QCD phase structure under rotation

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QCD Theory Seminar Series
October 14, 2022

Content

- Introduction
- Where are rotating QCD matter?
- Can rotation affect chiral condensate?
- Can rotation affect confinement?
- Chiral condensate and confinement on rotating lattice
- Summary

Introduction

Rotation and magnetic field

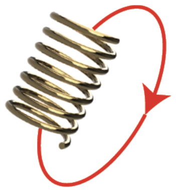
- Hints for possible rotation effect: comparison with B field

Spin:

Magnetic moment

$$H_{\mathrm{Zeeman}} = -\gamma \boldsymbol{S} \cdot \boldsymbol{B}$$

Magnetic field



Angular momentum

$$H_{\mathrm{spin}-\omega} = -\boldsymbol{S} \cdot \boldsymbol{\omega}$$

Rotation field

Orbital:

In magnetic field, Lorentz force:

$$\boldsymbol{F} = e(\dot{\boldsymbol{x}} \times \boldsymbol{B})$$

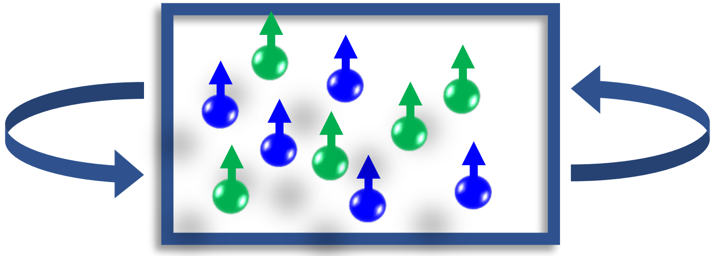
Larmor theorem: $e\boldsymbol{B} \sim 2m\boldsymbol{\omega}$

In rotating frame, Coriolis force:

$$\boldsymbol{F} = 2m(\dot{\boldsymbol{x}} \times \boldsymbol{\omega}) + O(\boldsymbol{\omega}^2)$$

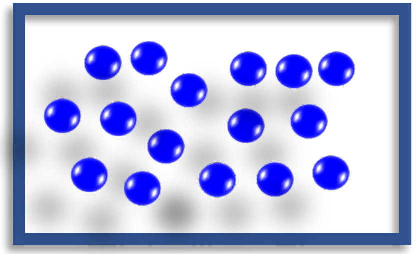
Rotation and chemical potential

- Hints for possible rotation effect: comparison with chemical potential



Rotation

$$H = H_0 - \omega J_z$$



Chemical potential

$$H = H_0 - \mu N$$

For massless Dirac fermions

$$P = \frac{7\pi^2}{180\beta^4} + \frac{(\omega/2)^2}{6\beta^2} + \frac{(\omega/2)^4}{12\pi^2}$$

(At rotating axis, for unbounded system)

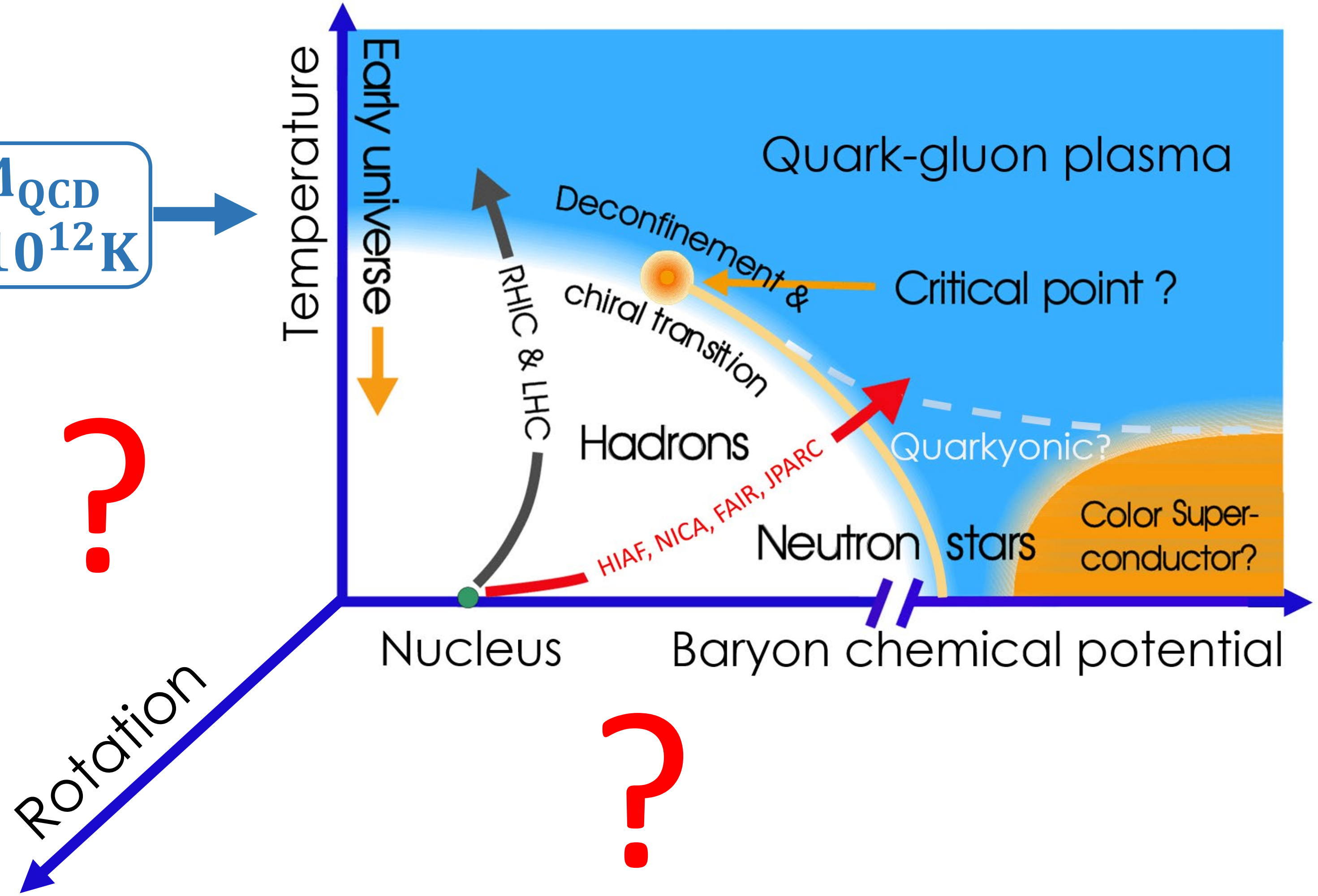
(Ambrus and Winstanley 2019; Palermo etal 2021)

$$P = \frac{7\pi^2}{180\beta^4} + \frac{\mu^2}{6\beta^2} + \frac{\mu^4}{12\pi^2}$$

>>> Both have sign problem on lattice

QCD phase diagram

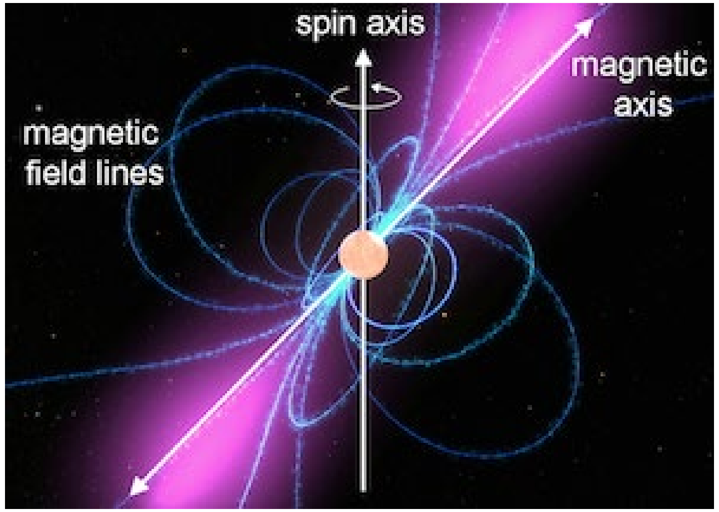
$T_c \sim \Lambda_{QCD}$
$\sim 10^{12}\mathrm{K}$



- Where rotating QCD matter?
- Rotation affects chiral condensate and confinement?
- Rotation effects combined with finite densities?

