Heavy-quark spin polarization induced by the Kondo effect in a magnetic field

Daiki Suenaga (RIKEN, Nishina Center)

Based on

D. S., Y. Araki, K. Suzuki and S. Yasui; PRD 105 (2022) 7, 074028

Y. Araki, D. S., K. Suzuki and S. Yasui; PRRes. 3 (2021) 2, 023098

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1. Introduction

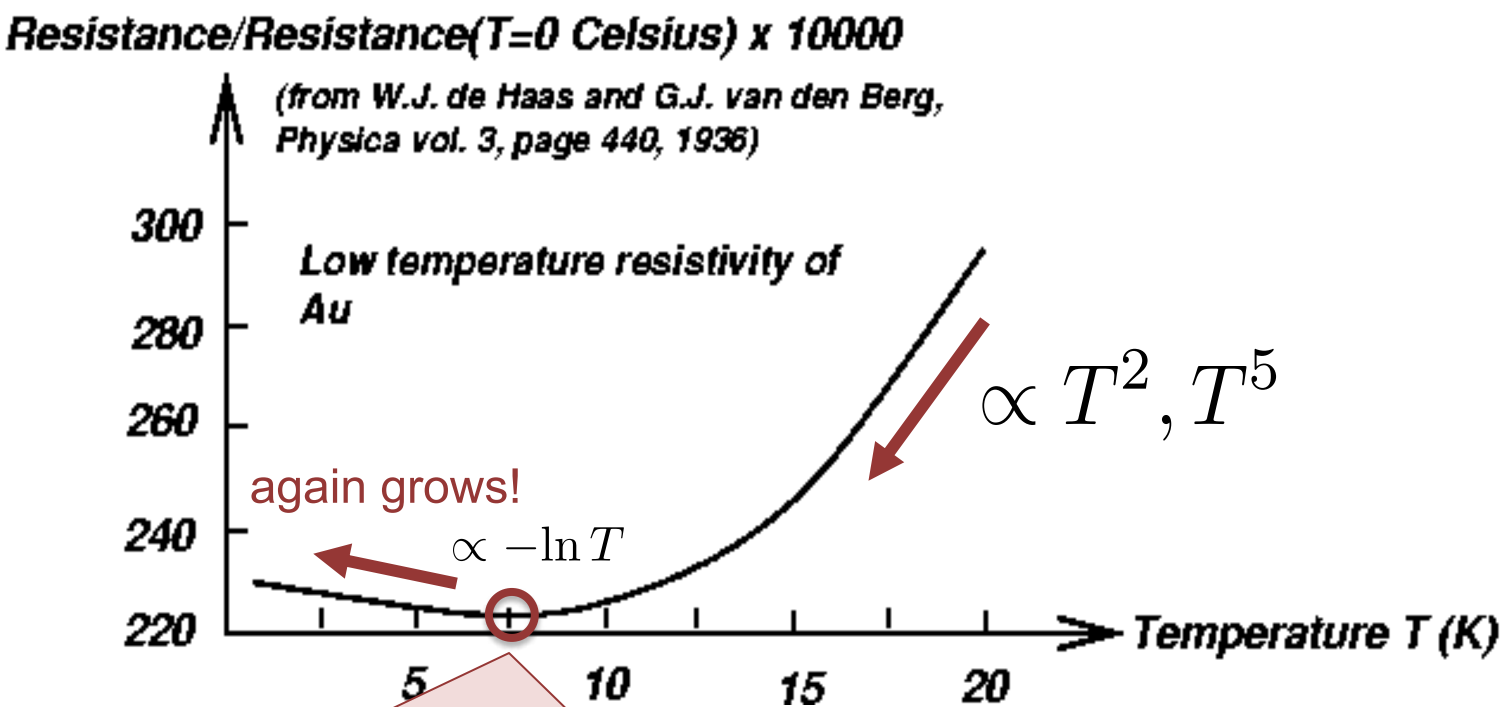
2. Heavy quark spin polarization

3. Spin polarization in metals

4. Conclusion

1. Introduction

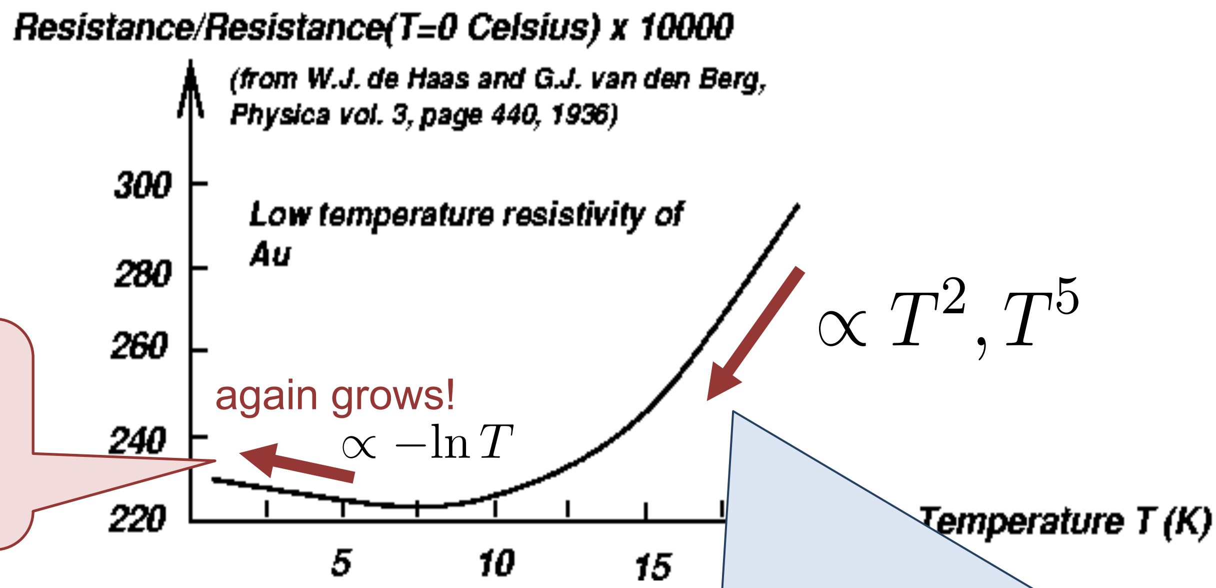
- **Puzzle of the electrical resistance**
  - In 1930s, in some metals a nonmonotonic change in electrical resistance (電気抵抗) with temperature was found



A mysterious minimum point was discovered

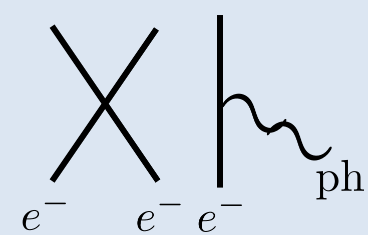
1. Introduction

- **Puzzle of the electrical resistance**
  - Electrical resistance is induced by scatterings of itinerant electrons
  (eg, Drude model, etc.)



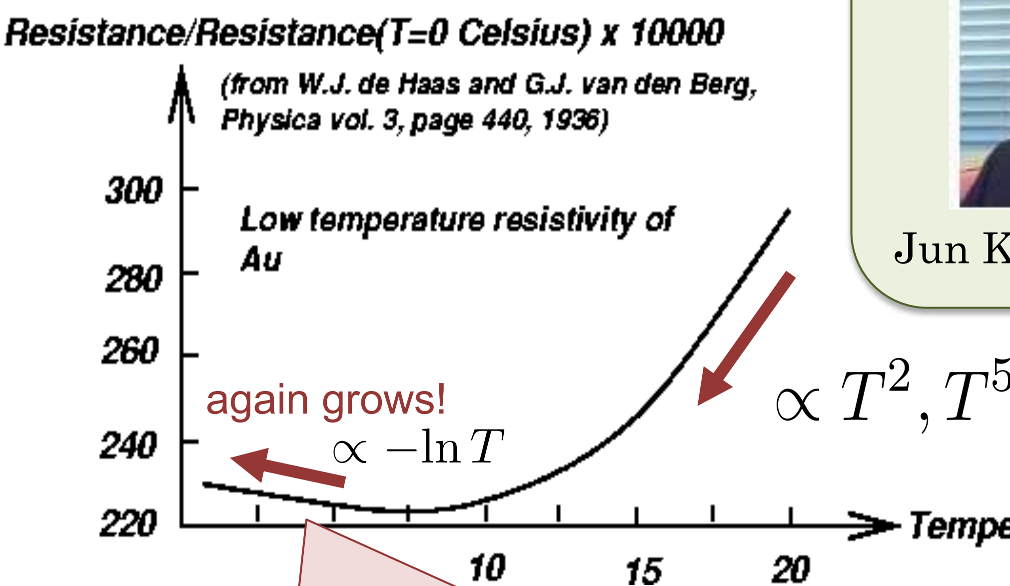
How can we explain it?
(Puzzle)

- The behavior can be understood by tree-level scatterings among a conduction electron or phonon
  [e.g. textbook by C. Kittel]



1. Introduction

- **Kondo effect**



Notable awards

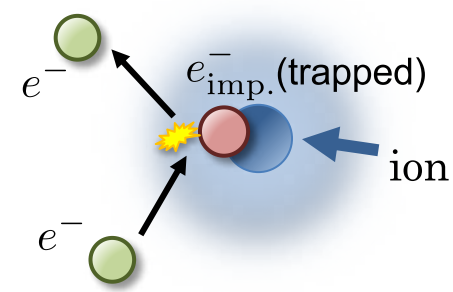
1973年 日本学士院恩賜賞

2003年 文化功労者

2020年 文化勲章

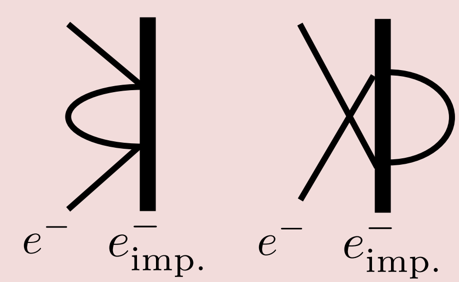
:

Jun Kondo (1930 – 2022/3/11)



- Kondo showed the increment is provided by 2nd-order scatterings with impurity electrons [Kondo effect]

