

Exotic hadrons with charm and bottom flavors:

X, Y, Z, P_c, T_{cc} and hadron interaction

- \bar{D} meson and nucleon interaction and charm nuclei -

Y. Yamaguchi, S. Y., A. Hosaka, Phys. Rev. D106, 094001 (2022)

Shigehiro YASUI

∈ Sasaki Lab. ⊂ SKCM² ⊂ Hiroshima University



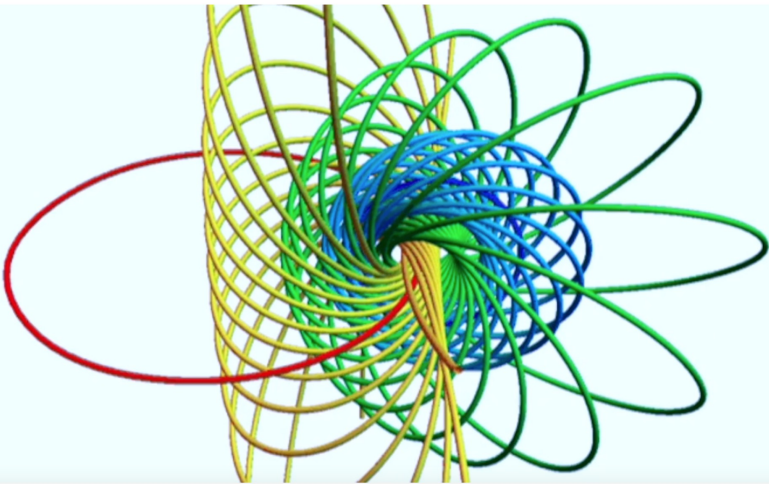
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World Premier International Research Center Initiative/WPI at Hiroshima University

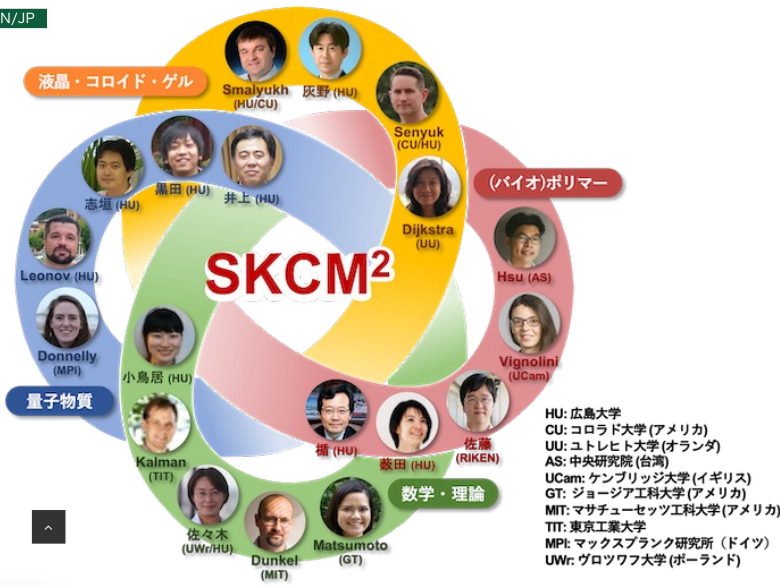


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Building a sustainable world.
knot by knot
The International Institute for Sustainability with Knotted Chiral Meta Matter



- ✓ Cross-pollinates mathematical knot theory and chirality knowledge across disciplines and scales
- ✓ Creation of designable artificial knot-like particles that exhibit highly unusual and technologically useful properties

Hadron & nuclear physics group

PI: Kenta SHIGAKI (HU, ALICE member)

PI: Chihiro SASAKI (HU, Uni. of Wroclaw)

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coPI: Muneto NITTA (HU, Keio Uni.)

Charm/bottom exotic hadrons



Contents

0. Introduction to exotic hadrons
1. Why \bar{D} meson and nucleon?
2. \bar{D} meson and nucleon potential
3. B meson and nucleon potential
4. Summary

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0. Introduction to exotic hadrons

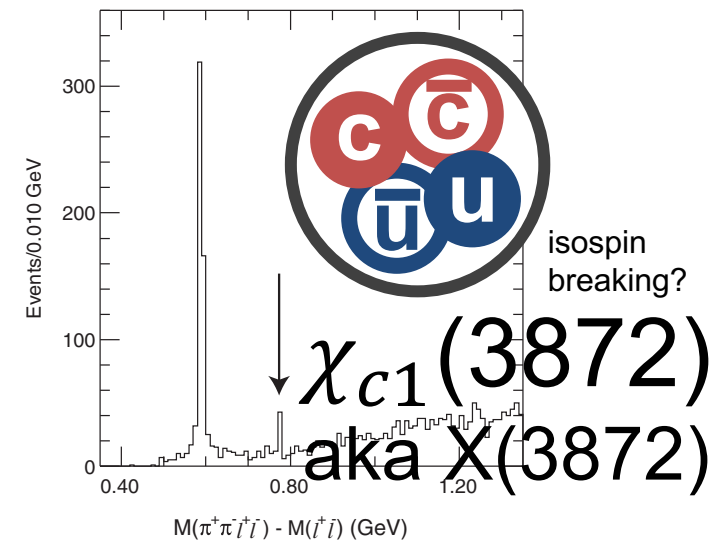
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0. Introduction to exotic hadrons

S. L. Olsen, T. Skwamicki, D. Ziemninska,
 Rev. Mod. Phys. 90, 015003 (2018)

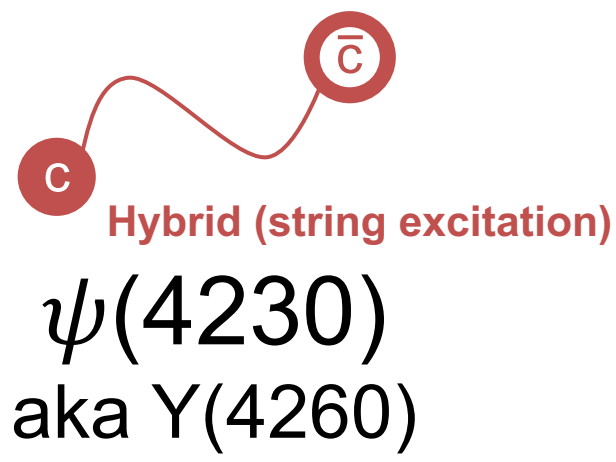
State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)	Experiment
$X(3872)$	3871.69 ± 0.17	< 1.2	1^{++}	$B \rightarrow K(J/\psi \pi^+ \pi^-)$ $p\bar{p} \rightarrow (J/\psi \pi^+ \pi^-) + \dots$ $B \rightarrow K(J/\psi \pi^+ \pi^- \pi^0)$ $B \rightarrow K(D^0 \bar{D}^0 \pi^0)$ $B \rightarrow K(J/\psi \gamma)$ $B \rightarrow K(\psi' \gamma)$ $pp \rightarrow (J/\psi \pi^+ \pi^-) + \dots$ $e^+ e^- \rightarrow \gamma(J/\psi \pi^+ \pi^-)$	Belle (Choi <i>et al.</i> , 2003, 2011), BABAR (Aubert <i>et al.</i> , 2005c), LHCb (Aaij <i>et al.</i> , 2013a, 2015d) CDF (Acosta <i>et al.</i> , 2004; Abulencia <i>et al.</i> , 2006; Aaltonen <i>et al.</i> , 2009b), D0 (Abazov <i>et al.</i> , 2004) Belle (Abe <i>et al.</i> , 2005), BABAR (del Amo Sanchez <i>et al.</i> , 2010a) Belle (Gokhroo <i>et al.</i> , 2006; Aushev <i>et al.</i> , 2010b), BABAR (Aubert <i>et al.</i> , 2008c) BABAR (del Amo Sanchez <i>et al.</i> , 2010a), Belle (Bhardwaj <i>et al.</i> , 2011), LHCb (Aaij <i>et al.</i> , 2012a) BABAR (Aubert <i>et al.</i> , 2009b), Belle (Bhardwaj <i>et al.</i> , 2011), LHCb (Aaij <i>et al.</i> , 2014a) LHCb (Aaij <i>et al.</i> , 2012a), CMS (Chatrchyan <i>et al.</i> , 2013a), ATLAS (Aaboud <i>et al.</i> , 2017) BESIII (Ablikim <i>et al.</i> , 2014d)
$X(3915)$	3918.4 ± 1.9	20 ± 5	0^{++}	$B \rightarrow K(J/\psi \omega)$	Belle (Choi <i>et al.</i> , 2005), BABAR (Aubert <i>et al.</i> , 2008b; del Amo Sanchez <i>et al.</i> , 2010a) Belle (Uehara <i>et al.</i> , 2010), BABAR (Lees <i>et al.</i> , 2012c)
$X(3940)$	3942_{-8}^{+9}	37_{-17}^{+27}	$0^{-+} (?)$	$e^+ e^- \rightarrow J/\psi(D^* \bar{D})$ $e^+ e^- \rightarrow J/\psi(\dots)$	Belle (Pakhlov <i>et al.</i> , 2008) Belle (Abe <i>et al.</i> , 2007)
$X(4140)$	$4146.5_{-5.3}^{+6.4}$	83_{-25}^{+27}	1^{++}	$B \rightarrow K(J/\psi \phi)$ $p\bar{p} \rightarrow (J/\psi \phi) + \dots$	CDF (Aaltonen <i>et al.</i> , 2009a), CMS (Chatrchyan <i>et al.</i> , 2014), D0 (Abazov <i>et al.</i> , 2014), LHCb (Aaij <i>et al.</i> , 2017a, 2017d) D0 (Abazov <i>et al.</i> , 2015)
$X(4160)$	4156_{-25}^{+29}	139_{-65}^{+113}	$0^{-+} (?)$	$e^+ e^- \rightarrow J/\psi(D^* \bar{D}^*)$	Belle (Pakhlov <i>et al.</i> , 2008)
$Y(4260)$	See $Y(4220)$ entry		1^{--}	$e^+ e^- \rightarrow \gamma(J/\psi \pi^+ \pi^-)$	BABAR (Aubert <i>et al.</i> , 2005a; Lees <i>et al.</i> , 2012b), CLEO (He <i>et al.</i> , 2006), Belle (Yuan <i>et al.</i> , 2007; Liu <i>et al.</i> , 2013)
$Y(4220)$	4222 ± 3	48 ± 7	1^{--}	$e^+ e^- \rightarrow (J/\psi \pi^+ \pi^-)$ $e^+ e^- \rightarrow (h_c \pi^+ \pi^-)$ $e^+ e^- \rightarrow (\chi_{c0} \omega)$ $e^+ e^- \rightarrow (J/\psi \eta)$ $e^+ e^- \rightarrow (\gamma X(3872))$ $e^+ e^- \rightarrow (\pi^- Z_c^+(3900))$ $e^+ e^- \rightarrow (\pi^- Z_c^+(4020))$	BESIII (Ablikim <i>et al.</i> , 2017c) BESIII (Ablikim <i>et al.</i> , 2017a) BESIII (Ablikim <i>et al.</i> , 2015g) BESIII (Ablikim <i>et al.</i> , 2015c) BESIII (Ablikim <i>et al.</i> , 2014d) BESIII (Ablikim <i>et al.</i> , 2013a), Belle (Liu <i>et al.</i> , 2013) BESIII (Ablikim <i>et al.</i> , 2013b)
$X(4274)$	4273_{-9}^{+19}	56_{-16}^{+14}	1^{++}	$B \rightarrow K(J/\psi \phi)$	CDF (Aaltonen <i>et al.</i> , 2017), CMS (Chatrchyan <i>et al.</i> , 2014), LHCb (Aaij <i>et al.</i> , 2017a, 2017d)
$X(4350)$	$4350.6_{-5.1}^{+4.6}$	$13.3_{-10.0}^{+18.4}$	$(0/2)^{++}$	$e^+ e^- \rightarrow e^+ e^- (J/\psi \phi)$	Belle (Shen <i>et al.</i> , 2010)
$Y(4360)$	4341 ± 8	102 ± 9	1^{--}	$e^+ e^- \rightarrow \gamma(\psi' \pi^+ \pi^-)$ $e^+ e^- \rightarrow (J/\psi \pi^+ \pi^-)$	BABAR (Aubert <i>et al.</i> , 2007; Lees <i>et al.</i> , 2014), Belle (Wang <i>et al.</i> , 2007, 2015) BESIII (Ablikim <i>et al.</i> , 2017c)
$Y(4390)$	4392 ± 6	140 ± 16	1^{--}	$e^+ e^- \rightarrow (h_c \pi^+ \pi^-)$	BESIII (Ablikim <i>et al.</i> , 2017a)
$X(4500)$	4506_{-21}^{+16}	92_{-30}^{+30}	0^{++}	$B \rightarrow K(J/\psi \phi)$	LHCb (Aaij <i>et al.</i> , 2017a, 2017d)
$X(4700)$	4704_{-45}^{+17}	120_{-45}^{+52}	0^{++}	$B \rightarrow K(J/\psi \phi)$	LHCb (Aaij <i>et al.</i> , 2017a, 2017d)
$Y(4660)$	4643 ± 9	72 ± 11	1^{--}	$e^+ e^- \rightarrow \gamma(\psi' \pi^+ \pi^-)$ $e^+ e^- \rightarrow \gamma(\Lambda_c^+ \Lambda_c^-)$	Belle (Wang <i>et al.</i> , 2007, 2015), BABAR (Aubert <i>et al.</i> , 2007; Lees <i>et al.</i> , 2014) Belle (Pakhlova <i>et al.</i> , 2008)

