Torsion-induced chiral magnetic current in equilibrium

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(TA and Y. Nishida, arXiv:2204.13415)

Outline

1. Introduction
    - Chiral transport phenomena
    - Torsion

2. Torsion-induced chiral magnetic effect

3. Torsion-induced transport in Weyl semimetals

4. Summary

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Chiral transport phenomena

✓ Chiral transport phenomena

- occurs in **massless fermionic systems**
- relevant to the quantum anomaly

✓ Chiral magnetic effect (CME)[1]

$$\boldsymbol{j} = \frac{\mu_5}{2\pi^2}\boldsymbol{B}$$

Axial chemical potential

$$\mu_5 := (\mu_R - \mu_L)/2$$

✓ Chiral vortical effect (CVE)[2]

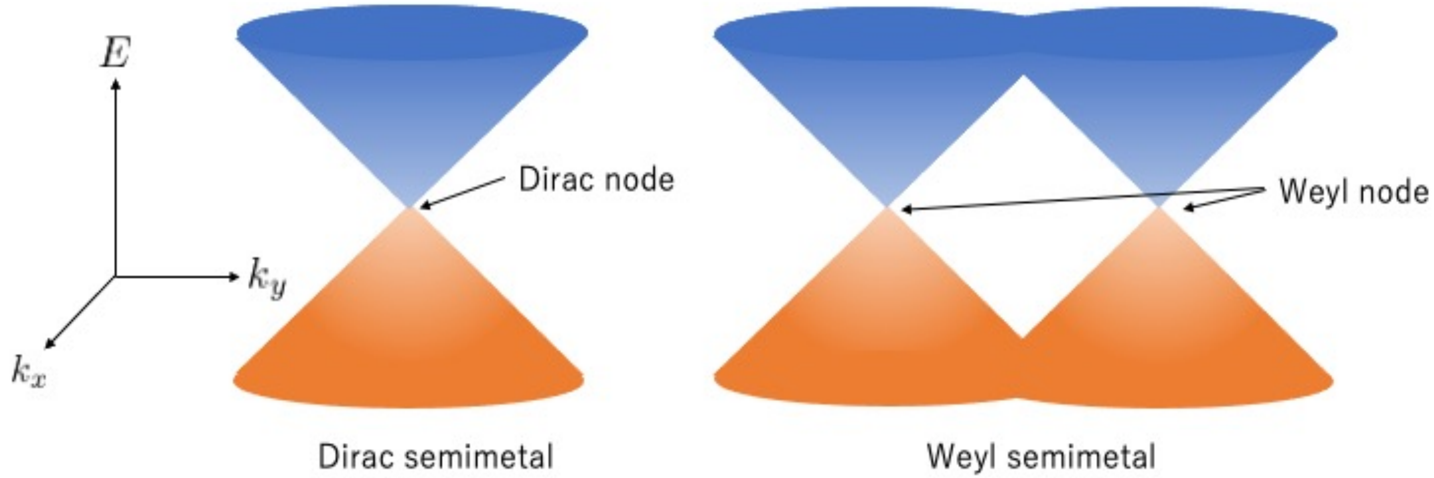
$$\boldsymbol{j} = \frac{\mu\mu_5}{\pi^2}\boldsymbol{\omega}$$

[1] K. Fukushima et al., PRD, 78, 074033 (2008)
[2] M. A. Stephanov and Y. Yin, PRL, 109, 162001 (2012)

Chiral transport phenomena

✓ Realization of the CME

- Quark-gluon plasmas
- Neutron stars
- **Dirac and Weyl semimetals**



➡ The chiral magnetic effect

has attracted wide interest

Chiral transport phenomena

The CME is absent in equilibrium.

= The CME cannot be activated simply

by applying the magnetic field.

✓ The way to activate the CME in WSMs

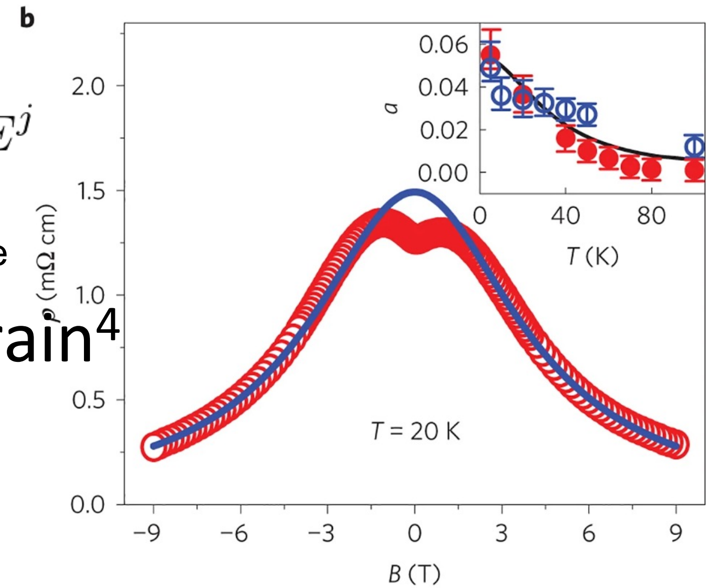
- To apply the electric field[3]

$$\delta\mu_5 \sim \frac{\tau}{\chi} C \boldsymbol{E}\cdot\boldsymbol{B} \Rightarrow j^i = \left(\sigma_{\mathrm{ohm}}\delta^{ij} + \frac{\tau}{\chi}C^2 B^i B^j\right) E^j$$