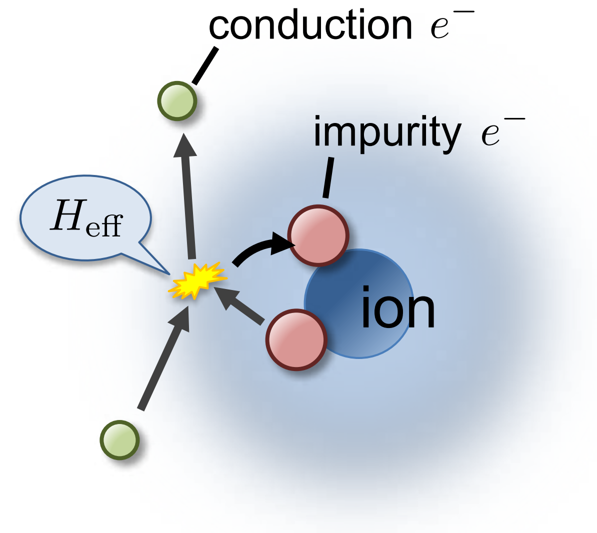
J. Kondo, PTP,32 (1964) 37



1. Introduction

- **Theoretical detail**

J. Kondo, PTP,32 (1964) 37



- Effective Hamiltonian for Kondo effect

$$H_{\rm eff} = J \sum_{kk'} \vec{s}_{kk'} \cdot \vec{S} \quad (J > 0)$$

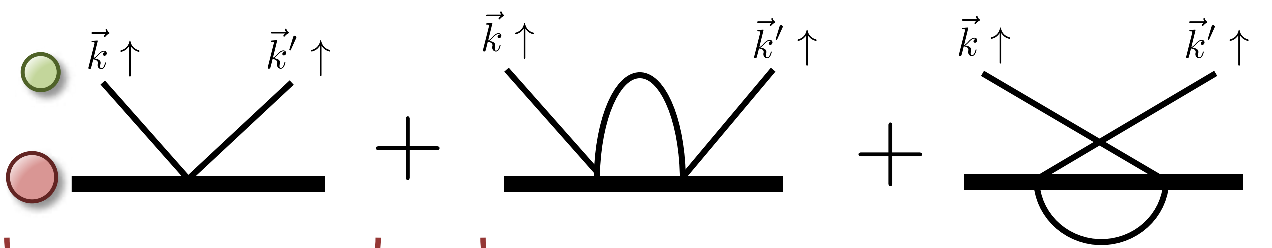
with $\vec{s}_{kk'} = c^\dagger_{k's'} \frac{\vec{\sigma}_{s's}}{2} c_{ks}$ spin operator of (green)

$\vec{S} = f^\dagger_{s'} \frac{\vec{\sigma}_{s's}}{2} f_s$ spin operator of (red)

[$H_{\rm eff}$ is driven from Anderson model with Rayleigh-Schrodinger perturbation method]

Schrieffer-Wolff, PR149 (1966) 491

- Scattering amplitude up to 2nd order can be evaluated



JS_z

$\sim -J^2 D_{\rm F} \ln\left(\frac{T}{\Lambda}\right) [S_-, S_+]$ **(→non-Abelian)**

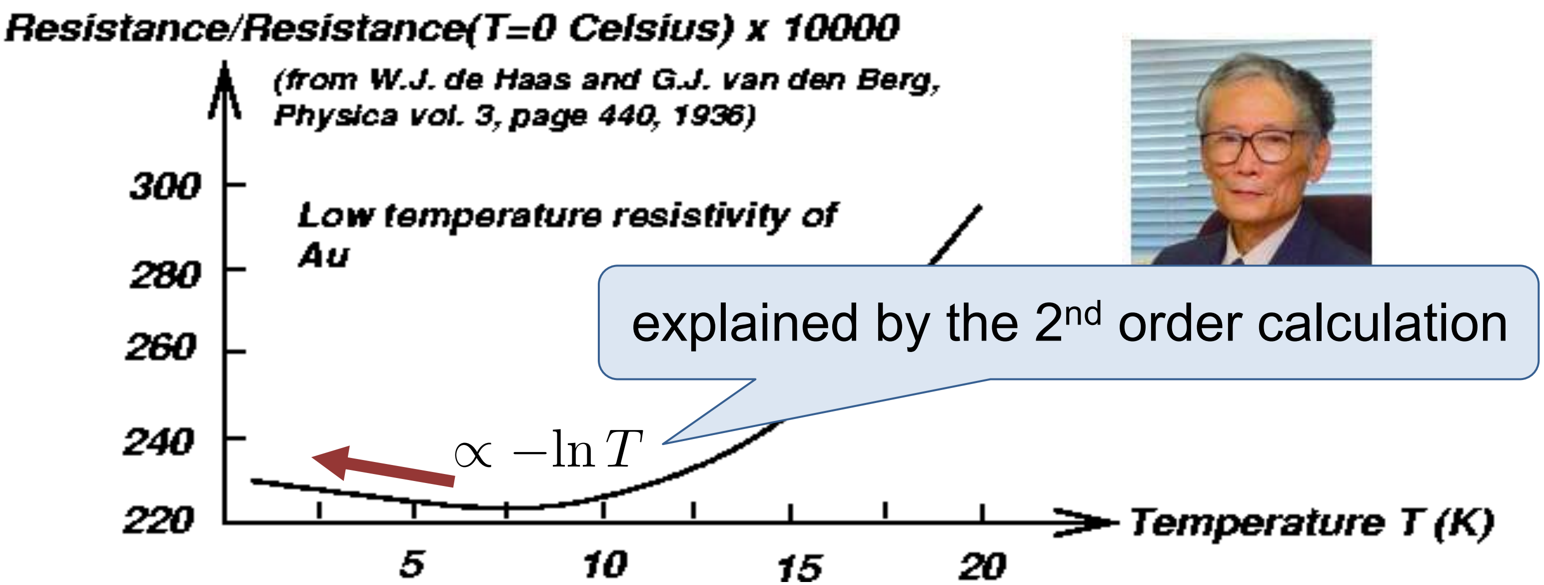
⇒ $\ln T$ appears!

D_F : DOS at Fermi surface

Λ : UV cutoff (bandwidth)

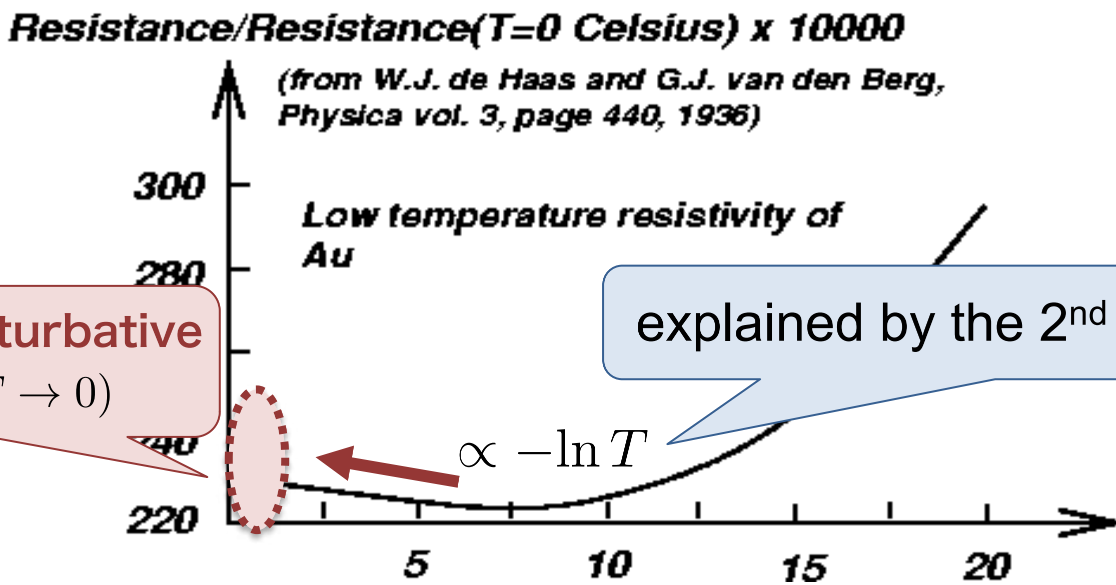
1. Introduction

- Nonperturbative nature



1. Introduction

- **Nonperturbative nature**

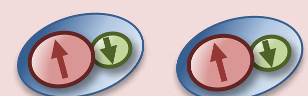


super nonperturbative

$\ln T \to \infty \; (T \to 0)$

explained by the 2nd order calculation

- Yosida showed that itinerant and impurity electrons form a **spin-singlet condensate** called Yosida singlet state



K.Yosida (1966)

cf, Wilson's RGE shows a Landau pole at $T_K = \Lambda \exp\left(-1/JD_F\right)$ P W Anderson (1970)

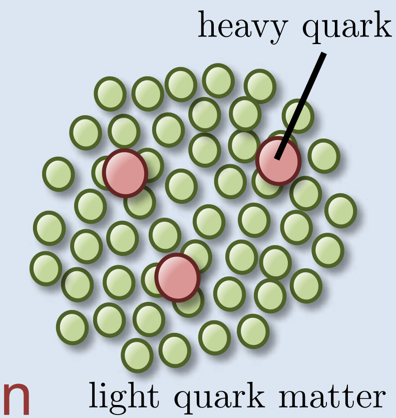
Derivation of T_K by a resummation Abrikosov (1965)

1. Introduction

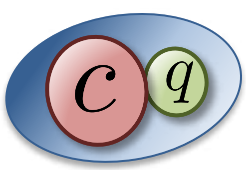
- **Kondo effect in QCD**

- Conditions for Kondo effect in QCD

1. Heavy impurity = heavy quark
2. Fermi surface = quark matter
3. Non-Abelian interaction = color interaction



- Nowadays the Kondo effect is referred to as quantum phenomena driven by the heavy-light condensate (Kondo condensate)



heavy-light condensate

= sibling of chiral condensate(χSB), diquark condensate(CSC) ...

• NJL analysis of Kondo effect

- NJL-type model for the Kondo effect is

Yasui-Suzuki-Itakura(2019)

$$\mathcal{L} = \bar{\psi}(i\partial_\mu \gamma^\mu + \mu\gamma_0)\psi + \Psi^\dagger i\partial_0 \Psi + \mathcal{L}_{\mathrm{eff}} \quad \text{with} \quad \mathcal{L}_{\mathrm{eff}} = G(\bar{\psi}\gamma^\mu T^a \psi)(\Psi^\dagger \gamma_\mu T^a \Psi)$$

ψ : light fermion

Ψ : heavy fermion within HQET

c.f. $H_{\mathrm{eff}} = J \sum_{kk'} \vec{s}_{kk'} \cdot \vec{S}$

for original Kondo problem

- The Fierz transformation at $\mathcal{O}(1/N_c^0)$ yields

$$\mathcal{L}_{\mathrm{eff}} = G\left[|\bar{\psi}\Psi|^2 + |\bar{\psi}i\gamma_5\Psi|^2 + |\bar{\psi}\vec{\gamma}\Psi|^2 + |\bar{\psi}\vec{\gamma}\gamma_5\Psi|^2\right]$$

- The ansatz of mean fields (in momentum space) is

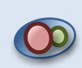
$$G\langle\bar{\psi}\Psi\rangle = \Delta$$
$$G\langle\bar{\psi}\vec{\gamma}\Psi\rangle = \Delta\hat{k}$$

$\Leftrightarrow$

$$G\langle\bar{\psi}\Lambda_{\mathrm{p}}\Psi\rangle = \Delta/2$$
with $\Lambda_{\mathrm{p}} = (1 + \hat{k}\cdot\vec{\alpha})/2$

only positive-energy parts of ψ and Ψ join

($\vec{k}$ is momentum of the fermions)

$\Delta \sim$  (Kondo condensate)

• Fermionic excitations

Yasui-Suzuki-Itakura(2019)

