IQuS InQubator for Quantum Simulation

UNIVERSITY of WASHINGTON

QCD meets Quantum Information Science: Thermalization of Gauge Theories from their Entanglement Spectrum

Niklas Mueller
University of Washington

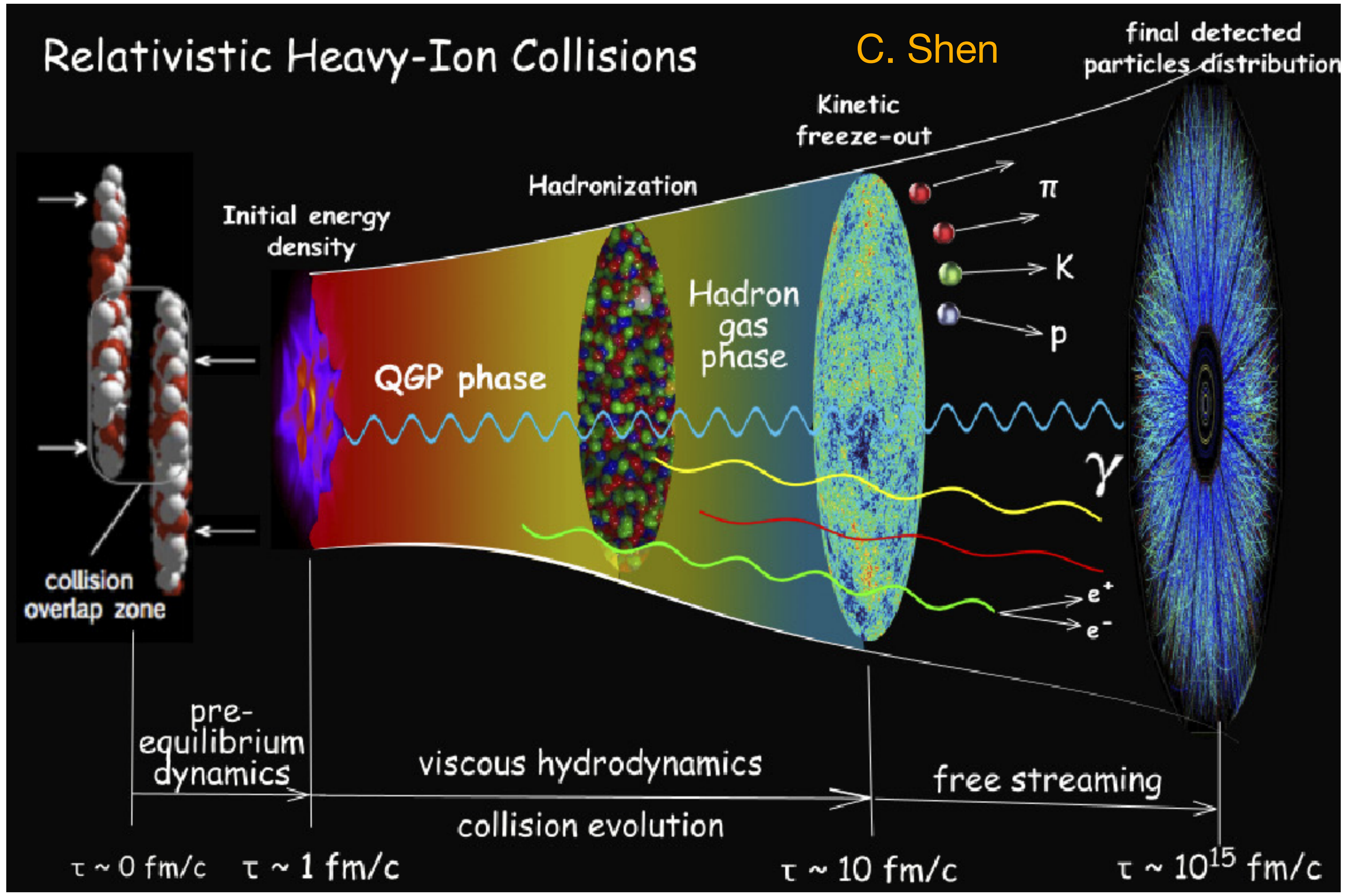
based on NM, Torsten Zache, Robert Ott,
Phys. Rev. Lett. 129, 011601 (2022)

Torsten Zache, Robert Ott,
Innsbruck

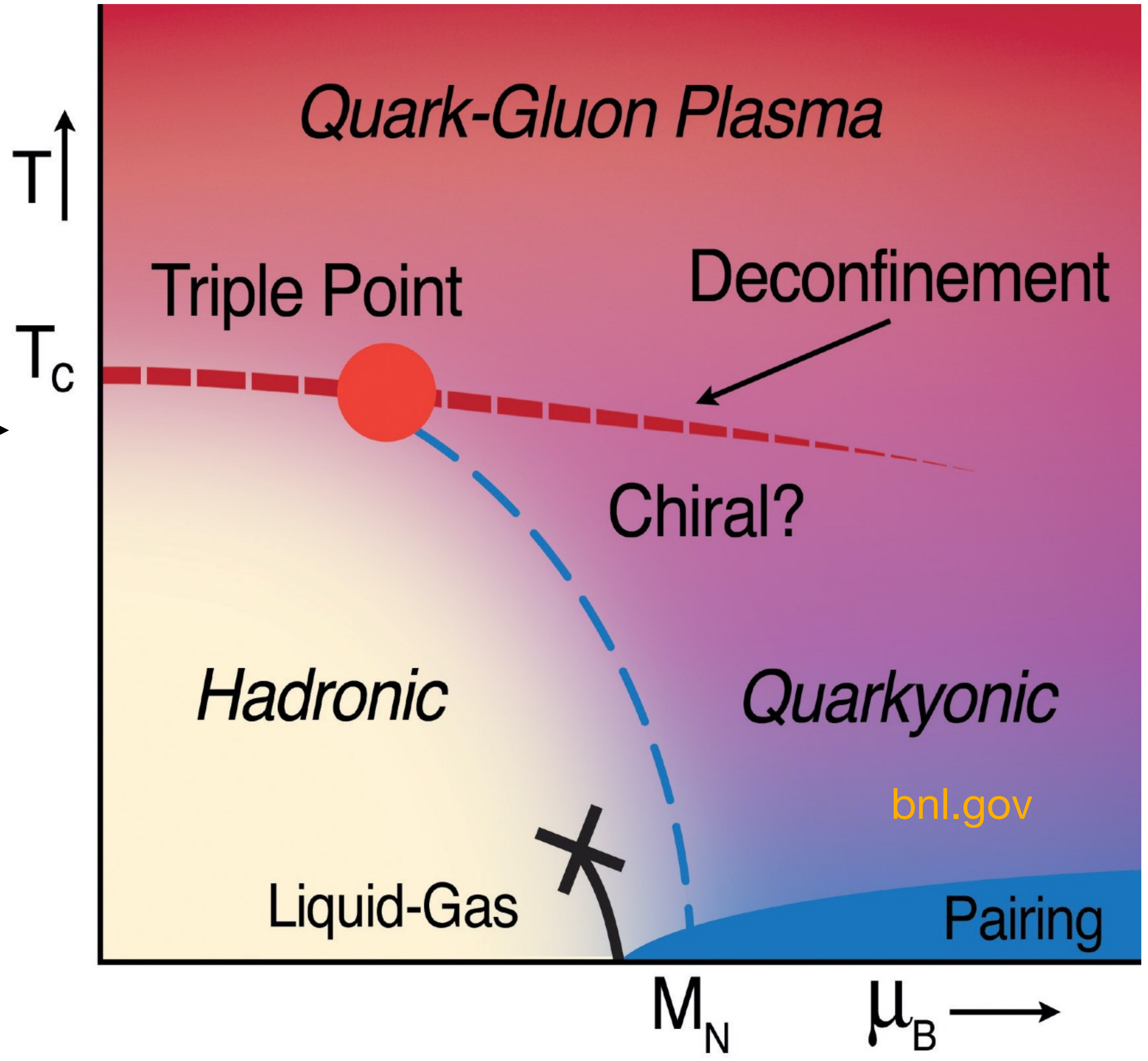
QCD Theory Seminar

Nov 13/14, 2022

Motivation: How does QCD thermalize?

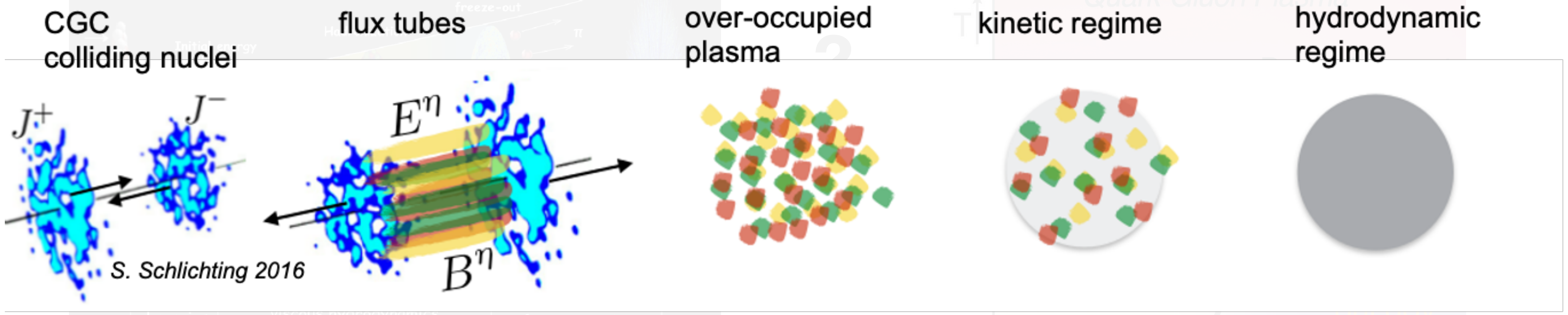


?



Motivation: How does QCD thermalize?

Issue #1: No first-principle (QCD) approach!



- "piecewise" descriptions: QCD kinetic theory, hydrodynamics

Arnold, Moore, Yaffe; JHEP 11 (2000) & 05 (2003)
Baier, Mueller, Schiff, Son; PLB 502, 51 (2001)
Berges, Heller, Mazeliauskas, Venugopalan, Rev. Mod. Phys. 93 (2021), 035003
Keegan, Kurkela, Romatschke, van der Schee, JHEP 2016(4), 1

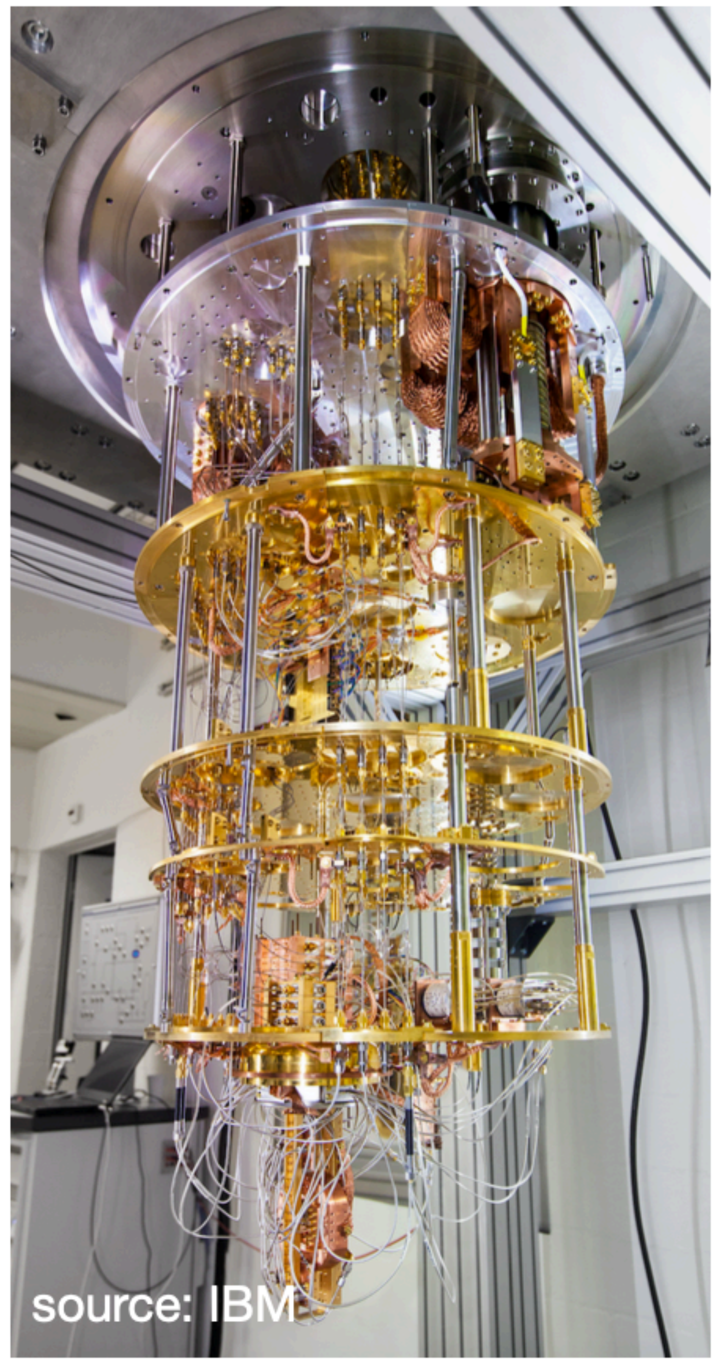
- Problem: Real-time dynamics, ~~MC-based lattice QCD~~

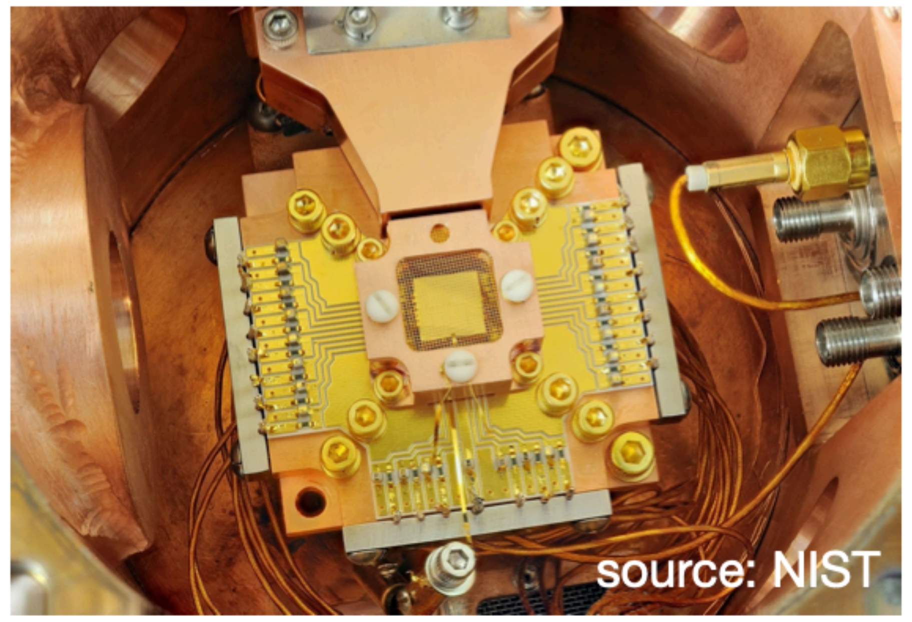
Issue #2: What actually means thermalization?

