Jet broadening in a non-equilibrium QCD medium

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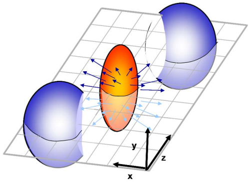
Phys.Rev.C 105 (2022) 1, 014914, arXiv:2109.04575
and recent work

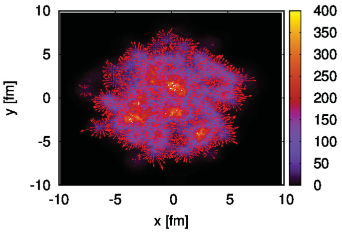
Heavy-ion collisions

- Quark-gluon plasma (QGP) is high-temperature QCD matter
- Create QGP by colliding atomic nuclei at ultrarelativistic energies.
- The QGP behaves like a fluid.
  - Relativistic hydrodynamics describes hadrons in experiments.
    [E.g. Romatschke, Romatschke (2007); Schenke, Jeon, Gale (2010)]
- Simulations show that shear viscosity is low

$$\frac{\eta}{s} \gtrsim \frac{1}{4\pi}$$

[Policastro, Son, Starinets (2001)]

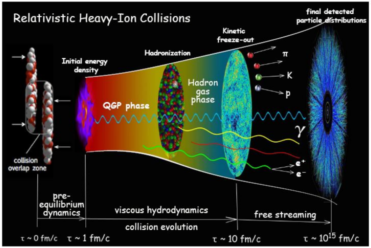




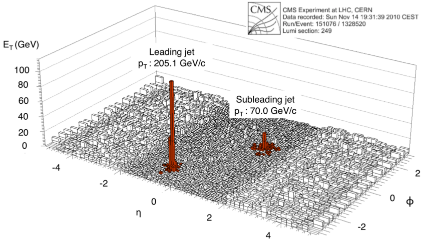
[McDonald (2016)]

Early stages of heavy-ion collisions

- How does far-from-equilibrium QCD medium reach hydrodynamic stage?
- Earliest stages: Highly occupied gluon fields (glasma).
- Different stages of collisions: Glasma → Hydrodynamics
  [See e.g. Berges, Heller, Mazeliauskas, Venugopalan (2020)]
- Need experimental probes of non-equilibrium physics, e.g. jets.



[Chun Shen, 2014]



Jet broadening

- Total transverse momentum broadening:

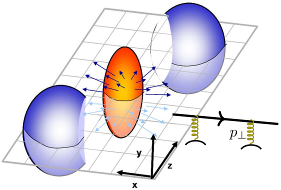
$$\widehat{q} = \frac{d\langle \mathbf{p}_\perp^2 \rangle}{dt}$$

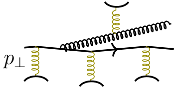
- Broadening allows for medium-induced gluon emission.

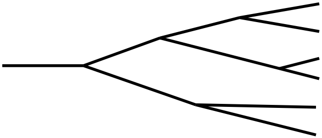
$$\Gamma \sim \frac{g^2\sqrt{\widehat{q}}}{\sqrt{E}}$$

- Wavepackets of partons overlap for a long time during emission.
- This process determines whole jet structure.

[For vacuum-like emission see e.g. Majumder (2018); Wang, Guo (2001)]

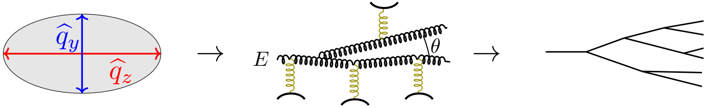






This talk

- How does anisotropy in broadening affect jet evolution?



  - Get polarization in jet.

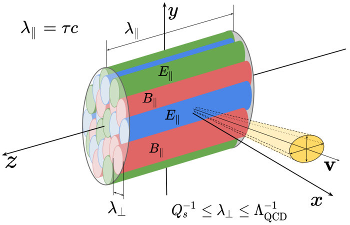
- What does momentum broadening look like after glasma stage?

  - Does pressure anisotropy lead to anisotropy in broadening?