Negative magnetoresistance

- To distort the WSMs with the strain[4]

⇒ Another way?



Cited from Ref. [5]

[3] D. T. Son and B. Z. Spivak, PRB 88, 104412 (2013)
[4] A. Cortijo, et al., PRB 94, 241405(R) (2016)
[5] Q. Li et al., Nature Physics 12, 550 (2016)

Chiral transport phenomena

In this work, we activate the CME

by using a TORSION.

✓ **Torsion**

- Physical quantity introduced in the curved spacetime
- Torsion couples with the (3+1)d fermions as **an axial gauge field**[6].

➡ It is expected that the torsion plays the role of the axial chemical potential and induces the CME!

[6] I. L. Shapiro, PR 357, 113 (2002)

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Torsion

✓ Riemann geometry

- Torsion tensor

Curved spacetime is described by the metric $g_{\mu\nu}$

and the affine connection $\Gamma^{\lambda}{}_{\mu\nu}$.

A torsion is defined as $T^{\lambda}{}_{\mu\nu} := \Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu}$.

- Metric compatibility condition

$$\nabla_{\lambda} g_{\mu\nu} = 0$$

$$\Rightarrow \quad \Gamma^{\lambda}{}_{\mu\nu} = \underline{\mathring{\Gamma}^{\lambda}{}_{\mu\nu}} + \underline{K^{\lambda}{}_{\mu\nu}}$$

Levi-Civita connection   Contortion tensor

$$K_{\lambda\mu\nu} := \frac{1}{2}(T_{\mu\lambda\nu} + T_{\nu\lambda\mu} + T_{\lambda\mu\nu})$$

Torsion

✓ Cartan formalism

- Vielbein and spin connection

We can choose the basis as $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \eta_{ab}e^a e^b.$

$$\Rightarrow \quad e^a = e^a{}_\mu dx^\mu$$

$e^a{}_\mu$: **vielbein** (vierbein in 4d)

$$g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab}$$

We cannot determine the vielbein uniquely.

$$e^a_\mu \to e'^a_\mu = \Lambda^a{}_b e^b{}_\mu$$

$$\Lambda^a{}_c \Lambda^b{}_d \eta_{ab} = \eta_{cd}$$

$\Lambda^a{}_b$: Local Lorentz (LL) transf.

⇒ We introduce the gauge field for LL symmetry.

= **spin connection** $\omega_{ab\mu}$

Torsion

✓ Cartan formalism

- Affine connection

$$\Gamma^{\lambda}{}_{\mu\nu} = e_a{}^{\lambda}(\partial_\mu e^a{}_\nu + \omega^a{}_{b\mu} e^b{}_\nu) \Leftarrow \nabla_\mu e^a{}_\nu = 0$$

- Torsion

$$T^a_{\mu\nu} = \partial_\mu e^a{}_\nu + \omega^a{}_{b\mu} e^b{}_\nu - (\mu \leftrightarrow \nu)$$

We decompose the spin connection as

$$\omega_{ab\mu} = \underline{\overset{\circ}{\omega}_{ab\mu}} + \underline{K_{a\mu b}} \, .$$

Torsion-less spin connection　　Contortion tensor

$$\overset{\circ}{\omega}_{ab\mu} = \frac{1}{2} e^c{}_\mu (\gamma_{cab} - \gamma_{abc} - \gamma_{bca})$$

$$\gamma^c{}_{ab} := (e_a{}^\mu e_b{}^\nu - e_a{}^\nu e_b{}^\mu)\partial_\mu e^c{}_\nu$$

Torsion

✓ Spinor in curved spacetime

- Local Lorentz transformation

$$\psi \to S(\Lambda)\psi$$

$$S(\Lambda) = \exp\left[-\frac{i}{4}\epsilon^{ab}\sigma_{ab}\right]$$: Spinor representation of LL transf.

$$\sigma_{ab} := \frac{i}{2}[\gamma_a, \gamma_b]$$

- Covariant derivative w.r.t LL transf.

$$D_\mu \psi = (\partial_\mu + \omega_\mu)\psi$$

Under the LL transf.