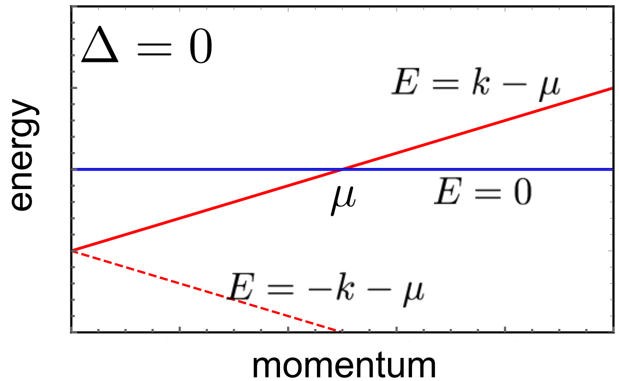
- Dispersion relation of the fermions are obtained as

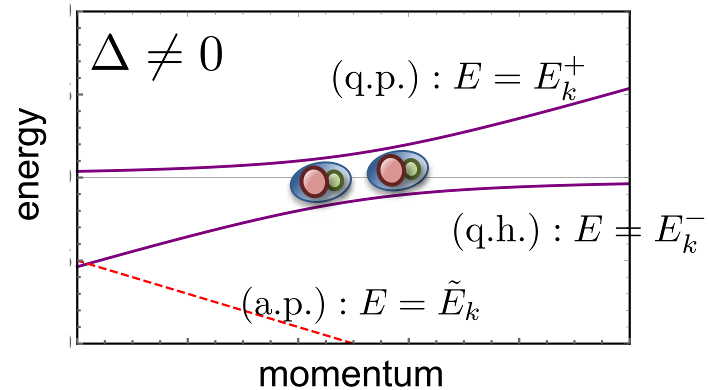
light: $E = k - \mu$ → $E_k^+ = \frac{1}{2}\left(k - \mu + \sqrt{(k-\mu)^2 + 8|\Delta|^2}\right)$ quasi-particle (q.p.)

heavy: $E = 0$ → $E_k^- = \frac{1}{2}\left(k - \mu - \sqrt{(k-\mu)^2 + 8|\Delta|^2}\right)$ quasi-hole (q.h.)

light: $E = -k - \mu$ → $\tilde{E}_k = -k - \mu$ anti-particle (a.p.)

- Schematic picture is drawn as





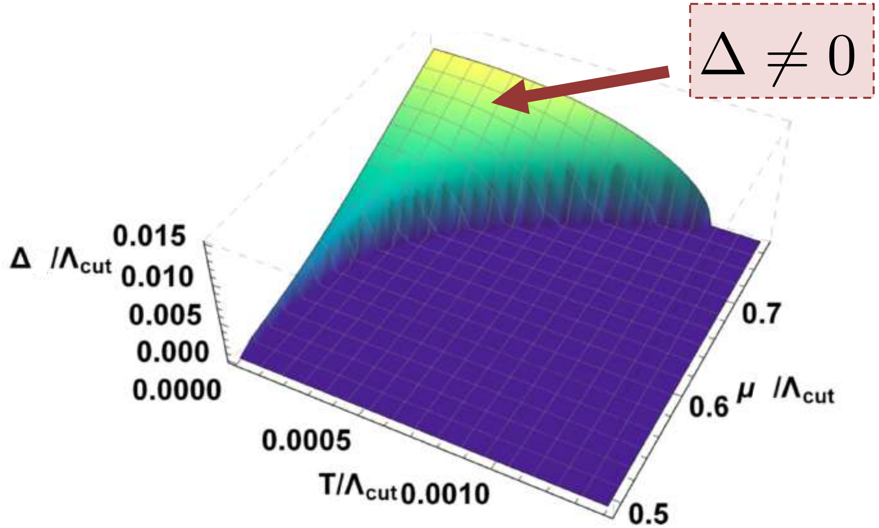
- **Phase diagram**

Yasui-Suzuki-Itakura(2019), DS-Suzuki-Araki-Yasui(2020)

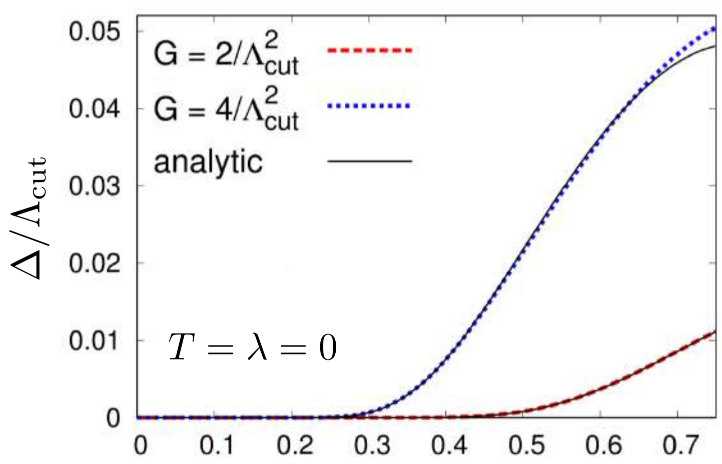
- Thermodynamic potential at finite temperature is

$$\Omega/V = -\int \frac{d^3k}{(2\pi)^3}\left\{\tilde{E}_k + E_k^+ + E_k^- + 2T\ln\left(1+\mathrm{e}^{-\beta\tilde{E}_k}\right)\left(1+\mathrm{e}^{-\beta E_k^+}\right)\left(1+\mathrm{e}^{-\beta E_k^-}\right)\right\} + \frac{2}{G}|\Delta|^2$$

- Solving a gap equation yields



phase diagram at $T=0$



- The **Kondo phase** at low temperature and high density emerges

1. Introduction

- **Previous works**

Yasui, K. Sudoh, PRC88, 015201 (2013)
K. Hattori, K. Itakura, S. Ozaki, S.Yasui, PRD92, 065003 (2015)
S. Ozaki, K. Itakura, Y. Kuramoto, PRD94, 074013 (2016)
T. Kanazawa, S. Uchino, PRD94, 114005 (2016)
S.Yasui, PRC93, 065204 (2016)
S.Yasui, K. Sudoh, PRC95, 035204 (2017)
S.Yasui, PLB773, 428 (2017)
T. Kimura, S. Ozaki, J. Phys. Soc. Jpn. 86, 084703 (2017)
S.Yasui., K. Suzuki, K. Itakura, PRD96, 014016 (2017)
K. Suzuki, S.Yasui., K. Itakura, PRD96, 114007 (2017)
S.Yasui., S. Ozaki, PRD96, 114027 (2017)
S.Yasui., K. Suzuki, K. Itakura, NPA983, 90 (2019)
T. Kimura, S. Ozaki, Phys. Rev. D99, 014040 (2019)
K. Hattori, X.-G. Huang, R. D. Pisarski, PRD99, 094044 (2019)
S.Yasui, T. Miyamoto, PRC100, 045201 (2019)
R. Fariello, Juan C. Macías, F.S. Navarra, arXiv:1901.01623
D. S., K. Suzuki, S. Yasui, PRRes2, 023066 (2020)
T. Kanazawa, arXiv:2006.00200
Y. Araki, **D. S.**, K. Suzuki, S. Yasui, PRRes. 3 (2021) 013233
Y. Araki, **D. S.**, K. Suzuki, S. Yasui, PRRes. 3 (2021) 023098
D. S., Y. Araki, K. Suzuki, S. Yasui, PRD 103 (2021) 054041
D. S., Y. Araki, K. Suzuki, S. Yasui, PRD 105 (2022) 074028
K. Hattori, **D. S.**, K. Suzuki, S. Yasui, arXiv: 2211.16150
⋮

- Phase structures (with T, μ, μ_5)
- Kondo excitons (fluctuation modes)
- Wilson's RGE analysis
- Kondo effect under magnetic field
- Transport coefficients
- Interplay with chiral condensate
- Magnetic responses

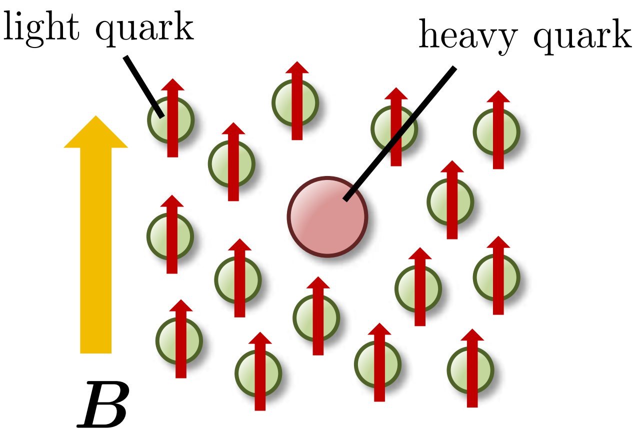
⋮

1. Introduction

- **Heavy quark spin polarization by Kondo effect**

  - Kondo effect is expected to lead to a novel mechanism of **heavy quark spin polarization (HQSP)**

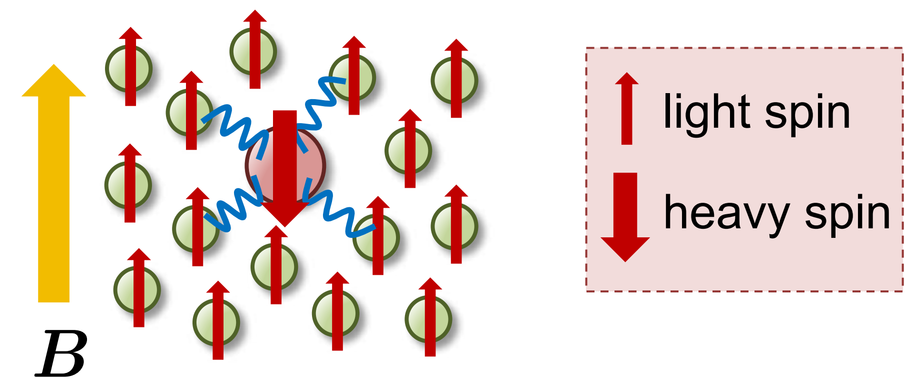
without Kondo effect



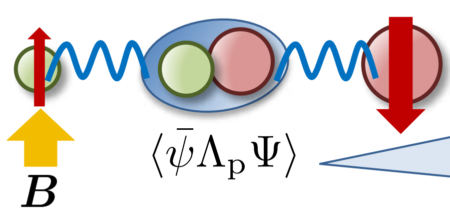
no HQSP

$$H_{\rm NR} = -\frac{\vec{\nabla}^2}{2m_Q} - \underbrace{\frac{e_Q}{m_Q}}_{0\,(m_Q \to \infty)} \vec{S}\cdot\vec{B} + \cdots$$

with Kondo effect

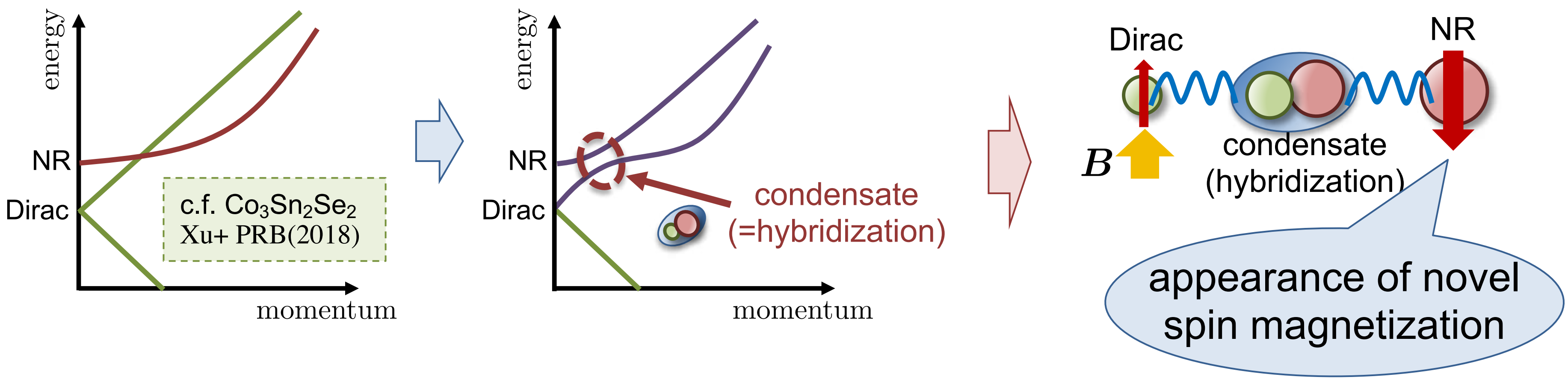


emergence of HQSP



light and heavy spins are correlated!
[$\hat{k}\cdot\vec{\alpha}$ term is essential]