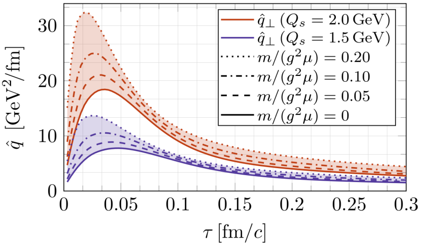
  - Other non-equilibrium effects?

Jet broadening in glasma

- Jet partons traverse heavily occupied gluon fields.
- Deflected by chromomagnetic and chromoelectric forces.
- As much broadening as during hydro stage!
  - $\Delta p_\perp^2\big|_{\text{glasma}} / \Delta p_\perp^2\big|_{\text{hydro}} \approx 0.9$
    [Carrington, Czajka, Mrowczynski (2022)]



[Carrington, Czajka, Mrowczynski (2022)]



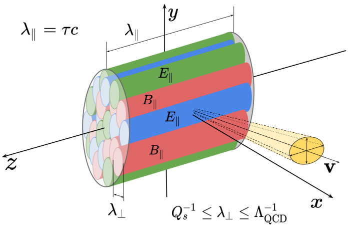
[Ipp, Muller, Schuh (2020)]

Jet broadening in glasma

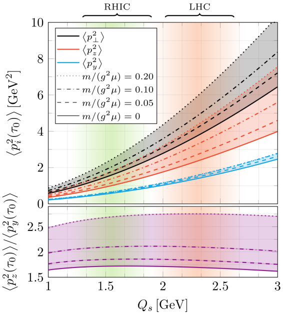
- Broadening can be anisotropic:
  - $\widehat{q}_z \neq \widehat{q}_y$ with $\widehat{q}_y = \frac{d\langle p_y^2 \rangle}{dt}$

- In glasma broadening is heavily anisotropic,

$$\widehat{q}_z \approx 2\widehat{q}_y$$



[Carrington, Czajka, Mrowczynski (2022)]

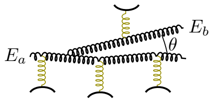


[Ipp, Muller, Schuh (2020)]

Jets in an isotropic plasma

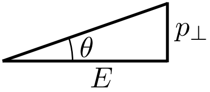
- Broadening brings parton off shell so it can radiate.

  [See e.g. review: Qin, Wang (2015)]

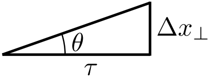


- Wavepackets overlap for a long time (LPM).

  [Landau, Pomeranchuk (1953); Migdal (1955)]



- Schematic estimate:
  - $\theta \sim \frac{p_\perp}{E} \sim \frac{\Delta x_\perp}{\tau}$
  - Uncertainty principle: $p_\perp \Delta x_\perp \sim 1$ so $\tau \sim \frac{E}{p_\perp^2} \sim \frac{E}{\hat{q}\tau}$



- Get rate $\Gamma \sim \alpha_s P(z)/\tau \sim \alpha_s\, P(z)\, \frac{\sqrt{\hat{q}}}{\sqrt{E}}$

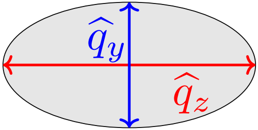
- $P_{\text{hard}}(z) = \frac{1+z^4+(1-z)^4}{z(1-z)}$ is splitting function; $z = E_b/E_a$.

Jets in an anisotropic plasma

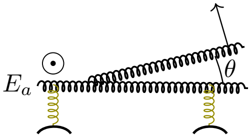
- Total rate is still

$$\Gamma \sim \alpha_s P(z) \frac{\sqrt{\hat{q}_y + \hat{q}_z}}{\sqrt{E}} \left[1 + \mathcal{O}\left(\left(\frac{\hat{q}_z - \hat{q}_y}{\hat{q}_z + \hat{q}_y} \right)^2 \right) \right]$$

  - Much splitting in glasma phase.



- Daughter parton has net polarization:
  - Opening angle θ preferably in z direction.
  - Daughter partons are preferably polarized in plane of θ.



- Want to quantify degree of polarization.