$$\omega_\mu \to S(\Lambda)\omega_\mu S(\Lambda)^{-1} - [\partial_\mu S(\Lambda)]S(\Lambda)^{-1}$$

$$\omega^a{}_{b\mu} \to \Lambda^a{}_c \omega^c{}_{d\mu} (\Lambda^{-1})^d{}_b - [\partial_\mu \Lambda^a{}_c](\Lambda^{-1})^c{}_b$$

⇒ $$D_\mu \psi = (\partial_\mu + \omega_\mu)\psi = \left(\partial_\mu - \frac{i}{4}\omega_{ab\mu}\sigma^{ab}\right)\psi$$

Torsion

✓ 4D fermions with torsion

- Action

$$S = \int d^4x e(\bar{\psi} i\gamma^a e^\mu{}_a D_\mu \psi + \bar{\psi} \overleftarrow{D}_\mu i\gamma^a e^\mu{}_a \psi)$$

$$\Rightarrow \int d^4x e \bar{\psi} i\gamma^a e^\mu{}_a \left(\partial_\mu + \omega_\mu + \frac{1}{2} T^\lambda{}_{\mu\lambda}\right) \psi$$

$$\Rightarrow \int d^4x e \bar{\psi} i\gamma^a e_a{}^\mu (\underbrace{\partial_\mu + \mathring{\omega}_\mu}_{\text{torsion-less}} + \underbrace{i S_\mu \gamma^5}_{\text{torsion}}) \psi$$

$$S_\mu := \frac{1}{8} \epsilon^{\mu\lambda\rho\sigma} T_{\lambda\rho\sigma}$$

Torsion couples with the 4D fermion as the **axial gauge field**!

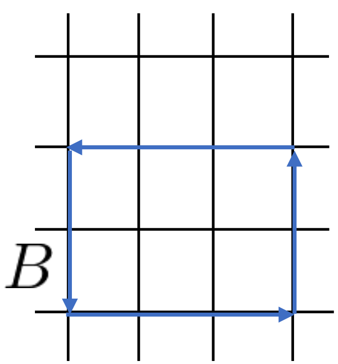
Torsion

✓ Torsion and dislocation

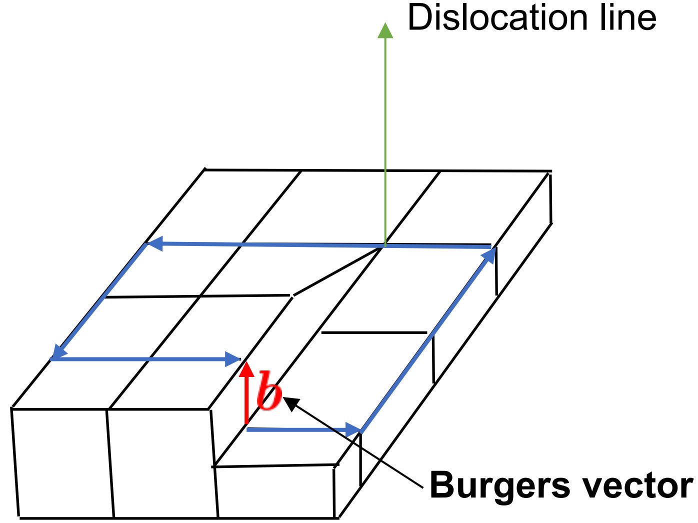
Torsion is set to be zero in general relativity

In condensed matter systems, we can realize the torsion by the **lattice dislocation**[7]**.**

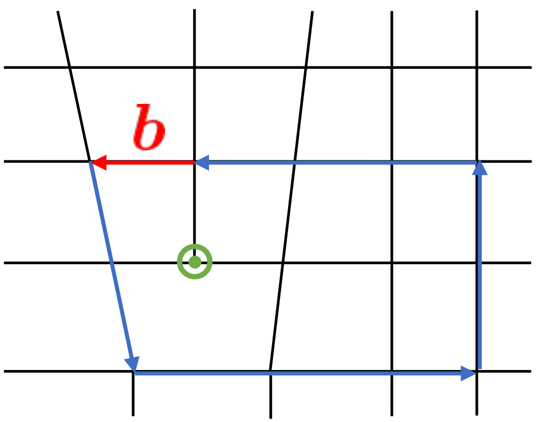
- Dislocation



Ideal lattice



screw dislocation



edge dislocation

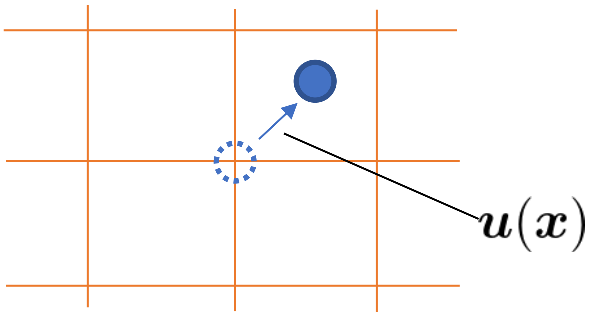
[7] H. Kleinert, *Multivalued Fields in Condensed Matter, Electromagnetism, and Gravitation,* (2008)

Torsion

✓ Torsion and dislocation

- Displacement vector

$$u^i(\boldsymbol{x}) = x^i - \bar{x}^i(\boldsymbol{x})$$



- Burgers vector

$$b^i = -\oint du^i = -\iint dx^j \wedge dx^k (\partial_j \partial_k - \partial_k \partial_j) u^i(\boldsymbol{x})$$

The deformation is regarded as a coordinate transformation $\bar{x}^i(\boldsymbol{x}) \to x^i$.

