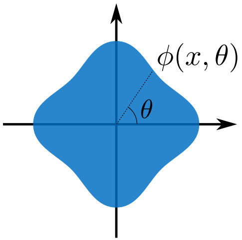
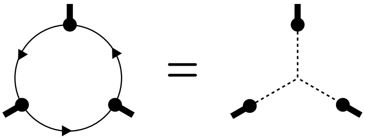
Nonlinear Bosonization of Fermi Liquids

Luca Delacrétaz

University of Chicago





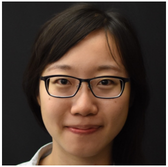
arXiv:**2203.05004**

Virtual QCD Theory Seminar

May 18, 2022

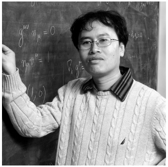
Nonlinear Bosonization of Fermi Liquids

Luca Delacrétaz
University of Chicago



Yi-Hsien Du

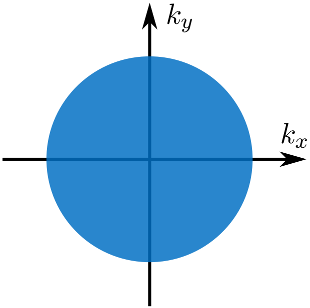
Umang Mehta



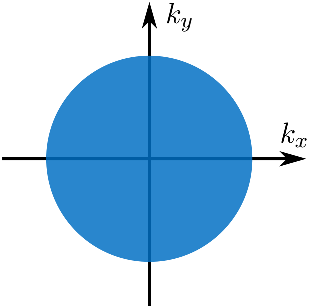
Dam Thanh Son

arXiv:**2203.05004**

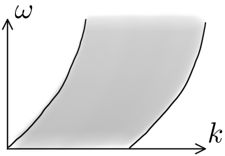
Virtual QCD Theory Seminar
May 18, 2022


k_y
k_x

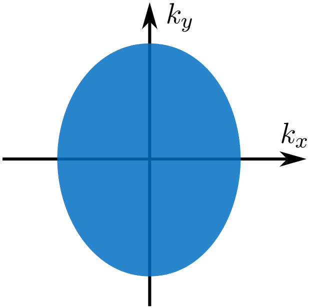
Fermi Gas



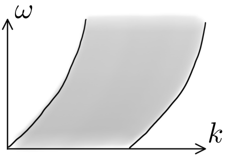
Very gapless system, compared to 'vacuum' QFT

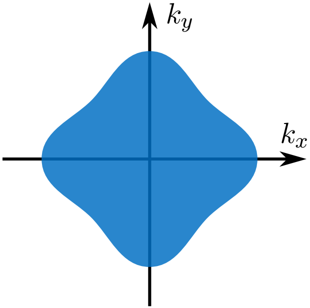


Fermi Gas

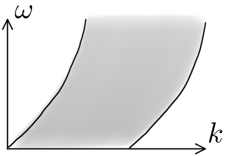


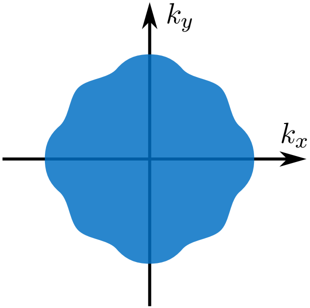
Very gapless system, compared to 'vacuum' QFT



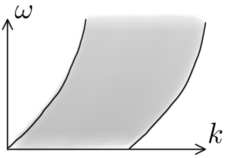


Very gapless system, compared to 'vacuum' QFT

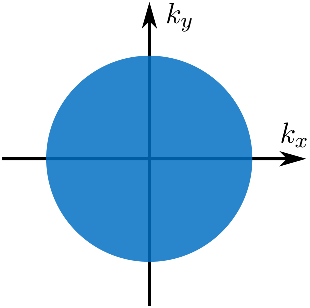




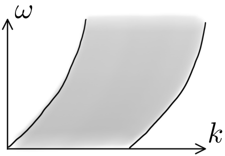
Very gapless system, compared to 'vacuum' QFT