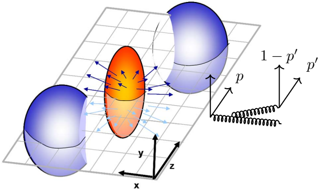
Jets in an anisotropic plasma

- Ensemble of gluons: Probability p of polarization in beam direction.
- Daughter parton has

$$p' - \frac{1}{2} = f(z)\left(p - \frac{1}{2}\right) + \frac{c}{4}\, g(z)\, \frac{\widehat{q}_z - \widehat{q}_y}{\widehat{q}_z + \widehat{q}_y}$$

$$f(z) = \frac{z^2 + z(1-z)^2 + z\sqrt{(1-z)^2 + z^2 + z^2(1-z)^2}}{2((1-z)^2 + z^2 + z^2(1-z)^2)}, \quad g(z) = \frac{(1-z)^2}{(1-z)^2 + z^2(1-z)^2 + z^2}$$

- Isotropic:
  Polarization reduced at each splitting.
- Anisotropic:
  Unpolarized mother radiates polarized daughter!
- Two competing effects.

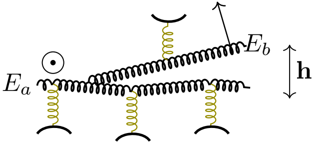


Formalism for jet splitting

- Isotropic case has been analyzed widely:
  [E.g. Baier, Dokshitzer, Peigné, Schiff, Mueller (1996); Zakharov (1997)
  Arnold, Moore, Yaffe (2002); Hauksson, Jeon, Gale (2018)]
- Rate of branching is

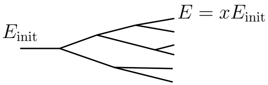
$$\frac{d\Gamma_{z\to z}}{dz} \sim \alpha_s \mathrm{Re} \int d^2h\; \mathbf{h}\cdot\mathbf{F}(\mathbf{h}) \left[\cos^4\phi\, \mathcal{F}_{\mathrm{in}\to\mathrm{in,in}}(z) + \sin^4\phi\, \mathcal{F}_{\mathrm{out}\to\mathrm{out,in}}(z) + \cdots \right]$$

- Here
  $\mathbf{h} = ih^2\mathbf{F}(\mathbf{h}) - \left(\widehat{q}_z\,\partial^2_{h_z} + \widehat{q}_y\,\partial^2_{h_y}\right)\mathbf{F}(\mathbf{h})$



- Solve by expanding in $\frac{\widehat{q}_z - \widehat{q}_y}{\widehat{q}_z + \widehat{q}_y}$. Gives details of radiation pattern.
- Join with polarized splitting functions $\mathcal{F}(z)$, $z = E_b/E_a$.

Evolution of spin polarization



- Consider total evolution of jet in medium with constant $\frac{\widehat{q}_z - \widehat{q}_y}{\widehat{q}_z + \widehat{q}_y}$.
- Use $x := E/E_{\text{init}} \ll 1$

$$\frac{dD_{\text{tot}}(x,\tau)}{d\tau} = \int_x^1 dz\, \mathcal{K}_0(z)\sqrt{\frac{z}{x}}\, D_{\text{tot}}\left(\frac{x}{z},\tau\right) - \int_0^1 dz\, \mathcal{K}_0(z)\, \frac{z}{\sqrt{x}}\, D_{\text{tot}}(x,\tau)$$

$$\frac{d\widetilde{D}(x,\tau)}{d\tau} = \int_x^1 dz\, \mathcal{M}_0(z)\sqrt{\frac{z}{x}}\, \widetilde{D}\left(\frac{x}{z},\tau\right) - \int_0^1 dz\, \mathcal{K}_0(z)\, \frac{z}{\sqrt{x}}\, \widetilde{D}(x,\tau) + \int_x^1 dz\, \mathcal{L}_0(z)\sqrt{\frac{z}{x}}\, D_{\text{tot}}\left(\frac{x}{z},\tau\right).$$

$$\mathcal{K}_0(z) \approx \frac{1}{z^{3/2}(1-z)^{3/2}}, \qquad \mathcal{M}_0(z) \approx z\mathcal{K}_0(z), \qquad \mathcal{L}_0(z) \approx \frac{c}{2}\,\frac{\widehat{q}_x - \widehat{q}_y}{\widehat{q}_x + \widehat{q}_y}\mathcal{K}_0(z)$$

- Here $D_{\text{tot}} = x\frac{d(N_z + N_y)}{dx}$ is total energy spectrum, and $\widetilde{D} = x\frac{d(N_z - N_y)}{dx}$ is polarization.