← Firstly discovered tetraquark



S. K. Choi et al. [Belle Collaboration],
 Phys. Rev. Lett. 91, 262001 (2003)

← Hybrid mesons (gluon excitation)



0. Introduction to exotic hadrons

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Rev. Mod. Phys. 90, 015003 (2018)

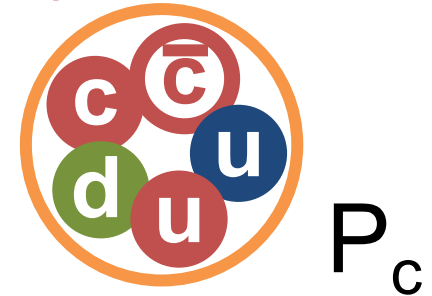
State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)	Experiment
$Z_c^{+0}(3900)$	3886.6 ± 2.4	28.1 ± 2.6	1^{+-}	$e^+e^- \rightarrow \pi^-^0(J/\psi\pi^+)^0$ $e^+e^- \rightarrow \pi^-^0(D\bar{D}^*)^{+0}$	BESIII (Ablikim <i>et al.</i> , 2013a, 2015f), Belle (Liu <i>et al.</i> , 2013) BESIII (Ablikim <i>et al.</i> , 2014b, 2015e)
$Z_c^{+0}(4020)$	4024.1 ± 1.9	13 ± 5	$1^{+-} (?)$	$e^+e^- \rightarrow \pi^-^0(h_c\pi^+)^0$ $e^+e^- \rightarrow \pi^-^0(D^*\bar{D}^*)^{+0}$	BESIII (Ablikim <i>et al.</i> , 2013b, 2014c) BESIII (Ablikim <i>et al.</i> , 2014a, 2015d)
$Z^+(4050)$	4051_{-43}^{+24}	82_{-55}^{+51}	$?^{2+}$	$B \rightarrow K(\chi_{c1}\pi^+)$	Belle (Mizuk <i>et al.</i> , 2008), BABAR (Lees <i>et al.</i> , 2012a)
$Z^+(4200)$	4196_{-32}^{+35}	370_{-149}^{+99}	1^+	$B \rightarrow K(J/\psi\pi^+)$ $B \rightarrow K(\psi'\pi^+)$	Belle (Chilikin <i>et al.</i> , 2014) LHCb (Aaij <i>et al.</i> , 2014b)
$Z^+(4250)$	4248_{-45}^{+185}	177_{-72}^{+321}	$?^{2+}$	$B \rightarrow K(\chi_{c1}\pi^+)$	Belle (Mizuk <i>et al.</i> , 2008), BABAR (Lees <i>et al.</i> , 2012a)
$Z^+(4430)$	4477 ± 20	181 ± 31	1^+	$B \rightarrow K(\psi'\pi^+)$ $B \rightarrow K(J\psi\pi^+)$	Belle (Choi <i>et al.</i> , 2008; Mizuk <i>et al.</i> , 2009), Belle (Chilikin <i>et al.</i> , 2013), LHCb (Aaij <i>et al.</i> , 2014b, 2015b) Belle (Chilikin <i>et al.</i> , 2014)
$P_c^+(4380)$	4380 ± 30	205 ± 88	$(\frac{3}{2} / \frac{5}{2})^\mp$	$\Lambda_b^0 \rightarrow K(J/\psi p)$	LHCb (Aaij <i>et al.</i> , 2015c)
$P_c^+(4450)$	4450 ± 3	39 ± 20	$(\frac{5}{2} / \frac{3}{2})^\pm$	$\Lambda_b^0 \rightarrow K(J/\psi p)$	LHCb (Aaij <i>et al.</i> , 2015c)
$Y_b(10860)$	$10891.1_{-3.8}^{+3.4}$	$53.7_{-7.8}^{+7.2}$	1^{--}	$e^+e^- \rightarrow (\Upsilon(nS)\pi^+\pi^-)$	Belle (Chen <i>et al.</i> , 2008; Santel <i>et al.</i> , 2016)
$Z_b^{+0}(10610)$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$Y_b(10860) \rightarrow \pi^-^0(\Upsilon(nS)\pi^+)^0$ $Y_b(10860) \rightarrow \pi^-(h_b(nP)\pi^+)$ $Y_b(10860) \rightarrow \pi^-(B\bar{B}^*)^+$	Belle (Bondar <i>et al.</i> , 2012; Garmash <i>et al.</i> , 2015), Belle (Krokovny <i>et al.</i> , 2013) Belle (Bondar <i>et al.</i> , 2012) Belle (Garmash <i>et al.</i> , 2016)
$Z_b^+(10650)$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$Y_b(10860) \rightarrow \pi^-(\Upsilon(nS)\pi^+)$ $Y_b(10860) \rightarrow \pi^-(h_b(nP)\pi^+)$ $Y_b(10860) \rightarrow \pi^-(B^*\bar{B}^*)^+$	Belle (Bondar <i>et al.</i> , 2012; Garmash <i>et al.</i> , 2015) Belle (Bondar <i>et al.</i> , 2012) Belle (Garmash <i>et al.</i> , 2016)

← **Genuine tetraquark**



Electrically charged state (+)

← **Pentaquark**



Is that all ?

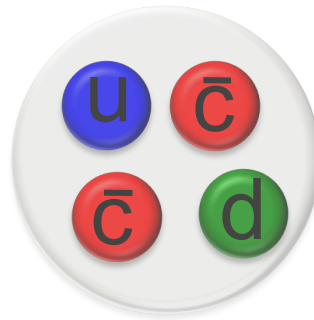
New type discovered!

0. Introduction to exotic hadrons

J.P. Ader, J.M. Richard and P. Taxil,
Phys. Rev. D25, 2370 (1982)

T_{cc}

Double charm tetraquark



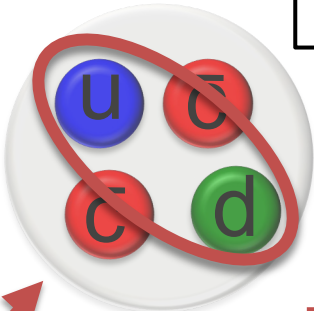
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J.P. Ader, J.M. Richard and P. Taxil,
Phys. Rev. D25, 2370 (1982)

T_{cc}
Double charm tetraquark

- Diquark
- Color confinement

almost uniquely
determined
 $I(J^P)=0(1^+)$



strong **ud**
attraction



strong decay?
(fall-apart)

T_{cc} can be stable!

Gluon exchange force induces color-spin interaction

$$H_{int} = \sum_{i>j} \frac{C_H}{m_i m_j} \vec{s}_i \cdot \vec{s}_j \quad C_H = v_0 \vec{\lambda}_i \cdot \vec{\lambda}_j \langle \delta(r_{ij}) \rangle$$

ud pair ~ 1 dominant attraction ($\bar{3}_c, I=0, {}^1S_0$)

$\bar{c}u$ pair $\sim 1/m_c$ suppressed

$\bar{c}\bar{c}$ pair $\sim 1/m_c^2$ more suppressed ($\bar{3}_c, {}^3S_1$)

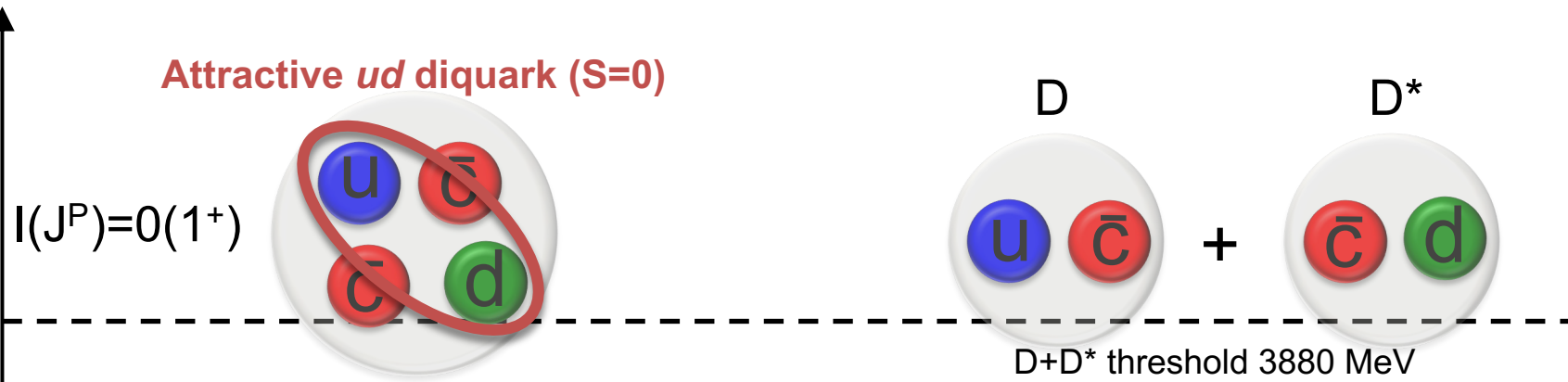
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T_{cc}

Double charm tetraquark

J.P. Ader, J.M. Richard and P. Taxil,
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mass



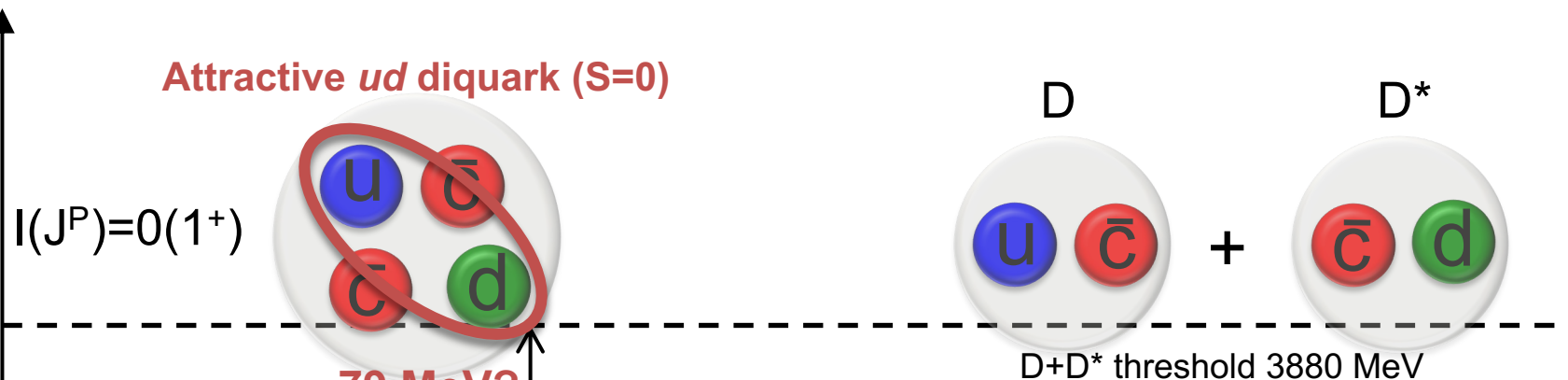
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T_{cc}

Double charm tetraquark

J.P. Ader, J.M. Richard and P. Taxil,
Phys. Rev. D25, 2370 (1982)

mass



T_{cc}^1	$ud\bar{c}\bar{c}$	$us\bar{c}\bar{c}$	$ds\bar{c}\bar{c}$
	-79.3	-8.7	-8.7
	$D^0 + D^{*-}, \bar{D}^{*0} + D^-$	$\bar{D}^0 + D_s^{*-}$	$D^- + D_s^{*-}$
T_{bb}^1	$ud\bar{b}\bar{b}$	$us\bar{b}\bar{b}$	$ds\bar{b}\bar{b}$
	-124.3	-62.3	-62.3
	$B^+ + B^{*0}, B^{*+} + B^0$	$B^+ + B_s^{*0}$	$B^0 + B_s^{*0}$
T_{cb}^1	$ud\bar{c}\bar{b}$	$us\bar{c}\bar{b}$	$ds\bar{c}\bar{b}$
	-59.0	$+2.9$	$+2.9$
	$B^{*+} + D^-, B^{*0} + \bar{D}^0$	$B_s^{*0} + D^0$	$B_s^{*0} + D^-$

weak decay:
 $D \rightarrow K\pi$

$D^{*-} K^+ \pi^-$

S.-H. Lee, S. Yasui, Eur. Phys. J. C64,283 (2009)
S.-H. Lee, S. Yasui, W. Liu, C.-M. Ko, Eur. Phys. J. C54, 259 (2008)