(spoiler: Entanglement has to do with it)

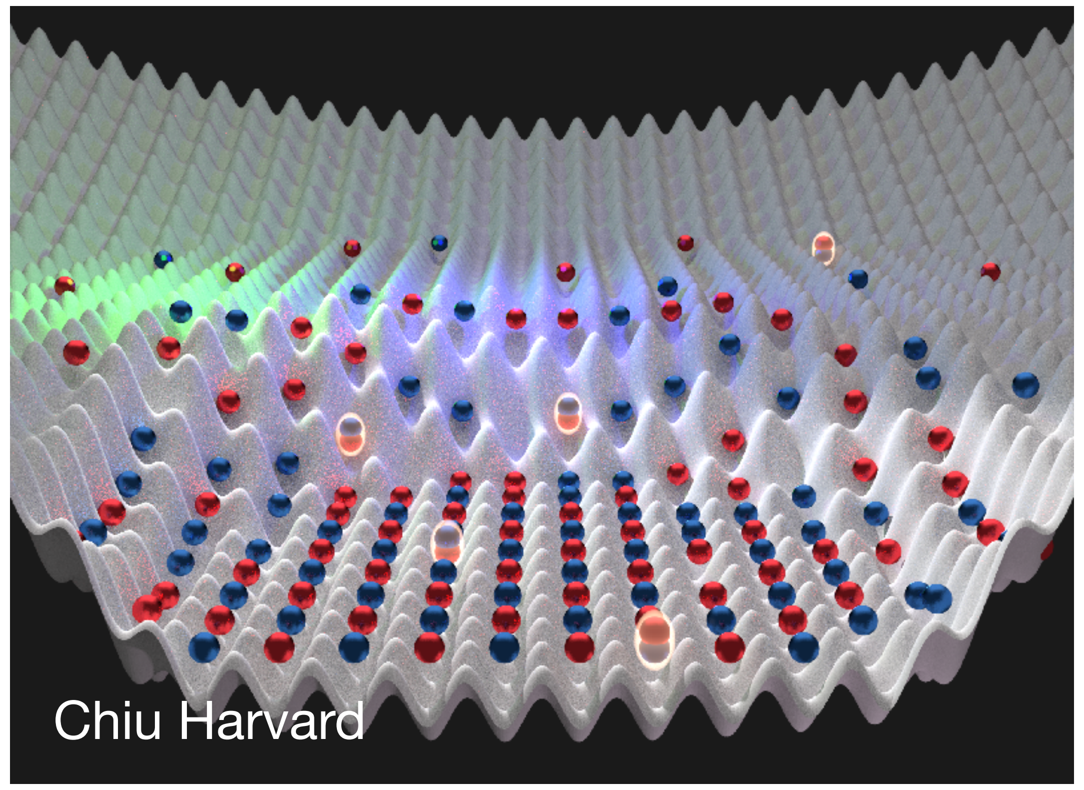
The Nobel Prize in Physics 2022

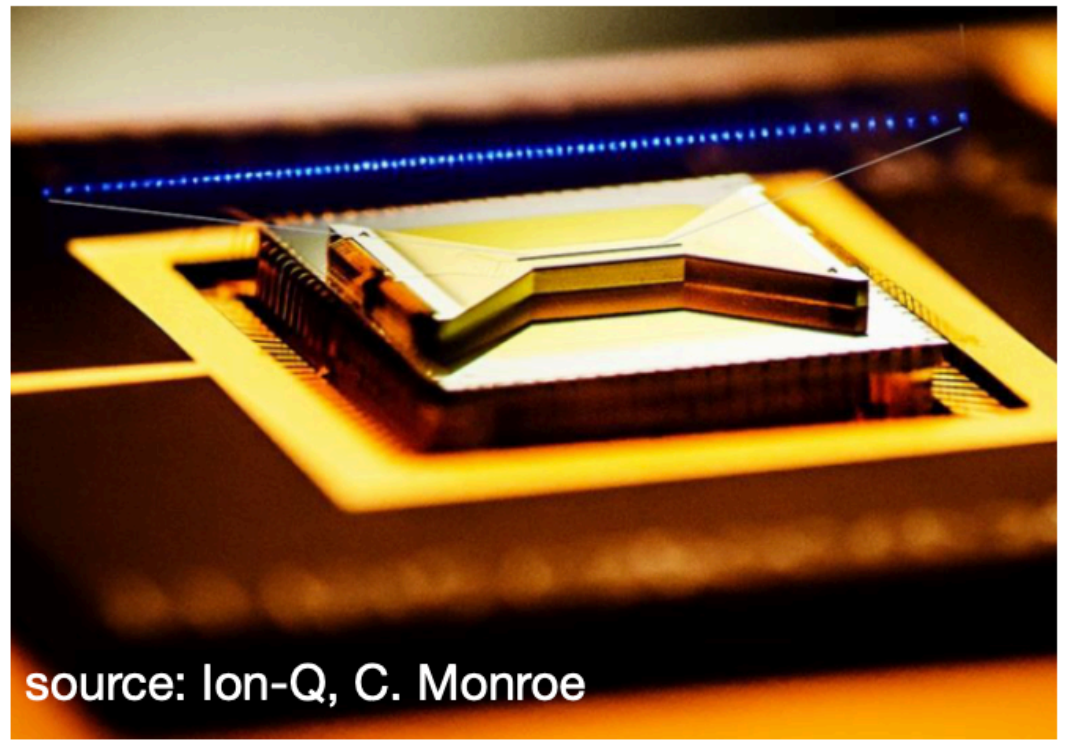
issue #1, perhaps soon no longer an issue?

