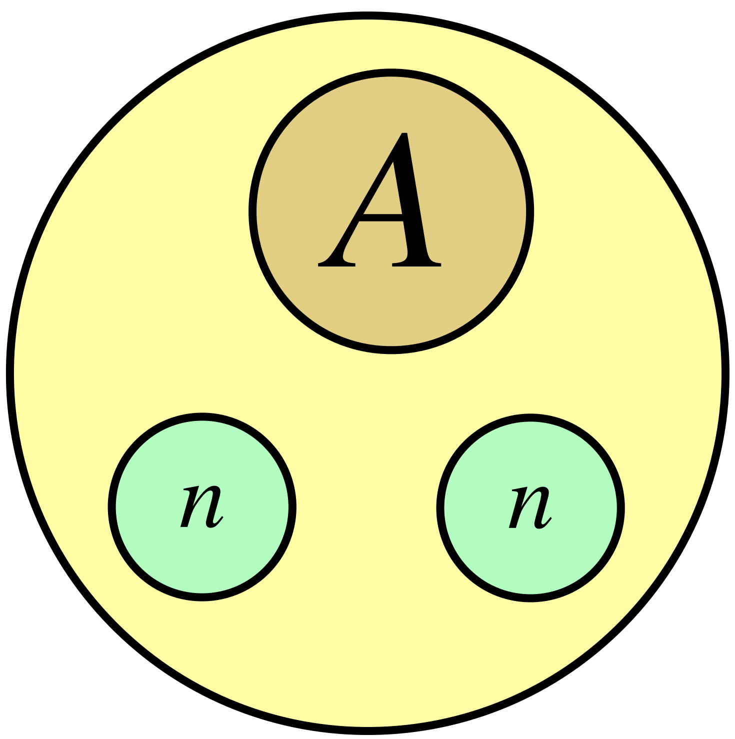
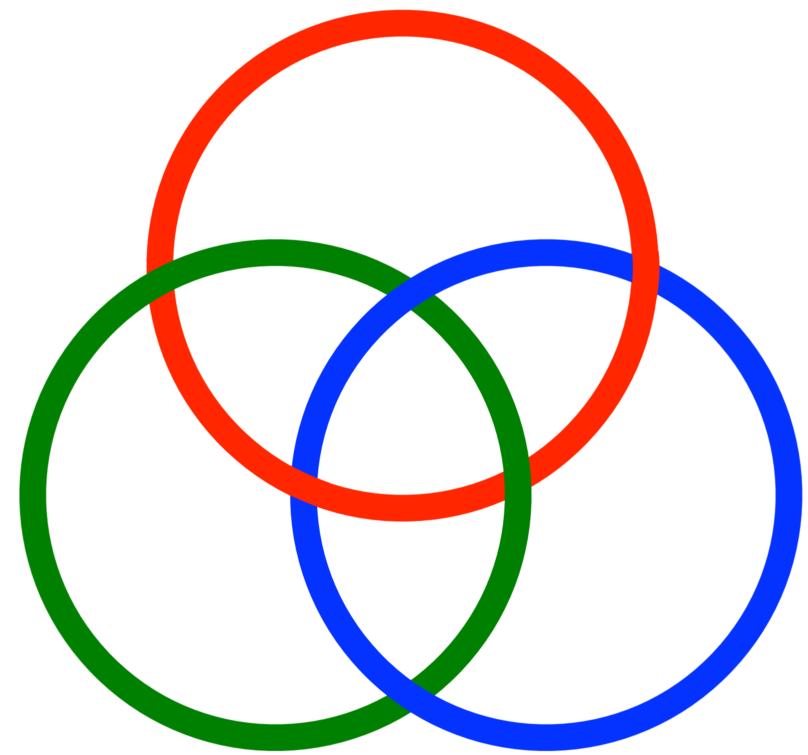
Effective field theoretical approach to weakly bound Borromean nuclei





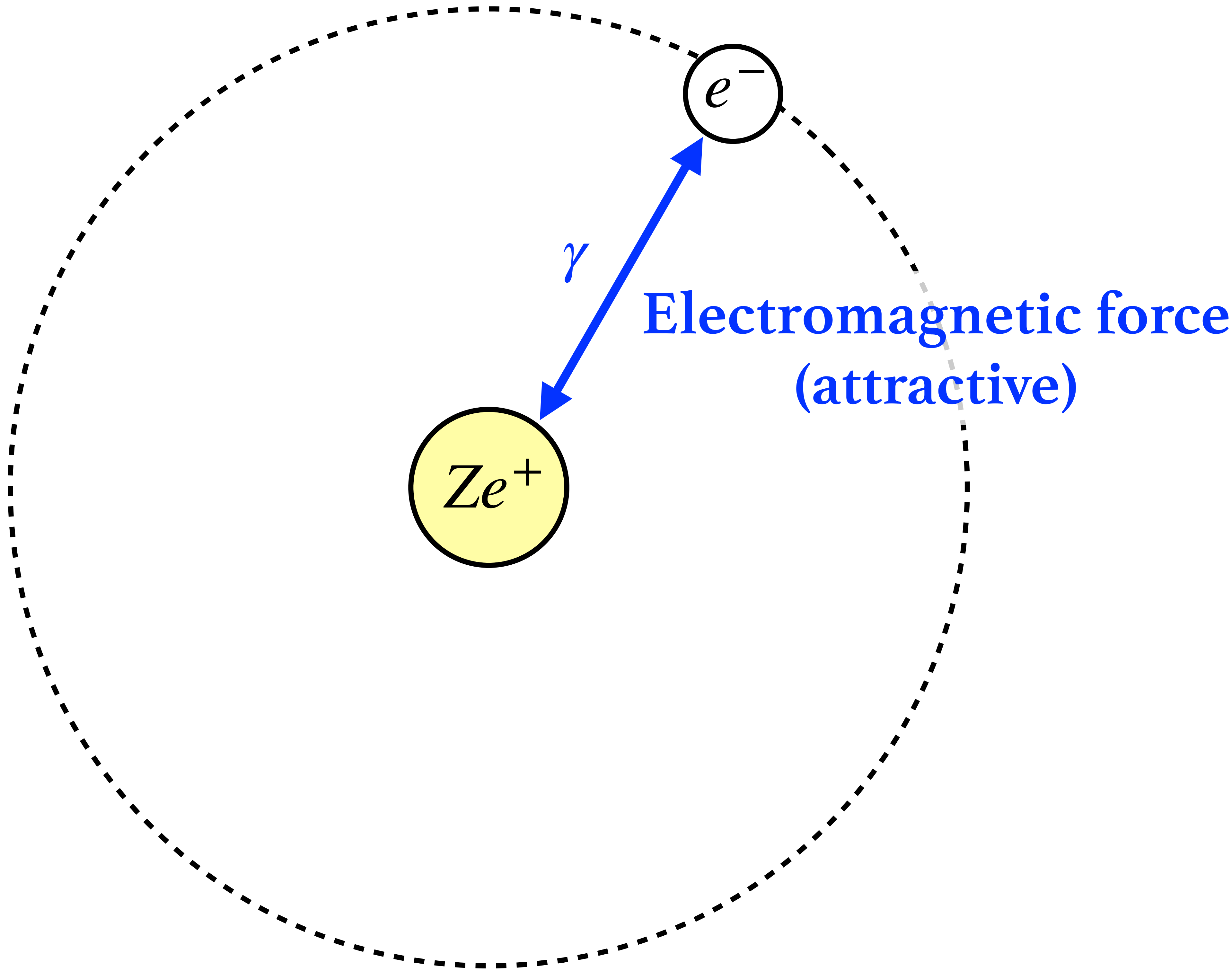
Masaru Hongo (Niigata University)

2022/04/18, QCD theory Seminars

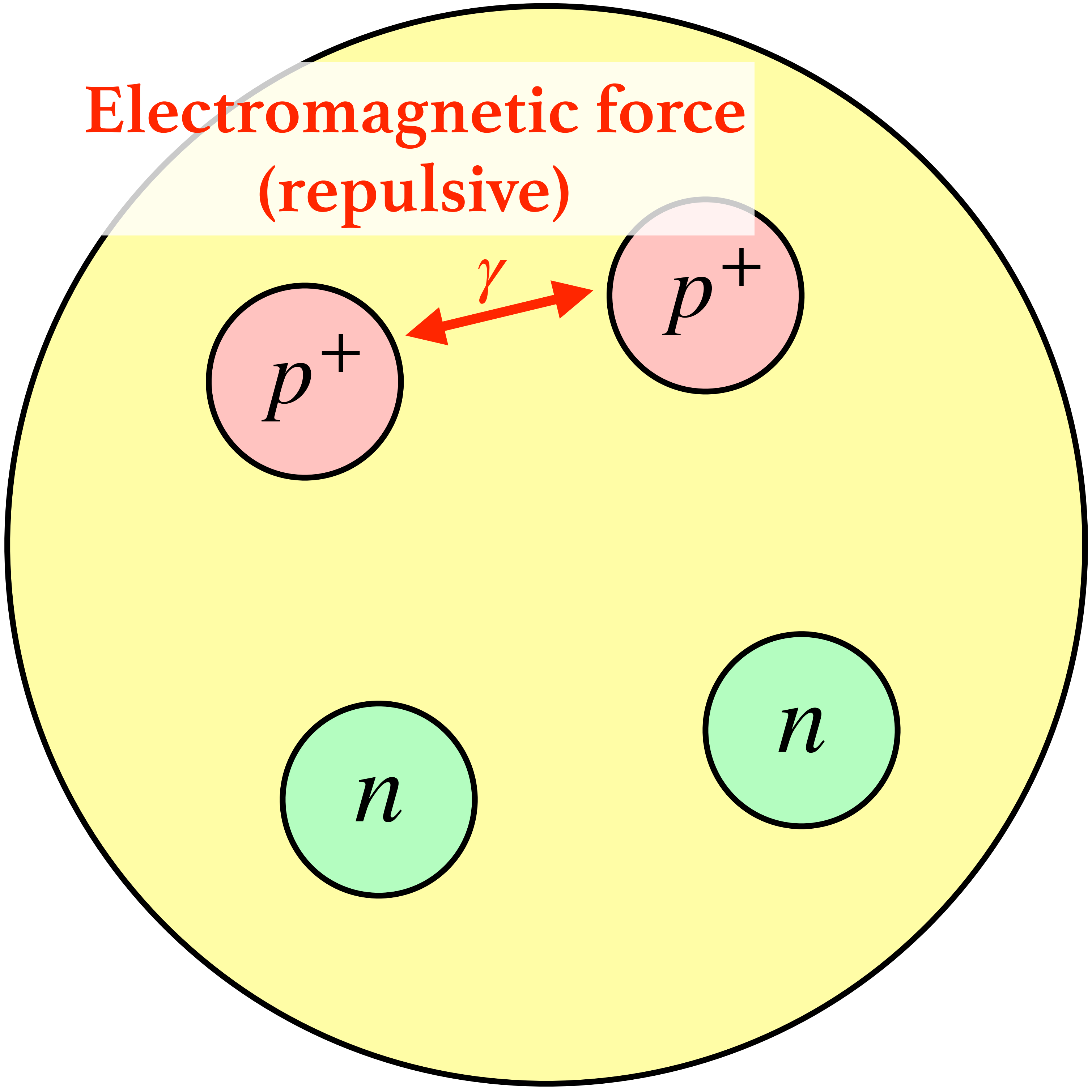
M. Hongo, D. T. Son, arXiv:2201.09912 [nucl-th]

Introduction:
What is Borromean nuclei?

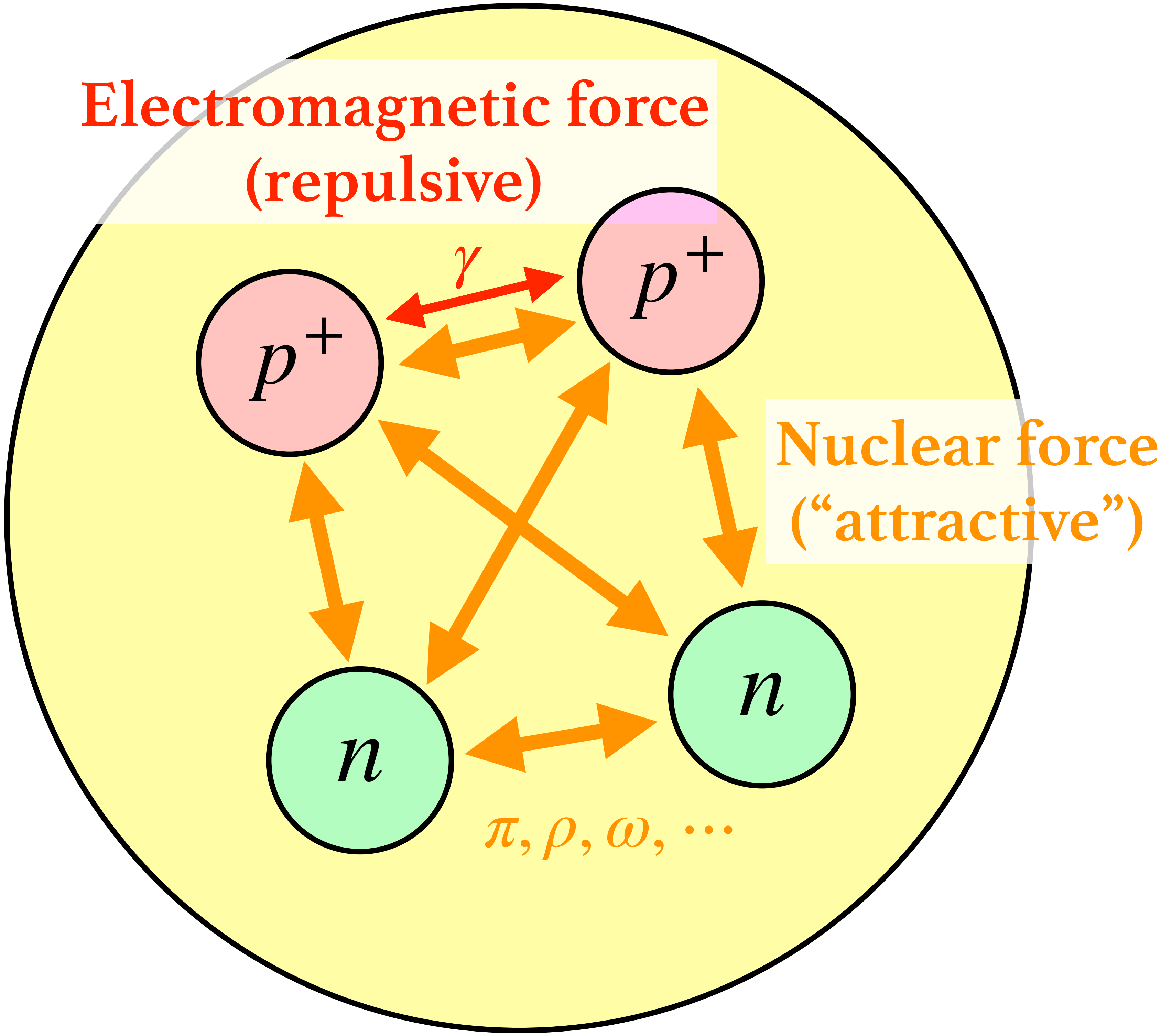
Atom = Nuclei + Electrons



Nuclei = Proton + Neutron?