<u>Where rotating QCD matter?</u>

Rotating neutron stars



Isolated pulsars can have $\omega \sim 10^3 s^{-1}$

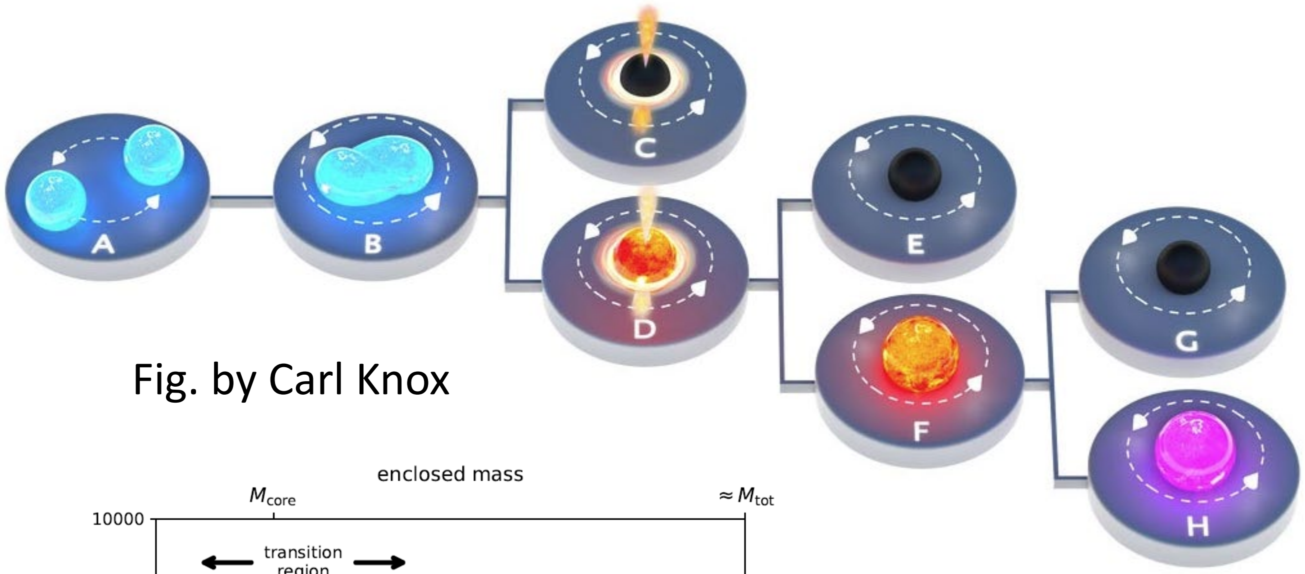
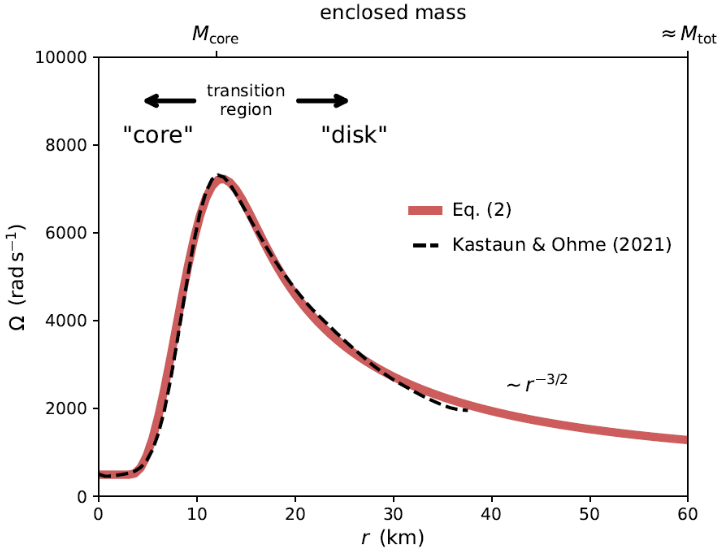


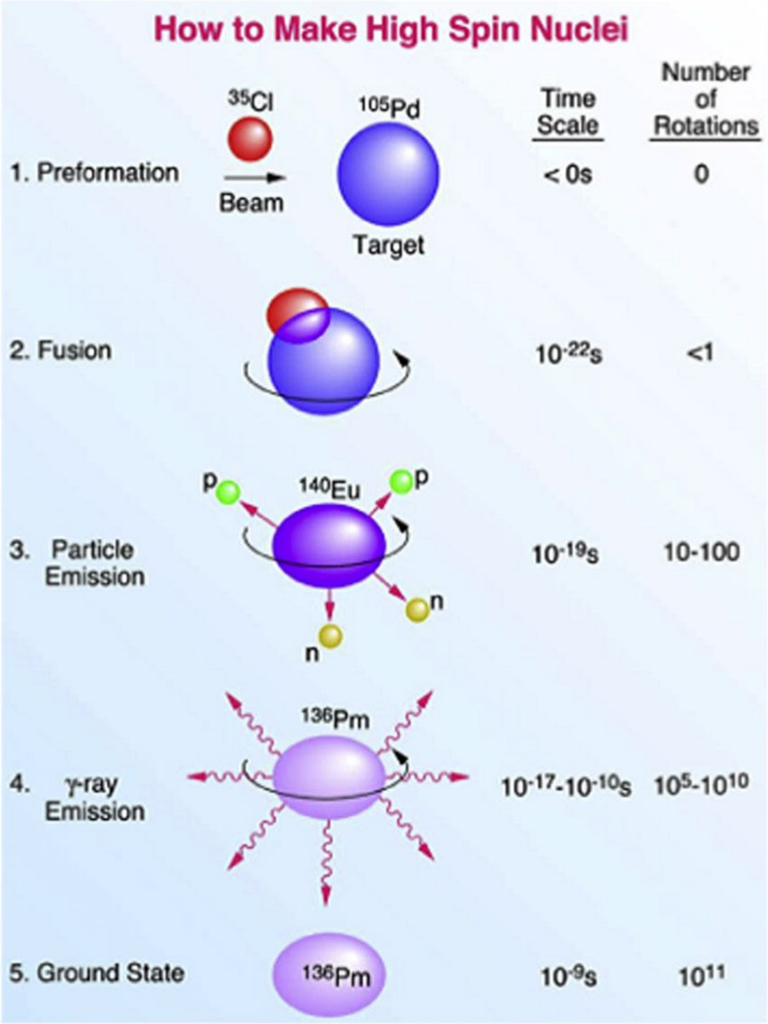
Fig. by Carl Knox

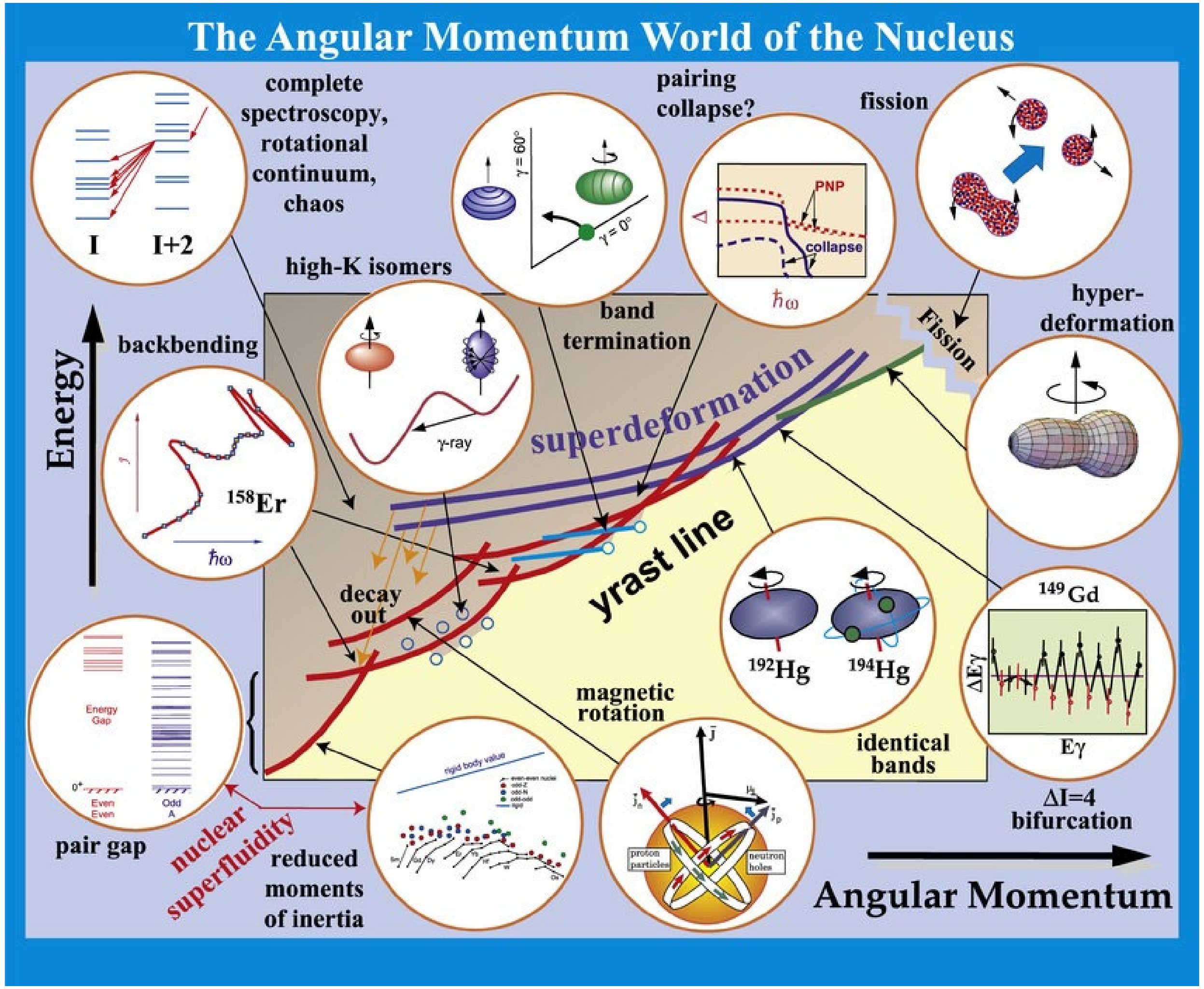


(Margalit etal 2206.10645)

