# Surprises on the way to the QCD phase diagram

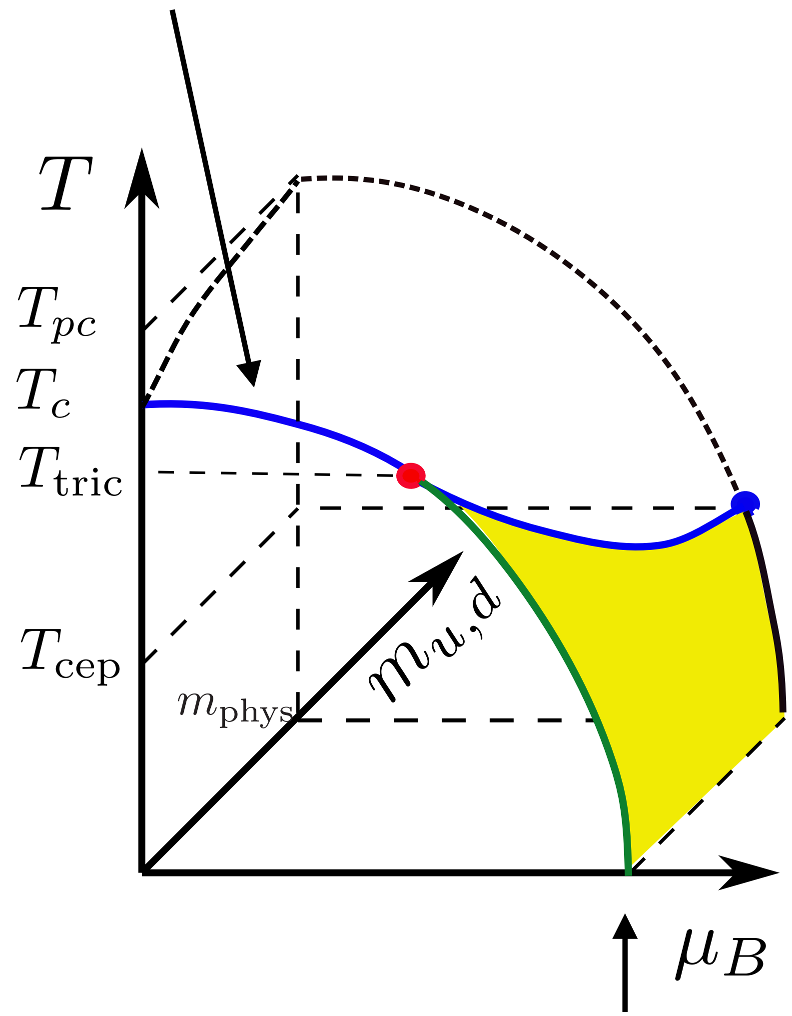
Owe Philipsen

- Chiral phase transition in massless limit constrains the QCD phase diagram
- ~40 years of "common wisdom" + inconclusive lattice results:
  Resolution with a surprise
- Emergent chiral spin symmetry:
  Additional evidence from screening masses+spectral functions
- A modified phase diagram

# The critical point "derived from" the chiral limit

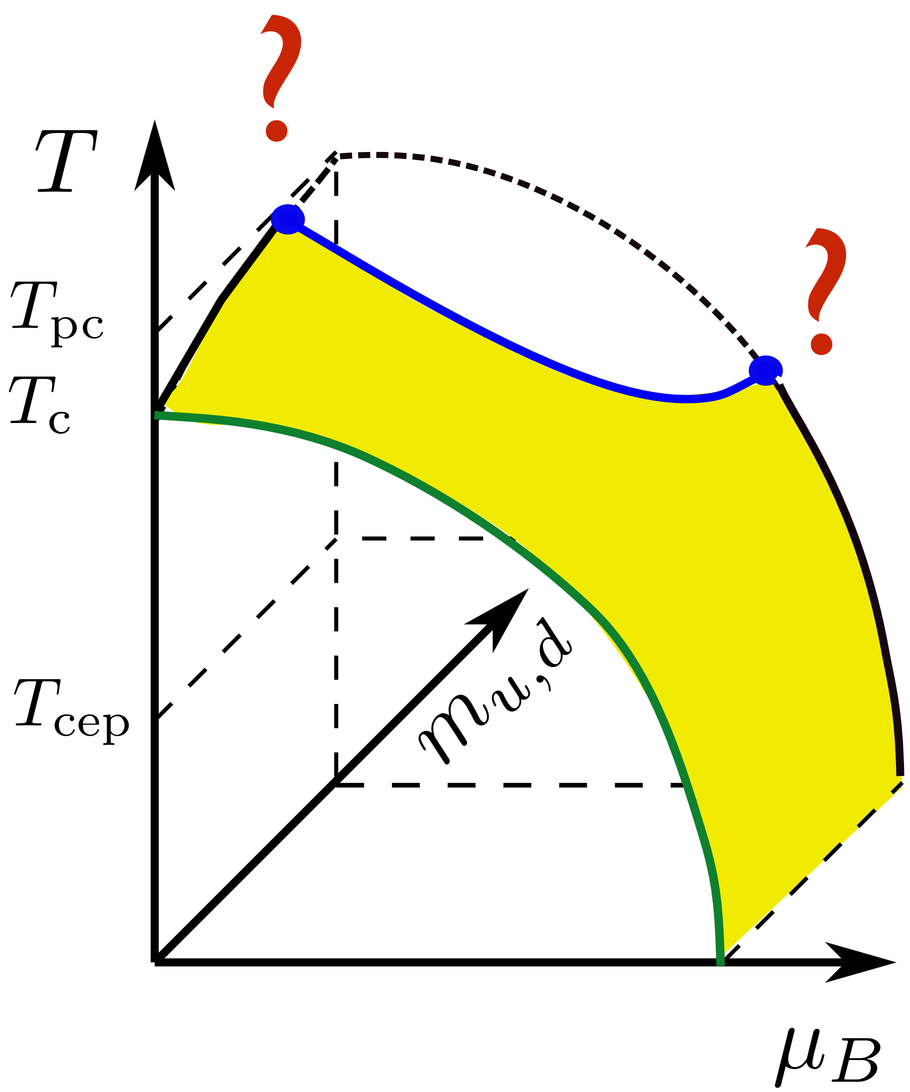
[Halasz et al., PRD 98; Rajagopal, Wilczek 00, Hatta, Ikeda, PRD 03,…]

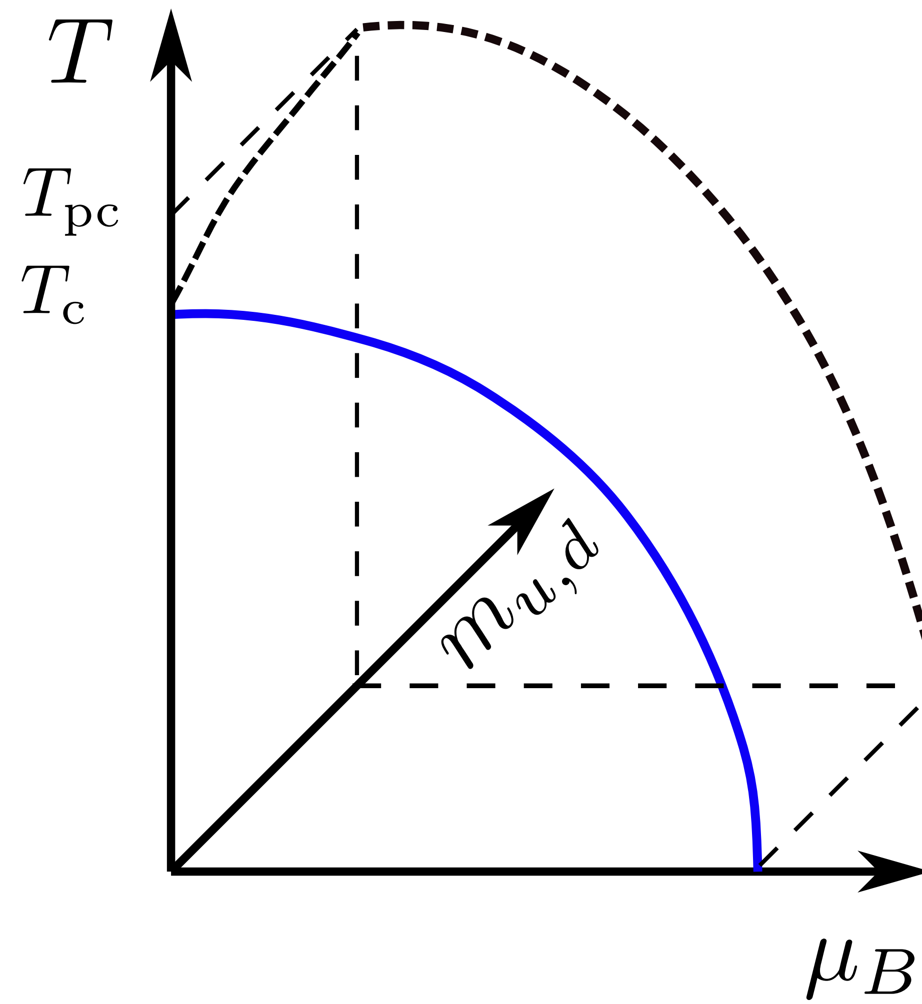
Importance of the chiral limit!




model predictions, no QCD information

# Other possibilities, mostly ignored

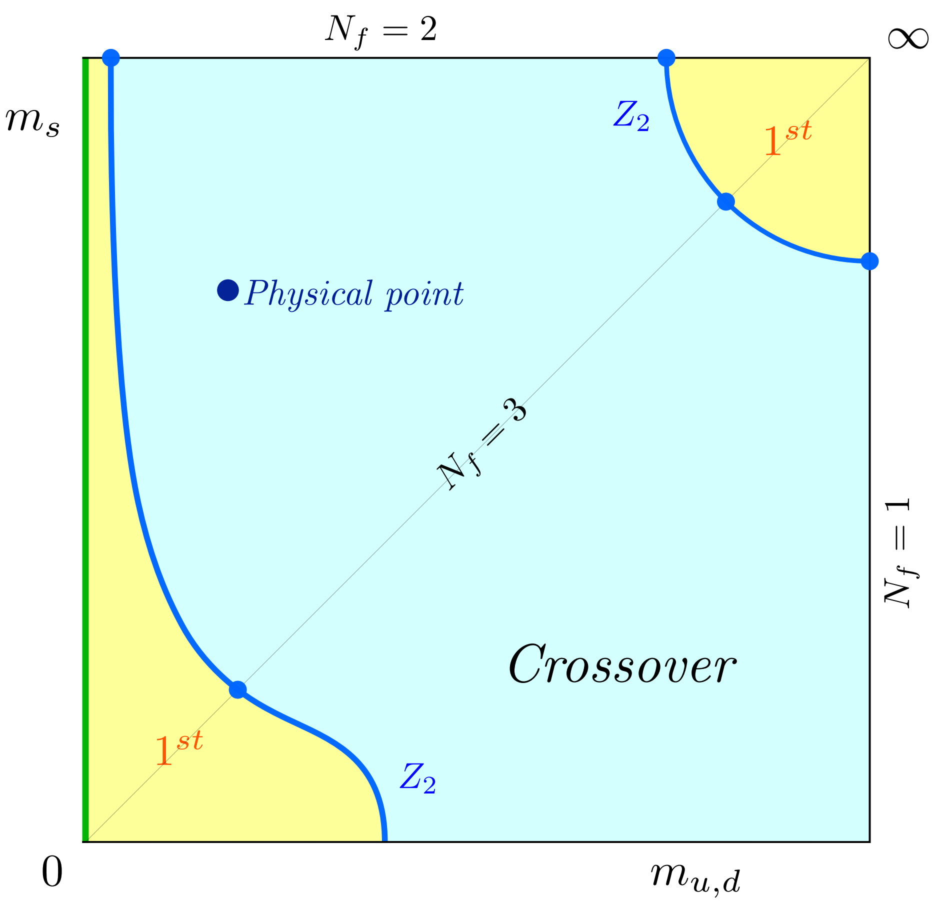



Note: universality is NOT an argument to deduce the QCD phase diagram from models!

e.g. 3d Ising model vs. water: same universality at critical point, different phase diagrams

# The nature of the QCD thermal transition, $\mu_B = 0$



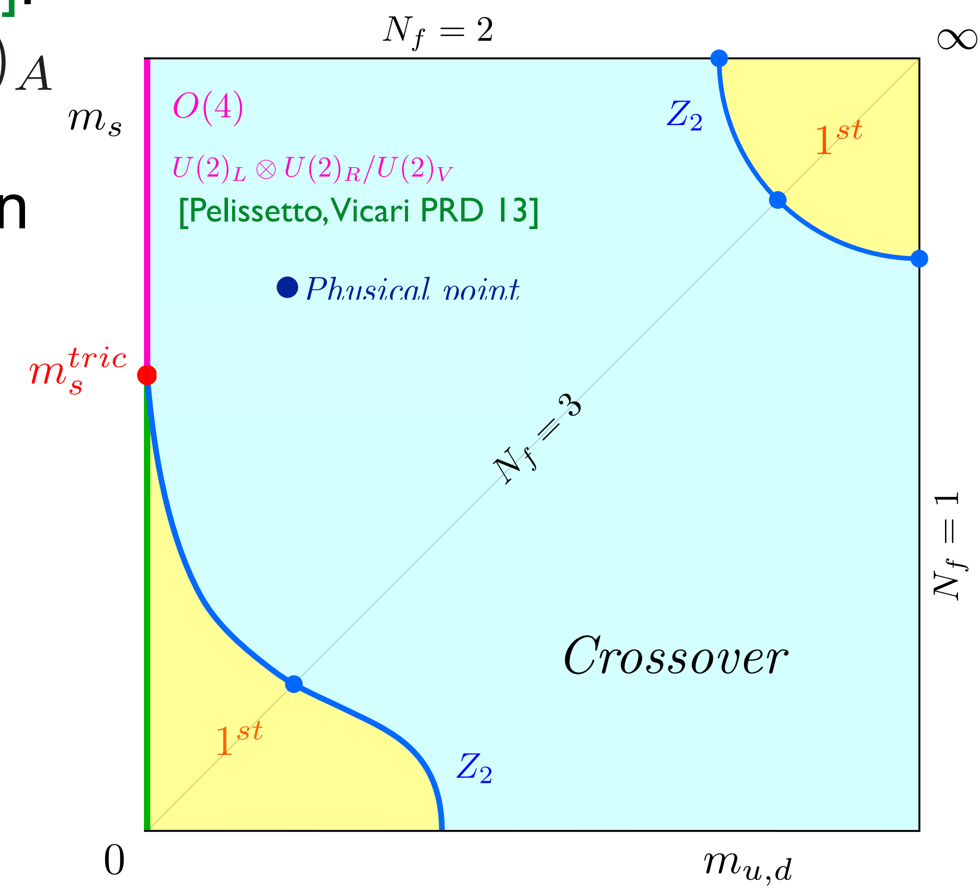

[Pisarski, Wilczek, PRD 84]:

$N_f = 2$ depends on $U(1)_A$

restored

broken

$N_f \geq 3$ 1st order




chiral p.t.

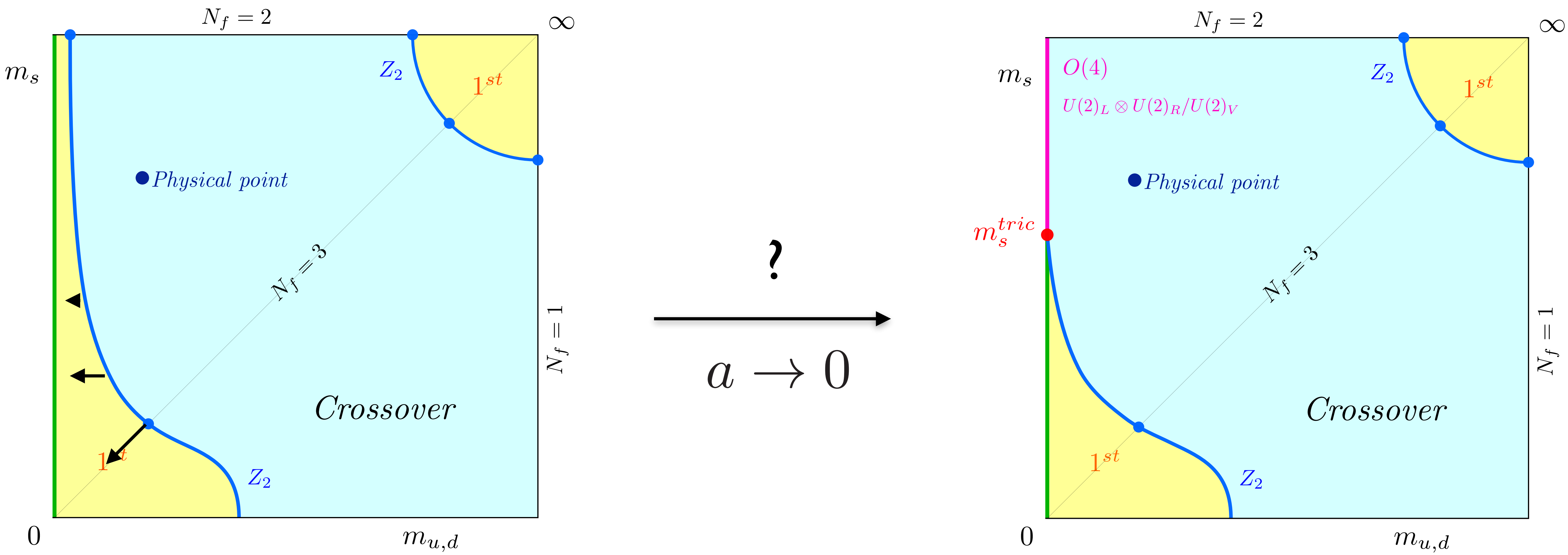
restoration of global symmetry in flavour space

$SU(2)_L \times SU(2)_R \times U(1)_A$

anomalous

# The nature of the QCD chiral transition

…is elusive, massless limit not simulable!