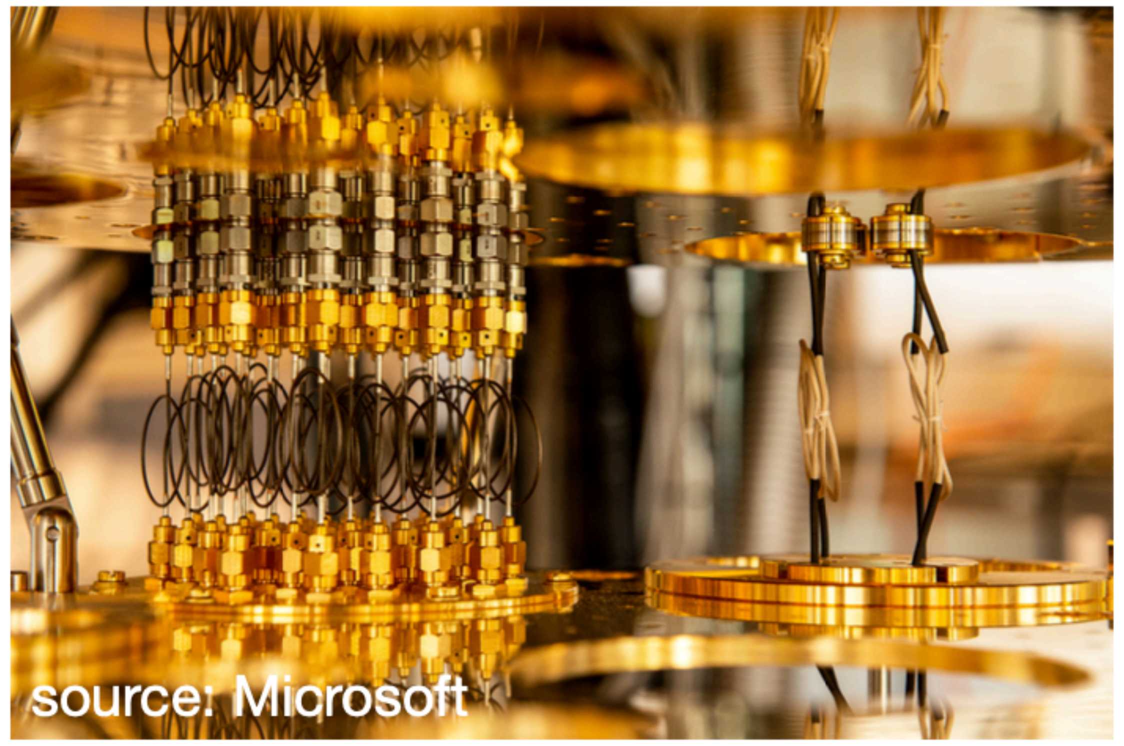

source: IBM

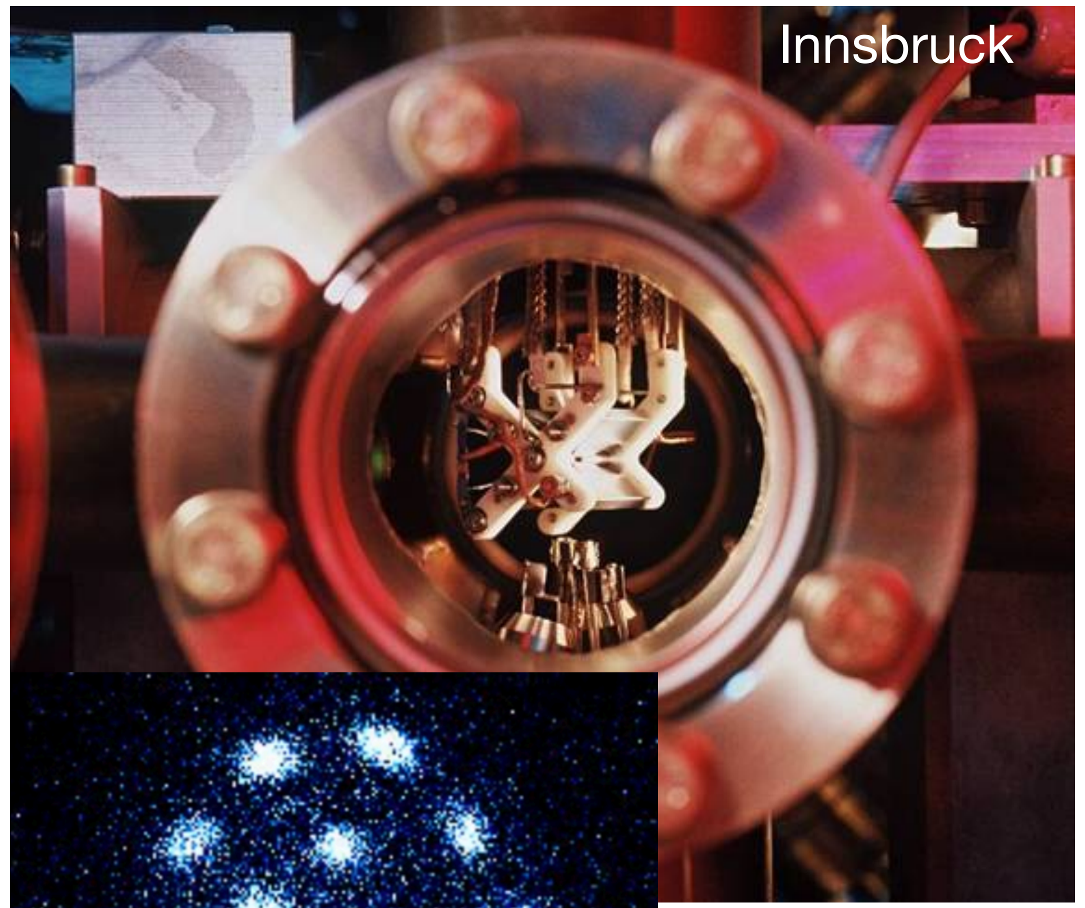

source: NIST

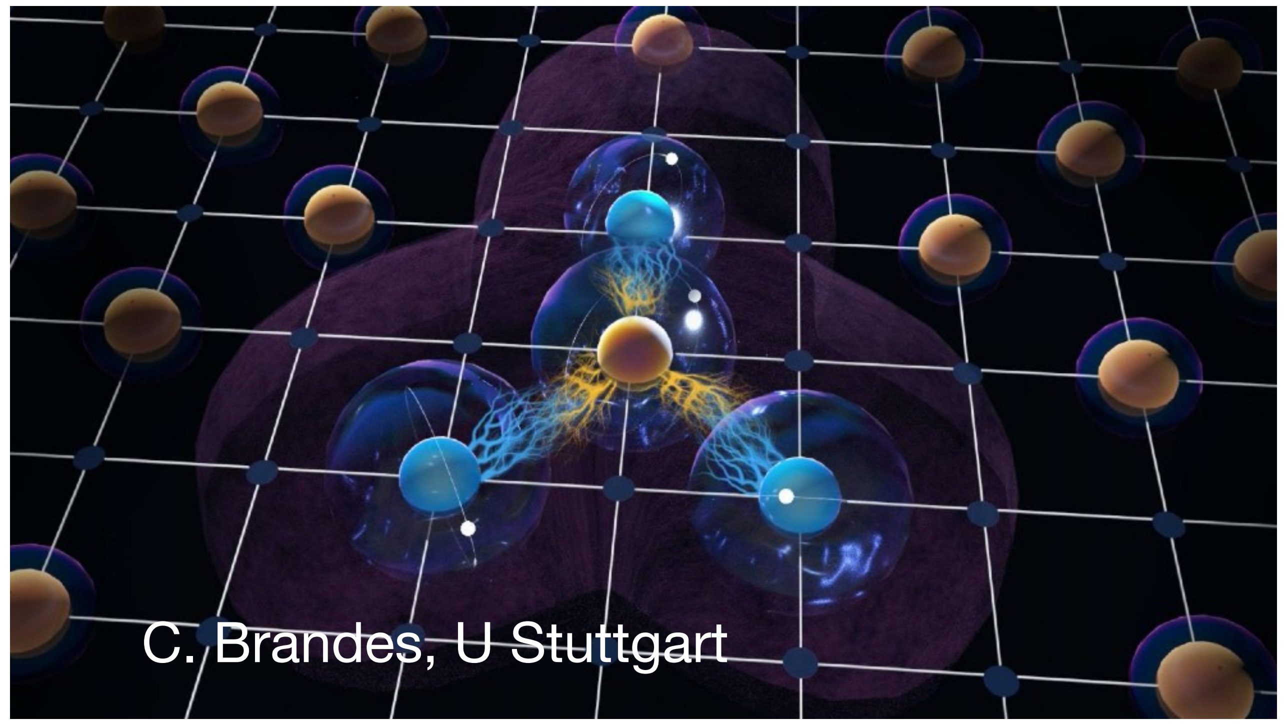

source: Google

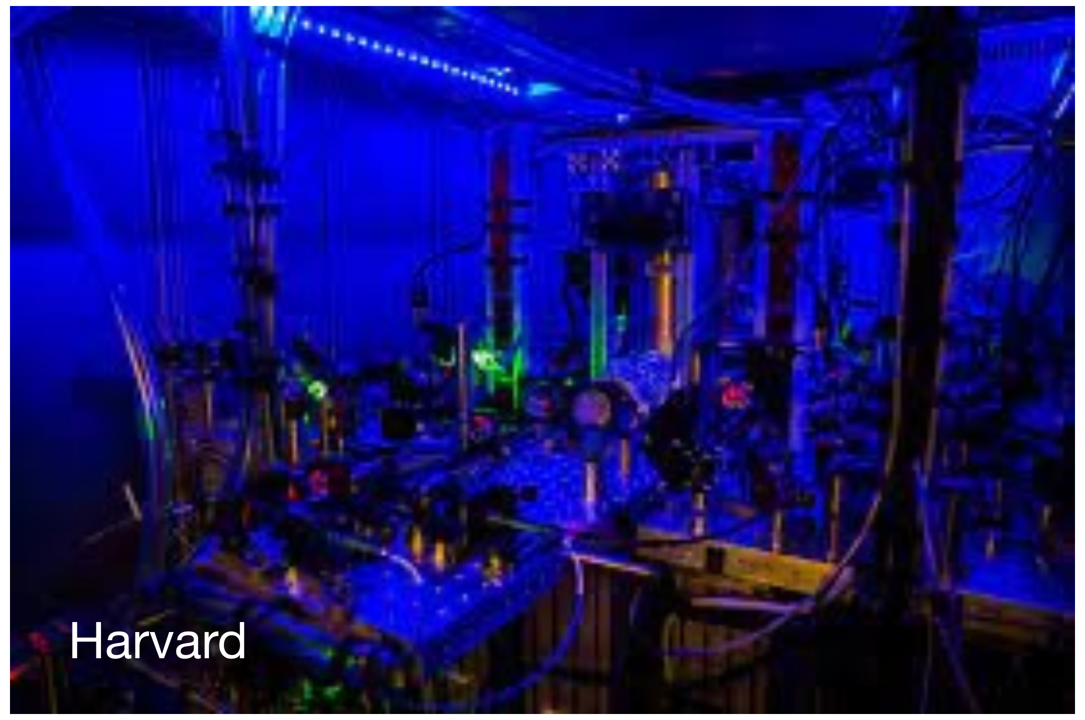

Chiu Harvard

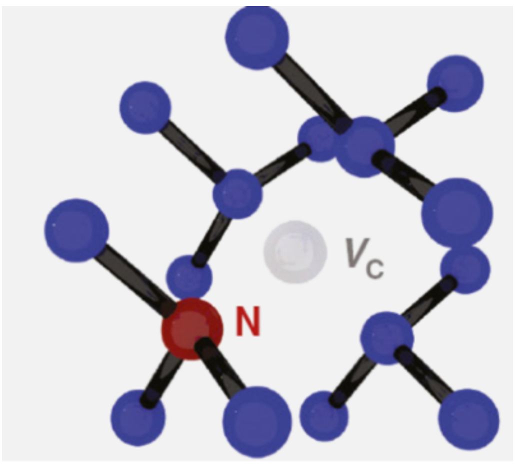

source: Ion-Q, C. Monroe


source: Microsoft

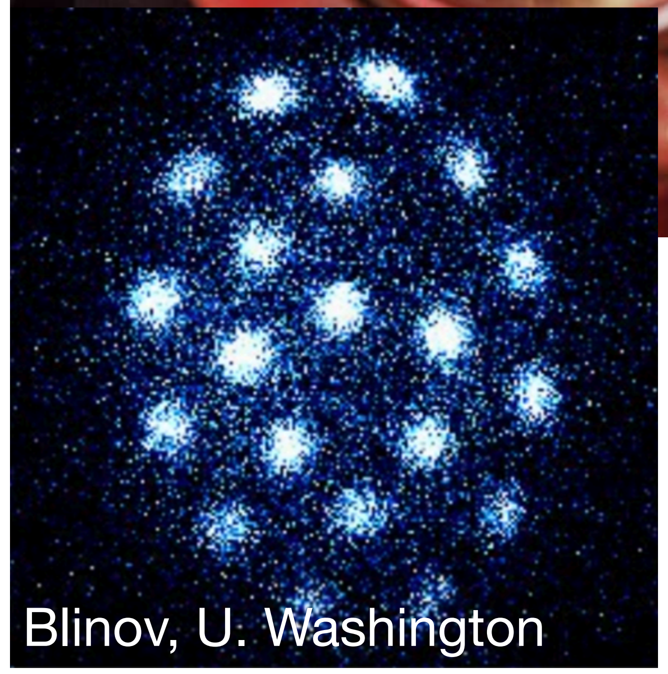

Innsbruck


C. Brandes, U Stuttgart


Harvard



Fu, U Washington


Blinov, U. Washington

This talk: Mostly issue #2

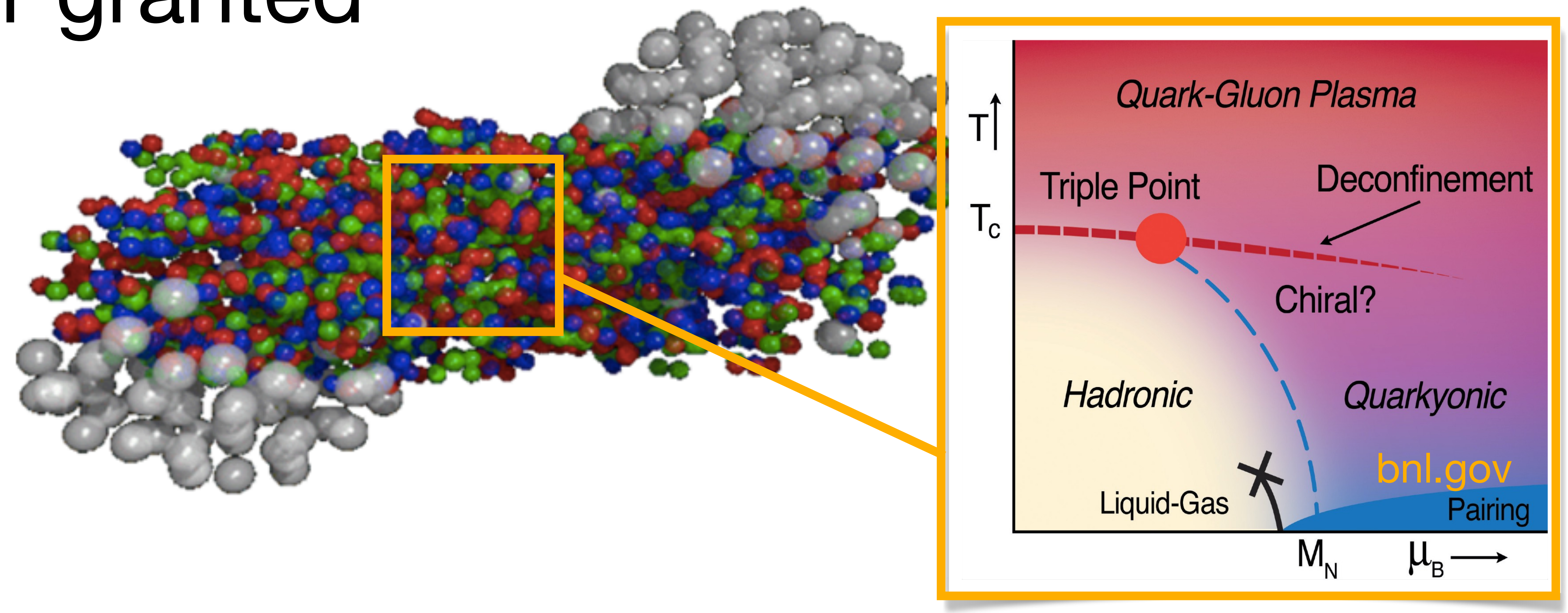
Thermalization of Gauge Theories

Thermal equilibrium often taken for granted

- "Hydrodynamics works"
  = local thermal equilibrium?

- Weak-coupling: QCD kinetic theory,
  "should thermalize … eventually"

Arnold, Moore, Yaffe; JHEP 11 (2000) & 05 (2003)
Baier, Mueller, Schiff, Son; PLB 502, 51 (2001)

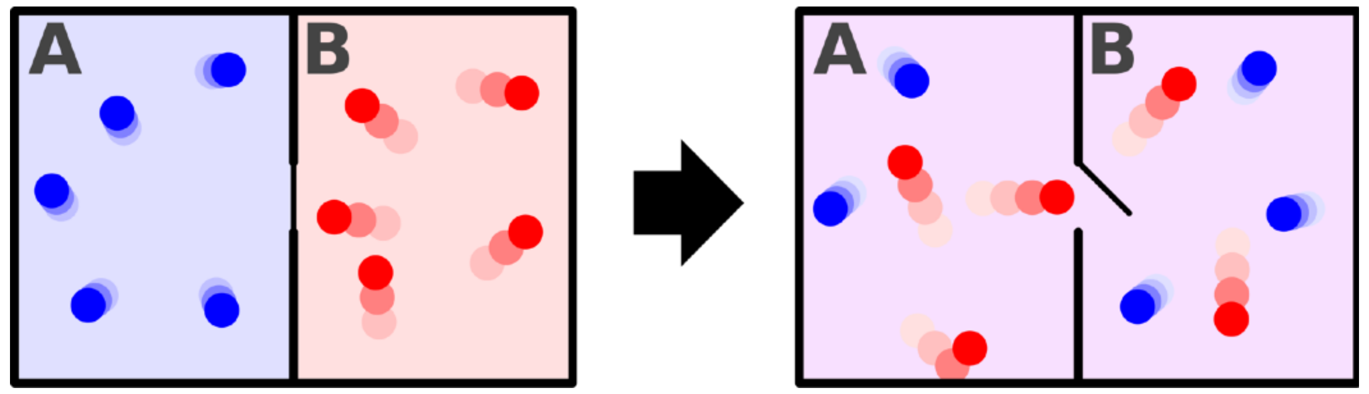


What means thermal equilibrium?

thermal equilibrium = system behaves statistically, i.e. according to the laws of thermodynamics

- **thermal expectation values**
  expectation values of physical variable predicted by Gibbs ensemble

- **fluctuation-dissipation relation**
  fluctuations of physical variable ↔ response ("admittance") to external change



$$Z \sim \sum_k e^{-\beta E_k} \qquad \beta = 1/k_B T$$

$$\langle E \rangle = -\frac{\partial Z}{\partial \beta}$$

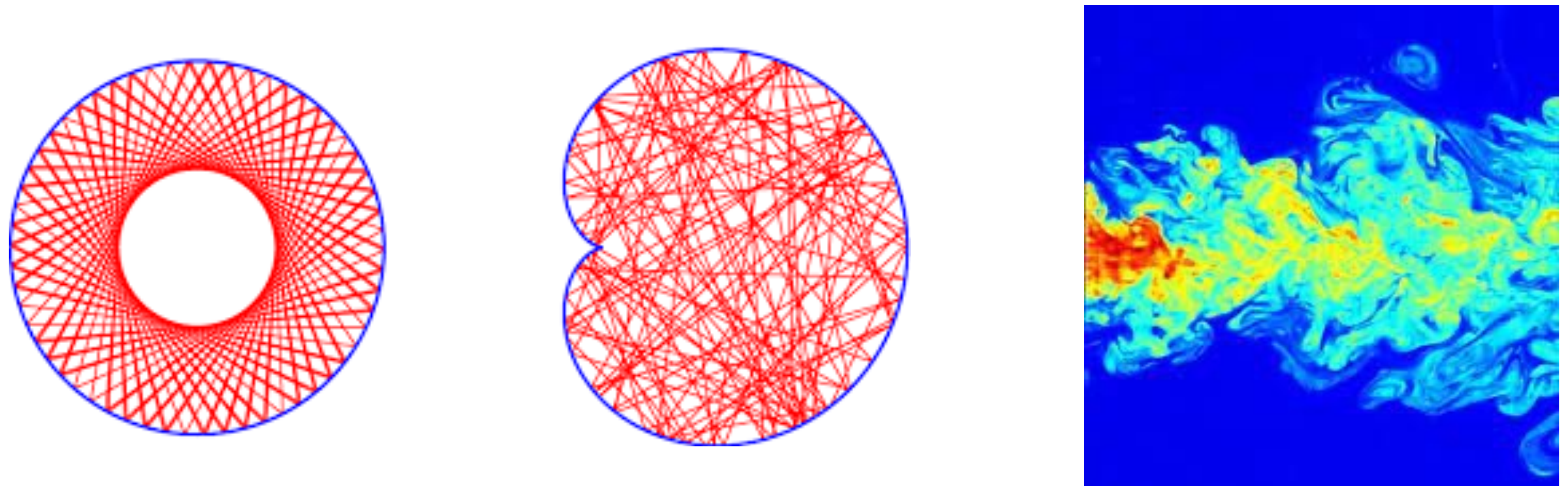
Thermalization of Gauge Theories

How to reach thermal equilibrium?

Initial state characterized by lots of information → "loss of information"? → **Thermal state "**Gibbs ensemble" only characterized by: temperature, chemical potential …

Classical systems

- ergodicity & chaos
  small systems can relax to micro-canonical ensemble



Quantum systems

- formally, isolated quantum systems cannot
  quantum mechanics is unitary

$$i\partial_t|\psi\rangle = H|\psi\rangle \qquad |\psi\rangle \not\to e^{-\beta H}$$

- **Eigenstate Thermalization Hypothesis**

Deutsch, PRA 43, 2046 (1991); Srednicki PRE 50, 888 (1994)

Typicality: von Neumann (1929)
Golstein, Lebowitz, Mastrodonato, Tumulk, Aznghi, Proc. R. Soc. A 466, 3203 (2010)
Eisert, Friesdorf, Gogolin, Nature Phys. 11, 124130 (2015)
Random Matrix Theory: Wigner, Dyson, Bohigas, Berry
Borgonovi, Izrailev, Santos, Zelevinsky, Phys. Rep. 626, 1 (2016)
Berry's conjecture
Berry, J Phys A 10, 2083 (1977)

Thermalization of Gauge Theories

- ETH

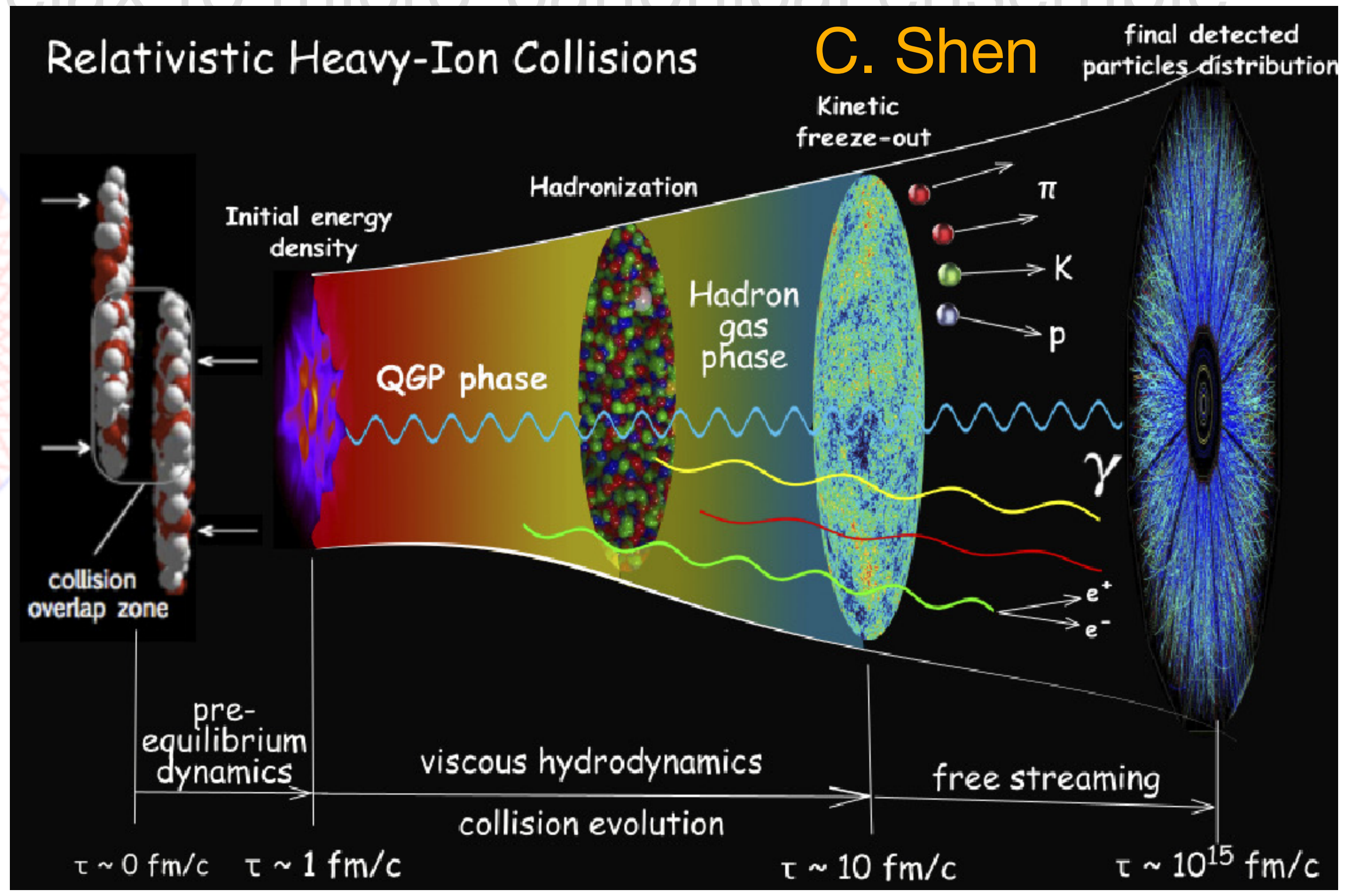
Finally we would like to comment on the much-discussed question of an appropriate definition for quantum chaos. Some time ago, van Kampen suggested that quantum chaos be defined as “that property that causes a quantum system to behave statistically [33].