Neutron star mergers

Rotating nuclei

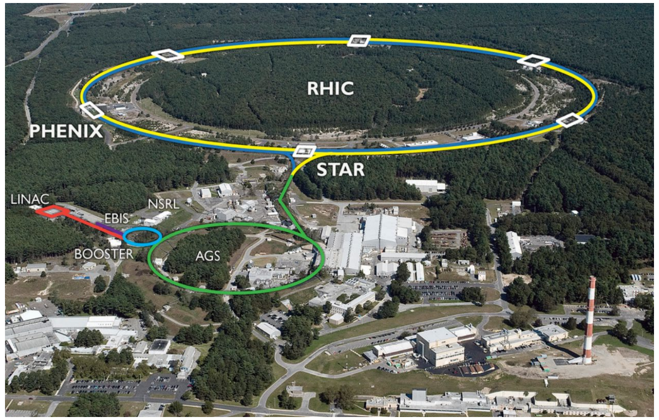


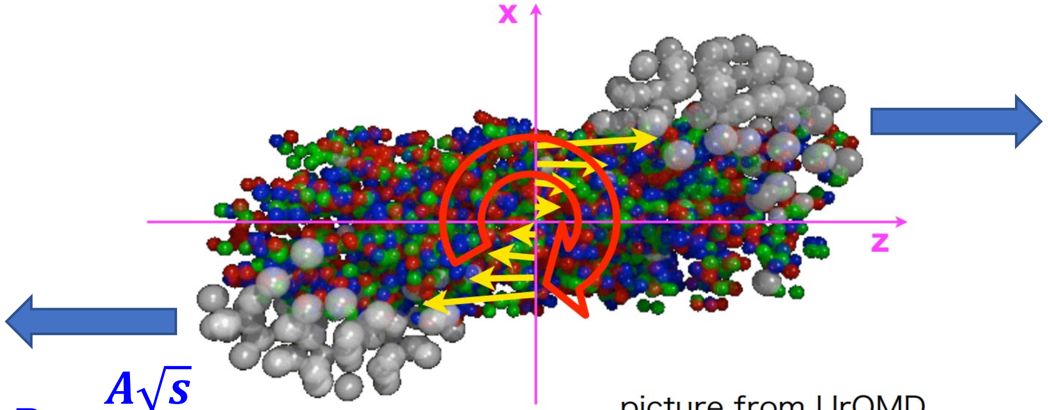


Rotation can reach $\omega \sim 10^{21} s^{-1}$

(M A Riley *et al* 2016 *Phys. Scr.* **91** 123002)

Rotating quark-gluon plasma



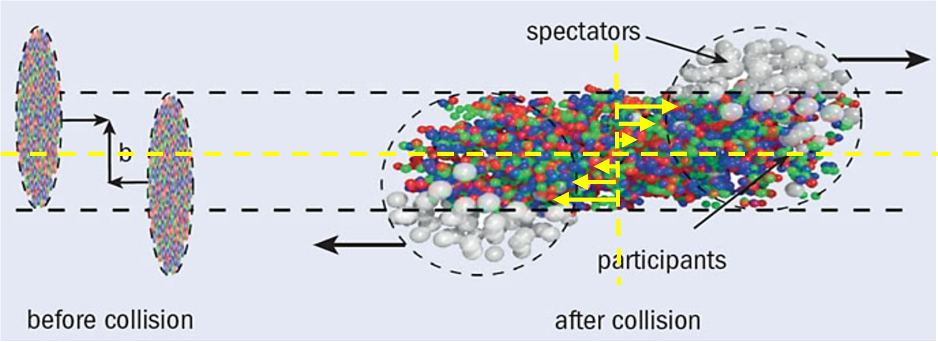


picture from UrQMD

$$P_z \sim \frac{A\sqrt{s}}{2}$$

Rotation by global angular momentum

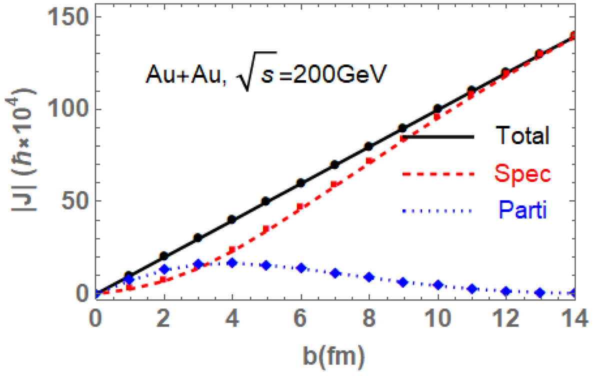
- Angular momentum conservation

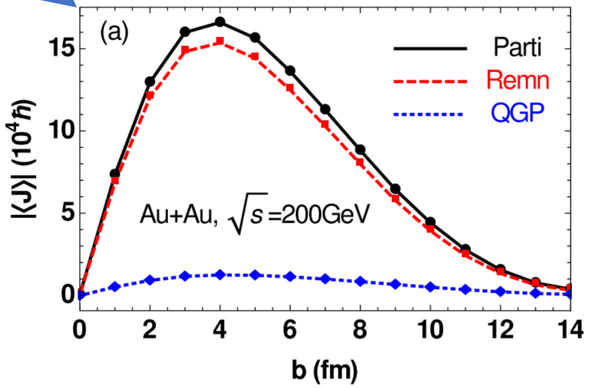


No rigid rotation*, but local fluid vorticity

$$\boldsymbol{\omega} = \frac{1}{2}\nabla \times \boldsymbol{v}$$

(Angular velocity of fluid cell)

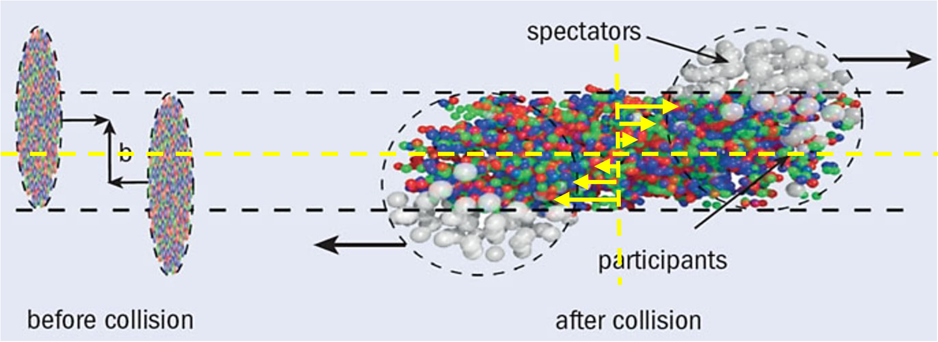




* At low energy, there is a possibility that two colliding nuclei fuse into a compound high-spin nucleus

Rotation by global angular momentum

- Angular momentum conservation



No rigid rotation*, but local fluid vorticity

$$\boldsymbol{\omega} = \frac{1}{2}\nabla \times \boldsymbol{v}$$

(Angular velocity of fluid cell)

- Estimation at low energy $\sqrt{s} \gtrsim 2m_N$

part of $J_0 \sim Ab(\sqrt{s} - 2m_N)$ **retained in the produced matter**:

$J = \int d^3\boldsymbol{x}\, I(\boldsymbol{x})\omega(\boldsymbol{x}) \approx \int d^3\boldsymbol{x}\, \varepsilon(\boldsymbol{x}) x_\perp^2 \bar{\omega} \sim 2m_N\, A\, R_A^2\, \bar{\omega}$ **for** $b < 2R_A$

$$\bar{\omega} \sim \frac{b}{R_A^2}\frac{\sqrt{s} - 2m_N}{2m_N} \sim 10^{22} s^{-1}$$

$(b = R_A, \sqrt{s} = 3\ \mathrm{GeV})$

- Estimation at high energy $\sqrt{s} \gg 2m_N$

part of $J_0 \sim Ab\sqrt{s}$ **retained in the produced matter**:

$J \approx \int d^3\boldsymbol{x}\, \gamma^2(\boldsymbol{x})\varepsilon(\boldsymbol{x}) x_\perp^2 \bar{\omega} \sim s\, A\sqrt{s}\, R_A^2\, \bar{\omega}/(2m_N)^2$ **for** $b < 2R_A$

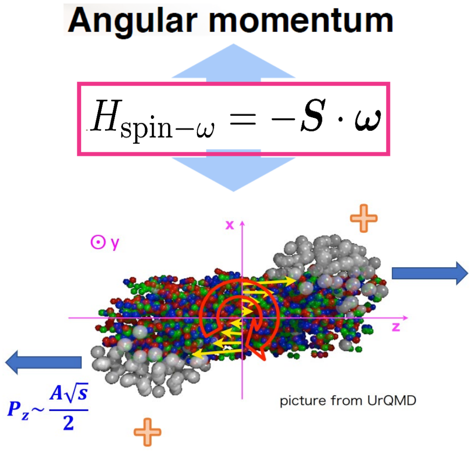
$$\bar{\omega} \sim \frac{b}{R_A^2}\left(\frac{2m_N}{\sqrt{s}}\right)^2 \sim 10^{19} s^{-1}$$

$(b = R_A, \sqrt{s} = 200\ \mathrm{GeV})$

* At low energy, there is a possibility that two colliding nuclei fuse into a compound high-spin nucleus

Measuring the vorticity: spin polarization

- From global angular momentum to vorticity to hyperon spin polarization

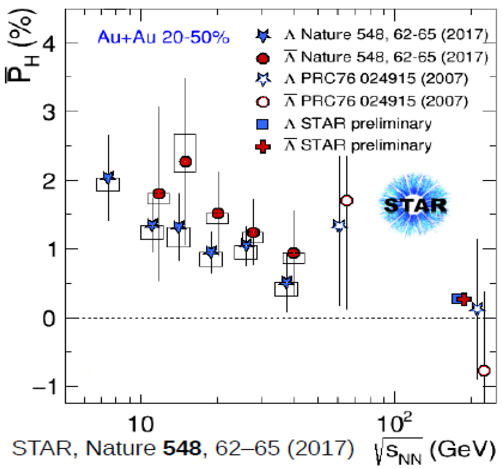


(at thermal equilibrium)

$$\frac{dN_s}{d\boldsymbol{p}} \sim e^{-(H_0 - \boldsymbol{\omega} \cdot \boldsymbol{S})/T}$$

$$\mathrm{P} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \sim \frac{\omega}{2T}$$