➡ The vierbein is given by $e^a{}_i(\boldsymbol{x}) = \dfrac{\partial \bar{x}^a}{\partial x^i} = \delta^a{}_i - \partial_i u^a$

$$b^i = \iint dx^j \wedge dx^k T^i{}_{jk} + \mathcal{O}(\partial u^2)$$

Torsion

✓ Torsion and dislocation

- Screw dislocation along the z axis

$$u^z(x) = \frac{b}{2\pi}\varphi = \frac{b}{2\pi}\tan^{-1}\frac{y}{x}$$

➡ $$e^z{}_x = -\frac{b}{2\pi}\frac{y}{r^2+\epsilon^2} \qquad e^z{}_y = \frac{b}{2\pi}\frac{x}{r^2+\epsilon^2}$$

➡ $$T^z{}_{xy} = \lim_{\epsilon\to 0}\frac{b}{2\pi}\frac{2\epsilon^2}{(r^2+\epsilon^2)^2} = b\delta(x)\delta(y)$$

➡ $$S_0 = -\frac{b}{4}\delta(x)\delta(y)$$

0^{th} component of axial gauge field = axial chemical potential

Torsion

✓ Short summary

- The torsion couples with the 4D fermions as the **axial gauge field.**
- Torsion = Dislocation = 0th component of axial gauge field

$$S_0 = -\frac{b}{4}\delta(x)\delta(y)$$

➡ We can use the torsion instead of μ_5

$$\underline{j^i \propto S_0(\boldsymbol{x})B^i}$$

(cf. $j^i \propto \mu_5 B^i$)

The axial chemical potential is spatially uniform.

⟷ The torsion depends on the space.

➡ <u>The CME may be induced locally even in the equilibrium.</u>

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Torsion-induced CME

✓ Set up

- Equilibrium state ⬅ Imaginary-time formalism
- Massless fermionic system

  with the torsion and the magnetic field

$$S = \int d^4x \gamma^\mu \bar{\psi}(x) \left(i\partial_\mu - A_\mu(x) - S_\mu(x)\gamma^5 \right) \psi(x)$$

$$A^\mu = (0, \boldsymbol{A}(\boldsymbol{x})) \quad S^\mu = (S^0(\boldsymbol{x}), \boldsymbol{0})$$

- Magnetic field is spatially uniform.
- The spacetime is flat: ${e^a}_\mu = {\delta^a}_\mu$.

⬆

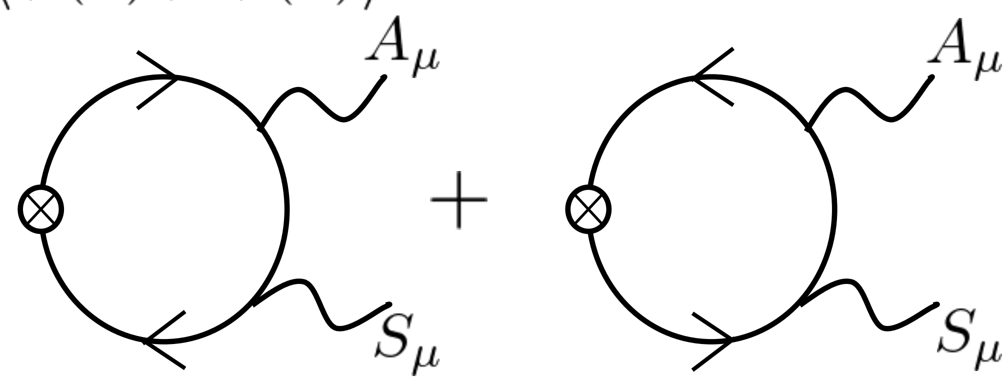
The generality is not lost since the torsion always appears as S_μ

