Higgs-Confinement continuity in light of particle-vortex statistics

QCD Theory Seminar

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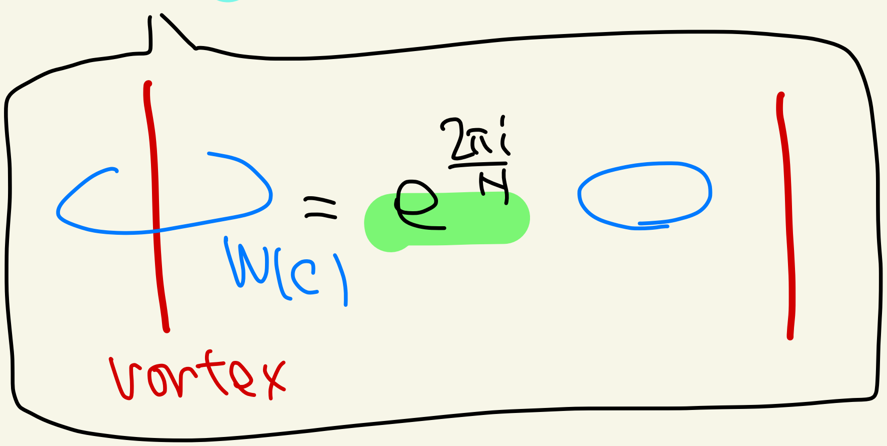
based on arXiv: 2203.02129

Short Summary

dense QCD

In superfluid gauge-Higgs systems :

Higgs ⟷ Conf



recently-debated issue

⇝ phase transition somewhere ?
or not ?

⇓

My claim :

the AB phase respects Higgs-confinement continuity

(∼ Fradkin-Shenker's sense)

Plan

1. Introduction

2. Continuity & AB phase

two examples

① Cherman-Jacobson-Sen-Yaffe's Abelian toy model

② SU(N) gauge + U(N) Higgs [N=3 ~ CFL] (+ neutral scalar)

3. Summary & Comments

1. Introduction

Nontrivial particle-vortex statistics (AB phase)

$\rightsquigarrow$ Higgs confinement transition ?

Introduction

superconductor

- <u>Higgs Mechanism</u> often appears in physics

Higgs condensation "$\langle\phi\rangle \neq 0$" $\rightsquigarrow$ gauge boson acquires mass

but,

- What is "Higgs phase"?

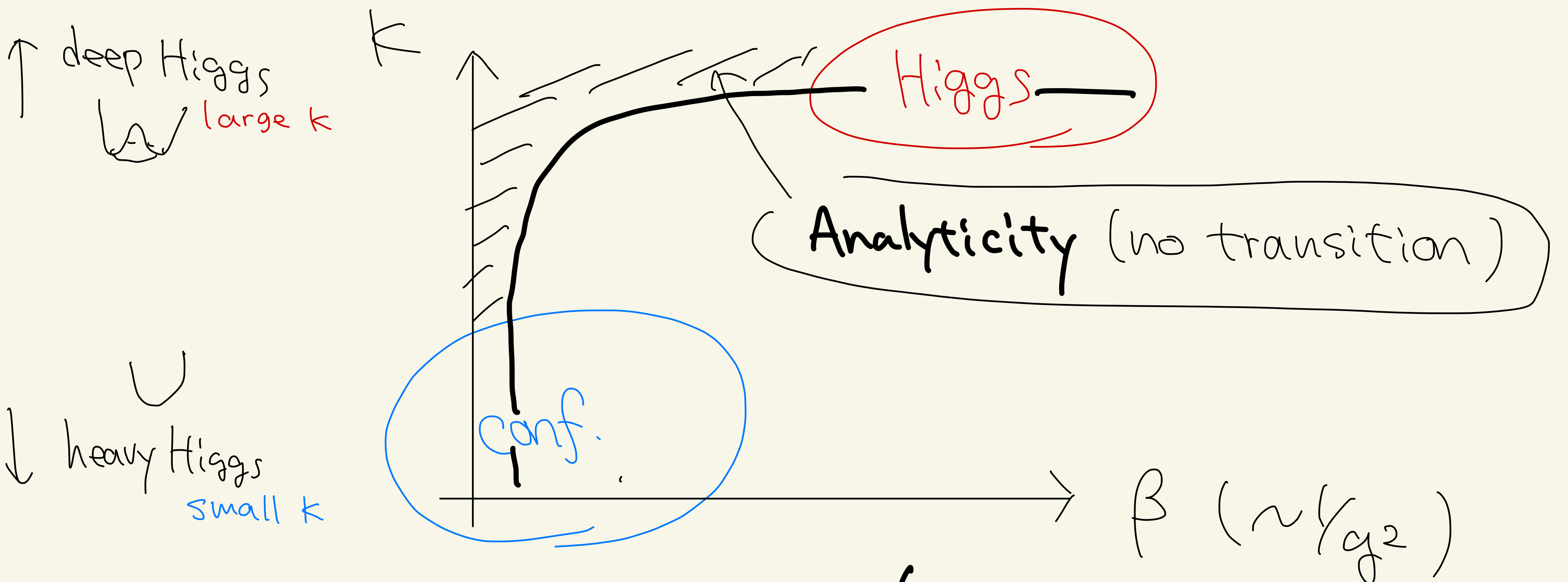
Spontaneous breaking of gauge symmetry? No.

never broken [Elitzur '75]

"Higgs - Confinement Continuity"

[Osterwalder-Seiler '78] [Fradkin & Shenker '79] [Banks-Rabinovici '79]

'Higgs phase and Confinement phase are continuously connected in fundamental gauge-Higgs models'

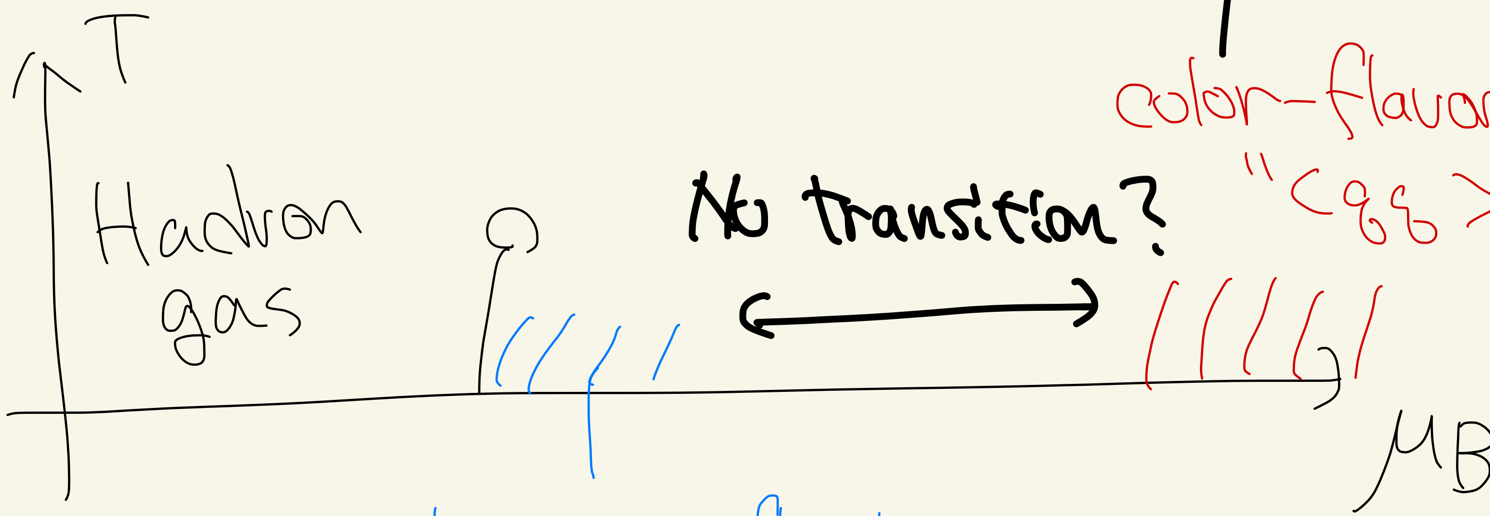


No transition!

A Lesson from Higgs-Confinement continuity.