If we replace “behave statistically” with “obey the laws of statistical mechanics,” then we have seen that the key feature is Berry’s conjectured properties of the energy eigenstates. In particular, properties of the energy eigenvalues (such as GOE rather than Poisson statistics for the unfolded level spacings [33]) have played no role at all in the present work.

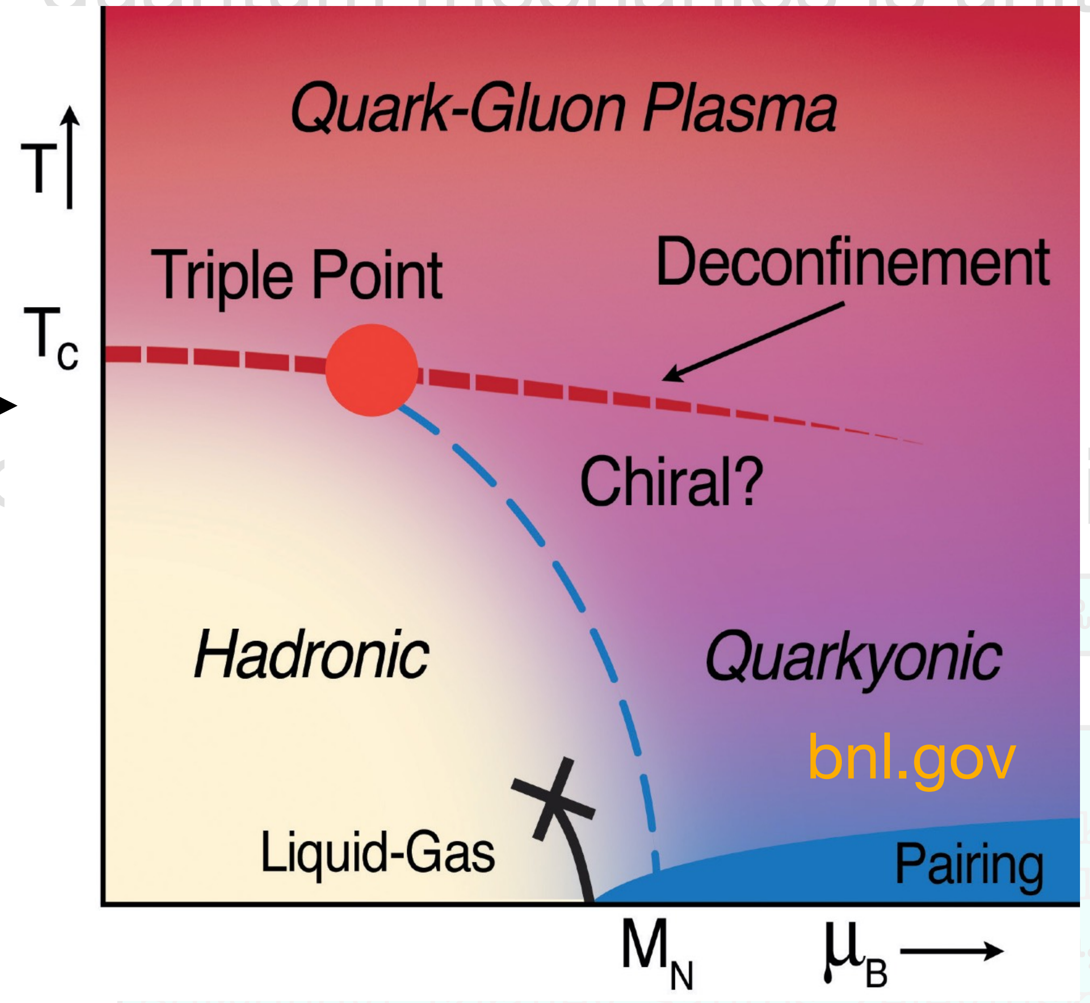
Steiner has suggested [21] that Berry’s conjecture be elevated to the status of the best definition of quantum chaos, a proposal which we see to be equivalent to (our version of) van Kampen’s. **More generally, in quantum mechanics, where time evolution is always linear and therefore essentially trivial, the only place to encode the complexities of the classical limit is in the energy eigenfunctions: that is where quantum chaos, like thermal behavior, must be sought.**

Srednicki PRE 50, 888 (1994)

- “complexities” & structure of quantum states: Quantify through **Entanglement**



?

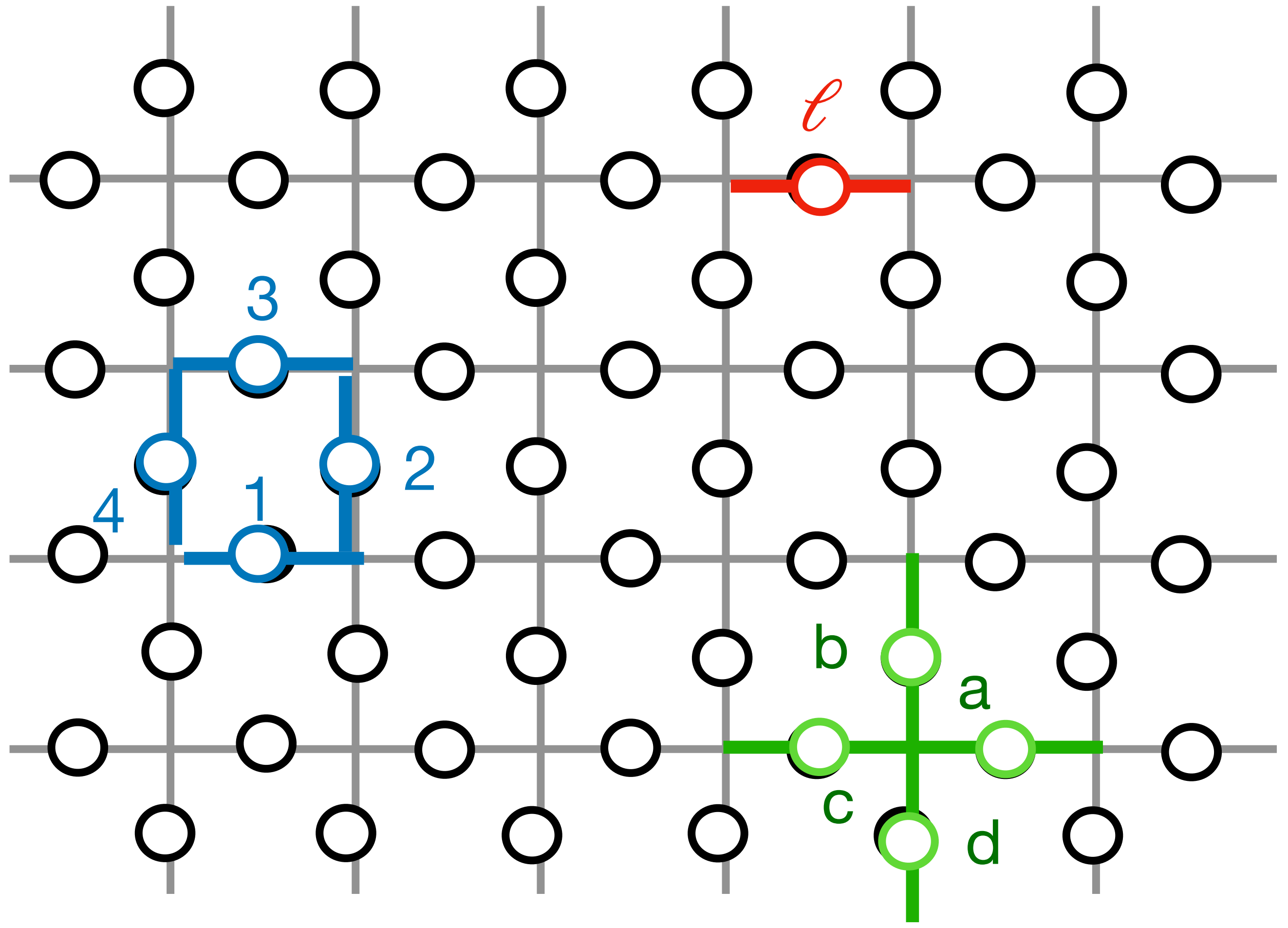


- has not been explored yet for gauge theories

Basics

What is a (lattice) gauge theory?

Z_2 Lattice Gauge Theory



$$H = -\sum_{\square} \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z - \epsilon \sum_{\ell} \sigma_\ell^x \qquad \ell \equiv (\mathbf{n}, i)$$

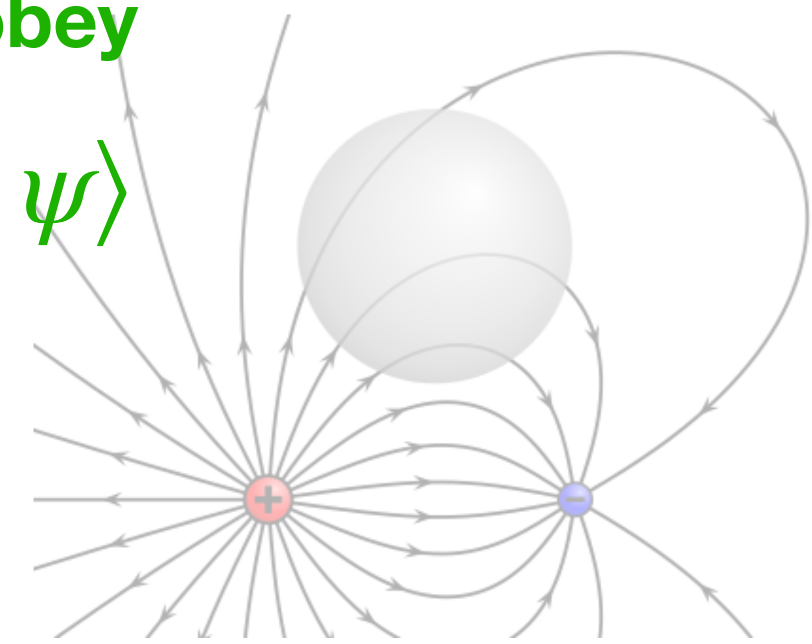
F. Wegner 1979
J. Math. Phys. 12, 2259

Gauss law

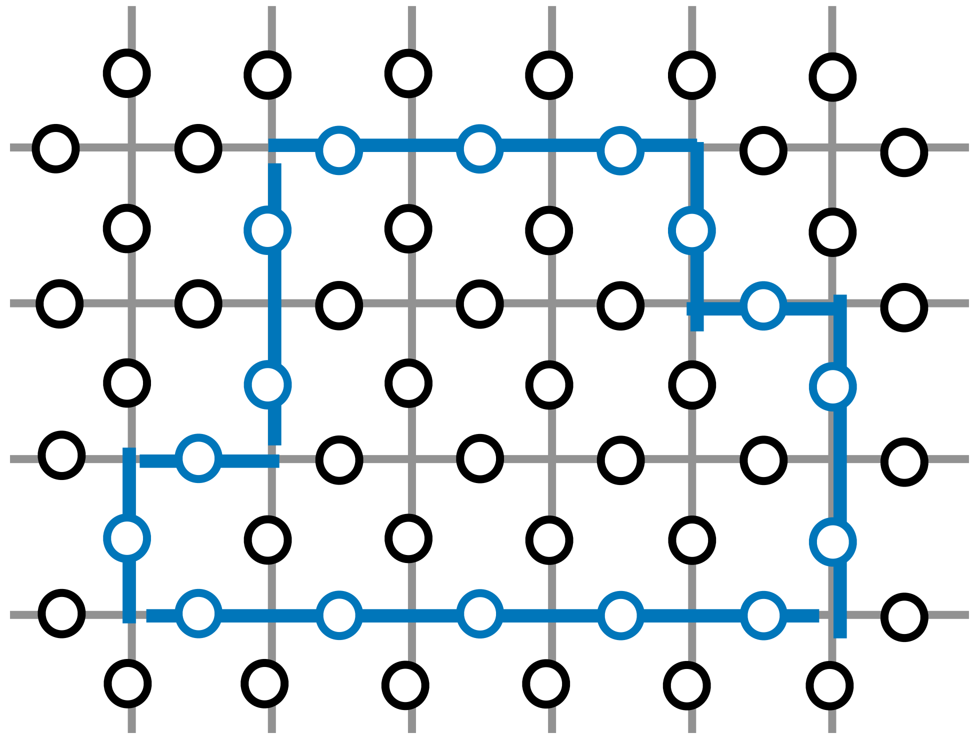
$$G_{\mathbf{n}} = \sigma_a^x \sigma_b^x \sigma_c^x \sigma_d^x \qquad [G_{\mathbf{n}}, H] = 0$$

physical states obey

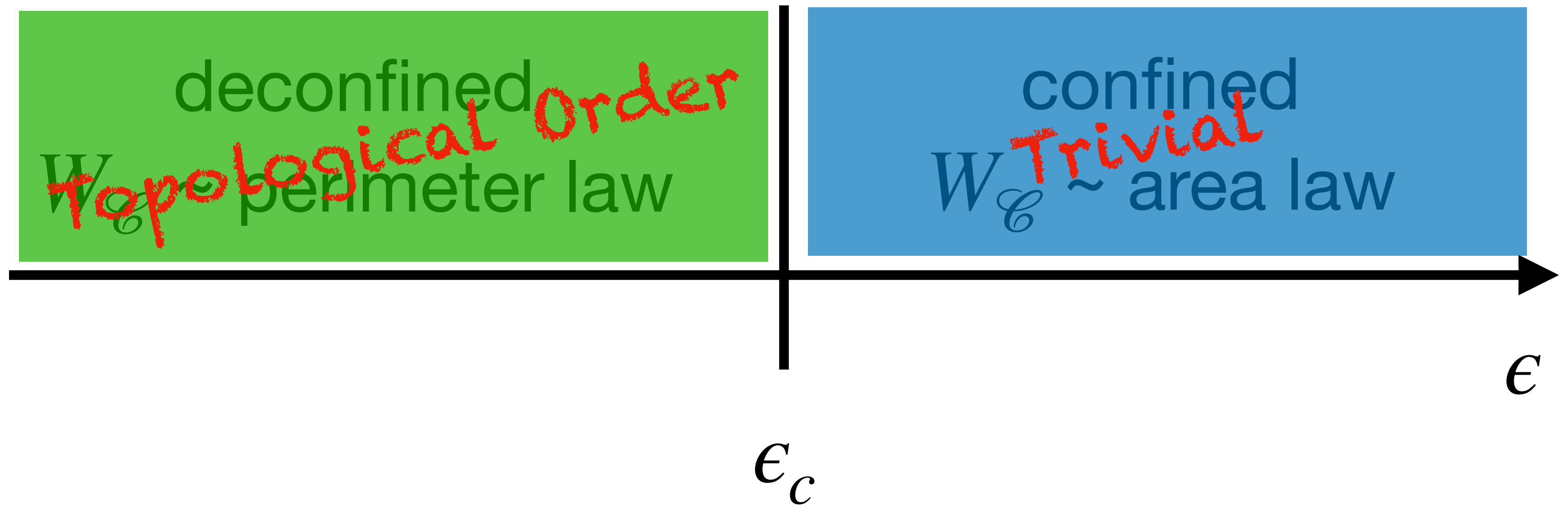
$$G_{\mathbf{n}} |\psi\rangle = 1 |\psi\rangle$$



