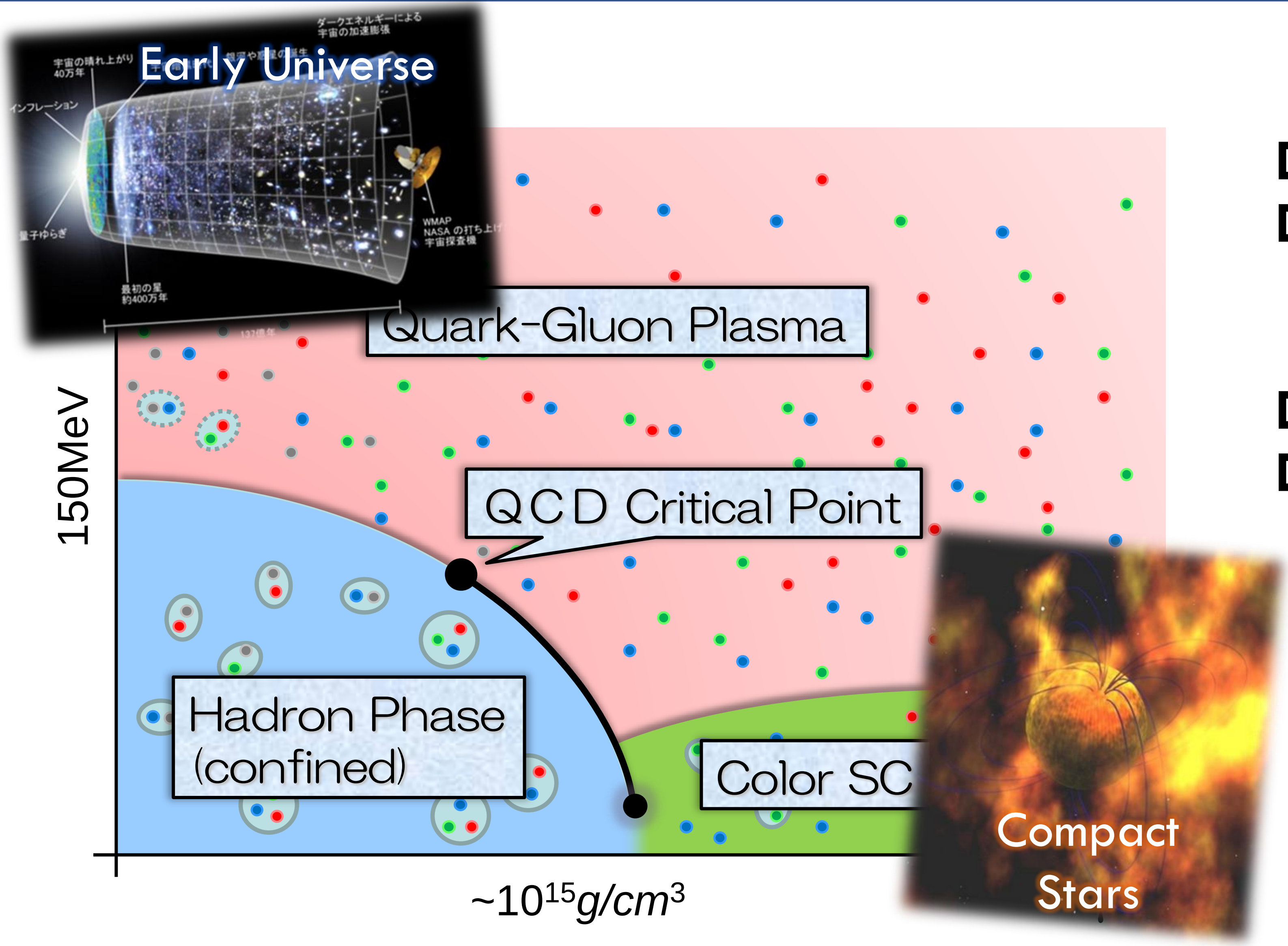
QCD Theory Seminars, 2022/Feb./14, online

Search for Phase Transitions in Dense QCD in Heavy-Ion Collisions

Masakiyo Kiazawa (Osaka University)