Fermi Gas



Very gapless system, compared to ‘vacuum’ QFT

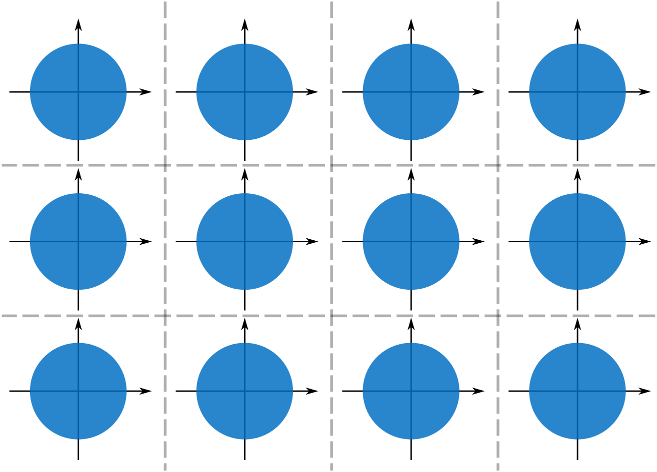


Important quantity: k_F.

Effective description for long distance physics $x \gg 1/k_F$?

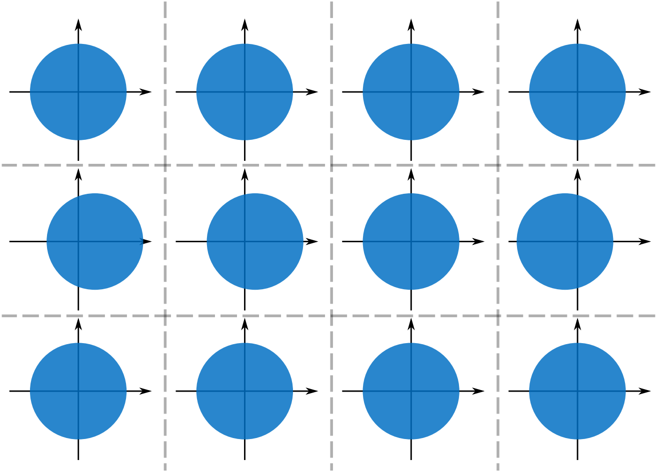
Semi-classical Fermi surface

Local Fermi surface in every volume of size $\xi \gg 1/k_F$



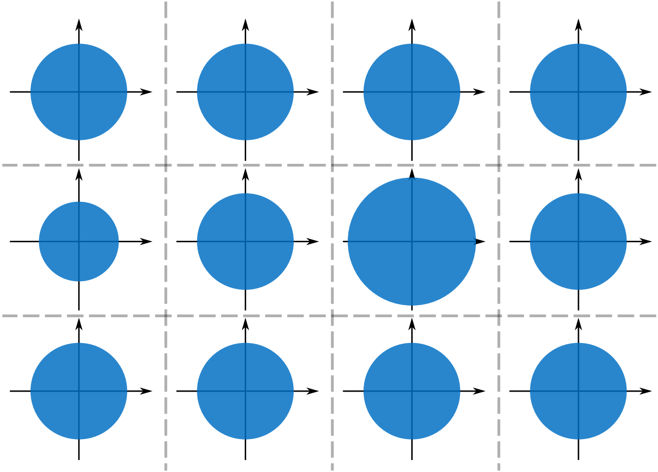
Semi-classical Fermi surface

Local Fermi surface in every volume of size $\xi \gg 1/k_F$



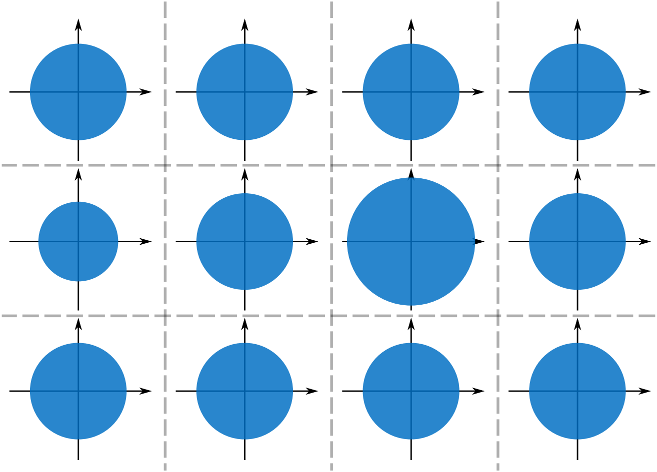
Semi-classical Fermi surface

Local Fermi surface in every volume of size $\xi \gg 1/k_F$



Semi-classical Fermi surface

Local Fermi surface in every volume of size $\xi \gg 1/k_F$

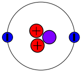


Described by kinetic equation

$$\partial_t f(t,x,p) + \vec{v}(p) \cdot \nabla_x f(t,x,p) = 0$$

Successes of FL theory

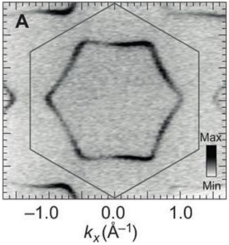
^{3}He



- Specific heat $c_V \propto T$
- Observation of collective excitations (e.g. zero sound)
- Measurement of Landau parameters
- Viscosity, sound attenuation, equilibration time $\tau \sim E_F/T^2$

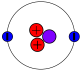
Metals

- Visualize Fermi surfaces with ARPES
- Magnetic oscillations, Hall coefficient, $\sigma_\perp(q) \sim k_F/q$
- $\rho_{\rm dc} \sim T^2$

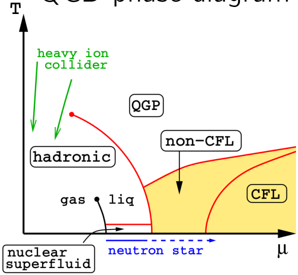


Successes of FL theory

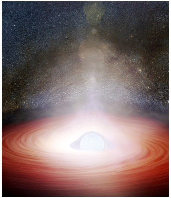
^{3}He



QCD phase diagram

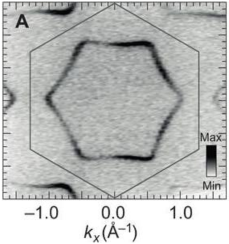


Neutron stars,
