Chiral anomalous MHD

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QCD Theory Seminar

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Plan

1. Non-equilibrium EFT formalism

2. Two approaches to strong field MHD

    (a) one-form symmetry (brief comments)

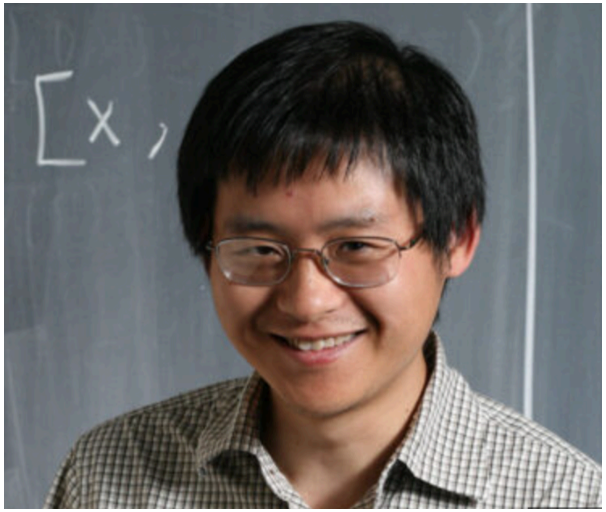
    (b) Partial Higgs

3. Chiral anomalous MHD

4. Future perspectives

Based on a paper with Hong Liu

arXiv:2212.09757



A systematic formulation of chiral anomalous magnetohydrodynamics

Builds on results from solo paper: arxiv:2101.02210

Effective field theory

Microscopic description

Identify ϕ : Low energy degrees of freedom

Integrate out the rest

$$\int D\phi \, e^{iS_{eff}[\phi]}$$

Renormalization group, universality

Macroscopic phenomena

Direct computation of $S_{eff}[\phi]$: not possible

Identify symmetries and constraints of $S_{eff}[\phi]$, Write down the most general theory consistent with the symmetries

EFT has been the main paradigm for studying equilibrium physics in many areas.

Conventional approaches to non-equilibrium physics:

microscopic:
e.g. Liouville equation, BGGKY hierarchy,
Boltzmann equation ……
(applicable to few body or dilute systems)

phenomenological:
hydrodynamics, stochastic systems
…

EFT provides nice middle ground, but its use for non-equilibrium systems has been limited.

Hydrodynamic approach

1. Observables are conserved quantities (e.g. stress-energy tensor or particle-number current)

2. Local equilibrium — constitutive relation relate conserved currents to local thermodynamic quantities

3. Impose local first law of thermodynamics — free energy

4. Impose local second law of thermodynamics — positivity constraints

5. Onsager relations — microscopic time-reversal

6. N-point Onsager relations?

7. Anything else? It is not clear if the above list is exhaustive

Challenges for non-equilibrium EFTs

1. What observables are of interest?

2. Nature of IR variables very different

   Often those identified in phenomenological approaches are not suitable for formulating an action principle

3. Symmetries

   e.g. what symmetries define a fluid?

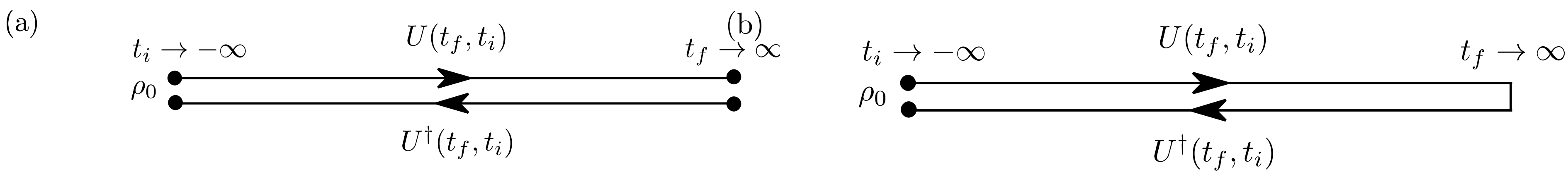
4. How to incorporate retardation and dissipation from first principle