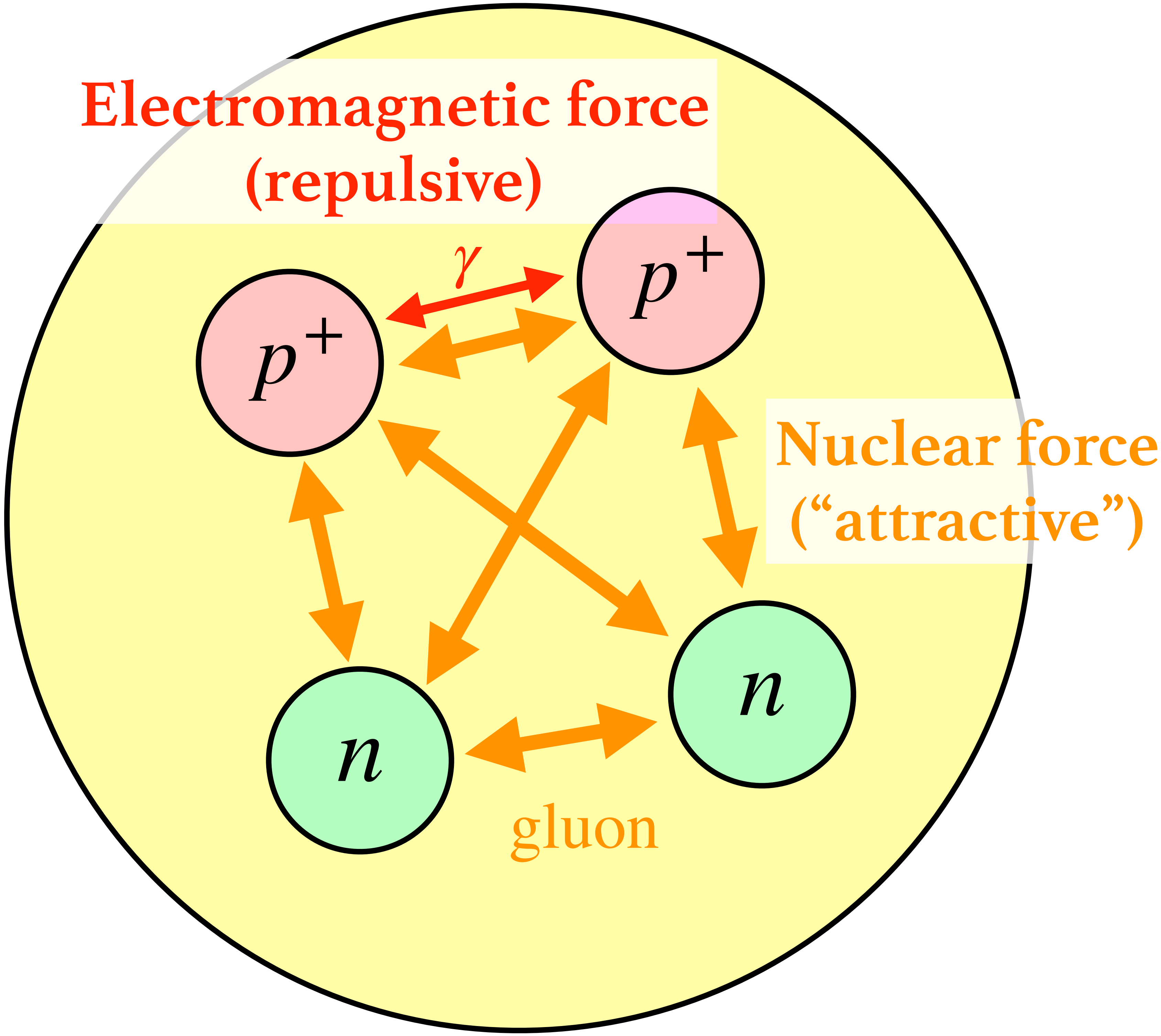


Nuclear (strong) force



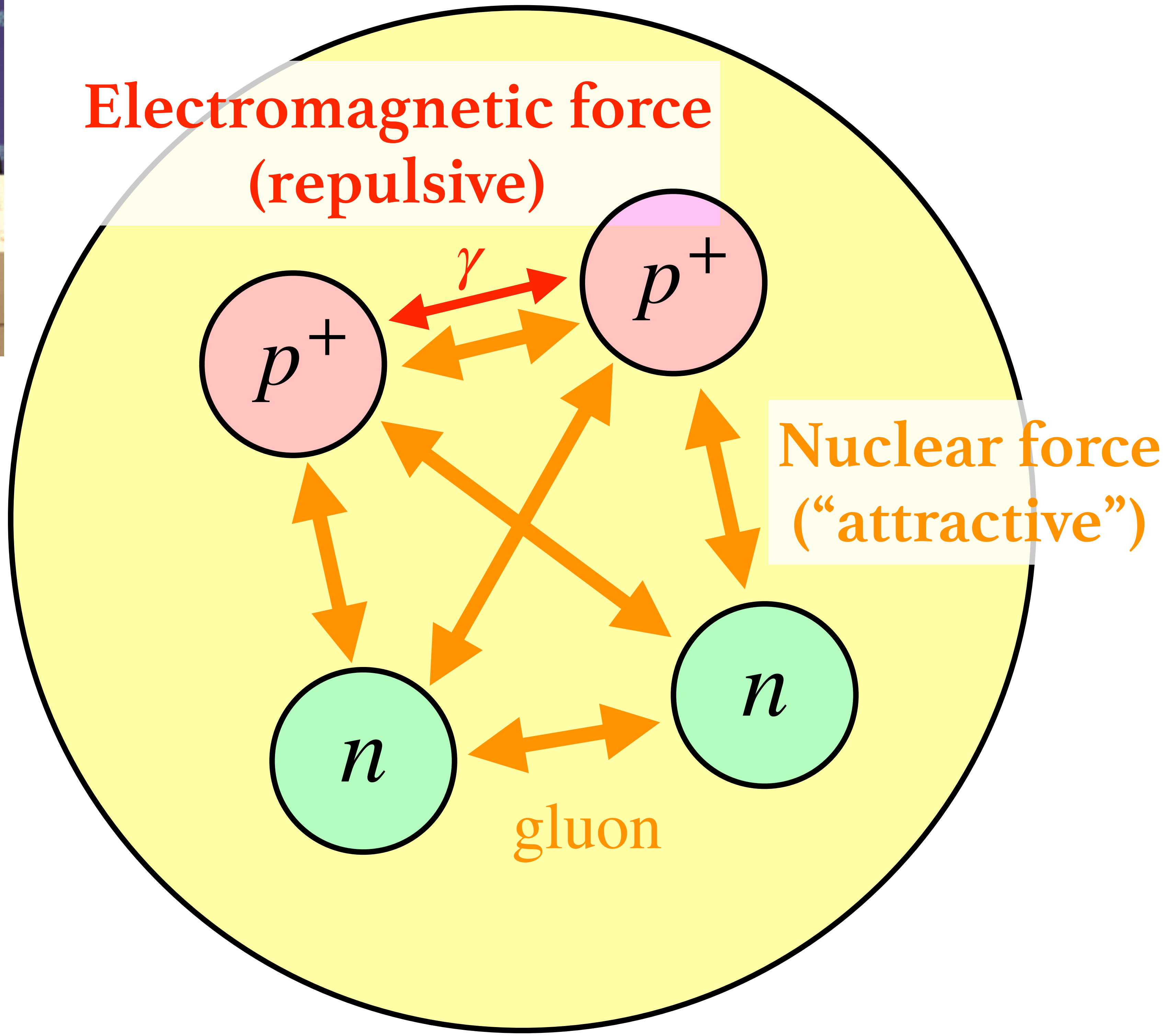
Yukawa theory (1935)

Nuclear (strong) force



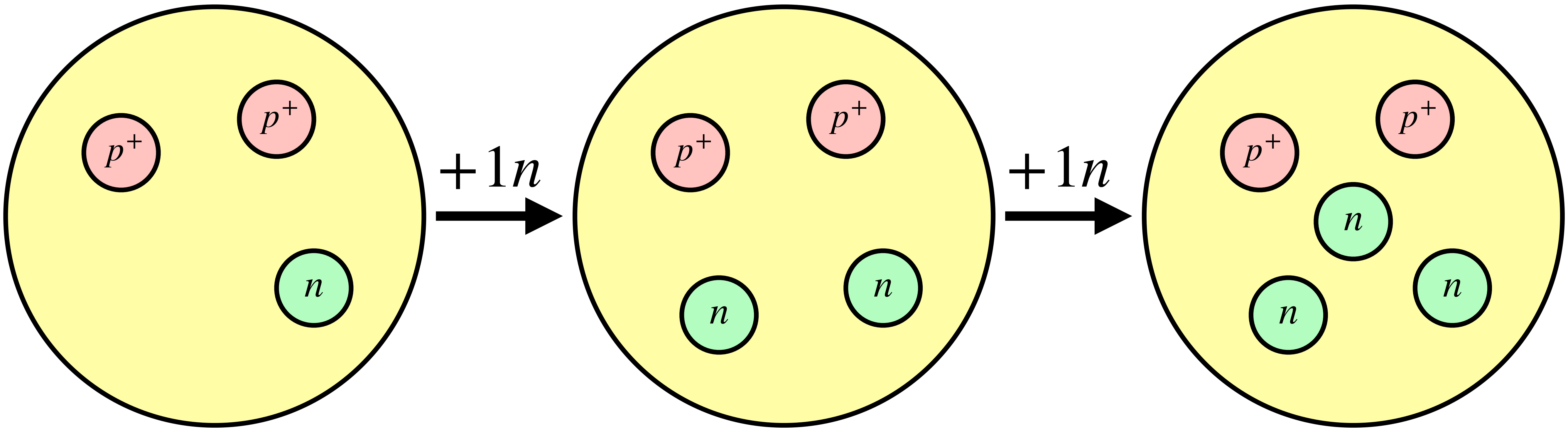
Yukawa theory (1935) → (QCD)

Nuclear (strong) force



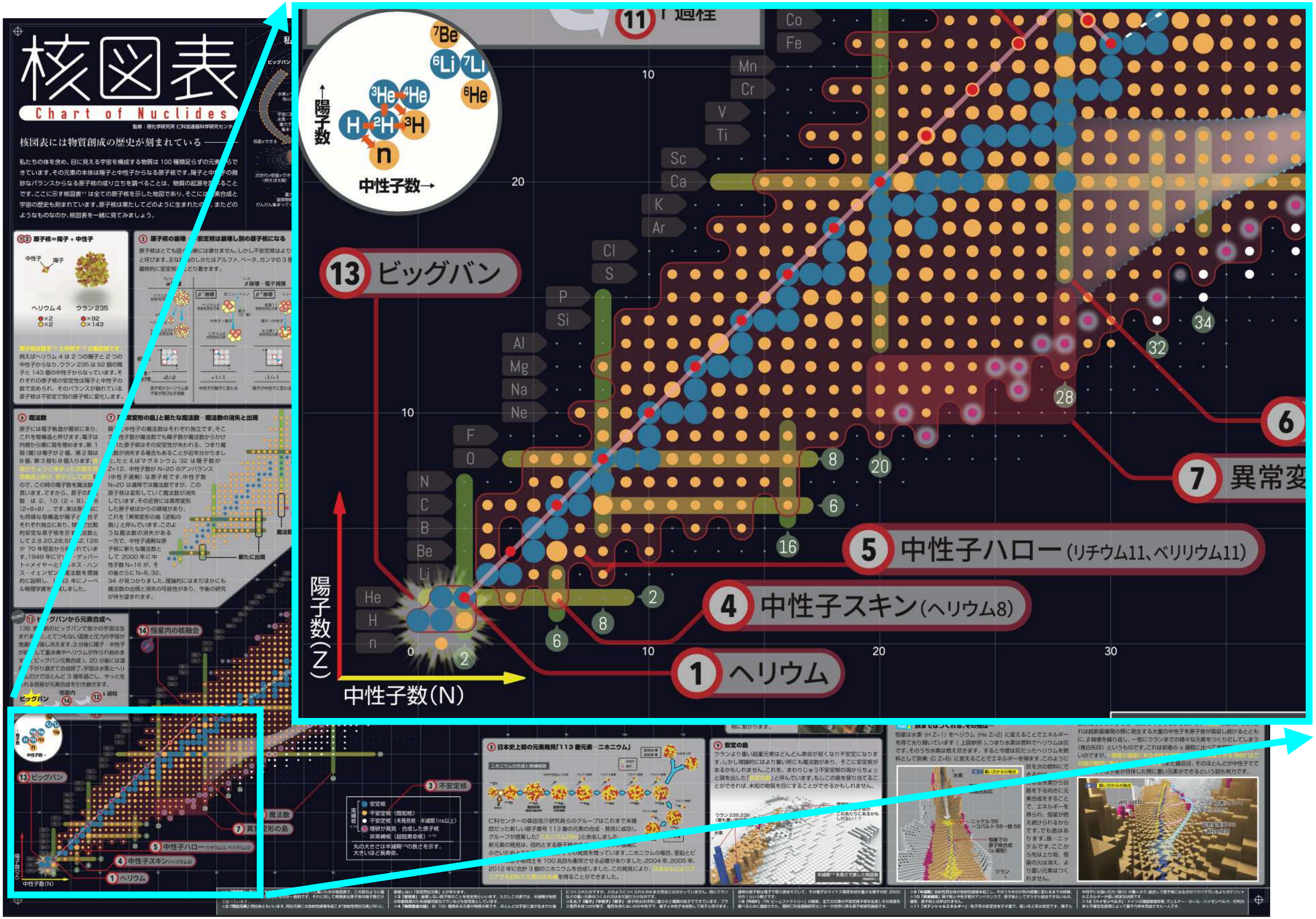
Yukawa theory (1935) → (QCD) → HAL QCD method (2007-)