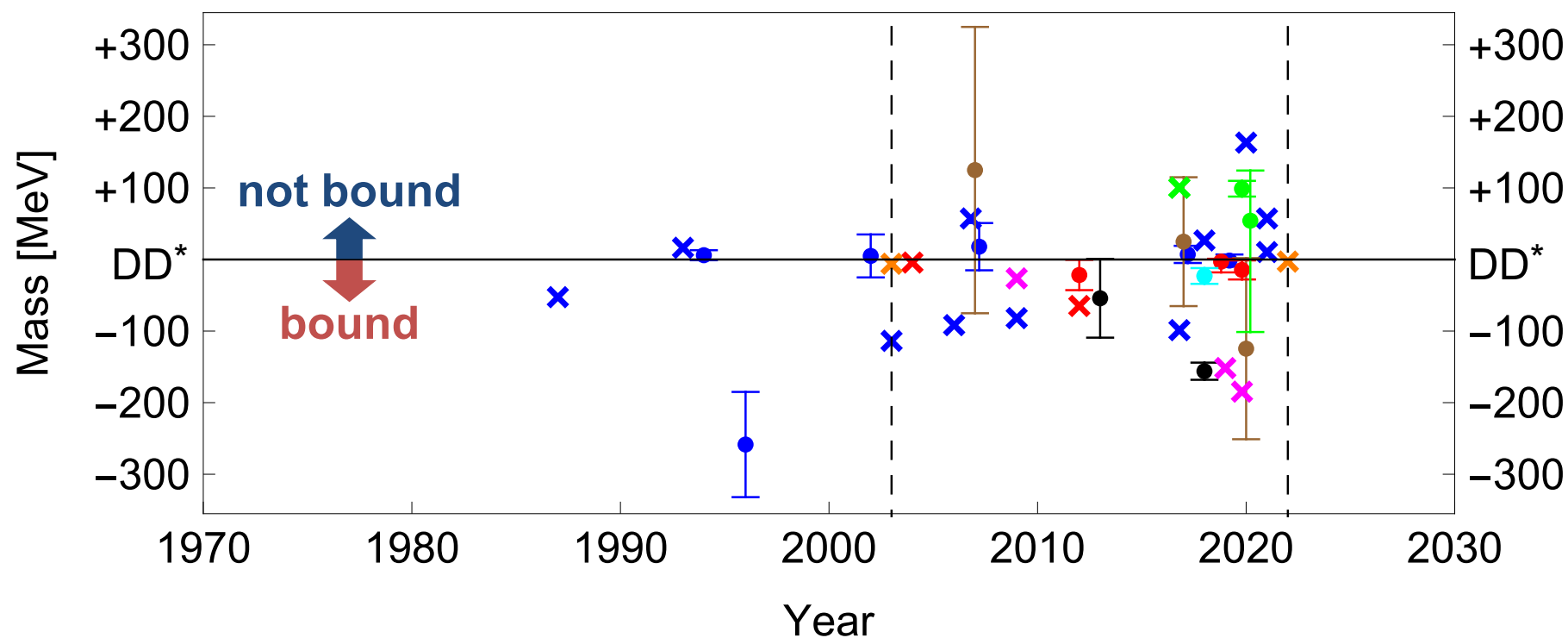
0. Introduction to exotic hadrons

Recent review:

H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu,
S.-L. Zhu, 2204.02649 [hep-ph]

T_{cc}
Double charm tetraquark

Summary of theoretical studies (chronological table)



bound or not bound ?

0. Introduction to exotic hadrons

T_{cc} Double charm tetraquark

PRL **119**, 202002 (2017)

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Heavy-Quark Symmetry Implies Stable Heavy Tetraquark Mesons $Q_i Q_j \bar{q}_k \bar{q}_l$

Estia J. Eichten* and Chris Quigg†

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(Received 8 August 2017; published 15 November 2017)

~100 MeV above DD^* threshold

State	J^P	j_e	$m(Q_i Q_j q_m)$ (c.g.)	HQS relation	$m(Q_i Q_j \bar{q}_k \bar{q}_l)$	Decay channel	\mathcal{Q} (MeV)
$\{cc\}[\bar{u}\bar{d}]$	1^+	0	3663 ^b	$m(\{cc\}u) + 315$	3978	$D^+ D^{*0}$ 3876	102
$\{cc\}[\bar{q}_k \bar{s}]$	1^+	0	3764 ^c	$m(\{cc\}s) + 392$	4156	$D^+ D_s^{*-}$ 3977	179
$\{cc\}\{\bar{q}_k \bar{q}_l\}$	$0^+, 1^+, 2^+$	1	3663	$m(\{cc\}u) + 526$	4146, 4167, 4210	$D^+ D^0, D^+ D^{*0}$ 3734, 3876	412, 292, 476
$[bc][\bar{u}\bar{d}]$	0^+	0	6914	$m([bc]u) + 315$	7229	$B^- D^+ / B^0 D^0$ 7146	83
$[bc][\bar{q}_k \bar{s}]$	0^+	0	7010 ^d	$m([bc]s) + 392$	7406	$B_s D$ 7236	170
$[bc]\{\bar{q}_k \bar{q}_l\}$	1^+	1	6914	$m([bc]u) + 526$	7439	$B^* D / B D^*$ 7190/7290	249
$\{bc\}[\bar{u}\bar{d}]$	1^+	0	6957	$m(\{bc\}u) + 315$	7272	$B^* D / B D^*$ 7190/7290	82
$\{bc\}[\bar{q}_k \bar{s}]$	1^+	0	7053 ^d	$m(\{bc\}s) + 392$	7445	$D B_s^*$ 7282	163
$\{bc\}\{\bar{q}_k \bar{q}_l\}$	$0^+, 1^+, 2^+$	1	6957	$m(\{bc\}u) + 526$	7461, 7472, 7493	$B D / B^* D$ 7146/7190	317, 282, 349
$\{bb\}[\bar{u}\bar{d}]$	1^+	0	10 176	$m(\{bb\}u) + 306$	10 482	$B^- \bar{B}^{*0}$ 10 603	-121
$\{bb\}[\bar{q}_k \bar{s}]$	1^+	0	10 252 ^c	$m(\{bb\}s) + 391$	10 643	$\bar{B} \bar{B}_s^* / \bar{B}_s \bar{B}^*$ 10 695/10 691	-48
$\{bb\}\{\bar{q}_k \bar{q}_l\}$	$0^+, 1^+, 2^+$	1	10 176	$m(\{bb\}u) + 512$	10 674, 10 681, 10 695	$B^- B^0, B^- B^{*0}$ 10 559, 10 603	115, 78, 136

No bound state for T_{cc} ?

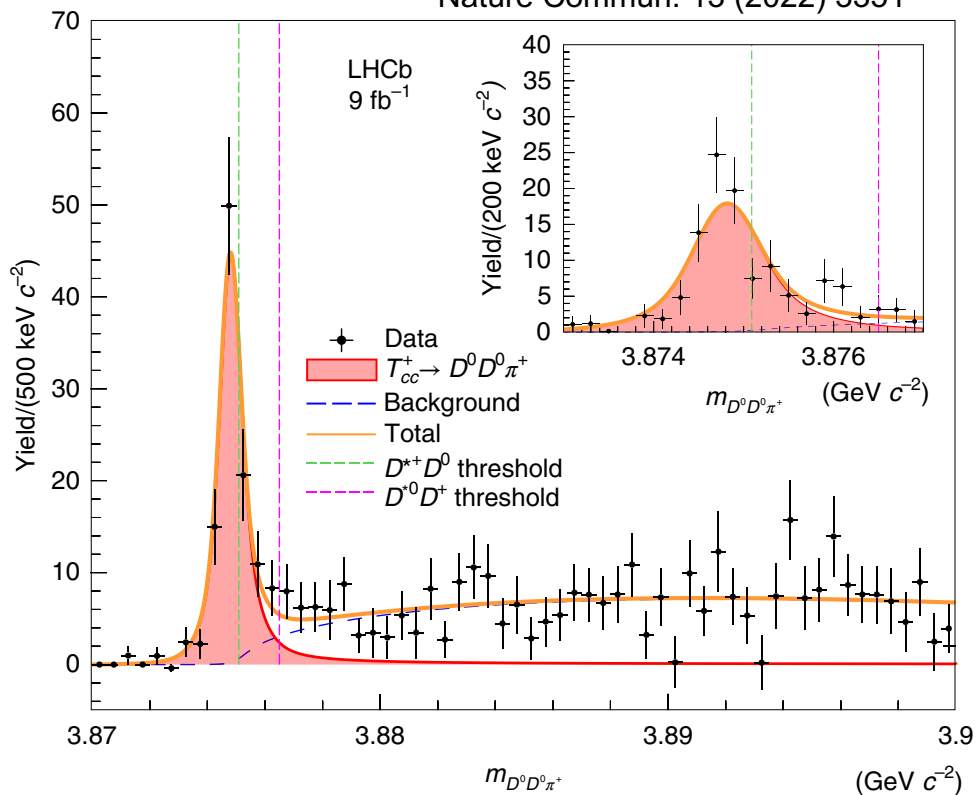
0. Introduction to exotic hadrons

OPEN

Observation of an exotic narrow doubly charmed tetraquark

LHCb Collaboration*

LHCb, Nature Phys. 18 (2022) 751,
Nature Commun. 13 (2022) 3351



Bound state below $D^{*+} D^0$ threshold

$$\delta m_{\text{BW}} = -273 \pm 61 \pm 5_{-14}^{+11} \text{ keV } c^{-2},$$

$$\Gamma_{\text{BW}} = 410 \pm 165 \pm 43_{-38}^{+18} \text{ keV},$$

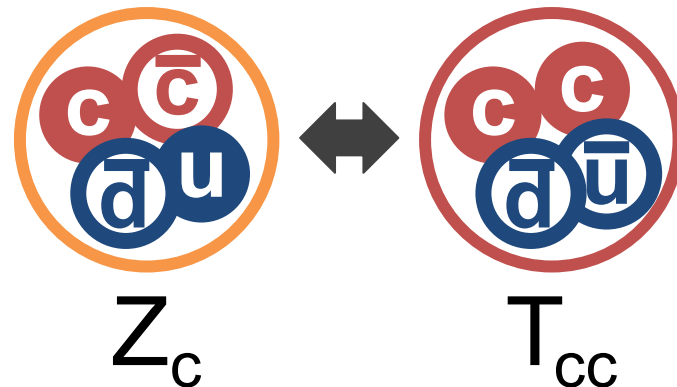
Note: Above
 $D^0 D^0 \pi^+$ threshold
 3869.25 MeV:
 "resonance(?)"

New type of exotics!

T_{cc} : doubly charmed tetraquark

$$|C| = 0$$

$$|C| = 2$$



T_{cc} has four quarks at least:
 genuinely exotic.

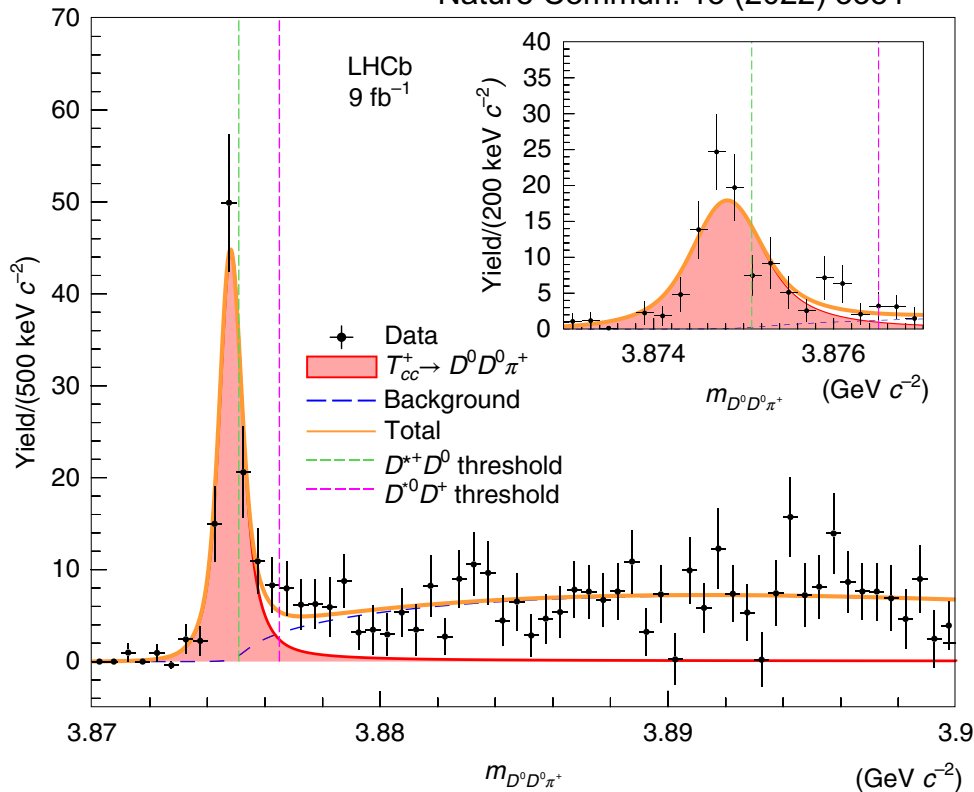
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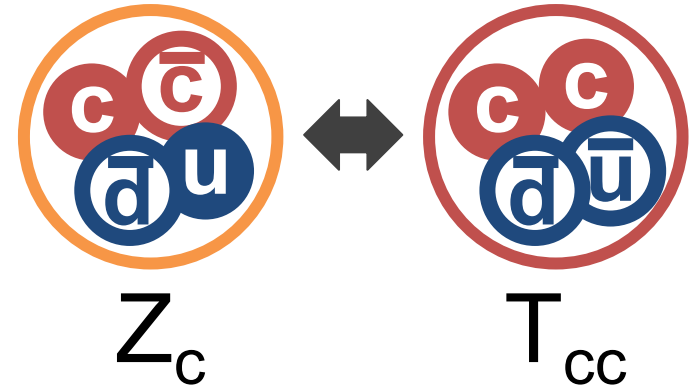
Note: Above $D^0 D^0 \pi^+$ threshold
 3869.25 MeV:
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New type of exotics!

