UNIVERSITY OF HELSINKI

Soft light-cone observables from electrostatic QCD ⌚

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QCD theory seminar, 07/2022

⌚ J. Ghiglieri, G. D. Moore, P. Schicho, and N. Schlusser, *The force-force-correlator in hot QCD perturbatively and from the lattice,* JHEP **02** (2022) 58 [2112.01407]

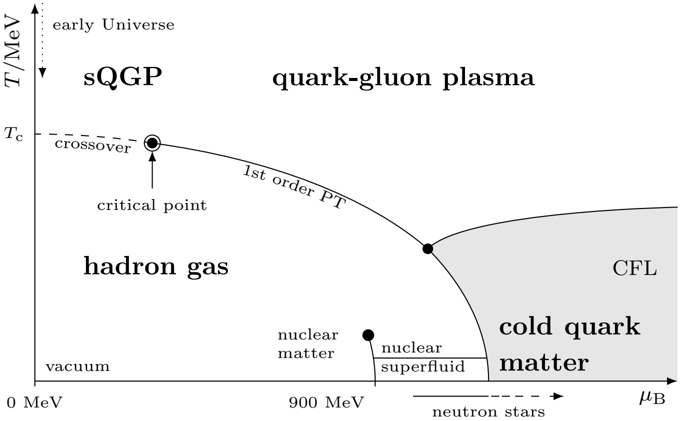
Motivation

What are the properties of the quark-gluon plasma (QGP)?

High-T: quark-gluon plasma in early universe or heavy-ion collision

High-μ: cold quark matter conjectured in neutron stars (NS)

Pressure (p) encodes bulk thermodynamics.



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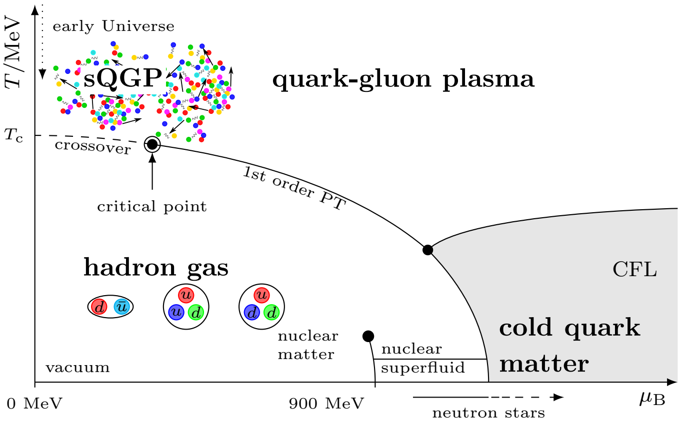
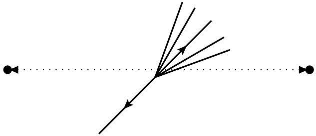
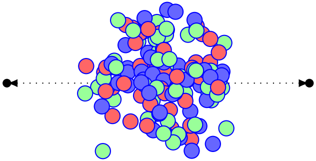


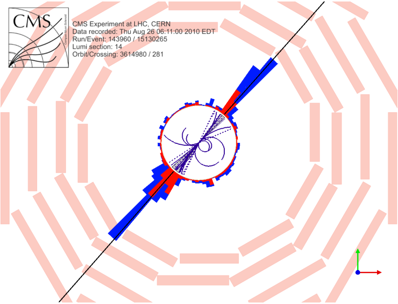
Probing the quark-gluon plasma with heavy-ion collisions

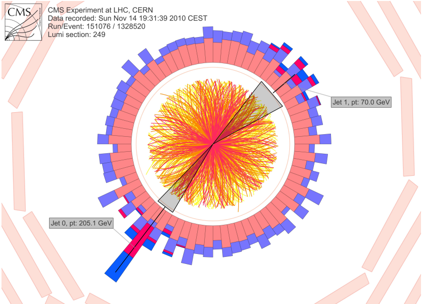
P + P-collisions (baseline)



Pb + Pb-collisions







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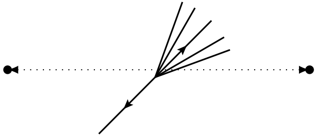
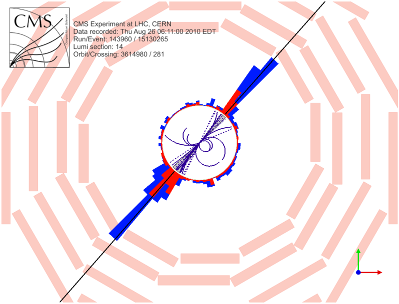
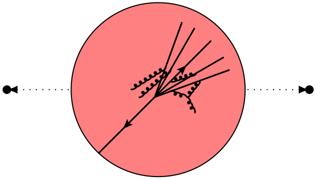
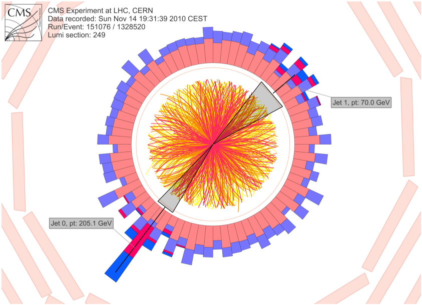




$Pb + Pb$-collisions





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QCD in thermal equilibrium

Preferred choice: First principle lattice methods. Fail at certain regimes:

- Intermediate chemical potential ($\uparrow \mu$)
- Implement chiral fermions on the lattice at finite T
- Incorporating hierarchy of scales

Near T_c non-perturbative modes dominate (show later) $\rightarrow$ Lattice.

In a weakly coupled electroweak theory at high T $\rightarrow$ Analytic methods.

$\Rightarrow$Interplay of both methods. Today: Weak-coupling.