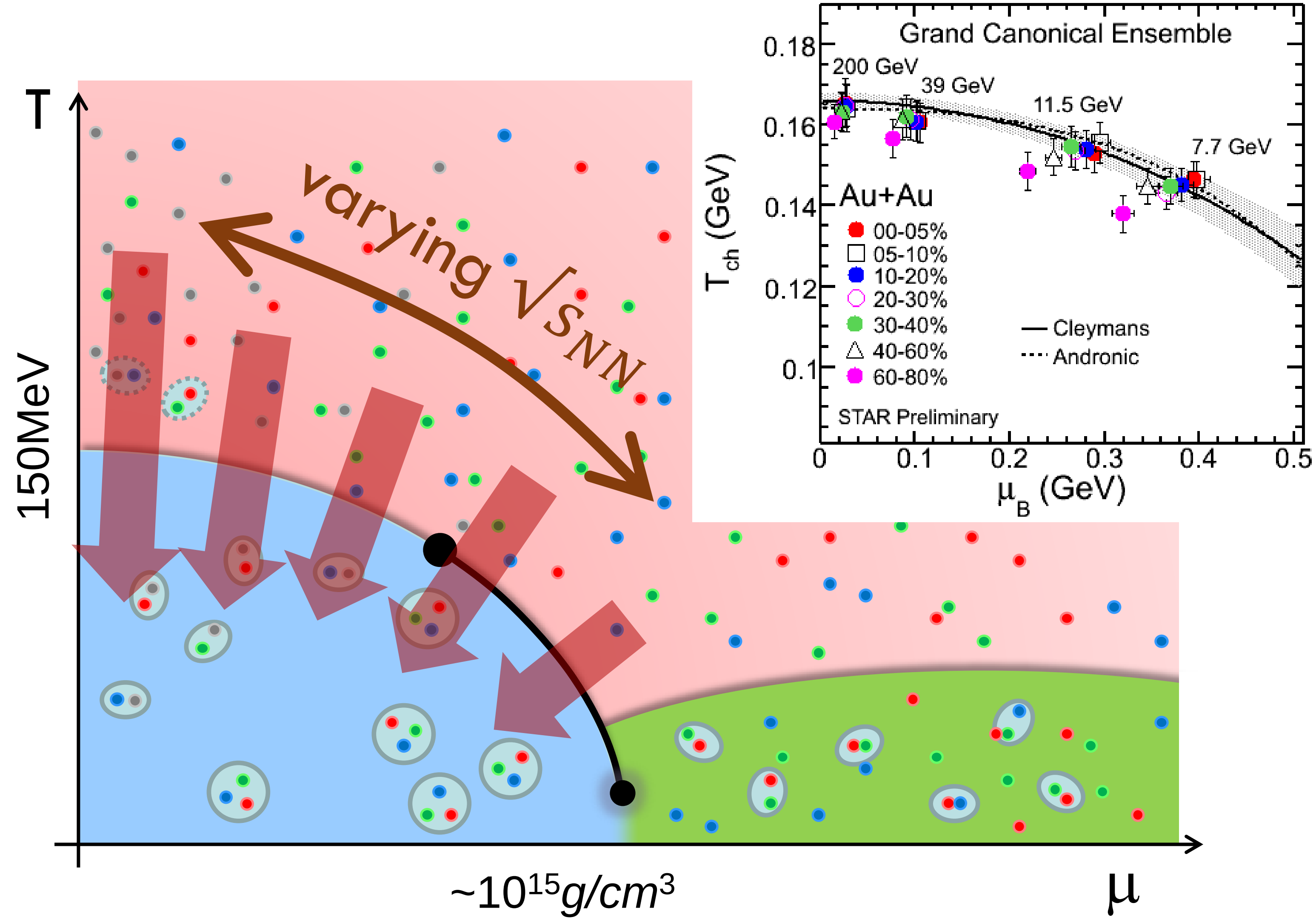
Nishimura, MK, Kunihiro, arXiv:2201.01963

QCD Phase Diagram



- Crossover at $\mu = 0$
- Possible first-order transition and QCD critical point in dense region
- Multiple QCD-CP? MK+ ('02)
- Color superconducting phases in dense and cold quark matter

Beam-Energy Scan in Heavy-Ion Collisions



- In HIC, T, μ can be changed by varying the collision energy.
- The "beam-energy scan" program is ongoing all over the world.
  - present: RHIC-BES, GSI-HADES, NA61/SHINE, …
  - future: NICA-MPD, GSI-FAIR, J-PARC-HI