Isotope and stability of nuclei

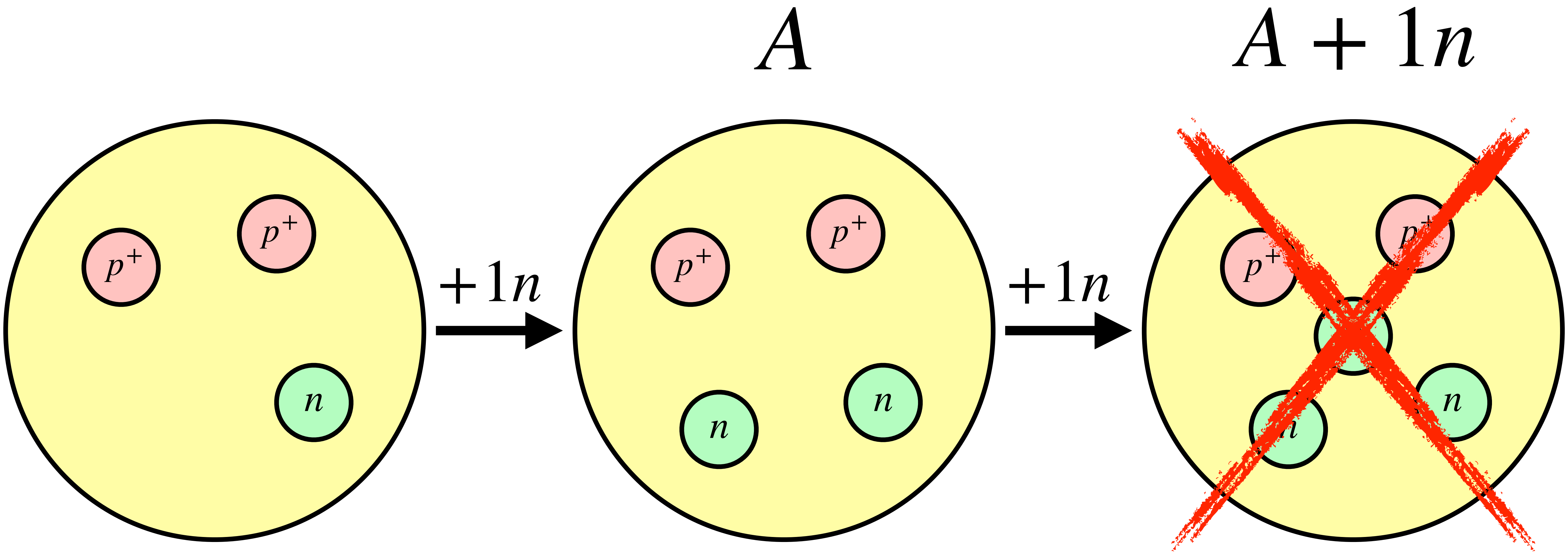


Given the atomic number Z,
how many neutrons can the nuclei have?

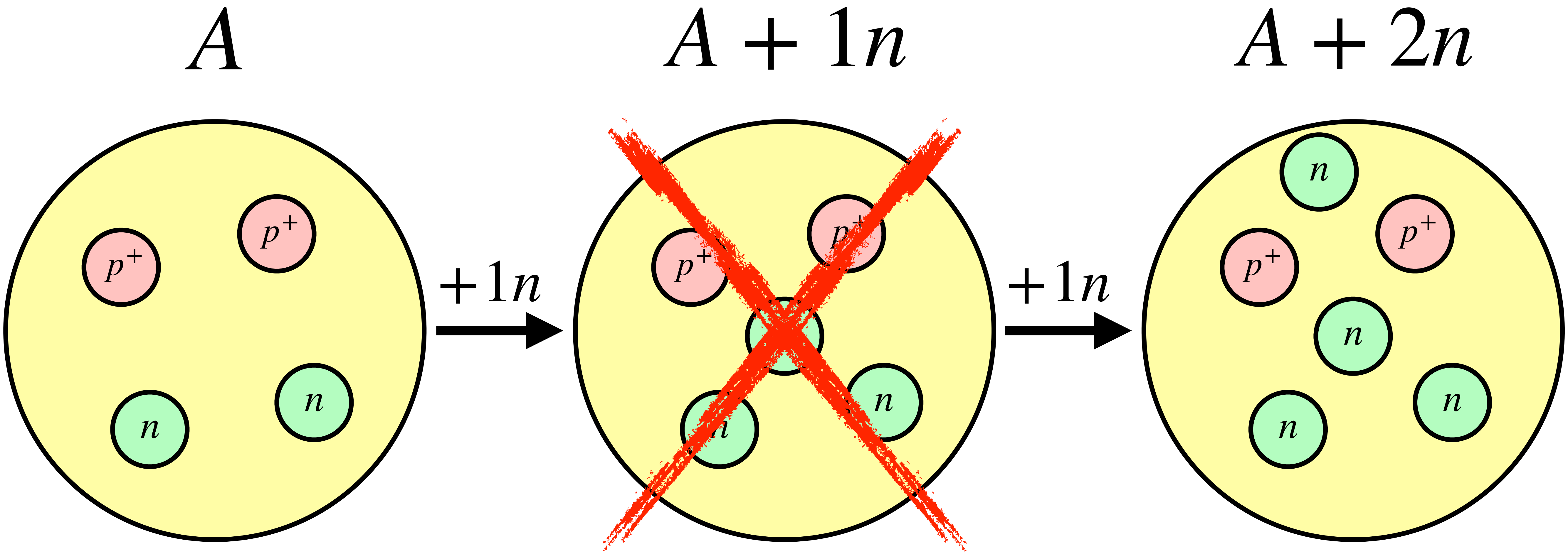
Chart of Nuclides



A naive expectation



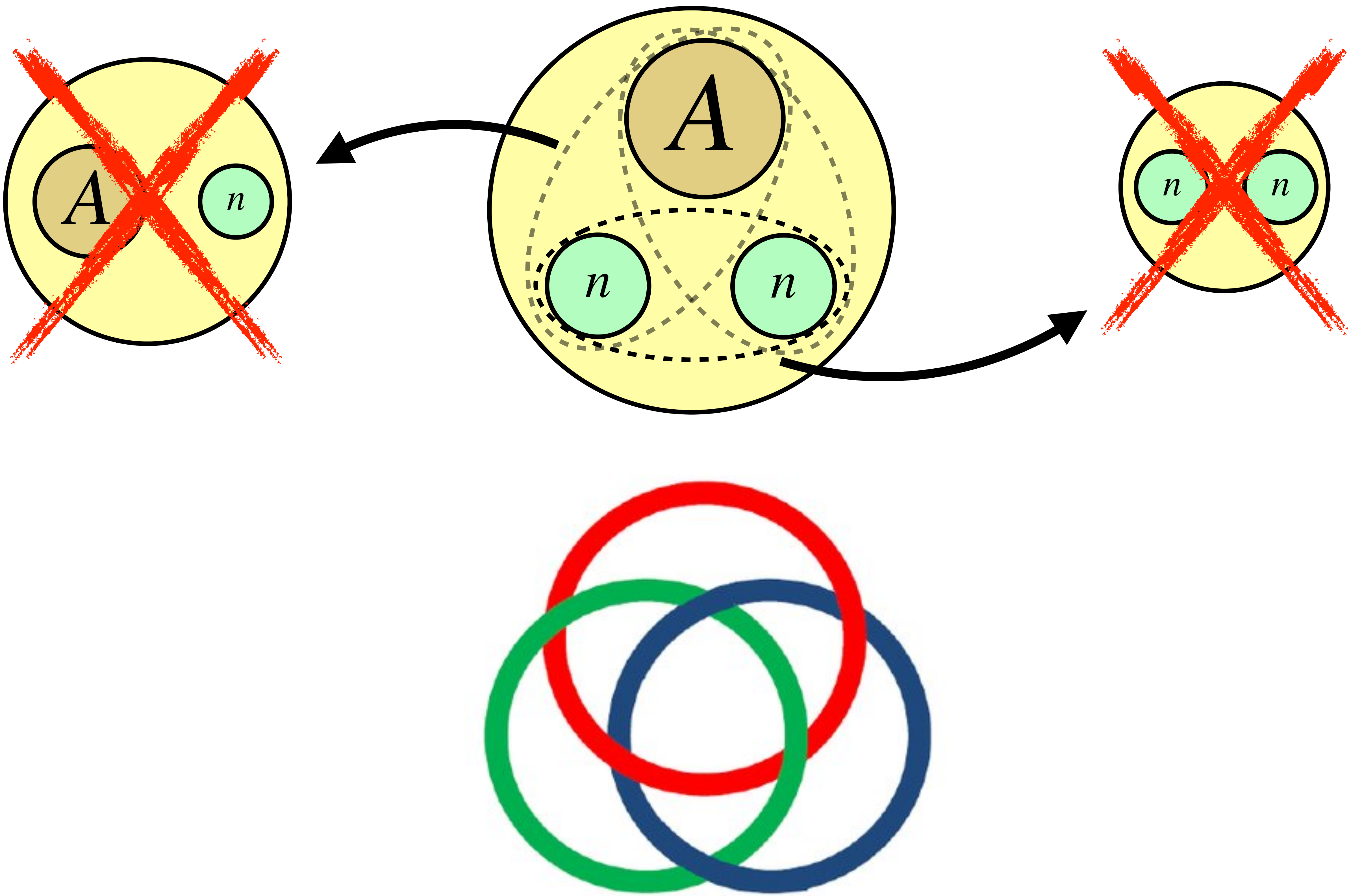
Exotic nuclei



$A + 1n$ **does not form the bound state.**

BUT, $A + 2n$ does form the bound state!!

Borromean nuclei



Examples of Borromean nuclei

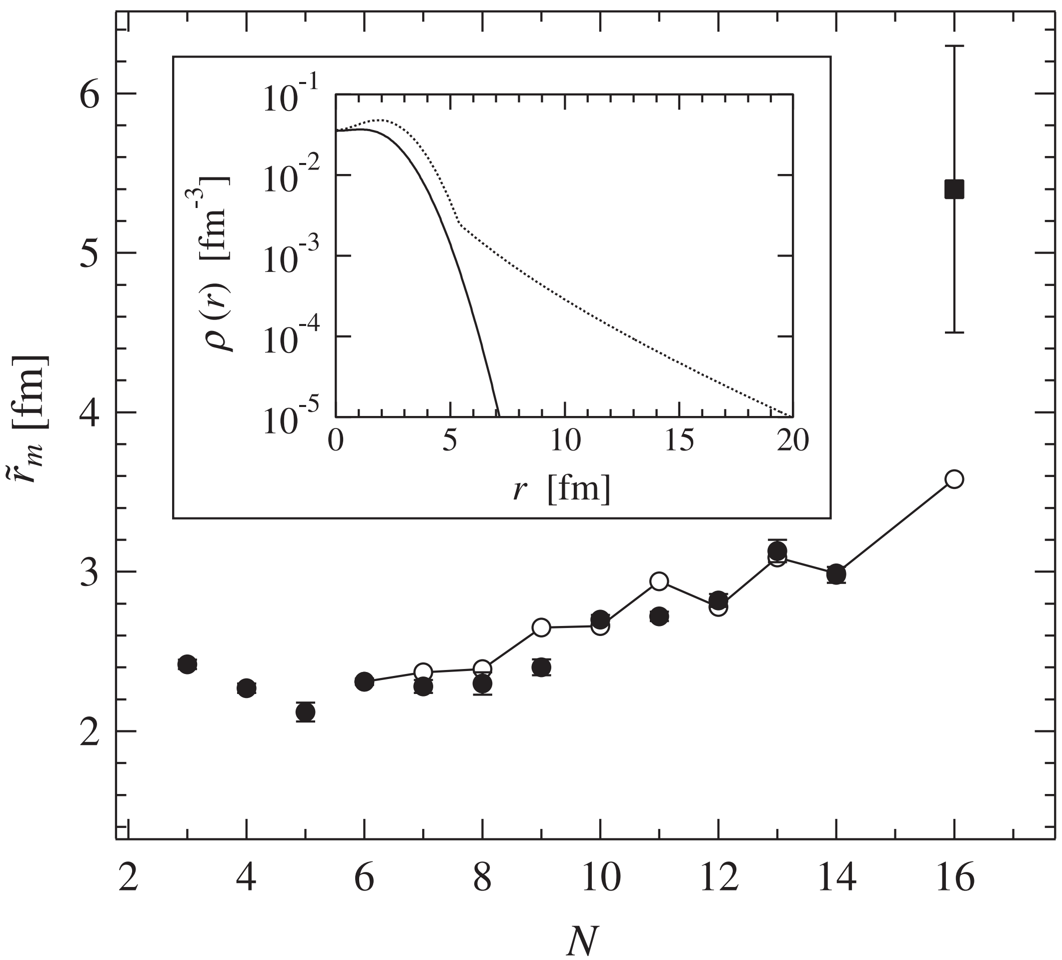
$$^{6}\mathrm{He} = {}^{4}\mathrm{He} + 2n$$