White dwarfs



Metals

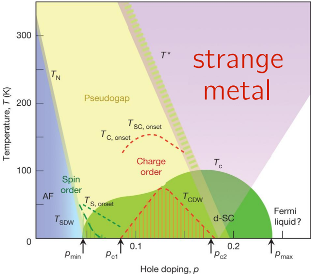
- Visualize Fermi surfaces with ARPES
- Magnetic oscillations, Hall coefficient, $\sigma_\perp(q) \sim k_F/q$
- $\rho_{\rm dc} \sim T^2$



Beyond FL

Experiment

High-T_c superconductors



[Keimer Kivelson Norman Uchida Zaanen '15]

Fermi surface, but thermodynamics and transport not FL-like

Theory

S_{FL} + gapless boson

NFL fixed point

Hertz '76, Halperin Lee Read '93, Nayak Wilczek '94, Altshuler Ioffe Millis '94, Lee '09, Metlitski Sachdev '10, ...

Beyond FL

Experiment

High-T_c superconductors

P.W. Anderson, 1989

"Know the enemy!"

[Keimer Kivelson Norman Uchida Zaanen '15]

Fermi surface, but thermodynamics and transport not FL-like

Theory

S_{FL} + gapless boson

NFL fixed point

Hertz '76, Halperin Lee Read '93, Nayak Wilczek '94, Altshuler Ioffe Millis '94, Lee '09, Metlitski Sachdev '10, ...

An EFT for FL

What is an Effective Field Theory (EFT)?

"A microscopics-insouciant description of a system based on general principles"

What is an Effective Field Theory (EFT)?

"A microscopics-insouciant description of a system based on general principles"

E.g.: spontaneous symmetry breaking

Pions in QCD

$$U \in \frac{SU(2)\times SU(2)}{SU(2)}$$

$$S = \int \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)$$

Spin waves in Ferromagnet

$$\vec{n} \in \frac{SU(2)}{U(1)}$$

$$S = \int (\partial_\mu \vec{n})^2$$

An EFT for FL

What is an Effective Field Theory (EFT)?