- Phase transition w/o local order parameter
  deconfinement vs. confinement



$$W_{\mathscr{C}} \equiv \prod_{\ell \in \mathscr{C}} \sigma_\ell^z$$



Entanglement of Gauge Theories

- Entanglement Structure

$$\rho_A = \mathrm{Tr}_B\{\rho\} \quad \equiv \exp\{-H_A\}$$

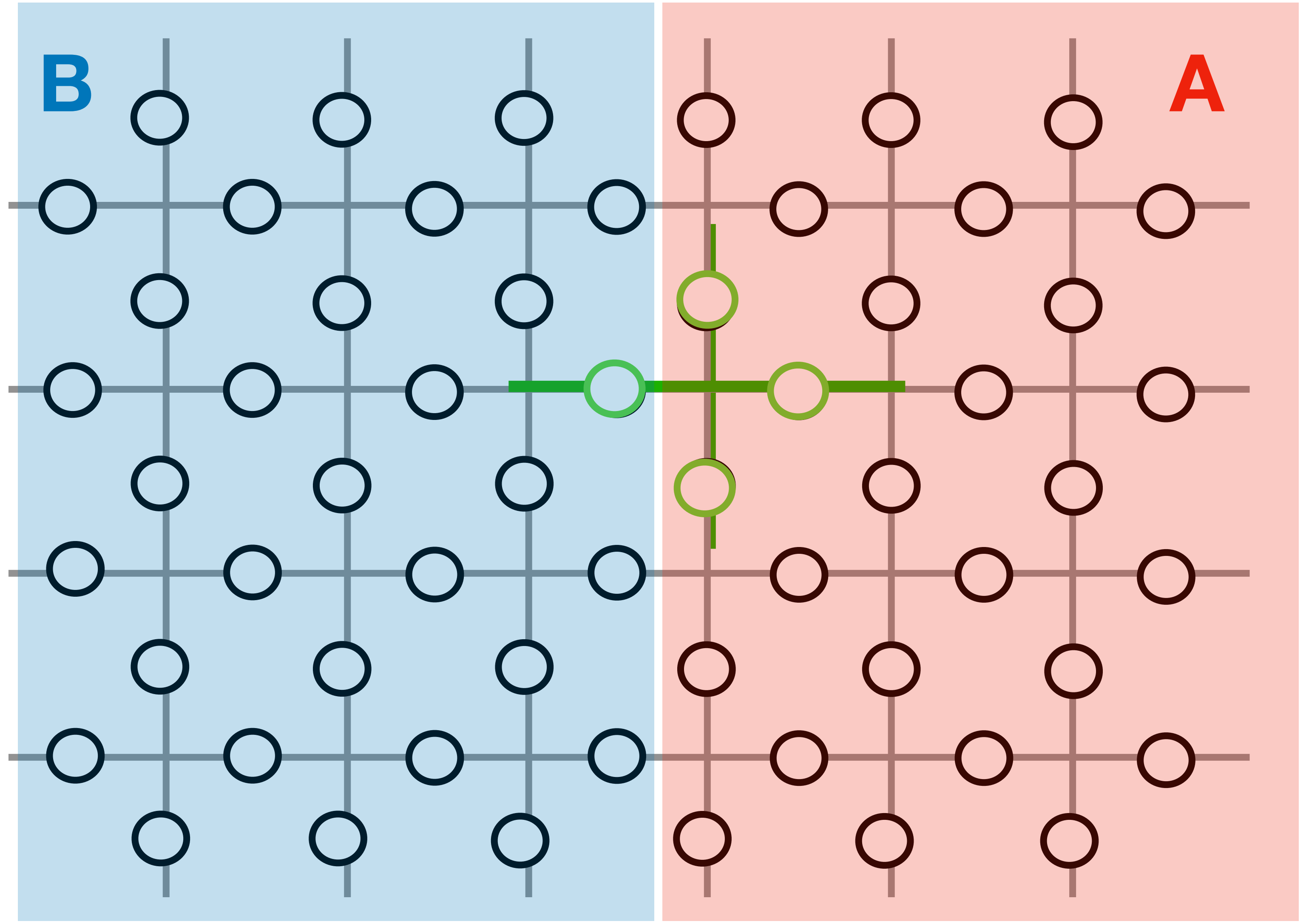
$H_A = -\log(\rho_A)$ "Entanglement Hamiltonian"

Entanglement Spectrum as a Generalization of Entanglement Entropy: Identification of Topological Order in Non-Abelian Fractional Quantum Hall Effect States

Hui Li and F. D. M. Haldane
Physics Department, Princeton University, Princeton, New Jersey 08544, USA
(Dated: July 3, 2008)

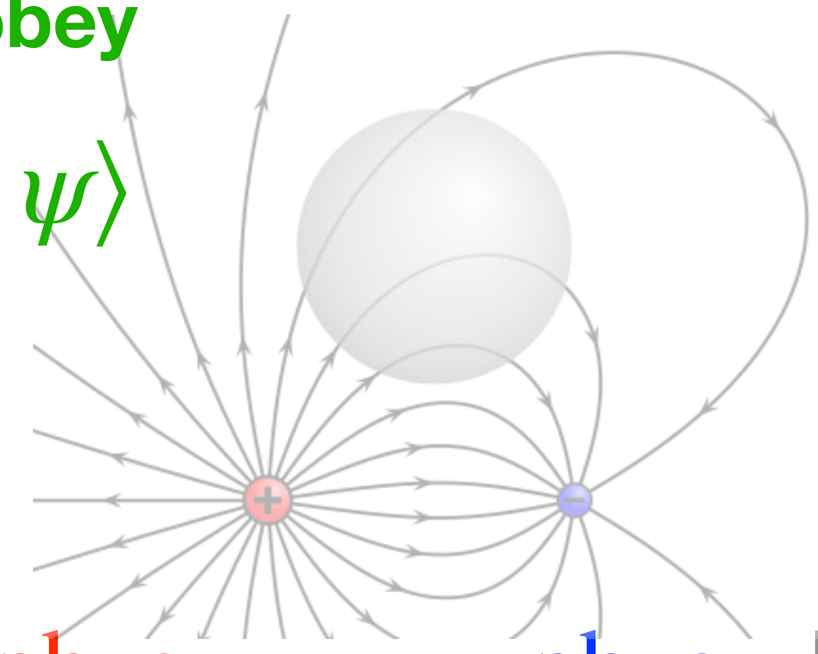
Li, Haldane, PRL 101, 010504 (2008)

- ρ_A for a gauge theory?



physical states obey

$$G_{\mathbf{n}}|\psi\rangle = 1|\psi\rangle$$



$$\mathscr{H}^{\mathrm{phys}} \neq \mathscr{H}_A^{\mathrm{phys}} \otimes \mathscr{H}_B^{\mathrm{phys}}$$

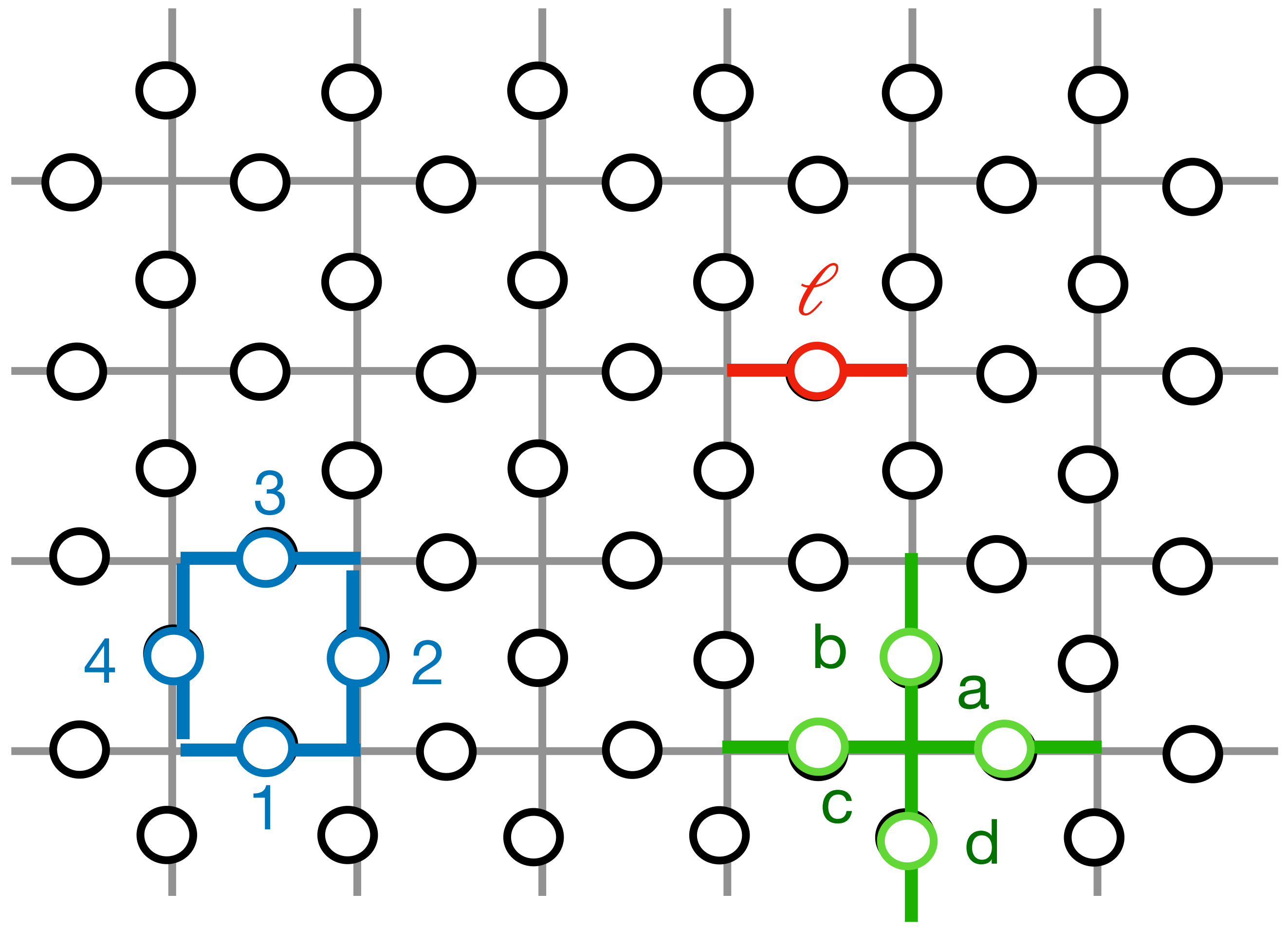
Buividovich. Polikarpov, PLB 670, 141 (2008),
Casini, Huerta Rosabal, PRD 89, 085012 (2014).
Aoki, Iritani, Nozaki, Numasawa, Shiba, Tasaki, JHEP 2015, 1 (2015).
Ghosh, Soni, Trivedi, JHEP, 1 (2015).
Van Acoleyen, Bultinck, Haegeman, Marien,
Scholz, Verstraete, PRL 117, 131602 (2016).
Lin and D. Radicevic, NPB 958, 115118 (2020)

Entanglement of Gauge Theories

- measurements determine state

$$\mathrm{Tr}(\rho O) = \langle O \rangle \qquad \rho = \sum_{O \in \mathscr{A}} \rho_O O \qquad \rho_O = \frac{\langle O \rangle}{2}$$

- Only gauge invariant operators have non-zero expectation value



$$H = -\sum_{\square} \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z \qquad \ell \equiv (\mathbf{n}, i)$$