$$^{11}\mathrm{Li} = {}^{9}\mathrm{Li} + 2n$$

$$^{22}\mathrm{C} = {}^{20}\mathrm{C} + 2n$$

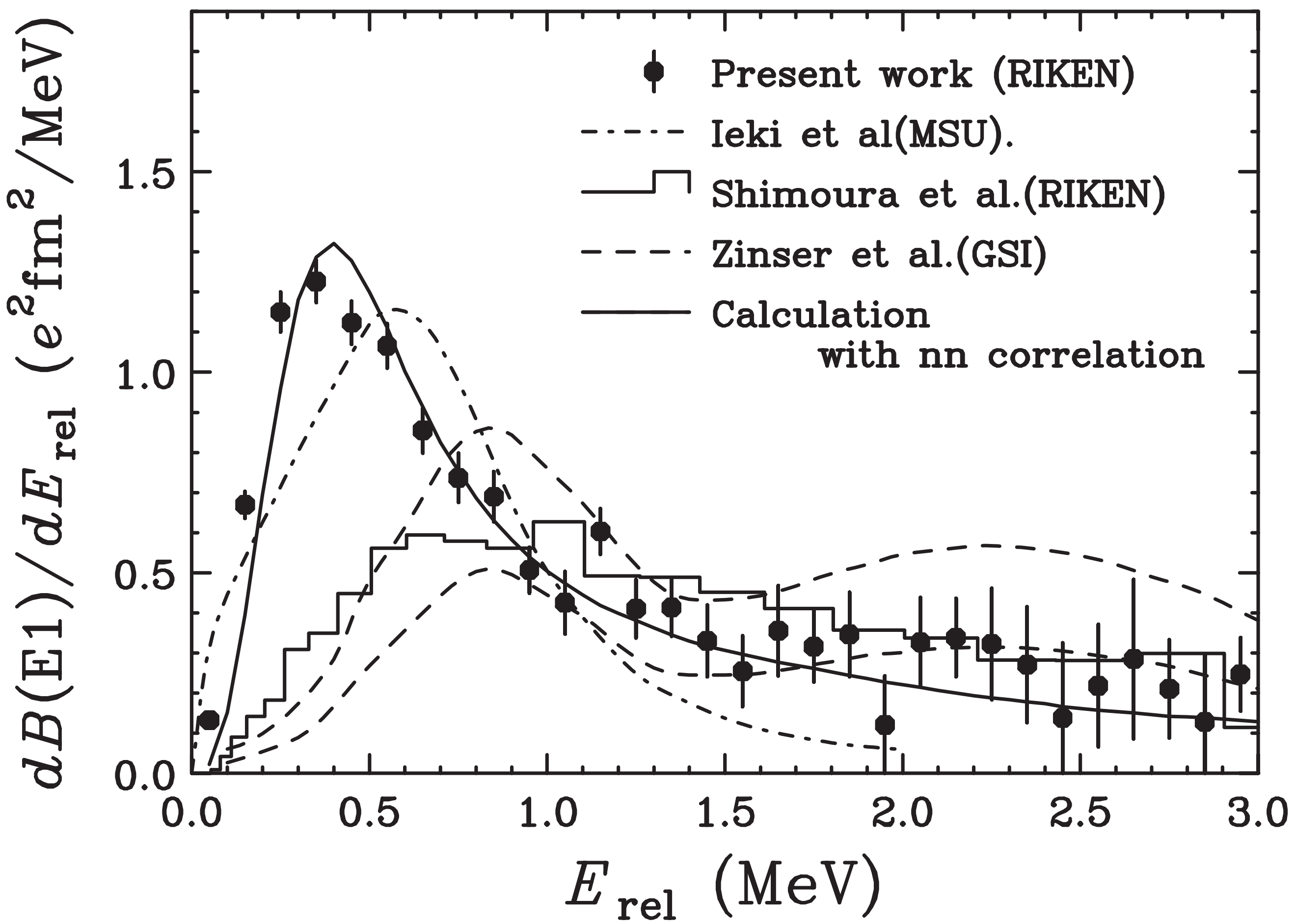
What is measured special?

◆ Large matter radius



Tanaka et al. PRL 104, 062701 (2010)

◆ Soft dipole resonance



Nakamura et al. PRL **96,** 252502 (2006)

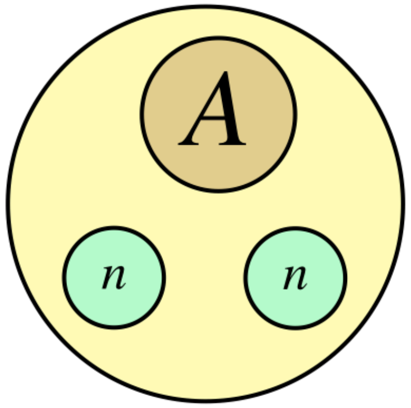
- Can we understand these phenomena from a simple EFT?
- Do we have certain universal relation for Borromean nuclei?

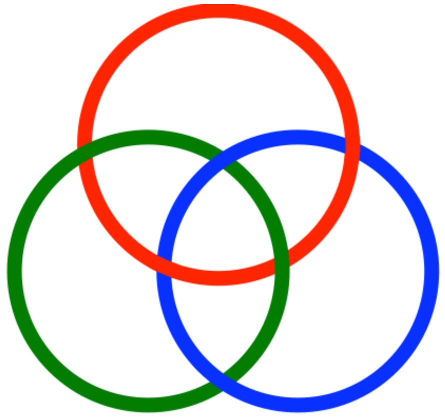
[M. Hongo, D. T. Son, arXiv:2201.09912 [nucl-th]]

Outline

Motivation:

Exotic (but universal) properties of Borromean nuclei?





Approach:

Effective field theory

Result:

(1) Ratio of the charge and matter radii

(2) E1 dipole strength function

Assumption for EFT to work

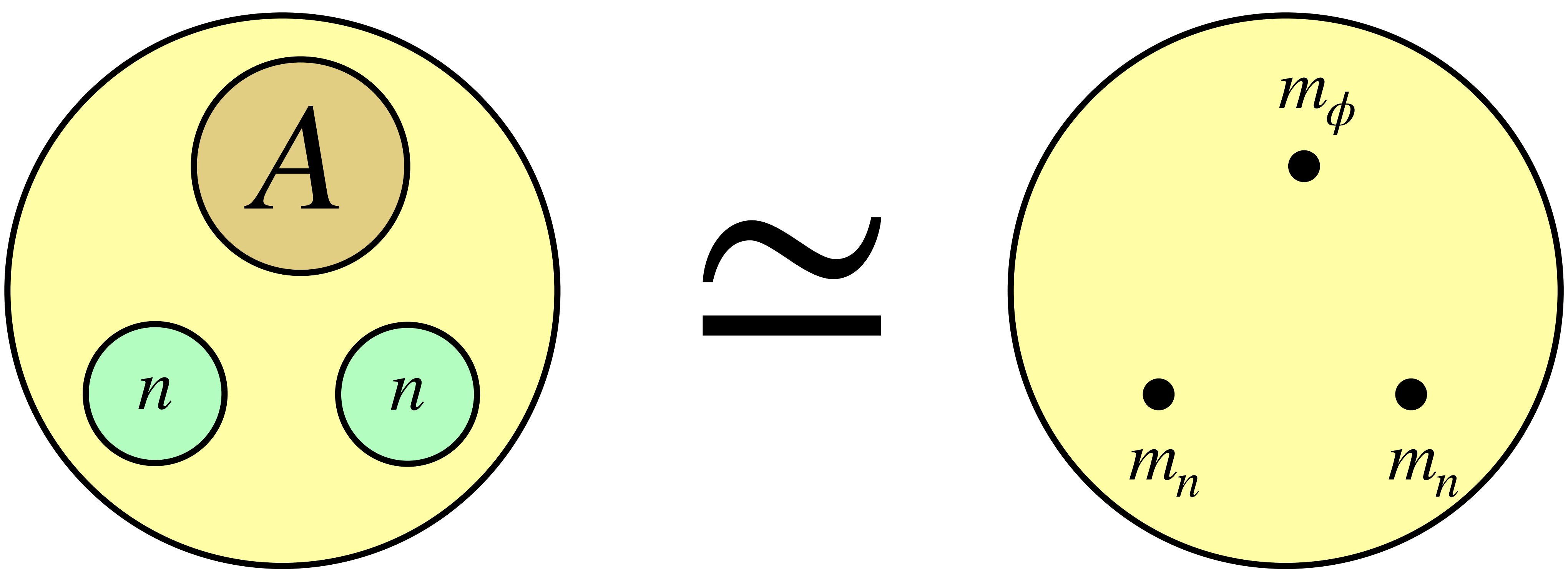
◆ **Two scales at present**

***s*-wave *n*eutron scattering length:** $a \simeq -19\,\mathrm{fm} \Leftrightarrow \epsilon_n = \dfrac{1}{m_n a^2} \simeq 120\,\mathrm{keV}$

Binding energy of Borromean: $B\,(=S_{2n}) \sim 100\,\mathrm{keV}$ for $^{22}\mathrm{C}$

We assume only these two scales are relevant!

(For instance, the neutron effective range is $r_0 \simeq 2.8\,\mathrm{fm} \ll |a|$)



Review on EFT of neutrons

◆ Effective Lagrangian

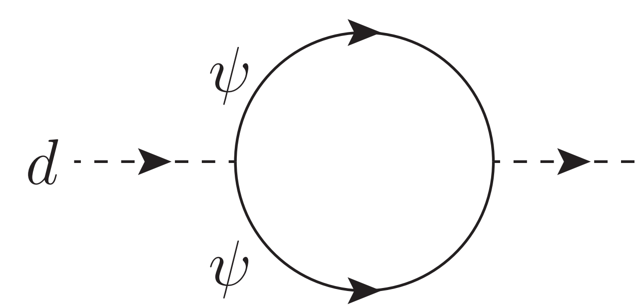
$$\mathcal{L}_n = \sum_\sigma \psi_\sigma^\dagger \left(\mathrm{i}\partial_t + \frac{\nabla^2}{2m_n} \right) \psi_\sigma + c_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

$$= \sum_\sigma \psi_\sigma^\dagger \left(\mathrm{i}\partial_t + \frac{\nabla^2}{2} \right) \psi_\sigma - \frac{1}{c_0} d^\dagger d + \psi_\uparrow^\dagger \psi_\downarrow^\dagger d + d^\dagger \psi_\downarrow \psi_\uparrow$$

[Neutron field: ψ_σ, Auxiliary dimer ("di-neutron") field: d]

◆ Green's function & scaling dimension of dimer

$$D(p) = -\frac{4\pi}{\sqrt{-p_0 + \frac{\boldsymbol{p}^2}{4} - \frac{1}{a}}} \qquad \left(\frac{1}{4\pi a} = -\frac{1}{c_0} + \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} \frac{1}{\boldsymbol{q}^2} \right)$$



$$\underbrace{D(p)}_{=-1} \equiv \int \underbrace{\mathrm{d}t\mathrm{d}^3x}_{=-2-3}\, \mathrm{e}^{\mathrm{i}(p^0 t - \boldsymbol{p}\cdot\boldsymbol{x})} \underbrace{\langle d(x) d^\dagger(0) \rangle}_{=2[d]} \;\Rightarrow\; [d] = 2$$

EFT for Borromean nuclei

◆ Effective Lagrangian

$$\mathcal{L} = h^\dagger\left(\mathrm{i}\partial_t + \frac{\nabla^2}{2m_h} + B\right)h + \phi^\dagger\left(\mathrm{i}\partial_t + \frac{\nabla^2}{2m_\phi}\right)\phi$$
$$+ g(h^\dagger\phi d + \phi^\dagger d^\dagger h) + \mathcal{L}_n + \text{counterterms}$$