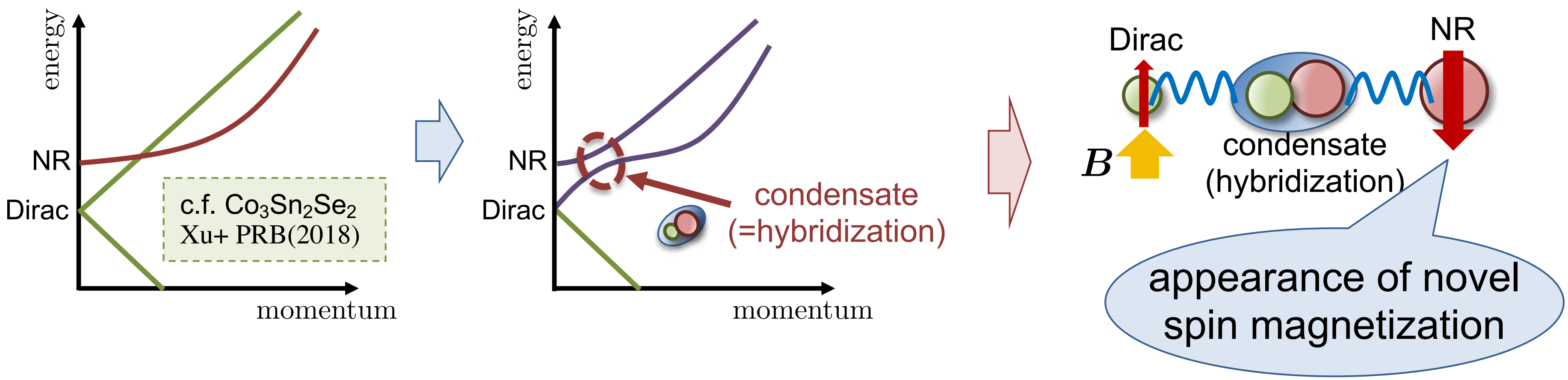
- **Application to metals**
  - The idea of HQSP can be applied to Dirac (Weyl) metals with nonrelativistic bands, in the presence of their condensate (=hybridization)

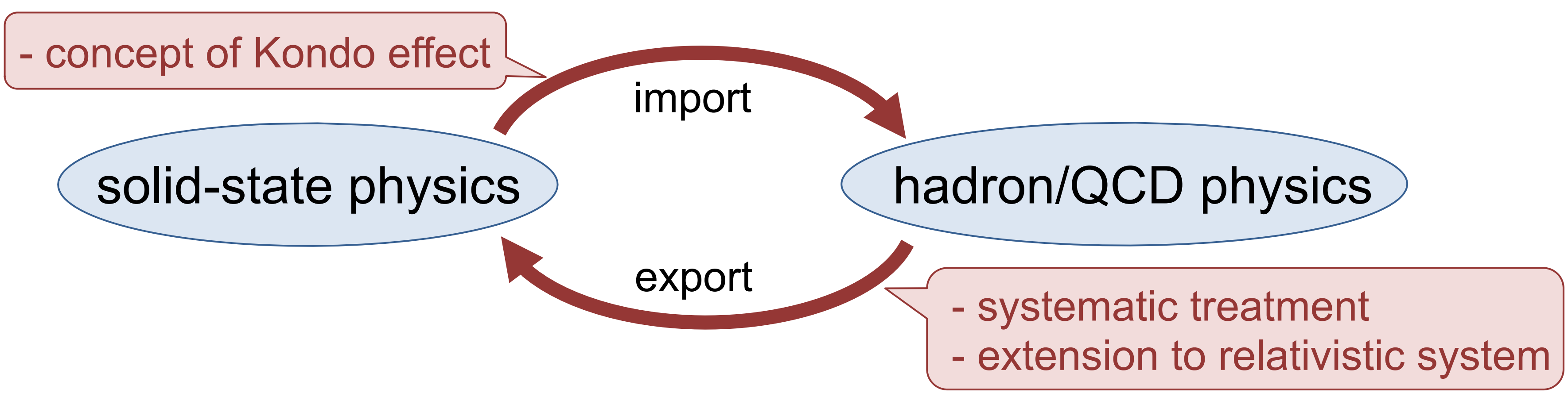


1. Introduction

- **Application to metals**

  - The idea of HQSP can be applied to Dirac (Weyl) metals with nonrelativistic bands, in the presence of their condensate (=hybridization)





2. Heavy quark spin polarization

- **Model**

- The NJL-type Lagrangian with the mean-field approximation is

$$\mathcal{L} = \bar{\psi}(i\not{D} + \mu\gamma_0)\psi + \Psi_v^\dagger iD_0\Psi_v$$
$$-g\left[\langle\bar{\psi}\Psi_v\rangle\Psi_v^\dagger\psi + \langle\bar{\psi}\gamma^i\Psi_v\rangle\Psi_v^\dagger\gamma^i\psi + (\text{h.c.})\right]$$
$$+g\left[\langle\bar{\psi}\Psi_v\rangle\langle\Psi_v^\dagger\psi\rangle + \langle\bar{\psi}\gamma^i\Psi_v\rangle\langle\Psi_v^\dagger\gamma^i\psi\rangle\right] ,$$

$$D_\mu\psi = (\partial_\mu + ie_qA_\mu)\psi$$
$$D_0\Psi_v = (\partial_0 + ie_QA_0)\Psi_v$$

⇩ assuming ansatz $g\langle\bar{\psi}\Lambda_{\mathrm{p}}\Psi_v\rangle = \Delta/2$ with $\Lambda_{\mathrm{p}} = (1 + \hat{p}\cdot\vec{\alpha})/2$

- The Lagrangian turns into

$$\mathcal{L} = \bar{\Phi}i\mathcal{G}_0^{-1}\Phi - e_q\bar{\psi}\not{A}\psi - e_Q\Psi_v^\dagger A_0\Psi_v + \frac{2|\Delta|^2}{g}$$

with the inverse of Green's function

$$i\mathcal{G}_0^{-1}(p_0,\boldsymbol{p}) = \begin{pmatrix} p_0+\mu & -\boldsymbol{p}\cdot\boldsymbol{\sigma} & \Delta^* \\ \boldsymbol{p}\cdot\boldsymbol{\sigma} & -p_0-\mu & -\Delta^*\hat{\boldsymbol{p}}\cdot\boldsymbol{\sigma} \\ \Delta & \Delta\hat{\boldsymbol{p}}\cdot\boldsymbol{\sigma} & p_0 \end{pmatrix}$$

2. Heavy quark spin polarization

- **Summary of Green's function**

$$\tilde{\mathcal{G}}_0(p_0,\boldsymbol{p}) = \begin{pmatrix} \tilde{\mathcal{G}}_0^{\bar{\psi}\psi}(p_0,\boldsymbol{p}) & \tilde{\mathcal{G}}_0^{\bar{\psi}\Psi_v}(p_0,\boldsymbol{p}) \\ \tilde{\mathcal{G}}_0^{\Psi_v^\dagger\psi}(p_0,\boldsymbol{p}) & \tilde{\mathcal{G}}_0^{\Psi_v^\dagger\Psi_v}(p_0,\boldsymbol{p}) \end{pmatrix}$$

with

$$\begin{cases} \tilde{\mathcal{G}}_0^{\bar{\psi}\psi}(p_0,\boldsymbol{p}) = i\left[\dfrac{U_+(\boldsymbol{p})}{p_0-E_{\boldsymbol{p}}^+} + \dfrac{U_-(\boldsymbol{p})}{p_0-E_{\boldsymbol{p}}^-}\right]\Lambda_{\mathrm{p}} + i\dfrac{U_{\mathrm{a}}(\boldsymbol{p})}{p_0-E_{\boldsymbol{p}}^{\mathrm{a}}}\Lambda_{\mathrm{a}}\,, \\ \tilde{\mathcal{G}}_0^{\bar{\psi}\Psi_v}(p_0,\boldsymbol{p}) = i\left[\dfrac{V_+^*(\boldsymbol{p})}{p_0-E_{\boldsymbol{p}}^+} + \dfrac{V_-^*(\boldsymbol{p})}{p_0-E_{\boldsymbol{p}}^-}\right]\Lambda_{\mathrm{pH}} \\ \tilde{\mathcal{G}}_0^{\Psi_v^\dagger\psi}(p_0,\boldsymbol{p}) = i\left[\dfrac{V_+(\boldsymbol{p})}{p_0-E_{\boldsymbol{p}}^+} + \dfrac{V_-(\boldsymbol{p})}{p_0-E_{\boldsymbol{p}}^-}\right]\Lambda_{\mathrm{Hp}} \\ \tilde{\mathcal{G}}_0^{\Psi_v^\dagger\Psi_v}(p_0,\boldsymbol{p}) = i\left[\dfrac{W_+(\boldsymbol{p})}{p_0-E_{\boldsymbol{p}}^+} + \dfrac{W_-(\boldsymbol{p})}{p_0-E_{\boldsymbol{p}}^-}\right]\mathbf{1}\,. \end{cases}$$

$$\begin{cases} U_+(\boldsymbol{p}) = \dfrac{2(|\Delta|^2+|\boldsymbol{p}|E_{\boldsymbol{p}}^+)}{(E_{\boldsymbol{p}}^- - E_{\boldsymbol{p}}^+)(E_{\boldsymbol{p}}^{\mathrm{a}} - E_{\boldsymbol{p}}^+)}\,, \\ U_-(\boldsymbol{p}) = \dfrac{2(|\Delta|^2+|\boldsymbol{p}|E_{\boldsymbol{p}}^-)}{(E_{\boldsymbol{p}}^+ - E_{\boldsymbol{p}}^-)(E_{\boldsymbol{p}}^{\mathrm{a}} - E_{\boldsymbol{p}}^-)}\,, \\ U_{\mathrm{a}}(\boldsymbol{p}) = 1\,, \\ V_+(\boldsymbol{p}) = \dfrac{\Delta}{E_{\boldsymbol{p}}^- - E_{\boldsymbol{p}}^+}\,, \quad V_-(\boldsymbol{p}) = \dfrac{\Delta}{E_{\boldsymbol{p}}^+ - E_{\boldsymbol{p}}^-}\,, \\ W_+(\boldsymbol{p}) = \dfrac{E_{\boldsymbol{p}}^+ - |\boldsymbol{p}| + \mu}{E_{\boldsymbol{p}}^+ - E_{\boldsymbol{p}}^-}\,, \quad W_-(\boldsymbol{p}) = \dfrac{E_{\boldsymbol{p}}^- - |\boldsymbol{p}| + \mu}{E_{\boldsymbol{p}}^- - E_{\boldsymbol{p}}^+} \end{cases}$$