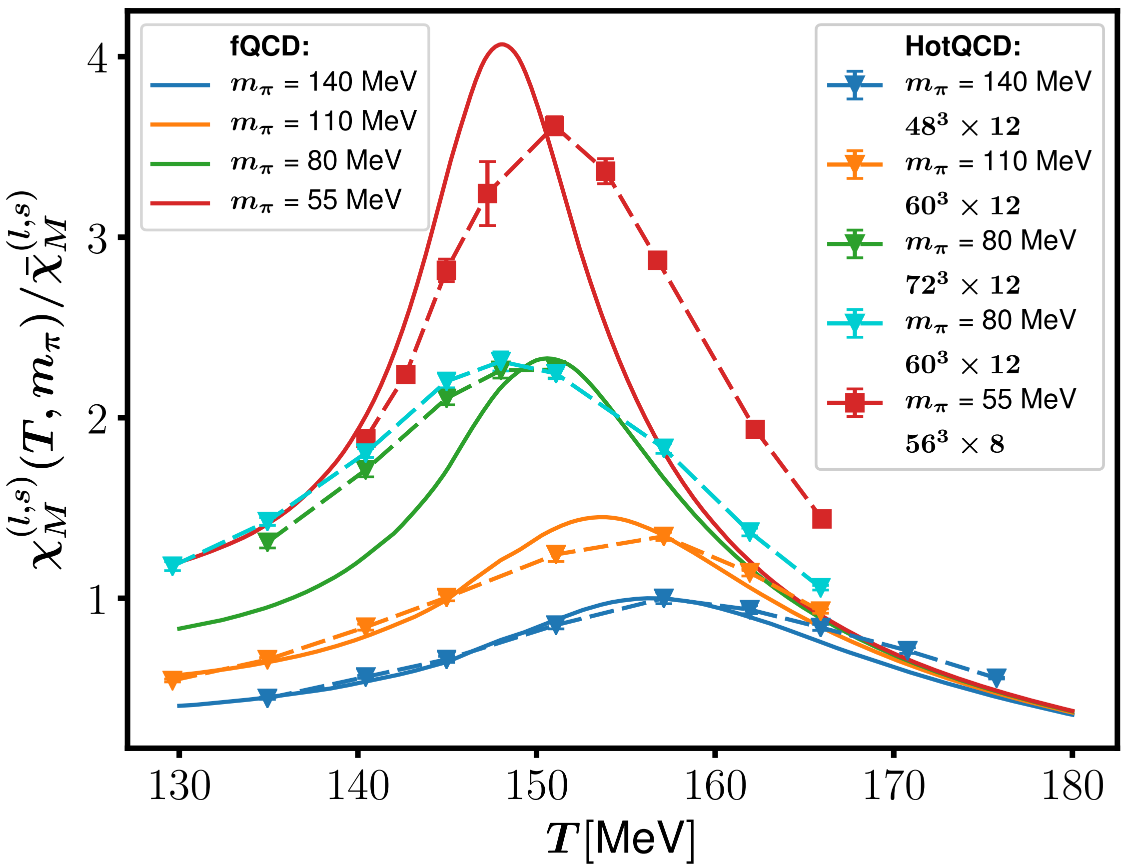


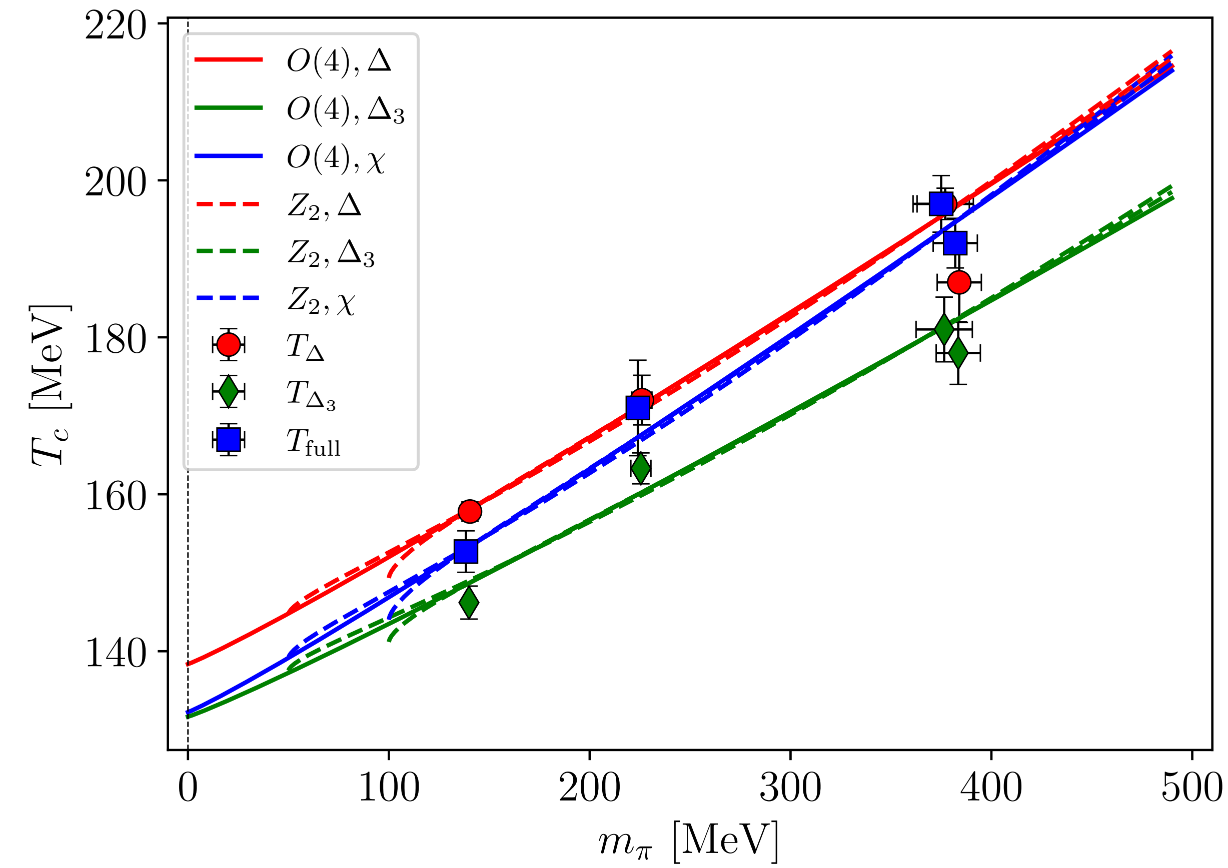

- Coarse lattices or unimproved actions: 1st order for $N_f = 2, 3$
- 1st order region shrinks rapidly as $a \to 0$
- Improved staggered actions: no 1st order region so far, even for $N_f = 3$ $m_{PS} > 45\text{MeV}$ [HotQCD PRL 19]

Details and reference list: [O.P., Symmetry 13, 21]

# From the physical point to the chiral limit

arXiv:2012.06231



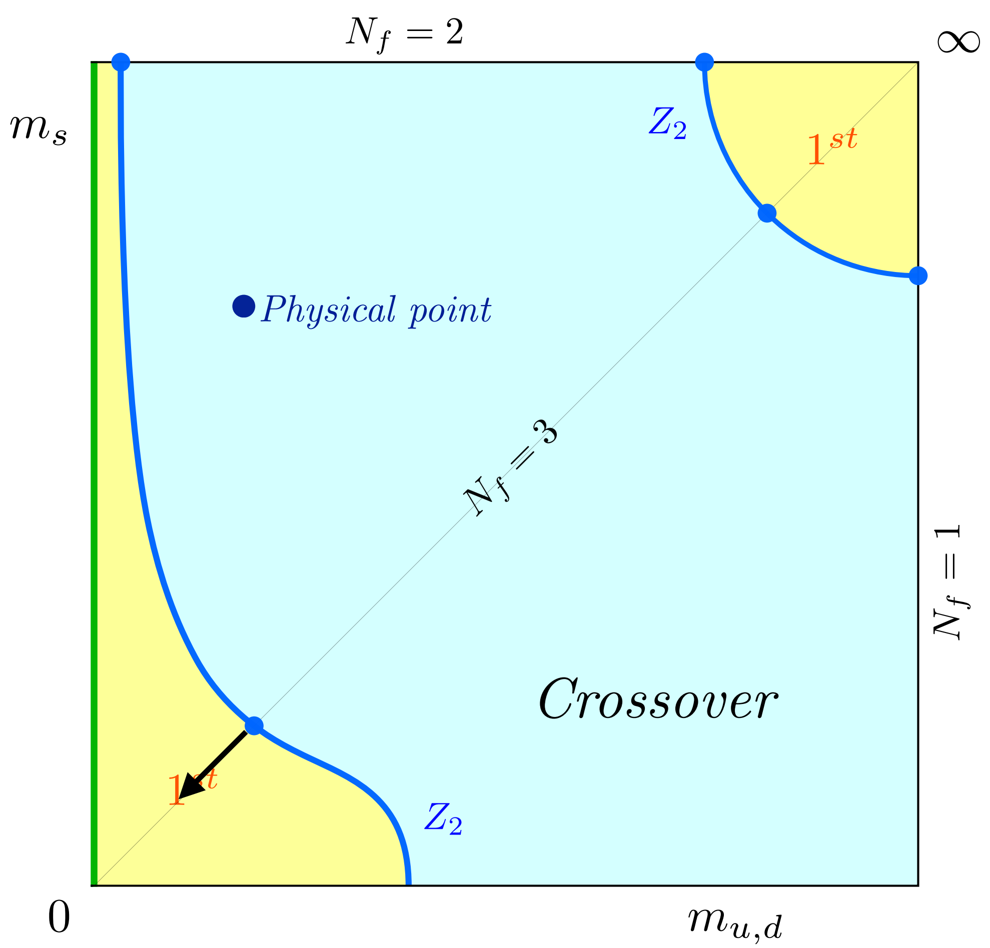

[HotQCD, PRL 19] HISQ (staggered)  [Kotov, Lombardo, Trunin, PLB 21] Wilson twisted mass

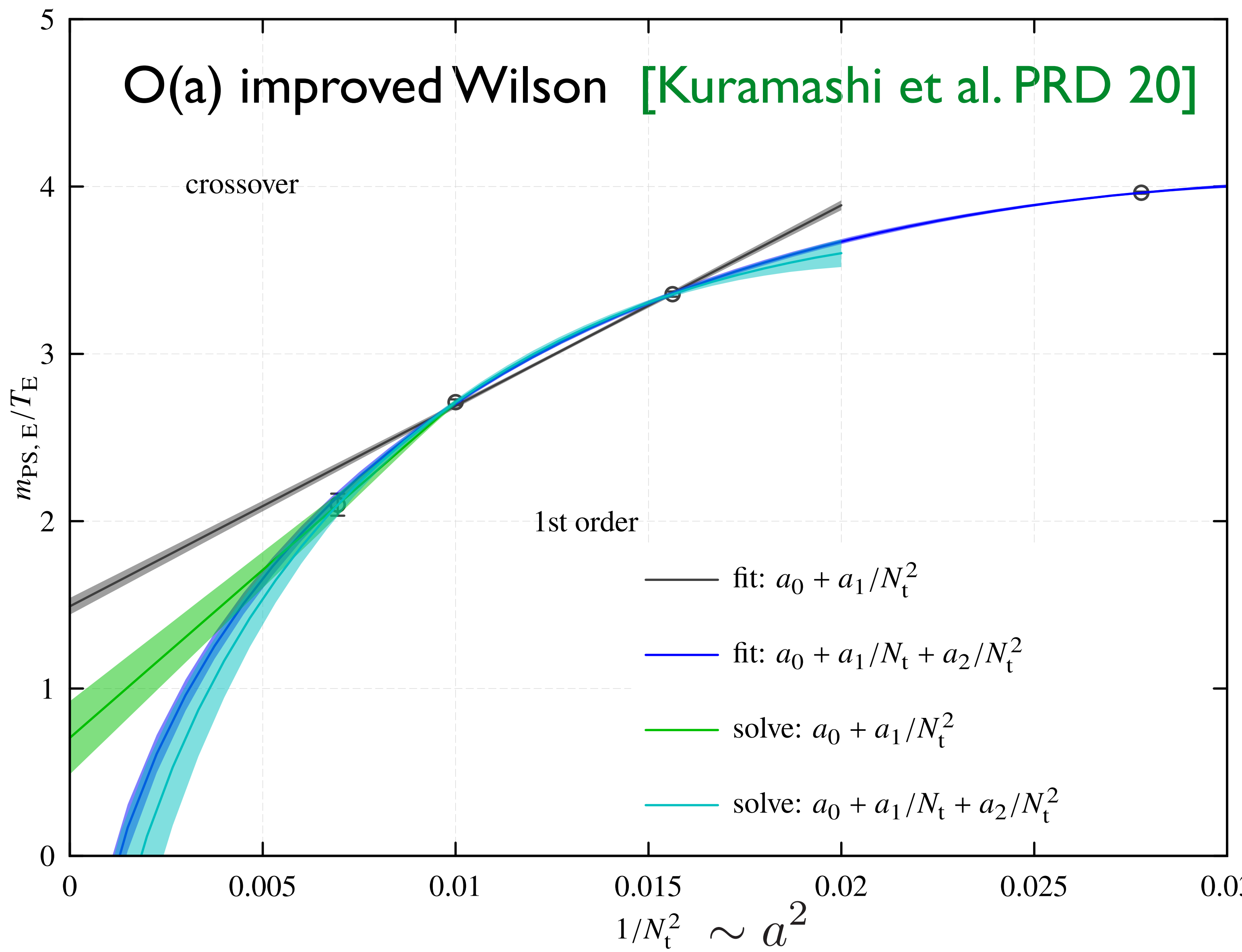
$$T_c^0 = 132^{+3}_{-6}\ \text{MeV} \qquad T_{pc}(m_l) = T_c^0 + K\, m_l^{1/\beta\delta} \qquad T_c^0 = 134^{+6}_{-4}\ \text{MeV}$$

- Keep strange quark mass fixed, crossover gets stronger as chiral limit approached
- Cannot distinguish between Z(2) vs. O(4) exponents, need exponential accuracy!
- Determination of chiral critical temperature possible, but not the order of the transition
- Comparison with fRG: $T_c^0 \approx 142\text{MeV},$ "most likely O(4)" [Braun et al., PRD 20,21]

# The nature of the QCD chiral transition, Nf=3

…has enormously large cut-off effects!

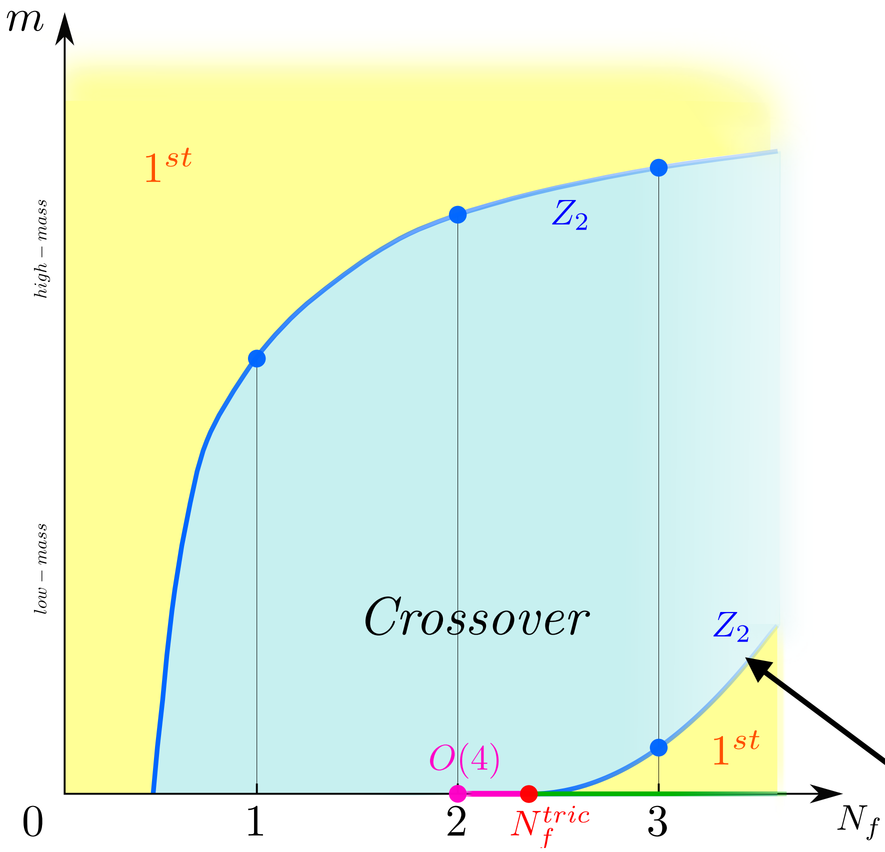


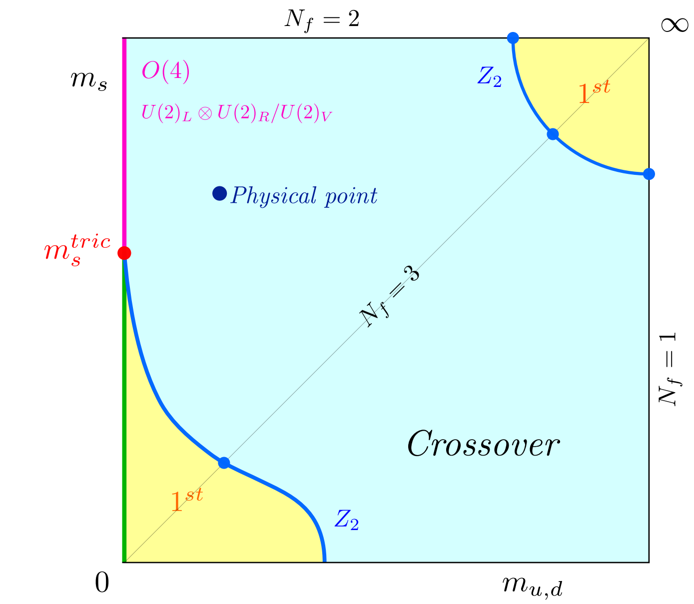




O(a)-improved Wilson:
1st order region shrinks for $a \to 0$

$m_\pi^c \leq 110$ MeV $\quad N_\tau = 4, 6, 8, 10, 12$

# Different view point: mass degenerate quarks



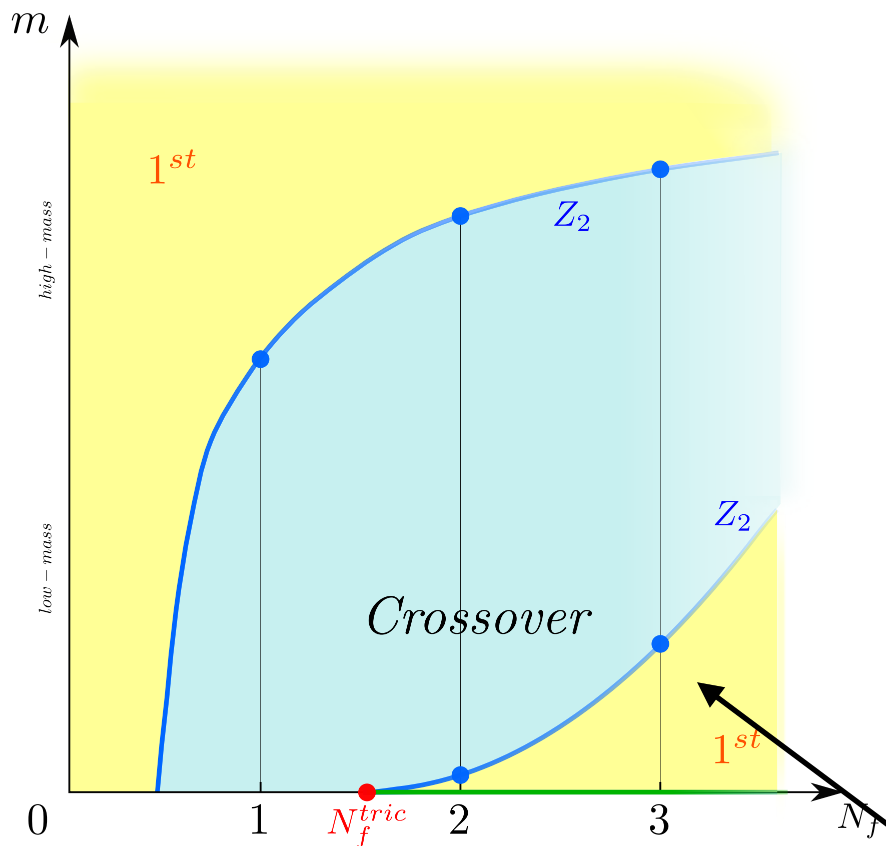

$$Z(N_\mathrm{f}, g, m) = \int \mathcal{D}A_\mu \, (\det M[A_\mu, m])^{N_\mathrm{f}} \, e^{-\mathcal{S}_\mathrm{YM}[A_\mu]}$$

