

$$O(\hbar)$$

$$\partial_\mu J^\mu \sim \vec{E}\cdot\vec{B} \quad \longleftrightarrow \quad \text{Berry monopole}$$

$$O(\hbar^2)$$

$$T^\mu{}_\mu \sim \beta(e) F_{\mu\nu}^2 \quad \longleftrightarrow \quad ???$$

Quantum kinetic theory

- classical

$$n_{\boldsymbol{p}}(t, \boldsymbol{x}, \boldsymbol{p})$$ no uncertainty

- quantum field theory (relativistic)

$$W(x,p) = \int_y e^{-ip\cdot y/\hbar} \langle \bar{\psi}(x + \tfrac{y}{2}) \psi(x - \tfrac{y}{2}) \rangle$$

uncertainty Green's function

- right-handed current density

$$\mathcal{R}^\mu(x,p) = \mathrm{tr}\left[\gamma^\mu \frac{1+\gamma^5}{2} W(x,p)\right]$$

Quantum kinetic theory

- classical

$$n_{\boldsymbol{p}}(t, \underline{\boldsymbol{x}, \boldsymbol{p}})$$ no uncertainty

- quantum field theory (relativistic)

$$W(x,p) = \int_y \underline{\mathrm{e}^{-\mathrm{i}p\cdot y/\hbar}} \langle \bar{\psi}(x+\tfrac{y}{2})\psi(x-\tfrac{y}{2})\rangle$$

uncertainty   Green's function

- right-handed current density

$$\mathcal{R}^\mu(x,p) \sim (n_{\boldsymbol{p}}, \boldsymbol{v} n_{\boldsymbol{p}}) + O(\hbar)$$

- chiral kinetic theory

EoMs for $\mathcal{R}^\mu = \mathcal{R}^\mu_{(0)} + \hbar \mathcal{R}^\mu_{(1)} + \hbar^2 \mathcal{R}^\mu_{(2)}$

EoMs (zeroth order)

- number conservation (kinetic equation) $$(\partial_\mu - F_{\mu\nu}\partial_p^\nu)\mathcal{R}^\mu = 0$$
- scale invariance $$p_\mu \mathcal{R}^\mu = 0$$
- AM conservation $$p^\mu \mathcal{R}^\nu - p^\nu \mathcal{R}^\mu = 0$$

current $\mathcal{R}^\mu(x,p) \sim (n_{\boldsymbol{p}}, \boldsymbol{v} n_{\boldsymbol{p}}) + O(\hbar)$

EM tensor $\mathcal{R}^\mu p^\nu$

Zeroth-order Wigner function

- general solution

$$\mathcal{R}^{\mu}_{(0)} = 2\pi \underline{\delta(p^2)} p^{\mu} \underline{f_{(0)}(x,p)}$$

on-shell condition   distribution function

- current

$$J^0_{(0)} = \int_{\boldsymbol{p}} n_{\boldsymbol{p}} \qquad J^i_{(0)} = \int_{\boldsymbol{p}} \frac{p^i}{|\boldsymbol{p}|} n_{\boldsymbol{p}}$$

$$n_{\boldsymbol{p}} = f_{(0)}(p_0 = |\boldsymbol{p}|) - f_{(0)}(p_0 = -|\boldsymbol{p}|)$$

Linear-order Wigner function

- general solution Hidaka, Pu, Yang (2016)

$$\mathcal{R}^\mu_{(1)} = 2\pi\delta(p^2)\left[p^\mu f_{(1)} + \left(\Sigma_n^{\mu\nu}\Delta_\nu - \frac{1}{p^2}\tilde{F}^{\mu\nu}p_\nu\right)f_{(0)}\right]$$

- spin tensor

$$\Sigma_n^{\mu\nu} = \frac{\varepsilon^{\mu\nu\rho\sigma}p_\rho n_\sigma}{2p\cdot n}$$

Chen, Son, Stephanov (2015)

e.g. $n^\mu = (1, \mathbf{0})$

$$(\Sigma^{23}, \Sigma^{31}, \Sigma^{12}) = \frac{\boldsymbol{p}}{2p_0} \sim \frac{\hat{\boldsymbol{p}}}{2}$$