Contents

1. Search for **QCD Critical Point**

   using fluctuation observables

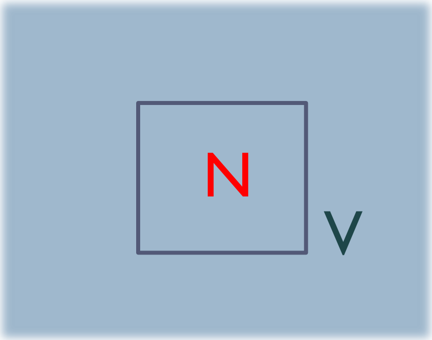
2. Search for **CSC Phase Transition**

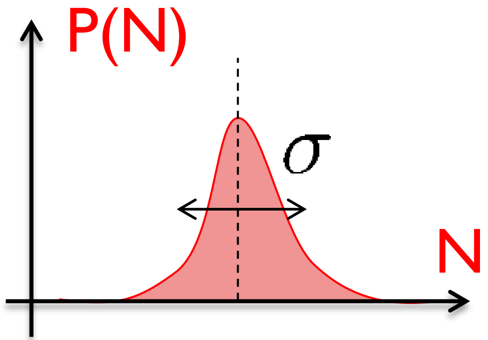
   using dilepton production rates

   Nishimura, MK, Kunihiro, arXiv:2201.01963

Thermal Fluctuations

Observables are fluctuating even in an equilibrated medium.





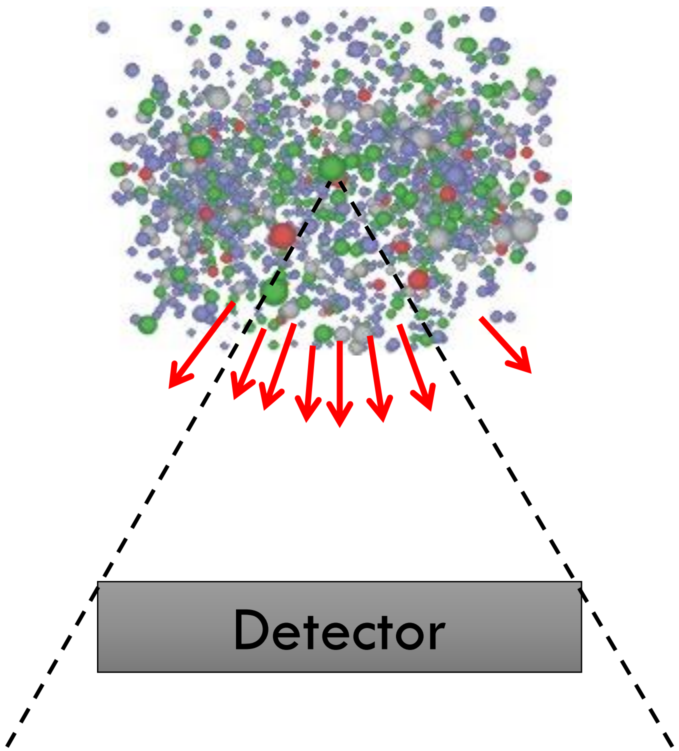
- ☐ Phase transition → Large fluctuation
- ☐ **Non-Gaussian fluctuations:** good observables of QCD-CP

Stephanov, PRL (2009); Asakawa, Ejiri, MK, PRL (2009)