"A microscopics-insouciant description of a system based on general principles"

E.g.: spontaneous symmetry breaking

Pions in QCD

$$U \in \frac{SU(2)\times SU(2)}{SU(2)}$$

$$S = \int \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + \mathrm{Tr}(U^\dagger dU)^5$$

Spin waves in Ferromagnet

$$\vec{n} \in \frac{SU(2)}{U(1)}$$

$$S = \int (\partial_\mu \vec{n})^2 + \vec{n}\cdot(\partial_\tau \vec{n} \times \partial_s \vec{n})$$

What is an Effective Field Theory (EFT)?

"A microscopics-insouciant description of a system based on general principles"

E.g.: spontaneous symmetry breaking

Pions in QCD

$$U \in \frac{SU(2)\times SU(2)}{SU(2)}$$

$$S = \int \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + \mathrm{Tr}(U^\dagger dU)^5 + \mathrm{Tr}(\partial^2 U^\dagger \partial^2 U) + \cdots$$

Spin waves in Ferromagnet

$$\vec{n} \in \frac{SU(2)}{U(1)}$$

$$S = \int (\partial_\mu \vec{n})^2 + \vec{n}\cdot(\partial_\tau \vec{n} \times \partial_s \vec{n}) + (\partial^2 \vec{n})^4 + \cdots$$

[Callan Coleman Wess Zumino '70]

An EFT for FL

What is S_{FL} ? EFT for Fermi liquids

Benfatto Gallavotti '90, Shankar '91, Polchinski '92

$$S = \int dt \int d^d p \, \psi_p^\dagger (\partial_t + \epsilon(p)) \psi_p + \int_{p_1 p_2 p_3} V(p_1, p_2, p_3, p_4) \psi_1^\dagger \psi_2^\dagger \psi_3 \psi_4 + \cdots$$

Landau parameters and BCS couplings as only marginal terms

An EFT for FL

What is S_{FL} ? EFT for Fermi liquids

Benfatto Gallavotti '90, Shankar '91, Polchinski '92

$$S = \int dt \int d^d p \, \psi_p^\dagger (\partial_t + \epsilon(p)) \psi_p + \int_{p_1 p_2 p_3} V(p_1, p_2, p_3, p_4) \psi_1^\dagger \psi_2^\dagger \psi_3 \psi_4 + \cdots$$

Landau parameters and BCS couplings as only marginal terms

Major drawback: Many subtle cancellations in perturbation theory

An EFT for FL

What is S_{FL} ? EFT for Fermi liquids

Benfatto Gallavotti '90, Shankar '91, Polchinski '92

$$S = \int dt \int d^d p \, \psi_p^\dagger (\partial_t + \epsilon(p)) \psi_p + \int_{p_1 p_2 p_3} V(p_1, p_2, p_3, p_4) \psi_1^\dagger \psi_2^\dagger \psi_3 \psi_4 + \cdots$$

Landau parameters and BCS couplings as only marginal terms

Major drawback: Many subtle cancellations in perturbation theory

There is an alternative: bosonization

Haldane '92, Castro Neto Fradkin '93, Houghton Kwon Marston '94, Khveshchenko '94

An EFT for FL

What is S_{FL} ? EFT for Fermi liquids

Benfatto Gallavotti '90, Shankar '91, Polchinski '92

$$S = \int dt \int d^d p\, \psi_p^\dagger (\partial_t + \epsilon(p)) \psi_p + \int_{p_1 p_2 p_3} V(p_1, p_2, p_3, p_4) \psi_1^\dagger \psi_2^\dagger \psi_3 \psi_4 + \cdots$$

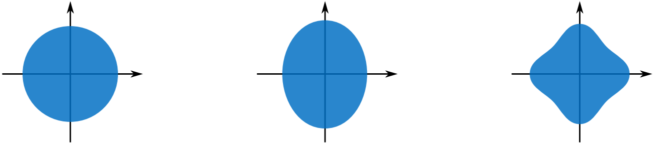
Landau parameters and BCS couplings as only marginal terms

Major drawback: Many subtle cancellations in perturbation theory

There is an alternative: bosonization

Haldane '92, Castro Neto Fradkin '93, Houghton Kwon Marston '94, Khveshchenko '94

Our strategy: write an EFT directly for the shape, or $f(x, p, t)$



Nonlinear completion of multidimensional bosonization

1. Introduction
2. Why bosonization?
3. S_{FL} from coadjoint orbits
4. Using it

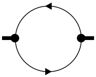
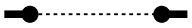
Why bosonization?