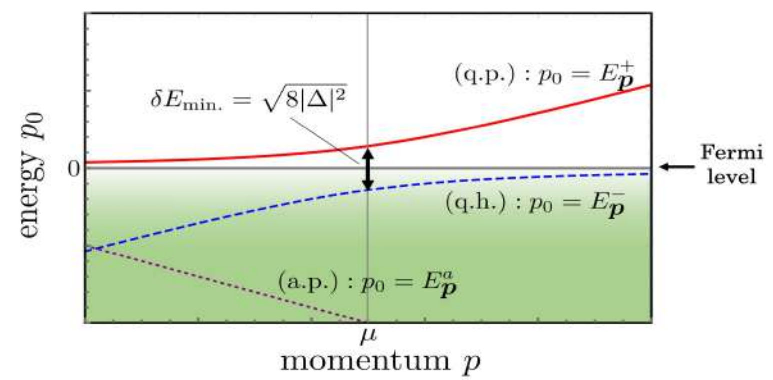
$$\begin{cases} \Lambda_{\mathrm{p}} \equiv \dfrac{1+\hat{\boldsymbol{p}}\cdot\boldsymbol{\alpha}}{2}\gamma_0\,, \quad \Lambda_{\mathrm{a}} \equiv \dfrac{1-\hat{\boldsymbol{p}}\cdot\boldsymbol{\alpha}}{2}\gamma_0 \\ \Lambda_{\mathrm{pH}} \equiv \begin{pmatrix} 1 \\ \hat{\boldsymbol{p}}\cdot\boldsymbol{\sigma} \end{pmatrix}\,, \quad \Lambda_{\mathrm{Hp}} \equiv \begin{pmatrix} 1 & -\hat{\boldsymbol{p}}\cdot\boldsymbol{\sigma} \end{pmatrix} \end{cases}$$

- The dispersion relations are

(q. p.) $E_{\boldsymbol{p}}^+ = \dfrac{1}{2}\left(|\boldsymbol{p}| - \mu + \sqrt{(|\boldsymbol{p}|-\mu)^2 + 8|\Delta|^2}\right)$

(q. h.) $E_{\boldsymbol{p}}^- = \dfrac{1}{2}\left(|\boldsymbol{p}| - \mu - \sqrt{(|\boldsymbol{p}|-\mu)^2 + 8|\Delta|^2}\right)$

(a. p.) $E_{\boldsymbol{p}}^{\mathrm{a}} = -|\boldsymbol{p}| - \mu\,.$

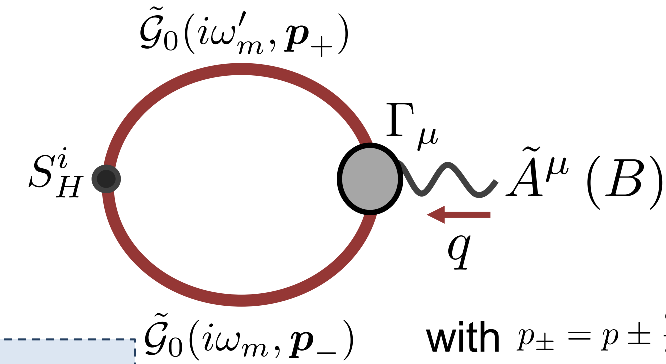


2. Heavy quark spin polarization

• Evaluation of HQSP

- The HQSP in the imaginary-time formalism within the linear response theory can be obtained via

$$\langle \tilde{S}^i_H(i\bar{\omega}_n, \boldsymbol{q})\rangle_\beta = -N_c T \sum_m \int^\Lambda \frac{d^3p}{(2\pi)^3}$$
$$\times \mathrm{tr}\left[S^i_H \tilde{\mathcal{G}}_0(i\omega'_m, \boldsymbol{p}_+)\Gamma_\mu \tilde{\mathcal{G}}_0(i\omega_m, \boldsymbol{p}_-)\right] \tilde{A}^\mu(i\bar{\omega}_n, \boldsymbol{q})$$



with $p_\pm = p \pm \frac{q}{2}$

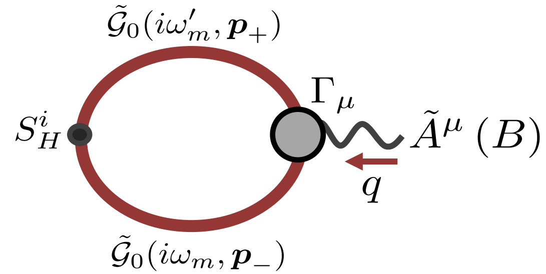
$\vec{S}_H = \begin{pmatrix} 0_{4\times 4} & 0_{4\times 2} \\ 0_{2\times 4} & \left(\frac{\vec{\sigma}}{2}\right)_{2\times 2} \end{pmatrix}$ is the heavy-quark spin operator

- The vertex Γ_μ must be corrected to satisfy the following Ward-Takahashi identity (WTI) of $U(1)_{EM}$ symmetry

$$q_\mu \Gamma^\mu = i\tilde{\mathcal{G}}_0^{-1}(p_0^+, \boldsymbol{p}_+)Q - Qi\tilde{\mathcal{G}}_0^{-1}(p_0^-, \boldsymbol{p}_-) \qquad \text{with} \quad Q \equiv \begin{pmatrix} e_q & 0 \\ 0 & e_Q \end{pmatrix}$$

2. Heavy quark spin polarization

- **Vertex corrections**



- The WTI for each component reads

$$\begin{cases} q_\mu \Gamma^\mu_{A\bar{\psi}\psi} = e_q \not{q} \ , \\ q_\mu \Gamma^\mu_{A\bar{\psi}\Psi_v} = \Delta^* \begin{pmatrix} e_Q - e_q \\ -e_Q \hat{\boldsymbol{p}}_+ \cdot \boldsymbol{\sigma} + e_q \hat{\boldsymbol{p}}_- \cdot \boldsymbol{\sigma} \end{pmatrix} \ , \\ q_\mu \Gamma^\mu_{A\Psi_v^\dagger \psi} = \Delta \begin{pmatrix} e_q - e_Q & e_q \hat{\boldsymbol{p}}_+ \cdot \boldsymbol{\sigma} - e_Q \hat{\boldsymbol{p}}_- \cdot \boldsymbol{\sigma} \end{pmatrix} \\ q_\mu \Gamma^\mu_{A\Psi_v^\dagger \Psi_v} = e_Q q_0 \mathbf{1} \ . \end{cases}$$

with $\Gamma^\mu \equiv \begin{pmatrix} \Gamma^\mu_{A\bar{\psi}\psi} & \Gamma^\mu_{A\bar{\psi}\Psi_v} \\ \Gamma^\mu_{A\Psi_v^\dagger\psi} & \Gamma^\mu_{A\Psi_v^\dagger\Psi_v} \end{pmatrix}$

⇨ The off-diagonal elements must be corrected (the diagonal ones need not)

- The approximate solutions for small $\boldsymbol{q}$ relevant to external $\vec{B}$ are

$$\begin{cases} \Gamma^\mu_{A\Psi_v^\dagger\psi} = -\dfrac{\Delta(e_q + e_Q)}{2|\boldsymbol{p}|^3} \delta^{\mu j} \begin{pmatrix} 0 & (\boldsymbol{p}_- \cdot \boldsymbol{p})\sigma^j - (\boldsymbol{p}_- \cdot \boldsymbol{\sigma}) p^j \end{pmatrix} \\ \qquad - \delta^{\mu j} \hat{\Gamma}^+_{A\Psi_v^\dagger\psi} \begin{pmatrix} 0 & (\boldsymbol{p} \cdot \boldsymbol{q})\sigma^j - (\boldsymbol{q} \cdot \boldsymbol{\sigma}) p^j \end{pmatrix} \ , \\ \Gamma^\mu_{A\bar{\psi}\Psi_v} = \dfrac{\Delta^*(e_q + e_Q)}{2|\boldsymbol{p}|^3} \delta^{\mu j} \begin{pmatrix} 0 \\ (\boldsymbol{p}_+ \cdot \boldsymbol{p})\sigma^j - (\boldsymbol{p}_+ \cdot \boldsymbol{\sigma}) p^j \end{pmatrix} \\ \qquad + \delta^{\mu j} \hat{\Gamma}^-_{A\bar{\psi}\Psi_v} \begin{pmatrix} 0 \\ (\boldsymbol{p} \cdot \boldsymbol{q})\sigma^j - (\boldsymbol{q} \cdot \boldsymbol{\sigma}) p^j \end{pmatrix} \ , \end{cases}$$

⇨ For simplicity we take $\hat{\Gamma}^-_{A\bar{\psi}\Psi_v} = \hat{\Gamma}^+_{A\Psi_v^\dagger\psi} = 0$

NOTE: NG modes for charged Δ do not contribute by the longitudinal nature

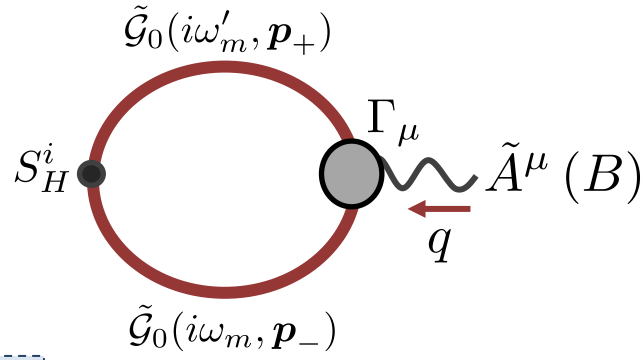
2. Heavy quark spin polarization

- **HQSP response function**

  - The HQSP response function $\tilde{\Pi}_H$ is defined by

$$\langle \tilde{S}^i_H(q_0, \boldsymbol{q}) \rangle_\beta = e\tilde{B}^i(q_0, \boldsymbol{q})\tilde{\Pi}_H(q_0, \boldsymbol{q})$$

(e is the elementary charge)



impurity: $e_Q = (+2/3)e$

quark matter: $e_q = (+2/3)e$ or $e_q = (-1/3)e$

  - I will investigate $\tilde{\Pi}_H$ in the following two limits

$$\tilde{\Pi}^{\rm dyn}_H \equiv \lim_{q_0 \to 0} \tilde{\Pi}_H(q_0, \boldsymbol{0})$$ (dynamical limit)

$$\tilde{\Pi}^{\rm sta}_H \equiv \lim_{q \to 0} \tilde{\Pi}_H(0, \boldsymbol{q})$$ (static limit)

2. Heavy quark spin polarization

- **Temperature dependence of** $\tilde{\Pi}_H^{\rm dyn} \equiv \lim_{q_0 \to 0} \tilde{\Pi}_H(q_0, \mathbf{0})$