[Equation for D_{tot}: Blaizot, Iancu, Mehtar-Tani (2013); Blaizot, Mehtar-Tani (2015); Fister, Iancu (2014); Iancu, Wu (2015); Escobedo, Iancu (2016)]

Evolution of spin polarization

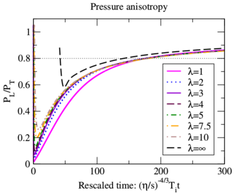
- Isotropic medium: $D_{\text{tot}}(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}} e^{-\pi\tau^2/(1-x)} \sim 1/\sqrt{x}$
  [Blaizot, Iancu, Mehtar-Tani (2013)]
  - Polarization decays away: $\widetilde{D} \sim \sqrt{x}$
- Anisotropic medium: $\widetilde{D} \sim \frac{\widehat{q}_x - \widehat{q}_y}{\widehat{q}_x + \widehat{q}_y} \frac{1}{\sqrt{x}}$
- Constant fraction of particles with spin polarization at all x!
  $\widetilde{D}/D_{\text{tot}} = \frac{c}{4} \frac{\widehat{q}_x - \widehat{q}_y}{\widehat{q}_x + \widehat{q}_y}$.
- Can this be measured?
  - Measurements of spin difficult.
  - Hydrodynamic phase more isotropic $\rightarrow$ Can wash out polarization.

Momentum broadening at later stages

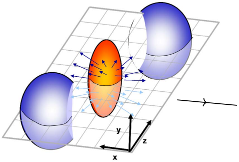
- How does QGP pressure anisotropy affect momentum broadening?
- How is momentum broadening in non-equilibrium QGP?
  - Go beyond simple models $\mathcal{C}(\mathbf{p}_\perp) \sim \frac{1}{(\mathbf{p}_\perp^2 + m_D^2)^2}$.
  - Important for physics of jets, photons and in kinetic theory.
- Describe using kinetic theory and HTL.
- Assume momentum distribution

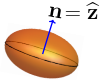
$$f(\mathbf{p}) = \sqrt{1+\xi}\, f_{\text{eq}}\left(\sqrt{p^2 + \xi\, p_z^2}\right)$$

[Romatschke, Strickland (2003)]

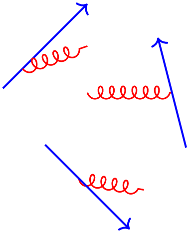


[Keegan, Kurkela, Romatschke, van der Schee, Zhu (2015)]





Non-equilibrium QGP



- Two scales in perturbative QGP:
  - Hard partons at energy Λ.
    [Arnold, Moore, Yaffe (2003)]

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = \mathcal{C}[f, A]$$

  - Soft gluon fields at energy $g\Lambda$
    [Braaten, Pisarski (1990)]

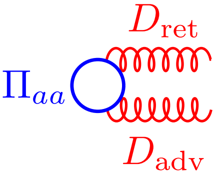
$$\mathcal{D}_\mu F^{\mu\nu} = j^\nu[f]$$

- Integrating out f gives HTL retarded correlator.
  [Blaizot, Iancu, 2001; Mrowczynski, Thoma, 2000; Anisotropic: Romatschke, Strickland (2003)]
- Have derived non-equilibrium density of soft gluons.
  [**Hauksson**, Jeon, Gale (2021)]

$$D^{\mu\nu}_{rr}(x,y) = \tfrac{1}{2}\langle\{A^\mu(x), A^\nu(y)\}\rangle$$

Non-equilibrium QGP

$$
\begin{aligned}
D_{rr}^{(\mu\nu)} = & - \tilde{D}^A_{\rm ret} \left(\tilde{D}^A_{\rm ret}\right)^* \\
& \times \Bigg[\left\{ \alpha \, |X|^2 - 2\delta R \, \mathrm{Re}\big(XW^*\big) + \gamma R \, |W|^2 \right\} P_L^{\mu\nu} \\
& + \left\{ \gamma \, |Z|^2 - 2\delta R \, \mathrm{Re}\big(ZW^*\big) + \alpha R \, |W|^2 \right\} C^{\mu\nu} \\
& + \Big\{ - \alpha \, \mathrm{Re}\big(XW^*\big) - \gamma \, \mathrm{Re}\big(ZW^*\big) \\
& + \delta \, \mathrm{Re}\big(XZ^*\big) + \delta R \, |W|^2 \Big\} D^{\mu\nu} \Bigg] \\
& - \tilde{D}^B_{\rm ret} \left(\tilde{D}^B_{\rm ret}\right)^* \beta E^{\mu\nu}.
\end{aligned}
$$