T_{cc} : doubly charmed tetraquark

$$|C| = 0$$

$$|C| = 2$$



T_{cc} has four quarks at least:
 genuinely exotic.

Important questions:

1. strong ud diquark attraction ?
2. $D(c\bar{u})D^*(c\bar{d})$ molecule ?
3. Are there other T_{cc} ?
4. Are there T_{bb} (double bottom) ?
 etc.

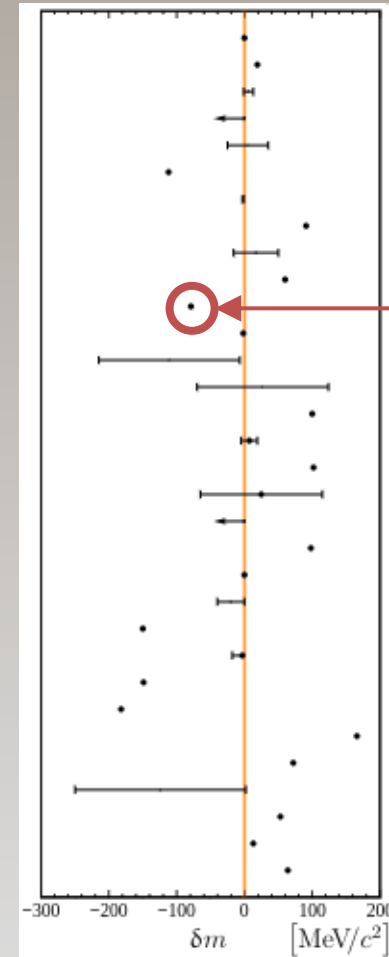
0. Introduction to exotic hadrons

T_{cc} has been studied over **35 years** in theories!

Theory predictions

Ivan Polyakov (2021)

Reference	Year	$\delta'm$ [MeV/c ²]	
J. Carlson, L. Heller and J. A. Tjon	36	1987	~ 0
B. Silvestre-Brac and C. Semay	37	1993	+19
C. Semay and B. Silvestre-Brac	38	1994	[-1, +13]
S. Pepin, F. Stancu, M. Genovese and J. M. Richard	39	1996	< 0
B. A. Gelman and S. Nussinov	40	2002	[-25, +35]
J. Vijande, F. Fernandez, A. Valcarce, A. and B. Silvestre-Brac	41	2003	-112
D. Janc and M. Rosina	42	2004	[-3, -1]
F. Navarra, M. Nielsen and S. H. Lee	43	2007	+91
J. Vijande, E. Weissman, A. Valcarce	44	2007	[-16, +50]
D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha	45	2007	+60
S. H. Lee and S. Yasui	46	2009	-79
Y. Yang, C. Deng, J. Ping and T. Goldman	47	2009	-1.8
G.-Q. Feng, X.-H. Guo and B.-S. Zou	48	2013	-215
Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. Inoue, N. Ishii, K. Murano, H. Nemura and K. Sasaki	49	2013	[-70, +124]
S.-Q. Luo, K. Chen, X. Liu, Y.-R. Liu and S.-L. Zhu	50	2017	+100
M. Karliner and J. Rosner	51	2017	$7 \pm 12 \rightarrow 1$
E. J. Eichten and C. Quigg	52	2017	+102
Z. G. Wang	53	2017	+25 \pm 90
G. K. C. Cheung, C. E. Thomas, J. J. Dudek and R. G. Edwards	54	2017	$\lesssim 0$
W. Park, S. Noh and S. H. Lee	55	2018	+98
A. Francis, R. J. Hudspith, R. Lewis and K. Maltman	56	2018	~ 0
P. Junnarkar, N. Mathur and M. Padmanath	57	2018	[-40, 0]
C. Deng, H. Chen and J. Ping	58	2018	-150
M.-Z. Liu, T.-W. Wu, V. Pavon Valderrama, J.-J. Xie and L.-S. Geng	59	2019	-3^{+4}_{-15}
G. Yang, J. Ping and J. Segovia	60	2019	-149
Y. Tan, W. Lu and J. Ping	61	2020	-182
Q.-F. Lü, D.-Y. Chen and Y.-B. Dong	62	2020	+166
E. Braaten, L.-P. He and A. Mohapatra	63	2020	+72
D. Gao, D. Jia, Y.-J. Sun, Z. Zhang, W.-N. Liu and Q. Mei	64	2020	[-250, +2]
J.-B. Cheng, S.-Y. Li, Y.-R. Liu, Z.-G. Si, T. Yao	65	2020	+53
S. Noh, W. Park and S. H. Lee	66	2021	+13
R. N. Faustov, V. O. Galkin and E. M. Savchenko	67	2021	+64



me!

Ivan Polyakov, Syracuse University

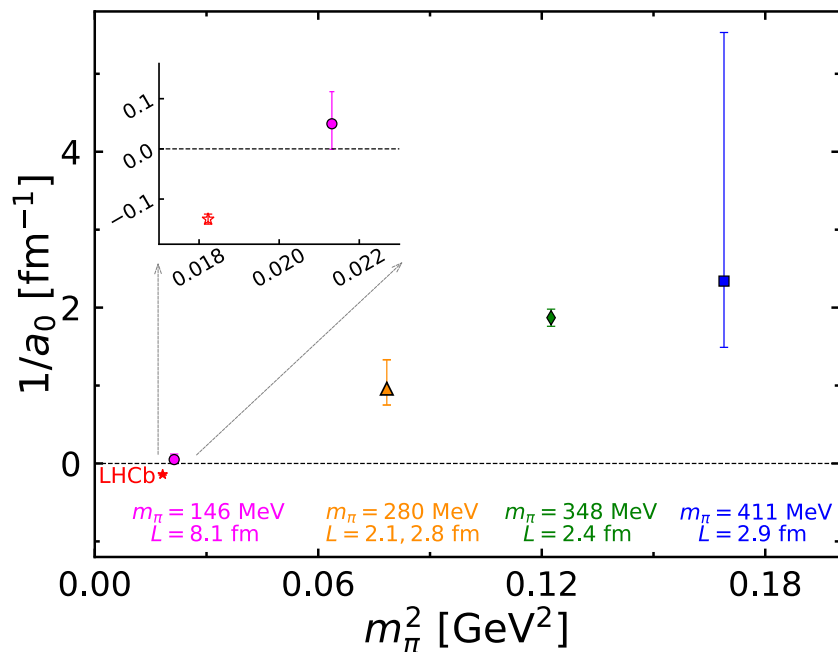
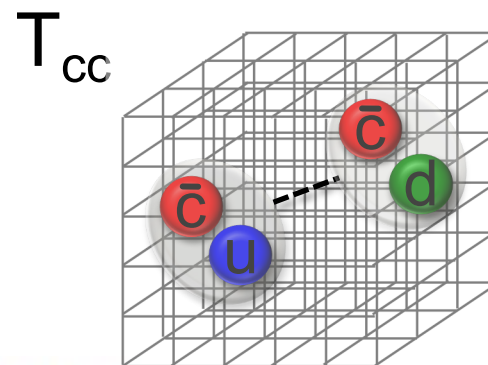
17

0. Introduction to exotic hadrons

Lattice QCD study of T_{cc} near physical point

Y. Ikeda, et al. (HAL Collaboration), PLB729, 85 (2014) : $m_\pi = 410, 700$ MeV

Y. Lyu, et al. (HAL Collaboration), 2302.04505: $m_\pi = 135$ MeV (near physical point)

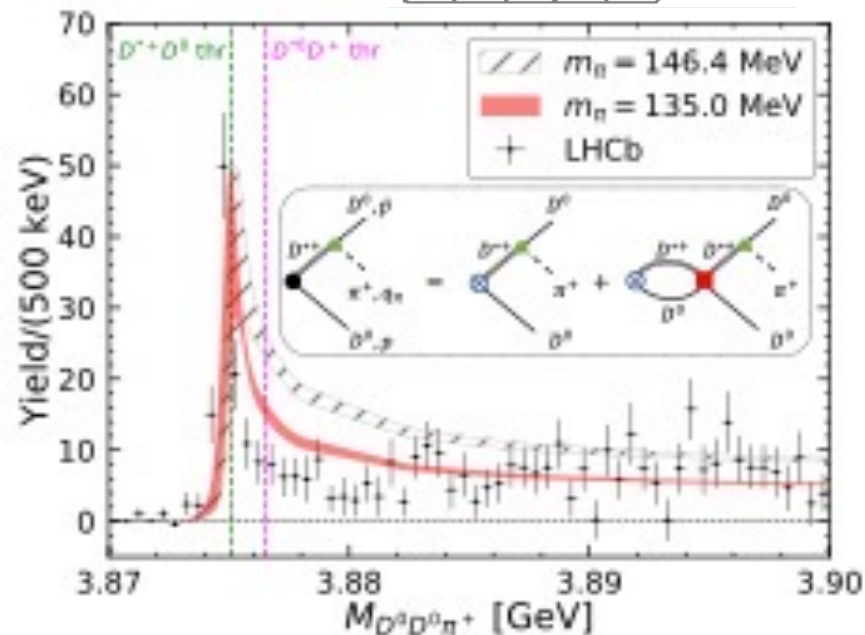


$E_{\text{pole}} = -45$ keV
 “virtual state”

Cf. LHCb (2022): below $D^{*+}D^0$ threshold

$$\delta m_{\text{BW}} = -273 \pm 61 \pm 5_{-14}^{+11} \text{ keV } c^{-2},$$

$$\Gamma_{\text{BW}} = 410 \pm 165 \pm 43_{-38}^{+18} \text{ keV},$$

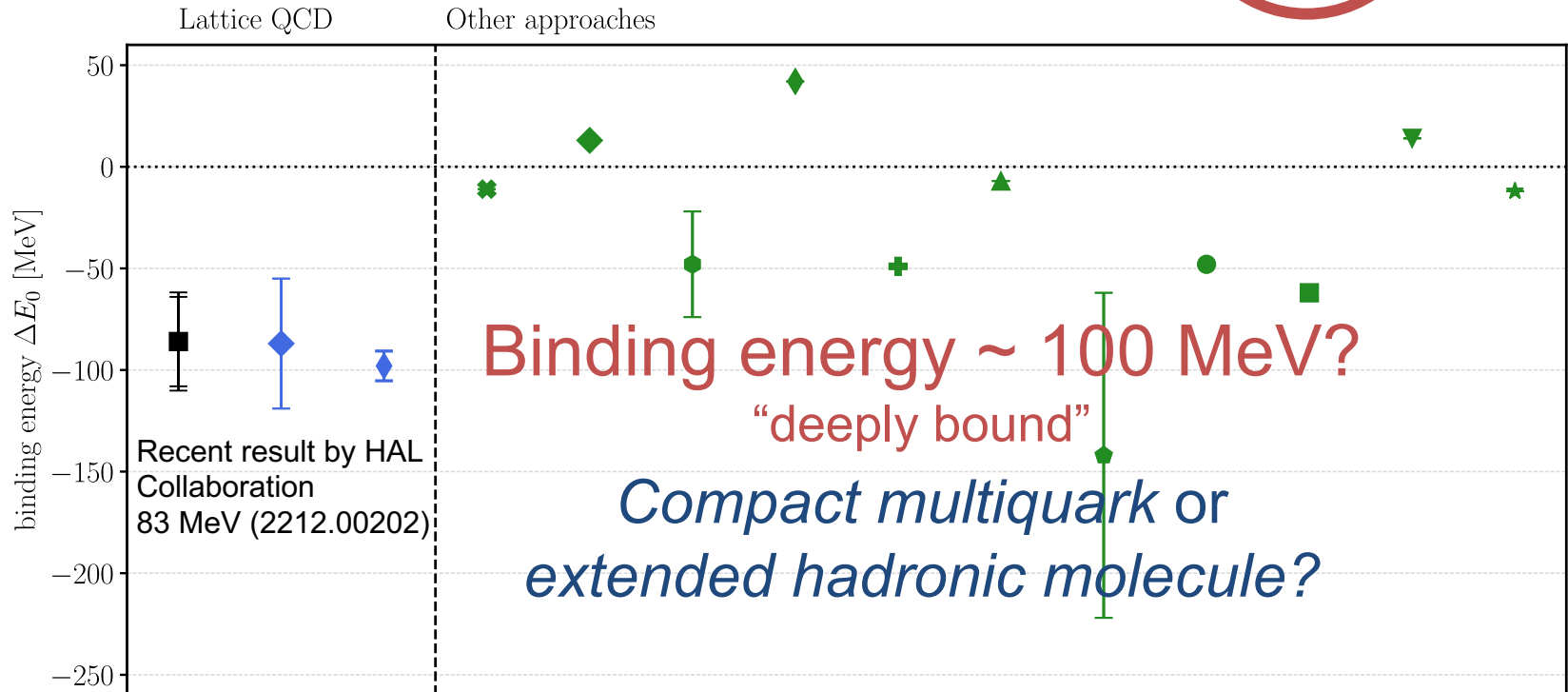


Mass spectrum

0. Introduction to exotic hadrons

Recent lattice QCD study on T_{bb}
Meinel, Pflaumer, Wagner,
Phys. Rev. D106, 034507 (2022)

T_{bb}
Doubly bottom tetraquark



■ This work	◆ Braaten et al. (2020)	● Eichten and Quigg (2017)
◆ Junnarkar et al. (2018)	◆ Lü et al. (2020)	■ Lee and Yasui (2009)
◆ Francis et al. (2016)	⊕ Deng et al. (2018)	▽ Ebert et al. (2007)
⊗ Dai et al. (2022)	▲ Park et al. (2018)	★ Silvestre-Brac and Semay (1993)
◆ Faustov et al. (2021)	◆ Wang (2017)	

Let us research T_{bb} in future experiments!