Torsion-induced CME

✓ Current density

$$j^\mu(x) = \langle \psi\bar{(}x)\gamma^\mu \psi(x)\rangle$$

$$=$$



$$= \frac{1}{\beta}\sum_m \int_{\boldsymbol{k},\boldsymbol{p},\boldsymbol{q}} \mathrm{tr}\left[\underbrace{\gamma^\mu \frac{1}{\not{k}+\not{p}}\gamma^\nu \frac{1}{\not{k}}\gamma^\rho\gamma^5 \frac{1}{\not{k}-\not{q}} + \gamma^\mu \frac{1}{\not{k}+\not{q}}\gamma^\rho\gamma^5 \frac{1}{\not{k}}\gamma^\nu \frac{1}{\not{k}-\not{p}}}_{=:\, I^{\mu\nu\rho}(p,q,k)}\right]$$

$$\times\, e^{i\boldsymbol{p}\cdot\boldsymbol{x}} A_\nu(\boldsymbol{p}) e^{i\boldsymbol{q}\cdot\boldsymbol{x}} S_\rho(\boldsymbol{q})$$

$$k_0 = i\omega_m + \mu \quad \omega_m := \frac{(2m+1)\pi}{\beta} \quad p_0 = q_0 = 0$$

k-integral seems to be linearly divergent

➡ Pauli-Villars regularization

Torsion-induced CME

✓ Pauli-Villars regularization

$$I^{\mu\nu\rho}(p,q,k) \rightarrow I^{\mu\nu\rho}_{\rm reg}(p,q,k)$$

$$:= \mathrm{tr}\left[\gamma^\mu \frac{1}{\not{k}+\not{p}}\gamma^\nu \frac{1}{\not{k}}\gamma^\rho\gamma^5 \frac{1}{\not{k}-\not{q}}\right] + \mathrm{tr}\left[\gamma^\mu \frac{1}{\not{k}+\not{q}}\gamma^\rho\gamma^5 \frac{1}{\not{k}}\gamma^\nu \frac{1}{\not{k}-\not{p}}\right]$$

$$- \mathrm{tr}\left[\gamma^\mu \frac{1}{\not{k}+\not{p}-M}\gamma^\nu \frac{1}{\not{k}-M}\gamma^\rho\gamma^5 \frac{1}{\not{k}-\not{q}-M}\right]$$

$$- \mathrm{tr}\left[\gamma^\mu \frac{1}{\not{k}+\not{q}-M}\gamma^\rho\gamma^5 \frac{1}{\not{k}-M}\gamma^\nu \frac{1}{\not{k}-\not{p}-M}\right]$$

We introduce the ghost field with infinite mass $M \to \infty$

$$j^\mu(\boldsymbol{x}) = \frac{1}{\beta}\sum_m \int_{\boldsymbol{k},\boldsymbol{p},\boldsymbol{q}} I^{\mu\nu\rho}_{\rm reg}(p,q,k) e^{i\boldsymbol{p}\cdot\boldsymbol{x}} A_\nu(\boldsymbol{p}) e^{i\boldsymbol{q}\cdot\boldsymbol{x}} S_\rho(\boldsymbol{q})$$

Torsion-induced CME

✓ Gradient expansion

We expand the integrand w.r.t p .

$$j^i(\boldsymbol{x}) = \frac{1}{\beta}\sum_m \int_{\boldsymbol{k},\boldsymbol{p},\boldsymbol{q}} I^{ij0}_{\text{reg}}(\boldsymbol{p},\boldsymbol{q},k) e^{i\boldsymbol{p}\cdot\boldsymbol{x}} A_j(\boldsymbol{p}) e^{i\boldsymbol{q}\cdot\boldsymbol{x}} S_0(\boldsymbol{q})$$

$$= \frac{1}{\beta}\sum_m \int_{\boldsymbol{k},\boldsymbol{q}} I^{ij0}_{\text{reg}}(\boldsymbol{0},\boldsymbol{q},k) A_j(\boldsymbol{x}) e^{i\boldsymbol{q}\cdot\boldsymbol{x}} S_0(\boldsymbol{q}) + \frac{i}{\beta}\sum_m \int_{\boldsymbol{k},\boldsymbol{q}} \left.\frac{\partial I^{ij0}_{\text{reg}}(\boldsymbol{p},\boldsymbol{q},k)}{\partial p_l}\right|_{\boldsymbol{p}=\boldsymbol{0}} \partial_l A_j(\boldsymbol{x}) e^{i\boldsymbol{q}\cdot\boldsymbol{x}} S_0(\boldsymbol{q}) + \mathcal{O}(\partial^2 A)$$

- These integrals are convergent without the regularization.

Is the regularization essential? ➡ **Yes.**

Because…

the 0th order term does not vanish without the reg.

➡ The regularization is essential for the gauge invariance.