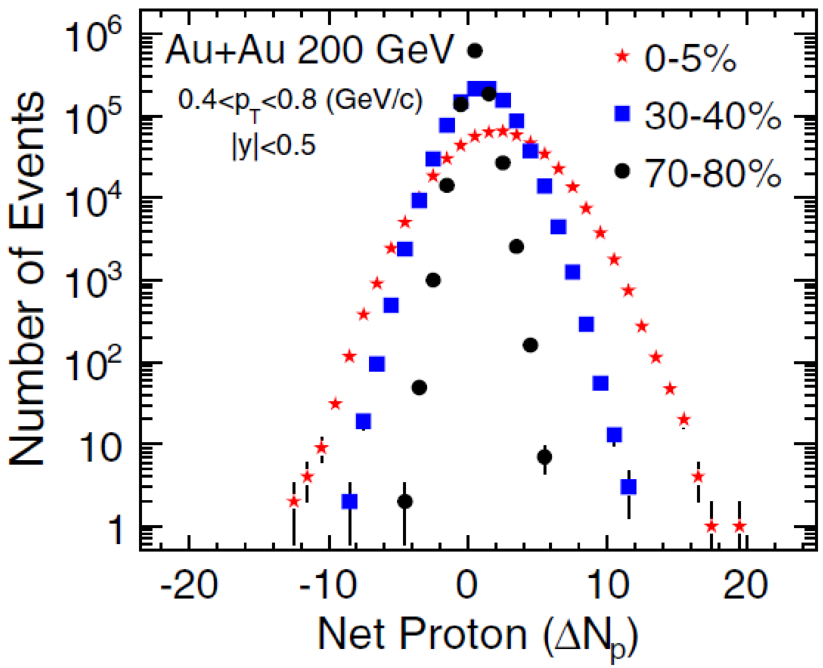
Event-by-Event Fluctuations in HIC

Review: Asakawa, MK, PPNP **90** (2016)

Fluctuations can be measured by e-by-e analysis in experiments.



STAR, PRL**105** (2010)

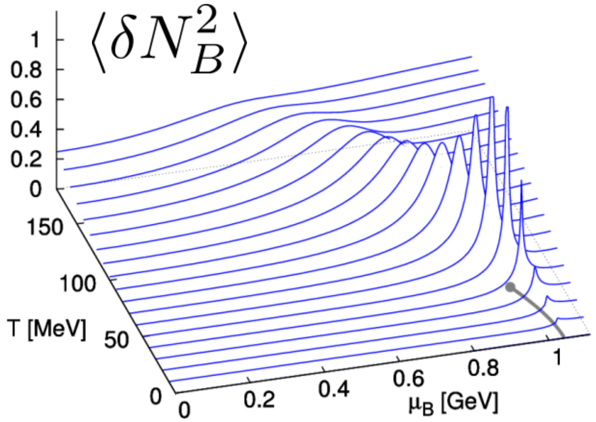


Cumulants

$$\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$$

Cumulants near QCD-CP

☐ Divergence at the CP

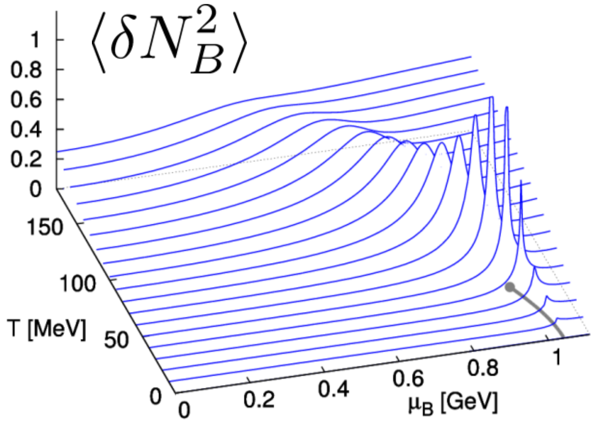


more singular behavior for higher order cumulants

Stephanov ('09)

Cumulants near QCD-CP

Divergence at the CP



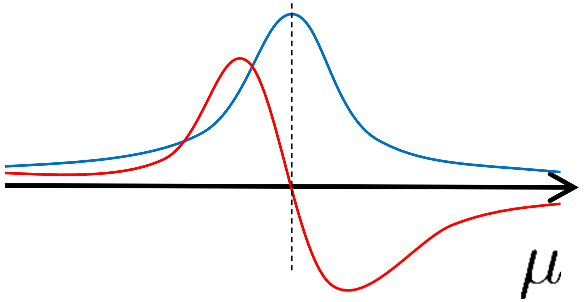
more singular behavior for higher order cumulants