"Condensation" of fundamental matter "$\langle \phi \rangle \neq 0$"
does not separate phases

⇝ Quark-Hadron continuity [Schäfer-Wilczek '99]



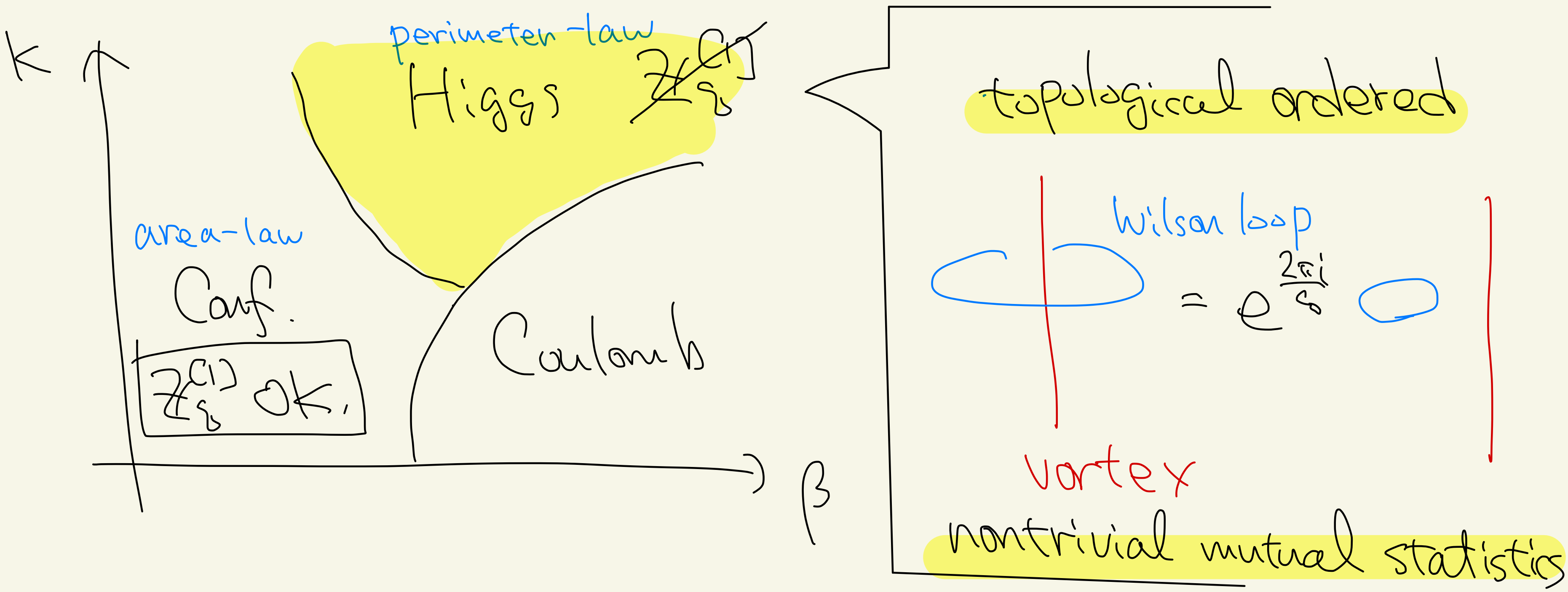
color-flavor locking
"$\langle qq \rangle \neq 0$"

nuclear superfluidity

condensate of
color (anti-) fundamental
diquark operator

When distinguishable?

e.g.) charge-q Abelian Higgs ($q \geq 2$)



Higgs $\cdots$ $\mathbb{Z}_q^{[1]}$ broken phase $\to$ top. order

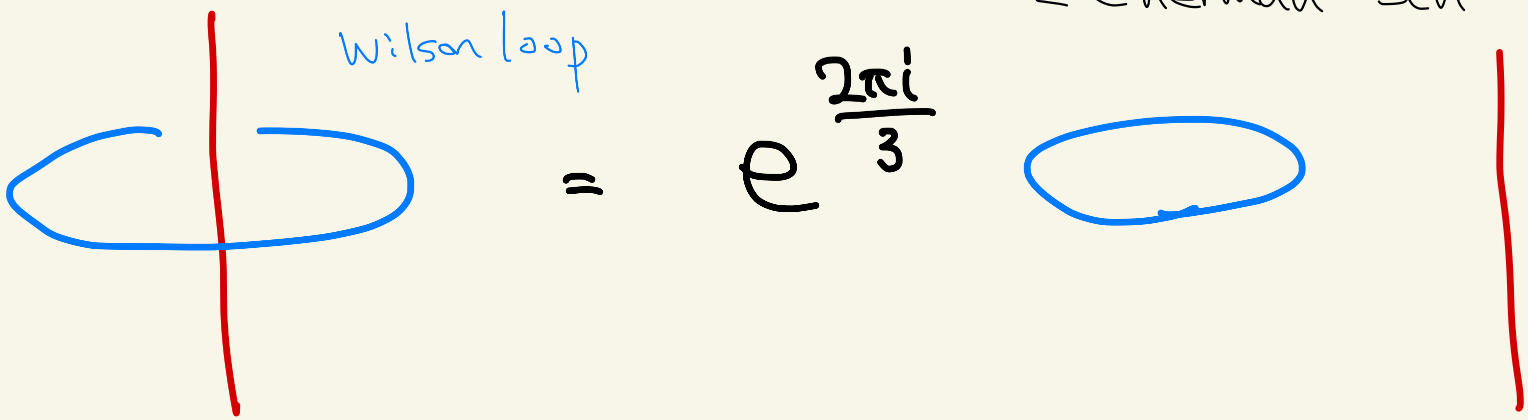
Confinement $\cdots$ $\mathbb{Z}_q^{[1]}$ unbroken phase

(※ $q=2$ $\to$ usual superconductor)

particle-vortex statistics in CFL phase

Nontrivial statistics $\longrightarrow$ $\exists$ transition ?

[Cherman-Sen-Yaffe '18]



minimal vortex
[non-abelian CFL vortex]

However, this does **NOT** mean topological order

[Hirono-Tanizaki '18 '19]

(CFL vortex is not topological)

AB phase can still be an order parameter?

toy model [Cherman-Jacobson-Sen-Yaffe '20]

$$\text{3d cpt. } U(1) \text{ gauge} + \begin{pmatrix} \text{charge-}(+1) \text{ scalar } \phi_+ \\ \text{charge-}(-1) \text{ scalar } \phi_- \end{pmatrix} + \text{neutral } \phi_0$$

$V_c(\phi_\pm)$ $= m_c^2 |\phi_\pm|^2 + \lambda_c |\phi_\pm|^4$

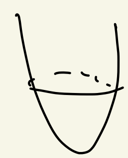
$(\phi_0 \phi_+ \phi_-)$-int.

dial m_c^2

m_c^2

$V_c(\phi_\pm)$

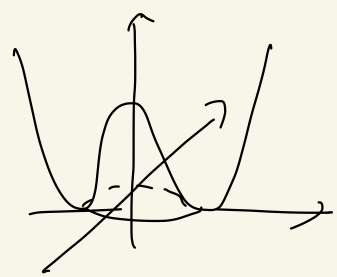
Higgs



$V_c(\phi_\pm)$

Conf.

$V_0(\phi_0)$



$\langle \phi_0 \rangle \neq 0$

potential

$$S = \int d^3x \left\{ \frac{1}{2e^2} |da|^2 + |D\phi_+|^2 + |D\phi_-|^2 + |d\phi_0|^2 + V_c(\phi_+) + V_c(\phi_-) + V_0(\phi_0) + \varepsilon\, \phi_+ \phi_- \phi_0 + \text{c.c.} \right.$$
$$\left. + (\text{monopole}) \right\}$$

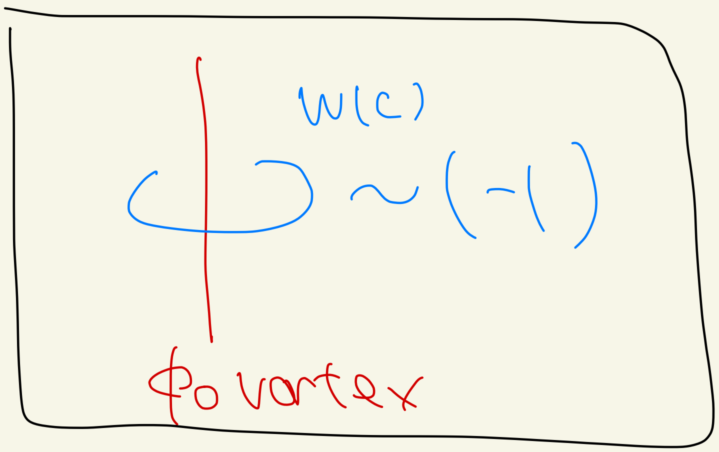
AB phase as an order parameter (?)