Torsion-induced CME

✓ Result

$$j^i(\boldsymbol{q}) = \frac{1}{4\pi^2}(B^i - (\boldsymbol{B}\cdot\hat{\boldsymbol{q}})\hat{q}^i)S_0(\boldsymbol{q})\left[1 + \int_0^\infty dk \frac{k}{2q}\ln\left(\frac{q+2k}{q-2k}\right)^2 N'_+(k)\right]$$

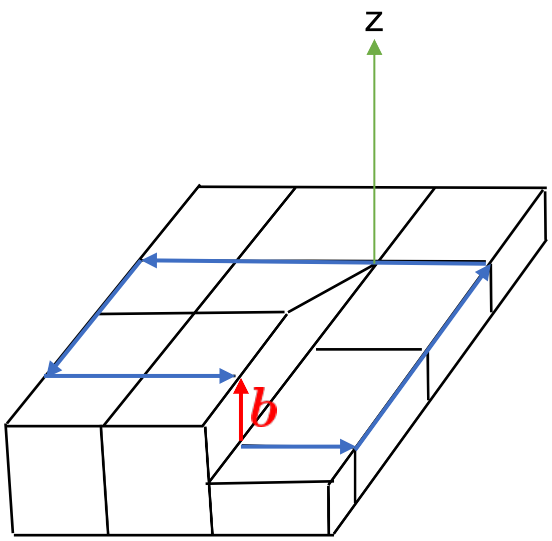
$$N_+(k) := n_F(k-\mu) + n_F(k+\mu)$$

✓ Conservation law

$$\boldsymbol{\nabla}\cdot\boldsymbol{j}(\boldsymbol{x}) \sim i\boldsymbol{q}\cdot\boldsymbol{j}(\boldsymbol{q}) = 0$$

➡ We analyze the current density

with
I. $S_0(\boldsymbol{x}) = -\mu_5$
II. $S_0(\boldsymbol{x}) = -\frac{b}{4}\delta(x)\delta(y)$.



Torsion-induced CME

I. The uniform case: $S_0(\boldsymbol{x}) = -\mu_5$

$$j^i(\boldsymbol{q}) = \frac{1}{4\pi^2}(B^i - (\boldsymbol{B}\cdot\hat{\boldsymbol{q}})\hat{q}^i)S_0(\boldsymbol{q})\left[1 + \int_0^\infty dk\frac{k}{2q}\ln\left(\frac{q+2k}{q-2k}\right)^2 N_+'(k)\right]$$

- Taking the zero frequency limit: $\boldsymbol{q} \to \boldsymbol{0}$

$$\lim_{q\to 0}\int_0^\infty dk N_+'(k)\frac{k}{q}\ln\left|\frac{2k+q}{2k-q}\right| = -1$$

- The CME vanishes in equilibrium.

$$j^i(\boldsymbol{x}) = 0$$

Torsion-induced CME

I. The uniform case: $S_0(\boldsymbol{x}) = -\mu_5$

The integral is convergent without the reg.

$$j^i(\boldsymbol{x}) = j^i_{\mathrm{D}}(\boldsymbol{x}) - j^i_{\mathrm{PV}}(\boldsymbol{x})$$

$$j^i_{\mathrm{D}}(\boldsymbol{x}) = -\frac{B^i}{2\pi^2} S_0(\boldsymbol{x}) + j^i(\boldsymbol{x})$$

$$j^i_{\mathrm{PV}}(\boldsymbol{x}) = -\frac{B^i}{2\pi^2} S_0(\boldsymbol{x})$$

- $j^i_{\mathrm{D}}(\boldsymbol{x})$ with $S_0(\boldsymbol{x}) = -\mu_5$ is the chiral magnetic current:

$$j^i_{\mathrm{D}} = \frac{\mu_5}{2\pi^2} B^i \, .$$

- $j^i_{\mathrm{PV}}(\boldsymbol{x})$ cancels the chiral magnetic current.

The regularization is essential

for the absence of the CME in eq.