- First measurement of Λ polarization by STAR@RHIC *



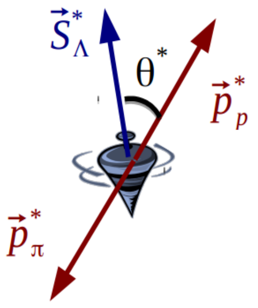
STAR, Nature **548**, 62–65 (2017)

parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

α: Λ decay parameter (α_Λ=0.732)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame



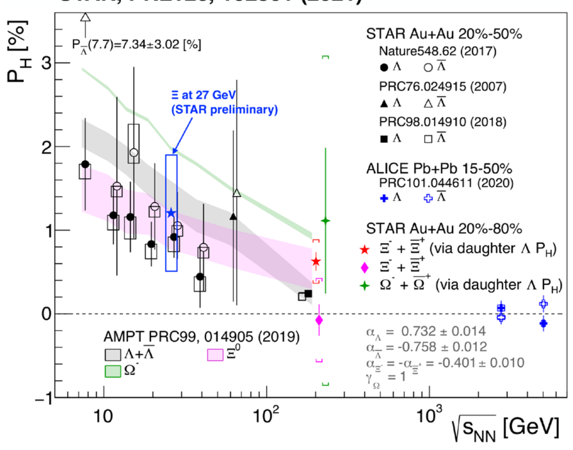
$\Lambda \to p + \pi^+$
(BR: 63.9%, cτ~7.9 cm)

(* First theoretical proposal: Liang and Wang 2004, later by Voloshin 2004)

Measuring the vorticity: spin polarization

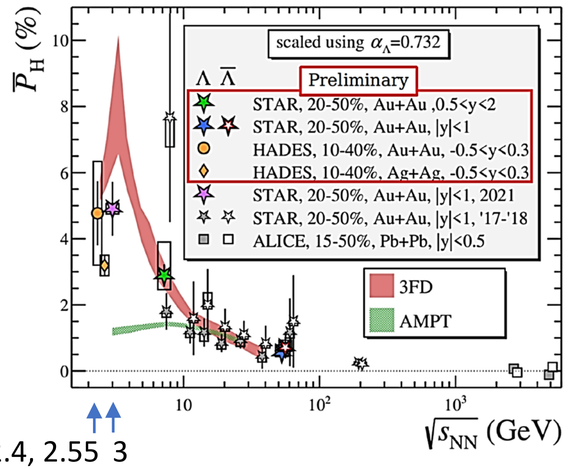
- More recent measurements: Ξ^-, Ω^- by STAR@RHIC, Λ by ALICE@LHC

STAR, PRL126, 162301 (2021)

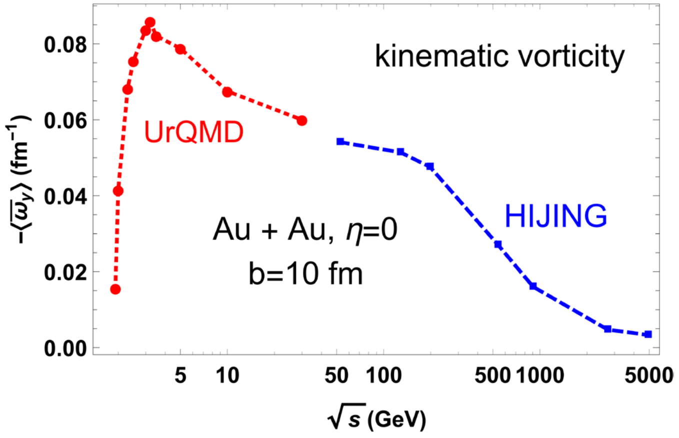


hyperon	decay mode	α_H	magnetic moment μ_H	spin
Λ (uds)	Λ→pπ⁻ (BR: 63.9%)	0.732	-0.613	1/2
Ξ⁻ (dss)	Ξ⁻→Λπ⁻ (BR: 99.9%)	-0.401	-0.6507	1/2
Ω⁻ (sss)	Ω⁻→ΛK⁻ (BR: 67.8%)	0.0157	-2.02	3/2

- Λ at low energy by STAR@RHIC 2021, HADES@GSI 2021



2.4, 2.55 3



(Deng-XGH 2016, Deng-XGH-Ma-Zhang 2020)

- "The most vortical fluid": $\omega \sim 10^{20} - 10^{21} s^{-1}$
- Relativistic suppression at high energies

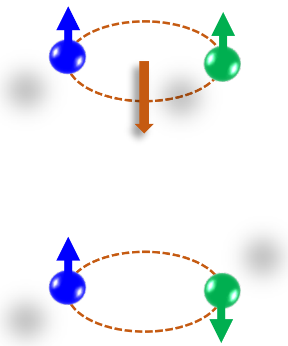
Can rotation affect chiral condensate?

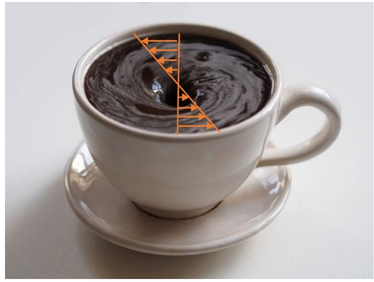
Angular momentum polarization

- Consider a scalar (or pseudoscalar) pair of fermions

J=0, L=1

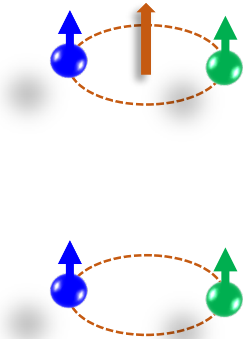
J=0, L=0



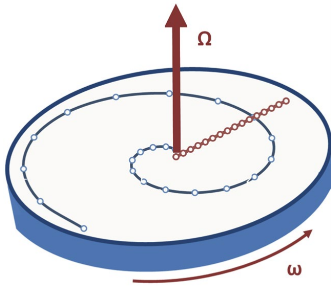


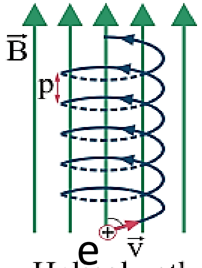
J=2, L=1

J=1, L=0



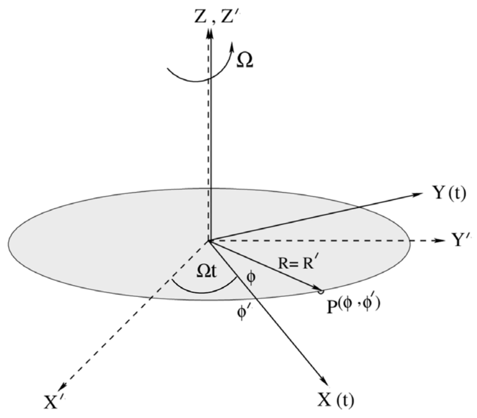
- Thus in general, rotation tends to suppress $\sigma, \Delta, \pi, \ldots\ldots$
- Compare with magnetic catalysis (dimensional reduction)





Rotating fermions

- Consider a rotating frame



$$\begin{cases} x' = x\cos\Omega t - y\sin\Omega t \\ y' = x\sin\Omega t + y\cos\Omega t \\ z' = z \\ t' = t \end{cases}$$

$$g_{\mu\nu} = \begin{pmatrix} 1-\Omega^2 r^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Fermion field

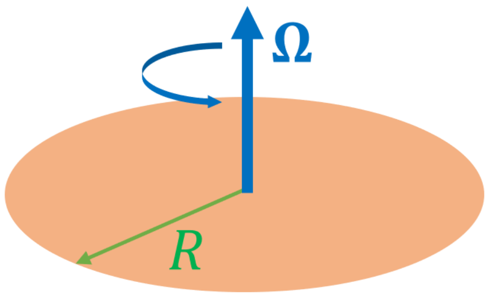
$$S = \int d^4x\sqrt{-g}\bar{\psi}\left(i\gamma^\mu\nabla_\mu - m_0\right)\psi$$

$$\nabla_\mu = \partial_\mu + i\hat{Q}A_\mu + \Gamma_\mu$$

$$H = \hat{Q}A_0 + m_0\beta + \boldsymbol{\alpha}\cdot\boldsymbol{\pi} - \boldsymbol{\Omega}\cdot(\mathbf{r}\times\boldsymbol{\pi} + \boldsymbol{\Sigma})$$

Rotating fermions

- Uniformly rotating system must be finite



$\boldsymbol{\Omega R \leq 1}$

- Boundary conditions for Dirac fermions in a cylinder
  - Dirichlet B.C. (No)
  - MIT B.C. (Yes)

$$[i\gamma^\mu n_\mu(\theta) - 1]\psi\Big|_{r=R} = 0 \quad \Rightarrow \quad j^\mu n_\mu = 0 \quad \text{at} \quad r = R$$

  - No-flux B.C. (Yes)

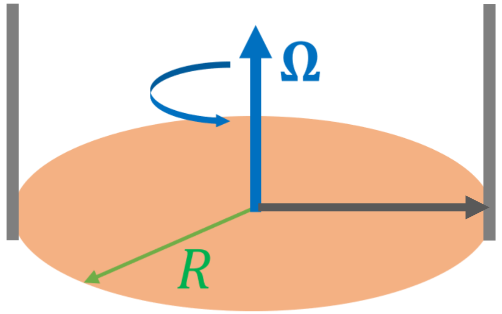
$$\int d\theta \bar{\psi}\gamma^r\psi\Big|_{r=R} = 0 \quad \Rightarrow$$

Minimum request for Hermiticity

(Ambrus-Winstanley 2015, Ebihara-Fukushima-Mameda 2016, Chen etal 2017, Chernodub-Gongyo 2016-2017)

Rotating fermions

- Consider no-flux B.C.