   Standard lore: Dissipative systems don't have an action formulation

$$m\ddot{x} + \nu\dot{x} = 0$$

Observables

A large class of non-equilibrium observables can be obtained from generating functionals on a closed time path (CTP) or Schwinger-Keldysh contour



$$\rho(t_f) = U(t_f, t_i)\rho_0 U^\dagger(t_f, t_i) \qquad\qquad \mathrm{Tr}\left(\rho_0 \cdots\right)$$

$$e^{W[\phi_{1i},\phi_{2i}]} = \mathrm{Tr}\left[\rho_0 \mathcal{P} e^{i\int dt\,(\mathcal{O}_{1i}(t)\phi_{1i}(t) - \mathcal{O}_{2i}(t)\phi_{2i}(t))}\right]$$

Captures all observables which do not need to run your lab backward in time.

Non-equilibrium EFT

Microscopic path integrals defined on a CTP

Integrate out the rest

Identify χ : relevant IR degrees of freedom, slow variables

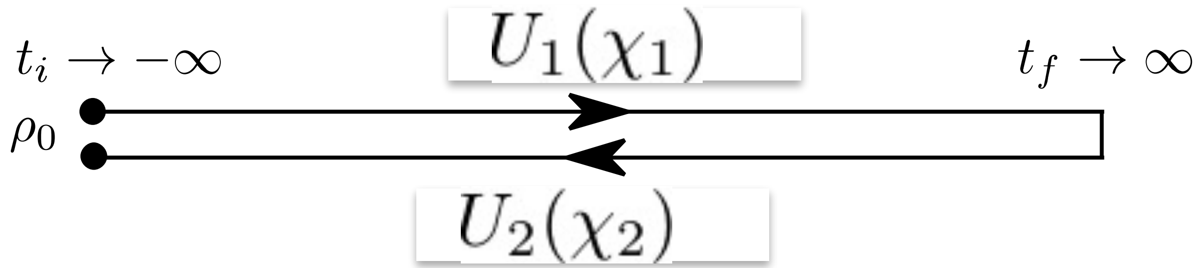
$$e^{W[\phi_1,\phi_2]} = \int D\chi_1 D\chi_2 \, e^{iS_{\rm EFT}[\chi_1,\phi_1;\chi_2,\phi_2;\rho_0]}$$

IR degrees of freedom are doubled.

The effective action automatically incorporates retardation and dissipative effects.

Constraints from unitarity time evolution

$$e^{iS_{\rm EFT}} =$$



1. $S^*_{\rm EFT}[\chi_1, \chi_2] = -S_{\rm EFT}[\chi_2, \chi_1]$ $\quad \mathrm{Tr}^*(U_1\rho_0 U_2^\dagger) = \mathrm{Tr}(U_2\rho_0 U_1^\dagger)$

   Terms symmetric in 1 <—> 2 must be purely imaginary

2. $\mathrm{Im}\, S_{\rm EFT} \geq 0$ $\quad \left|\langle\psi|U_2^\dagger U_1|\psi\rangle\right|^2 \leq 1$

3. $S_{\rm EFT}[\chi_1 = \chi, \chi_2 = \chi] = 0$ $\quad \mathrm{Tr}(U_1\rho_0 U_1^\dagger) = 1$

Not visible in Euclidean EFT

These constraints survive in the classical limit

Local equilibrium

Equilibrium correlation functions satisfy KMS relations (fluctuation-dissipation relations).

They also satisfy Onsager relations from microscopic time reversal.

Many non-equilibrium systems have local equilibrium, such as hydrodynamical systems.

How do we impose the system is in local equilibrium at the level of action principle?

Dynamical KMS symmetry

It turns out local equilibrium can be achieved by imposing an antilinear Z_2 symmetry, called dynamical KMS symmetry.