High-temperature effective theories

Equilibrium Thermodynamics: Imaginary Time Formalism

$\rho(\beta) = e^{-\beta\mathcal{H}} \to \mathcal{U}(t) = e^{-i\mathcal{H}t}$. Relating density operator to time evolution corresponds to path integral over imaginary-time $t \to -i\tau$,

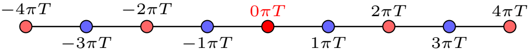
$$\mathcal{Z} = C \int_{\text{b.c.}} \mathcal{D}\phi \, \exp\left[- \int_0^{\beta=1/T} \mathrm{d}\tau \int_{\mathbf{x}} \mathcal{L}_{\mathrm{E}} \right] \,, \quad \phi(0,\mathbf{x}) = \pm\phi(\beta,\mathbf{x}) \,.$$

(Anti-)periodic bosonic(fermionic) fields at boundaries $\to$ compactified time direction: $\mathbb{R}^3 \times S^1_\beta$.

Finite-τ and (b.c.) induce a discrete Fourier sum for time component $P = (\omega_n, \mathbf{p})$ with Matsubara frequencies

$$\omega_n = \begin{cases} 2n\pi T & \text{bosonic} \\ (2n+1)\pi T & \text{fermionic} \end{cases}$$

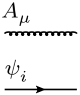
Absent zero mode $\omega_{n=0}$ for fermions:



Differences to zero temperature

$(d+1) \to d$-dimensional theory with infinite tower of massive modes.

Euclidean free-particle propagator for bosonic(fermionic) fields:



$$\left.\right\} \propto \frac{1}{P^2+m^2} = \frac{1}{\mathbf{p}^2+\omega_n^2+m^2}\,, \quad P = (\omega_n, \mathbf{p})\,.$$

Same diagrams as in zero temperature QFT, go over to Euclidean space, and substitute Euclidean frequency integrals by sums

$$\int \frac{\mathrm{d}^{d+1}p}{(2\pi)^{d+1}} f(p) \to T \sum_n \int \frac{\mathrm{d}^d p}{(2\pi)^d} f(\omega_n, \mathbf{p}) = \sum\!\!\!\!\!\!\!\int_P f(\omega_n, \mathbf{p})\,.$$

- Ultraviolet (UV) contained at $T=0$
- Infrared (IR) sensitivity worsened $\to$ field in reduced spacetime dimension

Multi-scale Hierarchy in hot gauge theories

Evaluate Matsubara sums yielding Bose(Fermi) distribution. At asymptotically high-T and weak $g \ll 1$ the effective expansion parameter

$$g^2 n_{\mathrm{B}}(|p|) = \frac{g^2}{e^{|p|/T} - 1} \approx \frac{g^2 T}{|p|}$$

differs from the weak coupling g^2. Fermions are IR-safe $g^2 n_{\mathrm{F}} |p| \sim g^2/2$.

Theory separates scales rigorously:

$$|p| \sim \begin{cases} \pi T & \textit{hard} \text{ scale} \\ gT & \textit{soft} \text{ scale} \\ g^2 T/\pi & \textit{ultrasoft} \text{ scale} \end{cases}$$

Limit: Confinement-like behavior in ultrasoft sector $g^2 n_{\mathrm{B}}(g^2 T) \sim \mathcal{O}(1)$. Light bosons are non-perturbative at finite T: Linde IR problem.[1]

[1] A. Linde, *Infrared problem in the thermodynamics of the Yang-Mills gas*, Phys. Lett. B **96** (1980) 289

Resummation

Dynamically generated masses through collective plasma effects

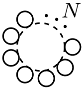
$$m_{T} = g^n T + m \ .$$

Evaluate Matsubara sums yielding Bose(Fermi) distribution. At asymptotically high-T and weak $g \ll 1$ the effective expansion parameter

$$g^2 n_{\mathrm{B}}(|p|) = \frac{g^2}{e^{|p|/T} - 1} \approx \frac{g^2 T}{|p|} \geq \frac{g^2 T}{m}$$

differs from the weak coupling g^2. Fermions are IR-safe $g^2 n_{\mathrm{F}} |p| \sim g^2/2$.

Cure IR sensittive contributions at $m_T \sim gT$ by thermal resummation:



$$\propto \quad g^{2N} \left[{m_{T}}^{3-2N} T \right] \left[\frac{T^2}{12} \right]^N \propto m^3 T \left[\frac{gT}{m_{T}} \right]^{2N}$$

For $m_T \leq g^2 T$ weak expansion breaks down. At finite T, light bosons are non-perturbative.

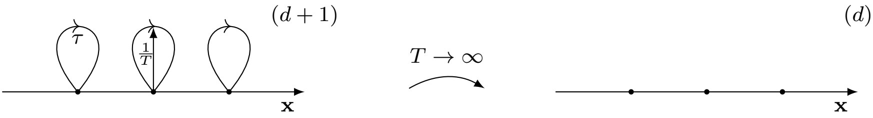
Effective Theory (EFT): Definition

Framework for theory with scale hierarchy: Effective Field Theory.

1. Identify soft degrees of freedom.
2. Construct most general low-energy Lagrangian.
3. *Match* Green's functions → determine EFT coupling constants.

Perturbative and IR safe: Matching in the IR (regime of mutual validity), UV incorporated in EFT coefficients.

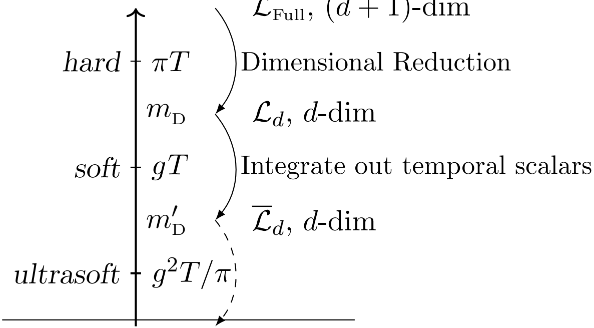
Modes with wavelengths $|\mathbf{x}|, |x_0| \gg \beta$ or $\omega_n^2 + m^2 \ll T^2$ *effectively* live in 3-dimensions.