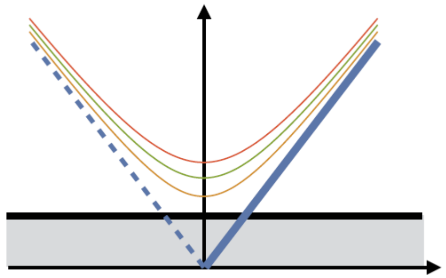


- $p_t = p_{l,k}$ discretized by $J_l(p_{l,k}R) = 0$
- $E = (p_{l,k}^2 + p_z^2 + m^2)^{1/2} > \Omega|l + \frac{1}{2}|$
- Vacuum does not rotate

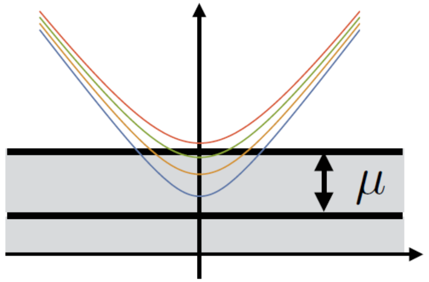
(Vilenkin 1979, Ebihara-Fukushima-Mameda 2016)

- To see uniform rotation effect, we need $\boldsymbol{T}$, $\boldsymbol{\mu}$, $\boldsymbol{B}$, ……

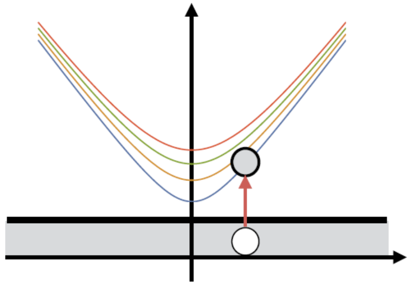
Figures drawn by K.Mameda



B: Chen etal 2015, Liu-Zahed 2017,Chen-Mameda-XGH 2019, Cao-He 2019, Tabatabaee etal 2021…



μ: XGH-Nishimura-Yamamoto 2017, Zhang-Hou-Liao 2018, Huang etal 2018, Nishimura etal 2020,2021…



T: Jiang-Liao 2016, Chernodub-Gongyo 2017, Wang etal 2019, Luo etal 2020, Jiang 2021, …

Rotating Nambu-Jona-Lasinio model

- Take a four-fermion model

$$Z = \int \mathcal{D}[\bar{\psi}, \psi] \exp\left(i \int d^4x \sqrt{-g} \mathcal{L}_{\rm NJL} \right)$$

$$\mathcal{L}_{\rm NJL} = \bar{\psi}(i\gamma^\mu \nabla_\mu - m_0)\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma^5 \boldsymbol{\tau} \psi)^2]$$

$$\nabla_\mu = \partial_\mu + i\hat{Q}A_\mu + \Gamma_\mu$$

- Mean-field approximation

$$V_{\rm eff} = \frac{1}{\beta V} \int d^4x_E \left\{ \frac{\sigma^2 + \boldsymbol{\pi}^2}{2G} - \sum_{\{\xi\}} \left[\frac{\varepsilon_{\{\xi\}}}{2} + \frac{1}{\beta} \ln(1 + e^{-\beta \varepsilon_{\{\xi\}}}) \right] \Psi^\dagger_{\{\xi\}} \Psi_{\{\xi\}} \right\}$$

$\varepsilon_{\{\xi\}}$ and $\Psi_{\{\xi\}}$: Eigen-energy and eigen-wavefunction with quantum numbers $\{\xi\}$

Rotating Nambu-Jona-Lasinio model

- Consider a simple case: massless, no pion modes, homogeneous

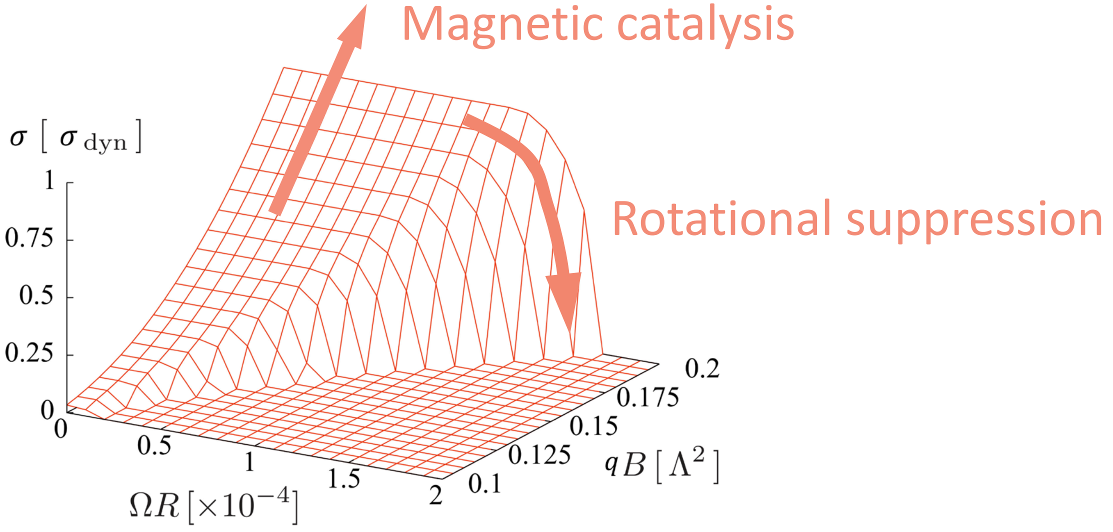
$$\varepsilon_{l,\pm} = \pm\sqrt{p_z^2 + p_t^2 + \sigma^2} - \Omega\left(l + \frac{1}{2}\right)$$

Ration

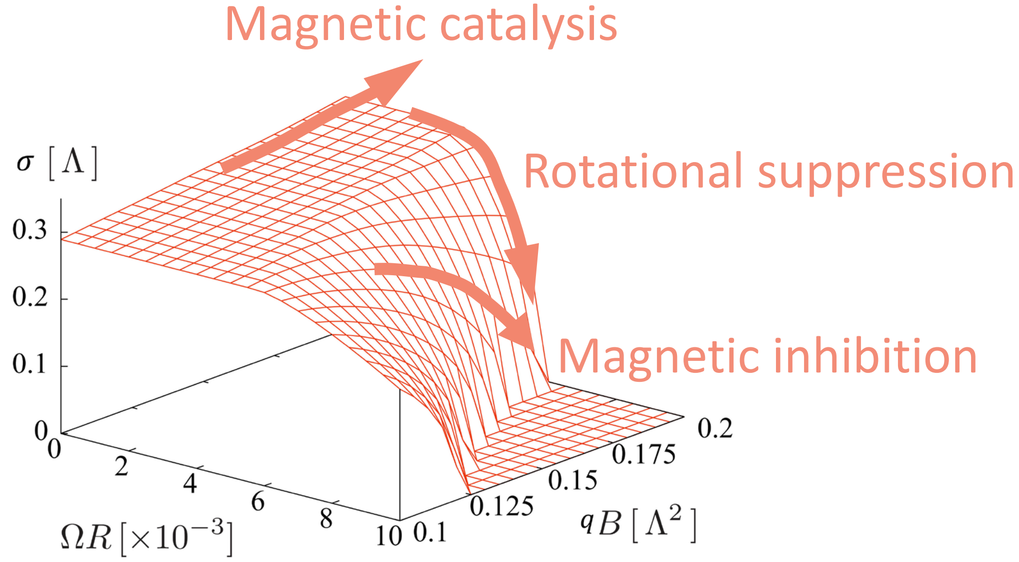
$$\varepsilon_{n,\pm} = \pm\sqrt{p_z^2 + \sigma^2 + 2nqB}$$

Magnetic field

- Chiral condensate vs rotation and/or magnetic field



$G < G_c$



$G > G_c$

(Chen-Fukushima-XGH-Mameda 2015)

Rotating Nambu-Jona-Lasinio model

- Consider a simple case: massless, no pion modes, homogeneous

$$\varepsilon_{l,\pm} = \pm\sqrt{p_z^2 + p_t^2 + \sigma^2} - \Omega\left(l + \frac{1}{2}\right)$$