The symmetry ensures that the resulting generating functional satisfies the Onsager relations as well as KMS relations for a thermal state.

Example (non-dynamical temperature) :

$$\tilde{\chi}_1(x) = \Theta\chi_1(t - i\theta, \vec{x}), \quad \tilde{\chi}_2(x) = \Theta\chi_2(t + i(\beta_0 - \theta), \vec{x}) \ .$$

Θ : any discrete operation involving time reversal

Microscopic time reversal and local equilibrium have to be imposed together.

Emergent entropy and the second law

Combination of unitarity constraints and dynamical KMS symmetry leads to a remarkable consequence:

One can construct a local current s^μ , the “charge” of which never decreases.

This provides a field theoretical definition of entropy

In derivative expansion, the divergence of the current is everywhere non-negative.

Also gives a universal expression for entropy production.

Classical limit

So far the discussion applies to a quantum many-body system.

When temperature is sufficiently high, expect quantum effects are not important.

We can define a systematic $\hbar \to 0$ limit, in which the formulation simplifies, including the dynamical KMS transformations.

Even in the classical limit, path integrals remain (capturing statistical fluctuations)

So far the discussion is completely general.

For example, the formulation can be applied to the critical Ising model to study dynamical critical phenomena (more systematic than the stochastic approach).

It can also be used to derive hydrodynamics, the universal theory of conserved quantities, based on symmetries and action principle:

- dynamical variable associated with each conserved quantity

  For any conserved current, the hydrodynamical variables are the Stückelberg variables for the corresponding symmetry.

- symmetries to impose on the action

  Depend on the conserved quantities and phases.

Hydrodynamics for a U(1) conserved current

Hydrodynamic variables for a conserved U(1) current are:

$$\phi = \frac{\phi_1 + \phi_2}{2}, \ \phi_a = \phi_1 - \phi_2$$

and in the presence of sources, the action has the form

$$I_{\text{hydro}} = I_{\text{hydro}}[A_\mu + \partial_\mu \phi, A_{a\mu} + \partial_\mu \phi_a]$$

- Impose unitarity constraints and dynamical KMS
- Choice of $\Theta = \mathcal{T}, \mathcal{PT}, \mathcal{CT}, \mathcal{CPT}$
- translational and rotational symmetries
- invariance under (U(1) not spontaneously broken)

$$\phi \to \phi + \chi(\vec{x}), \quad \phi_a \to \phi_a$$

- Other discrete symmetries

A universal theory of diffusion (nonlinear)

Magnetohydrodynamics (MHD)

MHD: a universal theory for charged fluids in the presence of dynamical EM field.

Dynamical variables: hydro variables for conserved quantities and magnetic field $\vec{B}$

Difficulty: lack of a general principle to write down constitutive relations at strong field.

A new insight: $\vec{B}$ are charge densities of a conserved current associated with a generalized global symmetry (1-form symmetry)

$$J^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho} \qquad \partial_\mu J^{\mu\nu} = 0$$

Grozdanov, Hofman, Iqbal, arXiv:1610.07392

$$B_i = J^{0i} \qquad E_i = -\frac{1}{2}\epsilon_{ijk}J_{jk}$$

One-form symmetry approach

Combining the non-equilibrium EFT formalism, and the one-form symmetry realization,

→ A systematic formulation of MHD based only on symmetries and action principle

Will neglect motion of the medium and temperature fluctuations.

Stückelberg variables for one-form symmetry:

Non-equilibrium EFT →

constitutive relations $\vec{B}(\mathcal{A}_\mu, \mathcal{A}_{a\mu}),\ \vec{E}(\mathcal{A}_\mu, \mathcal{A}_{a\mu})$

EOM: $\partial_\mu J^{\mu\nu} = 0$

$\mathcal{A}_\mu, \mathcal{A}_{a\mu}$ Not vector potential

Partial Higgs approach

Landry: arXiv: 2101.02210 &
Landry, Liu arXiv: 2212.09757

It turns out there is another approach.