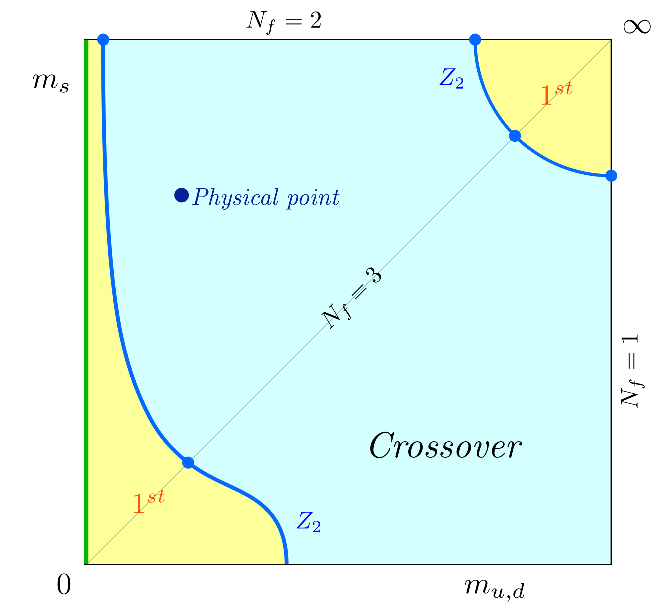
$$N_\mathrm{f}^c(am) = N_\mathrm{f}^\mathrm{tric} + \mathcal{B}_1 \cdot (am)^{2/5} + \mathcal{O}\left((am)^{4/5}\right)$$

- Consider analytic continuation to continuous $N_f$
- Tricritical point guaranteed to exist if there is 1st order at any $N_f$
- Known exponents for critical line entering tric. point!
- Continuation to $a \neq 0$: Z(2) surface ends in tricritical line

[Cuteri, O.P., Sciarra PRD 18]

# Different view point: mass degenerate quarks




$$Z(N_{\rm f}, g, m) = \int \mathcal{D}A_\mu \, (\det M[A_\mu, m])^{N_{\rm f}} \, e^{-\mathcal{S}_{\rm YM}[A_\mu]}$$

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[Cuteri, O.P., Sciarra PRD 18]

# Methodology to determine order of transition

Finite size scaling of generalised cumulants

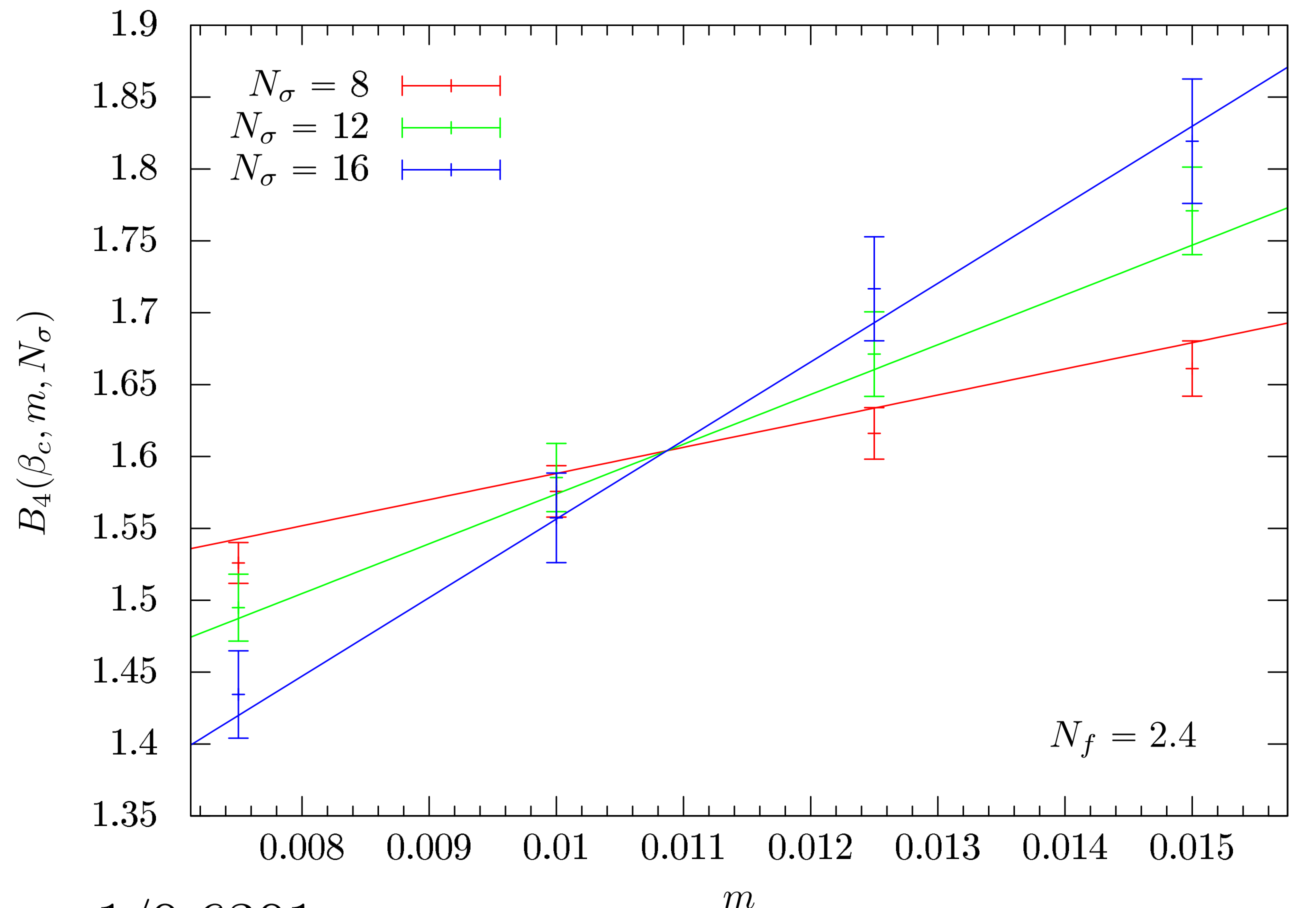
$$B_n = \frac{\langle (\bar{\psi}\psi - \langle \bar{\psi}\psi \rangle)^n \rangle}{\langle (\bar{\psi}\psi - \langle \bar{\psi}\psi \rangle)^2 \rangle^{n/2}}$$

Standard staggered fermions, bare parameters: $\beta, am, N_f, N_\tau$

(Pseudo-critical) phase boundary: $B_3 = 0$ 3d manifold

Second-order 3d Ising:

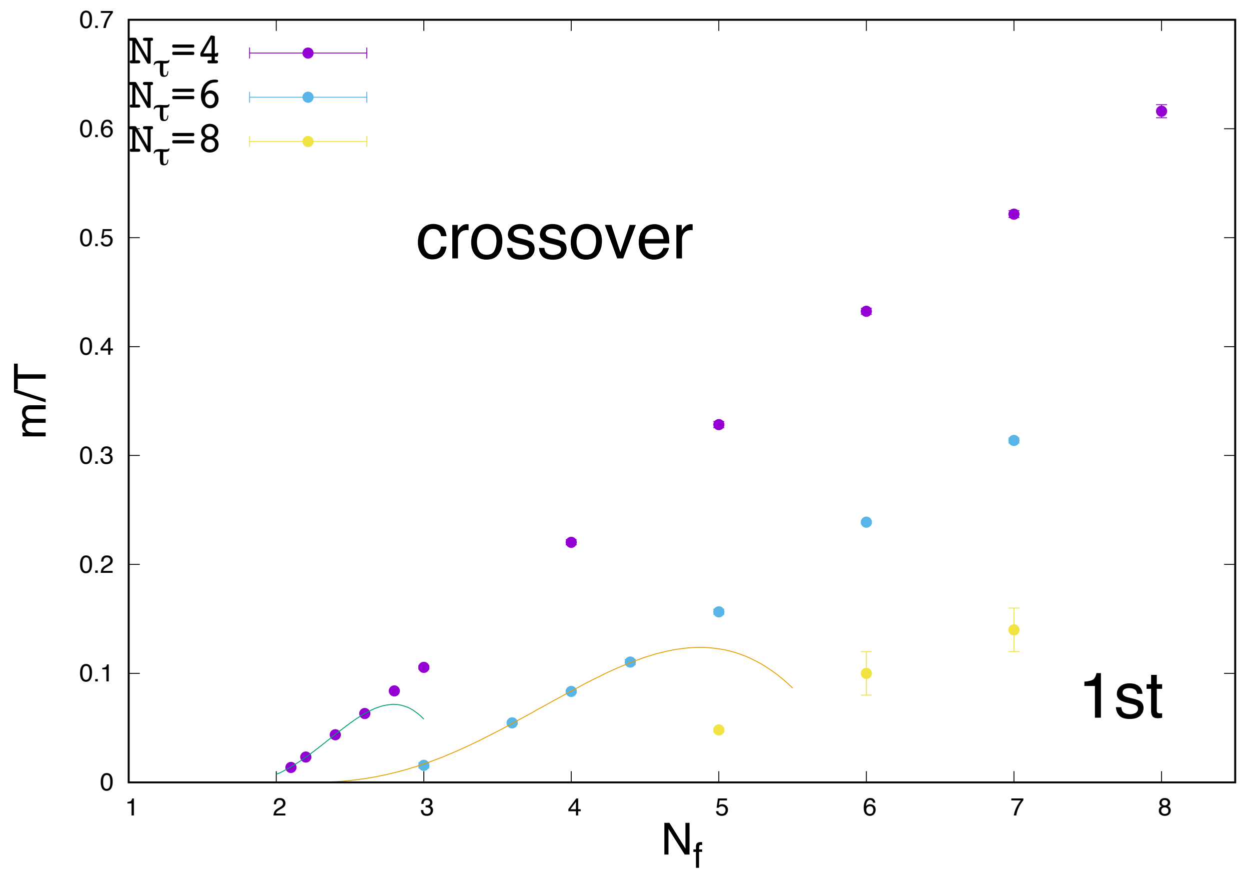
2d chiral critical surface separates 1st order from crossover

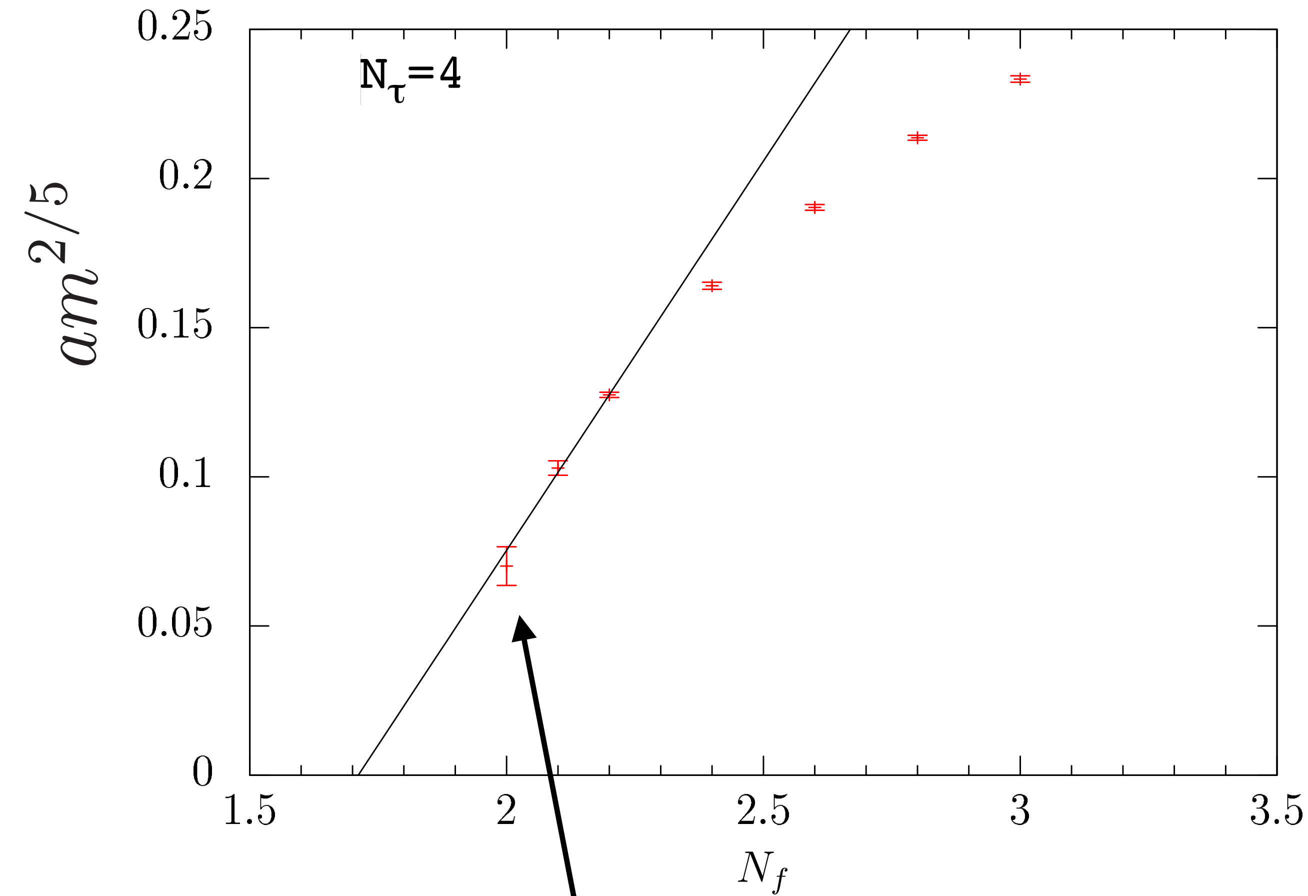



$$B_4(\beta_c, am, N_\sigma) \approx 1.604 + c\,(am - am_c)\,N_\sigma^{1/0.6301}$$

# Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra 21]  ~120 M Monte Carlo trajectories with light fermions, aspect ratios 3,4,5