[Cherman–Jacobson–Sen–Yaffe '20]

3d $U(1)$ gauge + charged (ϕ_+, ϕ_-) + $\underbrace{\text{neutral } \phi_0}_{\text{superfluid.}}$

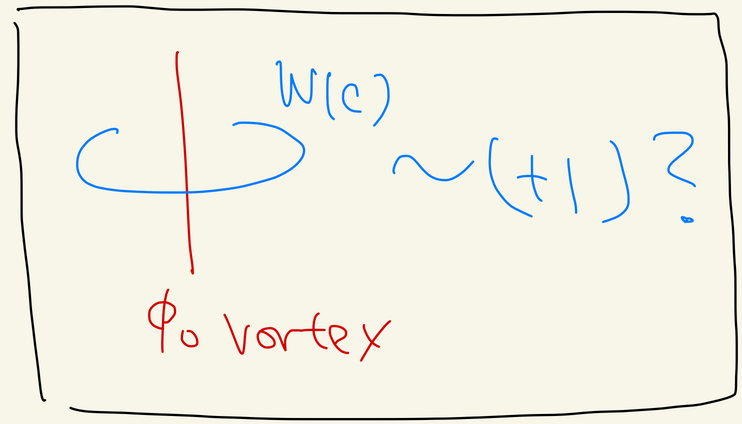
(superfluid) Higgs regime $\longrightarrow$ (superfluid) confining regime $\quad m_c^2$

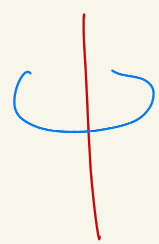
(discuss vortex config.) $\qquad$ (decouple $\phi_\pm$)



transition ?

$\longleftrightarrow$



$\left\{ \begin{array}{l} \phi_\pm \longrightarrow \phi_\mp \\ a \longrightarrow -a \end{array} \right.$ symmetry $\Rightarrow$  must be real ± 1

$\Rightarrow$ need jump (?)

Summary of Backgrounds

Continuity

CFL phase is not topological ordered.

How connected?

Transition

motivated by top. ordering

non-trivial particle-vortex statistics in CFL phase

AB phase can still be an order parameter

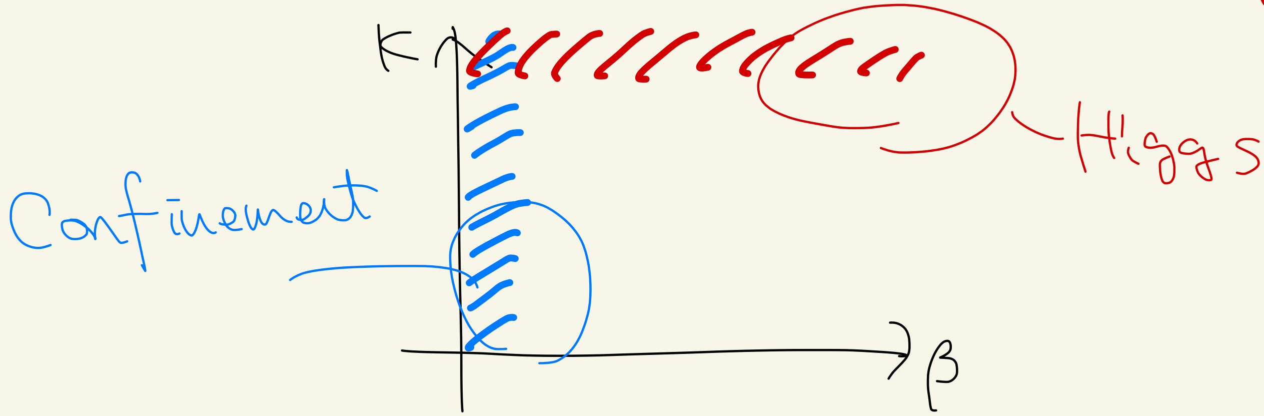
true?

2. Continuity & AB phase

answering these questions!

Claim & Strategy

For superfluid fundamental gauge-Higgs systems, the Aharonov-Bohm phase around a vortex is continuous (or constant, if protected by symmetry) in the strong-coupling/deep-Higgs regimes



$\Rightarrow$ $\exists$ region connecting conf. & Higgs regimes without jump in AB phase.

In this talk, we illustrate this claim in the following two models.

① the Abelian toy model by Cherman–Jacobson–Sen–Yaffe.

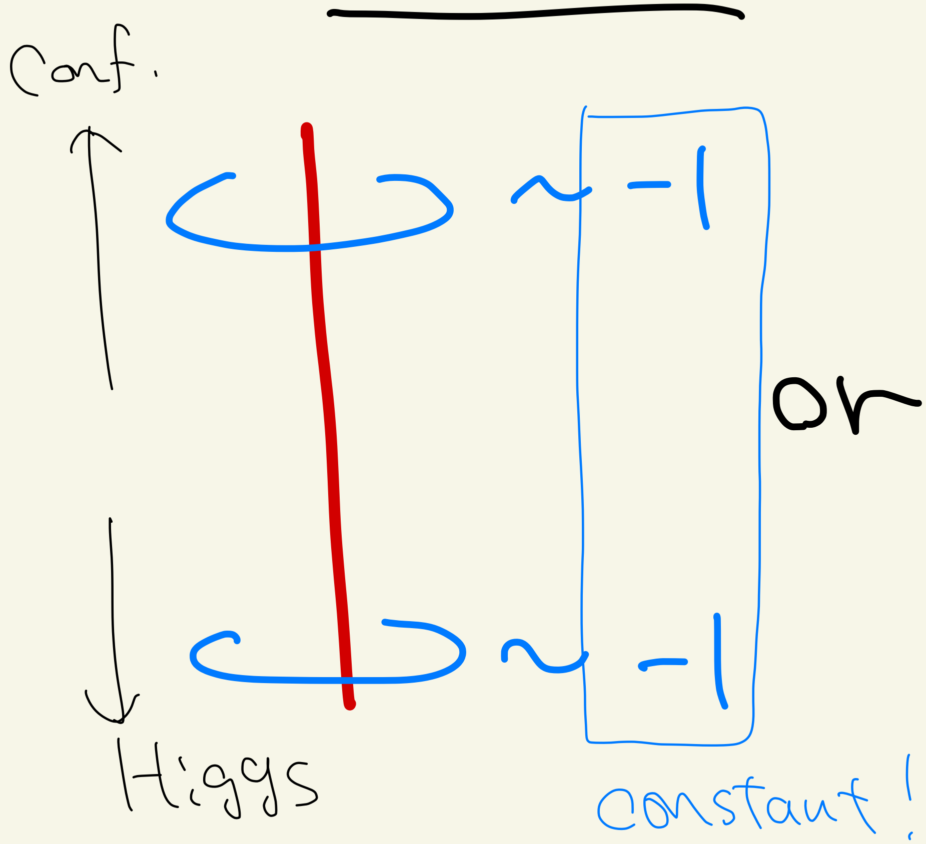
② $SU(N)$ gauge + $U(N)$ fundamental Higgs