Dimensional Reduction (DR)

Integrate out fast (hard) modes perturbatively → EFT for static modes.[2]

All-order thermal resummation to by-pass IR problem. Applied for thermodynamics of non-Abelian gauge theories such as (EW) phase transitions[3] and QCD.



[2] D. Bödeker, M. Sangel, and M. Wörmann, *Equilibration, particle production, and self-energy,* Phys. Rev. D **93** (2016) 045028 [1510.06742]

[3] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model,* Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379], K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis,* Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020]

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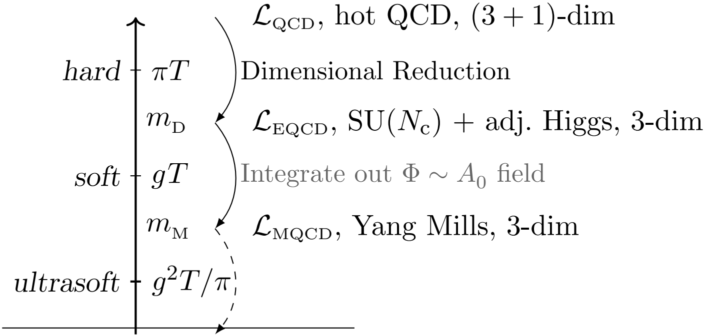

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Dimensionally reduced effective theory for hot QCD

QCD described by 3-dimensional super-renormalisable theory

$$S_{\rm EQCD} = \frac{1}{T}\int_{\mathbf{x}} \left\{ \mathcal{L}_{\rm EQCD} + \sum_{n\geq 5} \frac{\mathcal{O}_n}{(\pi T)^n} \right\} .$$

"Electrostatic QCD" (EQCD) at high T ($A_0^a \to \Phi^a$)

$$\mathcal{L}_{\rm EQCD} \equiv \frac{1}{2}{\rm Tr}\, F_{ij}F_{ij} + {\rm Tr}\,[D_i,\Phi][D_i,\Phi] + m_{\rm D}^2\,{\rm Tr}\,\Phi^2 + \lambda_{\rm E}({\rm Tr}\,\Phi^2)^2 \ ,$$

$D_i = \partial_i - i g_{\rm E} A_i$. Developed to study

- high-T thermodynamics,[4]
- soft light-cone observables.[5]

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[4] P. Ginsparg, *First and second order phase transitions in gauge theories at finite temperature,* Nucl. Phys. B **170** (1980) 388, T. Appelquist and R. D. Pisarski, *High-temperature Yang-Mills theories and three-dimensional quantum chromodynamics,* Phys. Rev. D **23** (1981) 2305

[5] S. Caron-Huot, *O(g) plasma effects in jet quenching,* Phys. Rev. D **79** (2009) 065039 [0811.1603], J. Ghiglieri, J. Hong, A. Kurkela, E. Lu, G. D. Moore, and D. Teaney, *Next-to-leading order thermal photon production in a weakly coupled quark-gluon plasma,* JHEP **2013** (2013) 10 [1302.5970]

EFT step 1: hot QCD → EQCD

DR step 1 fixes high-T EQCD. EFT for Electrostatic modes ($D_i = \partial_i - i g_{\rm E} A_i$), Describes hot QCD IR dynamics and contains UV in matching coefficients:[6]

$$g_{\rm E}^2 = \underbrace{\boxed{\begin{matrix}\text{tree-level}\\ T^2 g^2\end{matrix}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\begin{matrix}\text{1-loop}\\ \# g^4\end{matrix}}}_{\mathcal{O}(g^4)} + \underbrace{\boxed{\begin{matrix}\text{2-loop}\\ \# g^6\end{matrix}}}_{\mathcal{O}(g^6)} + \underbrace{\boxed{\begin{matrix}\text{3-loop}\\ \# g^8\end{matrix}}}_{\mathcal{O}(g^8)} + \mathcal{O}(g^{10}) \,,$$

$$m_{\rm D}^2 = \underbrace{\boxed{\text{tree-level} + \begin{matrix}\text{1-loop}\\ \# g^2 T^2\end{matrix}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\begin{matrix}\text{2-loop}\\ \# g^4\end{matrix}}}_{\mathcal{O}(g^4)} + \underbrace{\boxed{\begin{matrix}\text{3-loop}\\ \# g^6\end{matrix}}}_{\mathcal{O}(g^6)} + \mathcal{O}(g^8) \,,$$

$$\lambda_{\rm E} = \underbrace{\boxed{\text{tree-level}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\begin{matrix}\text{1-loop}\\ \# g^4\end{matrix}}}_{\mathcal{O}(g^4)} + \underbrace{\boxed{\begin{matrix}\text{2-loop}\\ \# g^6\end{matrix}}}_{\mathcal{O}(g^6)} + \mathcal{O}(g^8) \,.$$

[6] I. Ghişoiu, *Three-loop Debye mass and effective coupling in thermal QCD*, PhD thesis, Universität Bielefeld, Jan, 2013, I. Ghişoiu, J. Möller, and Y. Schröder, *Debye screening mass of hot Yang-Mills theory to three-loop order*, JHEP **2015** (2015) 121 [1509.08727], K. Kajantie, M. Laine, K. Rummukainen, and M. Shaposhnikov, *3d SU(N) + adjoint Higgs theory and finite temperature QCD*, Nucl. Phys. B **503** (1997) 357 [9704416]

EFT step 2: EQCD $\to$ MQCD

DR step 2 fixes high-T MQCD. EFT for Magnetostatic modes aka 3d pure Yang-Mills ($D_i = \partial_i - i g_{\mathrm{M}} A_i$). Describes EQCD IR dynamics and contains UV in matching coefficients:[7]

$$g_{\mathrm{M}}^2 = + \underbrace{\boxed{\begin{array}{c}\text{1-loop}\\ \# \frac{g_{\mathrm{E}}^4}{m_{\mathrm{D}}}\end{array}}}_{\mathcal{O}(g^3)} + \boxed{\begin{array}{c}\text{2-loop}\\ \# \frac{g_{\mathrm{E}}^6}{m_{\mathrm{D}}^2}\end{array}} + \boxed{\begin{array}{c}\text{3-loop}\\ \# \frac{g_{\mathrm{E}}^8}{m_{\mathrm{D}}^3}\end{array}} + \dots ,$$