where

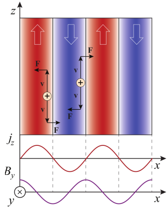
$$
\tilde{D}^A_{\rm ret} = \frac{1}{(Q^2 - \Pi_L)\,(Q^2 - \Pi_c) - R\,\Pi_d^2}
$$

and

$$
\tilde{D}^B_{\rm ret} = \frac{1}{Q^2 - \Pi_e}
$$

and $X = Q^2 - \Pi_c, Z = Q^2 - \Pi_L, W = -\Pi_d$

Instabilities in QCD plasma



- $\widehat{q} \sim \int d^4Q \, D_{rr}(Q)\delta(q^0 - q^z)$ is naively divergent in anisotropic plasma.

  [Romatschke (2007)]

  [Instabilities: Mrowczynski, Schenke, Strickland (2016)]

- Need to consider time evolution of instabilities.
- Separation of scales gives

$$D^{\mu\nu}_{rr}(t_x, t_y; \mathbf{k}) \approx \int \frac{dk^0}{2\pi} \, e^{-ik^0(t_x - t_y)} \left[\widehat{D}_{\rm ret} \, \Pi_{aa} \, \widehat{D}_{\rm adv}\right]^{\mu\nu} (k^0; \mathbf{k})$$
$$+ [A\Pi_{aa}(0)A^*]^{\mu\nu} \, \frac{e^{\gamma t_x} e^{\gamma t_y} - 1}{2\gamma}$$

  where

$$D_{\rm ret}(K) = \widehat{D}_{\rm ret}(K) + \frac{A}{q^0 - i\gamma}, \qquad D_{\rm ret}(t_x, t_y) \sim e^{\gamma(t_x - t_y)}$$

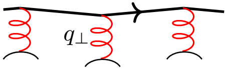
  [**Hauksson**, Jeon, Gale, Phys.Rev.C (2021)]

- For phenomenological prescription focus on fluctuating modes.

  [cf. Berges, Boguslavski, Schlicting, Venugopalan (2014)]

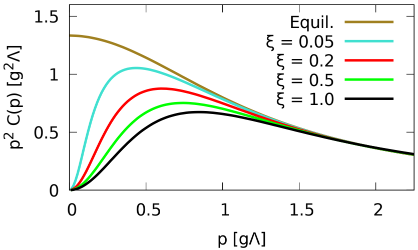
Momentum broadening in non-equilibrium QGP

- A parton receives transverse kicks from soft gluons,



$$\mathcal{C}(\mathbf{q}_\perp) = g^2 C_F \int \frac{dq^0 dq^z}{(2\pi)^2}\, D_{rr}^{\mu\nu}(Q) v_\mu v_\nu \, \delta(v\cdot Q)$$

$$\widehat{q} = \int \frac{d^2 q_\perp}{(2\pi)^2}\, \mathbf{q}_\perp^2 \mathcal{C}(\mathbf{q}_\perp)$$



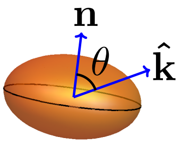
- Less momentum broadening due to increased screening in anisotropic plasma.

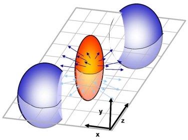
  [**Hauksson**, Jeon, Gale, Phys. Rev. C (2022)]

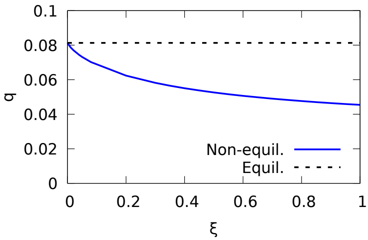
- $p_\perp^2 \frac{1}{p_\perp^2} \to \frac{p_\perp^2}{p_\perp^2 + m^2}$

Momentum broadening in non-equilibrium QGP

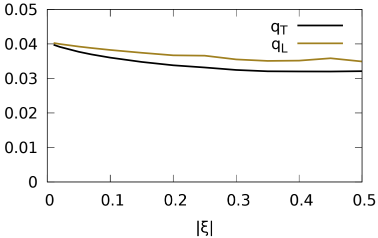
- Momentum broadening reduced in non-equilibrium plasma.
- Small anisotropy in broadening.