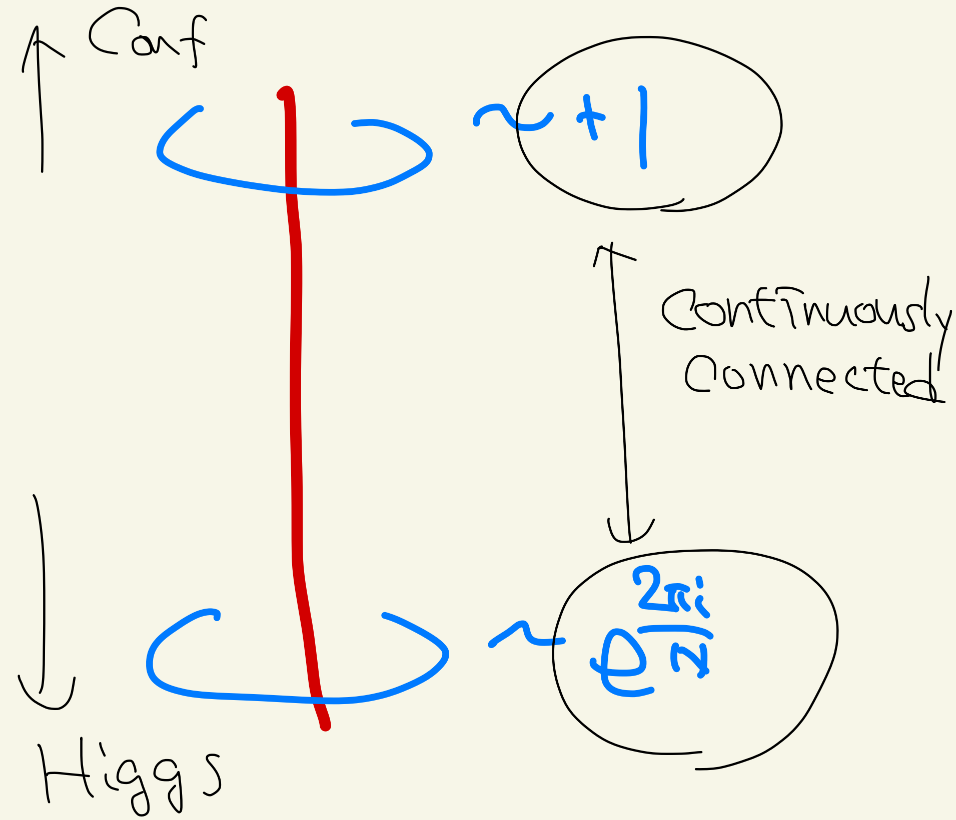
(+ neutral scalar)

$N=3$: gauged Ginzburg-Landau for CFL phase

(+ neutral scalar for hadronic superfluidity)

Results





Case (A)

∃ (symmetry) protecting AB phase

①, ②: $N = 2$

Case (B)

otherwise.

②: $N \geq 3$

① The toy model

$U(1) + \underbrace{\phi_+, \phi_-}_{\text{charged scalar}} + \phi_0$

neutral scalar

Lattice analog.

$$\begin{cases} U_\ell : U(1) \text{ link variable} \\ \phi_\pm, \phi_0 : U(1)\text{-valued site variable.} \end{cases}$$

with the action:

$$S = \beta \sum_\square U_\square + \kappa \sum_\ell \phi^*_{+,x} U_\ell \phi_{+,x'} + \kappa \sum_\ell \phi^*_{-,x} U^*_\ell \phi_{-,x'}$$

$$+ \underset{\sim}{\kappa_0} \sum_\ell \phi^*_{0,x} \phi_{0,x'} + \sum_x \varepsilon \phi_0 \phi_+ \phi_-$$

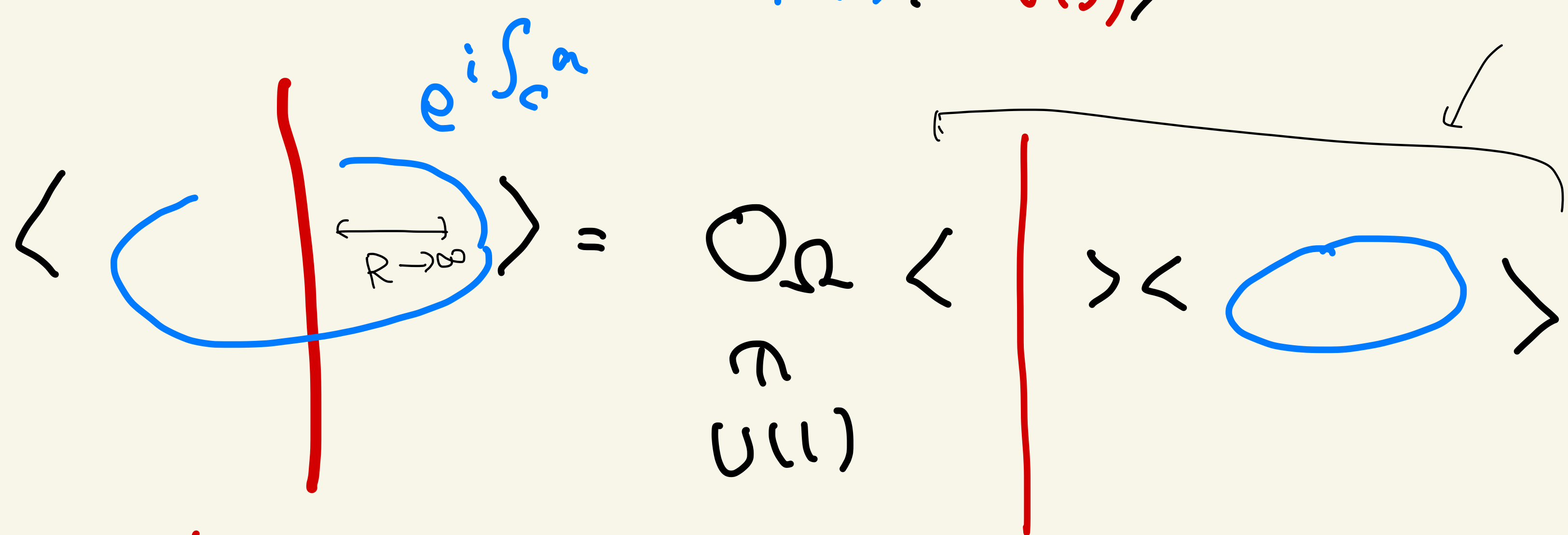
$$\boxed{\begin{array}{l} \text{large } \kappa \longrightarrow \text{Higgs} \\ \text{small } \kappa \longrightarrow \text{confining} \end{array}}$$

large $\kappa_0 \longrightarrow \cancel{U(1)}_{\text{global}}$

Aharonov-Bohm phase

[order parameter conjectured in Cherman-Jacobson-Sen-Yaffe '20]

$$O_\Omega := \lim_{|c|\to\infty} \frac{\langle W(c)\, V(S)\rangle}{\langle W(c)\rangle\langle V(S)\rangle}$$ ← normalization



ϕ_0 vortex

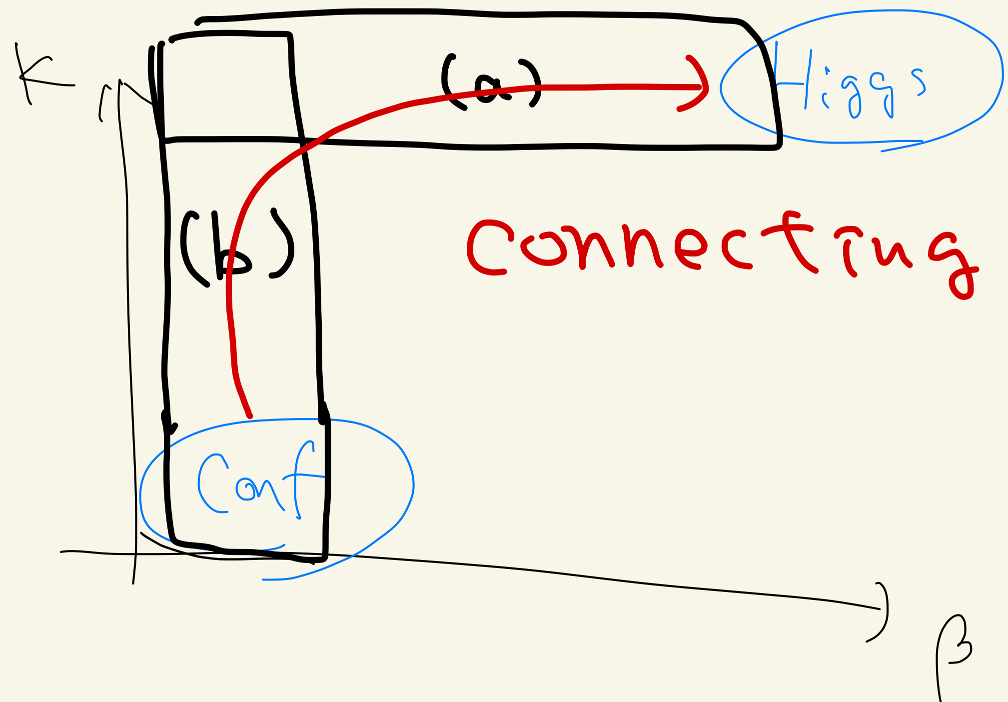
(minimal $U(1)_{global}$ vortex)

To show continuity,

let us look into:

$$\begin{cases} \text{(a) deep Higgs regime } (\kappa \to \infty) \\ \text{(b) strong coupling regime } (\beta \to 0) \end{cases}$$

Phase diagram (like Fradkin-Shenker)



Connecting path!