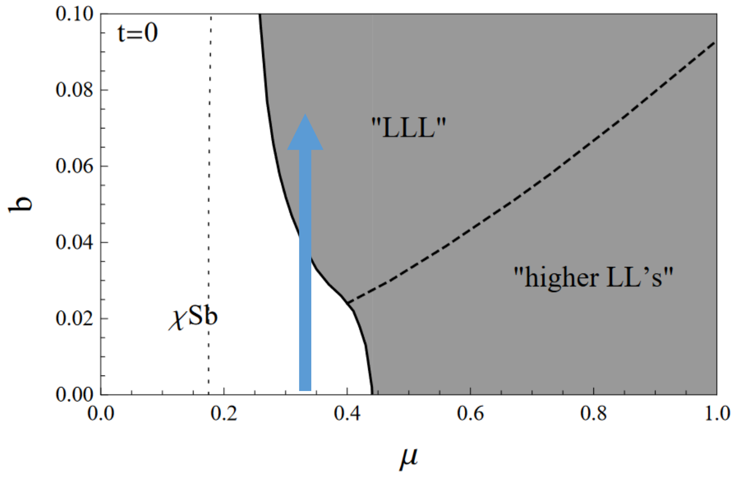
μ

Ration

$$\varepsilon_{n,\pm} = \pm\sqrt{p_z^2 + \sigma^2 + 2nqB}$$

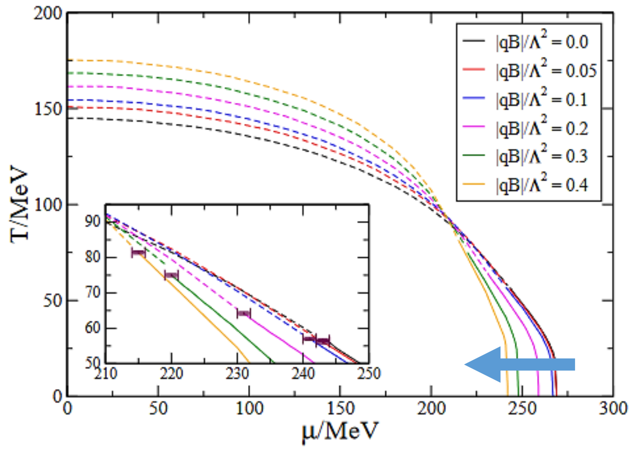
Magnetic field

- Compare with finite-density case:



Sakai-Sugimoto model

(Freis-Rebhan-Schmitt 2010)

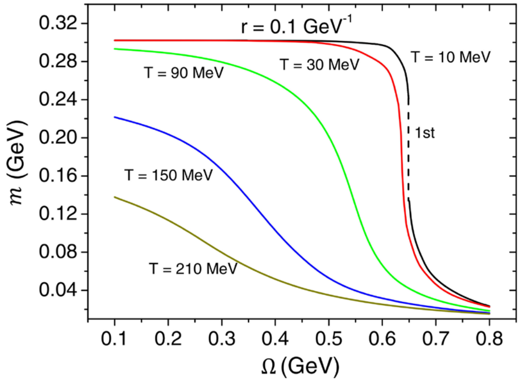


Quark-meson model

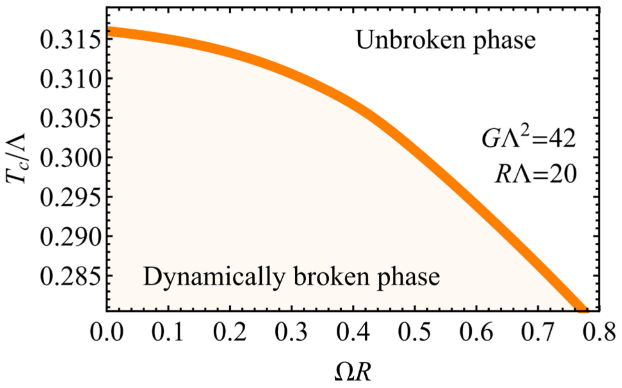
(Andersen-Tranberg 2012)

Rotating Nambu-Jona-Lasinio model

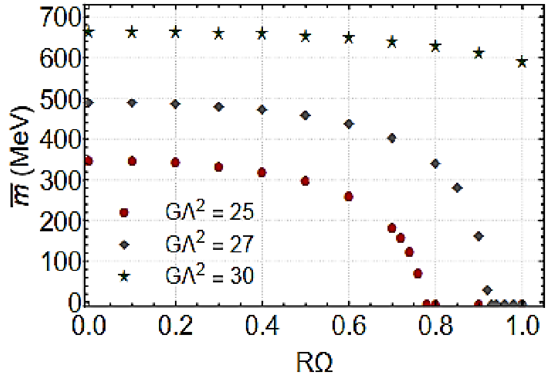
- Many model studies support that rotation suppresses chiral condensate



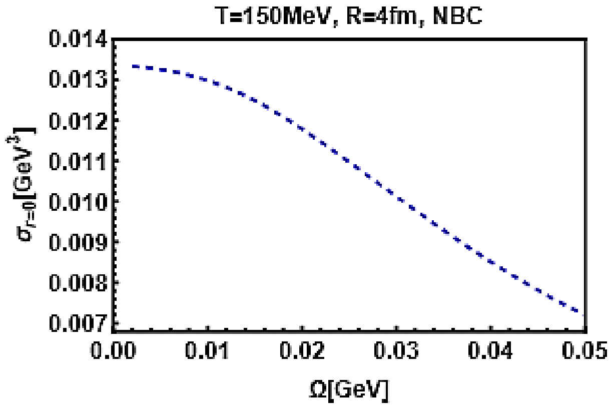
(Jiang-Liao 2016)



(Chernodub-Gongyo 2016)



(Sadooghi-Mehr-Taghinavaz 2022)

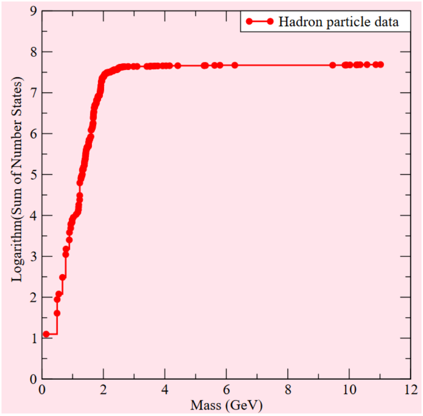


(Chen-Li-Huang 2022)

Can rotation affect confinement?

Confinement under rotation

- It is not easy to intuitively imagine the rotational effect on confinement
- Argument based on hadron resonance gas (HRG) model



$$\rho(m) = e^{m/T_H} \Rightarrow Z = \int dm\, \rho(m)\, e^{-m/T} \Rightarrow \text{diverges for } T > T_H$$

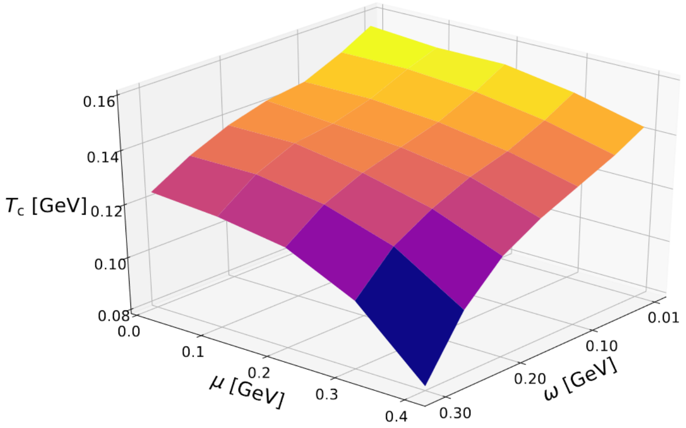
Interpreted as deconf. T

$$p(T,\mu,\omega;\Lambda) = \sum_{m;\, M_i \le \Lambda} p_m + \sum_{b;\, M_b \le \Lambda} p_b$$

$$p_{\mathrm{SB}} \equiv (N_c^2 - 1)\, p_g + N_c N_f\, (p_q + p_{\bar{q}})$$

$$\frac{p}{p_{\mathrm{SB}}}(T_c,\, \mu,\, \omega) = \gamma$$

Chosen to be indep. of rotation



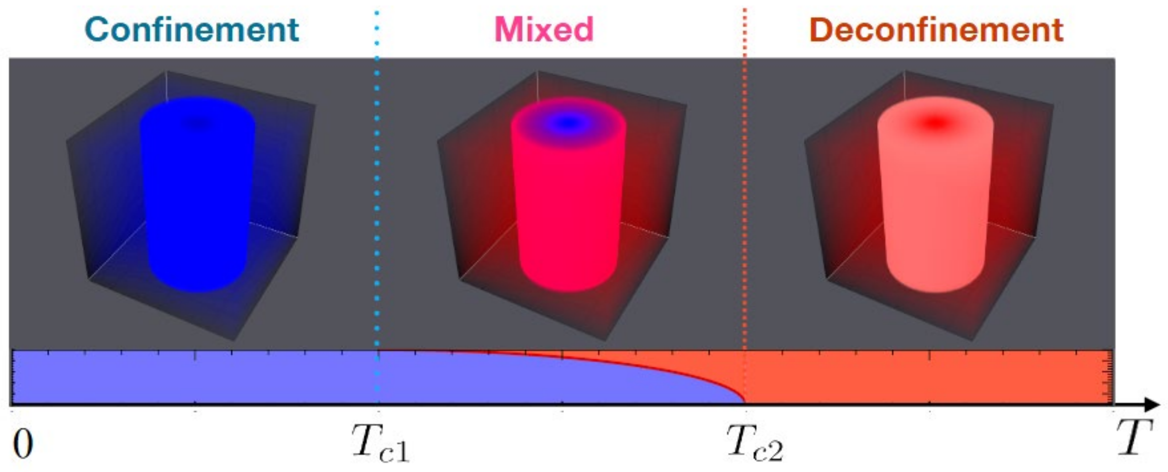
(Fujimoto-Fukushima-Hidaka 2021)

- Rotation favors deconfinement

Confinement under rotation

- It is not easy to intuitively imagine the rotational effect on confinement
- Argument based on Tolman-Ehrenfest temperature

$$\left.\begin{aligned} T(\boldsymbol{x})\sqrt{g_{00}(\boldsymbol{x})} &= T_0 \\ g_{00} &= 1-\rho^2\Omega^2 \end{aligned}\right\} \quad T(\rho) = \frac{T(0)}{\sqrt{1-\rho^2\Omega^2}}$$