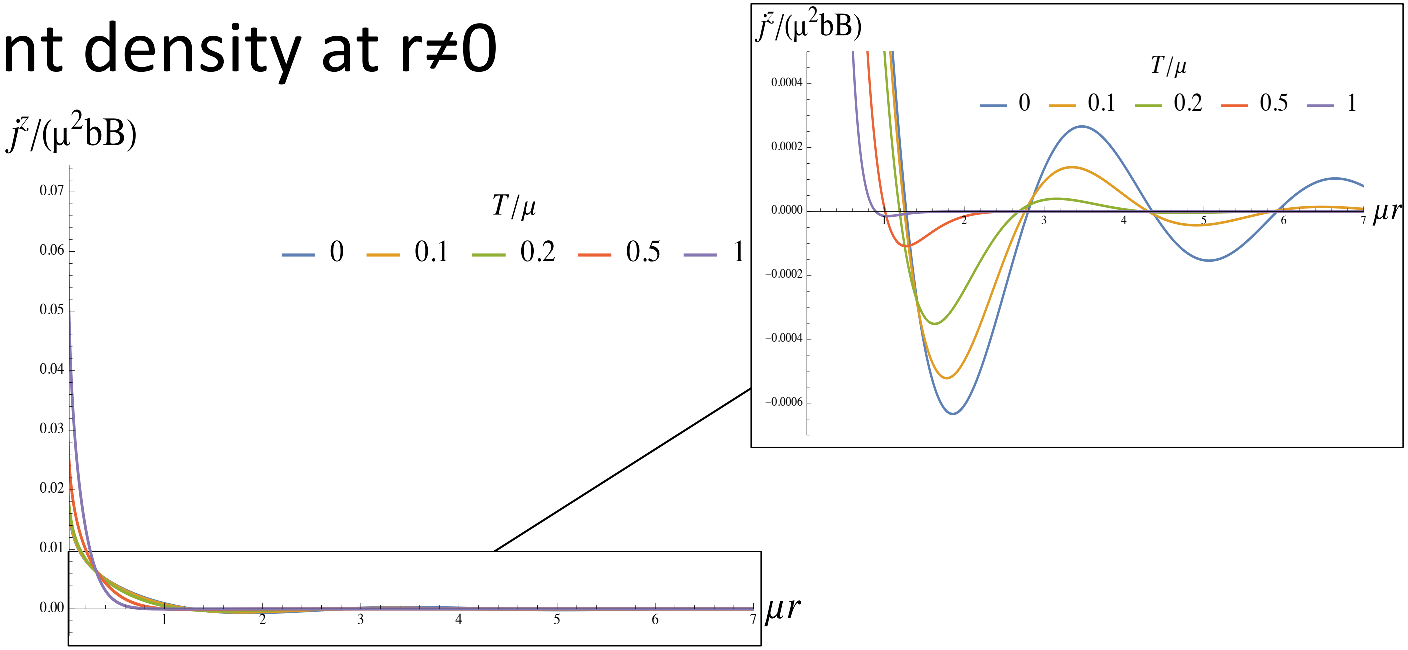
Torsion-induced CME

II. The case of the screw dislocation: $S_0(\boldsymbol{x}) = -\frac{b}{4}\delta(x)\delta(y)$

We apply the magnetic field

in parallel to the dislocation line: $\boldsymbol{B} = (0, 0, B_{||})$.

✓ Current density at r≠0



- The current is zero

$$J^z = \iint dx dy j^z(\boldsymbol{x}) = 0$$

Torsion-induced CME

II. The case of the screw dislocation: $S_0(\boldsymbol{x}) = -\frac{b}{4}\delta(x)\delta(y)$

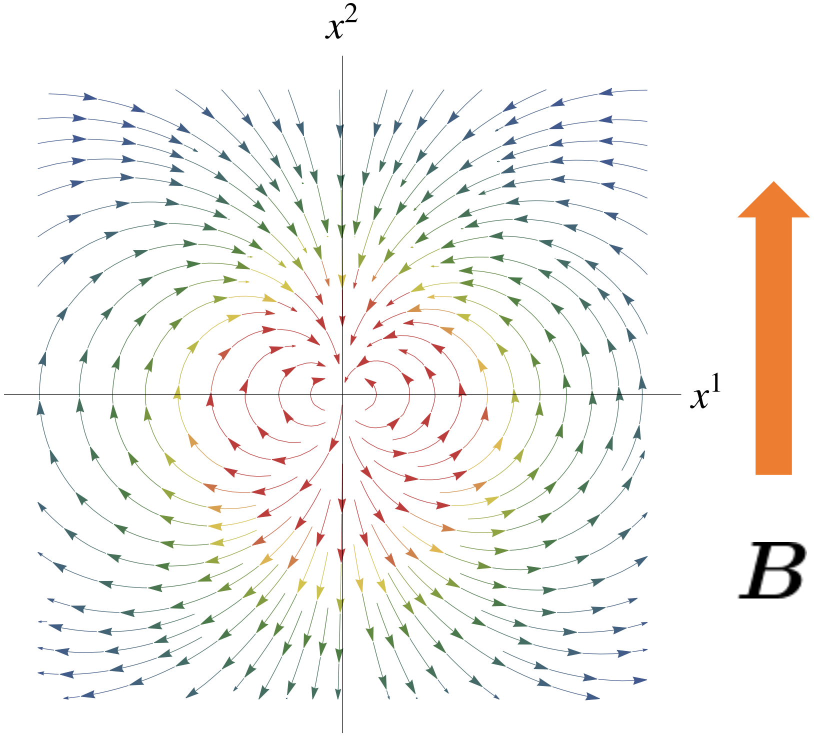
We apply the magnetic filed

in perpendicular to the dislocation line.

✓ Current density at T=μ=0

$$j^i(\boldsymbol{x}) = \frac{b}{32\pi^3}\frac{B^i - 2\hat{x}^i(\boldsymbol{B}\cdot\hat{\boldsymbol{x}})}{r^2}$$

- The circular current is induced.