- $\mathcal{R}^\mu_{(1)}$ should be frame-independent

Nonlinear-order Wigner function

- general solution   KM (2023)

$$\mathcal{R}^{(2)}_\mu = 2\pi\delta(p^2)\left[p_\mu f_{(2)} + \left(\Sigma^u_{\mu\nu}\Delta^\nu - \frac{1}{p^2}\tilde{F}_{\mu\nu}p^\nu\right)f_{(1)} - \Sigma^u_{\mu\nu}\varepsilon^{\nu\rho\sigma\lambda}\Delta_\rho\frac{n_\sigma}{2p\cdot n}\Delta_\lambda f_{(0)}\right]$$
$$+\frac{2\pi}{p^2}\left[-p_\mu Q\cdot p + 2p^\nu Q_{[\mu}p_{\nu]}\right]f_{(0)}\delta(p^2)$$
$$+2\pi\frac{\delta(p^2)}{p^2}\left(\frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}p^\nu\Delta^\rho + \frac{p_\mu p^\nu}{p^2}\tilde{F}_{\nu\sigma} - \tilde{F}_{\mu\sigma}\right)\left(\Sigma^{\sigma\lambda}_n\Delta_\lambda - \frac{1}{p^2}\tilde{F}^{\sigma\lambda}p_\lambda\right)f_{(0)}$$
$$+2\pi\frac{\delta(p^2)}{p^2}\Sigma^u_{\mu\nu}\left[\Delta_\alpha\Sigma^{\alpha\nu}_n + \frac{n_\alpha}{p\cdot n}\tilde{F}^{\alpha\nu} + \frac{1}{p^2}\tilde{F}^{\nu\lambda}p_\lambda\right]p\cdot\Delta f_{(0)}$$

- another frame vector? u^μ Hayata, Hidaka, KM (2021)

- $\mathcal{R}^\mu_{(2)}$ should be frame-independent

Equilibrium from Boltzmann Theory

$$f^{\mathrm{eq}} = f^{\mathrm{eq}}(g)$$

momentum

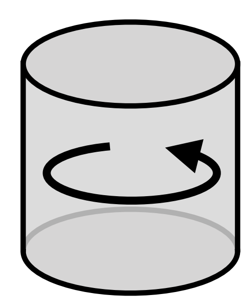
$$g = \underbrace{-\beta\mu}_{\text{charge}} + \beta\cdot p + \frac{\hbar}{2}\Sigma_n^{\mu\nu}\partial_\mu\beta_\nu$$

charge

total angular momentum

$$\beta^\mu = \beta(1, \boldsymbol{x}\times\boldsymbol{\omega})$$

e.g. $n^\mu = (1, \mathbf{0})$



$$g = \beta(p_0 - \mu - \boldsymbol{\omega}\cdot(\boldsymbol{x}\times\boldsymbol{p}) - \hbar\boldsymbol{\omega}\cdot\hat{\boldsymbol{p}}/2)$$

Equilibrium from Boltzmann Theory

$$f^{\mathrm{eq}} = f^{\mathrm{eq}}(g)$$

$$g = \underbrace{-\beta\mu}_{\text{charge}} + \underbrace{\beta \cdot p}_{\text{momentum}} + \frac{\hbar}{2}\Sigma_n^{\mu\nu}\partial_\mu\beta_\nu$$

charge — momentum — total angular momentum

↓

- ✓ $\mathcal{R}^{\mu}_{\mathrm{eq}}$ frame independent
- ✓ kinetic eq. satisfied

$$(\partial_\mu - F_{\mu\nu}\partial_p^\nu)\mathcal{R}^{\mu}_{\mathrm{eq}} = 0$$

Linear-order equilibrium Wigner function

$$O(\partial) \sim O(\hbar)$$

- one derivative   Hidaka, Pu, Yang (2016)

$$\mathcal{R}_\mu^{(1)} = \mathcal{R}_\mu^{(F)} + \mathcal{R}_\mu^{(\omega)}$$

- momentum integral at equilibrium

e.g. $$J^\mu_{(1)} = \frac{\mu}{4\pi^2} B^\mu + \left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12} \right) \omega^\mu$$

CME   CVE

- Nondissipative due to quantum anomaly

Equilibrium at nonlinear order

linear

$$f^{\rm eq} = f^{\rm eq}(g)$$

$$g = -\beta\mu + \beta\cdot p + \frac{\hbar}{2}\Sigma_n^{\mu\nu}\partial_\mu\beta_\nu$$