$\hat{e}_Q = \hat{e}_q = +2/3$

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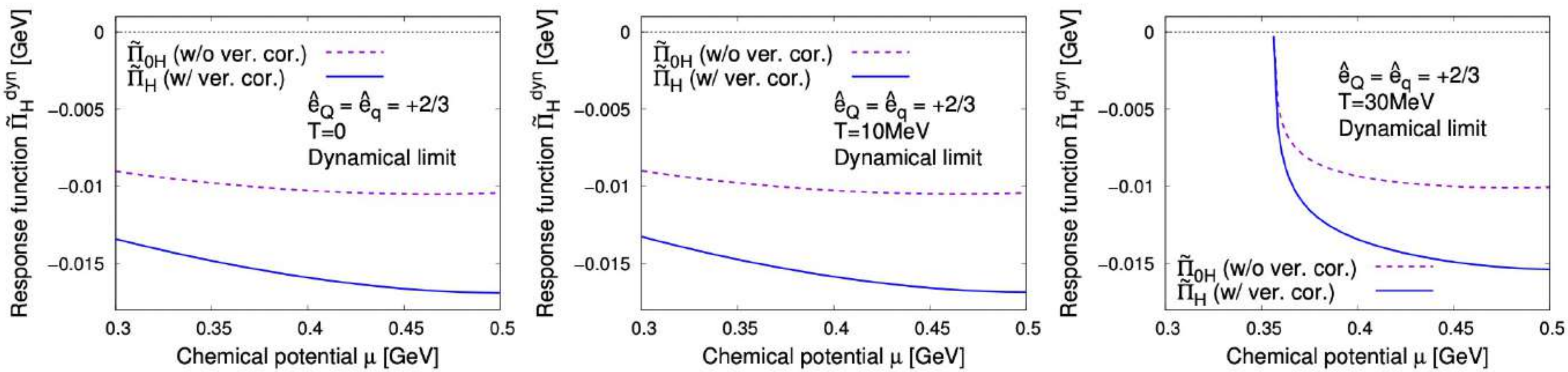
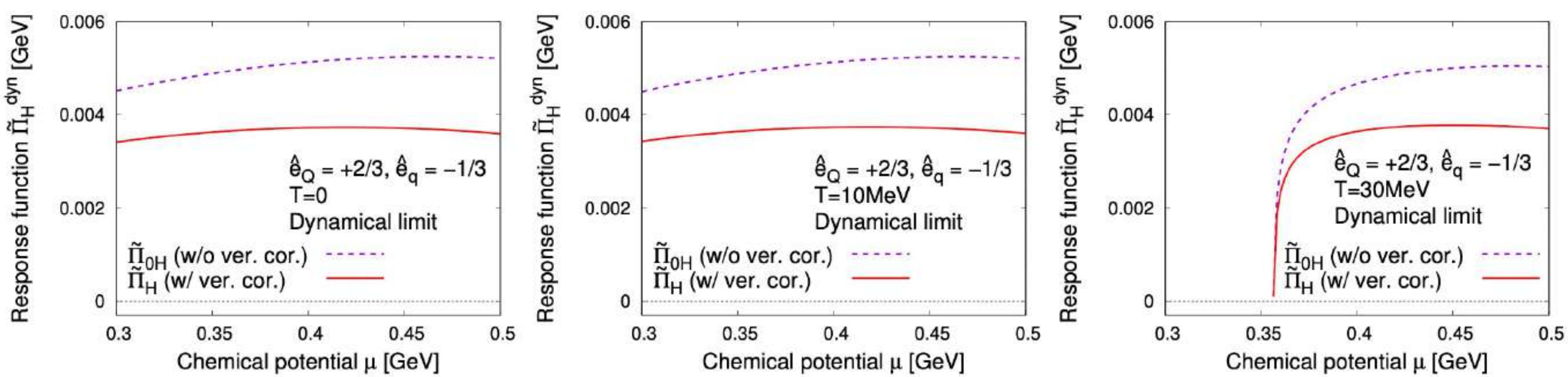


$\hat{e}_Q = +2/3\,,\ \hat{e}_q = -1/3$



2. Heavy quark spin polarization

- **Temperature dependence of** $\tilde{\Pi}_H^{\mathrm{sta}} \equiv \lim_{q \to 0} \tilde{\Pi}_H(0, \boldsymbol{q})$

$\hat{e}_Q = \hat{e}_q = +2/3$

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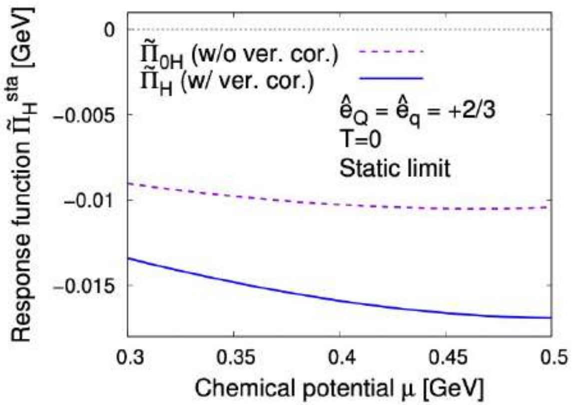
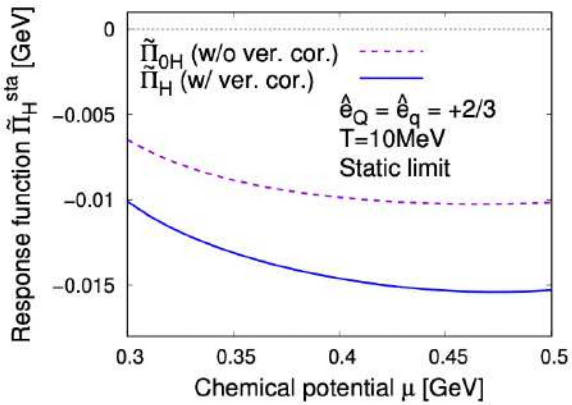
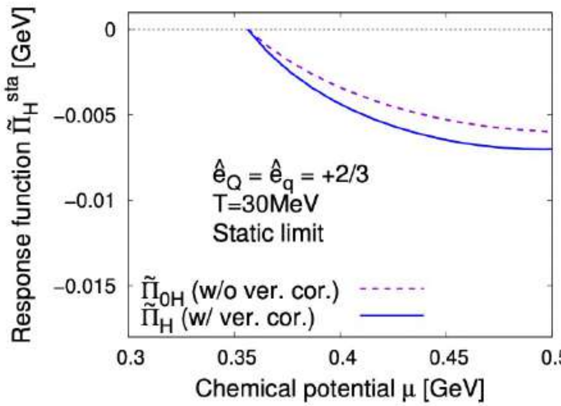
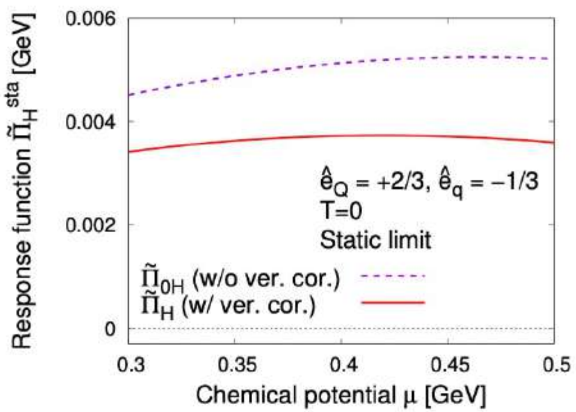
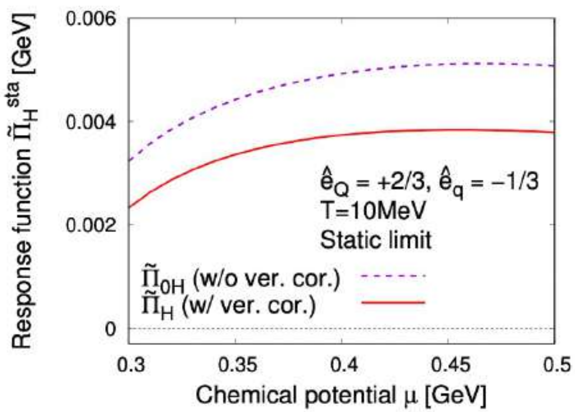


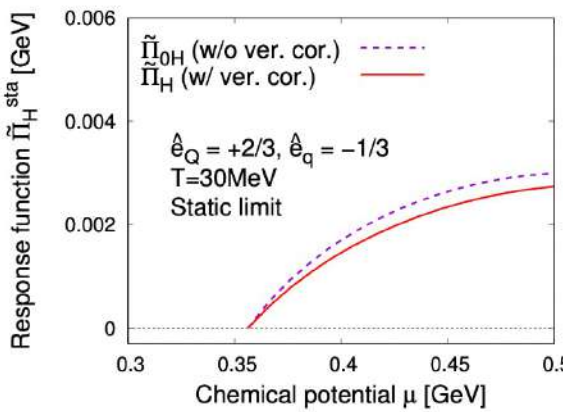




$\hat{e}_Q = +2/3\,,\ \hat{e}_q = -1/3$







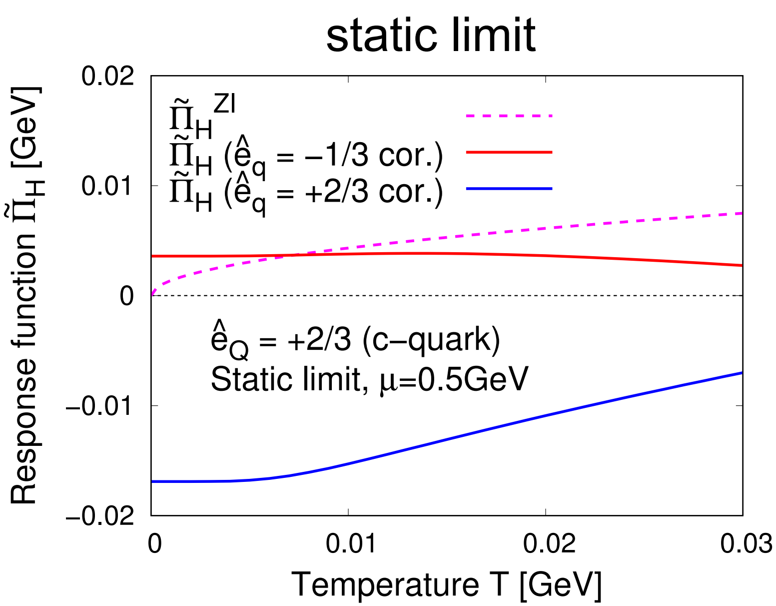
2. Heavy quark spin polarization

- **Comparison with HQSP from Zeeman effect**

- The HQSP response function $\tilde{\Pi}_H^{\mathrm{ZI}}$ by Zeeman effect is evaluated from

$$\mathcal{L}_{\mathrm{ZI}} = \Psi_v^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2m_Q} + \frac{e_Q}{m_Q} \boldsymbol{S}_h \cdot \boldsymbol{B} \right) \Psi_v \quad \text{with} \quad \boldsymbol{S}_h = \frac{\boldsymbol{\sigma}}{2} \text{ and } m_Q = 1.27\,\mathrm{GeV}$$

$$\left(\tilde{\Pi}_H^{\mathrm{ZI}}\right)^{\mathrm{sta}} = N_c \frac{\hat{e}_Q (2\pi m_Q T)^{1/2}}{8\pi^2} (1-\sqrt{2})\zeta(1/2) \qquad \left(\tilde{\Pi}_H^{\mathrm{ZI}}\right)^{\mathrm{dyn}} = 0$$