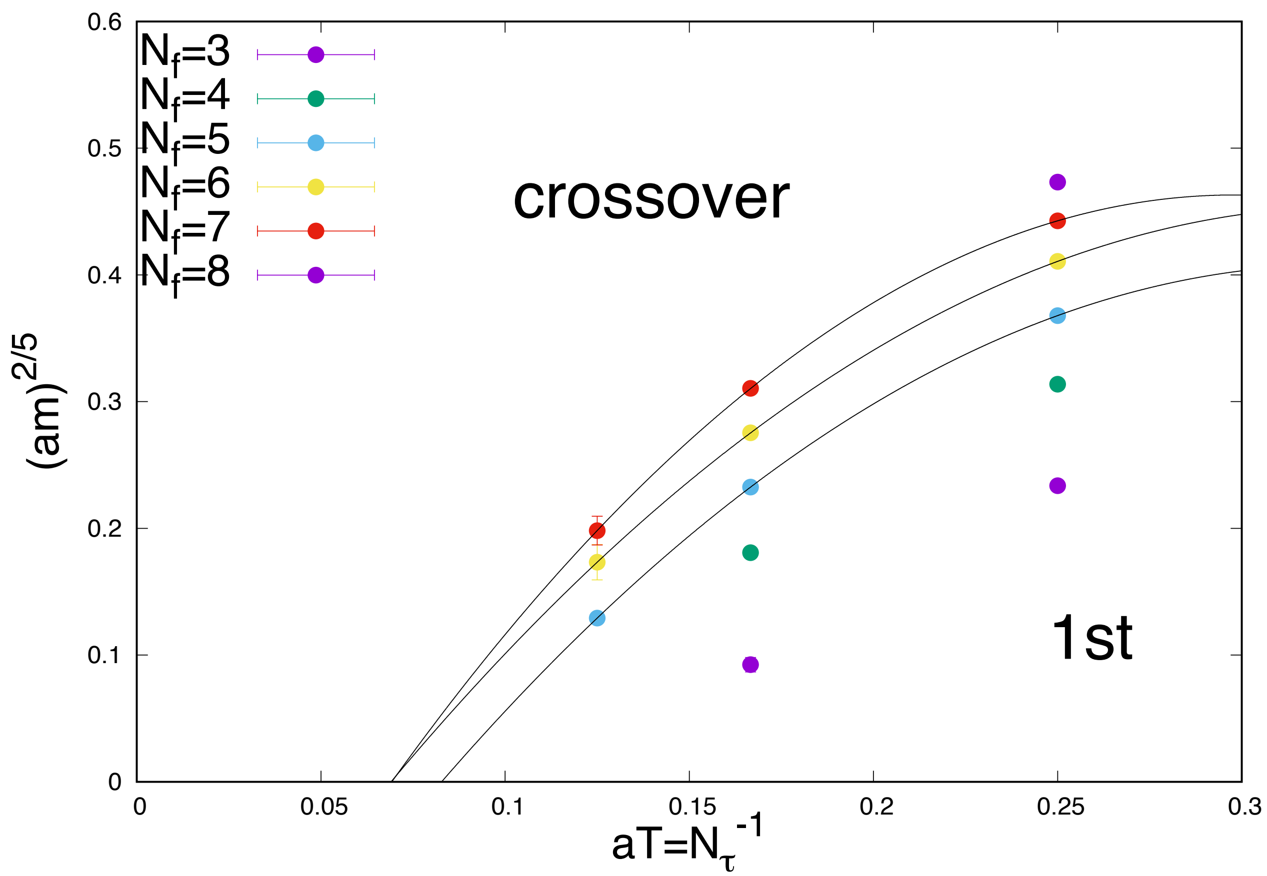



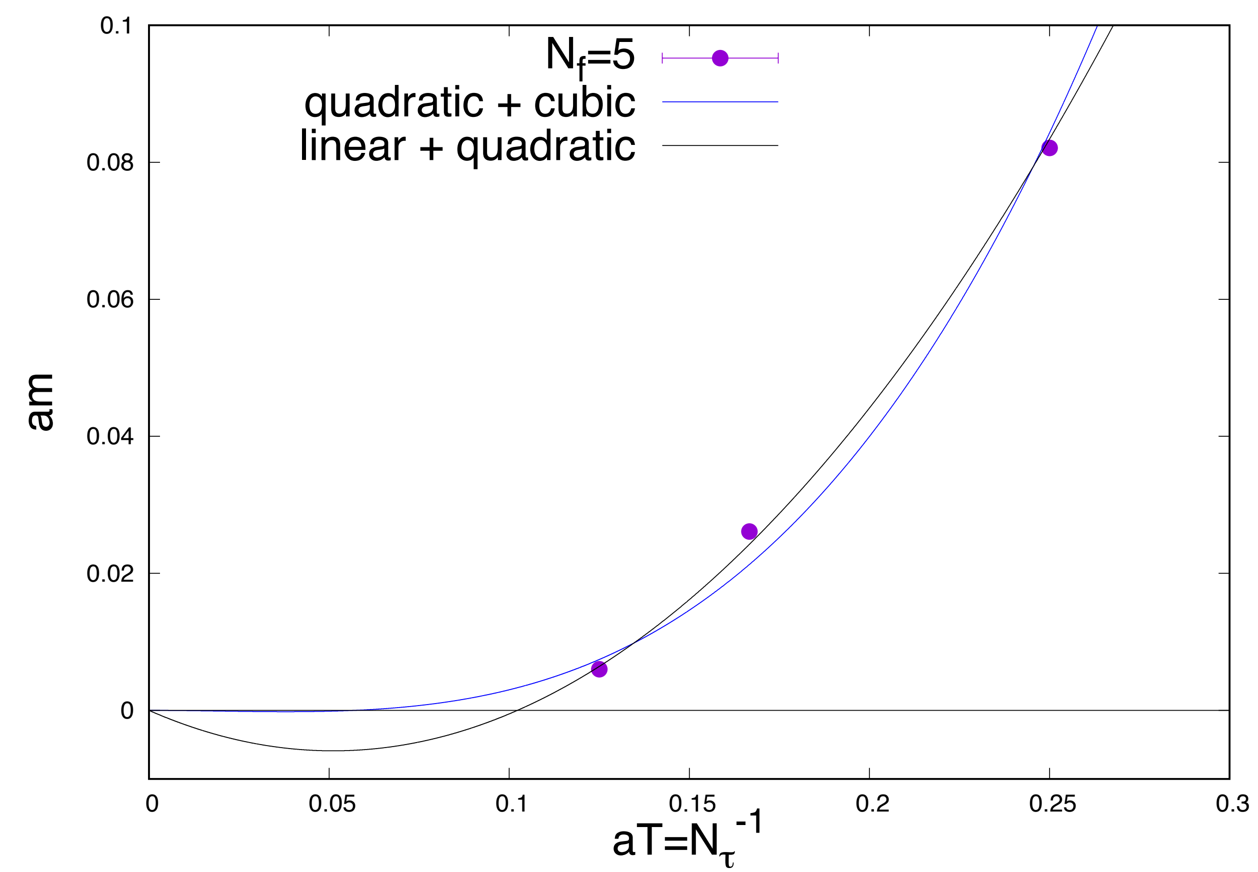

- Tricritical scaling observed in different variable pairings
- Consistent with tric. scaling from finite imaginary $\mu$  [Bonati et al. PRD 14]
- Old question: $m_c/T = 0 \text{ or } \neq 0$ ?  Answered for $N_f = 2$
- New question: will $N_f^{\text{tric}}$ slide beyond $N_f = 3$ ?

# Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra JHEP 21]

1st order scenario does not fit!



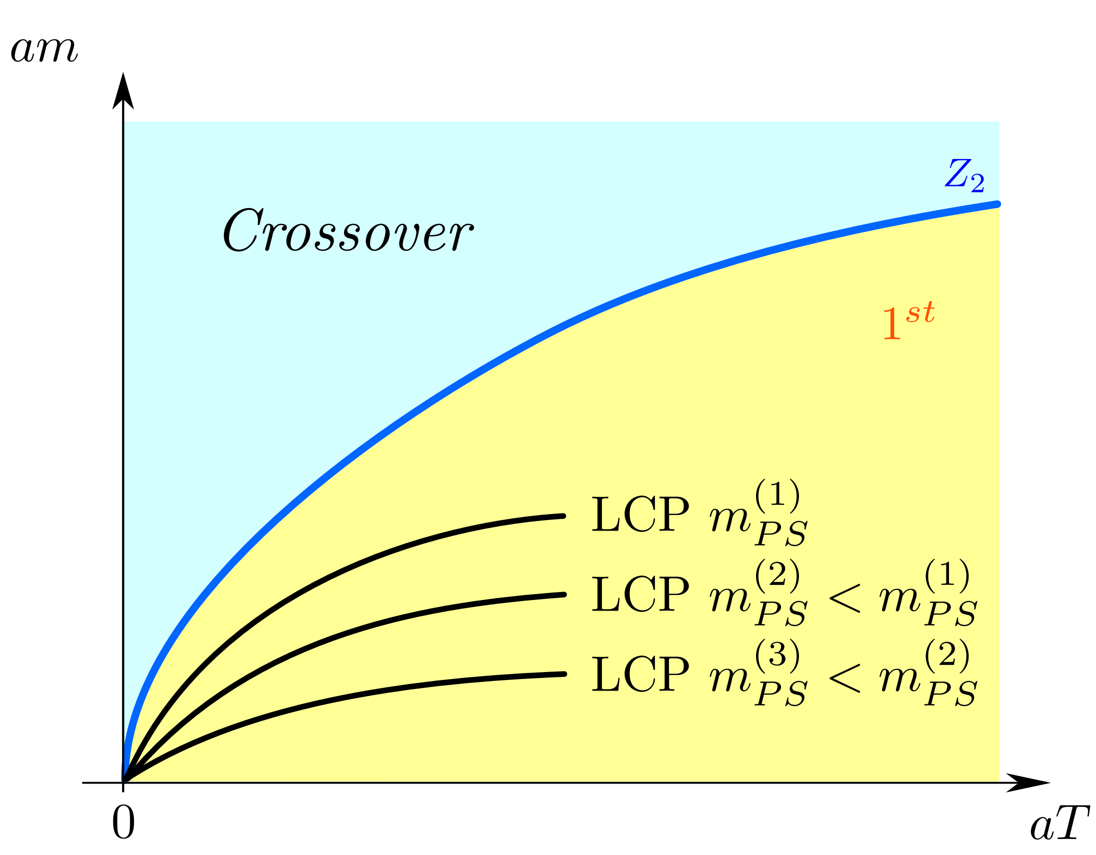




- Tricritical scaling observed also in plane of mass vs. lattice spacing
- Allows extrapolation to lattice chiral limit, tricritical points $N_\tau^{\rm tric}(N_f)$
- 1st order scenario: $m_c(a) = m_c(0) + c_1(aT) + c_2(aT)^2 + \ldots$

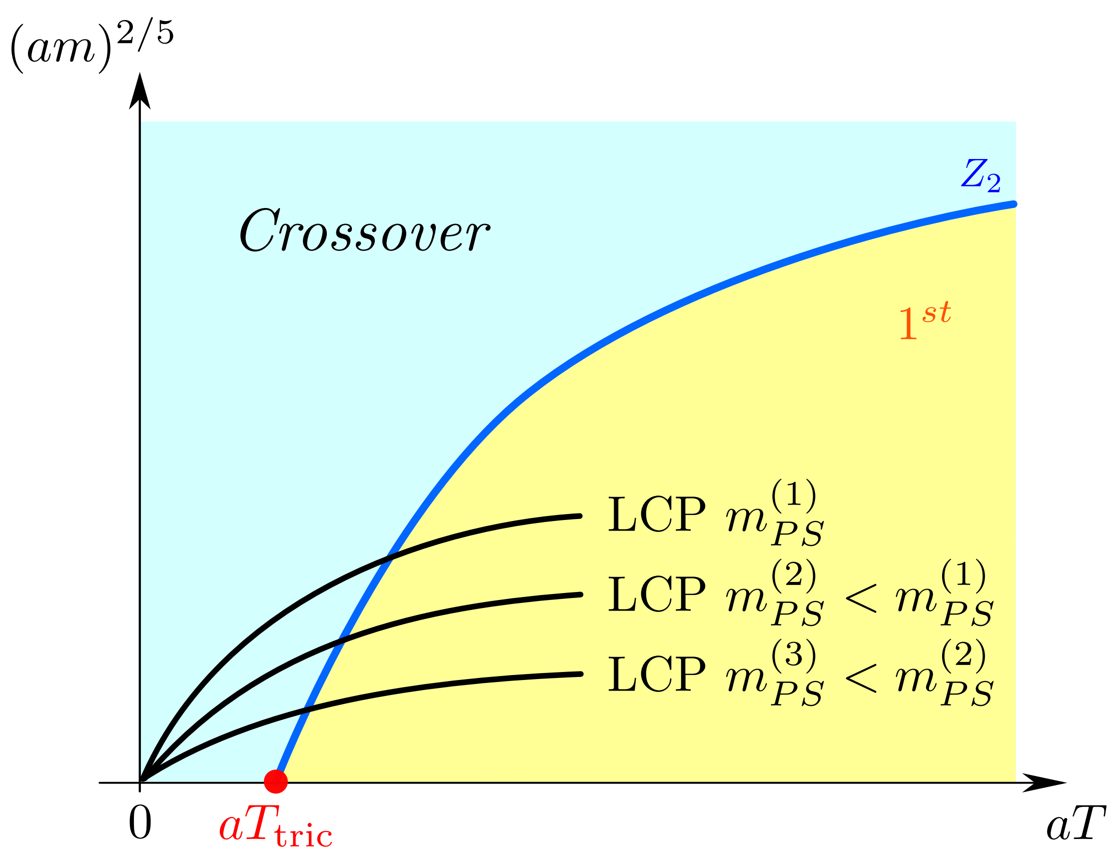
Incompatible with data! $\chi^2_{\rm dof} > 10$

# Implications for the continuum

- Finite $N_\tau^{\mathrm{tric}}(N_f)$ implies that 1st order transition is not connected to continuum
- Approaching continuum first, then chiral limit:
  Continuum chiral phase transition second-order!




1st order scenario

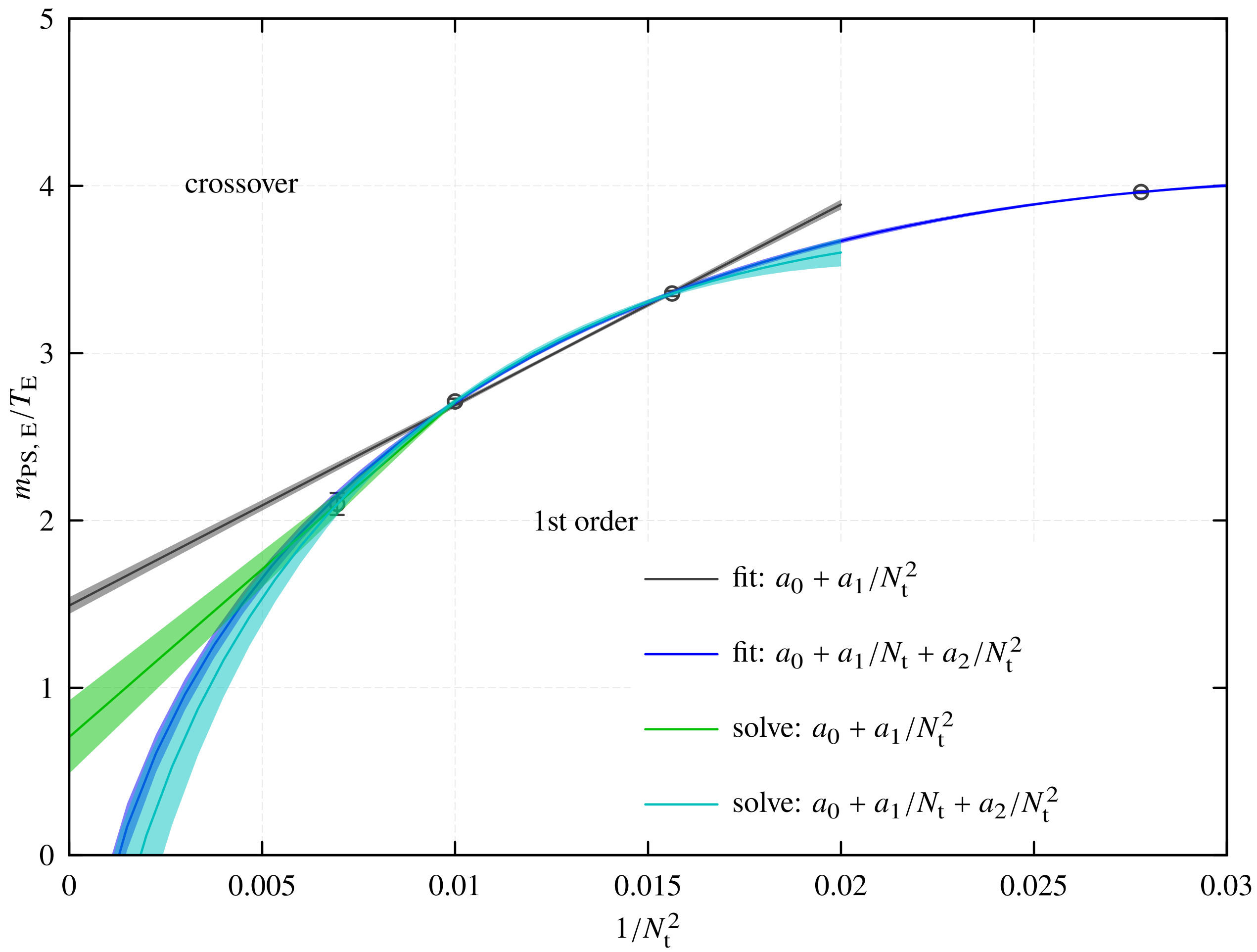


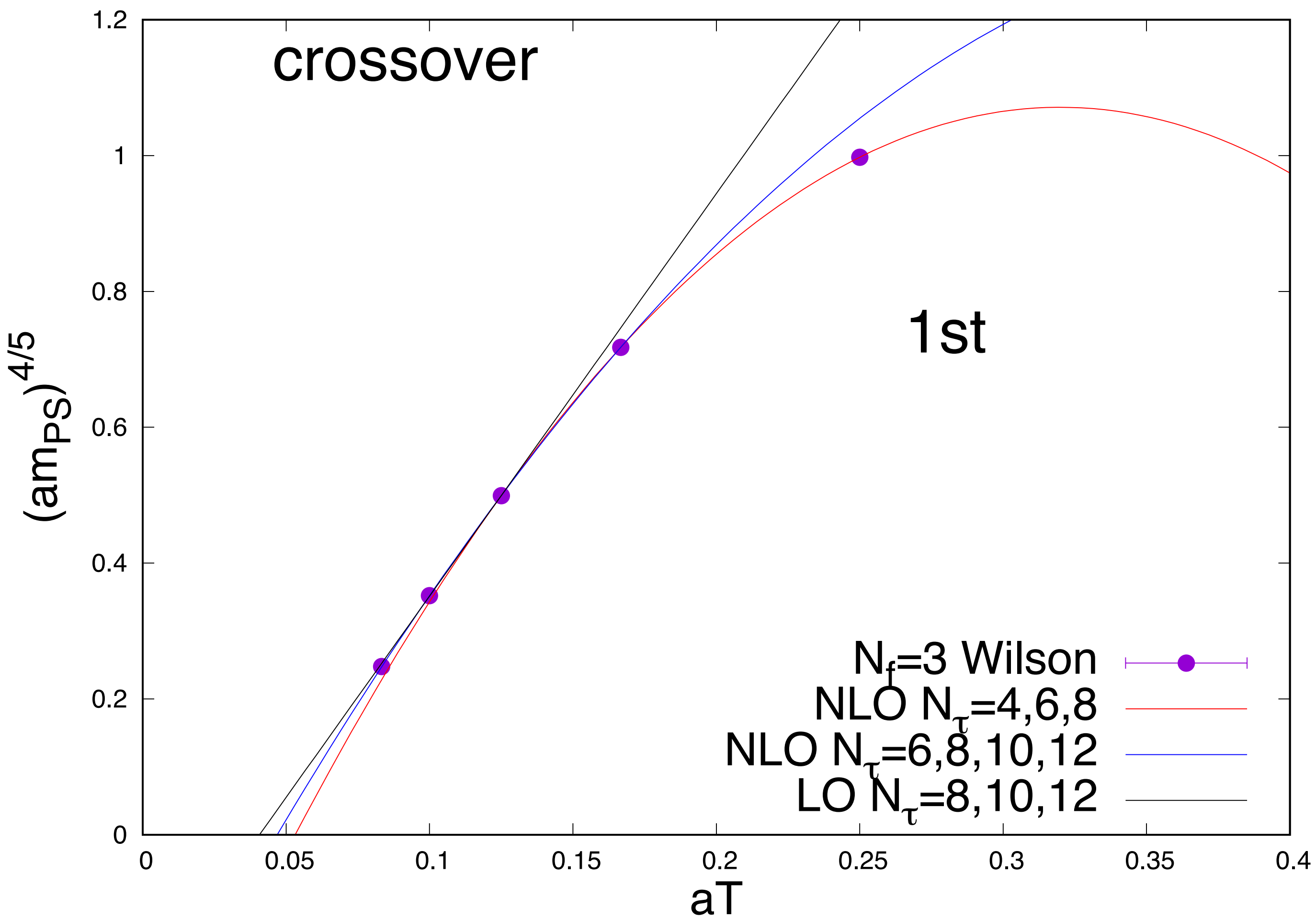

2nd order scenario

# Nf=3 O(a)-improved Wilson fermions

[Kuramashi et al. PRD 20] $m_\pi^c \leq 110\ \mathrm{MeV} \quad N_\tau = 4, 6, 8, 10, 12$