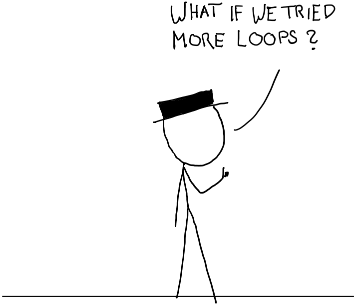
$$\mathcal{L}_{\mathrm{MQCD}} = \mathcal{L}_{\mathrm{3d\ Yang\text{-}Mills}} \equiv \frac{1}{2} \mathrm{Tr}\, F_{ij} F_{ij} \ .$$

[7] M. Laine and Y. Schröder, *Two-loop QCD gauge coupling at high temperatures*, JHEP **03** (2005) 067 [hep-ph/0503061], M. Laine, P. Schicho, and Y. Schröder, *Soft thermal contributions to 3-loop gauge coupling*, JHEP **2018** (2018) 37 [1803.08689]

Possible improvements:

▹ More precise computation of parameters.

Appelquist-Carazzone decoupling theorem[8] breaks down at finite-T[9].



[8] T. Appelquist and J. Carazzone, *Infrared singularities and massive fields,* Phys. Rev. D **11** (1975) 2856

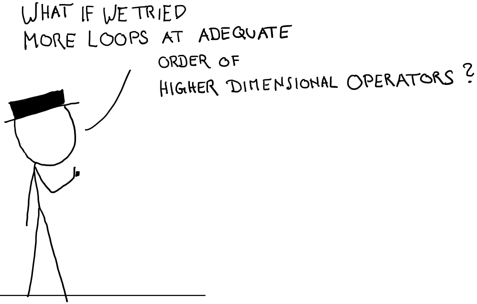
[9] N. Landsman, *Limitations to dimensional reduction at high temperature,* Nucl. Phys. B **322** (1989) 498

Possible improvements:

- More precise computation of parameters.
- Inclusion of higher-dimensional operators.

Appelquist-Carazzone decoupling theorem[8] breaks down at finite-T[9].

These are related!



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[9] N. Landsman, *Limitations to dimensional reduction at high temperature,* Nucl. Phys. B **322** (1989) 498

Dimension-six operators in EQCD

1-loop sum-integral yields finite contributions

$$\sum\!\!\!\!\!\!\int_P' \frac{1}{P^6} = \frac{\zeta_3}{128\pi^4 T^2}[1+\mathcal{O}(\epsilon)]\ .$$

Augment $\mathcal{L}_{\rm EQCD}$ by dim-6 operators[10] and colour trace in adjoint rep:[11]

$$\begin{aligned}\delta\mathcal{L}_{\rm EQCD}[A] = \left(\frac{2g_{\rm E}^2\zeta_3}{128\pi^4 T^2}\right) {\rm Tr}\Big\{ & c_1\,(D_\mu F_{\mu\nu})^2 + c_2\,(D_\mu F_{\mu 0})^2 \\ & + ig_{\rm E}\big[c_3\,F_{\mu\nu}F_{\nu\rho}F_{\rho\mu} + c_4\,F_{0\mu}F_{\mu\nu}F_{\nu 0} + c_5\,A_0(D_\mu F_{\mu\nu})F_{0\nu}\big] \\ & + g_{\rm E}^2\big[c_6\,A_0^2F_{\mu\nu}^2 + c_7\,A_0F_{\mu\nu}A_0F_{\mu\nu} + c_8\,A_0^2F_{0\mu}^2 + c_9\,A_0F_{0\mu}A_0F_{0\mu}\big] \\ & + g_{\rm E}^4\big[c_{10}A_0^6\big]\Big\}\ .\end{aligned}$$

Redundancies of coefficients leave physics invariant and are practical for cross-checks:

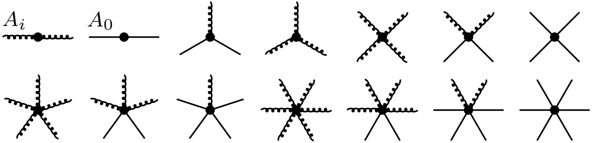
$$c_i^{\rm new} \equiv c_i + \delta c_i, \quad i = 4,\ldots,7\ .$$

[10] S. Chapman, *New dimensionally reduced effective action for QCD at high temperature,* Phys. Rev. D **50** (1994) 5308 [9407313]

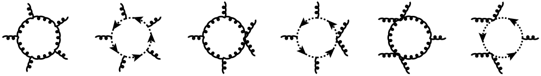
[11] ${\rm Tr}\,(AB) = A_{ab}B_{ba}$ with e.g. $(A_0)_{ab} = -if^{abc}A_0^c$ and $X^{abcd} = f^{m_4am_1}\cdots f^{m_3dm_4}$ etc.

Vertex structures and matching

$\mathcal{L}_{\text{EQCD}} + \delta\mathcal{L}_{\text{EQCD}}$ is non-super-renormalisable.



Determine coefficients $c_i(d)$ in d-dimensions in background field gauge[12]. Evaluate (2–6)-point vertices at one-loop order in hot YM → uniqueness.