In 1d, bosonization can solve interacting fermion problems

Even for free fermions, bosonization is useful

$$S = \int dtdx\, i\bar{\psi}\gamma^\mu \partial_\mu \psi \qquad \leftrightarrow \qquad S = \frac{1}{4\pi}\int dtdx\, (\partial_\mu \phi)^2$$

$$\rho = \psi^\dagger \psi \qquad \leftrightarrow \qquad \rho = \frac{1}{2\pi}\partial_x \phi$$

$\langle \rho\rho \rangle(\omega, q) =$  $\leftrightarrow$ $\langle \rho\rho \rangle(\omega, q) =$ 

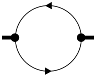
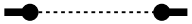
Why bosonization?

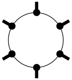
In 1d, bosonization can solve interacting fermion problems

Even for free fermions, bosonization is useful

$$S = \int dt dx \, i\bar{\psi}\gamma^\mu \partial_\mu \psi \qquad \leftrightarrow \qquad S = \frac{1}{4\pi}\int dt dx \, (\partial_\mu \phi)^2$$

$$\rho = \psi^\dagger \psi \qquad \leftrightarrow \qquad \rho = \frac{1}{2\pi}\partial_x \phi$$

$\langle \rho\rho \rangle(\omega, q) =$  $\leftrightarrow$ $\langle \rho\rho \rangle(\omega, q) =$ 

$\langle \rho\rho \cdots \rho \rangle =$  $=$???

Why bosonization?

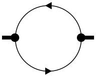
In 1d, bosonization can solve interacting fermion problems

Even for free fermions, bosonization is useful

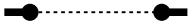
$$S = \int dt dx \, i\bar{\psi}\gamma^\mu \partial_\mu \psi \qquad \leftrightarrow \qquad S = \frac{1}{4\pi}\int dt dx \, (\partial_\mu \phi)^2$$

$$\rho = \psi^\dagger \psi \qquad \leftrightarrow \qquad \rho = \frac{1}{2\pi}\partial_x \phi$$

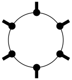
$$\langle \rho\rho \rangle(\omega, q) =$$



$$\leftrightarrow \qquad \langle \rho\rho \rangle(\omega, q) =$$



$$\langle \rho\rho\cdots\rho \rangle =$$



$$= ??? \qquad \leftrightarrow \qquad \langle \rho\rho\cdots\rho \rangle = 0 \;!$$

[Dzyaloshinskii Larkin '74]

Why bosonization?

Bosonization also helps study deformations of original problem

Example: Prove that Schwinger model (1+1d QED) is gapped

$$S = \int \bar{\psi}\gamma^{\mu}(\partial_{\mu} + eA_{\mu})\psi + (dA)^2$$

Relevant interaction!

Why bosonization?

Bosonization also helps study deformations of original problem

Example: Prove that Schwinger model (1+1d QED) is gapped

$$S = \int \bar{\psi}\gamma^{\mu}(\partial_{\mu} + eA_{\mu})\psi + (dA)^2 \quad \leftrightarrow \quad S = \int (\partial_{\mu}\phi + eA_{\mu})^2 + (dA)^2$$

Relevant interaction!

Why bosonization?

Bosonization also helps study deformations of original problem

Example: Prove that Schwinger model (1+1d QED) is gapped

$$S = \int \bar{\psi}\gamma^{\mu}(\partial_{\mu} + eA_{\mu})\psi + (dA)^2 \quad \leftrightarrow \quad S = \int (\partial_{\mu}\phi + eA_{\mu})^2 + (dA)^2$$

Relevant interaction!

Bosonized description directly gives $m_{\text{photon}} = e$.

Why bosonization?

In higher dimensions, or for $\epsilon(p) \neq p$, still *approximate* loop cancellations

Kopietz Hermisson Schönhammer '95, Metzner Castellani Di Castro '97

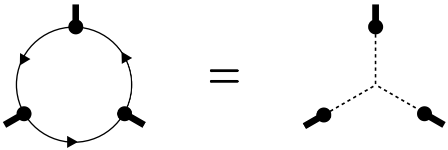
These make perturbative analyses of non-Fermi liquids difficult

Metlitski Sachdev '10, Holder Metzner '15

Why bosonization?