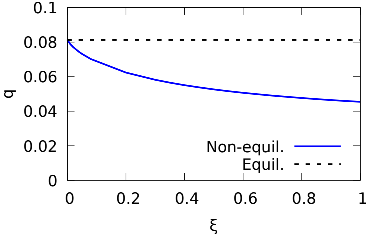
$E \sim \Lambda$, $\theta = 0$, Jet in beam direction



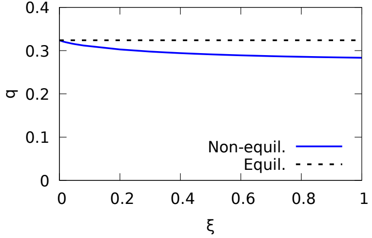
$E \sim \Lambda$, $\theta = \pi/2$ Jet at midrapidity

Momentum broadening in non-equilibrium QGP

- In HTL kinetic theory, $\widehat{q}$ needs UV cutoff:
  $\widehat{q} \sim g^4\Lambda^3 \int d^2p_\perp p_\perp^2 \left(\frac{1}{p_\perp^2}\right)^2 \sim g^4\Lambda^3 \log E/\Lambda$



$E \sim \Lambda,\, \theta = 0$



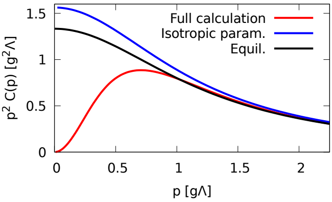
$E \sim 100\Lambda,\, \theta = 0$

- Different in glasma: Saturation scale is the cutoff.
  $\widehat{q} \sim g^2Q_s^3 + g^4Q_s^3 \log E/Q_s$

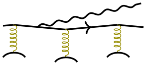
Example of consequences

- Results qualitatively different from ansatz used in kinetic theory simulations.

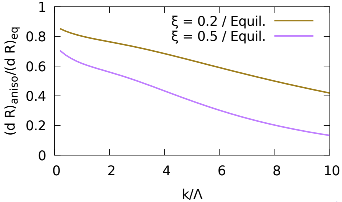
  [E.g. Kurkela, Zhu (2015)]



- Also important for collinear photon production.

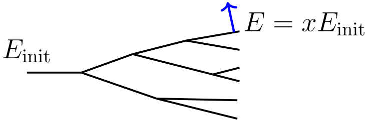


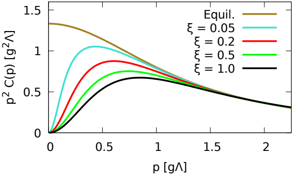
- Get reduced photon emission due to increased screening.



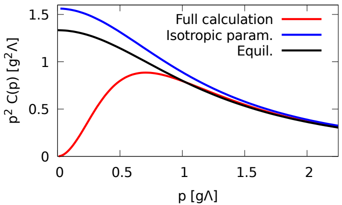
Conclusions

- Momentum broadening in heavy-ion collisions gives access to medium.
  - Determines rate of parton splitting and thus jet structure.
  - Simple models don't capture the medium.
- Glasma: Broadening independent of jet energy, heavily anisotropic.
  - Emitted jet gluons are polarized.
  - Get constant polarization at all x.
- Anisotropic QGP: Broadening depends on $\log E$, mildly anisotropic.
  - Much more screening in anisotropic plasma.





Momentum broadening in non-equilibrium QGP



- In thermal equilibrium ($\xi = 0$) have

$$p_\perp^2 \mathcal{C}(\mathbf{p}_\perp) \sim p_\perp^2 \left[\frac{1}{p_\perp^2} - \frac{1}{p_\perp^2 + m_D^2} \right]$$

  [Aurenche,Gelis,Zaraket (2002)]

- Here, even the magnetic mode has screening. Schematically,

$$\frac{1}{p_\perp^2} \to \frac{1}{p_\perp + m^2}$$

- Should give different results in e.g. kinetic theory.