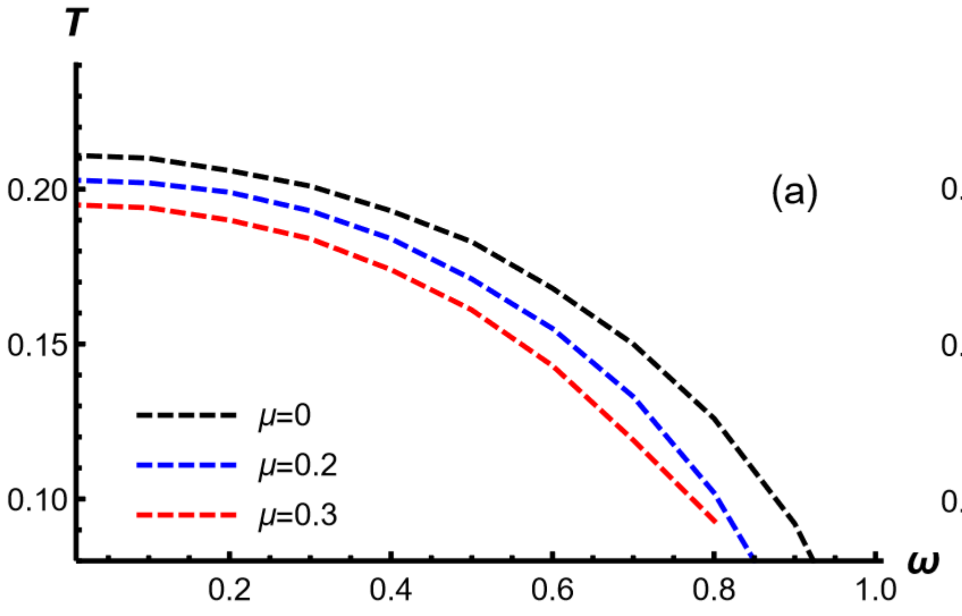


(Chernodub 2020)

- Rotation favors deconfinement

Confinement under rotation

- It is not easy to intuitively imagine the rotational effect on confinement
- Results based on holography

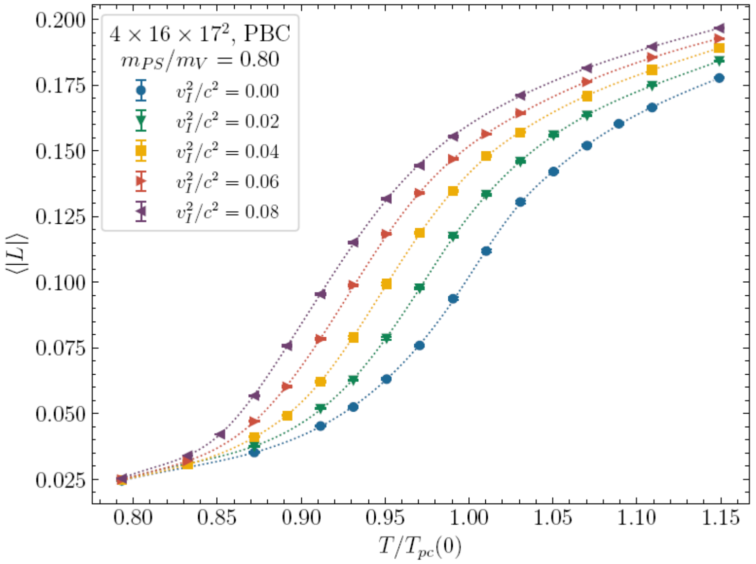


(Chen-Zhang-Li-Hou-Huang 2020)

- Rotation favors deconfinement

Confinement under rotation

- It is not easy to intuitively imagine the rotational effect on confinement
- Results based on lattice simulation of gluons



Imaginary rotation

$$v_I = \Omega_I (N_s - 1) a$$

$$\Omega_I = i\Omega$$

$$\frac{T_{pc}(v_I)}{T_{pc}(0)} = 1 - B_2 \frac{v_I^2}{c^2} \quad \Longrightarrow \quad \frac{T_{pc}(v)}{T_{pc}(0)} = 1 + B_2 \frac{v^2}{c^2}$$

- ~~Pseudo-critical~~ [Critical] temperature ~~decreases~~ [increases] due to ~~imaginary~~ [real] rotation

(Braguta etal 2021)

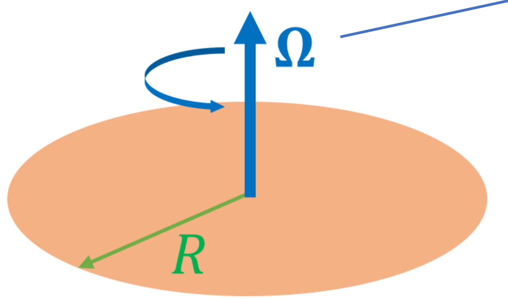
- Rotation disfavors deconfinement

Lattice calculation of rotating QCD

(Yang-XGH in preparation)

Formulate rotating lattice

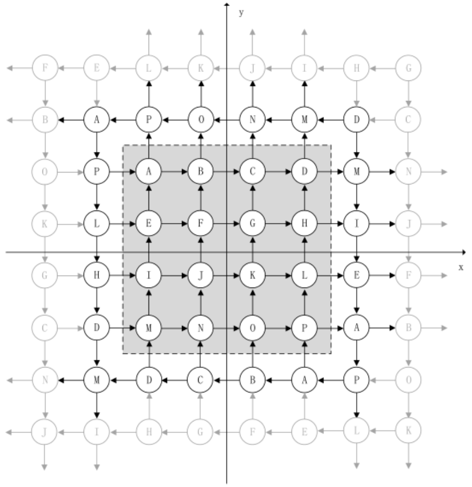
- Gluons and Wilson fermions (Angular momentum) (Yamamoto-Hirono 2013)
- Pure gluons (Polyakov loop) (Braguta etal 2021)
- We consider gluons and 2+1 flavor staggered fermions



Imaginary rotation: $\boldsymbol{\Omega} \rightarrow -i\boldsymbol{\Omega}$

No sign problem
No causality constraint

Projective-plane B.C for x-y plane
Periodic B.C. for t and z direction



- We measure: (imaginary) angular momentum

Ji decomposition

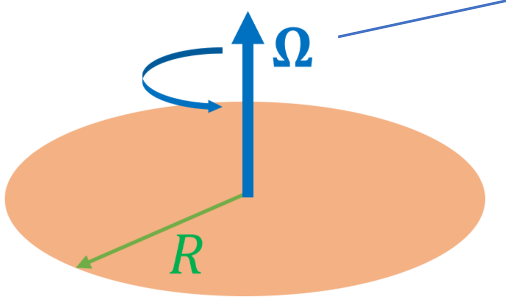
$$\mathbf{J} = \mathbf{J}_G + \mathbf{s}_q + \mathbf{L}_q$$

$$\begin{cases} \mathbf{J}_G = \sum_a \int d^3x \, \mathbf{r} \times (\mathbf{E}^a \times \mathbf{B}^a) \, , \\ \mathbf{s}_q = \int d^3x \, q^\dagger \frac{\boldsymbol{\Sigma}}{2} q, \\ \mathbf{L}_q = \frac{1}{i} \int d^3x \, q^\dagger \mathbf{r} \times \mathbf{D} q. \end{cases}$$

→ Chiral vortical effect

Formulate rotating lattice

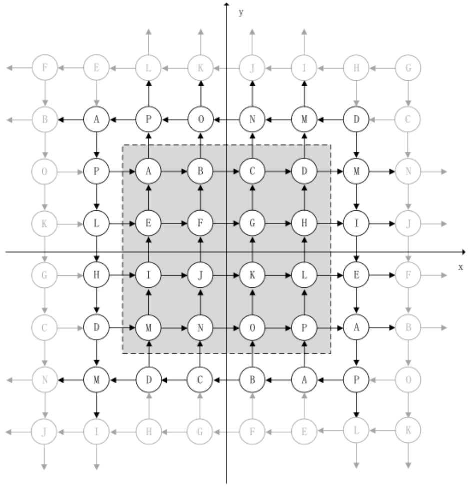
- Gluons and Wilson fermions (Angular momentum) (Yamamoto-Hirono 2013)
- Pure gluons (Polyakov loop) (Braguta etal 2021)
- We consider gluons and 2+1 flavor staggered fermions



Imaginary rotation: $\Omega \to -i\Omega$

No sign problem
No causality constraint

Projective-plane B.C for x-y plane
Periodic B.C. for t and z direction



- We measure: chiral condensate and Polyakov loop

$$\Delta_{l,s}(T,\Omega) = \frac{\langle\bar{\psi}_l\psi_l\rangle_{T,\Omega} - \frac{m_l}{m_s}\langle\bar{\psi}_s\psi_s\rangle_{T,0}}{\langle\bar{\psi}_l\psi_l\rangle_{0,0} - \frac{m_l}{m_s}\langle\bar{\psi}_s\psi_s\rangle_{0,0}}$$

$$L_{ren} = \exp(-N_\tau c(\beta) a/2) L_{bare}$$
$$L_{bare} = \mathrm{tr}\left[\sum_{\mathbf{n}} \prod_\tau U_\tau(\mathbf{n},\tau)\right] / 3N_x^3$$