These near cancellations are transparent in the bosonic picture

The ‘small’ remainder comes from ‘weak’ interactions

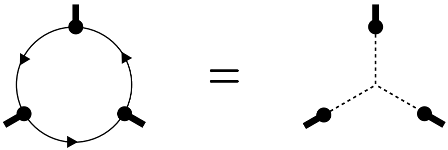


LVD Du Mehta Son ’22

Why bosonization?

These near cancellations are transparent in the bosonic picture

The ‘small’ remainder comes from ‘weak’ interactions

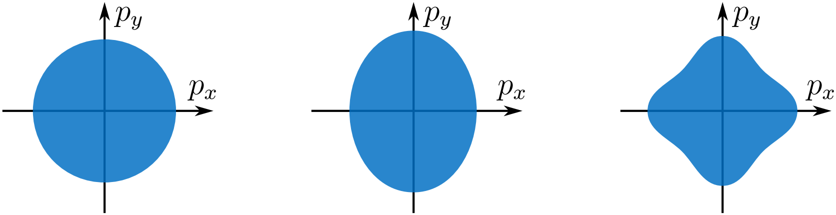


LVD Du Mehta Son ’22

Two sources of nonlinearities:

- $\epsilon(p) \neq p$
- Geometry of the Fermi surface

Our strategy: action for the *shape* of the Fermi surface



$$f(t,x,p) = \begin{cases} 1 & \text{if } p \in \\ 0 & \text{if } p \notin \end{cases}$$

We would like an action S_{FL} for $f(t,x,p)$

LVD Du Mehta Son '22

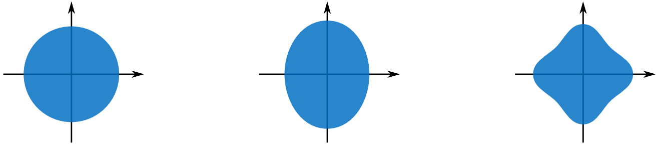
For free fermions with single particle $H = \epsilon(p) + V(x)$, the equation of motion should be the Boltzmann equation

$$\partial_t f + \nabla_p H \cdot \nabla_x f - \nabla_x H \cdot \nabla_p f = 0$$

For free fermions with single particle $H = \epsilon(p) + V(x)$, the equation of motion should be the Boltzmann equation

$$\partial_t f + \nabla_p H \cdot \nabla_x f - \nabla_x H \cdot \nabla_p f = 0$$

From $f_0(x,p) = \Theta(p_F - p)$, not every $f(x,p)$ can be reached

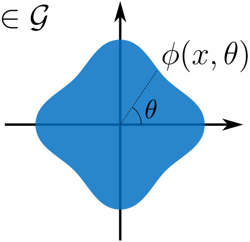


$$\partial_t f = [H, f] \qquad \Rightarrow \qquad f = e^{Ht} f_0 e^{-Ht}$$

allowed f's $\quad = \quad$ orbit of f_0 under $\mathcal{G}$

We can parametrize the degree of freedom as

$$f = U f_0 U^{-1}\,, \qquad \text{with} \quad U = e^{\phi} \in \mathcal{G}$$



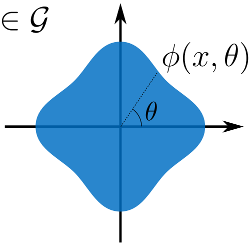
with $\phi = \phi(x,p)$

But: $\quad e^{\phi} \sim e^{\phi} e^{\alpha} \quad \Rightarrow \quad U = e^{\phi} \in \mathcal{G}/\mathcal{H}$

Can use this to choose $\phi = \phi(x,\theta)$

We can parametrize the degree of freedom as

$$f = U f_0 U^{-1}\,, \qquad \text{with} \quad U = e^{\phi} \in \mathcal{G}$$



with $\phi = \phi(x,p)$

But: $\quad e^{\phi} \sim e^{\phi} e^{\alpha} \quad \Rightarrow \quad U = e^{\phi} \in \mathcal{G}/\mathcal{H}$