(a) $K \to \infty$: deep Higgs limit

U_ℓ is pinned to maximize

$$K\left[\phi_+^* U_\ell \phi_+ + \phi_-^* U_\ell^* \phi_- + c.c.\right]$$

$$\Longrightarrow U_\ell \simeq u_\ell^*(\phi_+)\sqrt{u_\ell(\phi_+)\,u_\ell(\phi_-)}$$

$-\frac{\pi}{2} < \mathrm{Arg}\sqrt{\cdots} \le \frac{\pi}{2}$

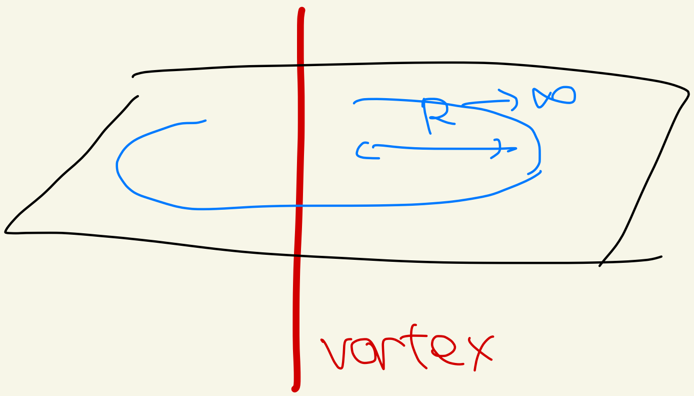
where $u_\ell(\phi_\pm) := \phi_{\pm,x}\,\phi_{\pm,x'}^* \in U(1)$ (pure gauge)

$$x \xrightarrow{\ \ell\ } x'$$

$$\Longrightarrow W(C) = \prod_{\ell \in C} U_\ell = \prod_{\ell \in C} \sqrt{u_\ell(\phi_+)\,u_\ell(\phi_-)}$$

In continuum, this means $a = \dfrac{d\varphi_+ - d\varphi_-}{2}$ with $\phi_\pm = e^{i\varphi_\pm}$

$\Longrightarrow$ In the presence of ϕ_0 vortex,



$r = 0$

$$\phi_0 \sim e^{i\theta} \quad (r \to \infty)$$

$$\Downarrow \quad \left\{ \begin{array}{l} \mathcal{L}(\phi_0 \phi_+ \phi_-) \text{ interaction} \\ \text{fix asymptotic winding} \end{array} \right.$$

$$(\phi_+ \phi_-) \sim e^{-i\theta} \quad (r \to \infty)$$

[otherwise suppressed by infrared divergence]

Then, the Wilson loop acquires nontrivial phase

$$\text{“}\langle W(C) V(S) \rangle\text{”} \sim \prod_{\ell \in C} \sqrt{u_\ell(\phi_+) u_\ell(\phi_-)} = -1$$

reproducing [Cherman-Jacobson-Sen-Yaffe '20]

$$\uparrow \quad (\phi_+ \phi_-) \sim e^{-i\theta}$$

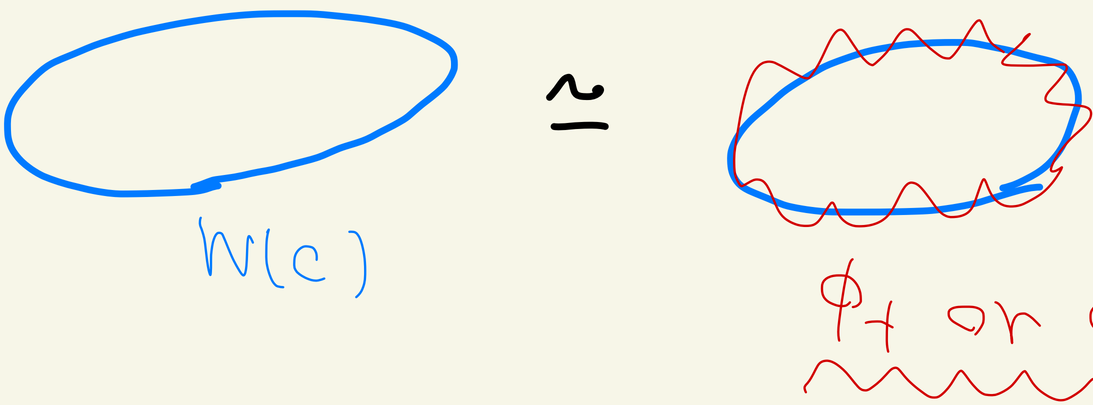
(b) $\beta \to 0$: strong coupling limit.

Naively, "confining $\Rightarrow$ decoupling of $\phi_\pm \Rightarrow O_0 = 1$."

However, this expectation is **not** correct.

Why?

Large Wilson loop $\longrightarrow$ screened by ϕ_+ & ϕ_-

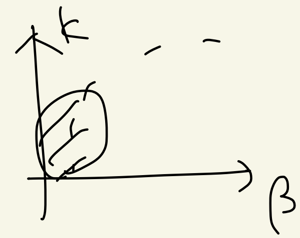


$W(C)$ $\simeq$ ϕ_+ or ϕ_-

can be affected by ϕ_0 vortex

(Large extended objects may be out of scope of naive low-energy EFT)

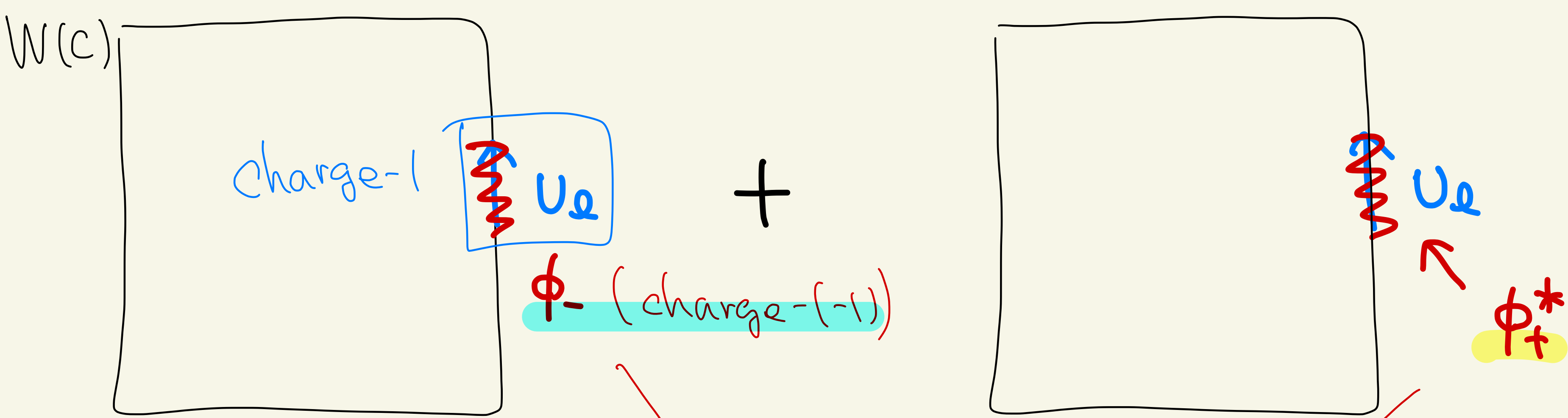
◻ Deep confining regime ($\beta \to 0$, small κ)