0. Introduction to exotic hadrons



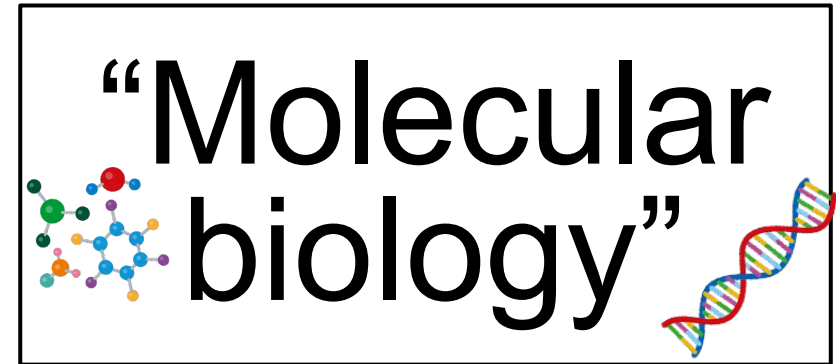
Personal view on significance of T_{cc} (possibly T_{bb})

Before...



Classification of exotic hadrons

After...



High-precision data of exotic hadrons

When (nearly) stable exotic states exist, studies for internal structures will become possible at high-precision for exotic hadrons in experiments, lattice QCD and phenomenology!

Internal property: charge radius, form factor, magnetic moment, quadrupole moment, parton structure, spin decomposition, ...

0. Introduction to exotic hadrons
1. Why \bar{D} meson and nucleon?
2. \bar{D} meson and nucleon potential
3. B meson and nucleon potential
4. Summary

0. Introduction to exotic hadrons

1. Why \bar{D} meson and nucleon?

2. \bar{D} meson and nucleon potential

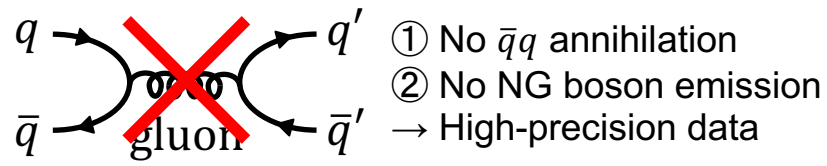
3. B meson and nucleon potential

4. Summary

1. Why \bar{D} meson and nucleon?

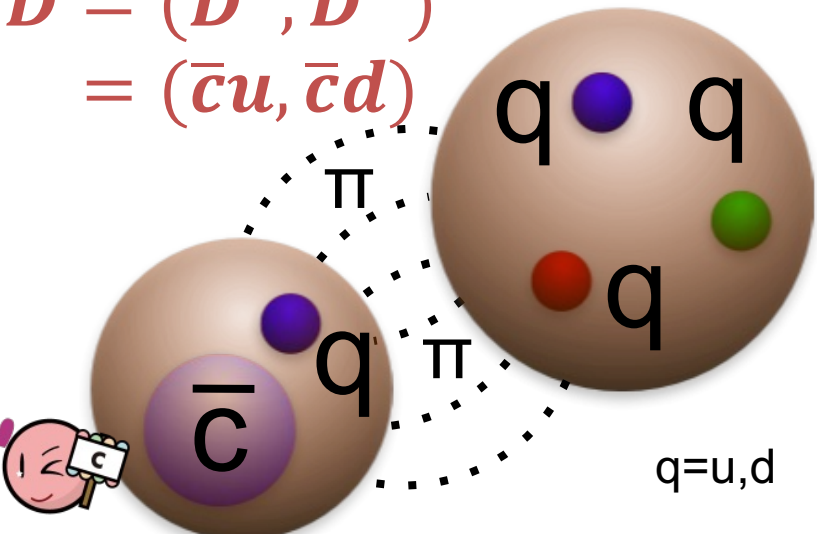
- \bar{D} meson and nucleon (pentaquark)

- ✓ $\bar{c}qqqq$ ($q = u, d$): no annihilation
- ✓ (Anti-)charm nuclei? Cf. Review paper: Hosaka, Hyodo, Sudoh, Yamaguchi, Yasui, PPNP 96, 88 (2017)
- ✓ Extension to B meson and nucleon



$$\bar{D} = (\bar{D}^0, D^-)$$

$$= (\bar{c}u, \bar{c}d)$$



\bar{D} meson
(anti D meson)

Nucleon

pentaquark (5 quark)

chiral + HQS symmetries:

Cohen, Hohler, Lebed, PRD72, 074010 (2005)

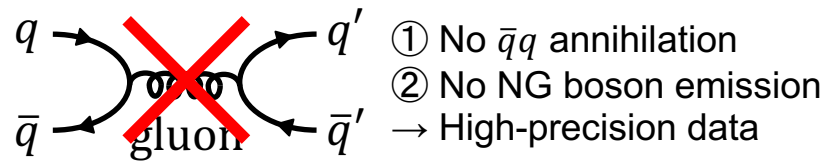
Yasui, Sudoh, PRD80, 034008 (2009)

Yamaguchi, Ohkoda, Yasui, Hosaka, PRD84, 014032 (2011), *ibid.* 85, 054003 (2012)

Cf. compact quarks: Gignoux, Silvestre-Brac, Richard, PLB193, 323 (1987), Lipkin, PLB195, 484 (1987)

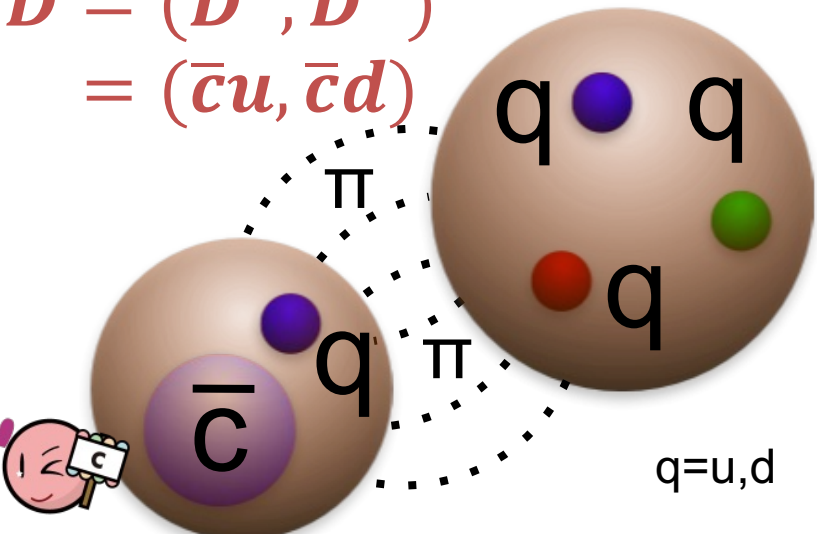
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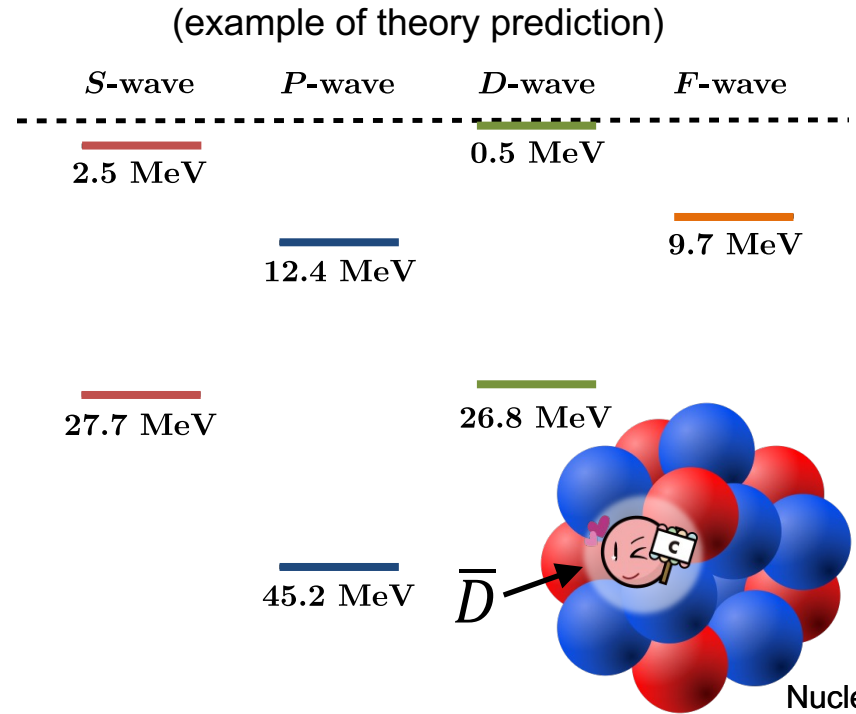
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 Cf. compact quarks: Gignoux, Silvestre-Brac, Richard, PLB193, 323 (1987), Lipkin, PLB195, 484 (1987)

(anti-)charm and bottom nuclei
 \bar{D} meson bound nuclei?



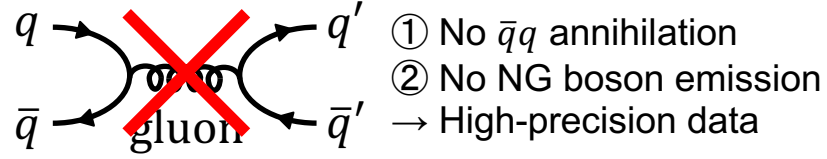
\bar{D} nucleus ($A = 16$ baryon #)
 Yamaguchi, Yasui, PTEP2017 (2017) 9, 093D02

Accessible to high-precision data of charm exotic hadrons and nuclei, if exist.

1. Why \bar{D} meson and nucleon?

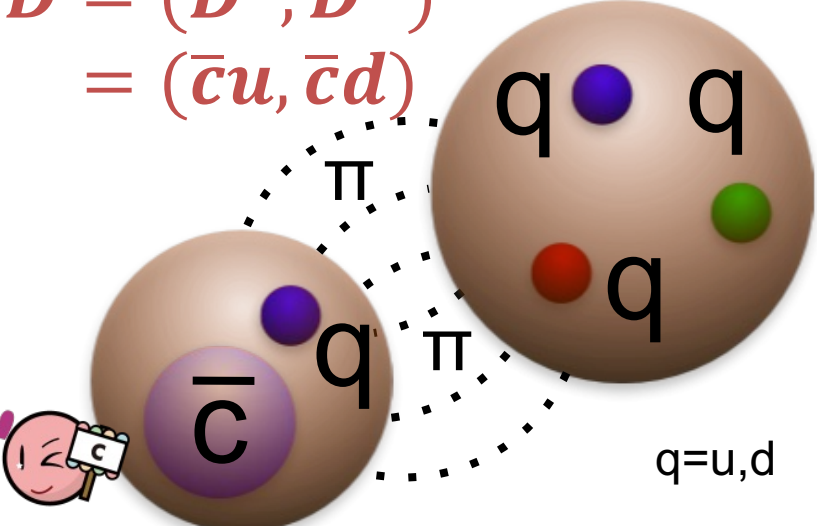
- \bar{D} meson and nucleon (pentaquark)

- ✓ $\bar{c}qqqq$ ($q = u, d$): no annihilation
- ✓ (Anti-)charm nuclei? Cf. Review paper: Hosaka, Hyodo, Sudoh, Yamaguchi, Yasui, PPNP 96, 88 (2017)
- ✓ Extension to B meson and nucleon



$$\bar{D} = (\bar{D}^0, D^-)$$

$$= (\bar{c}u, \bar{c}d)$$



\bar{D} meson
(anti D meson)