- ✓ $\mathcal{R}^\mu_{\rm eq}$ frame independent
- ✓ kinetic eq. satisfied

nonlinear

$$f^{\rm eq} = f^{\rm eq}(g)$$

minimal conditions

Equilibrium at nonlinear order

nonlinear-order f^{eq} exists only when

$$\partial_\mu(\beta\mu) + F_{\mu\nu}\beta^\nu = 0, \quad \partial_\mu\beta_\nu + \partial_\nu\beta_\mu = 0$$

with

$$1) \quad \beta\cdot\partial F_{\mu\nu} = 0, \qquad \beta\cdot\partial\beta_\mu = 0$$

or

$$2) \quad \partial_\lambda F_{\mu\nu} = 0$$

Yang et al. (2020)

Nonlinear-order equilibrium Wigner function

$$O(\partial) \sim O(\hbar)$$

- two derivatives Yang et al. (2020) KM (2023)

$$\mathcal{R}_\mu^{(2)} = \mathcal{R}_\mu^{(\partial F)} + \mathcal{R}_\mu^{(FF)} + \mathcal{R}_\mu^{(F\omega)} + \mathcal{R}_\mu^{(\omega\omega)}$$

- momentum integral at equilibrium

e.g. $$J^0_{(F\omega)} = -\frac{1}{8\pi^2} B \cdot \omega$$

- Nondissipative due to quantum anomaly

conformed from quantum field theory

UV divergence

- $f_{(0)}(x,p)$ contains vacuum contribution

$$\mathcal{R}^{\mu}_{(0)} = 2\pi\delta(p^2)p^{\mu}f_{(0)}(x,p)$$

- vacuum contribution to current

$$J_{(\partial F)} \sim \partial F \times \int_{\boldsymbol{p}} |\boldsymbol{p}|^{-3}\left[n_F(|\boldsymbol{p}|-\mu)+n_F(|\boldsymbol{p}|+\mu)-1\right]$$

+3 = +3 +0

- IR safe

$$n_F(|\boldsymbol{p}|-\mu)+n_F(|\boldsymbol{p}|+\mu) \longrightarrow 1$$

- UV logarithmic divergence

$$n_F(|\boldsymbol{p}|-\mu)+n_F(|\boldsymbol{p}|+\mu) \longrightarrow 0$$

Regularization problem

- Pauli-Villars $$\frac{1}{k^2 - m^2} \to \frac{1}{k^2 - m^2} - \frac{1}{k^2 - \Lambda^2}$$

inapplicable: no mass parameter in CKT

Yang et al. (2020)

- 't Hooft-Veltman (dimensional regularization) $$\int \mathrm{d}^4 k \to \int \mathrm{d}^{4-\epsilon} k$$

inapplicable: $\mathcal{R}^\mu = \mathrm{tr}\left[\gamma^\mu \frac{1+\gamma^5}{2} W\right]$ incorrect in $d \neq 4$

$$\gamma^\mu = (\gamma^0, \cdots, \gamma^3, \gamma^4, \cdots \gamma^{d-1})$$ commute with γ^5

Point-Splitting regularization in CKT

- charge current

$$J^\mu(x,y) = \mathrm{tr}\left\langle \bar\psi(x+\tfrac{y}{2})\gamma^\mu \frac{1+\gamma^5}{2}\psi(x-\tfrac{y}{2})\right\rangle = \int_p \mathrm{e}^{\mathrm{i}p\cdot y/\hbar}\mathcal{R}^\mu(x,p)$$

- UV divergence

$$\mathcal{J} = \int_0^{y^{-1}} \frac{\mathrm{d}p}{p}$$

$$p \sim \infty \longleftrightarrow y \sim 0$$

- regularized vacuum part

$$J^\mu_{(\partial F)\,\mathrm{vac}} = \frac{\mathcal{J}}{12\pi^2}\partial_\lambda F^{\mu\lambda}$$

+3 = +0 +3

$$T^{\mu\nu}_{(FF)\,\mathrm{vac}} = -\frac{\mathcal{J}}{12\pi^2}\left[F^\mu{}_\sigma F^{\nu\sigma} - \frac{1}{4}g^{\mu\nu}F^2_{\alpha\beta}\right]$$

+4 = +0 +4

Compensation of PS regularization

- gauge invariance unbroken

$$\partial_\mu J^\mu_{(2)} = 0$$

- translational invariance broken

$$\partial_\mu T^{\mu\nu}_{(2)} + F^{\mu\nu} J^{(2)}_\mu \neq 0$$

- conformal invariance unbroken

$$T^\mu{}_{\mu(2)} = 0$$

because of ~~classical EM fields~~? **regularization**

Yang et al. (2020)