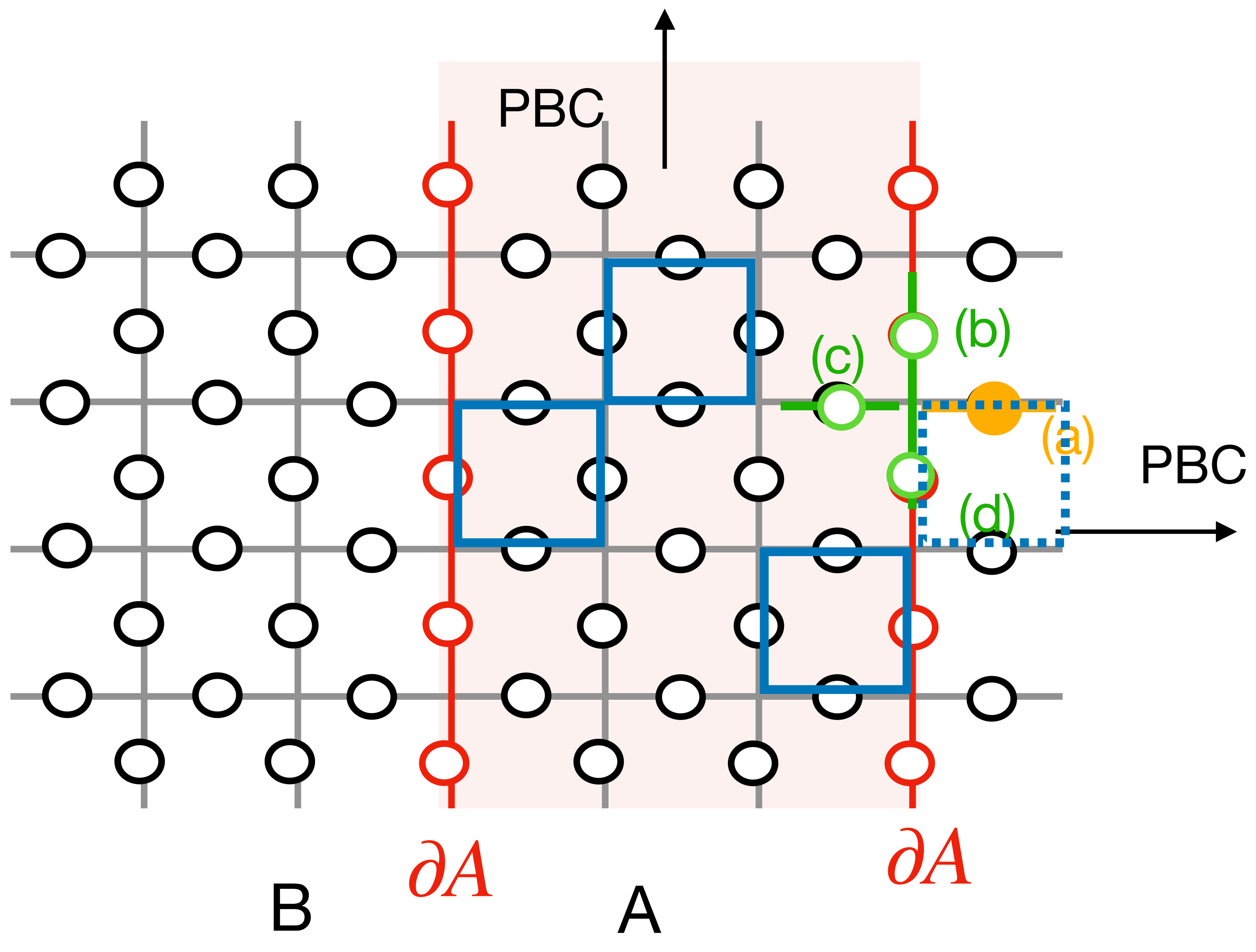
$$G_{\mathbf{n}} = \sigma_a^x \sigma_b^x \sigma_c^x \sigma_d^x \qquad [G_{\mathbf{n}}, H] = 0$$

- Example $\epsilon = 0$

$$\rho \sim \frac{1}{2^{\#}}\left[\mathbb{I} + \sum_{\square} W_{\square} + \sum_{\square \neq \square'} W_{\square} W_{\square'} + \ldots\right] \prod_{\mathbf{n}} \frac{1+G_{\mathbf{n}}}{2} = \prod_{\square} \frac{1+W_{\square}}{2} \prod_{\mathbf{n}} \frac{1+G_{\mathbf{n}}}{2} \qquad W_{\square} = \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z$$

Entanglement of Gauge Theories

- Consider subsystem A



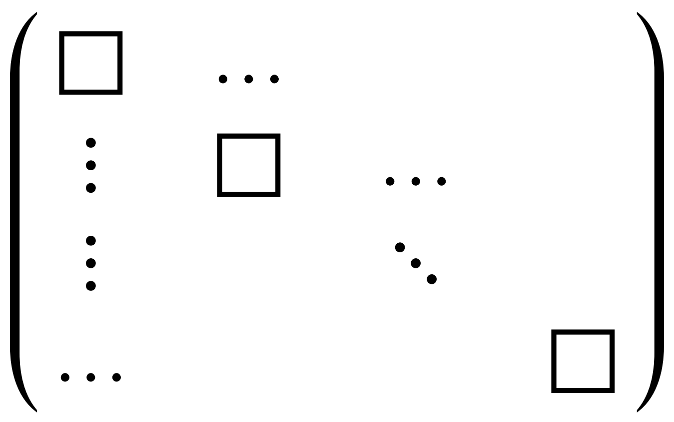
- Gauss law on boundary $\sigma_a^x = \sigma_b^x \sigma_c^x \sigma_d^x,$
- commutes with all operators in system A

$W_\square = \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z$

does not commute, but outside A

- $[\sigma_b^x \sigma_c^x \sigma_d^x \, , \rho_A] = 0$ defines symmetry

$$\rho_A = \prod_\square \frac{1 + W_\square}{2} \prod_{\mathbf{n} \neq \partial A} \frac{1 + G_\mathbf{n}}{2} \times \left(\frac{I^{\partial A}}{2^{2N_y}} \right)$$



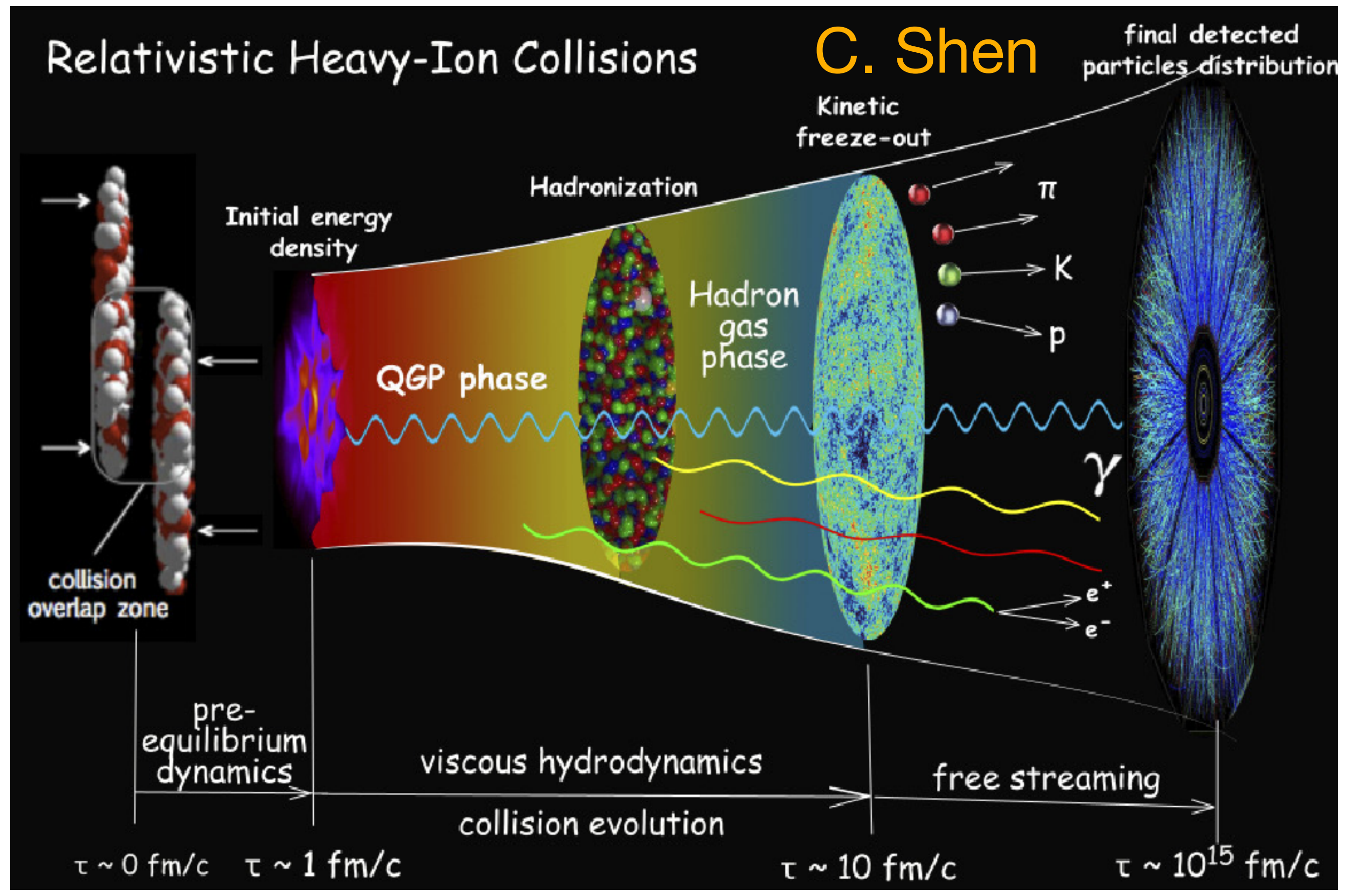
von Neumann entanglement entropy

$$S_E = 2N_y \log(2)$$

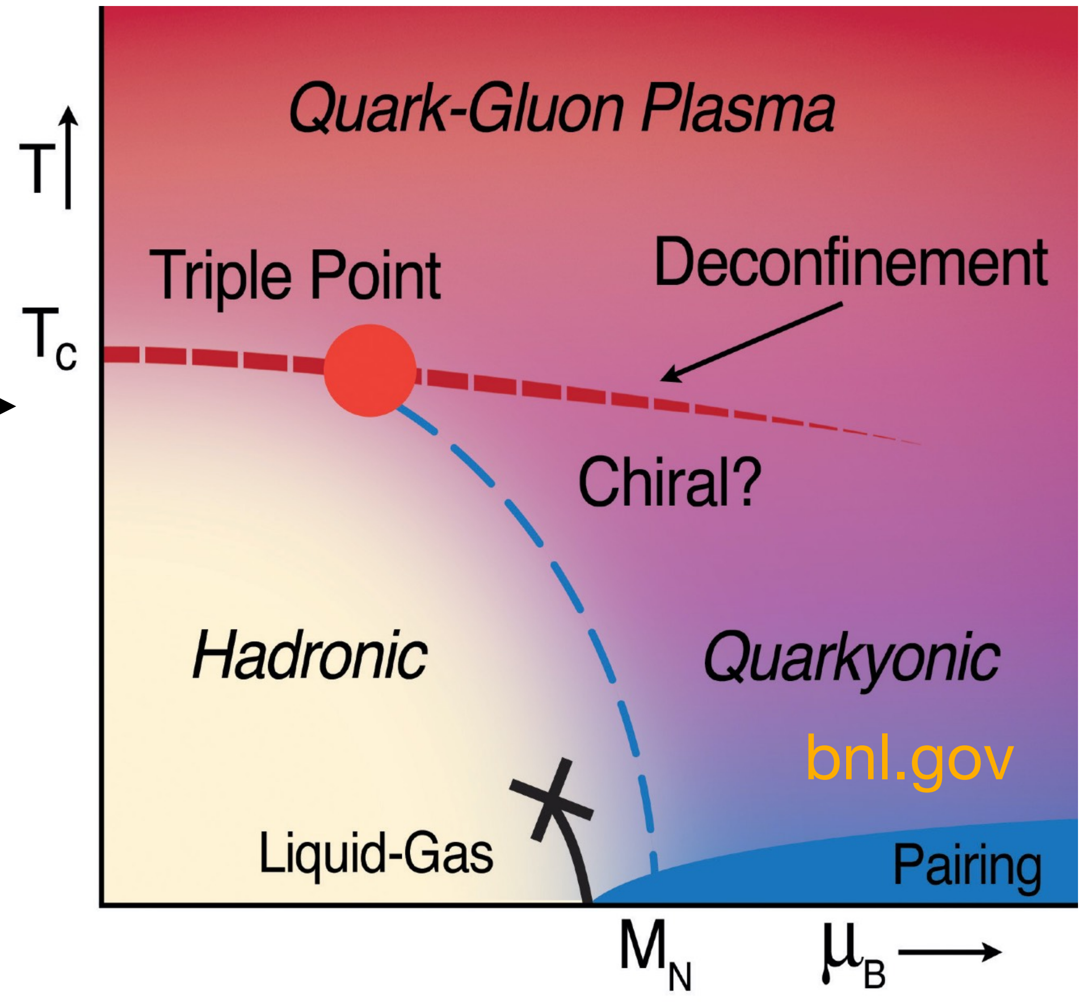
- Entanglement: distillable vs. symmetry/classical

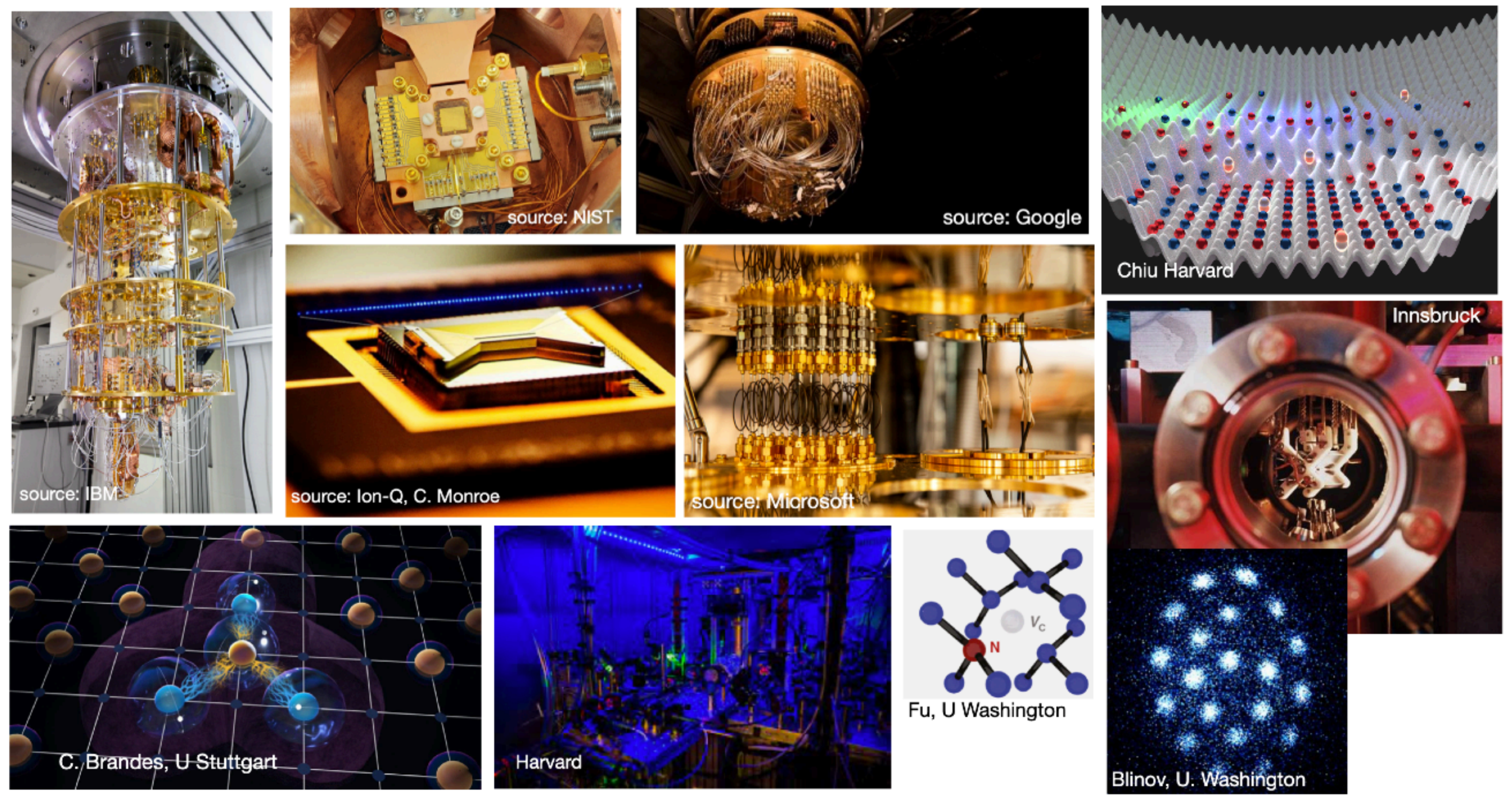
Casini, Huerta Rosabal, PRD 89, 085012 (2014).