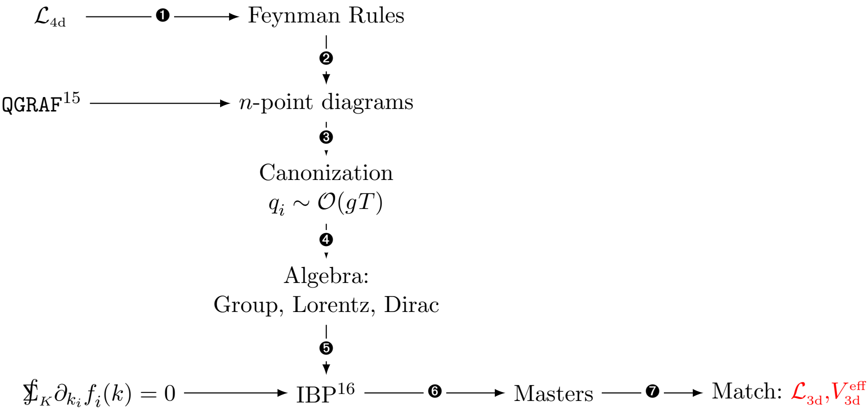
Done in d-dimensions.[13]

[12] L. Abbott, *The background field method beyond one loop,* Nucl. Phys. B **185** (1981) 189

[13] M. Laine, P. Schicho, and Y. Schröder, *Soft thermal contributions to 3-loop gauge coupling,* JHEP **2018** (2018) 37 [1803.08689]

Dimensional Reduction automated (DRalgo)

State-of-the-art **Mathematica** package **DRalgo**[14] for BSM and QCD.
Supply model Lagrangian $\mathcal{L}_{4\mathrm{d}}$:



[14] A. Ekstedt, P. Schicho, and T. V. I. Tenkanen, DRalgo: *a package for effective field theory approach for thermal phase transitions*, [2205.08815]

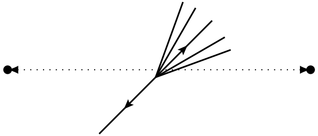
[15] P. Nogueira, *Automatic Feynman Graph Generation*, J. Comput. Phys. **105** (1993) 279

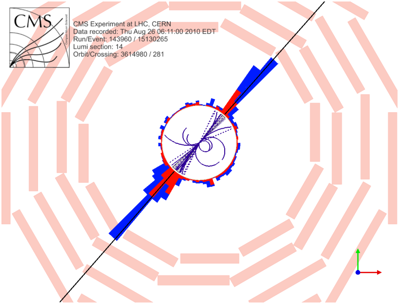
[16] S. Laporta, *High precision calculation of multiloop Feynman integrals by difference equations*, Int. J. Mod. Phys. A **15** (2000) 5087 [hep-ph/0102033]

Jet modifications in the QGP: Jet dispersion

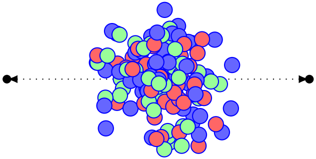
Probing the quark-gluon plasma with heavy-ion collisions

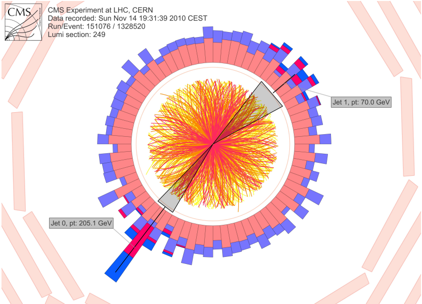
$P + P$-collisions (baseline)





$Pb + Pb$-collisions





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Probing the quark-gluon plasma with heavy-ion collisions

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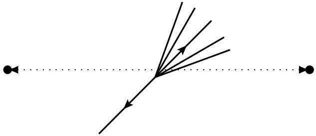

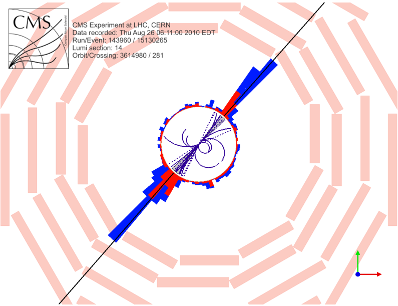

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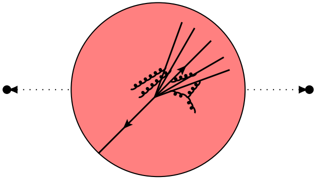

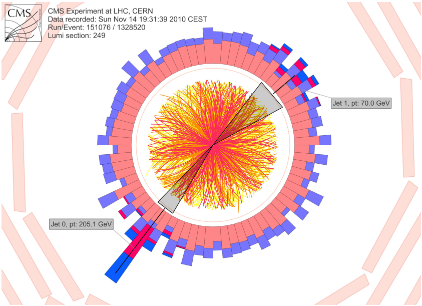

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Medium-induced radiation

Jet splitting rates in BDMPS-Z[17] formalism; $(a \to b + c)$:

$$\frac{\mathrm{d}P^a_{bc}}{\mathrm{d}k} \sim \mathrm{Re} \int_{t_1<t_2} \mathrm{d}t_1 \mathrm{d}t_2\, \nabla_{\boldsymbol{b}_1} \cdot \nabla_{\boldsymbol{b}_2} \left[\mathcal{K}(t_2, \boldsymbol{b}_2; t_1, \boldsymbol{b}_1)|_{\boldsymbol{b}_2=\boldsymbol{b}_1=0} - (\text{vac.}) \right] ,$$

where $\mathcal{K}$ is the Green's function of the Hamiltonian

$$\mathcal{H} = \boxed{-\frac{p\,\nabla^2_{\boldsymbol{b}}}{2k(p-k)} + \sum_i \frac{m_i^2}{2E_i}} - \boxed{i\mathcal{C}_3} ,$$

that encodes transverse diffusion.

Real part: Kinetic term for transverse propagation in the medium with in-medium masses m_i.

Imaginary part: Interaction with the medium; scattering kernel.