Euler-Heisenberg theory

- Effective Lagrangian

$$\mathcal{L}_{\mathrm{EH}} = -\mathcal{F} - \frac{e^2}{8\pi^2}\int_{s_0}^{\infty}\frac{\mathrm{d}s}{s}\,\mathrm{e}^{-sm^2}\,\frac{\mathrm{Re}\cosh\left[\hbar\, es\sqrt{2(\mathcal{F}+\mathrm{i}\mathcal{G})}\right]}{\mathrm{Im}\cosh\left[\hbar\, es\sqrt{2(\mathcal{F}+\mathrm{i}\mathcal{G})}\right]}\,\mathcal{G}$$

$$\mathcal{F} := F_{\alpha\beta}^2/4,\ \mathcal{G} := F^{\alpha\beta}\tilde{F}_{\alpha\beta}/4$$

Heisenberg, Euler (1936)
Schwinger (1950)

IR regulator

$$\sim n_F(p-\mu) + n_F(p+\mu)$$

Euler-Heisenberg theory

- Effective Lagrangian

$$\mathcal{L}_{\mathrm{EH}} = -\mathcal{F} - \frac{e^2}{8\pi^2}\int_{s_0}^{\infty}\frac{\mathrm{d}s}{s}\,\mathrm{e}^{-sm^2}\frac{\mathrm{Re}\cosh\left[\hbar es\sqrt{2(\mathcal{F}+\mathrm{i}\mathcal{G})}\right]}{\mathrm{Im}\cosh\left[\hbar es\sqrt{2(\mathcal{F}+\mathrm{i}\mathcal{G})}\right]}\mathcal{G}$$

$$\mathcal{F} := F_{\alpha\beta}^2/4,\ \mathcal{G} := F^{\alpha\beta}\tilde{F}_{\alpha\beta}/4$$

Heisenberg, Euler (1936)
Schwinger (1950)

UV regulator $\sim y^2$

- current & EM tensor

$$J^{\mu}_{\mathrm{EH}(2)}\Big|_{m\to 0} = \frac{\mathcal{J}}{6\pi^2}\partial_\lambda F^{\mu\lambda} = 2J^{\mu}_{(\partial F)\,\mathrm{vac}}$$

chirality

$$T^{\mu\nu}_{\mathrm{EH}(2)}\Big|_{m\to 0} = -\frac{\mathcal{J}}{6\pi^2}\left[F^{\mu}{}_{\sigma}F^{\nu\sigma} - \frac{1}{4}g^{\mu\nu}F_{\alpha\beta}^2\right] = 2T^{\mu\nu}_{(FF)\,\mathrm{vac}}$$

nonlinear CKT is consistent with EH theory

Physics in logarithm

- effective charge

$$\mathcal{L}_{\rm EH} = -\frac{1}{4}F_{\mu\nu}^2 \left(1 + \frac{e^2}{12\pi^2}\log\frac{s_0^{-1}}{m^2}\right) + \text{const.} + O\left(F^4\right)$$

$$= e_{\rm eff}^2(s_0)/e^2$$

Schwartz (2014)

EH theory

$$\log s_0^{-1/2}$$

nonlinear CKT

$$\log y^{-1}$$

charge renormalization

$$\beta(e_{\rm eff}) = M\frac{\mathrm{d}e_{\rm eff}(M)}{\mathrm{d}M} = \frac{e^3}{12\pi^2}$$

trace anomaly

$$T^{\mu}{}_{\mu} = \frac{\beta}{2e^2}F_{\mu\nu}^2 = \frac{e^2}{24\pi^2}F_{\mu\nu}^2$$

Summary

- Formulation of nonlinear CKT

- Verification for the nondissipativeness of $J^0 \sim \vec{B} \cdot \vec{\omega}$

- Regularization problem

  PV : no　　　DR : no　　　PS : relatively OK

- Consistency with Euler-Heisenberg theory

  $\log y^{-1}$ ⟷ charge renormalization, trace anomaly

- Potential developments

  meaning of another frame vector?

  nonlinear transport by Berry dipole Sodemann, Fu (2015)

  collisional effects Kadanoff, Baym (1962)

  Chiral plasma instability from dynamical gauge fields Akamatsu, Yamamoto (2013)