Consider first the U(1) symmetry global, then we have

$$I_{\text{hydro}} = I_{\text{hydro}}[A_\mu + \partial_\mu \phi, A_{a\mu} + \partial_\mu \phi_a]$$

Now make external fields dynamical (integrate over them)

The hydro fields are now eaten,

$$I_{\text{EFT}}[A_\mu, A_{a\mu}] = I_{\text{hydro}}[A_\mu, A_{a\mu}]$$

Gauge symmetries of EM fields reduce to

$$A_i \to A_i' = A_i + \partial_i \chi(\vec{x})$$

We call this partial Higgs mechanism.

EFT of EM fields in general media

This leads to an EFT for EM fields in general media:

$$I_{\rm EFT}[A_\mu, A_{a\mu}] = I_{\rm hydro}[A_\mu, A_{a\mu}]$$

To perform derivative expansion, still needs a power counting scheme, which reflects different physics:

1. Dielectric: $[A_\mu] = 0, \quad [\partial_\mu] = 1, \quad [E, B] = 1$

2. MHD:

$$[A_0] = 0,\ [A_i] = -1,\ [\partial_i] = 1,\ [\partial_0] = 2, [B] = 0,\ [E] = 1$$

………

Advantage: an action for physical EM fields

Strong field MHD

Both approaches give the same results: (only *T* conserved)

$$\partial_0 A_i = -r_{ij}(\epsilon_{jkl}\partial_k H_l + a_0 B_j)$$

a_0: constant
(topological)

$$H_i = -\frac{\partial F}{\partial B_i}$$

$F(A_0, B^2)$: Equilibrium free energy

$$\frac{\partial F}{\partial A_0} = 0 \quad \Rightarrow \quad A_0 = f(B^2)$$

$$r_{ij} = r_{\parallel} h_i h_j + r_{\perp}\Delta_{ij} + r_{\times}\epsilon_{ijk}h_k, \quad \Delta_{ij} = \delta_{ij} - h_i h_j$$

If only *CPT* is conserved: $a_0 = 0,\ f = 0,\ r_{\times} = 0$

In the case of neutron star: in principle all allowed.

One-form symmetry approach: misses a_0 term

Strong field MHD

$$\partial_0 A_i = -r_{ij}(\epsilon_{jkl}\partial_k H_l + a_0 B_j)$$

$$r_{ij} = r_{\parallel} h_i h_j + r_{\perp}\Delta_{ij} + r_{\times}\epsilon_{ijk}h_k, \quad \Delta_{ij} = \delta_{ij} - h_i h_j$$

$r_{\parallel}, r_{\perp}, r_{\times}$ B-dependent transport coefficients

$H_i = \dfrac{B_i}{\mu}$ μ : magnetic permeability. (B-dependent)

Weak field: $r_{\parallel} = r_{\perp} = \dfrac{1}{\sigma}, \quad r_{\times} = 0$

$$\mathbf{E} = c_\eta\,\mathbf{j}_B + c_a\,(\mathbf{B}\cdot\mathbf{j})\,\mathbf{B} + c_H\,\mathbf{j}_B \times \mathbf{B} + \nabla f,$$

$$\mathbf{j}_B \equiv \mathbf{j} - \nabla \ln\mu \times \mathbf{B} \qquad \mathbf{j} = \nabla \times \mathbf{B}$$

Goldreich-Reisenegger: all coefficients constants including μ

Magnetic diffusion

When linearized around a constant magnetic field, we get one more parameter than Goldreich-Reisenegger:

from first derivative w.r.t. B of μ

Leads to some additional non-isotropic effects in magnetic diffusion.

Effects are quantitative.

Chiral matter and ABJ anomaly

Now consider including chiral matter with ABJ anomaly:

$$\partial_\mu J_5^\mu = \frac{c}{4}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \qquad (*)$$

Chiral charge density not conserved, strictly speaking drops out hydrodynamics.