Two ways to screen $W(C)$

① ϕ_- line

② ϕ_+ line (reversed)



$$\Rightarrow U_\ell \longrightarrow \kappa\left[u_\ell(\phi_-) + u_\ell(\phi_+^*)\right] + O(\kappa^3)$$

$$S \supset \kappa\, U_\ell^* \left(\phi_{+,x}\phi^*_{+,x'} + \phi^*_{-,x}\phi_{-,x'}\right)$$

Its phase is,

$$\left[u_\ell(\phi_-) + u_\ell(\phi_+^*)\right] \propto u_\ell(\phi_+^*)\sqrt{u_\ell(\phi_-)u_\ell(\phi_+)}$$

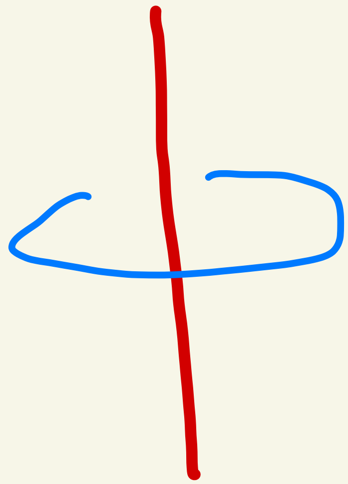
same as the deep Higgs limit!

$\Rightarrow$ nontrivial AB phase $O_\Omega = -1$
as in the Higgs regime!

This phase is robust under

- κ corrections (actually, it can be computed w/o κ expansion)
- β corrections ($\rightsquigarrow$ small deformation of $\mathbb{C}$)

Summary ①



reproduction

Higgs

pinned at

$$a \sim \frac{d\phi_+ - d\phi_-}{2}$$

$$W(C) \sim e^{i\int_C \frac{d\phi_+ - d\phi_-}{2}}$$

new

Conf.

screened

$$U_\ell \to [U_\ell(\phi_+^*) + U(\phi_-)]$$

$$W(C) \sim e^{i\int_C \frac{d\phi_+ - d\phi_-}{2}}$$

$$O_\Omega = -1 \ \text{(matched)}$$

② SU(N) gauge + U(N) Higgs model.

We want an effective model describing nuclear superfluid / CFL phases.

relevant d.o.f. :

CFL diquark $\Phi^{ai} = \varepsilon^{abc} \varepsilon^{ijk} q^{b\,t}_{j} C\gamma^5 q^c_k$

($a, b, c, \ldots$: color
$i, j, k, \ldots$: flavor)

- 3×3 matrix
- color & flavor antifundamental
- "$\langle \Phi \rangle \sim \mathbb{1}$" in CFL phase

$$\text{“}\; \frac{SU(3)_{\text{color}} \times SU(3)_f \times U(1)_q}{\mathbb{Z}_3 \times \mathbb{Z}_3} \longrightarrow \frac{SU(3)_{c+f} \times \mathbb{Z}_6}{\mathbb{Z}_3 \times \mathbb{Z}_3} \;\text{”}$$

"Color-flavor locked" (gauge-fixed description) gauged-Ginzburg-Landau description

$$S_{eff}[a,\Phi] = \frac{1}{2g^2}|f|^2 + |D\Phi|^2 + V(\Phi^\dagger\Phi)$$

minimum at $\Phi^\dagger\Phi = v^2 \mathbb{1}$.

$(\Phi/v) \in U(3)$

<u>A simple lattice analog</u>:

SU(3) gauge + U(3)-valued (anti-) fundamental Higgs

$$S = \beta \sum_\square \mathrm{tr}\, U_\square + \kappa \sum_\ell \mathrm{tr}(\phi_x^\dagger U_\ell \phi_{x'}) + \text{c.c.}$$

+ We want ~~U(1)~~ confining regime

$\longrightarrow$ add superfluid neutral scalar ϕ_0

[dibaryon]

$\rightsquigarrow$ Lattice analog of gauged GL model.
for nuclear superfluidity (CFL

$\begin{cases} U_\ell : \text{SU(3)-valued link variable} \\ \phi : \text{U(3)-valued field (diquark)} \\ \phi_0 : \text{U(1)-valued field} \end{cases}$

(ϕ_0: dibaryon, breaks $U(1) \to \mathbb{Z}_3$) $\subset U(3)$

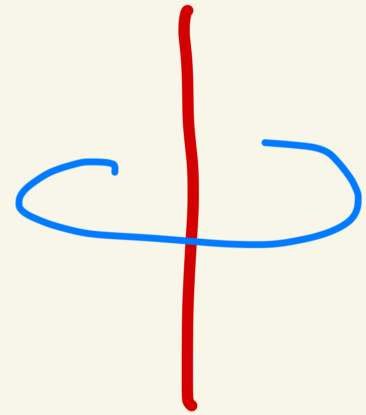
General N

Model [SU(N) gauge + U(N) Higgs]

$$\boxed{\begin{aligned} S = {} & \beta \sum_{\square} \mathrm{tr}\, U_\square + \kappa \sum_{\ell} \mathrm{tr}\left(\phi_x^\dagger U_\ell \phi_{x'}\right) + \kappa_0 \sum_{\ell} \phi_{0,x}^\dagger \phi_{0,x'} \\ & + \varepsilon \sum_x \phi_{0,x}^\dagger \left(\det \phi_x\right) + \text{c.c.} \end{aligned}}$$

We consider

$$O_\Omega = \lim_{|C|\to\infty} \frac{\langle W(C) V(S) \rangle}{\langle W(C)\rangle \langle V(S)\rangle}$$



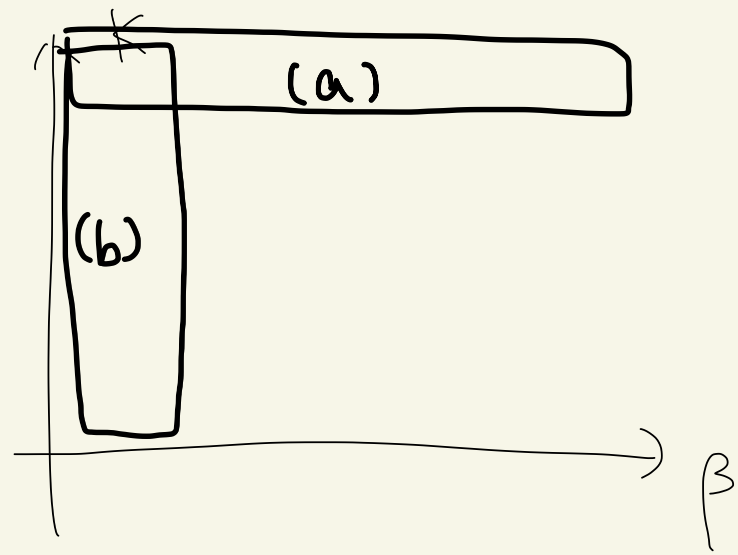
$V(S)$: minimal to vortex

corresponds to
- hadronic $\longrightarrow$ superfluid vortex
- CFL $\longrightarrow$ non-Abelian vortex

As before, we examine

(a) deep Higgs region

(b) strong coupling region



(a) deep Higgs limit

$\kappa \to \infty$ fixes U_ℓ to maximize

$$\kappa \sum_\ell \mathrm{tr}[\phi_x^\dagger U_\ell \phi_{x'}] + c.c.$$

$$\Downarrow$$

$-\frac{2\pi}{2N} < \mathrm{Arg}\, \textcircled{?}^{-1/N} < \frac{2\pi}{2N}$

$$U_\ell = (\det \phi_x \phi_{x'}^\dagger)^{-1/N} \phi_x \phi_{x'}^\dagger$$

$$\Downarrow$$

$$\prod_{\ell \in C} U_\ell = \prod_{\ell \in C} (\det \phi_x \phi_{x'}^\dagger)^{-1/N}$$

ϕ_0 vortex $\Rightarrow$ $\det \phi \sim e^{i\theta}$ $\Rightarrow$ $O_\Omega = e^{\frac{2\pi i}{N}}$