[17] R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne, and D. Schiff, *Radiative energy loss and p(T) broadening of high-energy partons in nuclei,* Nucl. Phys. B **484** (1997) 265 [hep-ph/9608322], B. G. Zakharov, *Radiative energy loss of high-energy quarks in finite-size nuclear matter and quark-gluon plasma,* J. Exp. Theor. Phys. Lett. **65** (1997) 615 [9704255]

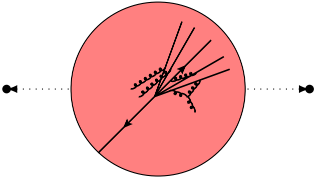
Jet modifications are infrared sensitive

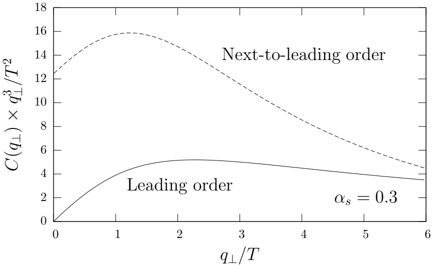
$$g^2 n_{\mathrm{B}}(p) \approx g^2 T/p \qquad \left[g^2 n_{\mathrm{F}}(p) \approx g^2/2\right]$$

$p \sim T$ Hard particles carry most of the stress-energy tensor.

$p \sim gT$ Classical soft medium modes.

Jet-medium interactions in the quark-gluon plasma (QGP) can have large non-perturbative IR contributions.[18]





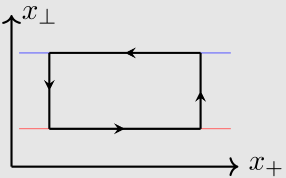
[18] S. Caron-Huot, *O(g) plasma effects in jet quenching*, Phys. Rev. D **79** (2009) 065039 [0811.1603]

Important quantities

Collision kernel

$$C(q_\perp) = \frac{\mathrm{d}\Gamma}{\mathrm{d}^2 q_\perp \mathrm{d}L}$$

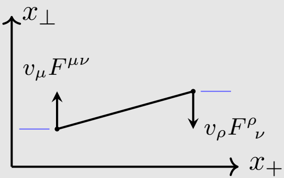
Wilson loop[19]



Asymptotic masses

$$m_\infty^2 = C_\mathrm{R}(Z_\mathrm{g} + Z_\mathrm{f})$$

Force-force-correlator[20]



Time-independent and Euclidean Gluon zero modes.[21] Calculate non-perturbative contributions in lattice **electrostatic QCD** (EQCD).

[19] J. Casalderrey-Solana and D. Teaney, *Transverse Momentum Broadening of a Fast Quark in a N=4 Yang Mills Plasma*, JHEP **04** (2007) 039 [hep-th/0701123]

[20] E. Braaten and R. D. Pisarski, *Simple effective Lagrangian for hard thermal loops*, Phys. Rev. D **45** (1992) R1827

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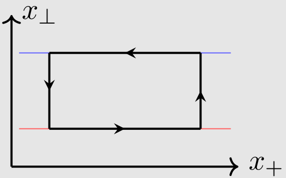

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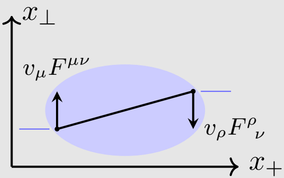

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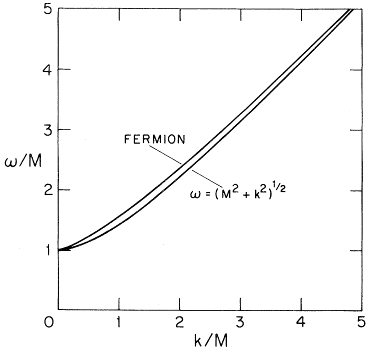
Asymptotic masses

Integrate out jet energy scale $E \gg T$.
Truncate $\frac{T}{E}$-series: LO correlators[22]

$$m_\infty^2 = C_\mathrm{R}(Z_\mathrm{g} + Z_\mathrm{f})$$

$$Z_\mathrm{f} \equiv \frac{1}{2d_\mathrm{R}} \left\langle \overline{\psi} \frac{v_\mu \gamma^\mu}{v \cdot D} \psi \right\rangle$$

$$Z_\mathrm{g} \equiv -\frac{1}{d_\mathrm{A}} \left\langle v_\mu F^{\mu\nu} \frac{1}{(v \cdot D)^2} v_\rho F^\rho{}_\nu \right\rangle$$



[22] E. Braaten and R. D. Pisarski, *Simple effective Lagrangian for hard thermal loops,* Phys. Rev. D **45** (1992) R1827, S. Caron-Huot, *O(g) plasma effects in jet quenching,* Phys. Rev. D **79** (2009) 065039 [0811.1603]

figure by H. A. Weldon, *Effective Fermion Masses of Order gT in High Temperature Gauge Theories with Exact Chiral Invariance,* Phys. Rev. D **26** (1982) 2789

Condensates of the asymptotic masses

In QCD rewrite detour through the medium as[23]

$$Z_{\rm g} = -\frac{1}{d_{\rm A}} \int_0^{\infty} {\rm d}x^+ x^+ \Big\langle v_\mu F_a^{\mu\nu}(x^+)\, U_{\rm A}^{ab}(x^+;0)\, v_\rho F^{\rho}_{b\,\nu}(0) \Big\rangle \;,$$

and match also operator onto EQCD

$$Z_{\rm g}^{\rm 3d} = -\frac{4T}{d_{\rm A}} \int_0^{\infty} {\rm d}L\, L \Big(-\langle EE\rangle + \langle BB\rangle + i\langle EB\rangle \Big) \;.$$

Correlator splits into electro- and magneto-static contributions:

$$\langle EE\rangle \equiv \frac{1}{2}\Big\langle (D_x\Phi(L))^a\, \tilde{U}_{\rm A}^{ab}(L,0)\, (D_x\Phi(0))^b \Big\rangle \;,$$

$$\langle BB\rangle \equiv \frac{1}{2}\Big\langle F^a_{xz}(L)\, \tilde{U}_{\rm A}^{ab}(L,0)\, F^b_{xz}(0) \Big\rangle \;,$$

$$i\langle EB\rangle \equiv \frac{i}{2}\Big\langle (D_x\Phi(L))^a\, \tilde{U}_{\rm A}^{ab}(L,0)\, F^b_{xz}(0) \Big\rangle + [BE] \;.$$

[23] $U_{\rm A}(x^+;0)$ is an adjoint, light-like Wilson line.