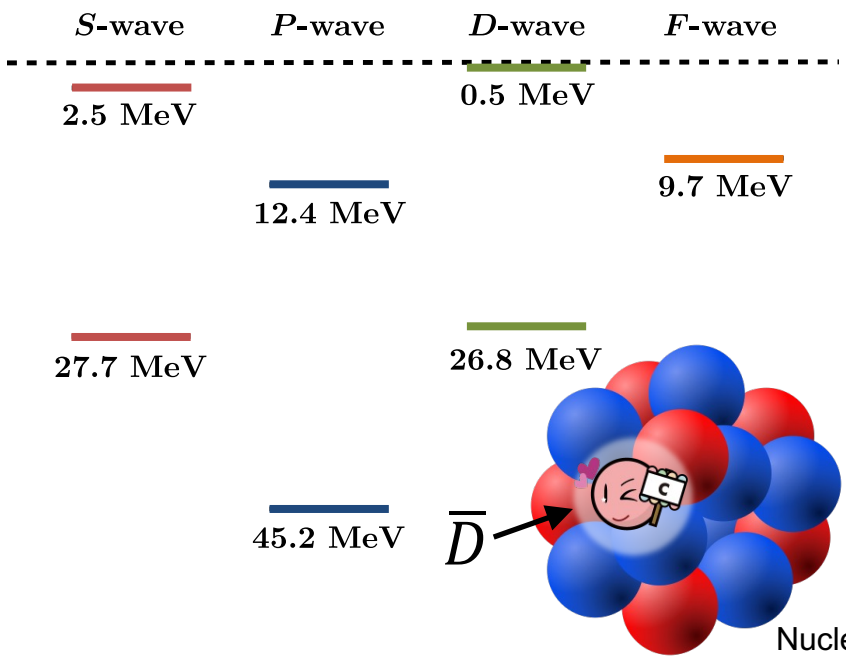
Nucleon

pentaquark (5 quark)

chiral + HQS symmetries:
 Cohen, Hohler, Lebed, PRD72, 074010 (2005)
 Yasui, Sudoh, PRD80, 034008 (2009)
 Yamaguchi, Ohkoda, Yasui, Hosaka, PRD84, 014032 (2011), *ibid.* 85, 054003 (2012)
 Cf. compact quarks: Gignoux, Silvestre-Brac, Richard, PLB193, 323 (1987), Lipkin, PLB195, 484 (1987)

(anti-)charm and bottom nuclei
 \bar{D} meson bound nuclei?

(example of theory prediction)



\bar{D} nucleus ($A = 16$ baryon #)
 Yamaguchi, Yasui, PTEP2017 (2017) 9, 093D02

Can (anti-)charm nuclei exist in our nature?

1. Why \bar{D} meson and nucleon?

- Charm (bottom) nuclei? **Flavor nuclei (diversity of matter)**
 - ✓ Can charm (bottom) nuclei exist as stable states?
 - ✓ Binding energy of \bar{D} mesons in nuclear medium?

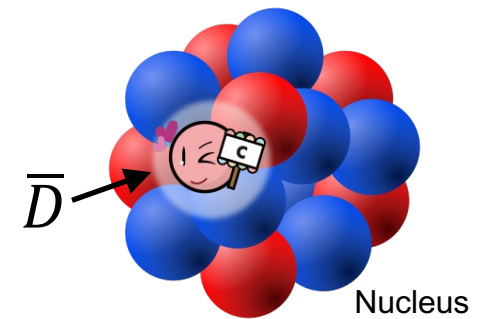


TABLE I. List of the mass shifts of the \bar{D} meson in nuclear medium in previous works: quark meson coupling (QMC) model, QCD sum rule, coupled channel analysis, and chiral effective model.

Analysis	Ref.	Mass shift of \bar{D} (MeV)	Density ρ (fm $^{-3}$)
QMC model (QMC: quark-meson coupling)	[18]	-62 attractive	0.15
QCD sum rule	[19]	-48 ± 8 attractive	0.17
	[23]	+45 (averaged mass shift of D and \bar{D}) repulsive	0.15
	[28]	-46 ± 7 (averaged mass shift of D and \bar{D}) attractive	0.17
	[30]	-72 (averaged mass shift of D and \bar{D}) attractive	0.17
	[31]	+38 repulsive	0.17
Coupled channel analysis	[21]	+18 repulsive	0.17
	[22]	+(11-20) repulsive	0.16
	[26]	+35 repulsive	0.17
	[15]	$\simeq -(20-27)$ attractive	0.17
Chiral effective model	[20]	$\simeq -(30-180)$ attractive	0.15
	[25]	-27.2 attractive	0.15
	[16]	-35.1 attractive	0.17
	[37]	+97 (parity doublet model), +120 (skyrmion crystal) repulsive	0.16
Our result*		+74 repulsive	0.095

*D. Suenaga, S. Yasui., M. Harada, Phys. Rev. C96, 015204 (2017) [See this paper for the reference numbers.]

(Anti-)charm nuclei:
Is $\bar{D}N$ interaction *attractive* or *repulsive*?

1. Why \bar{D} meson and nucleon? My contributions since 2009...

PHYSICAL REVIEW D **80**, 034008 (2009)

Exotic nuclei with open heavy flavor mesons

Shigehiro Yasui^{1,*} and Kazutaka Sudoh^{2,†}

PHYSICAL REVIEW D **84**, 014032 (2011)

Exotic baryons from a heavy meson and a nucleon: Negative parity states

Yasuhiro Yamaguchi,¹ Shunsuke Ohkoda,¹ Shigehiro Yasui,² and Atsushi Hosaka¹

PHYSICAL REVIEW D **85**, 054003 (2012)

Exotic baryons from a heavy meson and a nucleon: Positive parity states

Yasuhiro Yamaguchi,¹ Shunsuke Ohkoda,¹ Shigehiro Yasui,² and Atsushi Hosaka¹

Physics Letters B 727 (2013) 185–189

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb



PHYSICAL REVIEW D **91**, 034034 (2015)

Heavy quark symmetry in multihadron systems

Yasuhiro Yamaguchi,¹ Shunsuke Ohkoda,¹ Atsushi Hosaka,^{1,2} Tetsuo Hyodo,³ and Shigehiro Yasui^{4,5,*}

Spin degeneracy in multi-hadron systems with a heavy quark

Shigehiro Yasui^{a,*}, Kazutaka Sudoh^b, Yasuhiro Yamaguchi^c, Shunsuke Ohkoda^c,
Atsushi Hosaka^c, Tetsuo Hyodo^{d,1}



Nuclear Physics A 927 (2014) 110–118

www.elsevier.com/locate/nuclphysa

PHYSICS A

Exotic dibaryons with a heavy antiquark

Yasuhiro Yamaguchi^{a,*}, Shigehiro Yasui^b, Atsushi Hosaka^{a,c}

Prog. Theor. Exp. Phys. **2017**, 093D02 (14 pages)

DOI: 10.1093/ptep/ptx112

PTEP

Mesic nuclei with a heavy antiquark

Yasuhiro Yamaguchi^{1,2,*} and Shigehiro Yasui³

PHYSICAL REVIEW C **87**, 015202 (2013)

\bar{D} and B mesons in a nuclear medium

S. Yasui^{*}

KEK Theory Center, Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization, 1-1 Oho,
Ibaraki 305-0801, Japan

K. Sudoh

PHYSICAL REVIEW C **89**, 015201 (2014)

Probing gluon dynamics by charm and bottom mesons in nuclear theory with $1/M$ corrections

S. Yasui^{1,*} and K. Sudoh²

Progress in Particle and Nuclear Physics 96 (2017) 88–153

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp



Review

Heavy hadrons in nuclear matter

Atsushi Hosaka^{a,b}, Tetsuo Hyodo^c, Kazutaka Sudoh^d, Yasuhiro Yamaguchi^{c,e},
Shigehiro Yasui^{f,*}



1. Why \bar{D} meson and nucleon?

$\bar{D}N$ (BN) potential; the *latest* version

PHYSICAL REVIEW D **106**, 094001 (2022)

Open charm and bottom meson-nucleon potentials *à la* the nuclear force

Yasuhiro Yamaguchi^{*}

*Department of Physics, Nagoya University, Nagoya 464-8602, Japan
and Advanced Science Research Center, Japan Atomic Energy Agency (JAEA),
Tokai 319-1195, Japan*

Shigehiro Yasui[†]

*Research and Education Center for Natural Sciences, Keio University,
Hiyoshi 4-1-1, Yokohama, Kanagawa 223-8521, Japan*

Atsushi Hosaka[‡]

*Research Center for Nuclear Physics (RCNP), Ibaraki, Osaka 567-0047, Japan;
Advanced Science Research Center, Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195, Japan
and Theoretical Research Division, Nishina Center, RIKEN, Hirosawa, Wako, Saitama 351-0198, Japan*

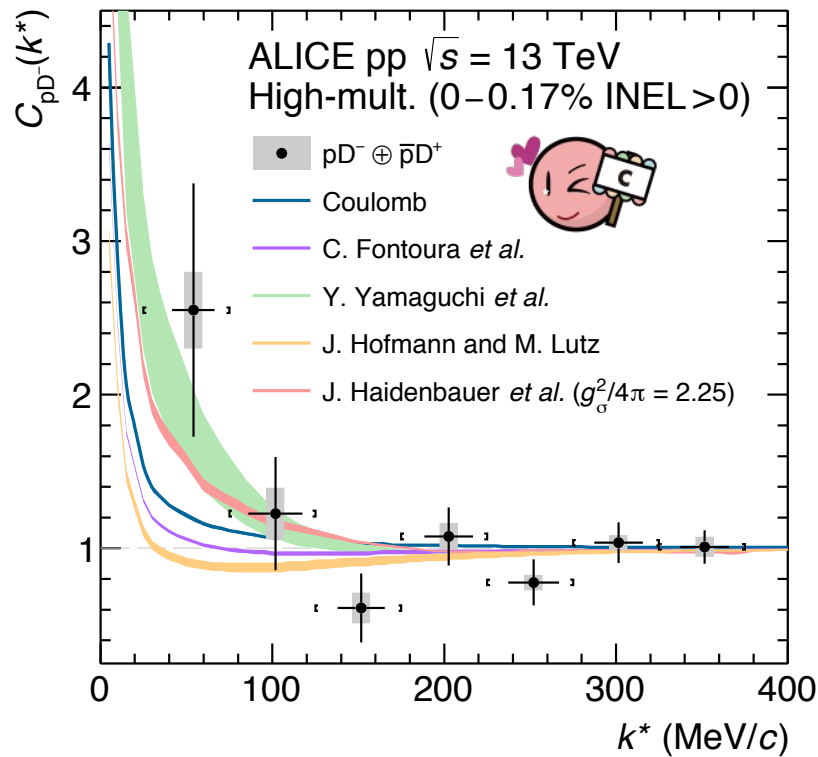
I talk on this.

1. Why \bar{D} meson and nucleon?

- 2022: First experiment for $\bar{D}N$ interaction!

- ✓ ALICE at LHC Phys. Rev. D106, 052010 (2022) ← analysis by Kamiya, Hyodo, **Ohnishi**
- ✓ D^-p ($\bar{D}N$) correlation function from proton-proton collisions
- ✓ Attraction suggested? (Cf. KN is repulsive.)

Cf. Hyperon interaction: Ohnishi et al., Nucl. Phys. A954, 294 (2016)



Model	f_0 (I = 0)	f_0 (I = 1)	n_σ
Coulomb			(1.1–1.5)
attraction Haidenbauer et al. [21]			
$-g_\sigma^2/4\pi = 1$	0.14	-0.28	(1.2–1.5)
$-g_\sigma^2/4\pi = 2.25$	0.67	0.04	(0.8–1.3)
repulsion Hofmann and Lutz [22]	-0.16	-0.26	(1.3–1.6)
attraction (bound) Yamaguchi et al. [24]	-4.38	-0.07	(0.6–1.1)
attraction Fontoura et al. [23]	0.16	-0.25	(1.1–1.5)

[21] Haidenbauer, Krein, Meißner, Sibirtsev, EPJ. A33, 107 (2007)
 [22] Hofmann, Lutz, NPA763, 90 (2005)
 [24] Yamaguchi, Ohkoda, Yasui, Hosaka, PRD84, 014032 (2011)
 [23] Fontoura, Krein, Vizcarra, PRC87, 025206 (2013)

We should explore \bar{D} meson and nucleon interaction
more seriously!