But if c is sufficiently small, its relaxation scale can be much smaller than microscopic relaxation scales

⇒ Chiral anomalous MHD

Many previous works on this, but all were in weak field regime

In one-form symmetry approach (*) is hard to realize as F itself should be determined dynamically.

Chiral anomalous MHD from partial Higgs

Using partial Higgs approach, the generalization is straightforward

φ, φ_a : hydro variables for the chiral current

$$J_5^i = (a_{50} - 2cA_0)B_i - \kappa_{ij}\partial_j\mu_5 - \lambda_{ij}^-\partial_t A_j + \epsilon_{ijk}\partial_j(mB_k)$$

$$\partial_t A_i = -r_{ij}(\epsilon_{jkl}\partial_k H_l + 2c\hat{\mu}_5 B_j + \lambda_{kj}^+\partial_k\mu_5)$$

$$n_5 = \frac{\partial F}{\partial \mu_5}, \quad H_i = -\frac{\partial F}{\partial B_i} \quad F(A_0, \mu_5, B^2)$$ = Equilibrium free energy

$$\hat{\mu}_5 \equiv \mu_5 - \frac{a_{00}}{2c} \quad \frac{\partial F}{\partial A_0} = 0 \Rightarrow A_0 = f(B^2, \mu_5)$$

Homogeneous equilibrium solutions

(1) $\hat{\mu}_5 = \text{const}, \quad B_i = 0$

(2) $\hat{\mu}_5 = 0, \quad B_i = \text{const}$

The first branch has an unstable mode, while the second branch is stable. If $\hat{\mu}_5 = \text{const}, \; B_i = \text{const}$

$$\hat{\mu}_5(\vec{x}, t) = f_0 \, e^{-\Gamma t} \qquad \Gamma \equiv \frac{(2cB_0)^2 r_{\parallel}}{\chi_5}$$

Chiral Magnetic Electric Separation Wave

For the branch $\hat{\mu}_5 = 0, \quad B_i = \text{const}$

Find decaying wave solution $\hat{\mu}_5(\vec{x}, t) = f_0(\vec{x}_\perp, z - v_z t) e^{-\Gamma t}$

$$\Gamma \equiv \frac{(2cB_0)^2 r_\parallel}{\chi_5} \qquad\qquad v_z \equiv \frac{4cB_0 r_\parallel \lambda_\parallel}{\chi_5}$$

There is a similar chiral wave for B in a direction perpendicular to the field.

Relies on CESE, not CSE — it is distinct from the chiral magnetic wave discussed by Kharzeev and Yee

CMW exists no matter choice of Θ, while
CMESW only sometimes exists; fails to exist for Θ = *CPT*

Chiral instability

$$\hat{\mu}_5 = \text{const}, \quad B_i = 0$$

Helical unstable mode:

$$B_x = \mathcal{B}(t)\cos kz, \quad B_y = \mathcal{B}(t)\sin kz, \quad \mathcal{B}(t) = B_0 e^{2cr\hat{\mu}_5 kt - D_B k^2 t}$$

first pointed out by Akamatsu and Yamamoto (2013)

Assuming all transport coefficients are isotropic and independent of B, we get a set of nonlinear equations which can be solved exactly

Depending on the sign of $2c + \eta_5 k$ $\quad \dfrac{\partial n_5}{\partial B_i} = \eta_5 B_i$

(1) An equilibrium state of constant $\hat{\mu}_5, \mathcal{B}$

(2) $\hat{\mu}_5, \mathcal{B}$ grows without bound

Solution unstable — requires numerics

Future perspectives

Important to understand field-dependence of various transport coefficients (e.g. in heavy ion collisions, Dirac and Weyl semimetals, electroweak plasmas and neutron stars)

and their effects on time evolution.

For the latter question, holographic models may help.

Full implications of these theories will await full nonlinear simulations.

Thank You