EFT matching with full QCD

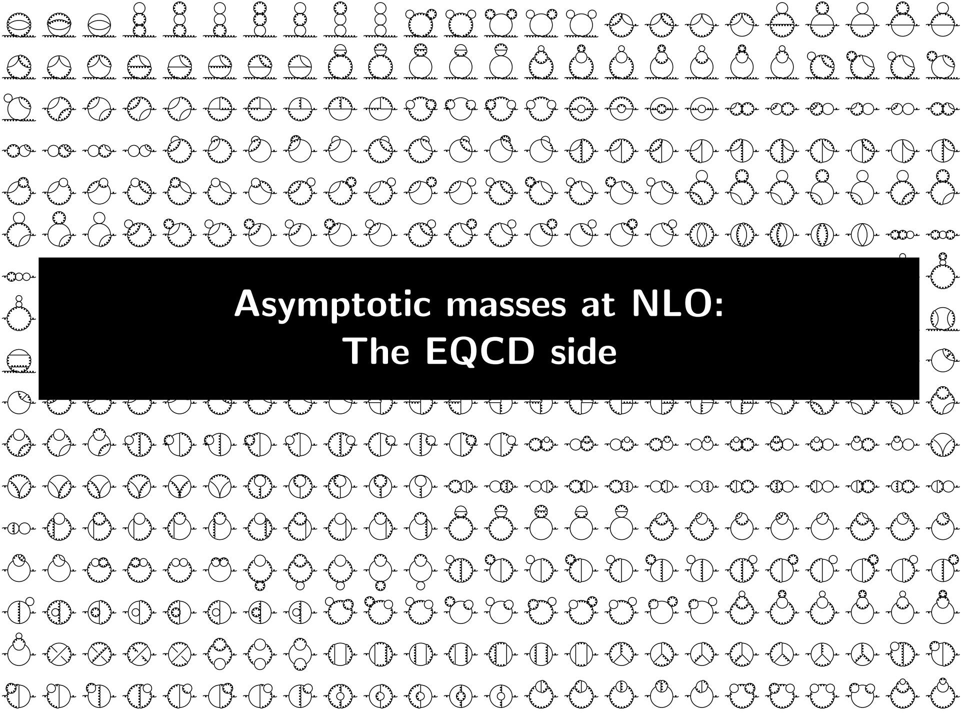
Strategy:

$$C_{\mathrm{QCD}}(x) = \underbrace{\boxed{C_{\mathrm{QCD}}(x) - C_{\mathrm{EQCD}}(x)}}_{\text{UV dominated}} + \underbrace{\boxed{C_{\mathrm{EQCD}}(x)}}_{\text{lattice}}$$

- Done[24] for $C(q_\perp)$.
- Partially done[25] for m_∞^2.
  - Missing full QCD contribution.

[24] P. Arnold and W. Xiao, *High-energy jet quenching in weakly coupled quark-gluon plasmas,* Phys. Rev. D **78** (2008) 125008 [0810.1026], J. Ghiglieri and H. Kim, *Transverse momentum broadening and collinear radiation at NLO in the* $\mathcal{N} = 4$ *SYM plasma,* JHEP **2018** (2018) 49 [1809.01349], G. D. Moore, S. Schlichting, N. Schlusser, and I. Soudi, *Non-perturbative determination of collisional broadening and medium induced radiation in QCD plasmas,* JHEP **10** (2021) 059 [2105.01679], S. Schlichting and I. Soudi, *Splitting rates in QCD plasmas from a non-perturbative determination of the momentum broadening kernel* $C(q_\perp)$*,* [2111.13731]

[25] J. Ghiglieri, G. D. Moore, P. Schicho, and N. Schlusser, *The force-force-correlator in hot QCD perturbatively and from the lattice,* JHEP **02** (2022) 58 [2112.01407]



Asymptotic masses at NLO: The EQCD side

$Z_{\rm g}$ receives IR contributions already at $\mathcal{O}(g)$[26]

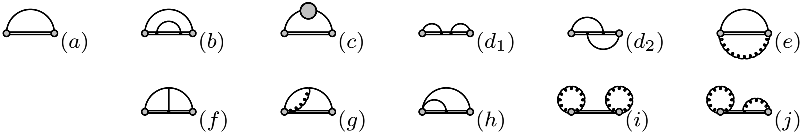
scale T (hard) scale gT (soft) scale g^2T (ultrasoft)

$$
\begin{aligned}
Z_{\rm g} = \Big[& \frac{T^2}{6} - \frac{T\mu_{\rm h}}{\pi^2} \Big] \\
+ \Big[& - \frac{Tm_{\rm D}}{2\pi} + \frac{T\mu_{\rm h}}{\pi^2} \Big] \\
+ \Big[& c^{\ln}_{\rm hard} \ln \frac{T}{\mu_{\rm h}} + c_T + c^{\ln}_{\rm hard} \ln \frac{\mu_{\rm h}}{m_{\rm D}} + c^{\ln}_{\rm soft} \ln \frac{m_{\rm D}}{\mu_{\rm s}} + c_{gT} + c^{\ln}_{\rm soft} \ln \frac{\mu_{\rm s}}{g^2 T} + c_{gT^2} \Big] \\
+ & \mathcal{O}(g^3) \; .
\end{aligned}
$$

Scheme-dependent at NLO; use intermediate regulators $T \gg \mu_{\rm h} \gg gT$ and $gT \gg \mu_{\rm s} \gg g^2 T$.