Preliminary results

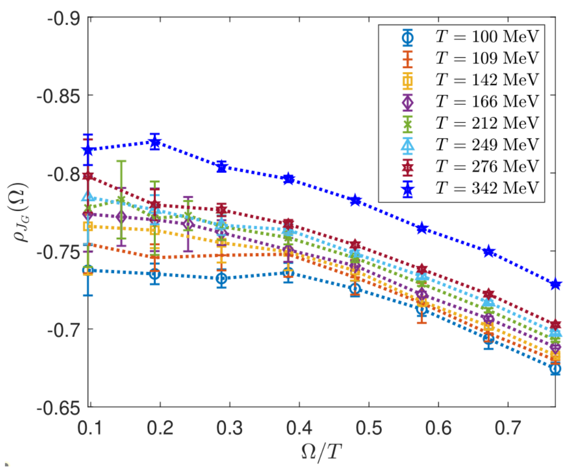
- Angular momentum
- J_G and L_q approximately $\propto r^2$, and s_q approximately independent of r, thus

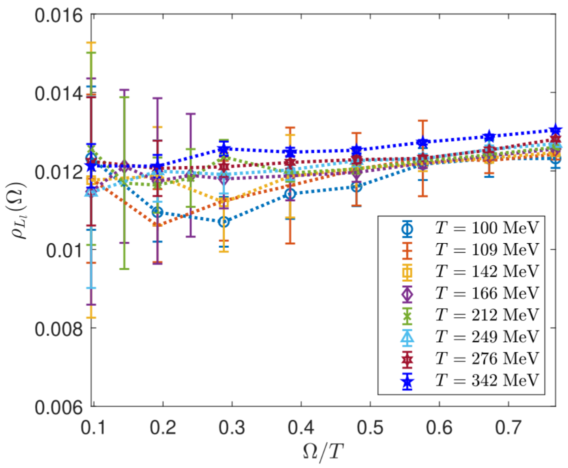
$$\rho_J = \frac{1}{N_{taste} N_{r_{max}}} \sum_{n_x^2+n_y^2<r_{max}^2} \frac{\langle J(n) \rangle}{a\Omega (a^{-1}r)^2}$$

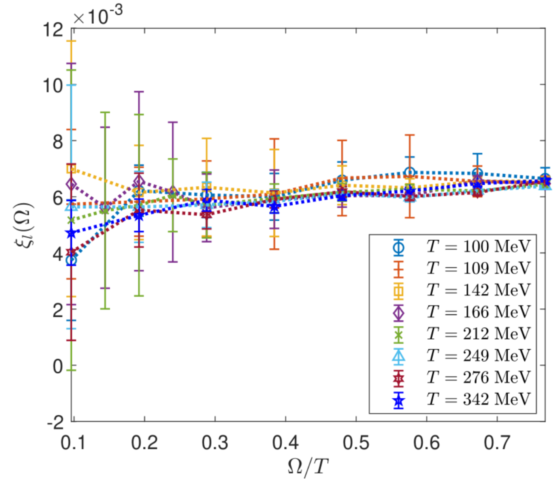
Moment of inertia

$$\xi_q = \frac{1}{4N_{r_{max}}} \sum_{n_x^2+n_y^2<r_{max}^2} \frac{\langle s_q(n) \rangle}{a\Omega}$$

Quark spin susceptibility

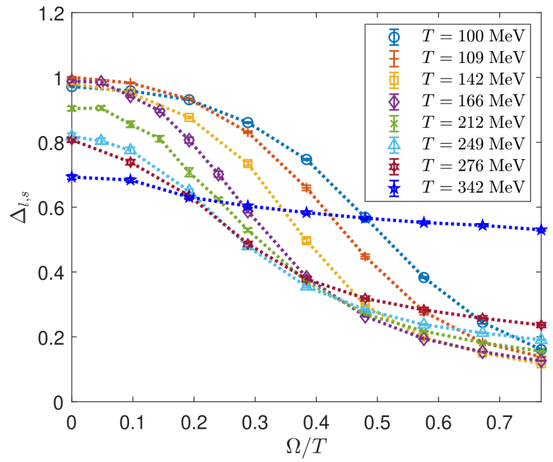


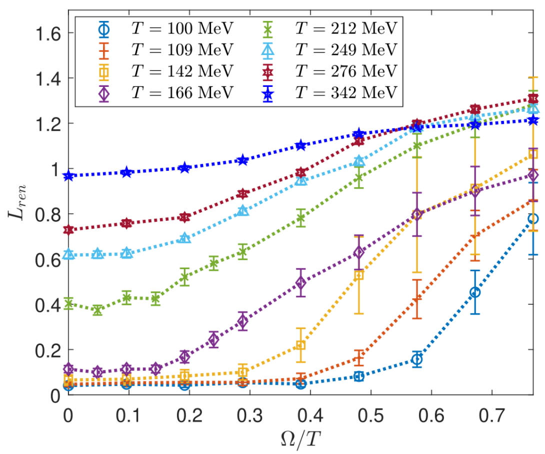




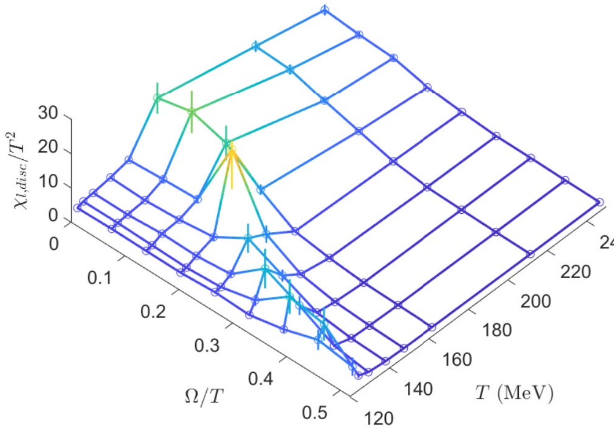
Preliminary results

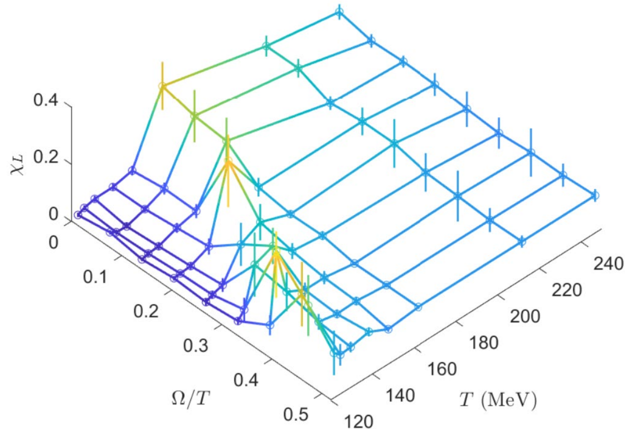
- Chiral condensate and Polyakov loop





- Chiral and Polyakov loop susceptibilities



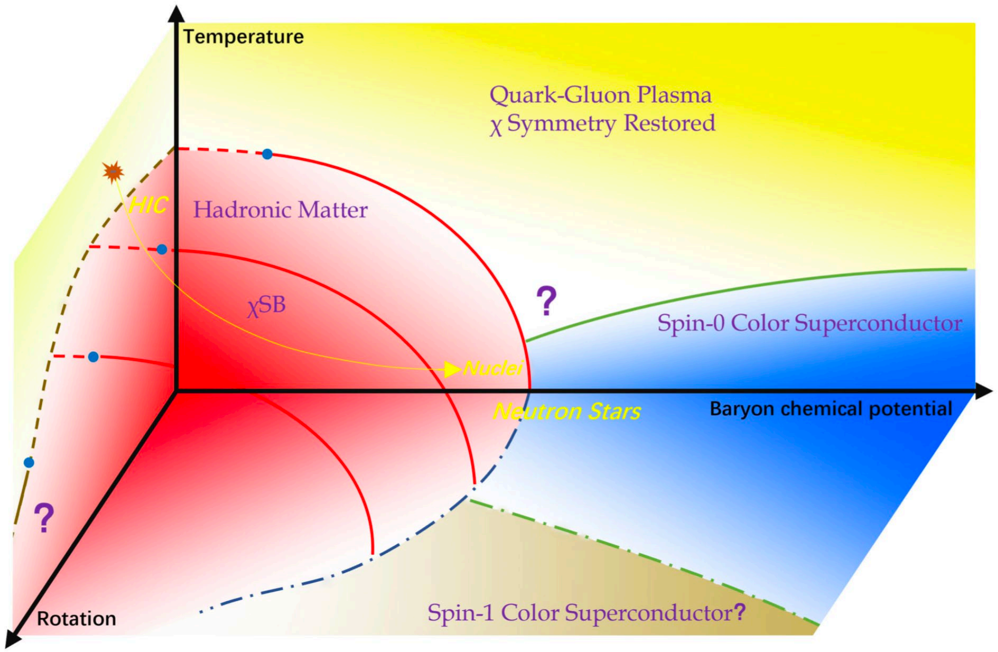


- Imaginary rotation tends to melt chiral condensate and deconfine the system
- The phase transition lines on $T-\Omega$ plane coincide
- Since they are even function of rotation, if we naively shift to real rotation, this implies rotational catalysis of chiral breaking and confinement
- Opposite to effective models

Summary and outlooks

Summary and outlooks

- It is NOT understood how rotation modifies chiral and deconfinement phase transitions of QCD.
- Outlooks:
  - More lattice simulations for imaginary rotation
  - Cross check torsion effect on chiral condensate and confinement on lattice (Yamamoto 2020)
  - Study of the analytical continuation from imaginary to real rotations (Chen-Fukushima-Shimada 2022)
  - More model studies
  - … …



Thank you!