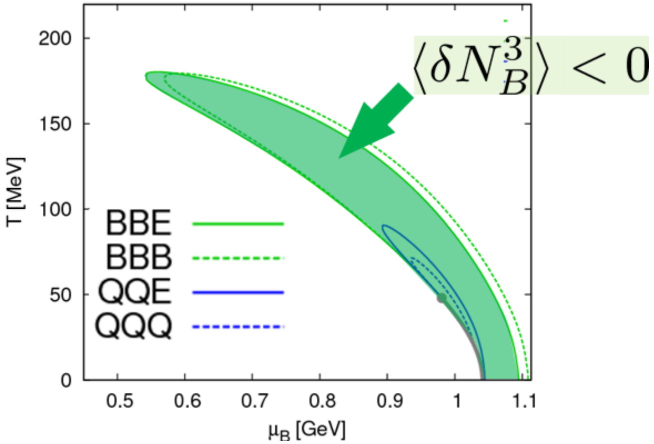
Stephanov ('09)

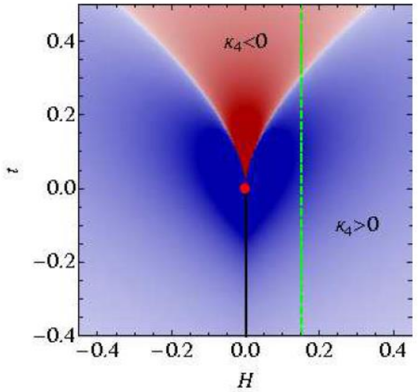
Sign change

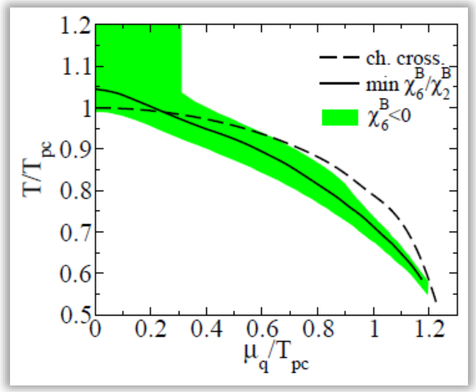
Asakawa, Ejiri, MK, 2009

$$\langle \delta N^m \rangle = T \frac{\partial \langle \delta N^{m-1} \rangle}{\partial \mu}$$