- HQSP at $T \approx 0$ in the static limit is dominated by that from Kondo effect

- HQSP in the dynamical limit is **entirely** dominated by that from Kondo effect

- **Significance of Kondo effect for HQSP was shown**

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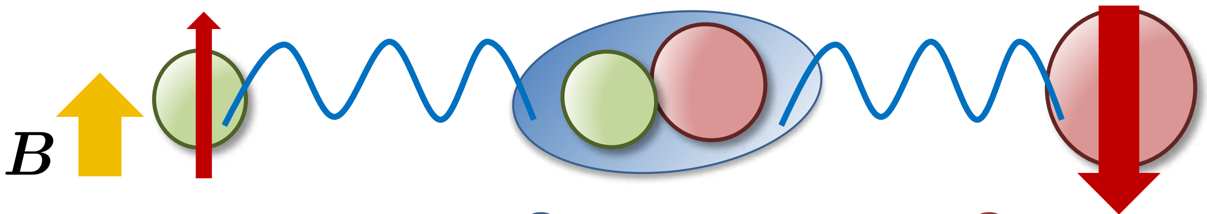
3. Spin magnetization in metals

- **Application to solid-state physics**

  - The emergence of HQSP was triggered by

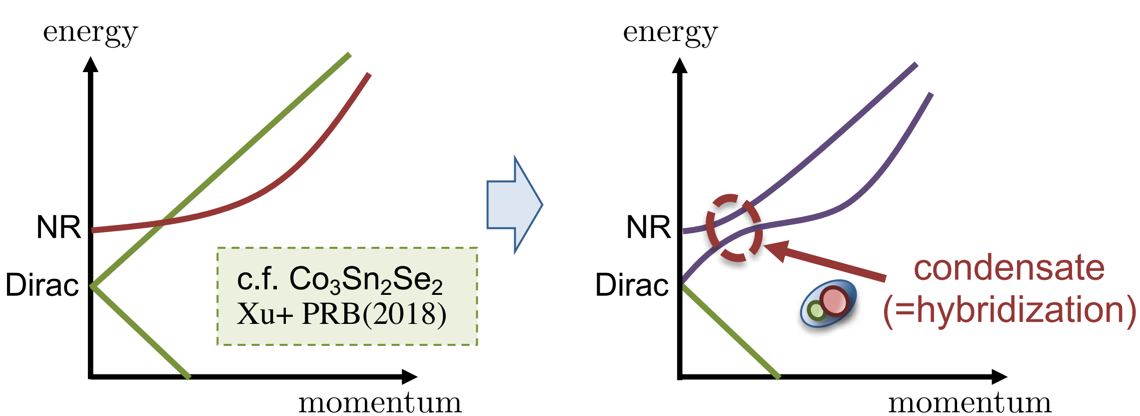


① relativistic fermion with Fermi surface

② condensate (spin correlation)

③ spin polarization of heavy fermion

  - A similar system can be realized in Dirac (Weyl) metal where nonrelativistic bands coexist in solid-state physics



When condensate occurs

Novel spin magnetization is expected

3. Spin magnetization in metals

- **Toy model**

- The Hamiltonian is $\mathcal{H} = \mathcal{H}_{\rm Dirac} + \mathcal{H}_{\rm NR} + \mathcal{H}_{\rm hyb}$ with

$$\mathcal{H}_{\rm Dirac} = \int d^3\boldsymbol{r}\ \Psi^\dagger_{\rm Dirac}(\boldsymbol{r})(-iv_{\rm F}\boldsymbol{\nabla}\cdot\boldsymbol{\alpha})\Psi_{\rm Dirac}(\boldsymbol{r})$$

$$\mathcal{H}_{\rm NR} = \int d^3\boldsymbol{r}\ \Psi^\dagger_{\rm NR}(\boldsymbol{r})\left[\frac{-\nabla^2}{2m}+\epsilon_0\right]\Psi_{\rm NR}(\boldsymbol{r}).$$

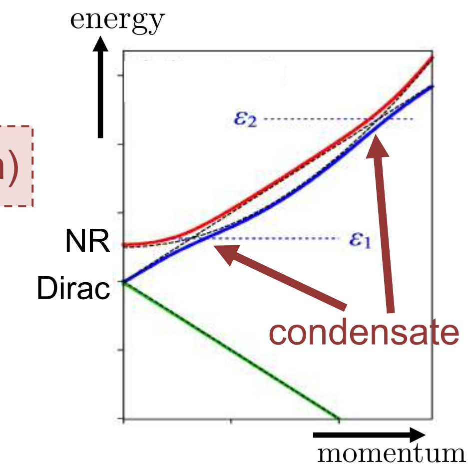
$$\mathcal{H}_{\rm hyb} = h\int d^3\boldsymbol{r}\sum_{s=\uparrow,\downarrow}\left[\psi^\dagger_{{\rm R},s}\psi_{{\rm NR},s}+\psi^\dagger_{{\rm L},s}\psi_{{\rm NR},s}+{\rm H.c.}\right]$$

$$\begin{cases}\Psi_{\rm Dirac} = (\psi_{\rm R\uparrow},\psi_{\rm R\downarrow},\psi_{\rm L\uparrow},\psi_{\rm L\downarrow})^T \\ \Psi_{\rm NR} = (\psi_{\rm NR\uparrow},\psi_{\rm NR\downarrow})^T\end{cases}$$

in a matrix form

$h \sim$ condensate (hybridization)

$$\mathcal{H} = \sum_{\boldsymbol{k}}\Psi^\dagger(\boldsymbol{k})H(\boldsymbol{k})\Psi(\boldsymbol{k}) \quad \text{with} \quad H(\boldsymbol{k}) = \begin{pmatrix} v_{\rm F}\boldsymbol{k}\cdot\boldsymbol{\sigma} & 0 & \underline{h} \\ 0 & -v_{\rm F}\boldsymbol{k}\cdot\boldsymbol{\sigma} & \underline{h} \\ \underline{h} & \underline{h} & \frac{k^2}{2m}+\epsilon_0\end{pmatrix}$$



- Magnetic interactions are obtained by gauging the fermions

magnetic interactions of nonrelativistic fermion are assumed to be absent

3. Spin magnetization in metals

- **Spin magnetization (spin polarization)**

- The spin magnetization is defined by

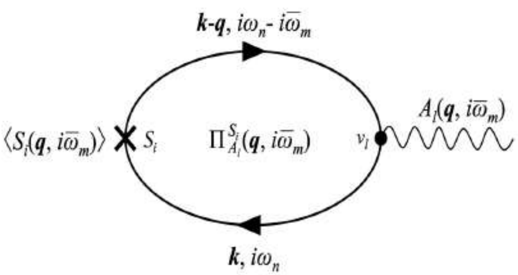
$$\boldsymbol{M}^{\mathrm{S}}(q_0, \boldsymbol{q}) = -\gamma \langle \boldsymbol{S}(q_0, \boldsymbol{q}) \rangle = \chi^{\mathrm{SO}}(q_0, \boldsymbol{q}) \boldsymbol{B}(q_0, \boldsymbol{q})$$

with the gyromagnetic ratio $\gamma = g\mu_B$

g : g-factor
μ_B : Bohr magneton

χ^{SO} is called the spin-orbital (SO) crossed susceptibility ($\simeq$ spin response function)

- χ^{SO} is evaluated within the linear response theory



by field-theoretical (diagrammatical) calculation

or

by solving the Schrodinger equation perturbatively
and using (spin) Berry connection, etc.

intuitive interpretation
with wave packet

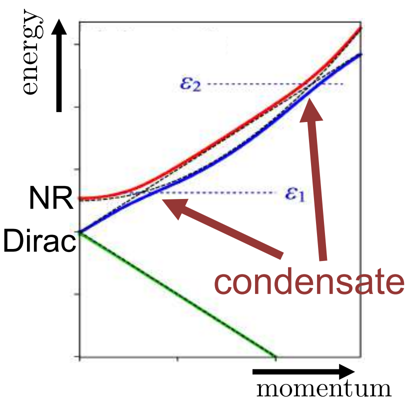
Xiao-Yao-Fang-Niu PRL(2006)

3. Spin magnetization in metals

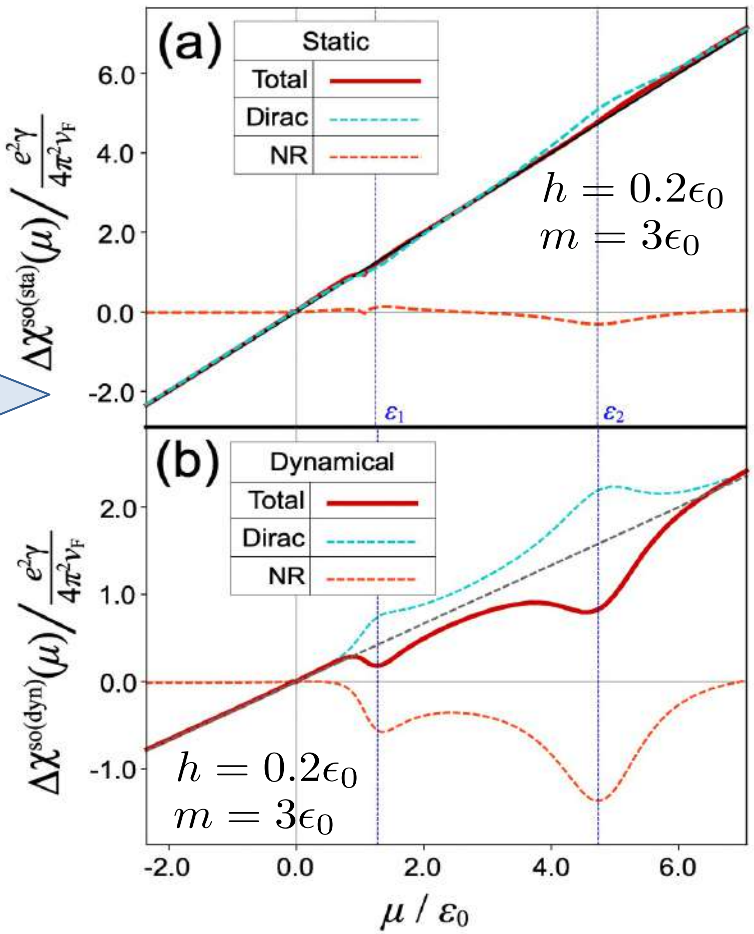
- **SO crossed susceptibility** $\Delta\chi^{\mathrm{SO}} = \chi^{\mathrm{SO}}(\mu) - \chi^{\mathrm{SO}}|_{\mu=0}$

static limit $\lim_{q\to 0} \Delta\chi^{\mathrm{SO}}(0, \boldsymbol{q})$

dynamical limit $\lim_{q_0\to 0} \Delta\chi^{\mathrm{SO}}(q_0, \boldsymbol{0})$



$$\boldsymbol{S}_{\mathrm{Total}} = \mathrm{diag}(\boldsymbol{\sigma}, \boldsymbol{\sigma}, \boldsymbol{\sigma})/2$$
$$\boldsymbol{S}_{\mathrm{Dirac}} = \mathrm{diag}(\boldsymbol{\sigma}, \boldsymbol{\sigma}, 0)/2$$
$$\boldsymbol{S}_{\mathrm{NR}} = \mathrm{diag}(0, 0, \boldsymbol{\sigma})/2$$