Can use this to choose $\phi = \phi(x,\theta)$

There is a WZW term (Kirillov-Kostant-Souriau form)

$$S_{\text{WZW}} = \int dt\ \text{Tr}\left(f_0 U^{-1}\partial_t U\right)$$

Kirillov '78, Khveshchenko '94

Otherwise, any action that is invariant under $\mathcal{H}$ is allowed

$$S = S_{\text{WZW}} - \int dtdxdp\ f(x,p,t)\left(\epsilon(p) + V(x)\right) + \cdots$$

In the spirit of EFT, one should include all possible terms

$$
\begin{aligned}
S[f] = S_{\mathrm{WZW}} &- \int_{txp} f(x,p,t)\,\epsilon(p) \\
&+ \int_{txp_1p_2} f(x,p_1,t)f(x,p_2,t)\,F^{(2)}(p_1,p_2) \\
&+ \int_{txp_1p_2p_3} f(x,p_1,t)f(x,p_2,t)f(x,p_3,t)F^{(3)}(p_1,p_2,p_3) \\
&+ \cdots
\end{aligned}
$$

Fluctuations

$$S \simeq -\frac{p_F}{2} \int dt d^2x d\theta \; \hat{n}(\theta) \cdot \nabla\phi \left[\dot{\phi} + v_F \left(\hat{n}(\theta) \cdot \nabla\phi\right)\right]$$

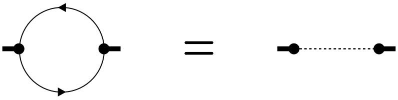
(recover Haldane '92, Castro Neto Fradkin '93, Houghton Kwon Marston '94)

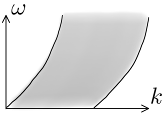
The density operator is

$$\rho(t, x) \simeq p_F \int d\theta \; \hat{n}(\theta) \cdot \nabla\phi(t, x, \theta)$$

'chiral boson on the Fermi surface'

$$\langle\rho\rho\rangle(\omega, q) = p_F \int d\theta \frac{\hat{n}(\theta) \cdot q}{\omega - v_F \hat{n}(\theta) \cdot q} = \frac{p_F}{v_F}\left[1 - \frac{|\omega|}{\sqrt{\omega^2 - v_F^2 q^2}}\right]$$



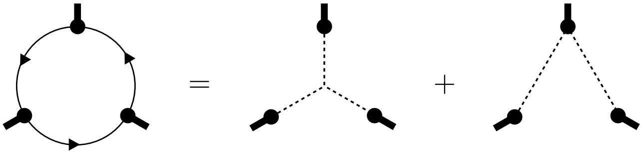


Nonlinear response

$$S_{\text{WZW}} = -p_F \int_{tx\theta} \dot{\phi} \hat{n} \cdot \nabla \phi + \frac{2}{3} \frac{1}{p_F} \hat{n} \cdot \nabla \phi \left((\partial_\theta \hat{n}) \cdot \nabla \phi \partial_\theta \dot{\phi} \right) + \cdots$$

$$S_H = -p_F \int_{tx\theta} \epsilon' (\hat{n} \cdot \nabla \phi)^2 + \frac{1}{3} \frac{1}{p_F} \left(\epsilon' + \epsilon'' p_F \right) (\hat{n} \cdot \nabla \phi)^3 + \cdots$$

$$\rho = p_F \int_\theta \hat{n} \cdot \nabla \phi + \frac{1}{2} \frac{1}{p_F} (\partial_\theta \hat{n}) \cdot \nabla \left(\partial_\theta \phi \hat{n} \cdot \nabla \phi \right) + \cdots$$



Scaling manifest! $\langle \rho(\omega_1, q_1) \cdots \rho(\omega_n, q_n) \rangle \sim 1$

HDL

Alternatively, obtain $\langle \rho\rho\cdots\rangle$ by solving kinetic equation in the presence of a source $A_0(t,x)$

$$(\partial_t + v(p)\cdot\nabla_x + E\cdot\nabla_p)\, f(t,x,p) = 0\,, \qquad f = \Theta(p_F(t,x,\theta) - p)$$

Used in Manuel '95 to compute HDLs in QCD

(see also Blaizot Iancu '93, Kelly Liu Lucchesi Manuel '94 for HTLs)