compute Entanglement of LGTs ✓



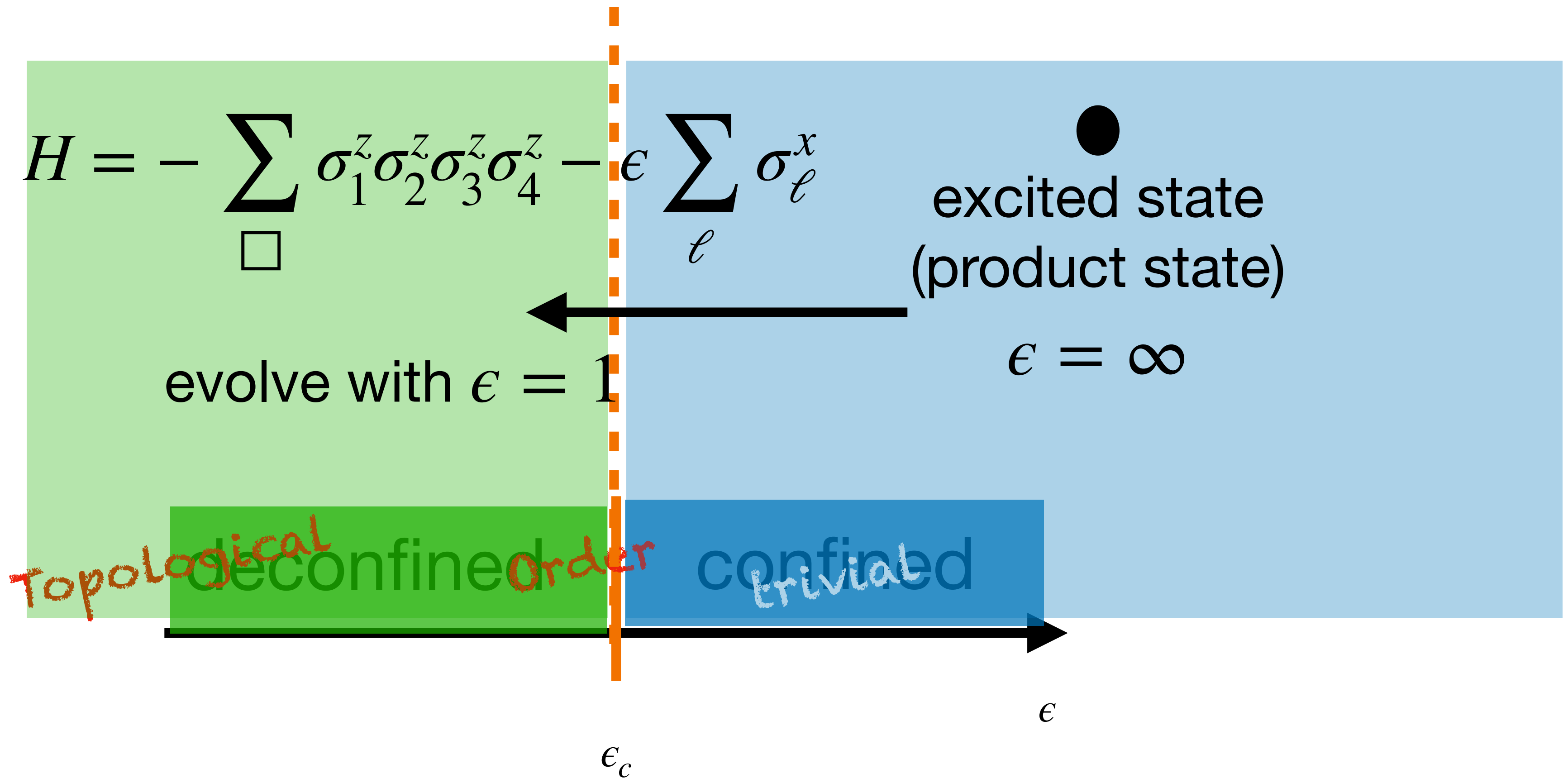
?





cannot compute a heavy ion collision ✗ (yet)

- "Quantum Quench"



Gauge Theory Thermalization

Stage I: Maximization of Schmidt rank

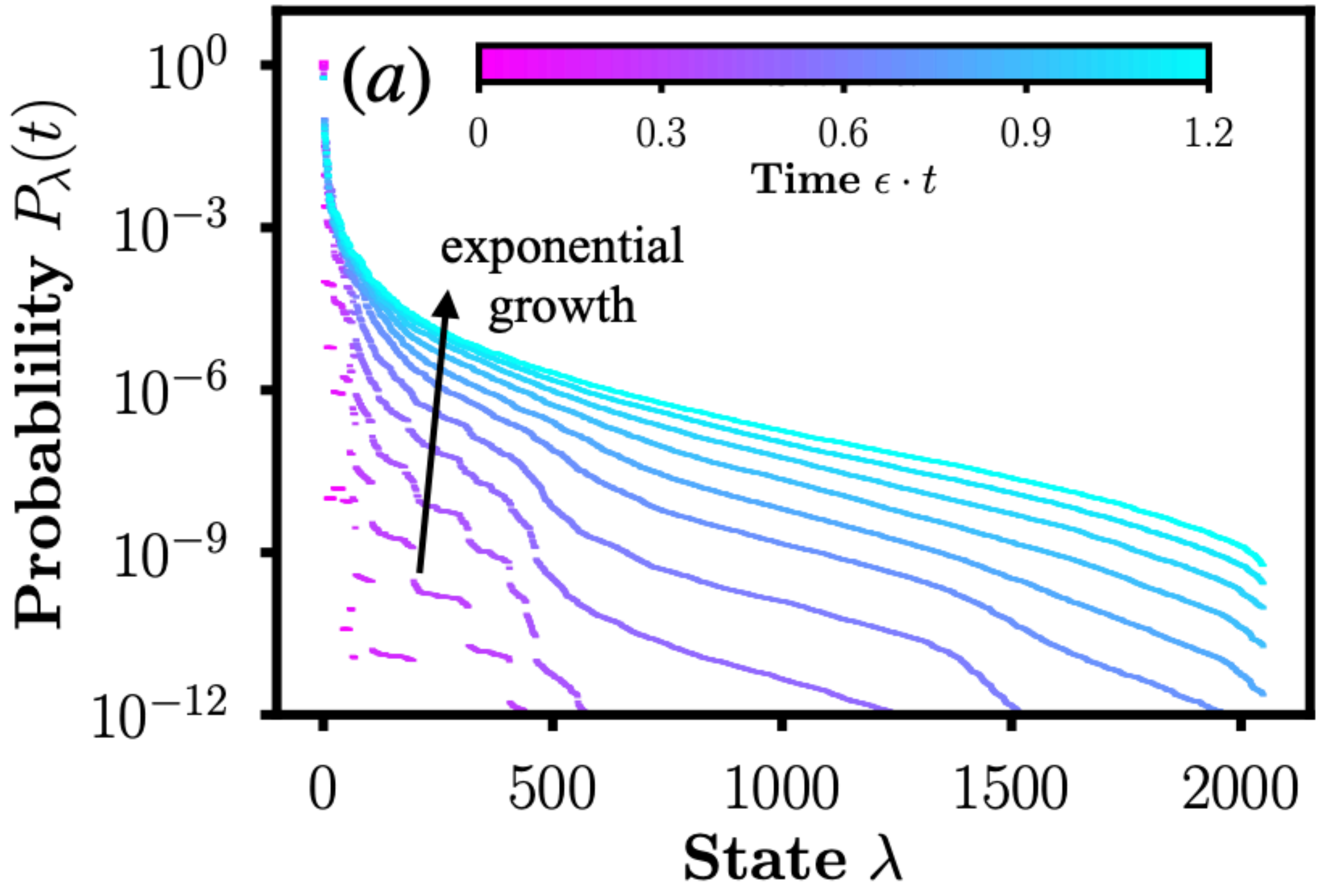
Schmidt decomposition

$$\rho_A \equiv \sum_{\lambda=0}^{\chi-1} P_\lambda |\psi_{\lambda_A}\rangle\langle\psi_{\lambda_A}|$$

Entanglement Structure

$$\rho_A(t) \equiv e^{-H_A(t)}$$

$$P_\lambda(t) \equiv e^{-\xi_\lambda(t)}$$

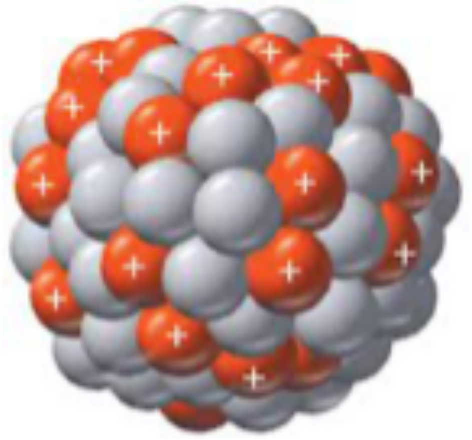


NM, Zache, Ott, PRL 129, 011601 2022)

Gauge Theory Thermalization

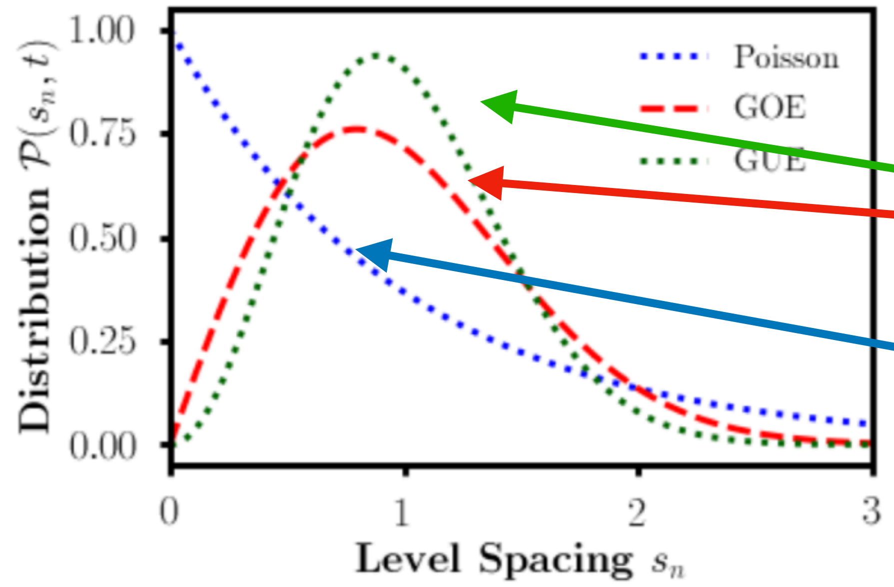
Stage II: Spreading of level repulsion

quantum chaos



$$\hat{H} \doteq \begin{bmatrix} \varepsilon_1 & \frac{V}{\sqrt{2}} \\ \frac{V^*}{\sqrt{2}} & \varepsilon_2 \end{bmatrix}$$

"Wigner's surmise"