- The emergence of significant spin magnetization is indeed realized
- The dynamical limit is more remarkable

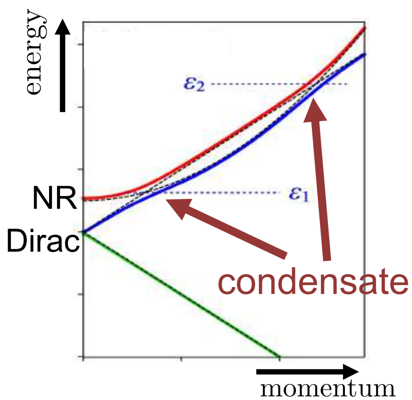
Y. Araki, **D. S.**, K. Suzuki, S. Yasui, PRRes. 3 (2021) 023098

3. Spin magnetization in metals

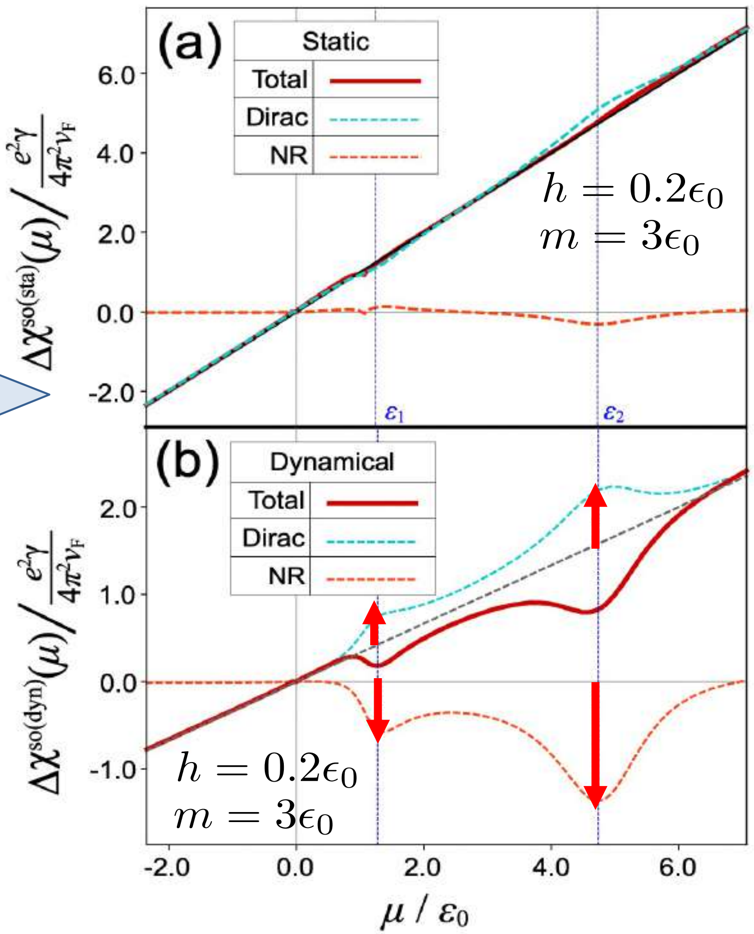
- **SO crossed susceptibility** $\Delta\chi^{\mathrm{SO}} = \chi^{\mathrm{SO}}(\mu) - \chi^{\mathrm{SO}}|_{\mu=0}$

static limit $\lim_{q\to 0} \Delta\chi^{\mathrm{SO}}(0, \boldsymbol{q})$

dynamical limit $\lim_{q_0\to 0} \Delta\chi^{\mathrm{SO}}(q_0, \boldsymbol{0})$



$$\boldsymbol{S}_{\mathrm{Total}} = \mathrm{diag}(\boldsymbol{\sigma}, \boldsymbol{\sigma}, \boldsymbol{\sigma})/2$$
$$\boldsymbol{S}_{\mathrm{Dirac}} = \mathrm{diag}(\boldsymbol{\sigma}, \boldsymbol{\sigma}, 0)/2$$
$$\boldsymbol{S}_{\mathrm{NR}} = \mathrm{diag}(0, 0, \boldsymbol{\sigma})/2$$



- The emergence of significant spin magnetization is indeed realized
- The dynamical limit is more remarkable

- Understood by (half-)relativistic version of Van Vlack paramagnetism

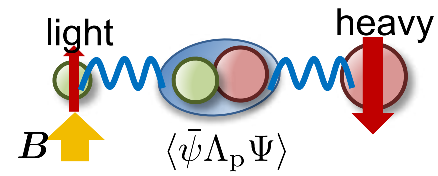
textbook by Kittel

Y. Araki, **D. S.**, K. Suzuki, S. Yasui, PRRes. 3 (2021) 023098

4. Conclusion

• Summary of HQSP

- I showed a new mechanism of heavy quark spin polarization (HQSP) induced by the Kondo effect in a magnetic field



- Comparison with the Zeeman effect was done
  - HQSP at $T \approx 0$ in the static limit is mostly dominated from Kondo effect
  - HQSP in the dynamical limit is **entirely** dominated from Kondo effect

- The HQSP is a new dense-QCD phenomena in B like CME, CSE, etc.

- The same effects (spin magnetization) in Dirac/Weyl metals with nonrelativistic bands were demonstrated

no sign problem

- Future **lattice simulations** in QC_2D are expected to measure the HQSP

⇨ Quantitative understanding of Kondo effect (my hope)

Back up

• Dynamical limit/static limit

(i) $\tilde{\Pi}_H^{++}$ (ii) $\tilde{\Pi}_H^{--}$ (iii) $\tilde{\Pi}_H^{+-}$ (iv) $\tilde{\Pi}_H^{-+}$

q.p. → q.p. scattering | q.h. → q.h. scattering | q.p. + q.h. pair annihilation | q.p. + q.h. pair creation

intra-band process | inter-band process

$$\mathcal{I}_{0H}^{\zeta\zeta}(\boldsymbol{p}_+;\boldsymbol{p}_-) \approx -\frac{V_\zeta(\boldsymbol{p})V_\zeta(\boldsymbol{p})}{i\bar{\omega}_n - \frac{\partial E_{\boldsymbol{p}}^\zeta}{\partial|\boldsymbol{p}|}(\hat{\boldsymbol{p}}\cdot\boldsymbol{q})}\frac{\partial f_F(E_{\boldsymbol{p}}^\zeta)}{\partial|\boldsymbol{p}|}(\hat{\boldsymbol{p}}\cdot\boldsymbol{q})$$

$= 0$ (dynamical limit)
(first $\boldsymbol{q}\to 0$, next $i\bar{\omega}_n \to 0$)

$\neq 0$ (static limit)
(first $i\bar{\omega}_n \to 0$, next $\boldsymbol{q}\to 0$)

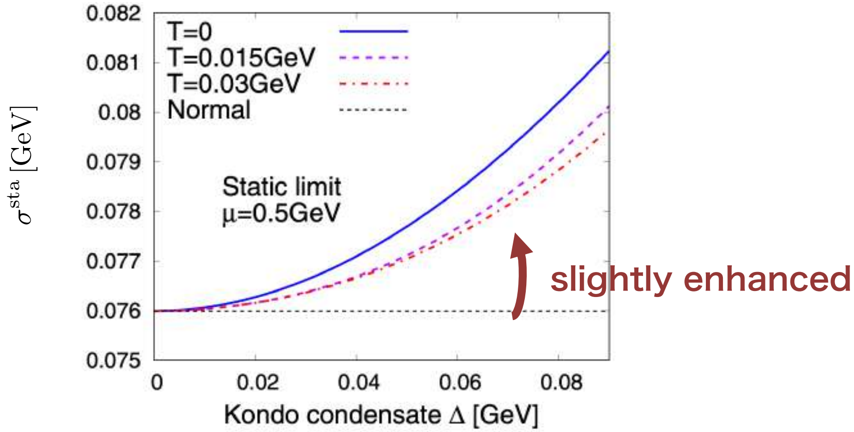
$$\mathcal{I}_{0H}^{\zeta\zeta'}(\boldsymbol{p}_+;\boldsymbol{p}_-) \approx \frac{V_\zeta(\boldsymbol{p})V_{\zeta'}(\boldsymbol{p})}{i\bar{\omega}_n - E_{\boldsymbol{p}}^\zeta + E_{\boldsymbol{p}}^{\zeta'}}\left[f_F(E_{\boldsymbol{p}}^{\zeta'}) - f_F(E_{\boldsymbol{p}}^\zeta)\right]$$

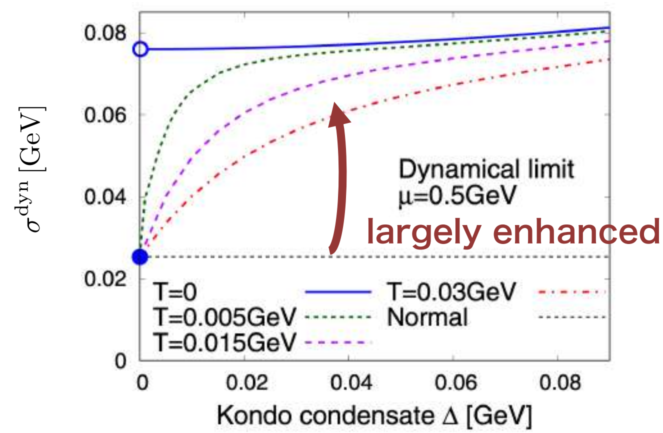
$\neq 0$ (dynamical limit)

$\neq 0$ (static limit)

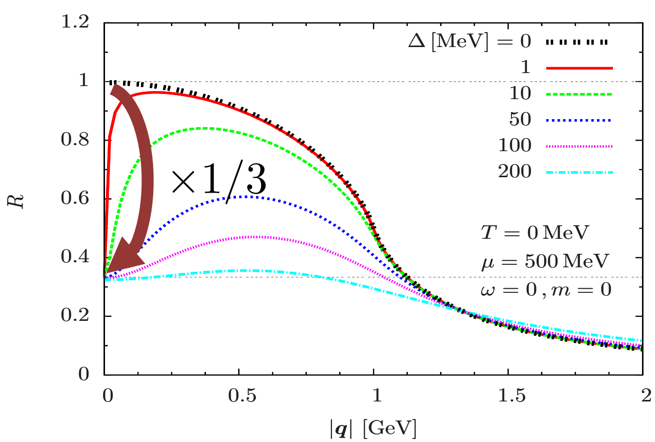
- **CSE conductivity with Kondo effect**

DS-Araki-Suzuki-Yasui PRD(2021)





- **CSE conductivity with diquark condensate (in QC_2D)**



DS-Kojo PRD(2021)

$\lim_{q\to 0} R$ (static limit) for $\Delta \neq 0$
becomes **exactly 1/3!**
[changed from $N_c\mu/(2\pi^2)$]