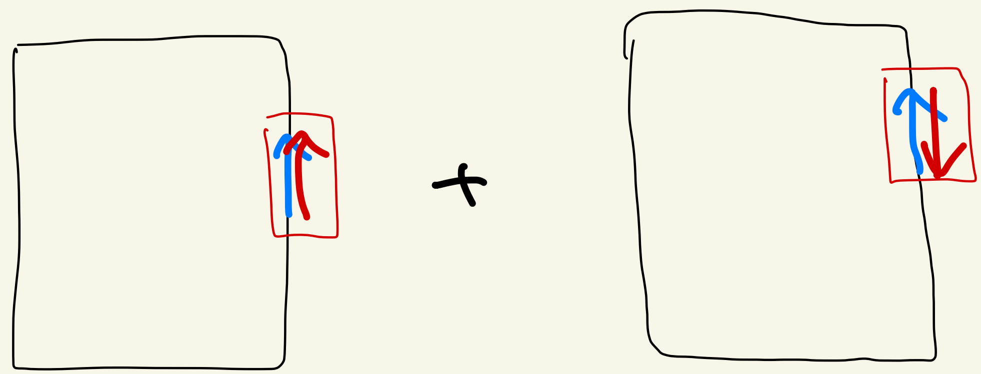
$(\phi_0 \sim e^{i\theta})$

reproduces [Cherman-Sen-Yaffe '18]

(b) strong coupling

$N = 2$

Deep confining $[\beta \to 0,\ \text{small } \kappa]$



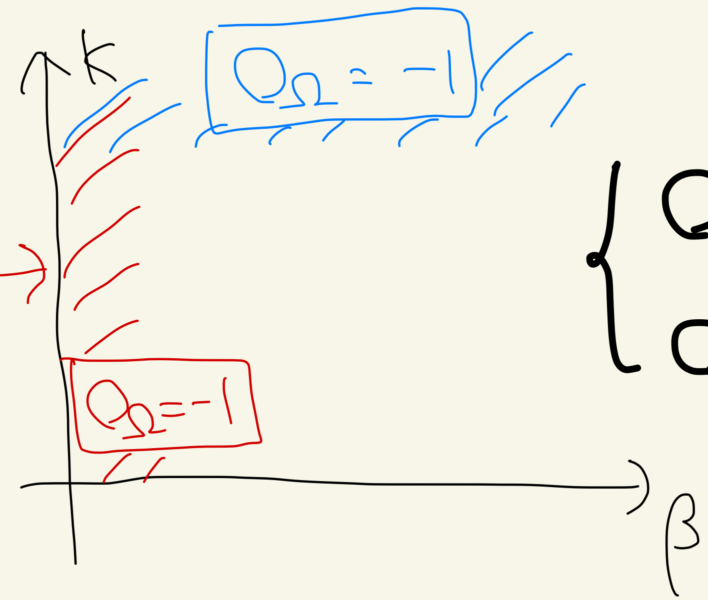
Both ϕ line & $\phi^\dagger$ line can screen $W(C)$

$$\int dU_l\, U_l\, \kappa \left[\mathrm{tr}(\phi^\dagger_{x'} U_l \phi_x) + \mathrm{tr}(\phi^\dagger_x U^\dagger_l \phi_{x'}) \right] + O(\kappa^2)$$

$$= \phi_x \left(\boxed{1 + \det \phi_x \det \phi^\dagger_x} \right) \phi^\dagger_{x'}$$

$$\propto \left(\det \phi_x \det \phi^\dagger_x \right)^{1/2}$$

$$\Rightarrow O_\Omega = -1$$



$$\begin{cases} Q_\Omega = -1 & \text{(deep Higgs)} \\ Q_\Omega = -1 & \text{(strong coupling)} \end{cases}$$

actually,

$\langle W(C) \rangle \sim \prod_{\ell \in C} \left(\det \phi_x^\dagger \phi_{x'} \right)^{1/2} = -1$ for all κ at $\beta \to 0$

At $N=2$, $\exists$ ($\mathbb{Z}_2$ symmetry)

$$\Phi \longrightarrow i\sigma^2 \Phi, \quad U_\ell \longrightarrow U_\ell^*$$

imposes $Q_\Omega = \pm 1$ (protecting AB phase)

[case (A) ; as in ①]

For $N \geq 3$, no such symmetry

$\mathcal{O}(k)$ $\uparrow\downarrow$ $\cancel{\uparrow\uparrow}$

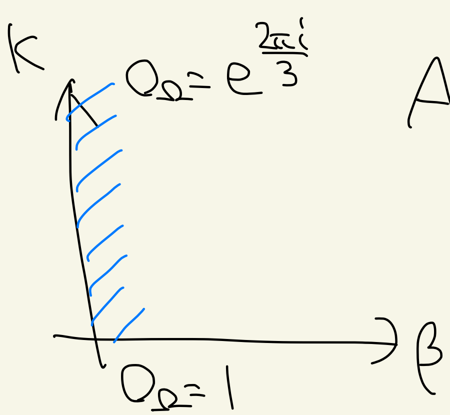
ϕ^{+} line can screen U_ℓ

We need ϕ^{N-1} to screen

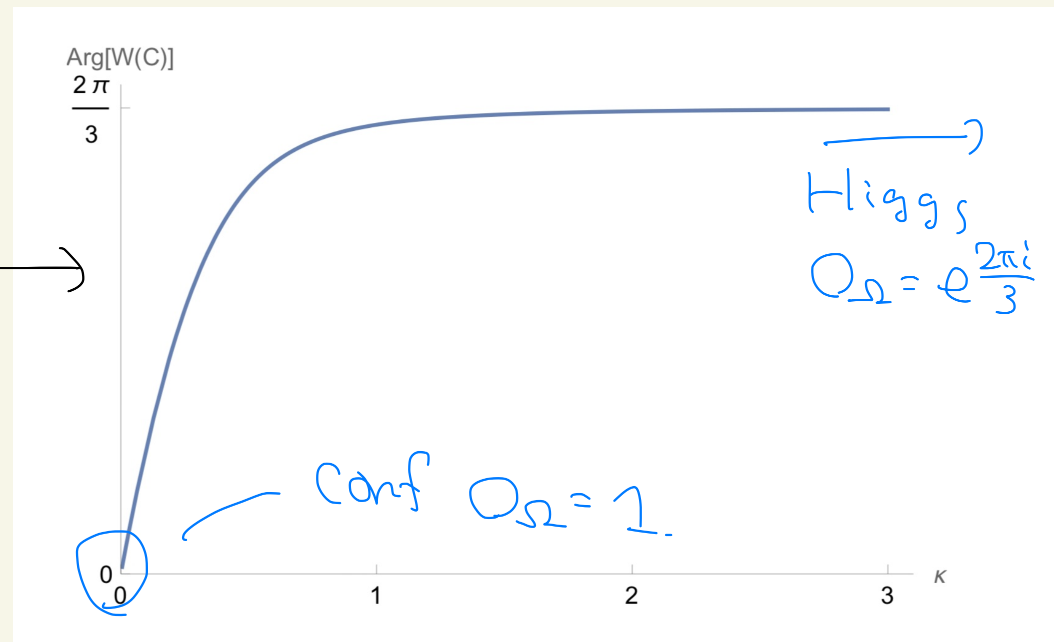
$\Rightarrow$ $O_\Omega = +1$ in deep confining region

Instead, O_Ω is continuous.