Re-analysis using: $am_{PS}^2 \propto am_q$



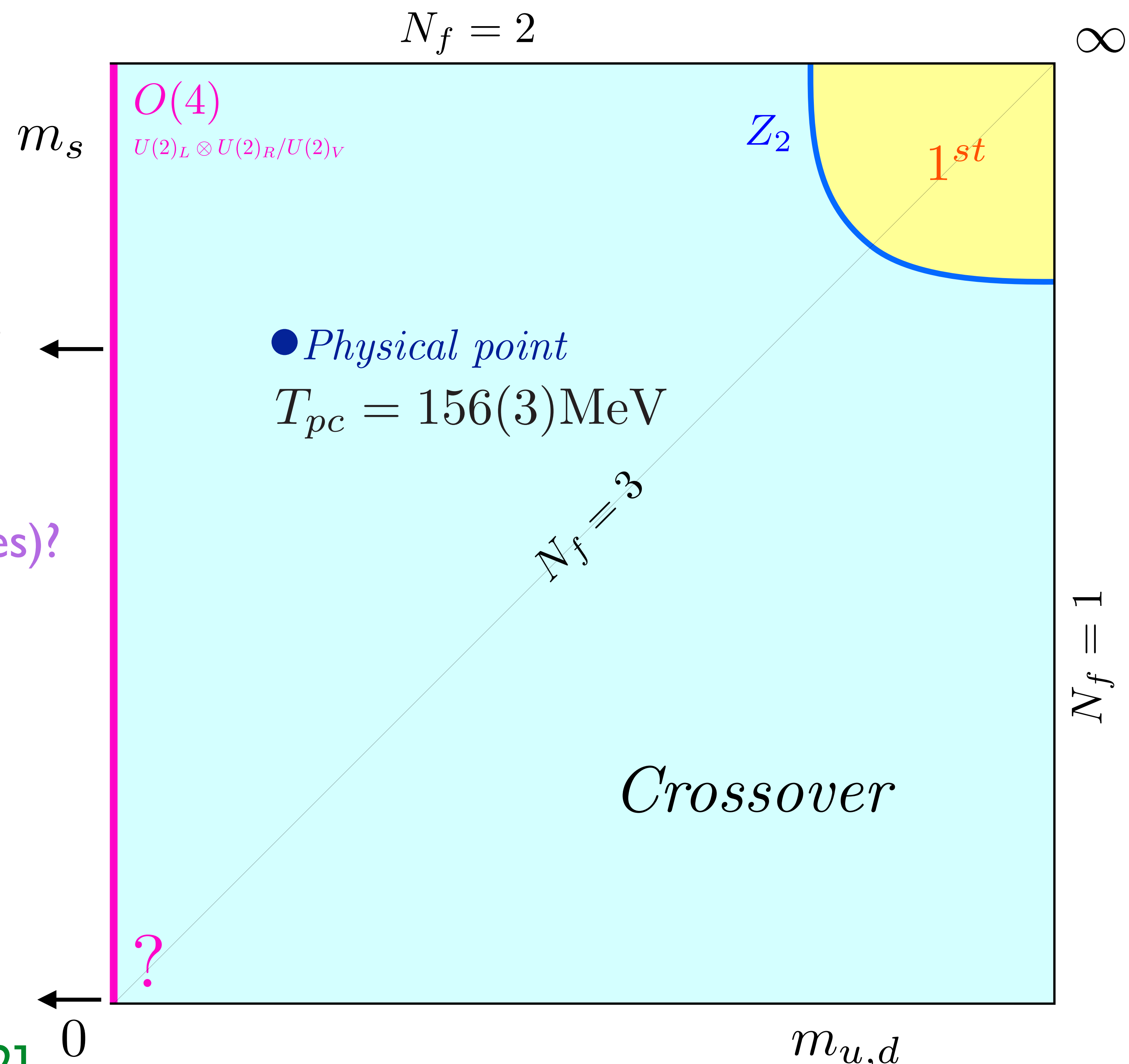




Tricritical scaling! [Cuteri, O.P., Sciarra, JHEP 21]

Nf=3 consistent with staggered, 2nd order in chiral continuum limit!

# The Columbia plot in the continuum

[Cuteri, O.P., Sciarra JHEP 21]




$T_c^0 = 135(8)\text{MeV}$ ←
[HotQCD PRL 19]

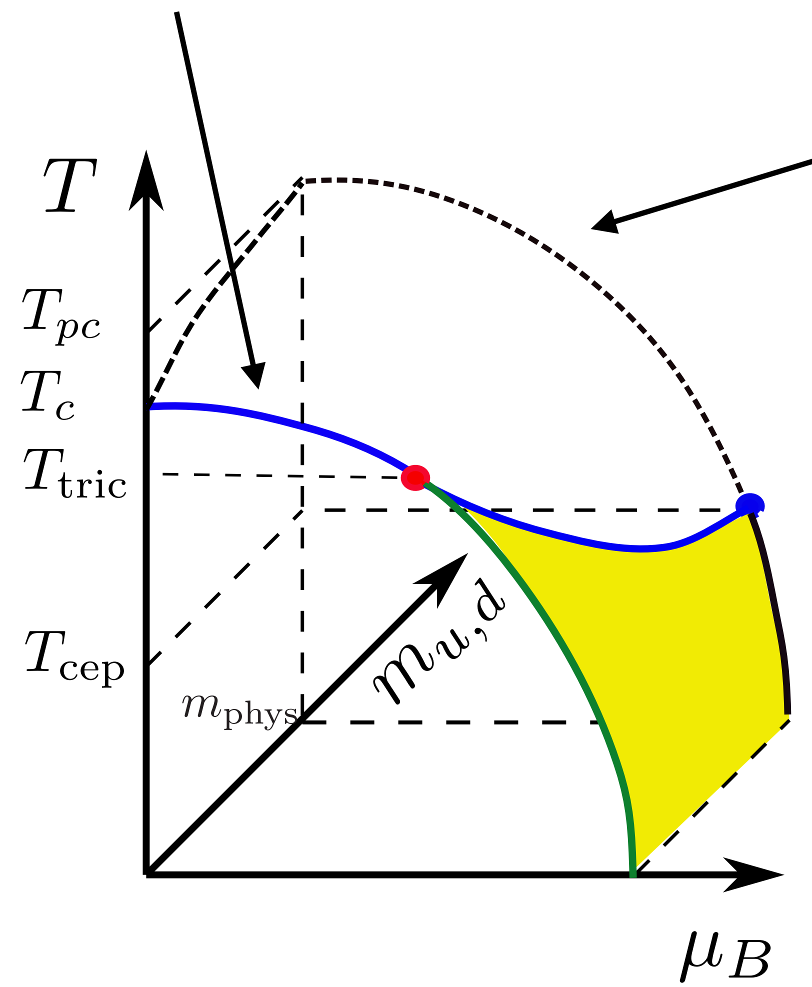
Universality class(es)?

$T_c^0 = 98(6)\text{MeV}$ ←
[HotQCD PRD 22]

# From the chiral limit to the physical point

The "standard scenario": [Halasz et al., PRD 98; Hatta, Ikeda, PRD 03…]

Importance of the chiral limit!



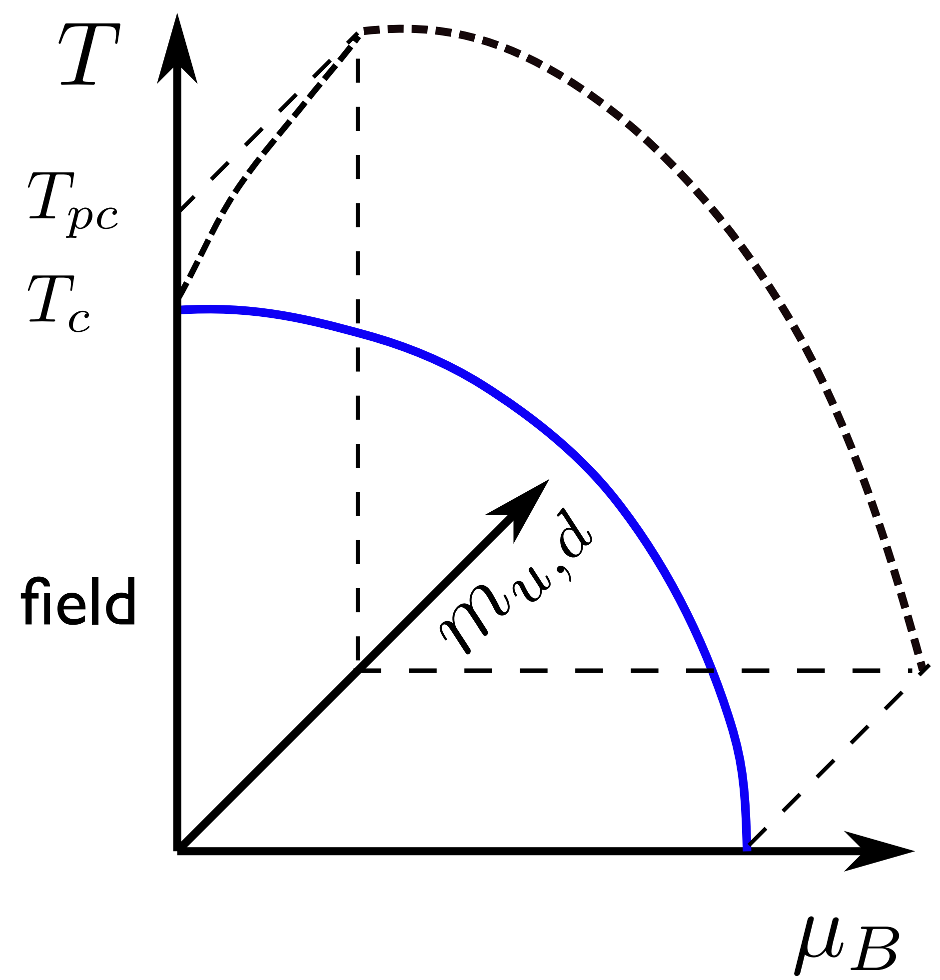

$$\frac{T_{pc}(\mu_B)}{T_{pc}(0)} = 1 - \kappa_2 \left( \frac{\mu_B}{T_{pc}(0)} \right)^2 + \ldots$$

| $\kappa_2$ | Action | |
|---|---|---|
| 0.0158(13) | imag. $\mu$, stout-smeared staggered | [Bellwied et al, PLB 15] |
| 0.0135(20) | imag. $\mu$, stout-smeared staggered | [Bonati et al, NPA 19] |
| 0.0145(25) | Taylor, stout-smeared staggered | [Bonati et al, PRD 18] |
| 0.016(5) | Taylor, HISQ | [HotQCD, PLB 19] |

$T_{pc} > T_c > T_{\rm tric} > T_{\rm cep}$ → $\mu_B^{\rm cep} > 3.1\ T_{pc}(0) \approx 485$ MeV

# But what if…… ?!?

- ….the chiral limit is all second order?
- Then for physical masses: all crossover!
- So far consistent with all available lattice results
- Predicted by nucleon-meson models, beyond mean field [Brandes, Kaiser, Weise, EPJA 21]
- Need to rule out one or the other scenario!



# A surprise: emergent chiral spin symmetry

Chiral spin transformation, $SU(2)_{CS}$: $\psi \to \psi' = \exp\left(i\frac{\varepsilon^n \Sigma^n}{2}\right)\psi$ $\Sigma^n = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}$

$$SU(2)_{CS} \otimes SU(2)_V \simeq SU(4) \supset SU(2)_L \times SU(2)_R \times U(1)_A$$

QCD quark action, chiral limit: $\bar{\psi}\gamma^\mu D_\mu \psi = \bar{\psi}\gamma^0 D_0 \psi + \bar{\psi}\gamma^i D_i \psi$

$[\Sigma^a, \gamma_0] = 0, [\Sigma^a, \gamma_i] \neq 0,$

CS invariant ($\bar{\psi}\gamma^0 D_0 \psi$) breaks CS ($\bar{\psi}\gamma^i D_i \psi$)

Necessary condition for approximate CS symmetry:

Quantum effective action dynamically dominated by colour-electric interactions!

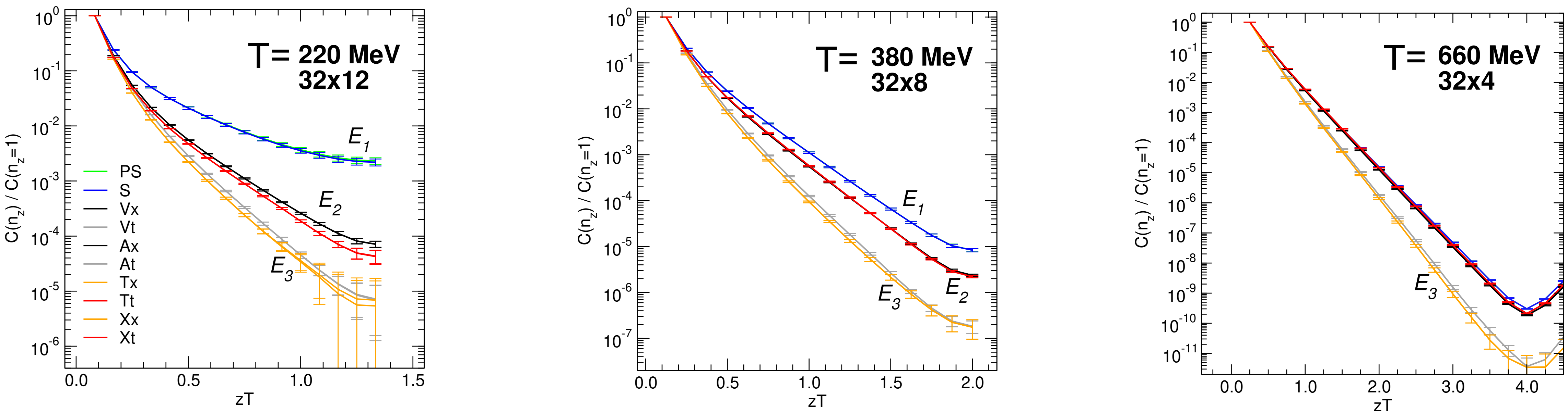
# CS-symmetry observed in meson correlators

JLQCD domain wall fermions at phys. point

Spatial correlators: [Rohrhofer et al., PRD 19]

confirmed by [Chiu, arxiv:2302.06073]

$$E_1: \quad PS \leftrightarrow S\,, \qquad U(1)_A$$
$$E_2: \quad V_x \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x\,, \qquad SU(4)$$
$$E_3: \quad V_t \leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t\,. \qquad SU(2)_L \times SU(2)_R \times U(1)_A$$

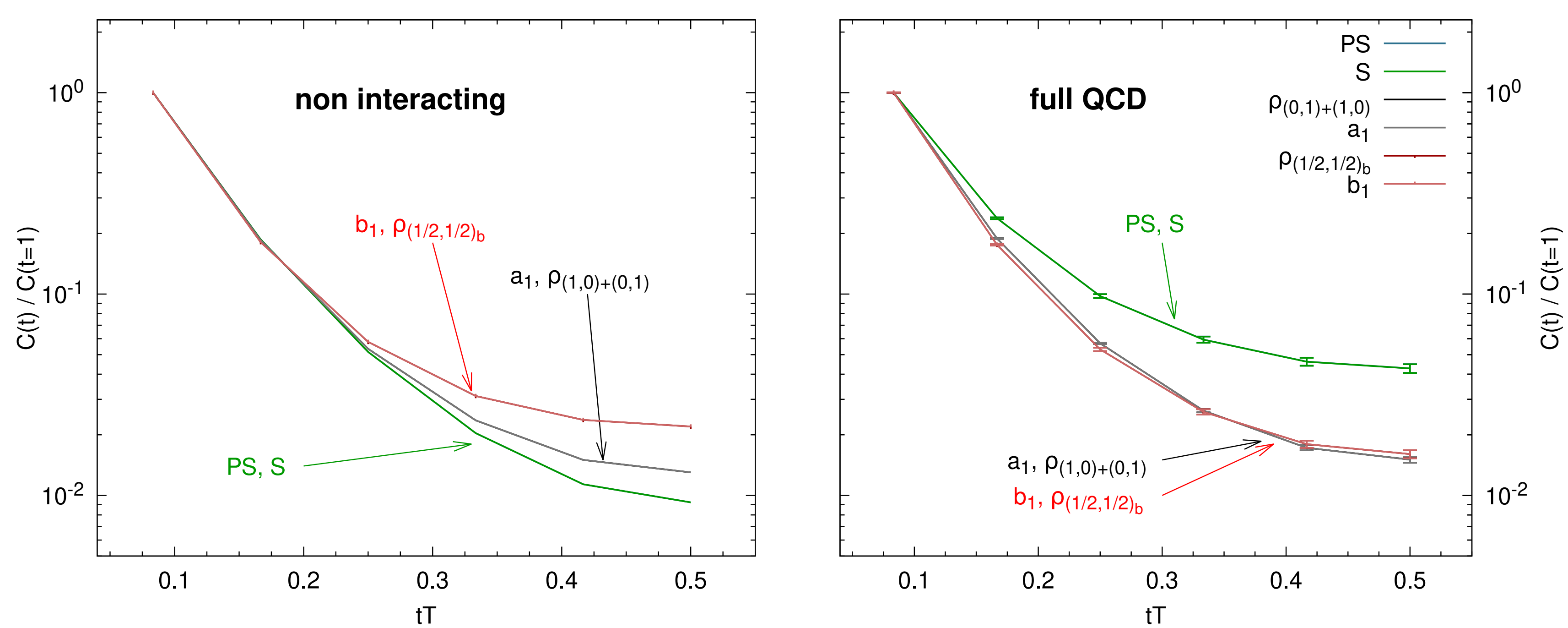



Temporal correlators:

[Rohrhofer et al., PLB 20]