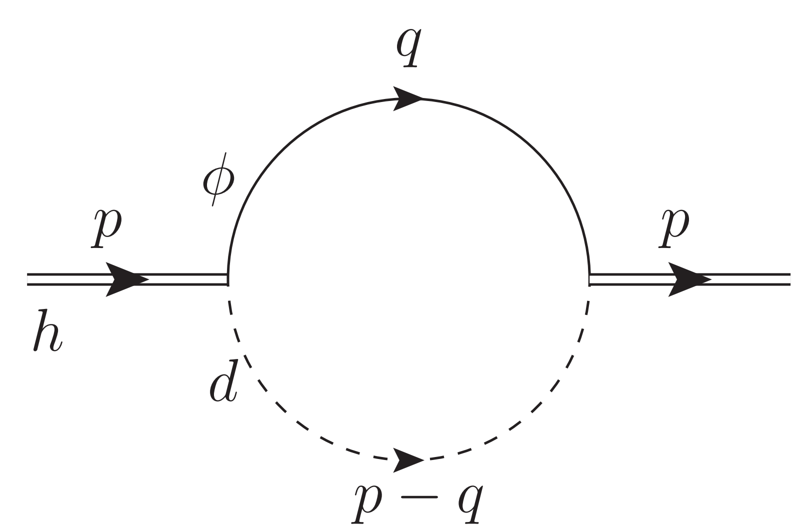
[Borromean nucleus: h, Core nucleus : ϕ with $m_h = (A+2)m_n$ and $m_\phi = Am_n$]

Noting that $[\mathcal{L}] = 5$ and $[\psi] = [\phi] = \dfrac{3}{2}$ and $[d] = 2$,

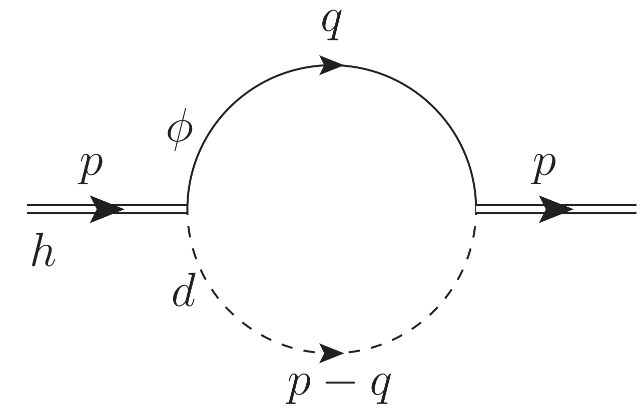
we find that the coupling constant g is dimensionless: $[g] = 0$

⬇

Need to renormalize by computing



Renormalization



◆ Green's function of Borromean nucleus

$$G_h^{-1}(p) = Z_h\left(p_0 - \frac{\boldsymbol{p}^2}{2m_h} + B_0\right) + 4\pi g^2 \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\, f_a\left(-p_0 + \frac{1}{2m_h}\boldsymbol{p}^2 + \frac{\boldsymbol{q}^2}{2\mu}\right)$$

with the field renormalization factor Z_h and $f_a(x) = \dfrac{1}{\sqrt{x} - \frac{1}{a}}$

◆ On-shell renormalization scheme

$$\left.\begin{aligned} G_h^{-1}(p_0, \mathbf{0})\Big|_{p_0=-B} &= 0 \\ \frac{\partial}{\partial p_0} G_h^{-1}(p_0, \mathbf{0})\Big|_{p_0=-B} &= 1 \end{aligned}\right\} \Leftrightarrow \left\{\begin{aligned} Z_h(B_0 - B) + 4\pi g^2 \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\, f_a(B_{\boldsymbol{q}}) &= 0 \\ Z_h - 4\pi g^2 \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\, f_a'(B_{\boldsymbol{q}}) &= 1 \end{aligned}\right.$$

Bare (physical) binding energy $B_0\,(B)$ and $B_q = B + \dfrac{q^2}{2\mu}$ and $\mu = \dfrac{2m_\phi}{m_h}$

Noting $Z_h = g^2/g_0^2$, we rewrite the second eq. as $g_0^2 = \dfrac{g^2}{1 + 4\pi g^2 \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\, f_a'(B_{\boldsymbol{q}})}$

RG equation and running coupling

Noting $Z_h = g^2/g_0^2$, we can rewrite the second condition as

$$Z_h - 4\pi g^2 \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} f_a'(B_{\boldsymbol{q}}) = 1 \quad \Leftrightarrow \quad g_0^2 = \frac{g^2}{1 + 4\pi g^2 \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} f_a'(B_{\boldsymbol{q}})}$$

◆ **RG equation and its solution**

$$\frac{\partial g}{\partial \ln E} = \beta(g) = \frac{2}{\pi}\left(\frac{A}{A+2}\right)^{3/2} g^3 \quad (>0)$$

$$g^2(E) = \frac{\pi}{4}\left(\frac{A+2}{A}\right)^{3/2} \frac{1}{\ln \frac{E_0}{E}} \quad (E_0 : \text{Energy of the Landau pole})$$

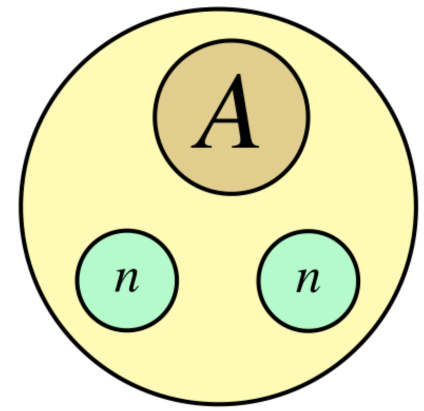
(i) Use experimental data to determine the running coupling at E

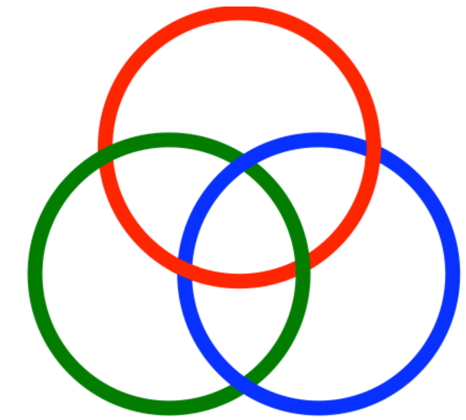
(ii) Compute ratio of two observables at the same order of g^2

Outline

Motivation:

Exotic (but universal) properties of Borromean nuclei?





Approach:

Effective field theory of point-like particles based on two relevant scales: binding energy B and scattering length a

Result:

(1) Ratio of the charge and matter radii

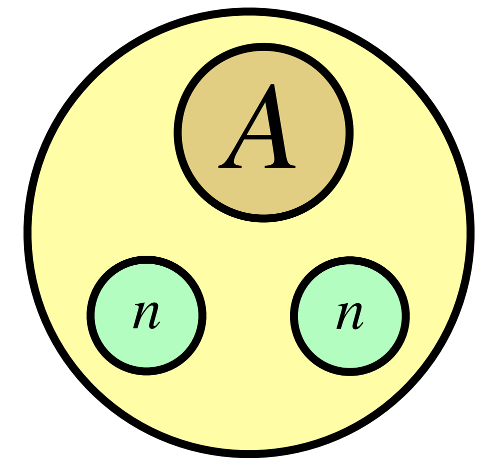
(2) E1 dipole strength function

(I) Charge and Matter radii

“Sizes” of Borromean nuclei

◆ Mean-square radii

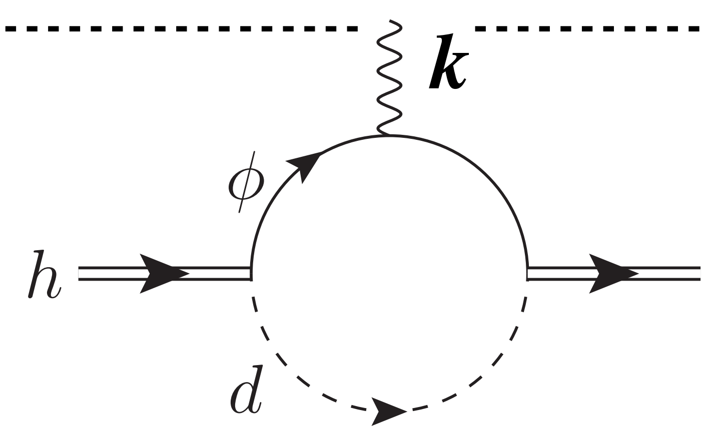
- Core size = Charge radius: $\langle r_c^2 \rangle$
- Di-neutron radius: $\langle r_n^2 \rangle$
- Matter radius: $\langle r_m^2 \rangle = \frac{2}{A+2}\langle r_n^2 \rangle + \frac{A}{A+2}\langle r_c^2 \rangle$



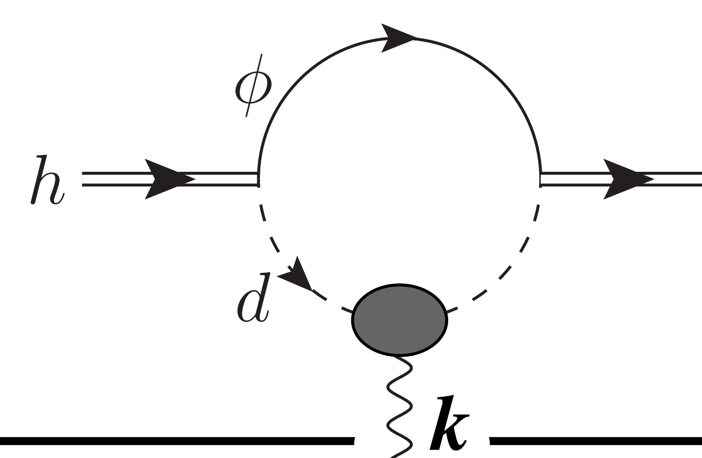
◆ Form factors and radii

$$F_a(\boldsymbol{k}^2) = 1 - \frac{1}{6}\boldsymbol{k}^2\langle r_a^2 \rangle + \cdots \text{ with } a = c, n$$

$F_c(\boldsymbol{k}^2) =$



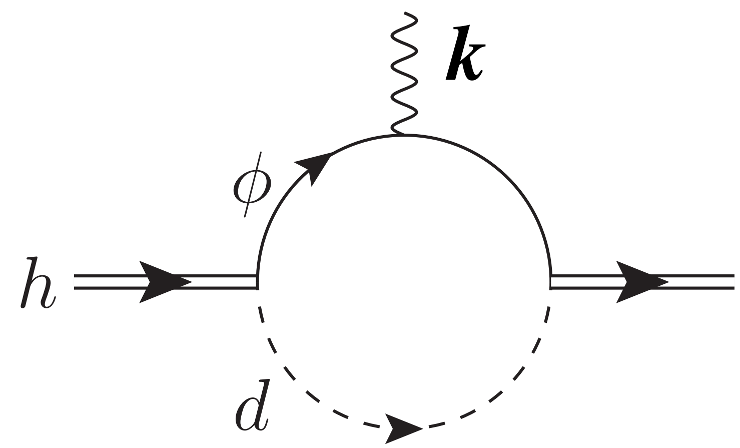
$F_n(\boldsymbol{k}^2) =$



Charge radius

◆ Charge radius of Borromean nuclei

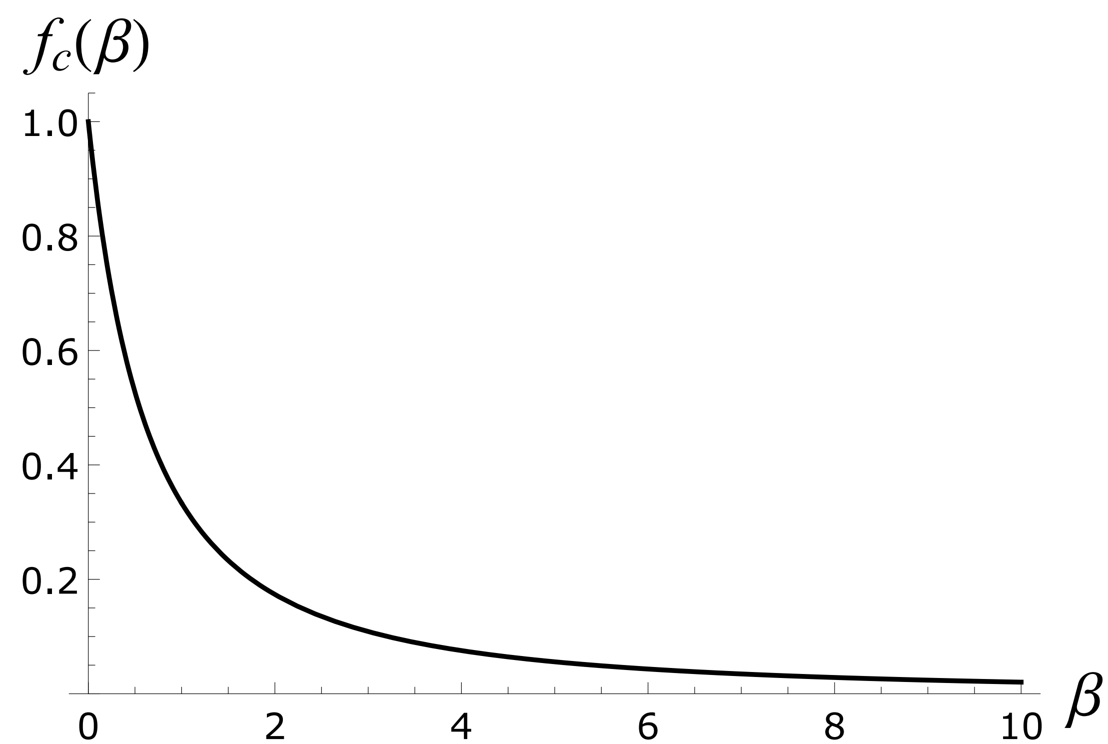
$$\langle r_c^2 \rangle = -6 \lim_{\boldsymbol{k}\to 0} \frac{\partial}{\partial \boldsymbol{k}^2}$$



$$= \frac{4}{\pi} \frac{A^{1/2}}{(A+2)^{5/2}} \frac{g^2}{B} f_c(\beta)$$

$$f_c(\beta) = \begin{cases} \dfrac{1}{1-\beta^2} - \dfrac{\beta \arccos \beta}{(1-\beta^2)^{3/2}}\,, & \beta < 1 \\[2ex] -\dfrac{1}{\beta^2 - 1} + \dfrac{\beta\, \mathrm{arccosh}\, \beta}{(\beta^2-1)^{3/2}}\,, & \beta > 1 \end{cases}$$