Indeed, at $N=3$



$\mathrm{Arg}\left(O_\Omega\right)_{\beta \to 0} \longrightarrow$

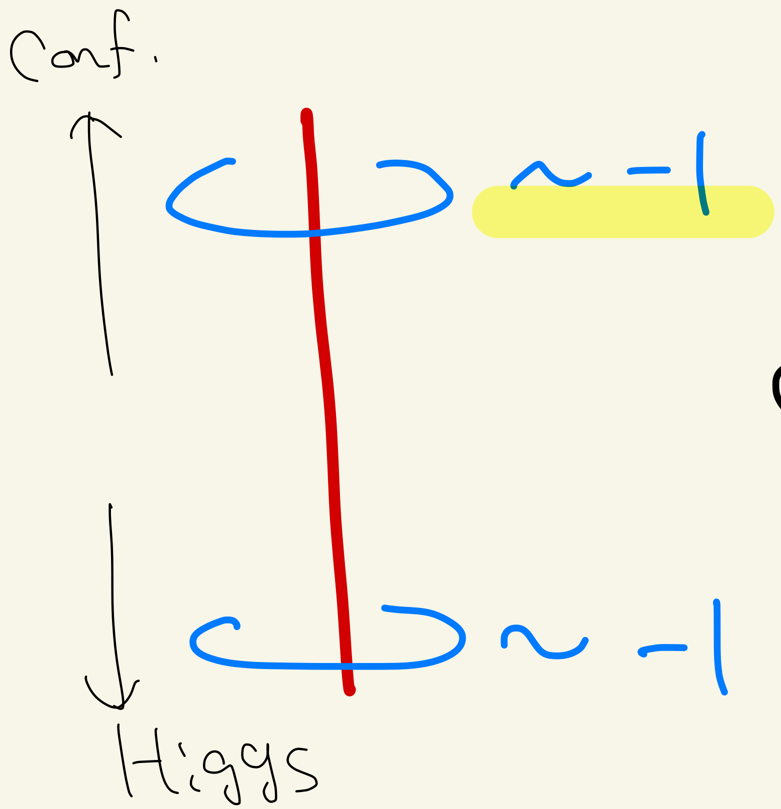


3. Summary & Comments

We have discussed AB phases in

1. Abelian toy model
2. $SU(N)$ YM + $U(N)$ Higgs
   [$N=3 \longrightarrow$ CFL effective model]

Case (A)



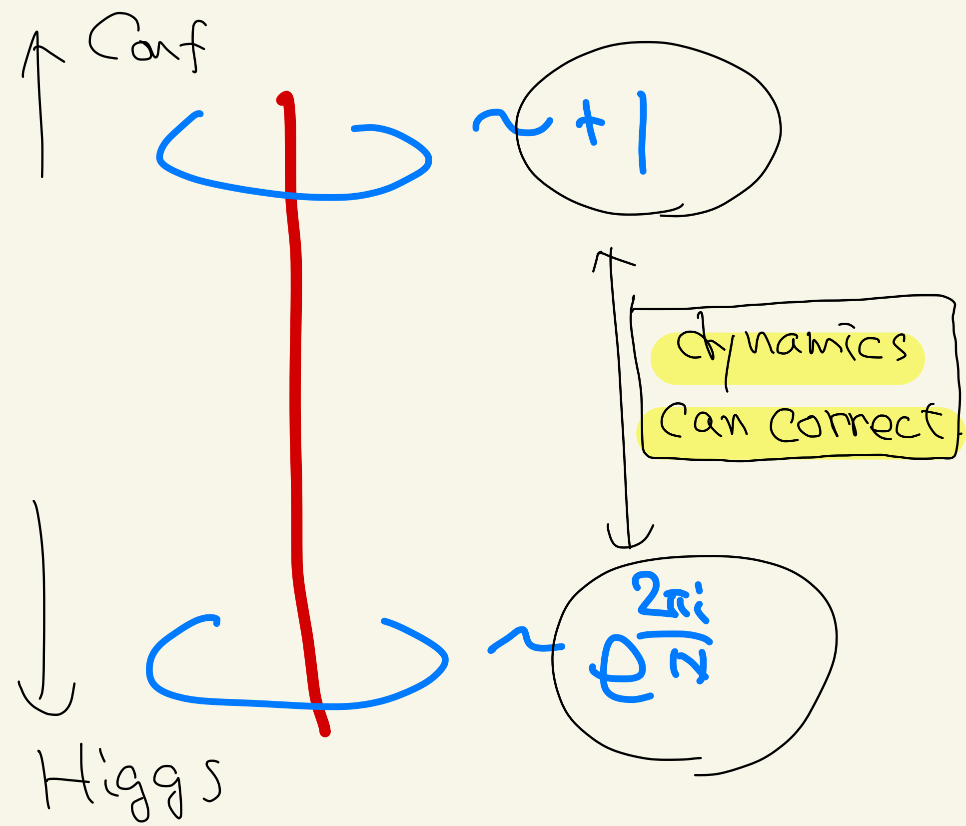
Constant AB phase

∃ (symmetry) protecting AB phase

①, ②: $N=2$

or

Case (B)



Continuous AB phase.

otherwise.

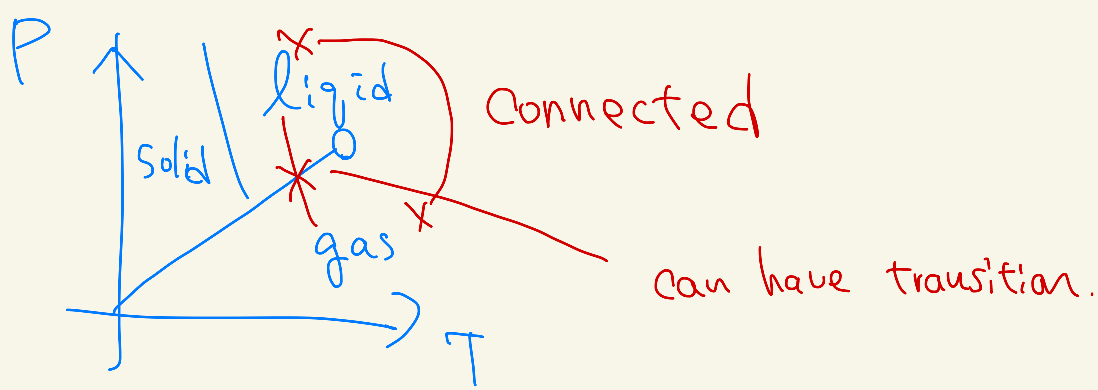
②: $N \geq 3$

Comments

(1) The Fradkin-Shenker-type argument claims $\exists$ (connecting region). This does not prove continuity in the actual QCD phase diagram.

there may be transition ocurred by dynamics

e.g. water phase diagram



Comments

(2) The perimeter-law Wilson loop generates (generally noninvertible) symmetry, acting on vortex surface

$$W_\rho(C)\, W_{\rho'}(C) = \sum_{\rho''} ⓐ\, W_{\rho''}(C)$$

common emergent symmetry between confining & Higgs regimes

Note. discrepancy in unbroken emergent symmetries does not distinguish phases.

trivially acting on the vacuum

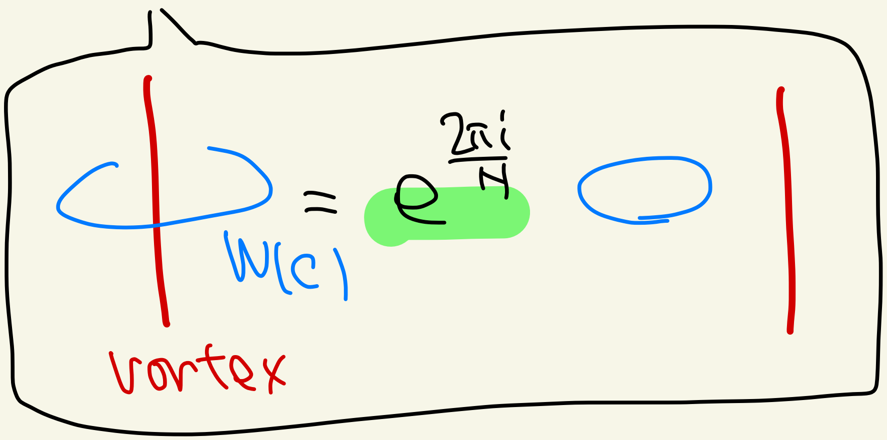
e.g.) U(1) Fradkin-Shenker
$U(1)_e^{[1]}$ vs. $U(1)_m^{[1]}$

Short Summary

nuclear superfluid
~ CFL phases

In superfluid fundamental gauge-Higgs systems :

Higgs ⟵⟶ Conf



recently-debated issue
⇝ phase transition somewhere ?
or not ?

⇓

My claim :

the AB phase respects Higgs-confinement continuity

⇒ quark-hadron continuity is a possible scenario.