$T = 220\text{MeV}\ (1.2T_c)$

$48^3 \times 12 \qquad (a = 0.075\ \text{fm})$



# Three temperature regimes of QCD

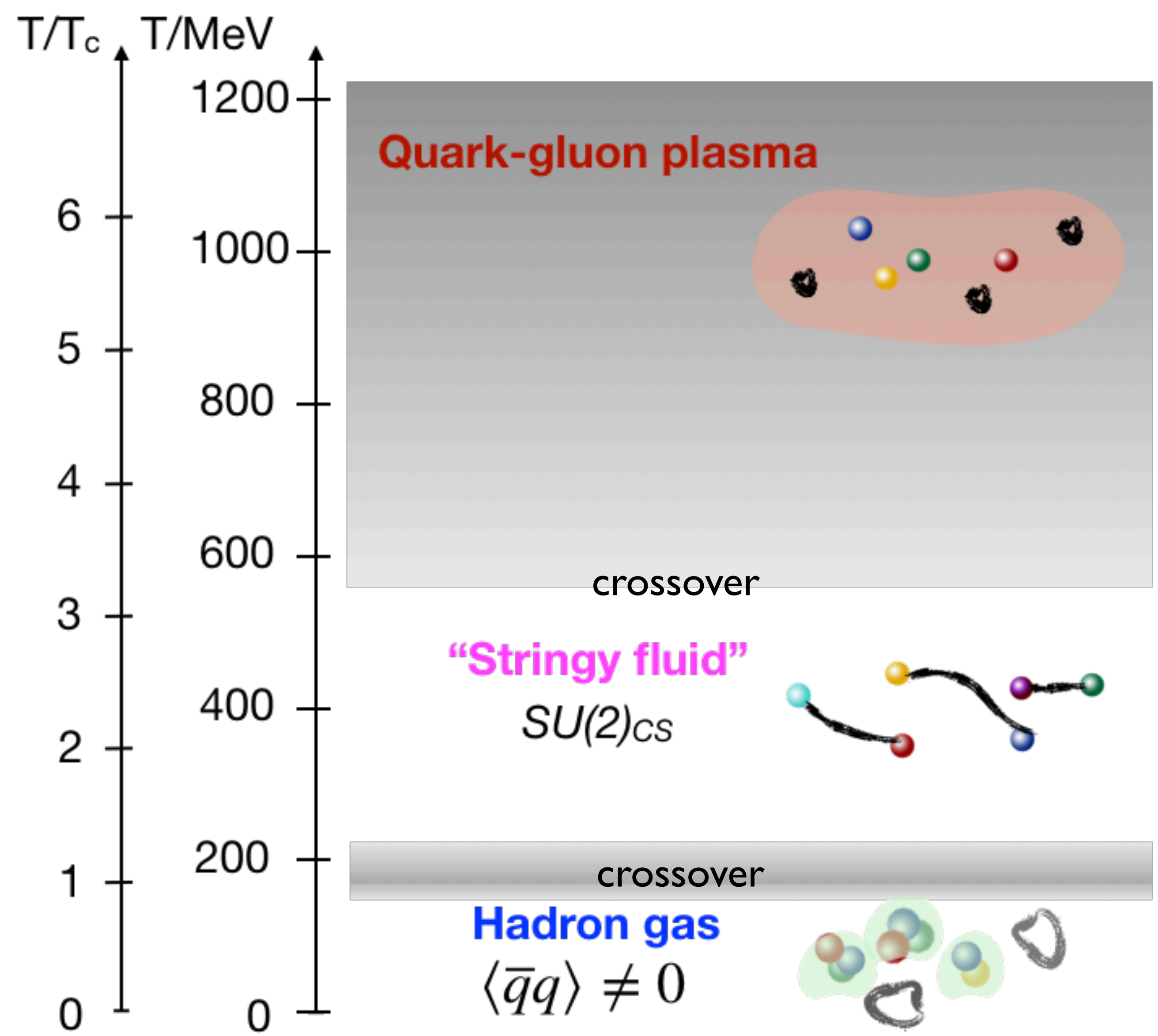
Symmetries (verified):

Degrees of freedom (to be verified):

Chiral symmetry (approximate)

Chiral spin symmetry (approximate)

Chiral symmetry broken




Rohrhofer et al., Phys. Rev. D100 (2019)

# Check well-studied observables: screening masses

$$C^s_\Gamma(z) = \sum_{x,y,\tau} C_\Gamma(\tau, \boldsymbol{x}) \quad \stackrel{z\to\infty}{\longrightarrow} \text{const. } e^{-m_{scr}z}$$

Directly related to the partition function and equation of state

by transfer matrices: $T = e^{-aH}, T_z = e^{-aH_z}$

$$e^{pV/T} = Z = \mathrm{Tr}(e^{-aHN_\tau})$$
$$= \mathrm{Tr}(e^{-aH_zN_z}) = \sum_{n_z} e^{-E_{n_z}N_z}$$

Screening masses: eigenvalues of $H_z$

For T=0 equivalent to eigenvalues of $H$, for $T \neq 0$ “finite size effect”

# Meson screening masses at high temperatures

[Dalla Brida et al., JHEP 22]

Nf=3, T=1 GeV -160 GeV

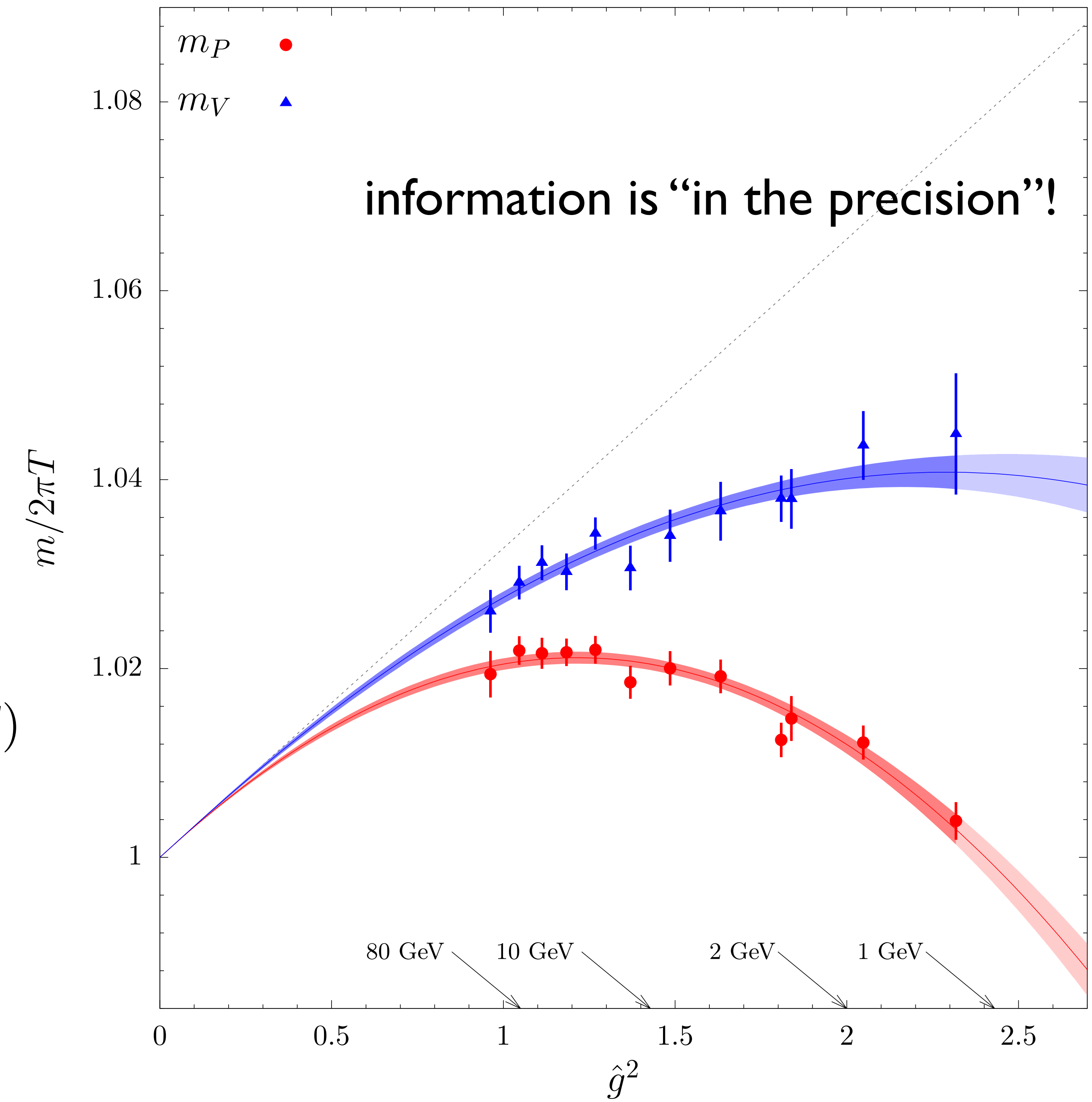
Highly non-trivial technically:
shifted b.c. + step-scaling techniques
(Alpha-Collaboration)

$$\frac{m_{PS}}{2\pi T} = 1 + p_2\,\hat{g}^2(T) + p_3\,\hat{g}^3(T) + p_4\,\hat{g}^4(T)$$
$$\frac{m_V}{2\pi T} = \frac{m_{PS}}{2\pi T} + s_4\,\hat{g}^4(T)$$

$p_2 = 0.032739961$
[Laine, Vepsäläinen., JHEP 04]

$p_3, p_4, s_4$ fitted, excellent $\chi^2_{\rm dof}$

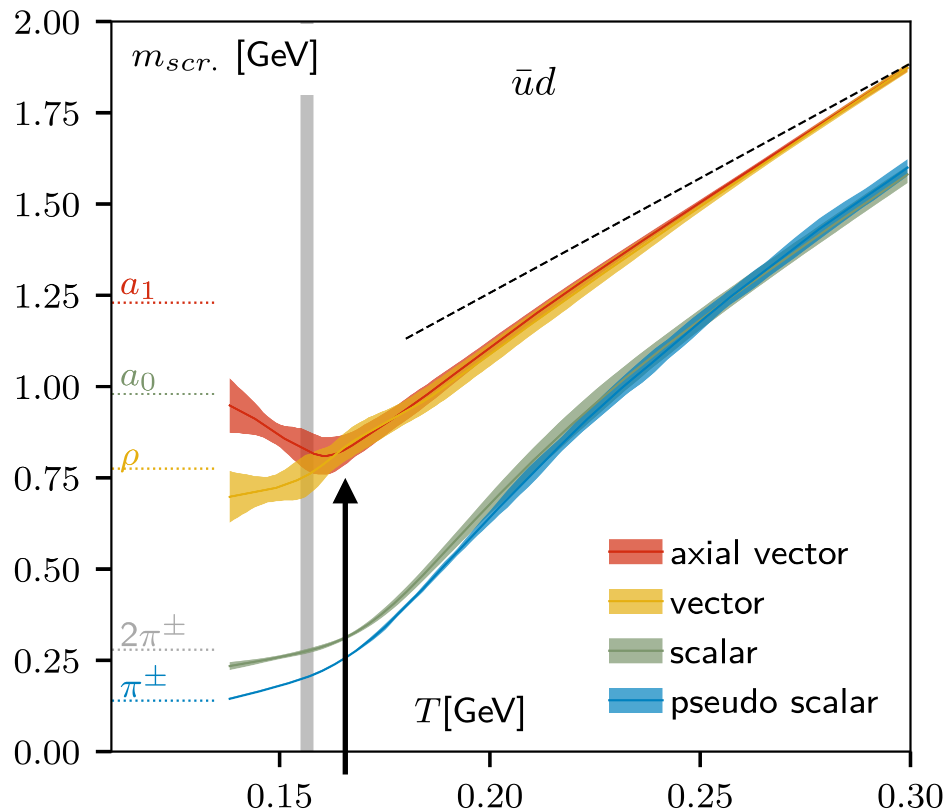
Quark hadron duality holds

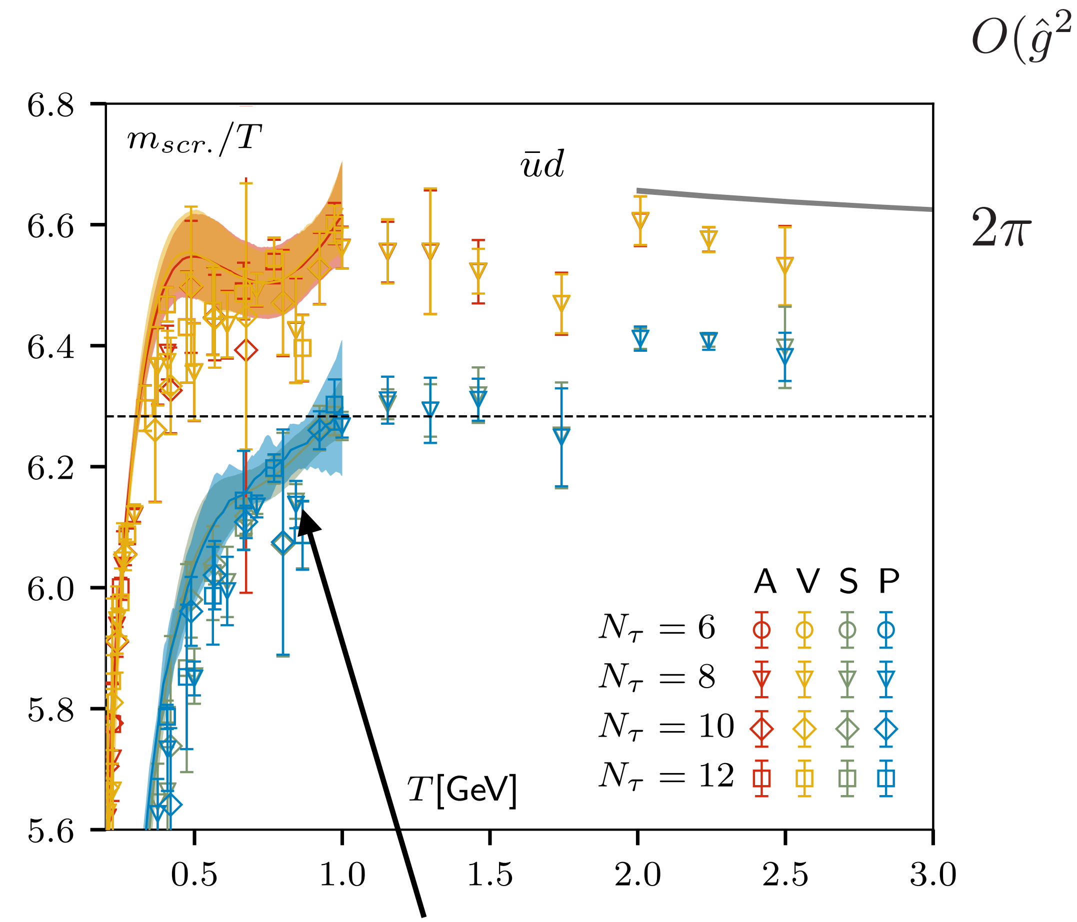



$$\frac{1}{\hat{g}^2(T)} \equiv \frac{9}{8\pi^2}\ln\frac{2\pi T}{\Lambda_{\overline{\rm MS}}} + \frac{4}{9\pi^2}\ln\left(2\ln\frac{2\pi T}{\Lambda_{\overline{\rm MS}}}\right)$$

# Meson screening masses at intermediate temperatures

[HotQCD, PRD 19]




Chiral symmetry restoration

Heavy chiral partners "come down" in all flavour combinations

➡ pressure increases

Drastic change: "vertical" - "horizontal"

Remember resummed pert. theory:

$$\frac{m_{PS}}{2\pi T} = 1 + p_2\,\hat{g}^2(T) + p_3\,\hat{g}^3(T) + p_4\,\hat{g}^4(T)\;,$$

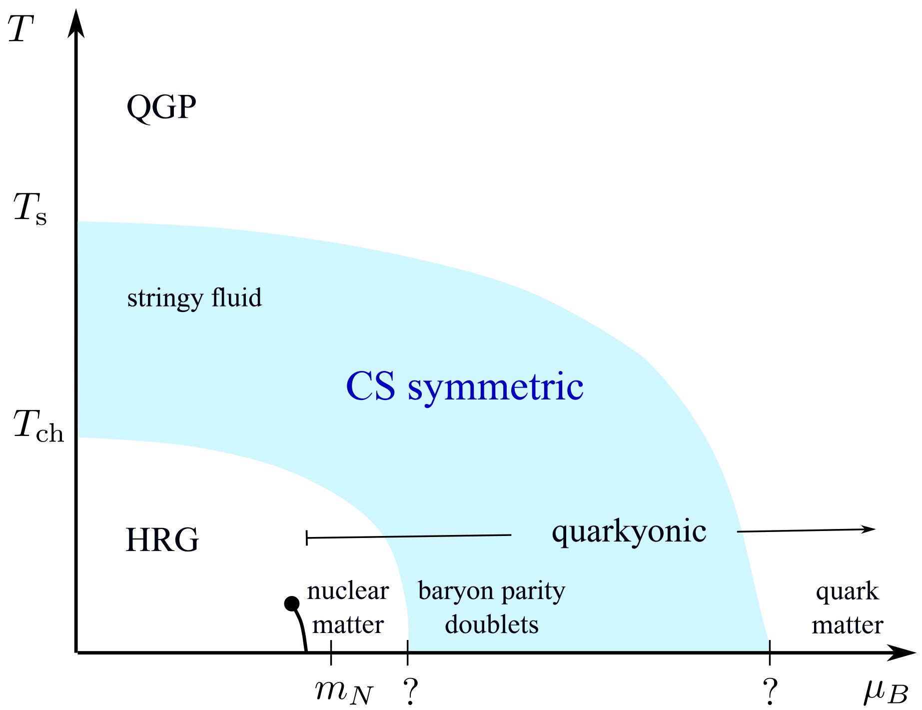
$$\frac{m_V}{2\pi T} = \frac{m_{PS}}{2\pi T} + s_4\,\hat{g}^4(T)\;,$$

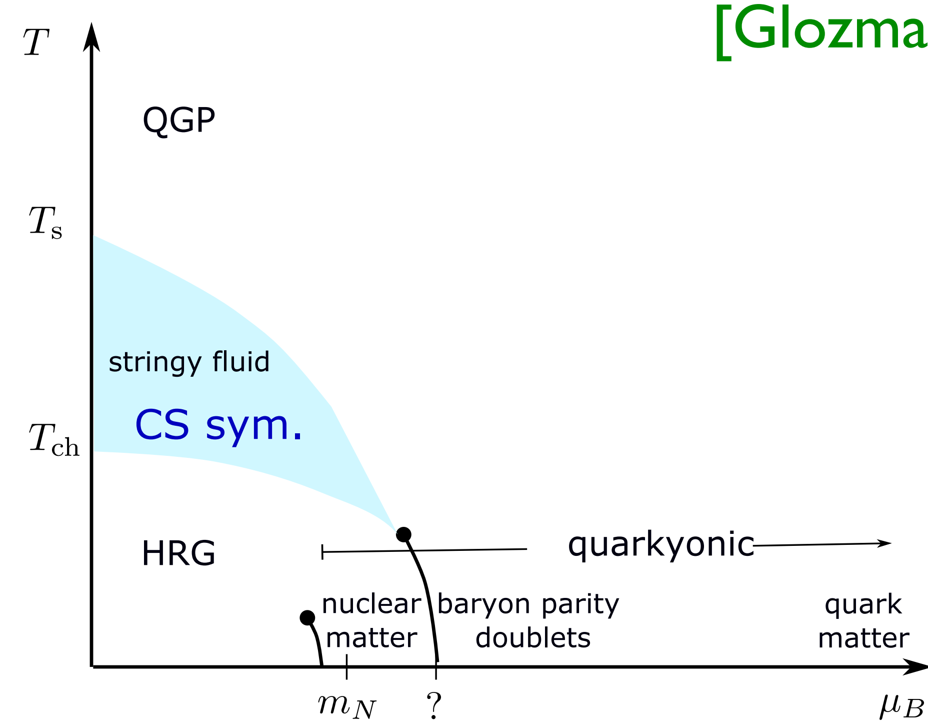
Cannot describe the "bend"

No quark hadron duality for T<0.5 GeV in 12 lightest meson channels! CS symmetry!