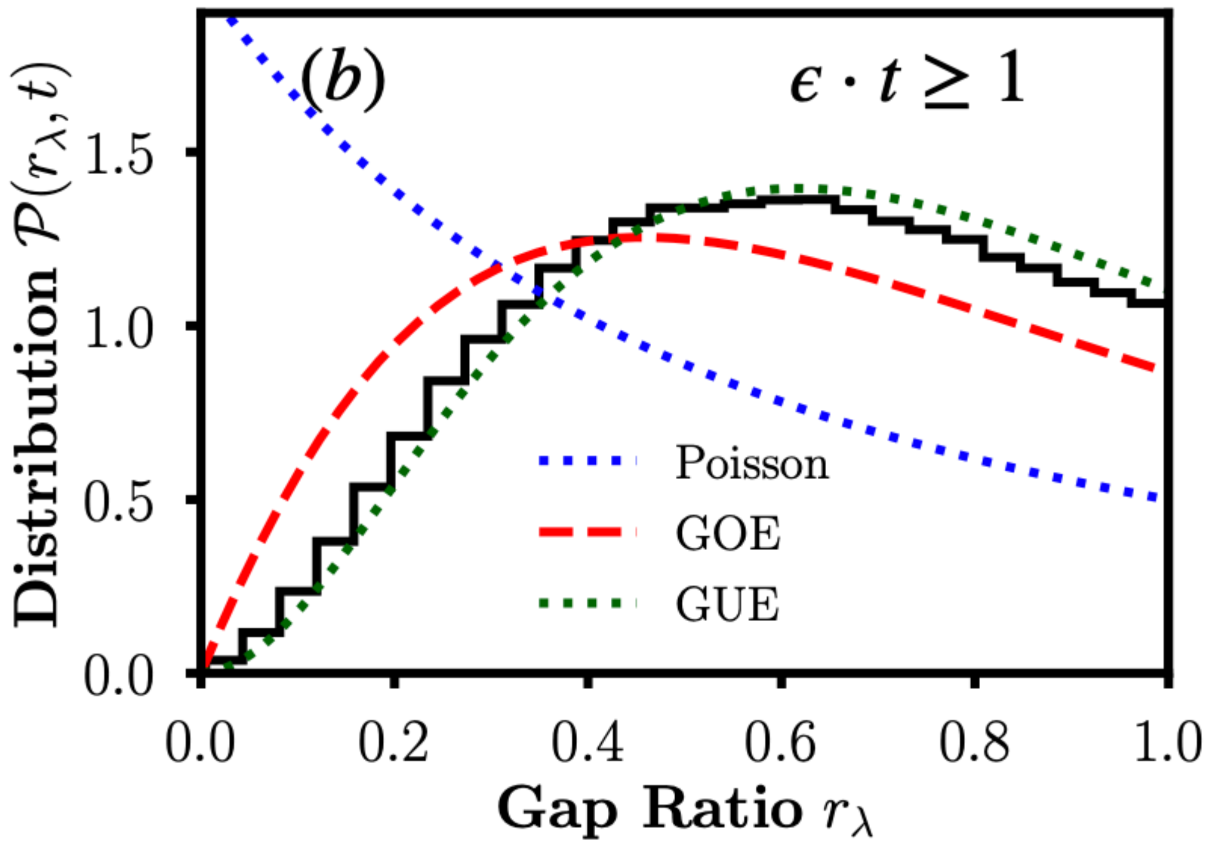
levels repulse
= chaos/thermalization

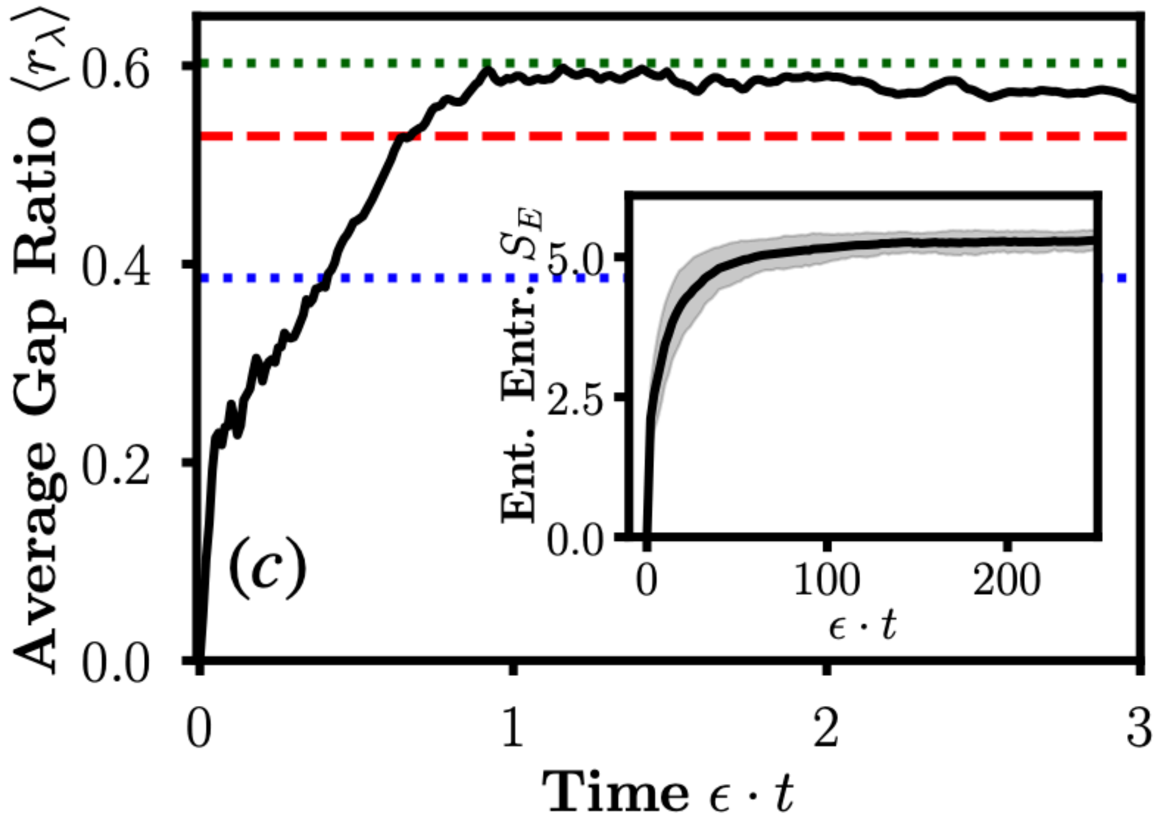
levels uncorrelated
= many-body localization
integrable

… seen in level spacings of Entanglement Hamiltonian

$H_A(t) \equiv -\log[\rho_A(t)]$

$\xi_\lambda(t) \equiv -\log[P_\lambda(t)]$





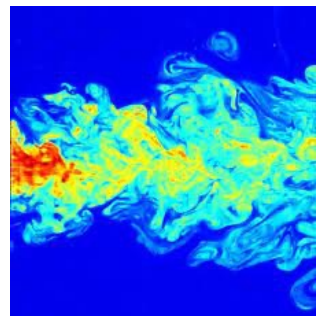
towards thermalization

MBL integrable

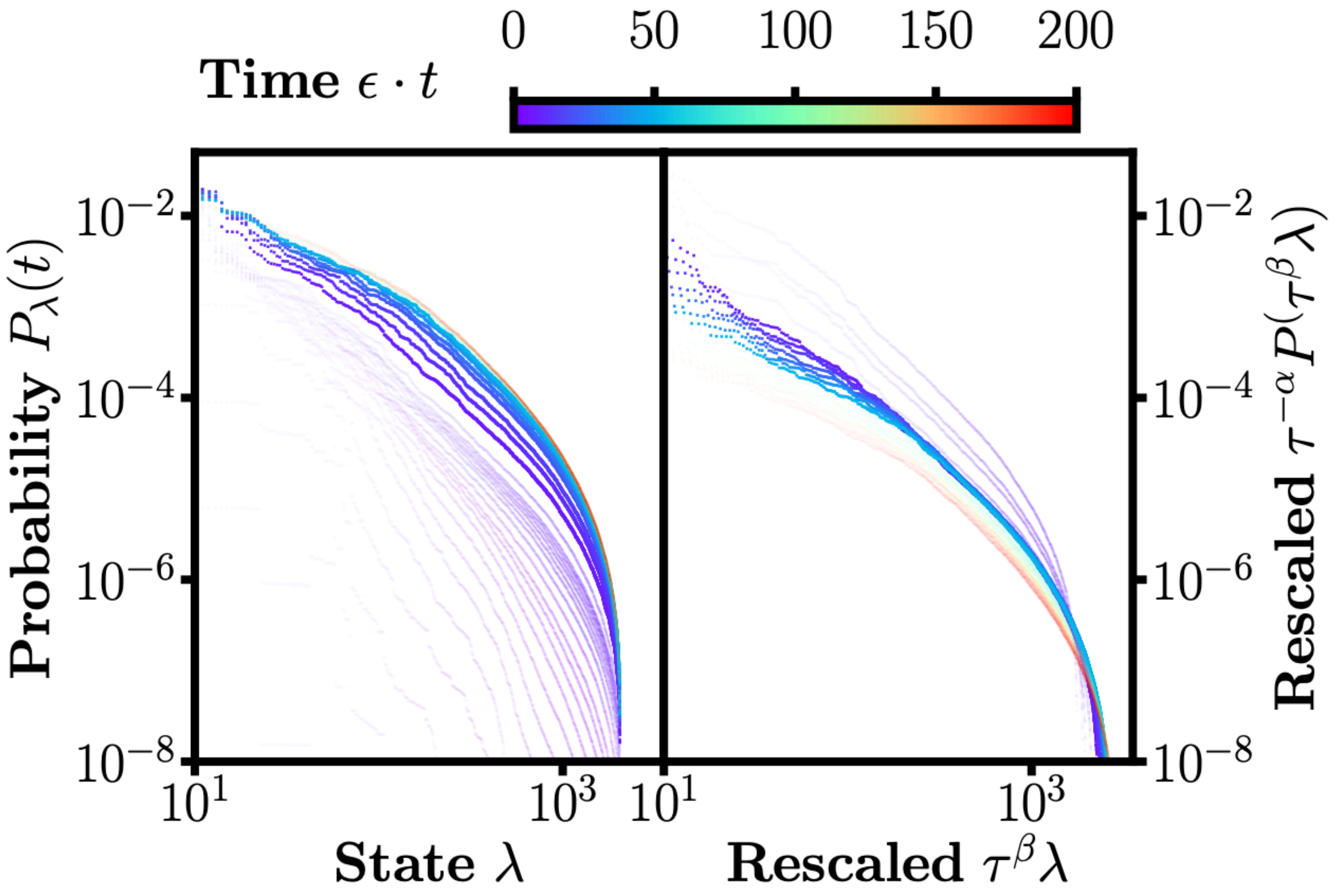
NM, Zache, Ott, PRL 129, 011601 2022)

Gauge Theory Thermalization

Stage III: Self Similar Evolution



"Quantum Turbulence"



- scaling form

$$P(\lambda, t) = \tau^{-\alpha} P(\tau^\beta \lambda)$$

$\tau = \epsilon(t - t_0)$

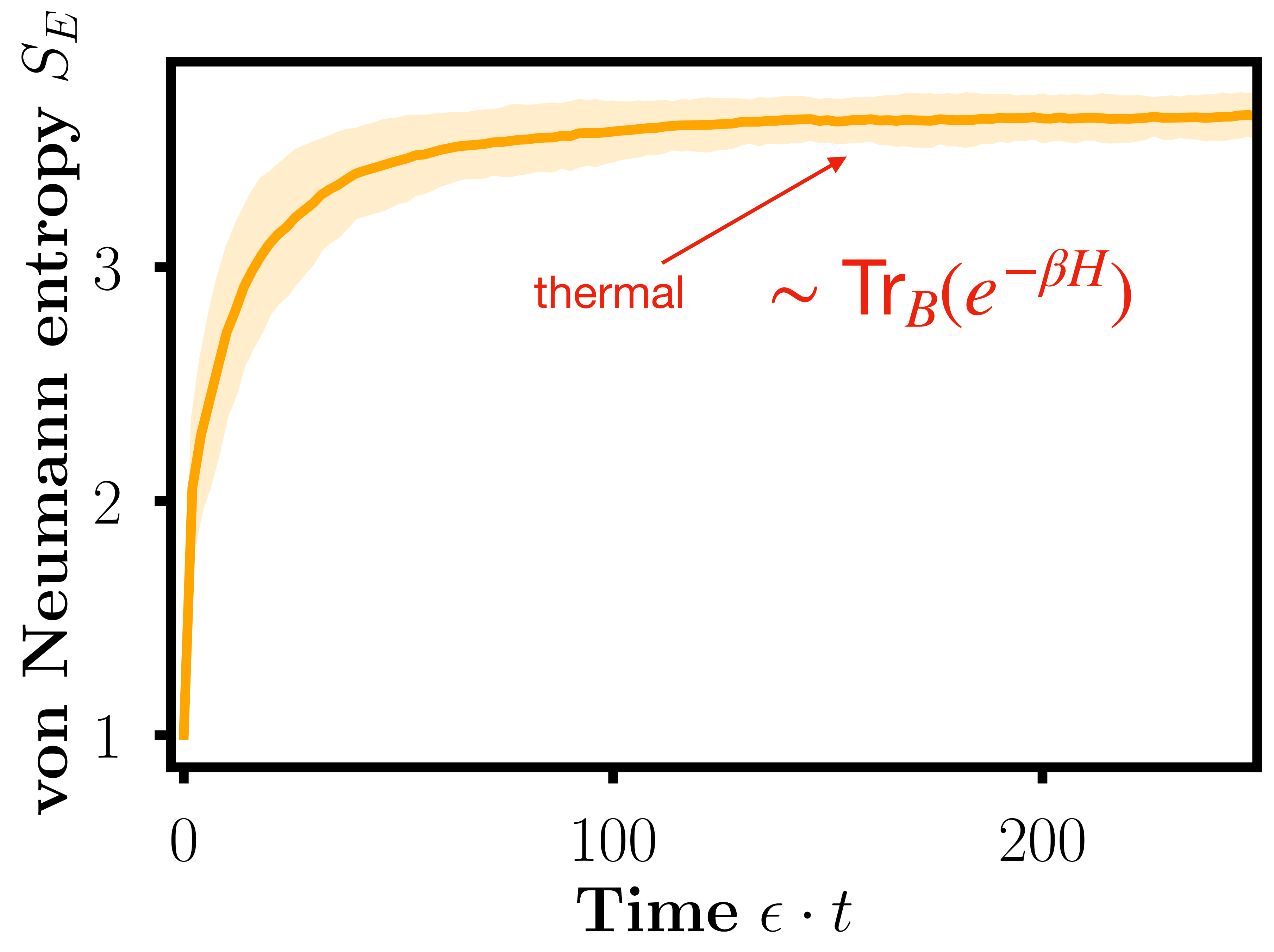
$\alpha = 0.8 \pm 0.1$

$\beta = 0.0 \pm 0.1$

NM, Zache, Ott, PRL 129, 011601 2022)

Gauge Theory Thermalization

Stage IV: Saturation of von Neumann entropy



von-Neumann entropy = thermal entropy

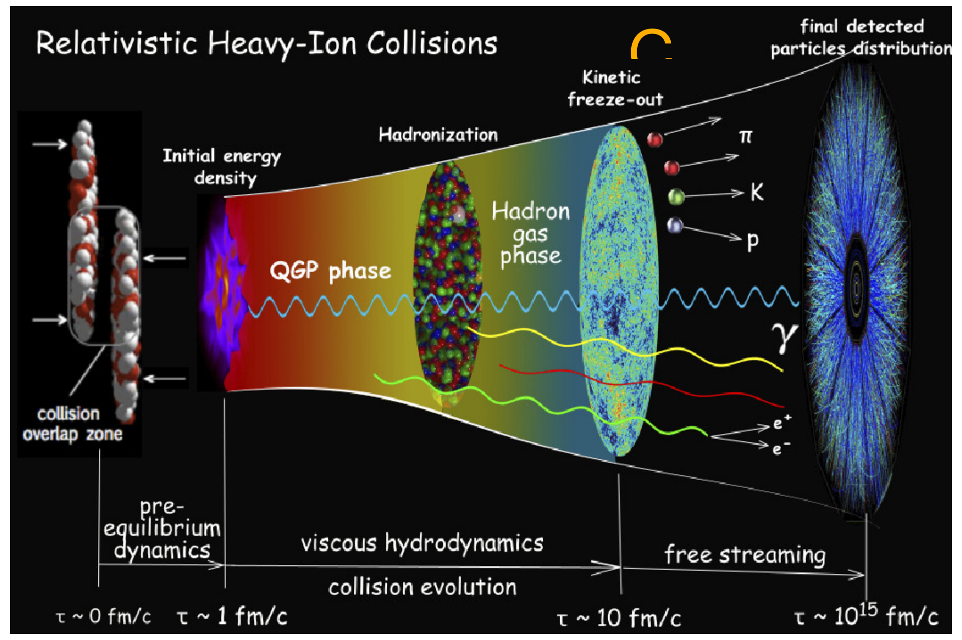
NM, Zache, Ott, PRL 129, 011601 2022)

Conclusions

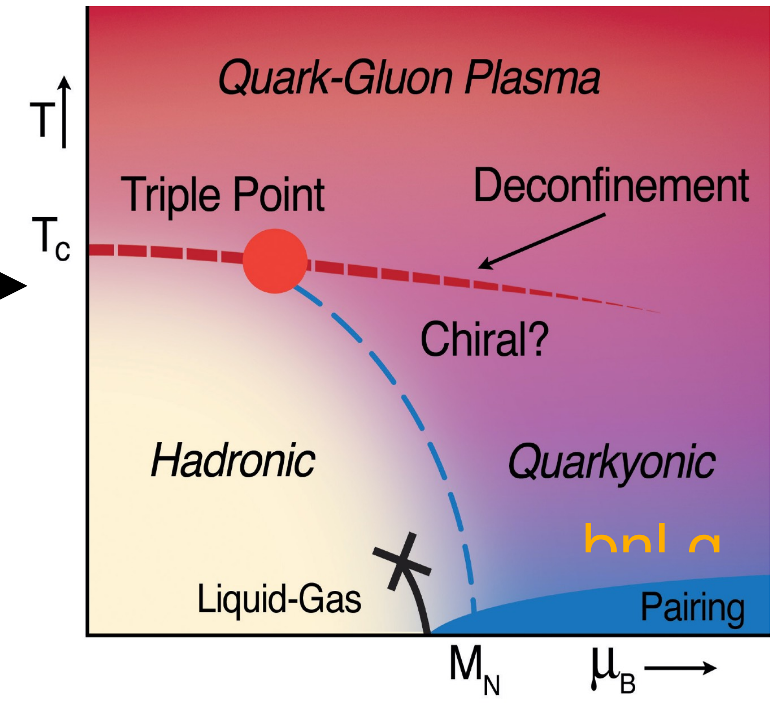
- Understanding gauge theory thermalization from entanglement structure.

- Characteristic sequence: quantum chaos, turbulence

- Consistent with ETH, but LGTs “not special”.



?



- $\mathbb{Z}_2^{2+1}$ —> QCD much harder.

- Great opportunity for
  Quantum Information Technology