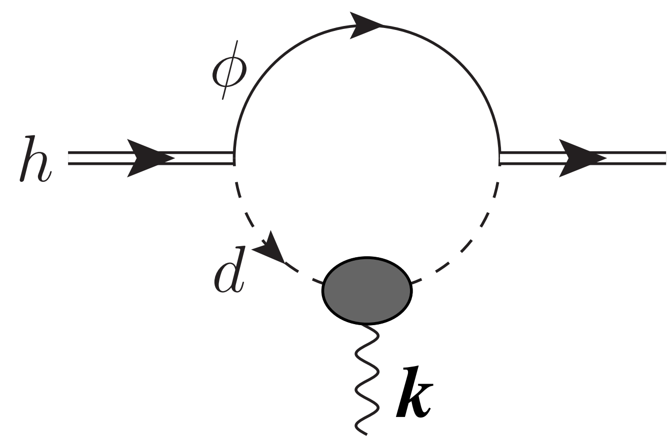
$$\beta = \frac{1}{-a\sqrt{B}} = \sqrt{\frac{\epsilon_n}{B}}$$



Di-neutron radius

◆ Di-neutron radius of Borromean nuclei

$$\langle r_n^2 \rangle = -6 \lim_{\boldsymbol{k}\to 0} \frac{\partial}{\partial \boldsymbol{k}^2}$$



$$\mathcal{L}_n = \sum_\sigma \psi_\sigma^\dagger \left(\mathrm{i}\partial_t + \frac{\nabla^2}{2} \right) \psi_\sigma - \frac{1}{c_0} d^\dagger d + \psi_\uparrow^\dagger \psi_\downarrow^\dagger d + d^\dagger \psi_\downarrow \psi_\uparrow$$

$$D(p) = -\frac{4\pi}{\sqrt{-p_0 + \frac{\boldsymbol{p}^2}{4}} - \frac{1}{a}}$$

The leading coupling between a dimer and "photon" is *not* given by a minimal gauge coupling!!

Effective dimer-photon vertex

◆ Analytic formula for effective vertex at small k^2

$$\Gamma_{dd\gamma}(k,p) = \Gamma_0(P_0) + k^2\Gamma_1(P_0) + K_0^2\Gamma_2(P_0)$$

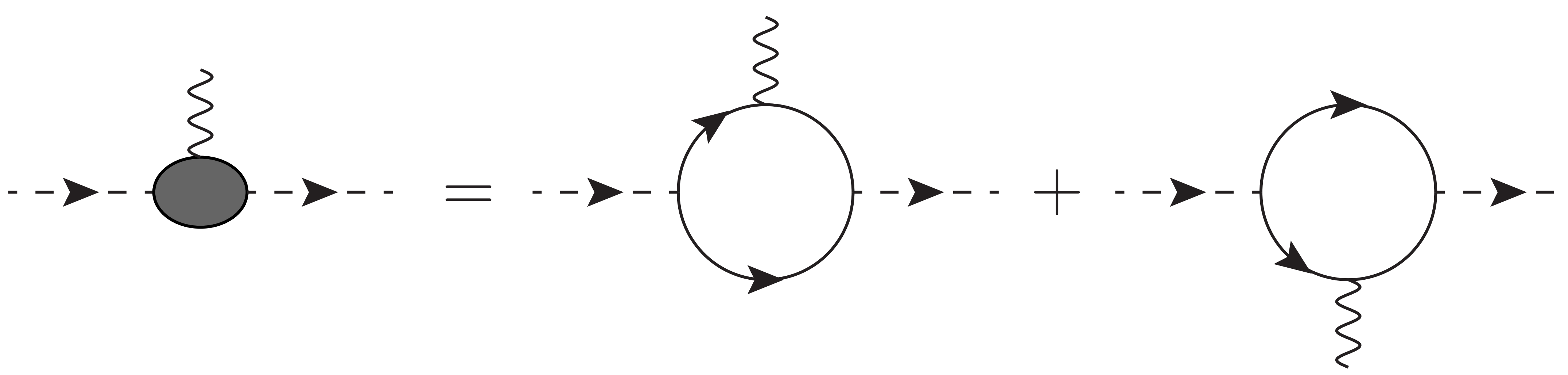
with

$$\Gamma_0(P_0) = \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\,\frac{2}{\left(P_0-\boldsymbol{q}^2\right)^2} = \frac{1}{4\pi}\frac{1}{\sqrt{-P_0}}\,,$$

$$\Gamma_1(P_0) = \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\left[\frac{1}{2}\frac{1}{(P_0-\boldsymbol{q}^2)^3} + \frac{1}{6}\frac{\boldsymbol{q}^2}{(P_0-\boldsymbol{q}^2)^4}\right] = -\frac{5}{384\pi}\frac{1}{(-P_0)^{3/2}}\,,$$

$$\Gamma_2(P_0) = \frac{1}{2}\int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\,\frac{1}{\left(P_0-\boldsymbol{q}^2\right)^4} = \frac{1}{128\pi}\frac{1}{(-P_0)^{5/2}}\,.$$

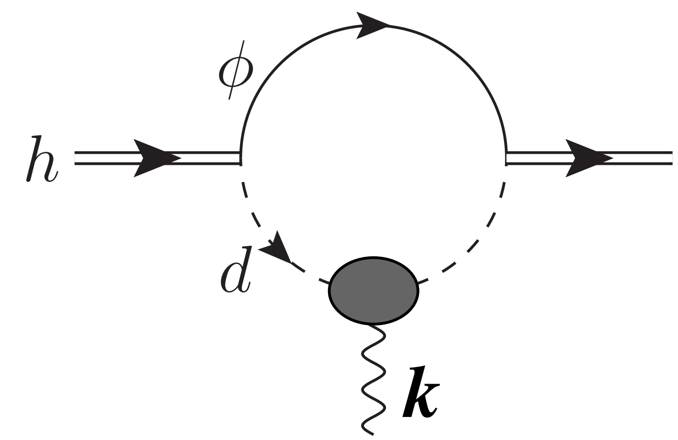
$$P_0 = p_0 - \frac{\boldsymbol{p}^2}{4}\,,\qquad K_0 = k_0 - \frac{\boldsymbol{p}\cdot\boldsymbol{k}}{2}\,,\qquad k = |\boldsymbol{k}|$$



Di-neutron radius

◆ Charge radius of Borromean nuclei

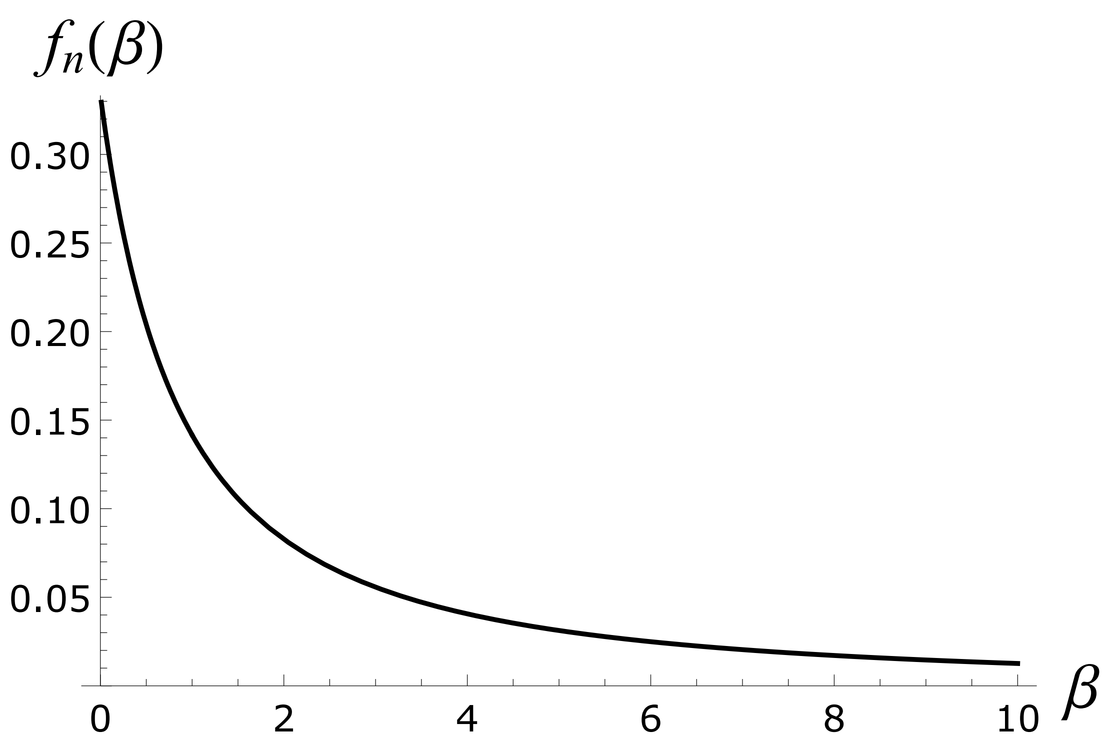
$$\langle r_n^2 \rangle = -6 \lim_{\boldsymbol{k}\to 0} \frac{\partial}{\partial \boldsymbol{k}^2}$$



$$= \frac{g^2}{\pi B} \left(\frac{A}{A+2} \right)^{3/2} \left[f_n(\beta) + \frac{A}{A+2} f_c(\beta) \right]$$

$$f_n(\beta) = \begin{cases} \dfrac{1}{\beta^3}\left[\pi - 2\beta + (\beta^2 - 2)\dfrac{\arccos\beta}{\sqrt{1-\beta^2}}\right], & \beta < 1 \\ \dfrac{1}{\beta^3}\left[\pi - 2\beta + (\beta^2 - 2)\dfrac{\operatorname{arccosh}\beta}{\sqrt{\beta^2-1}}\right], & \beta > 1 \end{cases}$$

$$\beta = \frac{1}{-a\sqrt{B}} = \sqrt{\frac{\epsilon_n}{B}}$$



Universal relation

Charge radius: $\langle r_c^2 \rangle = \frac{4}{\pi} \frac{A^{1/2}}{(A+2)^{5/2}} \frac{g^2}{B} f_c(\beta)$

Di-neutron radius: $\langle r_n^2 \rangle = \frac{g^2}{\pi B} \left(\frac{A}{A+2} \right)^{3/2} \left[f_n(\beta) + \frac{A}{A+2} f_c(\beta) \right]$

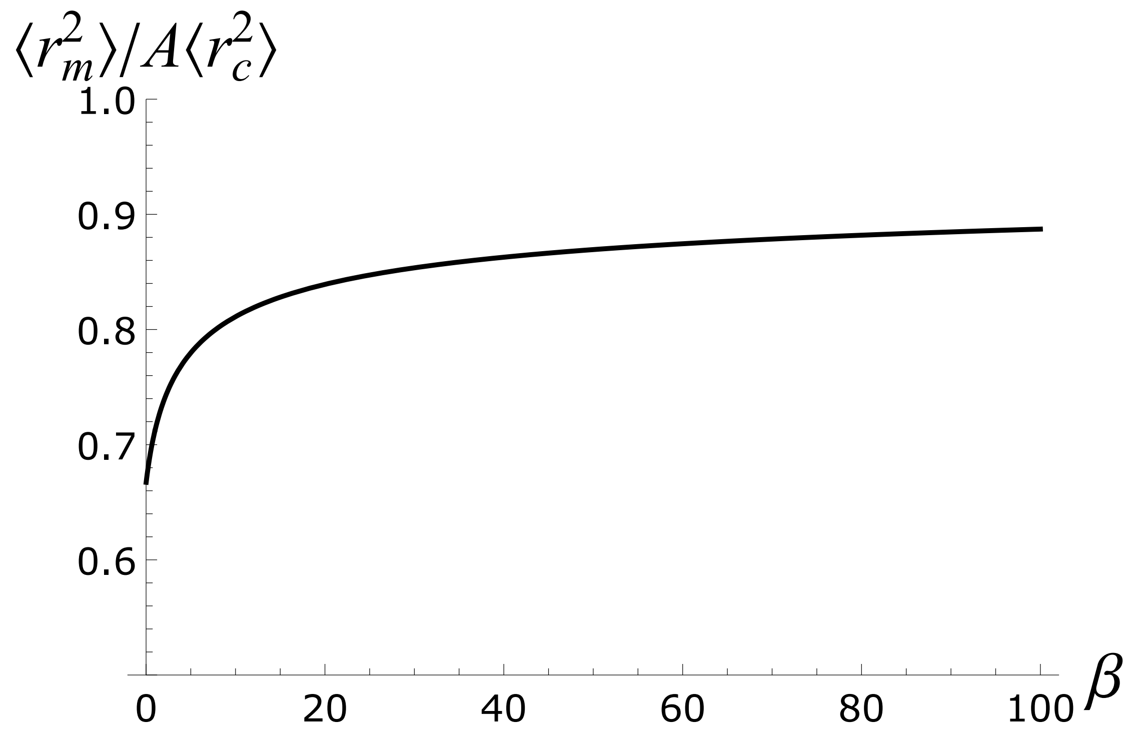
Matter radius: $\langle r_m^2 \rangle = \frac{2}{A+2} \langle r_n^2 \rangle + \frac{A}{A+2} \langle r_c^2 \rangle$