# The QCD phase diagram at finite density

[Glozman, O.P., Pisarski, EPJA 22]

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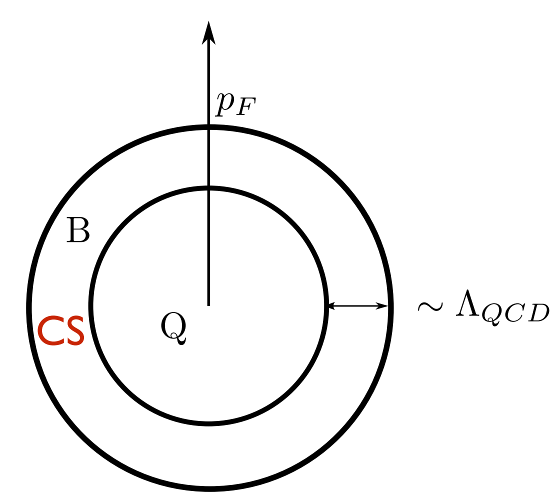
- Cold and dense candidate: baryon parity doublet models CS symmetric [Glozman, Catillo PRD 18]
- Quarkyonic matter [McLerran, Pisarski, NPA 07; O.P., Scheunert JHEP 19]
  Contains regime with chirally symmetric baryon matter
  Fully consistent with transient intermediate CS regime!
- Can be realized wit or without true chiral phase transition!



# Effective degrees of freedom…? → Spectral functions

Based on micro-causality of scalar, local quantum fields at finite T:

[Bros, Buchholz., NPB 94, Ann. Inst. Poincare Phys. Theor. 96]

$$\rho_{\mathrm{PS}}(p_0, \vec{p}) = \int_0^\infty ds \int \frac{d^3\vec{u}}{(2\pi)^2}\ \epsilon(p_0)\, \delta\big(p_0^2 - (\vec{p}-\vec{u})^2 - s\big)\, \widetilde{D}_\beta(\vec{u}, s)$$

Exact, goes to Källen-Lehmann representation for $T \to 0$

↑ thermal spectral density

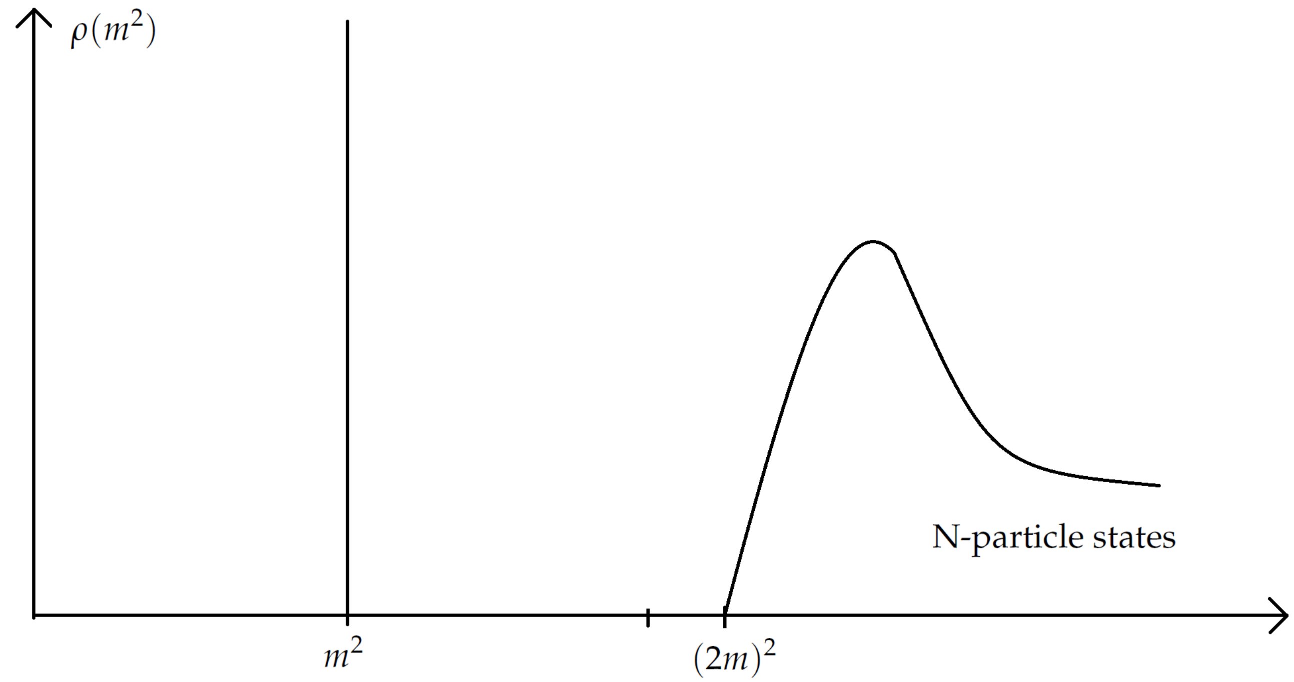
→ Relation between spatial correlators and thermal spectral density

$$C^s_{PS}(z) = \frac{1}{2}\int_0^\infty ds \int_{|z|}^\infty dR\, e^{-R\sqrt{s}} D_\beta(R, s)$$

[Lowdon, O.P., JHEP 22 ]

For stable massive particle with gap to continuum states (QCD pions!):

Vacuum spectral function:




Ansatz $$\widetilde{D}_\beta(\vec{u}, s) = \widetilde{D}_{m,\beta}(\vec{u})\,\delta(s - m^2) + \widetilde{D}_{c,\beta}(\vec{u}, s)$$ [Bros, Buchholz., NPB 02]

Analytic structure inherited from vacuum in absence of phase transition

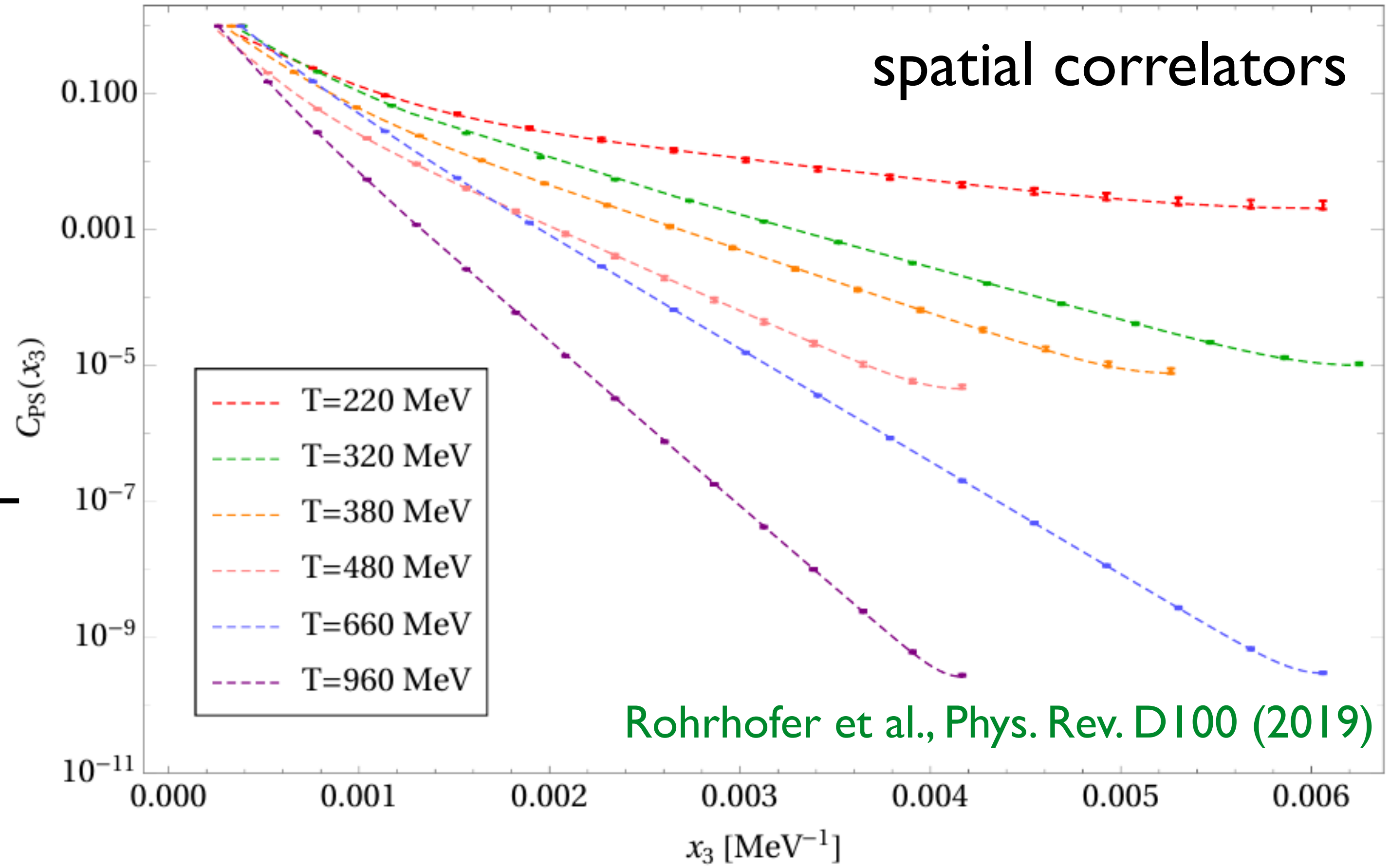
➡ low T behaviour dominated by vacuum particle states

# The pion spectral function

[Lowdon, O.P., JHEP 22]

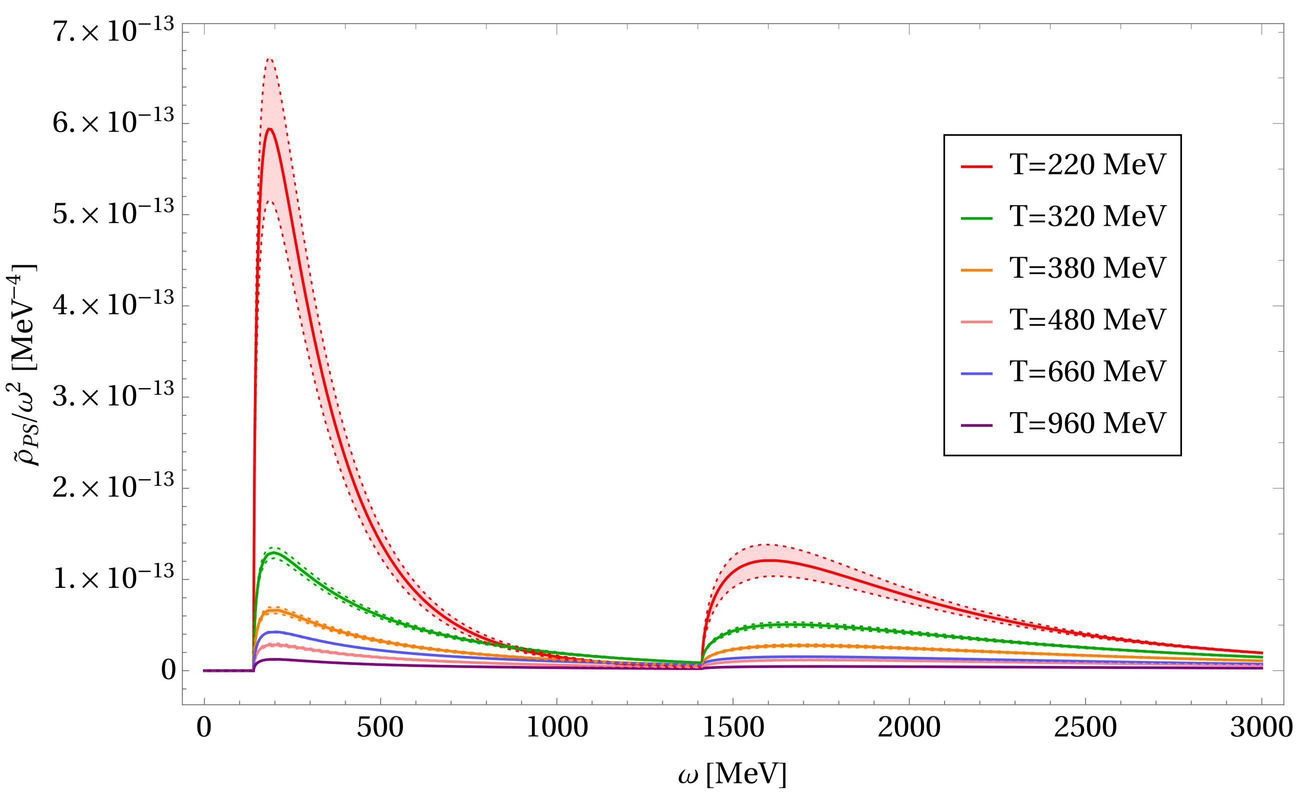
2-state fits $\pi, \pi^*$

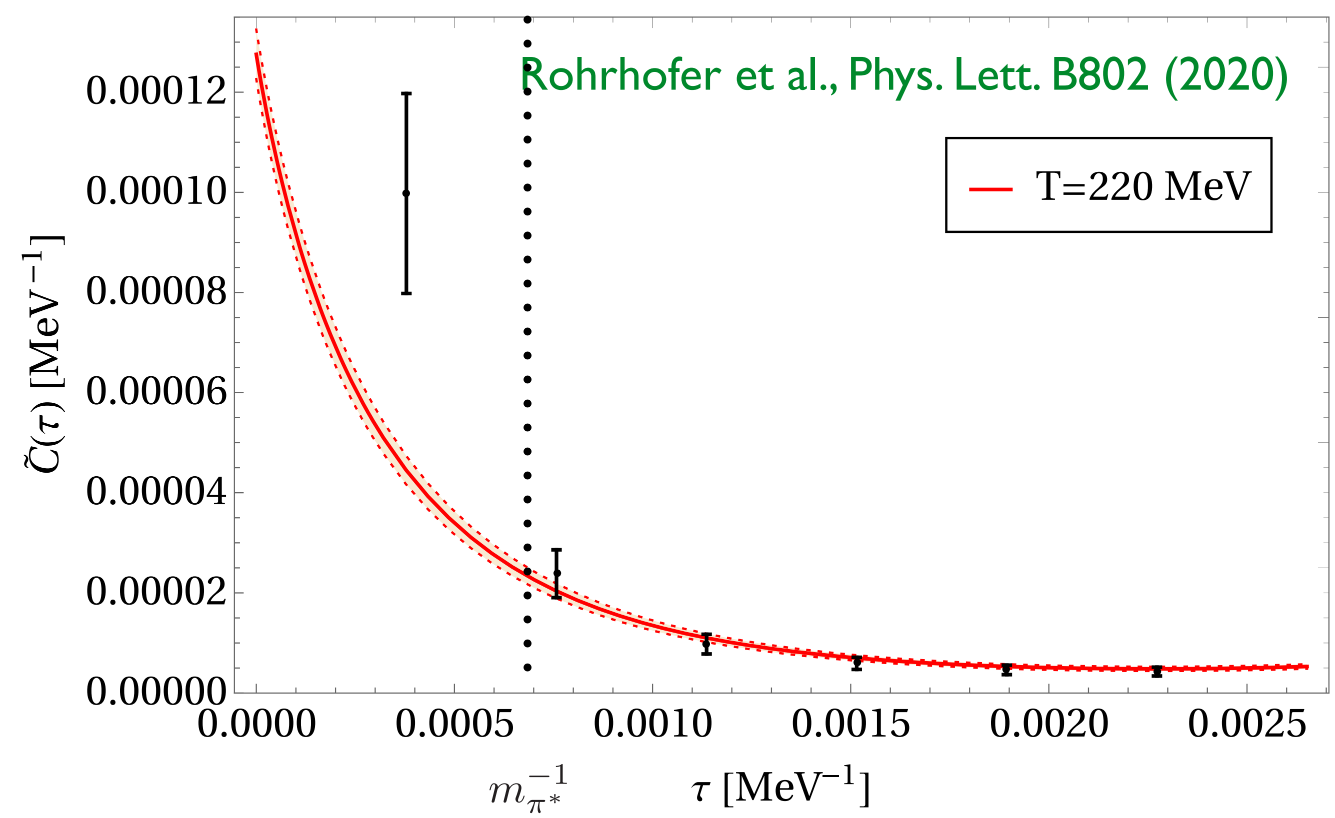
$$D_{m_{\pi^{(*)}},\beta} = \alpha_{\pi^{(*)}} e^{-\gamma_{\pi^{(*)}} x_3}$$