0. Introduction to exotic hadrons
1. Why \bar{D} meson and nucleon?
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0. Introduction to exotic hadrons
1. Why \bar{D} meson and nucleon?
- 2. \bar{D} meson and nucleon potential**
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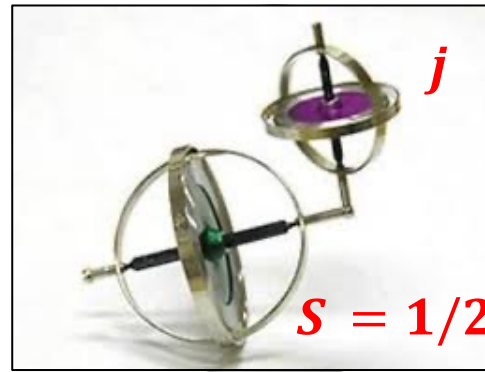
2. \bar{D} meson and nucleon potential

- Structure of \bar{D} meson: Heavy quark spin symmetry (HQS)

✓ HQS: $Q \rightarrow SQ$ with $S \in SU(2)_{\text{heavy quark spin}}$

✓ D and D^* mesons as HQS doublet

✓ B and B^* mesons also



HQS ($S = 1/2$)
conserved



Spin J
conserved



Light spin j
conserved!

$$\vec{J} = \vec{S} + \vec{j}$$

→ **Mass degeneracy for $J = j \pm 1/2$**

2. \bar{D} meson and nucleon potential

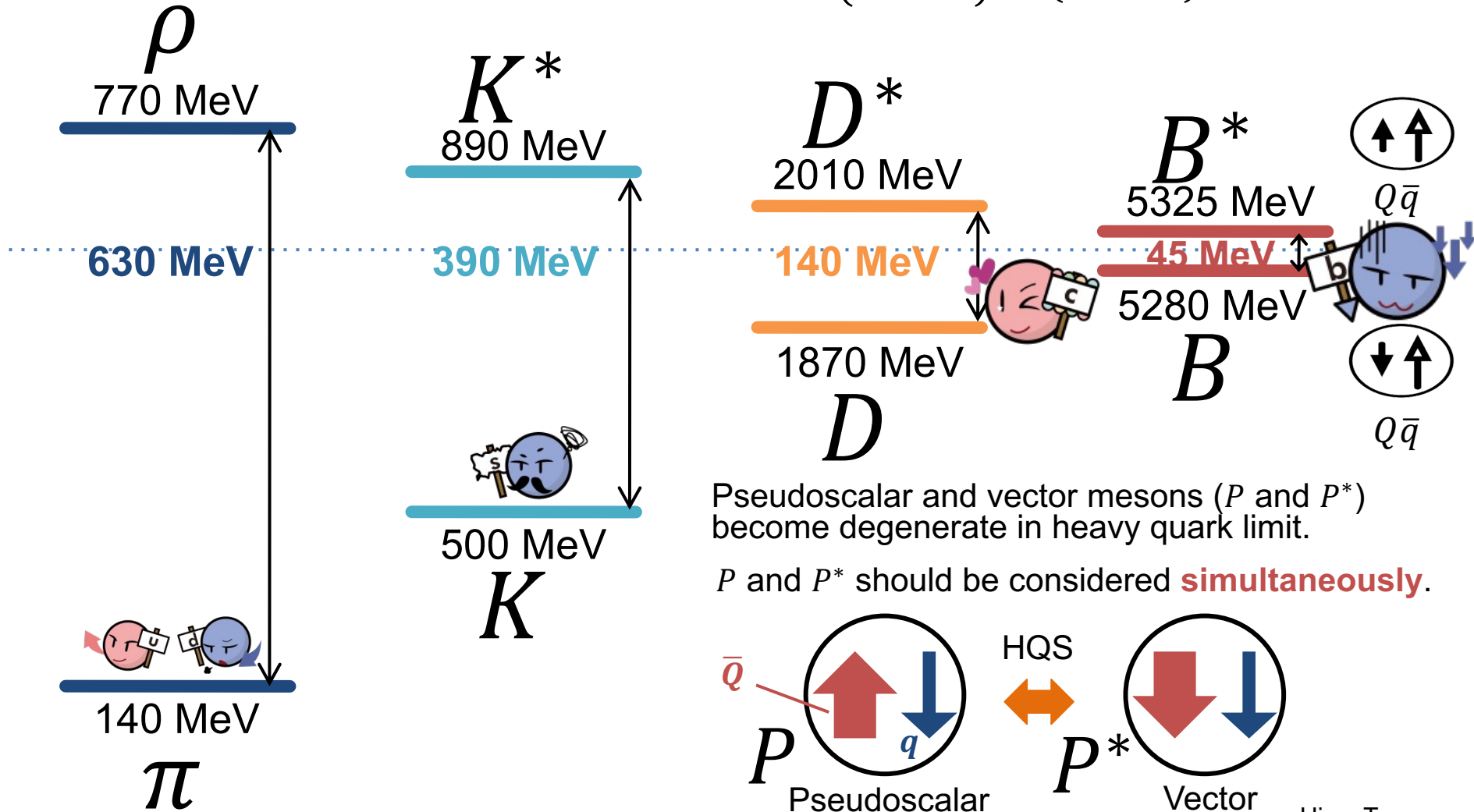
- Structure of \bar{D} meson: Heavy quark spin symmetry (HQS)

✓ HQS: $Q \rightarrow SQ$ with $S \in SU(2)$ heavy quark spin

✓ D and D^* mesons as HQS doublet $\bar{D} = (\bar{D}^0, D^-) = (\bar{c}u, \bar{c}d)$ u c t

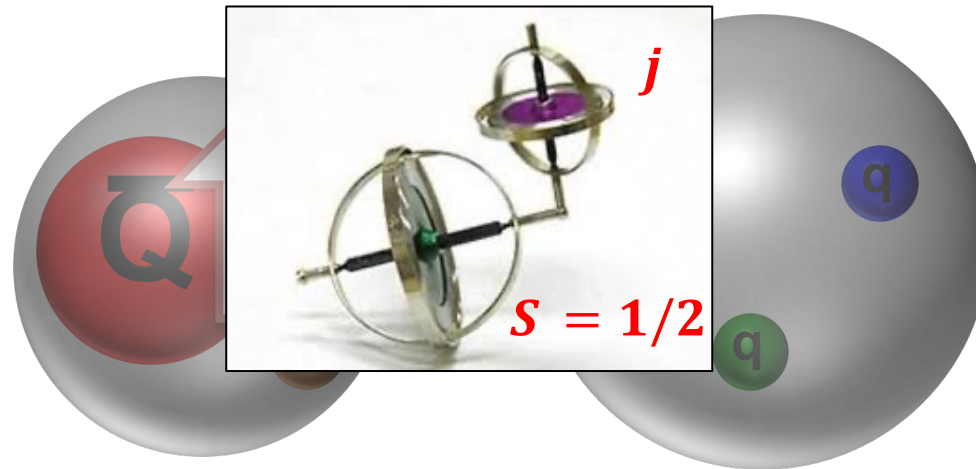
✓ B and B^* mesons also

$B = (B^+, B^0) = (\bar{b}u, \bar{b}d)$ d s b



2. \bar{D} meson and nucleon potential

- Structure of \bar{D} meson: Heavy quark spin symmetry (HQS)
 - ✓ HQS can be applied to arbitrary (exotic) heavy hadrons.
 - ✓ Pentaquark $\bar{Q}qqqq$ ($= \bar{Q}q + qqq$)



HQS ($S = 1/2$)
conserved

Spin J
conserved

Light spin j
conserved!

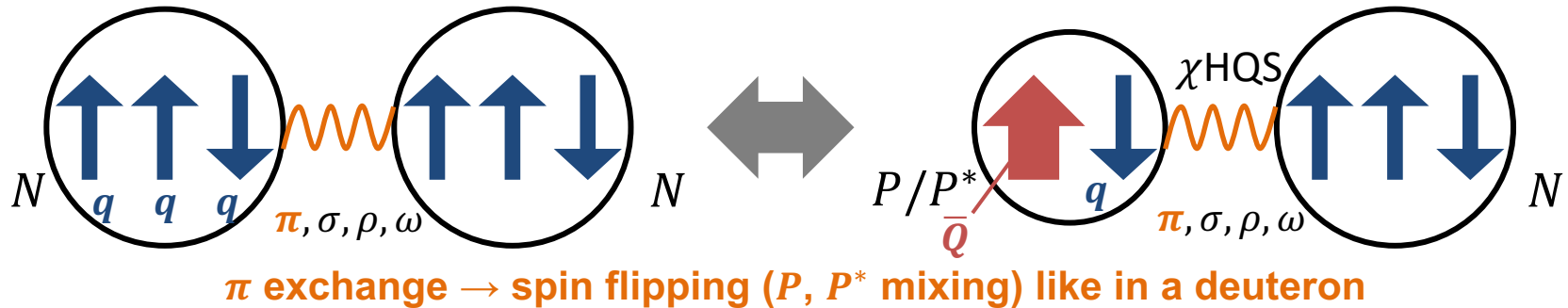
$$\vec{J} = \vec{S} + \vec{j}$$

→ **Mass degeneracy for $J = j \pm 1/2$**

We will see single-heavy hadronic molecules.

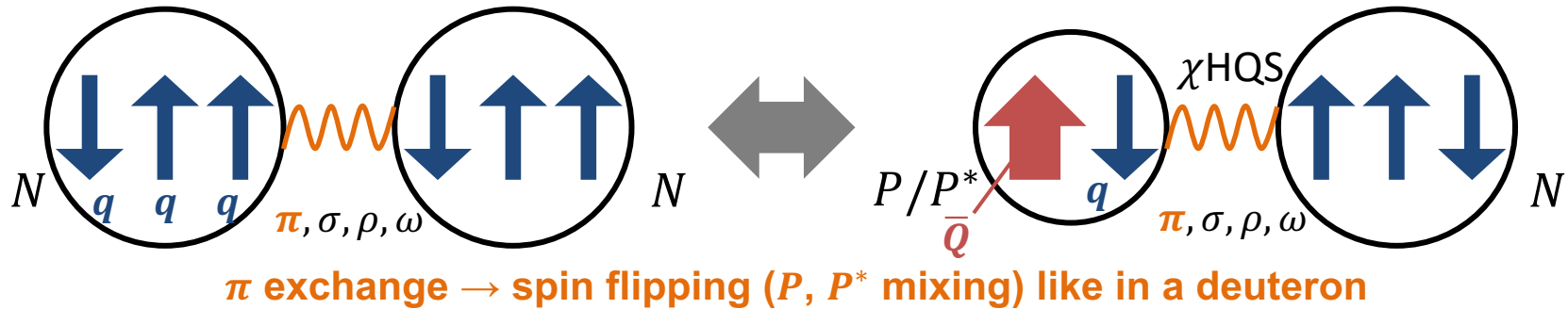
2. \bar{D} meson and nucleon potential

- \bar{D} meson and nucleon potential ($P = \bar{D}, P^* = \bar{D}^*$)
 - ✓ $PN - P^*N$ mixing (P and P^* are interchangeable.)
 - ✓ **Chiral (χ) symmetry + Heavy-quark spin (HQS) symmetry**
 - ✓ OPEP (one-pion exchange potential) $\leftarrow \chi + \text{HQS}$
 - ✓ Scalar (σ), vector (ρ, ω) exchanges
 - ✓ Analogy to nucleon-nucleon (NN) pot. (Note: $1/\sqrt{2}$ factor for $P^{(*)}P^{(*)}m$)



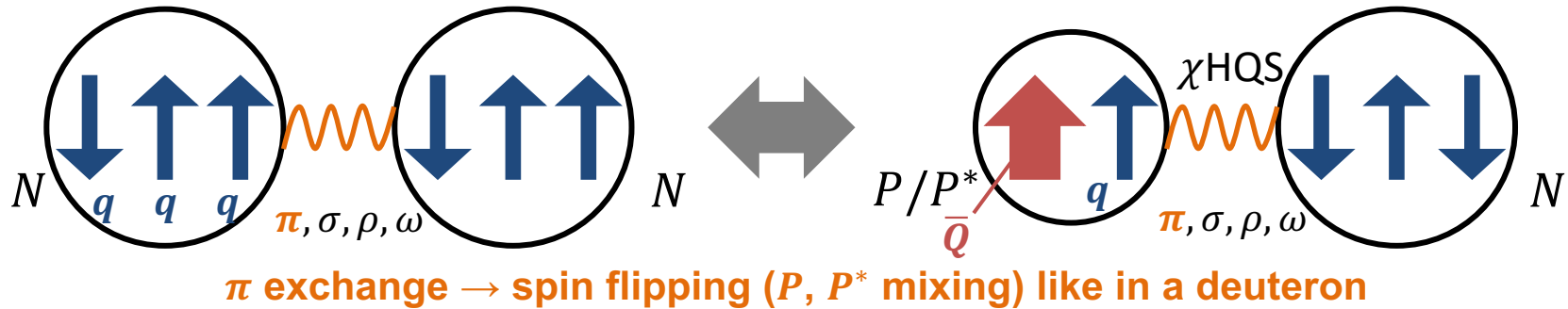
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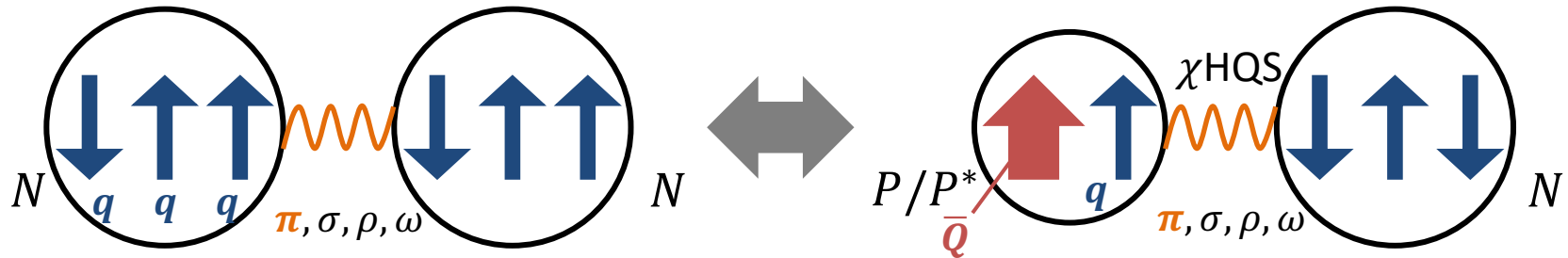
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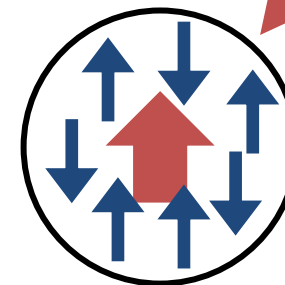