Each result is ***not*** universal because it is proportional to the running coupling, which is not expressed by B and a

◆ **Universal ratio of matter and charge radii**

$$\frac{\langle r_m^2 \rangle}{\langle r_c^2 \rangle} = \frac{A}{2} \left[1 + \frac{f_n(\beta)}{f_c(\beta)} \right]$$

Universal relation



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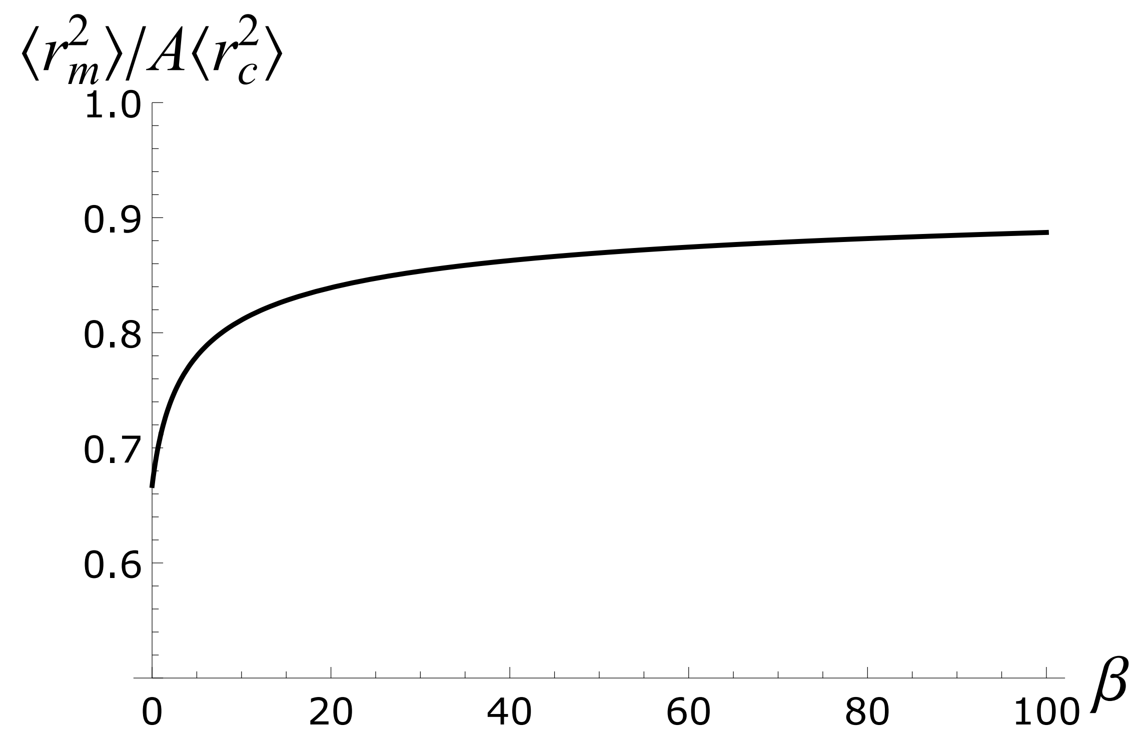
◆ **Universal ratio of matter and charge radii**

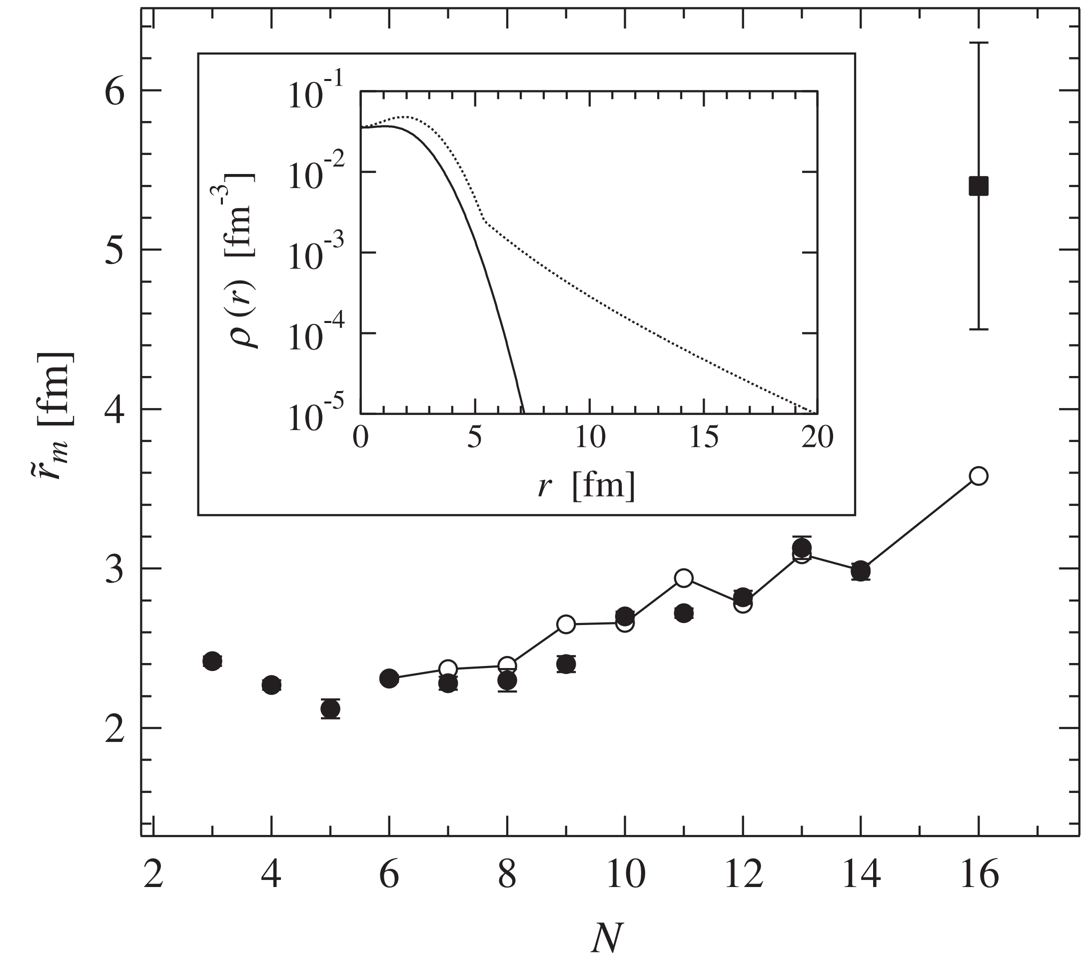
$$\frac{\langle r_m^2\rangle}{\langle r_c^2\rangle} = \frac{A}{2}\left[1+\frac{f_n(\beta)}{f_c(\beta)}\right] = \begin{cases} \frac{2}{3}A, & B \gg \epsilon_n \\ A, & B \ll \epsilon_n \end{cases}$$

Universal



Each result is ***not*** universal bec
the running coupling, which is

◆ Large matter radius



Tanaka et al. PRL 104, 062701 (2010)

◆ Universal ratio of matter and charge radii

$$\frac{\langle r_m^2\rangle}{\langle r_c^2\rangle} = \frac{A}{2}\left[1+\frac{f_n(\beta)}{f_c(\beta)}\right] = \begin{cases} \frac{2}{3}A, & B \gg \epsilon_n \\ A, & B \ll \epsilon_n \end{cases}$$

➡ **Suggest a large matter radius of Borromean nuclei?**

(2) Dipole strength function

Dipole strength function

◆ Definition

$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega}(\omega) = \sum_n |\langle n|\boldsymbol{\mathcal{M}}|0\rangle|^2 \delta(E_n - E_0 - \omega), \quad \boldsymbol{\mathcal{M}} = \sqrt{\frac{3}{4\pi}} Ze(\boldsymbol{r}_c - \boldsymbol{R}_{\mathrm{cm}})$$

[Coordinate of the core: $\boldsymbol{r}_c$, Coordinate of the cms: $\boldsymbol{R}_{\mathrm{cm}}$]

◆ Formula in terms of current correlation

$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = -\frac{3}{4\pi}\frac{1}{\pi\omega^2}\,\mathrm{Im}\,G_{JJ}(\omega), \quad \mathrm{i}G_{JJ}(\omega) = \int \mathrm{d}t\,\mathrm{e}^{\mathrm{i}\omega t}\langle 0|T\boldsymbol{J}(t)\boldsymbol{J}(0)|0\rangle$$

[Total electric current operator : $\boldsymbol{J}$]

Derivation.

Noting $\frac{\partial}{\partial t}\boldsymbol{\mathcal{M}} = \sqrt{\frac{3}{4\pi}}\boldsymbol{J}$, we find $\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = \frac{3}{4\pi}\frac{1}{\omega^2}\sum_n |\langle n|\boldsymbol{J}|0\rangle|^2 \delta(E_n - E_0 - \omega)$,

which is the spectral representation of the above formula.

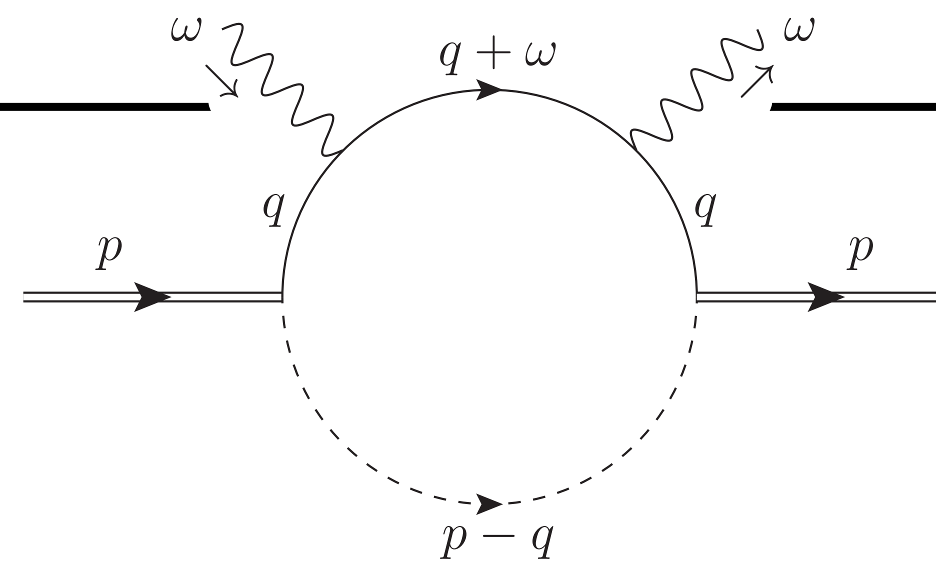
Result on dipole strength function

◆ Sum rules

$$\int_0^\infty \mathrm{d}\omega\, \frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = \frac{3}{4\pi} Z^2 e^2 \langle r_c^2 \rangle, \qquad \int_0^\infty \mathrm{d}\omega\, \omega \frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = \frac{3}{4\pi} Z^2 e^2 \frac{3}{A(A+2)}$$

◆ Analytic formula

$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = -\frac{3}{4\pi}\frac{1}{\pi\omega^2}\,\mathrm{Im}\, G_{JJ}(\omega) = -\frac{3}{4\pi}\frac{1}{\pi\omega^2}\,\mathrm{Im}$$



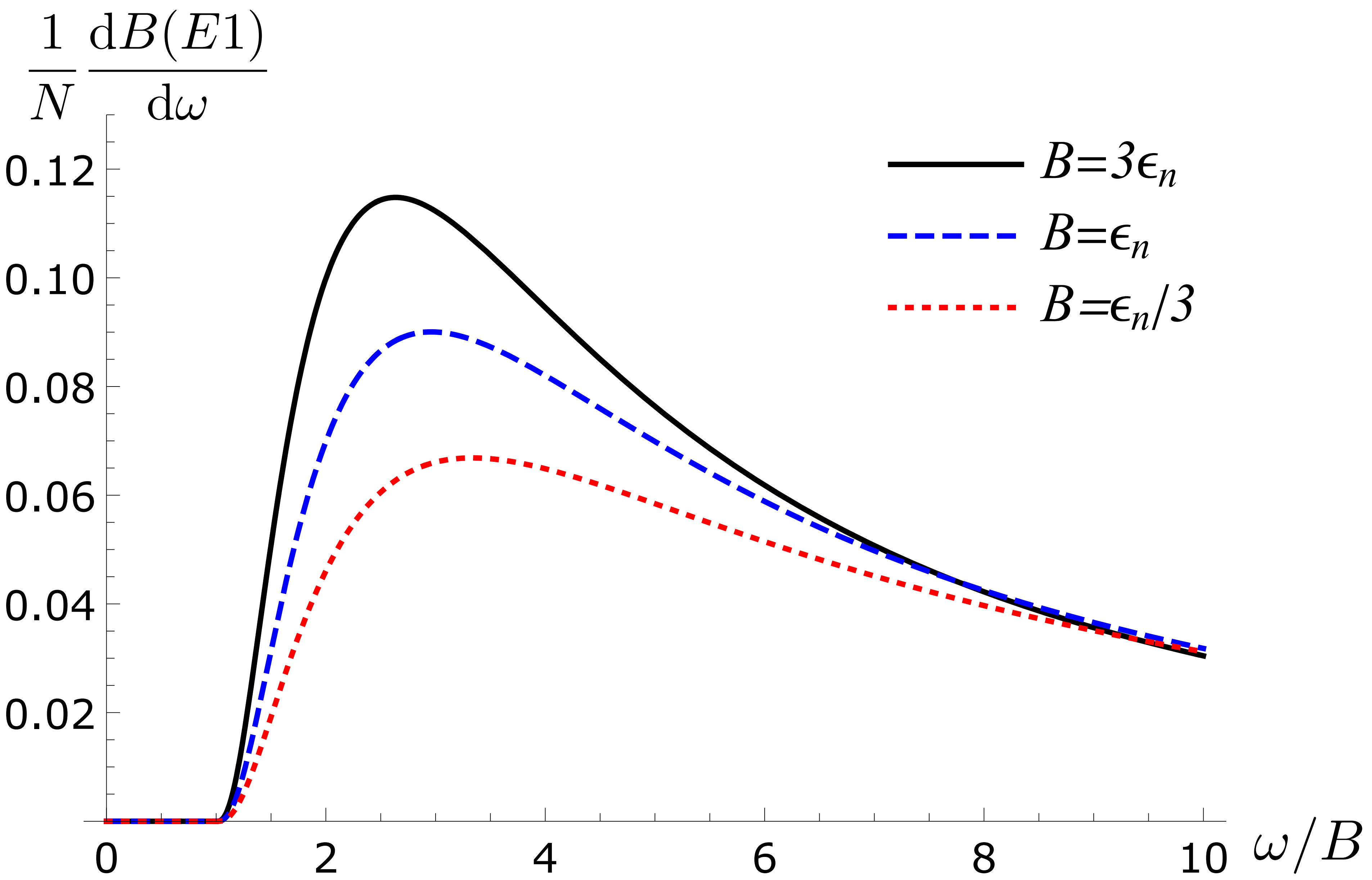
$$= -\frac{3}{4\pi}\frac{1}{\pi\omega^2}(Ze)^2 \frac{g^2}{m_\phi^2 \omega^2} \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\, \boldsymbol{q}^2\, \mathrm{Im}\, D\left(\omega - B - \frac{\boldsymbol{q}^2}{2m_\phi}, -\boldsymbol{q}\right)$$