[26] S. Caron-Huot, *O(g) plasma effects in jet quenching,* Phys. Rev. D **79** (2009) 065039 [0811.1603]

Diagrams contributing at LO + NLO to the force-force correlator Z_g in EQCD:



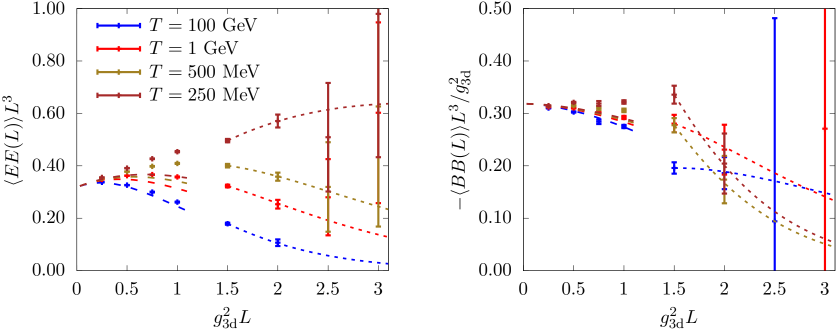
Example: LO colour-electric condensate $\langle EE \rangle$ – free solution

$$= 2 \times (a)^{\mathrm{EE}} = \partial_x \partial_{x'} \mathrm{Tr} \left\langle \Phi^a(x, L) \Phi^a(x', 0) \right\rangle \Big|_{x, x' \to 0}$$

$$= \frac{2 C_\mathrm{A} C_\mathrm{F}}{4\pi L^3} \epsilon^{-m_\mathrm{D} L} \left(1 + m_\mathrm{D} L\right)$$

Asymptotic masses (non-)perturbatively

Three different correlators contribute to $Z_{\mathrm{g}} \subset m_\infty^2$ in EQCD:



- small-L: NLO perturbative estimate
- large-L: Fit long L-tail to model[27]

[27] M. Laine and O. Philipsen, *Gauge-invariant scalar and field strength correlators in three dimensions*, Nucl. Phys. B **523** (1998) 267 [9711022]

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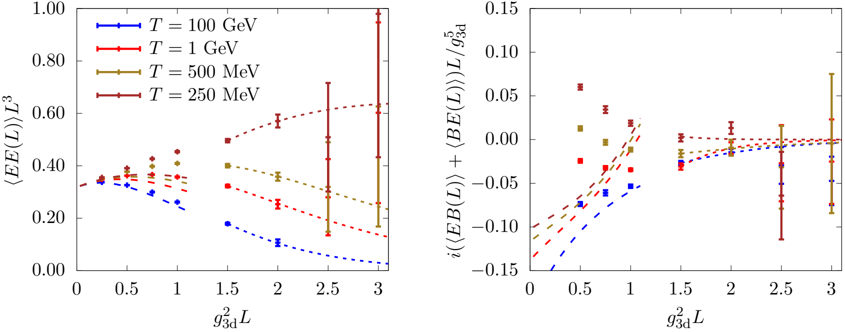

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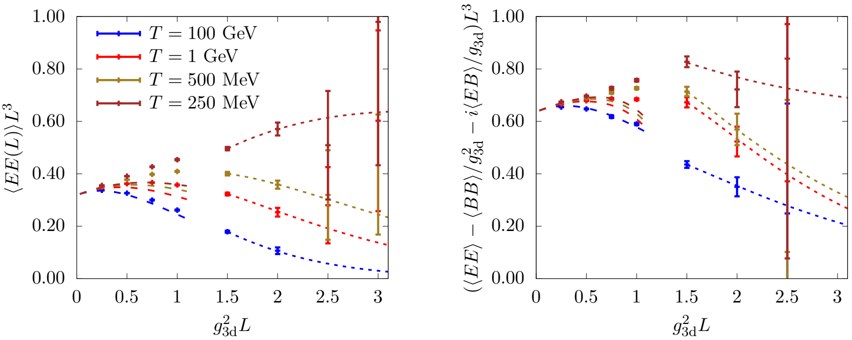

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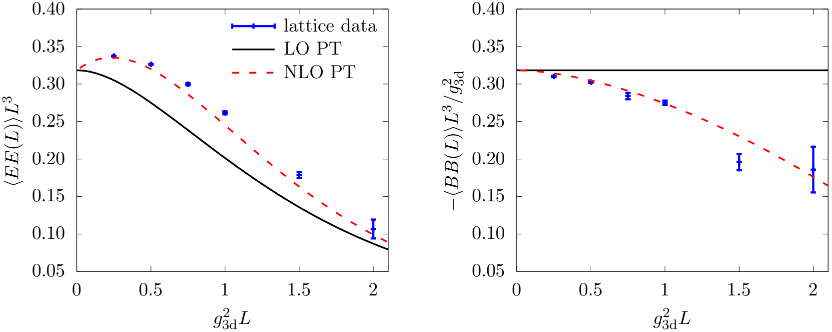
Asymptotic masses (non-)perturbatively

For $T = 100$ GeV and $N_{\rm f} = 5$, strong agreement between perturbative and non-perturbative $Z_{\rm g}$.



Conclusions

- Jet modifications (+other transport) involves soft IR QCD $\rightarrow$ (lattice) QCD
- Key quantities are $C(b_\perp)$ and asymptotic mass m_∞^2 from lattice EQCD

What's next for m_∞^2?

- ☆ Finalise matching computation to full QCD
- ☆ Jet splitting rates
- ☆ Input to effective kinetic theory AMY[28] $\rightarrow$ GMT[29]
- ☆ Ingredients for NNLO-transport
- ☆ Feed into event generator

[28] P. B. Arnold, G. D. Moore, and L. G. Yaffe, *Effective kinetic theory for high temperature gauge theories*, JHEP **01** (2003) 030 [hep-ph/0209353]

[29] J. Ghiglieri, G. D. Moore, and D. Teaney, *Jet-medium interactions at NLO in a weakly-coupled quark-gluon plasma*, JHEP **2016** (2016) 95 [1509.07773]