HDL

Alternatively, obtain $\langle \rho\rho\cdots\rangle$ by solving kinetic equation in the presence of a source $A_0(t,x)$

$$(\partial_t + v(p)\cdot\nabla_x + E\cdot\nabla_p)\, f(t,x,p) = 0\,, \qquad f = \Theta(p_F(t,x,\theta) - p)$$

Used in Manuel '95 to compute HDLs in QCD

(see also Blaizot Iancu '93, Kelly Liu Lucchesi Manuel '94 for HTLs)

Scaling of n-point functions different in QCD:

The algebra is extended to $\mathfrak{g}\otimes u(N)$

$$[F,G]^c = \{F^0, G^c\} - \{G^0, F^c\} + f_{ab}{}^c F^a G^b$$

$\rightsquigarrow$ lower derivative terms in the EFT: $S_{\text{WZW}} \sim \int f_{abc}\nabla\phi^a\dot{\phi}^b\phi^c$

$\rightsquigarrow$ different scaling $\langle \rho^{a_1}\cdots\rho^{a_n}\rangle \sim \frac{1}{q^{n-2}}$

Braaten Pisarski '92, Frenkel Taylor '92

Beyond Fermi Liquids

Couple the bosonized theory to a gapless boson

$$S = -\frac{p_F}{2}\int dtd^2xd\theta\ \hat{n}(\theta)\cdot\nabla\phi\left[\dot{\phi}+v_F\left(\hat{n}(\theta)\cdot\nabla\phi\right)\right]$$
$$+\int dtd^2xd\theta\,\lambda\Phi\hat{n}(\theta)\cdot\nabla\phi+\int dtd^2x\left(\nabla\Phi\right)^2$$

Find $z = 3$ at tree-level

$$\langle\Phi\Phi\rangle(\omega, q) = \frac{1}{q^2+\lambda^2\frac{|\omega|}{\sqrt{\omega^2-v_F^2q^2}}}$$

Lawler Barci Fernández Fradkin Oxman '06, Chubukov Khveshchenko '06

Also produces specific heat $c_V \sim T^{2/3}$

Beyond Fermi Liquids

Couple the bosonized theory to a gapless boson

$$S = -\frac{p_F}{2}\int dtd^2xd\theta\ \hat{n}(\theta)\cdot\nabla\phi\left[\dot{\phi} + v_F\left(\hat{n}(\theta)\cdot\nabla\phi\right)\right] + \int dtd^2xd\theta\,\lambda\Phi\hat{n}(\theta)\cdot\nabla\phi + \int dtd^2x\,(\nabla\Phi)^2$$

Find $z = 3$ at tree-level

$$\langle\Phi\Phi\rangle(\omega, q) = \frac{1}{q^2 + \lambda^2\frac{|\omega|}{\sqrt{\omega^2 - v_F^2 q^2}}}$$

Lawler Barci Fernández Fradkin Oxman '06, Chubukov Khveshchenko '06

Also produces specific heat $c_V \sim T^{2/3}$

Beyond tree-level: $z \neq 3$?

Holder Metzner '15

More...

- Spinful Fermi surfaces, BCS, $2k_F$ physics

- Which phases of matter can arise from CFT + μ ? Sachdev '12
  $\rightsquigarrow$ spectrum of CFT large charge operators
  Hellerman Orlando Reffert Watanabe '15, Monin Pirtskhalava Rattazzi Seibold '16
  $\rightsquigarrow$ UV/IR constraints as in relativistic EFTs?
  Adams Arkani-Hamed Dubovsky Nicolis Rattazzi '06, ...

- Future directions: Fermi Surface + X

More...

- Spinful Fermi surfaces, BCS, $2k_F$ physics

- Which phases of matter can arise from CFT + μ ? Sachdev '12
  $\rightsquigarrow$ spectrum of CFT large charge operators
  Hellerman Orlando Reffert Watanabe '15, Monin Pirtskhalava Rattazzi Seibold '16
  $\rightsquigarrow$ UV/IR constraints as in relativistic EFTs?
  Adams Arkani-Hamed Dubovsky Nicolis Rattazzi '06, ...

- Future directions: Fermi Surface + X

Thanks!

LVD@uchicago.edu