Torsion-induced CME

✓ Short summary

- Evaluation of a current density induced by the dislocation S^0 and the magnetic field B^i

$$j^i(\boldsymbol{q}) = \frac{1}{4\pi^2}(B^i - (\boldsymbol{B}\cdot\hat{\boldsymbol{q}})\hat{q}^i)S_0(\boldsymbol{q})\left[1 + \int_0^\infty dk \frac{k}{2q}\ln\left(\frac{q+2k}{q-2k}\right)^2 N'_+(k)\right]$$

- $\boldsymbol{B} = (0, 0, B_{\parallel})$ ➡ The current is induced along the dislocation line. (This is analog of the CME)
- $\boldsymbol{B} = (B_{\perp}, 0, 0)$ ➡ The circular current is induced around the dislocation line.

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Torsion-induced transport in WSMs

✓ Weyl semimetal

- WSMs have a couple of the Dirac cones
- The splitting of the Dirac cones is given by the axial gauge filed A_5^i

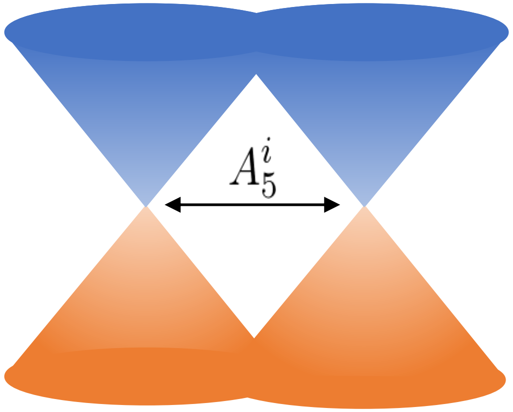
$$S = \int d^4x \bar{\psi}(x)(i\not{\partial} - \not{S}(x)\gamma^5 - \not{A}_5\gamma^5)\psi(x)$$

What is a possible current in this system?

$$j^i \propto \mu S_0 A_5^i$$

Cf. CME

$$j^i \propto S_0 B^i$$



This current is consistent with P,T,C.

In fact, such a current is numerically observed in a lattice calculation[8].

[8] K. Kodama and Y. Takane, J. Phys. Soc. Jpn. 88, 054715 (2019)

Torsion-induced transport in WSMs

✓ Set up

- Equilibrium state ⬅ Imaginary-time formalism
- Massless fermionic system

  with torsion and axial gauge field

$$S = \int d^4x \bar{\psi}(x)(i\partial\!\!\!/ - S\!\!\!/(x)\gamma^5 - A\!\!\!/_5\gamma^5)\psi(x)$$

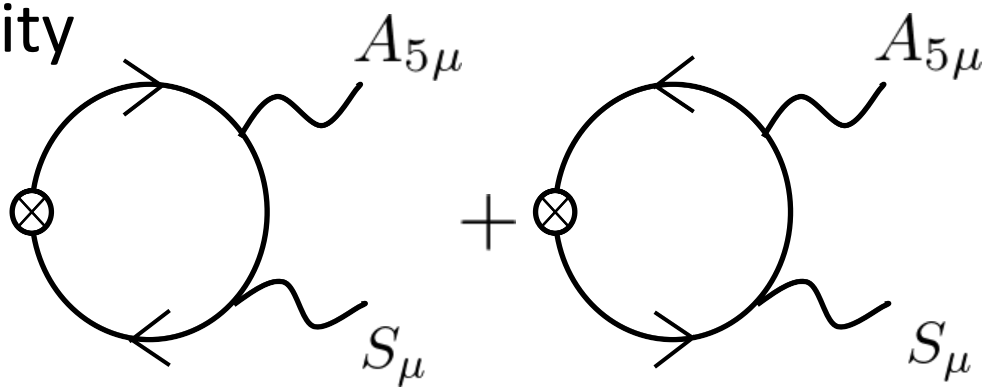
$$A_5^\mu = (0, \boldsymbol{A_5}) \qquad S^\mu = (S^0(\boldsymbol{x}), \boldsymbol{0})$$

- The axial gauge field A_5^i is spatially uniform
- The spacetime is flat $e^a{}_\mu = \delta^a{}_\mu$

Torsion-induced transport in WSMs

✓ Current density

$j^\mu(x) =$



- Pauli-Villars regularization

✓ Result

- $j^i \propto \mu S_0 A_5^i$ **is absent.**

$$j^i(\boldsymbol{x}) = 0$$

- This result is **NOT** consistent with the lattice calculation.

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Summary

✓ Evaluation of a current density induced

by the dislocation S^0 and the magnetic field B^i

➡ The local current is induced around the dislocation line even in the eq.

✓ Evaluation of a current density induced

by the dislocation S^0 and the Weyl node splitting A_5^i

➡ Such a current vanishes.

This result is NOT consistent with the lattice calculation.

✓ Future work

To resolve this inconsistency