$$= \frac{3}{4\pi} Z^2 e^2 \frac{12 g^2}{\pi} \frac{A^{1/2}}{(A+2)^{5/2}} \frac{(\omega - B)^2}{\omega^4} f_{E1}\left(\frac{1}{-a\sqrt{\omega - B}}\right)$$

with $f_{E1}(x) = 1 - \frac{8}{3} x (1+x^2)^{3/2} + 4x^2 \left(1 + \frac{2}{3} x^2\right)$

Normalized dipole strength function

$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = \frac{3}{4\pi} Z^2 e^2 \frac{12g^2}{\pi} \frac{A^{1/2}}{(A+2)^{5/2}} \frac{(\omega-B)^2}{\omega^4} f_{E1}\left(\frac{1}{-a\sqrt{\omega-B}}\right)$$

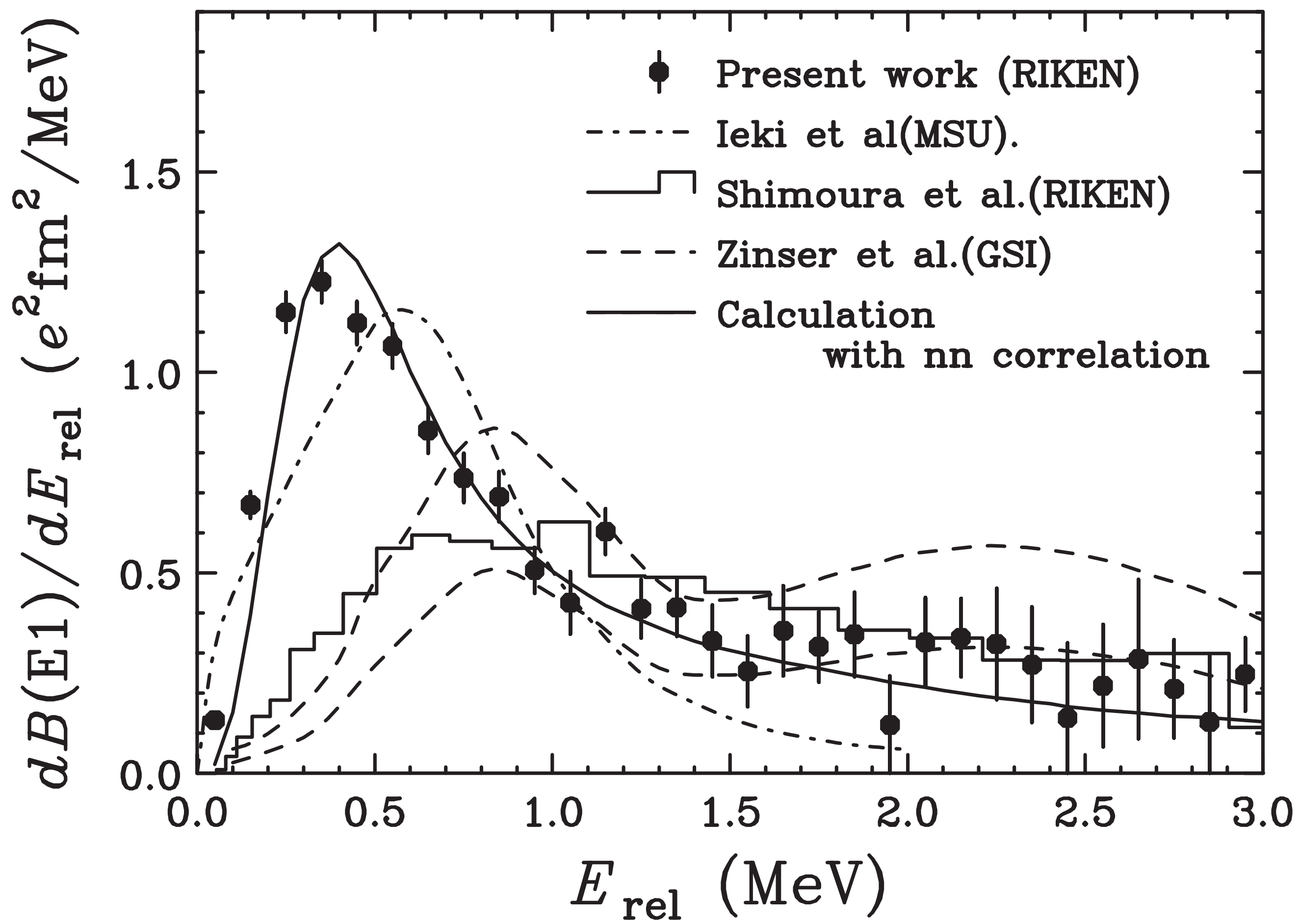


,

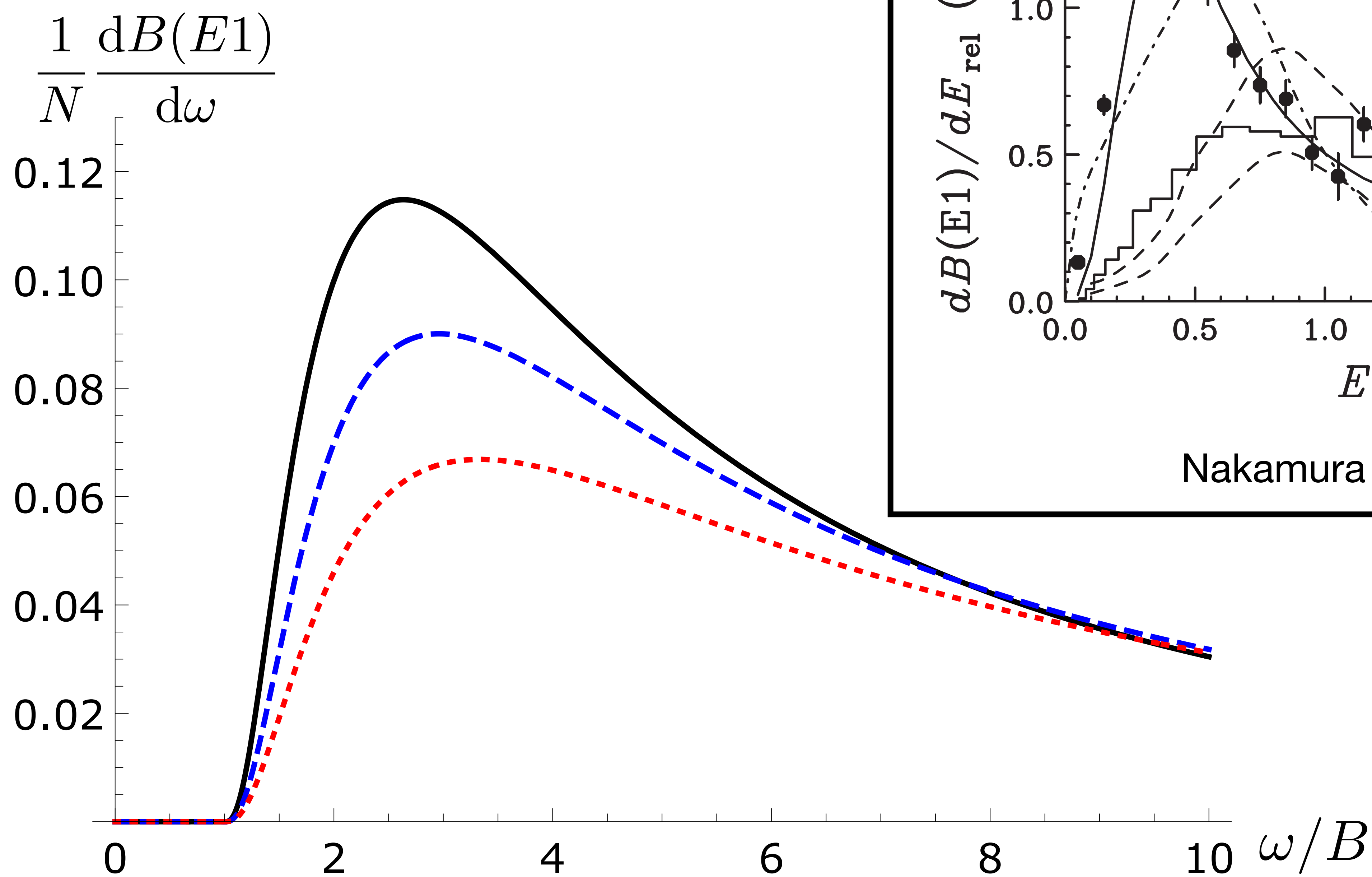
Normalized dipol

$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} = \frac{3}{4\pi}Z^2e^2\frac{12g^2}{\pi}\frac{A}{(A+}$$

◆ Soft Dipole resonance



Nakamura et al. PRL **96,** 252502 (2006)



➡ **Explain a soft dipole resonance of Borromean nuclei?**

Applicability of real systems

Examples of Borromean nuclei

$$^{6}\mathrm{He} = {}^{4}\mathrm{He} + 2n$$

$$^{11}\mathrm{Li} = {}^{9}\mathrm{Li} + 2n$$

$$^{22}\mathrm{C} = {}^{20}\mathrm{C} + 2n$$

Examples of Borromean nuclei

◆ **Two scales at present**

***s*-wave *n*eutron scattering length:** $a \simeq -19\,\text{fm} \Leftrightarrow \epsilon_n = \dfrac{1}{m_n a^2} \simeq 120\,\text{keV}$

Binding energy of Borromean: $B\,(=S_{2n}) \sim 100\,\text{keV}$ for ^{22}C

We assume only these two scales are relevant! ?

(For instance, the neutron effective range is $r_0 \simeq 2.8\,\text{fm} \ll |a|$)

$^{6}\text{He} = {}^{4}\text{He} + 2n\ :\ B \simeq 975\,\text{keV}$

$^{11}\text{Li} = {}^{9}\text{Li} + 2n\ :\ B \simeq 369\,\text{keV}$

Low-energy resonance seems to be present...

$^{22}\text{C} = {}^{20}\text{C} + 2n\ :\ B \sim 100\,\text{keV}$

Correction to our result from higher-order irrelevant terms are of order 20% or less!

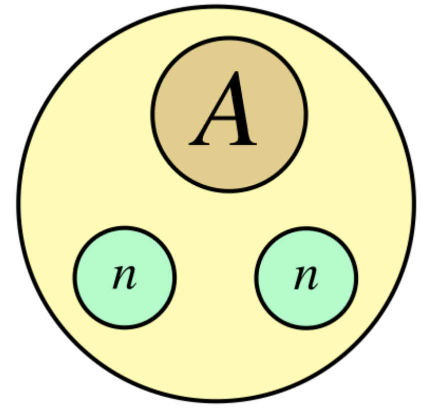
$\Delta\mathcal{L}_1 = a_{cn}\phi^\dagger\psi^\dagger\psi\phi$ ➡ Correction $\propto a_{cn}(2m_n B)^{1/2} \leq 0.2$ if $\begin{cases} B \leq 100\,\text{keV} \\ |a_{cn}| \leq 2.8\,\text{fm} \end{cases}$

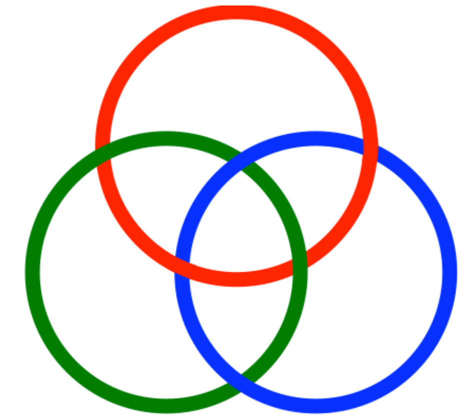
Summary

Summary

Motivation:

Exotic (but universal) properties of Borromean nuclei?





Approach:

Effective field theory of point-like particles based on two relevant scales: binding energy B and scattering length a

Result:

(1) Ratio of the charge and matter radii

(2) E1 dipole strength function