spectral functions

predict temporal correlators, compare with data



# Comparison with plasmon ansatz

Bros+Buchholz Ansatz

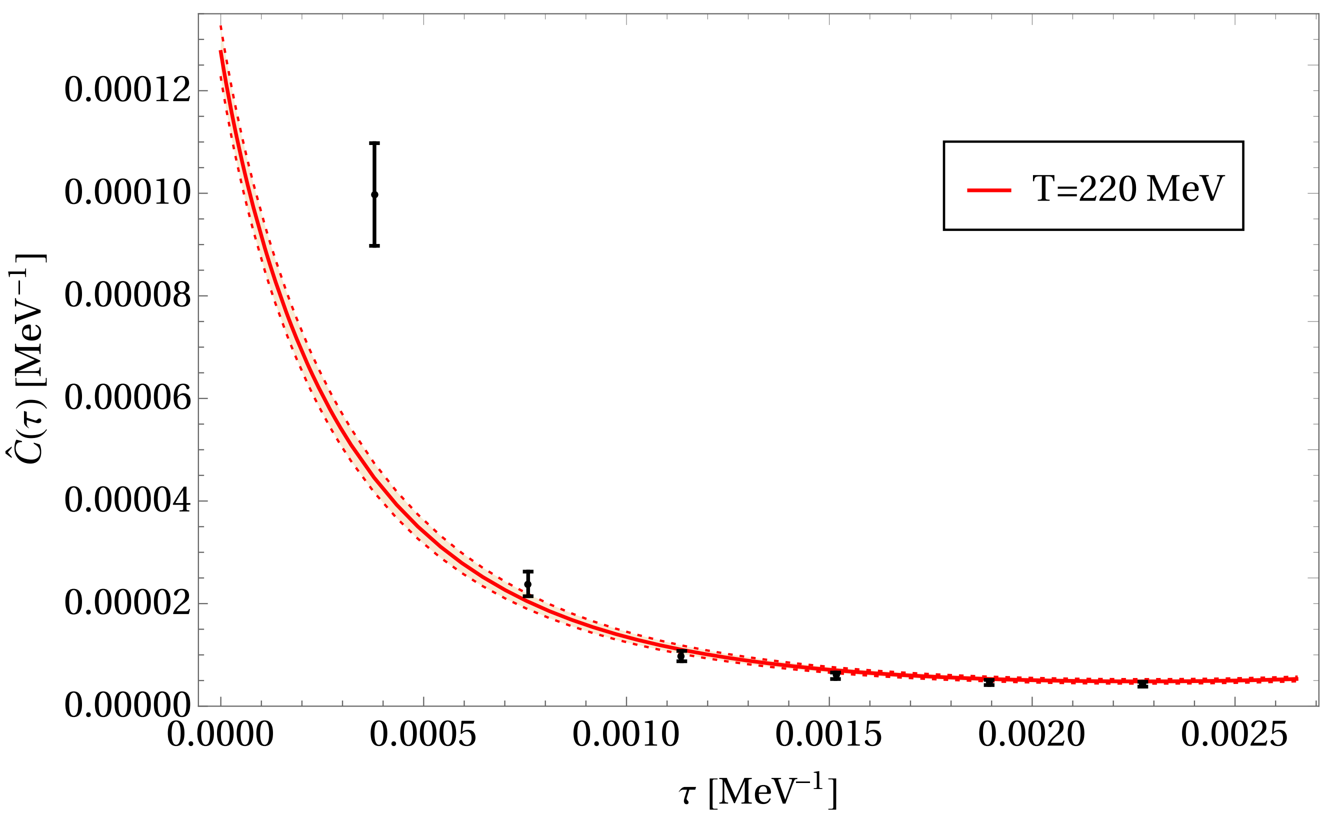
Perturbative plasmon: Breit-Wigner shape

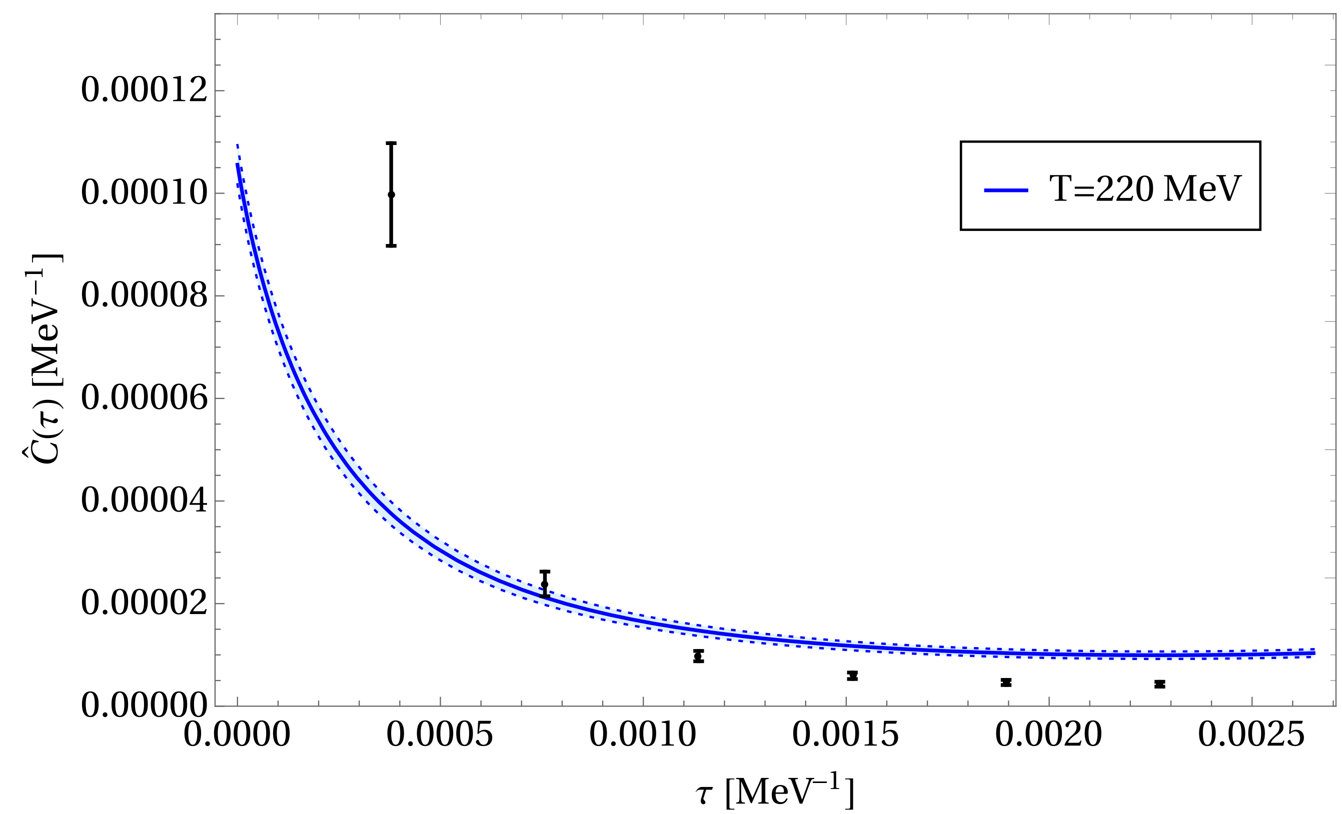
Both fit spatial correlator

$$\rho_{\rm PS}(\omega, \mathbf{p}=0) = \epsilon(\omega)\left[\theta(\omega^2 - m_\pi^2)\,\frac{4\,\alpha_\pi\,\gamma_\pi\sqrt{\omega^2 - m_\pi^2}}{(\omega^2 - m_\pi^2 + \gamma_\pi^2)^2} + \theta(\omega^2 - m_{\pi^*}^2)\,\frac{4\,\alpha_{\pi^*}\,\gamma_{\pi^*}\sqrt{\omega^2 - m_{\pi^*}^2}}{(\omega^2 - m_{\pi^*}^2 + \gamma_{\pi^*}^2)^2}\right]$$

$$\rho_{PS}^{BW}(\omega, \mathbf{p}=0) = \frac{4\alpha_\pi \omega \Gamma_\pi}{(\omega^2 - m_\pi^2 - \Gamma_\pi^2)^2 + 4\omega^2\Gamma_\pi^2} + \frac{4\alpha_\pi^* \omega \Gamma_{\pi^*}}{(\omega^2 - m_{\pi^*}^2 - \Gamma_{\pi^*}^2)^2 + 4\omega^2\Gamma_{\pi^*}^2}$$

Predicted temporal correlators:



# Conclusions

- Zero density, unimproved staggered, Nf=2-6, O(a)-improved Wilson Nf=3:

  1st order chiral transition region not connected to continuum limit

- Chiral transition second order (probably up to the conformal window)

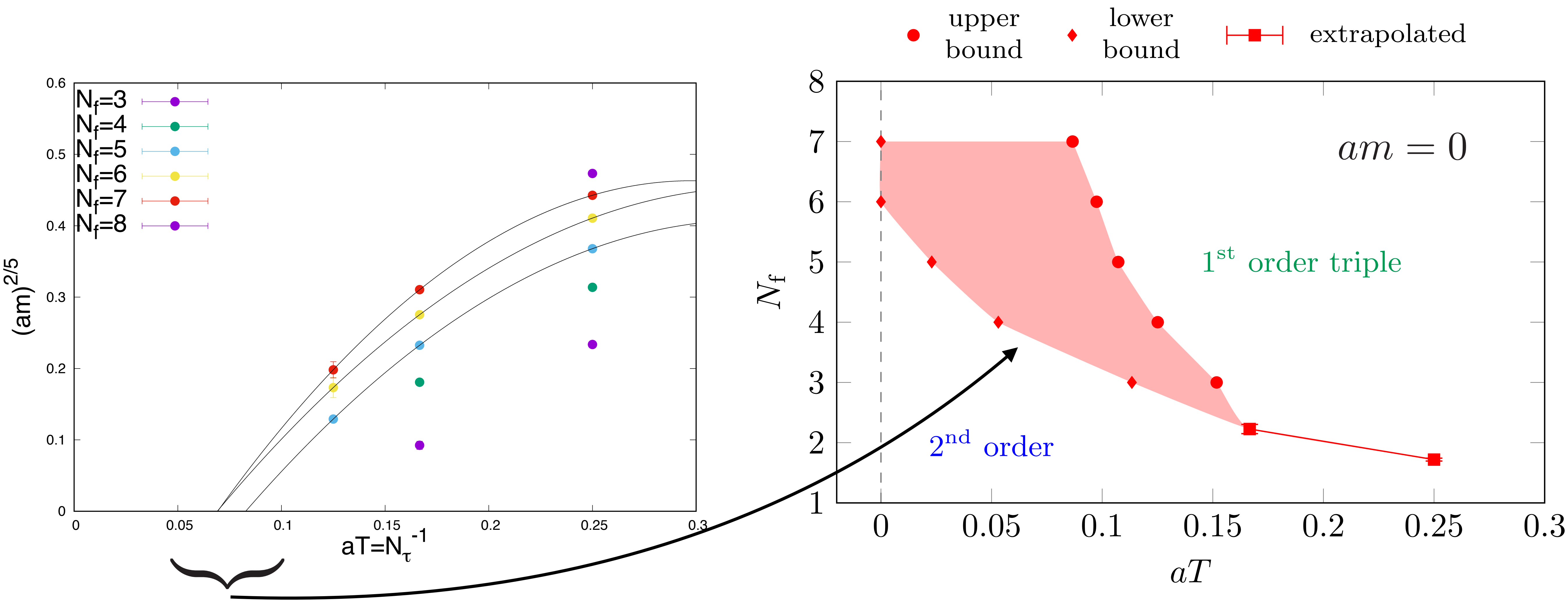
- Chiral spin symmetric regime in a band $T_{pc} < T < 3T_{pc}$

- CS-band must continue to finite density

- Lowest excitations in CS-band hadron/resonance - like,
  no perturbative/partonic description!

# Backup slides

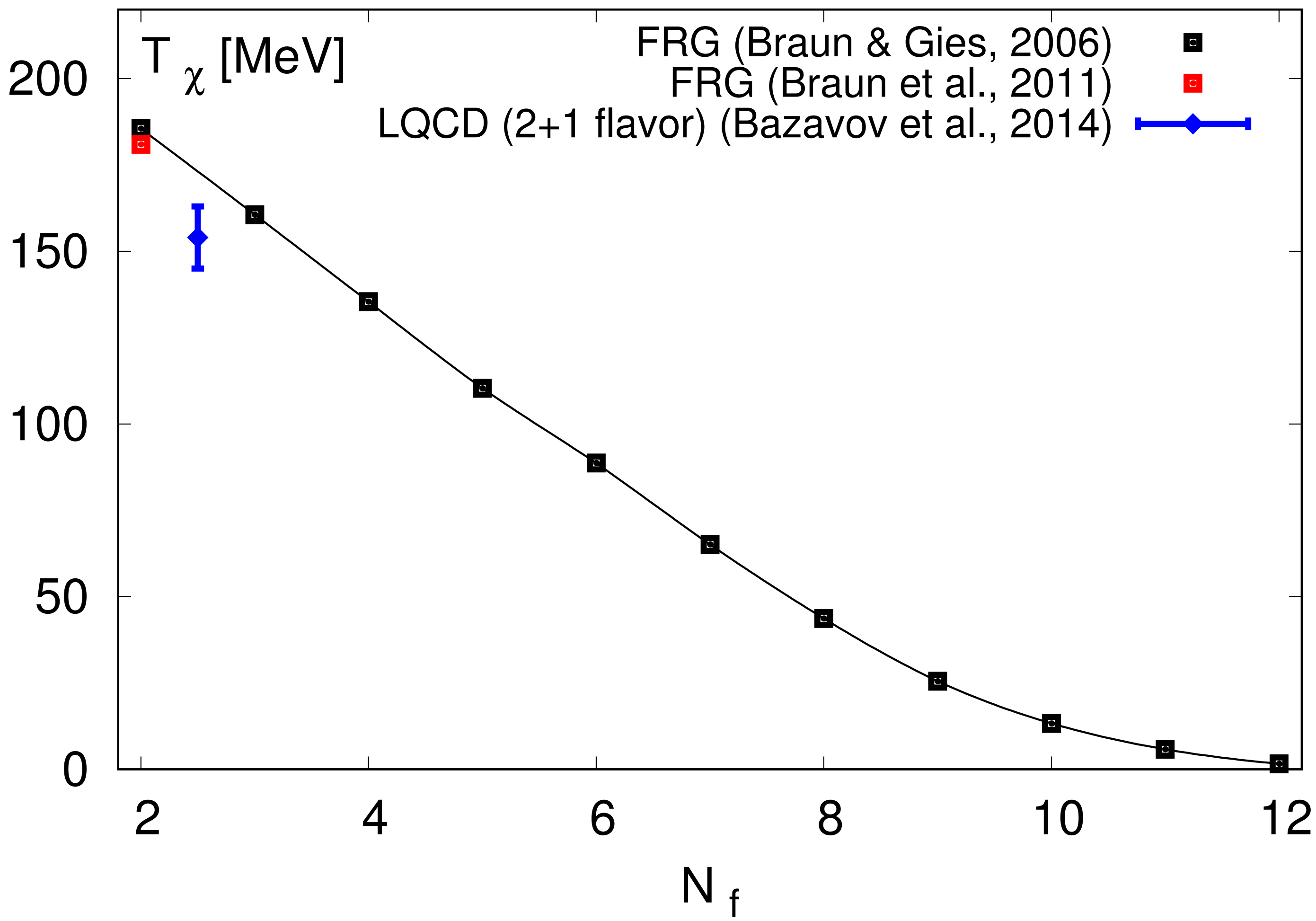
# Staggered: tricritical points as function of Nf




- $N_\tau^{\rm tric}(N_f)$ increasing function
- Tricritical line in the plane of the lattice chiral limit, separates 1st from 2nd

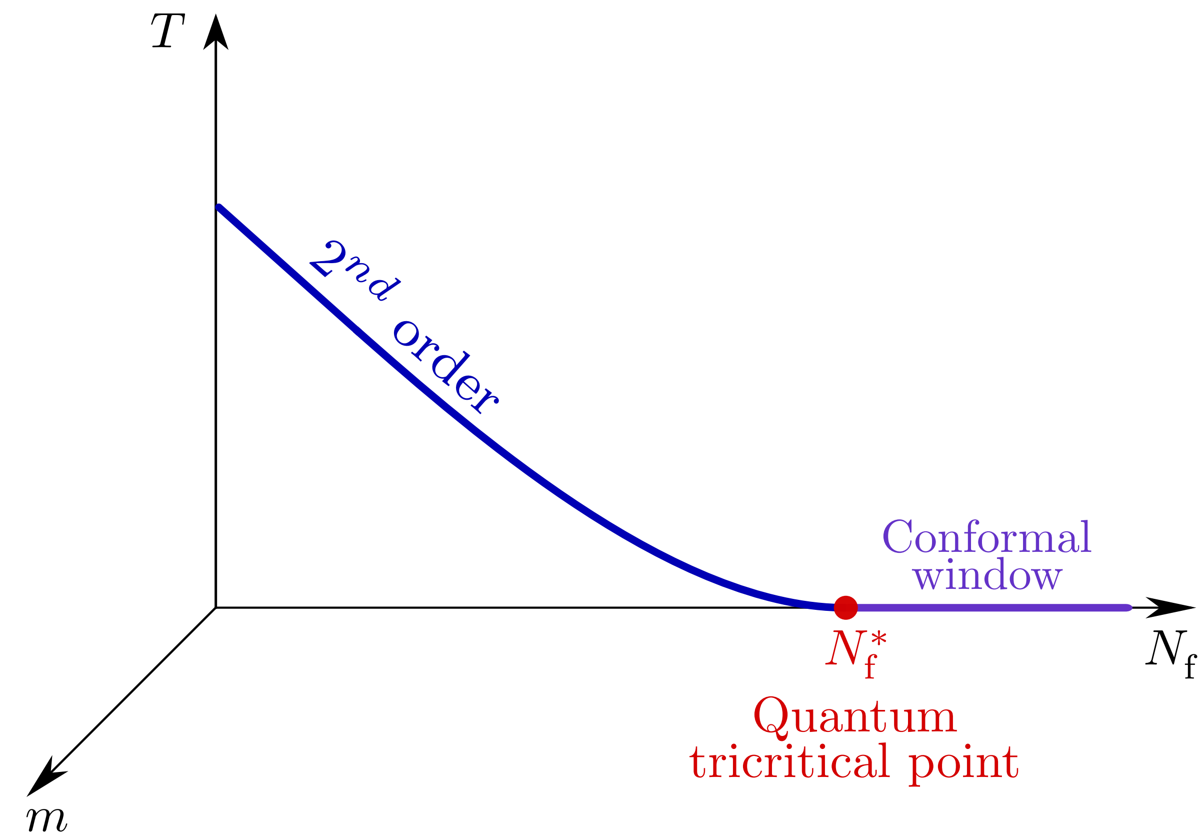
# The chiral phase transition for different $N_f$

Temperature dependence:




For lattice, see [Miura, Lombardo, NPB 13]

Order of the transition:




[Cuteri, O.P., Sciarra, JHEP 21]

The chiral phase transition in the massless limit is likely second-order for all $N_f$