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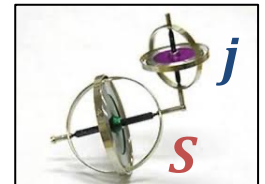


π exchange \rightarrow spin flipping (P, P^* mixing) like in a deuteron



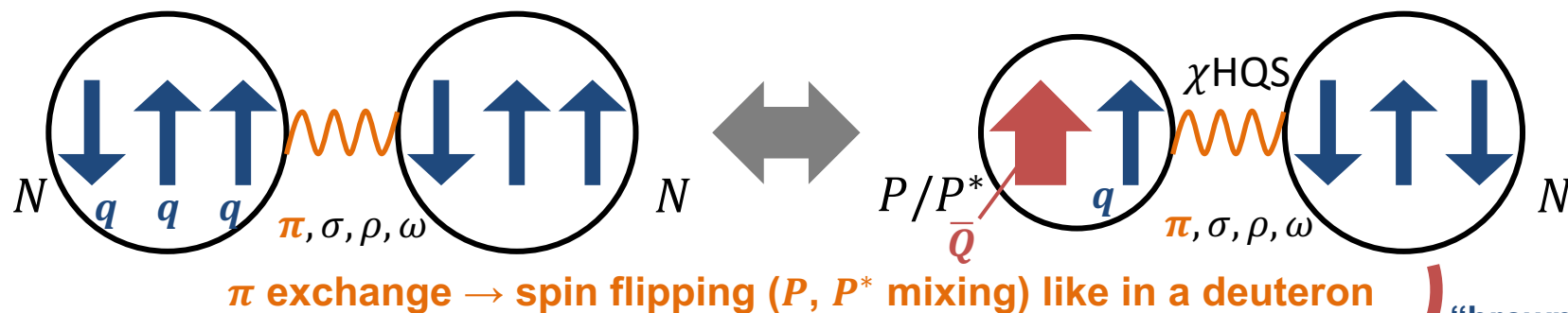
“brown muck”
light spin j

Spin decomposition
by light quarks and
gluons from heavy
quarks



2. \bar{D} meson and nucleon potential

- \bar{D} meson and nucleon potential ($P = \bar{D}$, $P^* = \bar{D}^*$)
 - ✓ $PN - P^*N$ mixing (P and P^* are interchangeable.)
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 - ✓ OPEP (one-pion exchange potential) $\leftarrow \chi + \text{HQS}$
 - ✓ Scalar (σ), vector (ρ, ω) exchanges
 - ✓ Analogy to nucleon-nucleon (NN) pot. (Note: $1/\sqrt{2}$ factor for $P^{(*)}P^{(*)}m$)



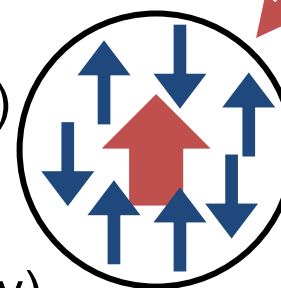
- Generality: spin-structure (q : light quark, N : nucleon)

✓ Recombination: $[\bar{Q}q]N = \bar{Q}[qN]$

✓ HQS multiplets: **which is realized in QCD?**

- **HQS singlet**: $q + N$ with $j = 0$ (total $J = 1/2$ only)

- **HQS doublet**: $q + N$ with $j = 1$ (total $J = 1/2, 3/2$ degenerate)



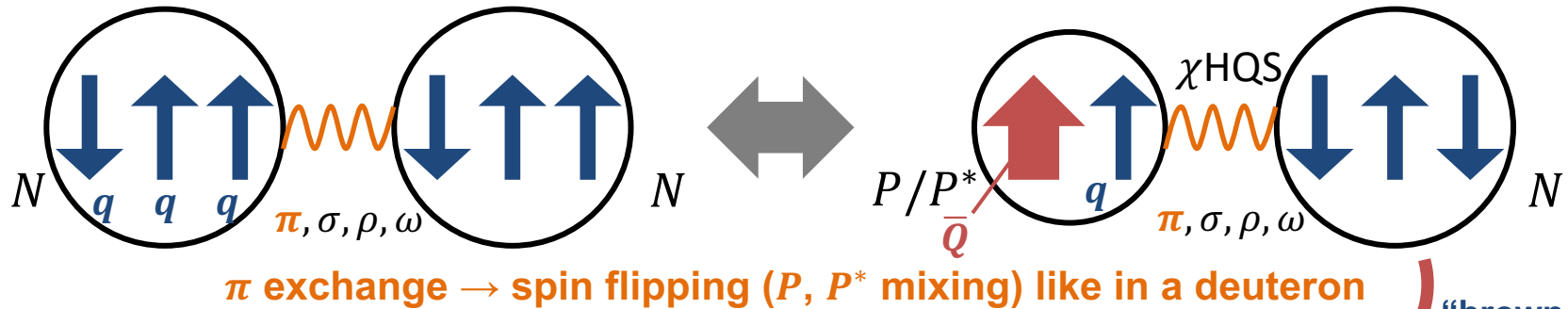
“brown muck”
light spin j

Spin decomposition
by light quarks and
gluons from heavy
quarks



2. \bar{D} meson and nucleon potential

- \bar{D} meson and nucleon potential ($P = \bar{D}$, $P^* = \bar{D}^*$)
 - ✓ $PN - P^*N$ mixing (P and P^* are interchangeable.)
 - ✓ **Chiral (χ) symmetry + Heavy-quark spin (HQS) symmetry**
 - ✓ OPEP (one-pion exchange potential) $\leftarrow \chi + \text{HQS}$
 - ✓ Scalar (σ), vector (ρ, ω) exchanges
 - ✓ Analogy to nucleon-nucleon (NN) pot. (Note: $1/\sqrt{2}$ factor for $P^{(*)}P^{(*)}m$)



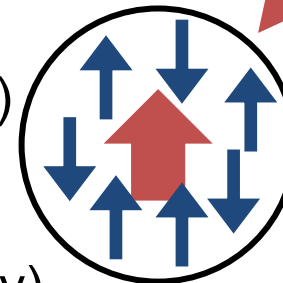
- Generality: spin-structure (q : light quark, N : nucleon)

✓ Recombination: $[\bar{Q}q]N = \bar{Q}[qN]$

✓ HQS multiplets: **which is realized in QCD?**

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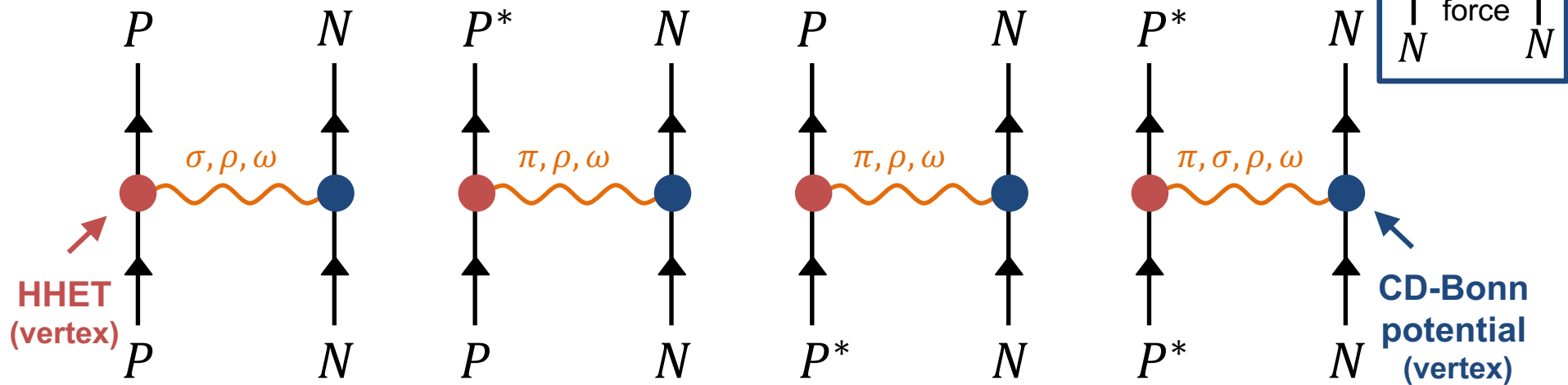


- We need to solve QCD in order to get the potential, but it's difficult.
 - ✓ We still rely on model calculations.

2. \bar{D} meson and nucleon potential

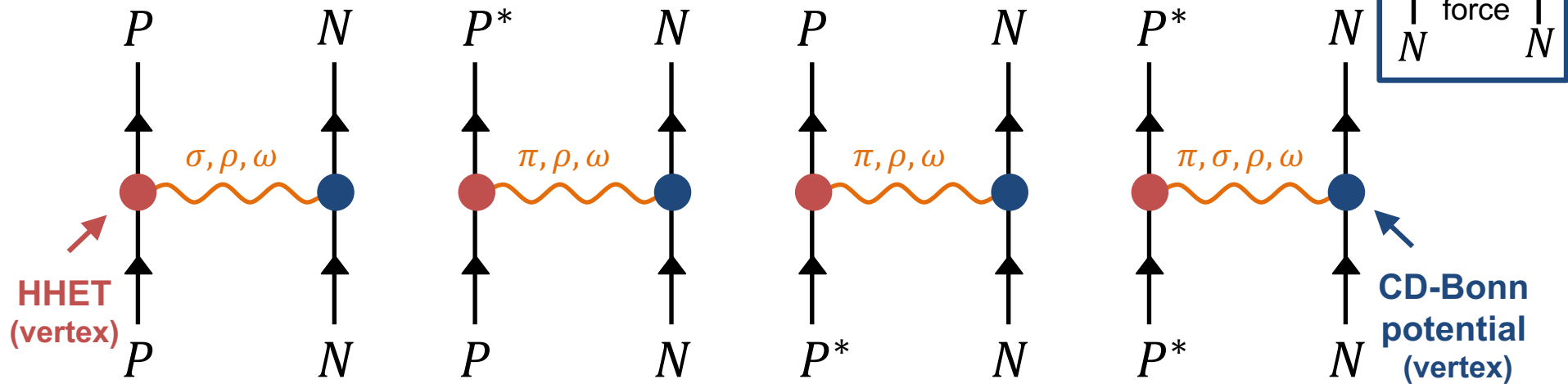
- $P^{(*)}N$ potential ($P = \bar{D}, B$ meson; $P^* = \bar{D}^*, B^*$ meson, N nucleon)

✓ $PN - P^*N$ channel mixing (multi-channel)



2. \bar{D} meson and nucleon potential

- $P^{(*)}N$ potential ($P = \bar{D}, B$ meson; $P^* = \bar{D}^*, B^*$ meson, N nucleon)
- ✓ $PN - P^*N$ channel mixing (multi-channel)



- **Heavy Hadron Effective Theory (HHET)** Luke, Manohar, Wise, Casalbuoni, ...

✓ Hadronic effective theory based on χ +HQS symmetries for P and P^*

✓ Effective field: $H_\alpha = (P_\alpha^{*\mu} \gamma_\mu + P_\alpha \gamma_5) \frac{1 - \psi}{2} H_\alpha \rightarrow \underset{\text{HQS}}{S} H_\beta \underset{\chi \text{ sym.}}{U^\dagger}_{\beta\alpha}$

✓ $P^{(*)}P^{(*)}m$ vertices are uniquely determined ($m = \pi, \sigma, \rho, \omega$)

$$\mathcal{L}_{\pi HH} = ig_\pi \text{tr}(H_\alpha \bar{H}_\beta \gamma_\mu \gamma_5 A_{\beta\alpha}^\mu)$$

$$\mathcal{L}_{\sigma_I HH} = g_{\sigma_I} \text{tr}(H \sigma_I \bar{H})$$

$$\begin{aligned} \mathcal{L}_{\nu HH} &= -i\beta \text{tr}(H_b v^\mu (\rho_\mu)_{ba} \bar{H}_a) \\ &+ i\lambda \text{tr}(H_b \sigma^{\mu\nu} (F_{\mu\nu}(\rho))_{ba} \bar{H}_a) \end{aligned}$$

Previous works:

π only: Yasui, Sudoh, PRD80, 034008 (2009)

π, ρ, ω : Yamaguchi, Ohkoda, Yasui, Hosaka, PRD84 014032 (2011),
ibid. 054003 (2012)