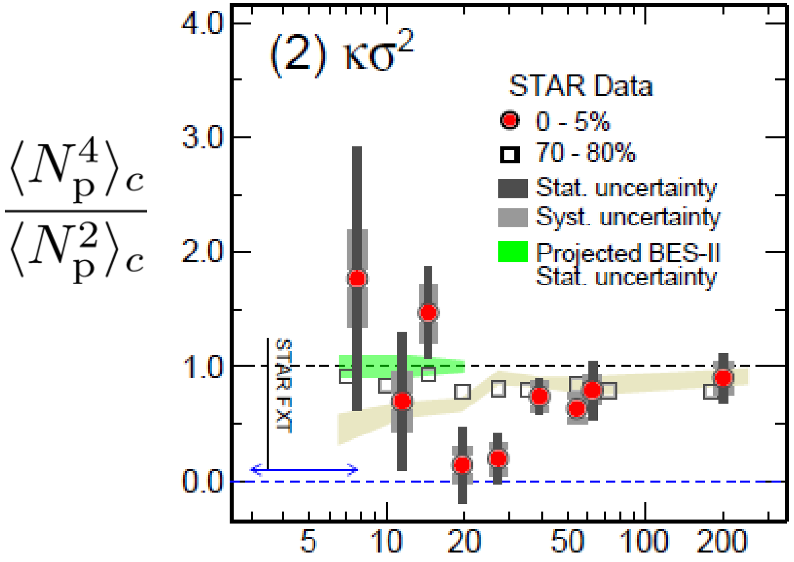


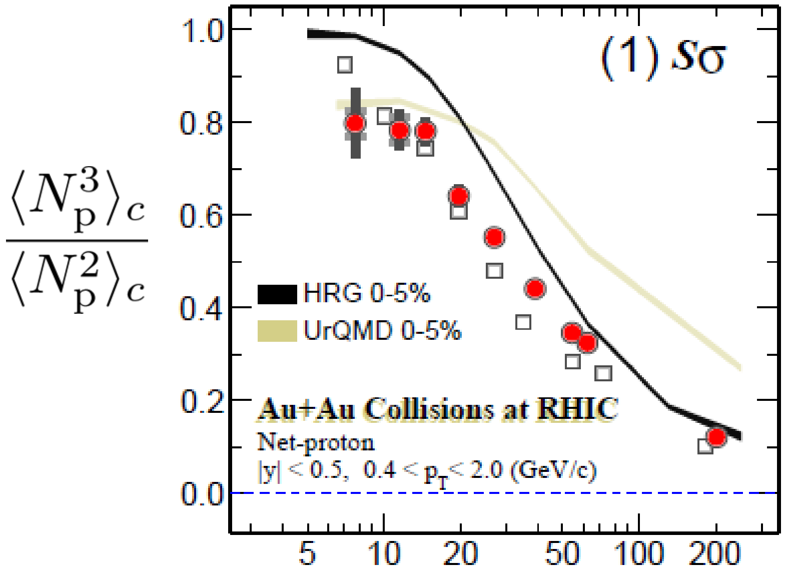




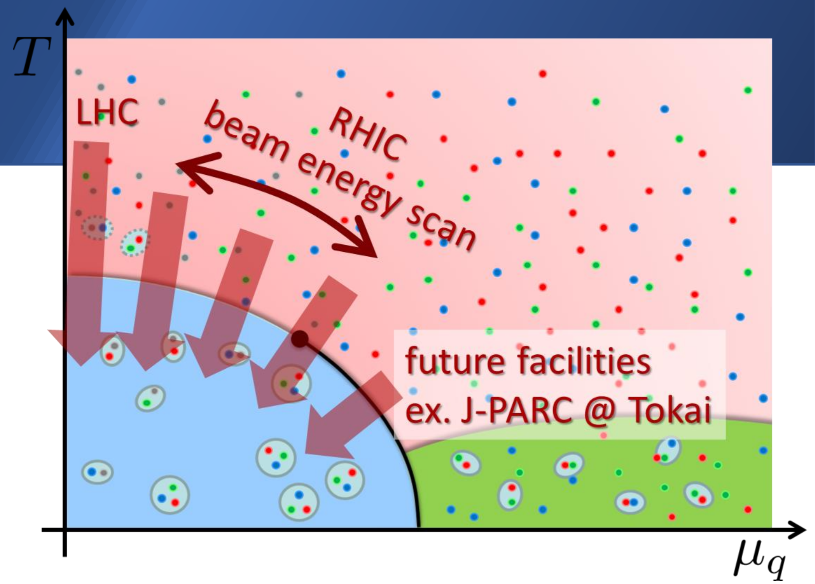
Stephanov, 2011;
Friman, Karsch, Redlich, Skokov, 2011; …

Experimental Results





STAR Collab. ('20)



Various Issues to be Resolved

- ☐ Volume (initial) fluctuations
- ☐ Difference b/w proton & baryon number
- ☐ Efficiency/acceptance correction
- ☐ Measurement in momentum space
- ☐ Dynamical evolution
- ☐ Resonance decays
- ☐ …

Proton vs Baryon Number Cumulants

MK, Asakawa, 2012; 2012

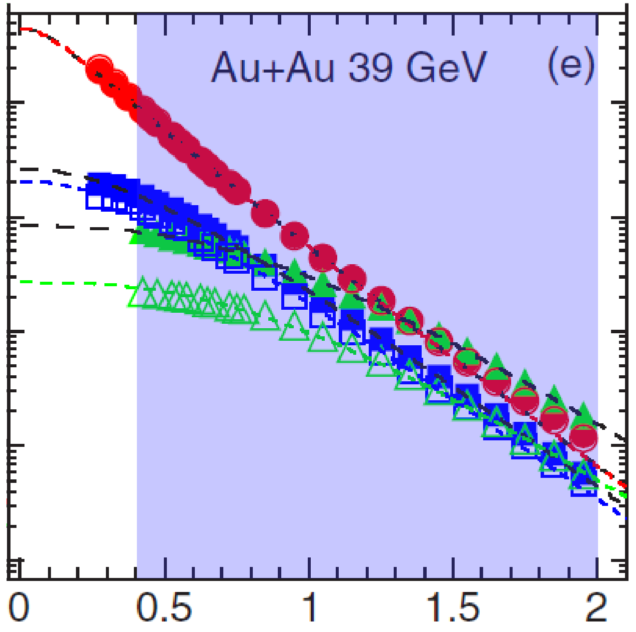
Experiments
proton number cumulants
$\langle N_p^n \rangle_c$

Many theories
baryon number cumulants
$\langle N_B^n \rangle_c$

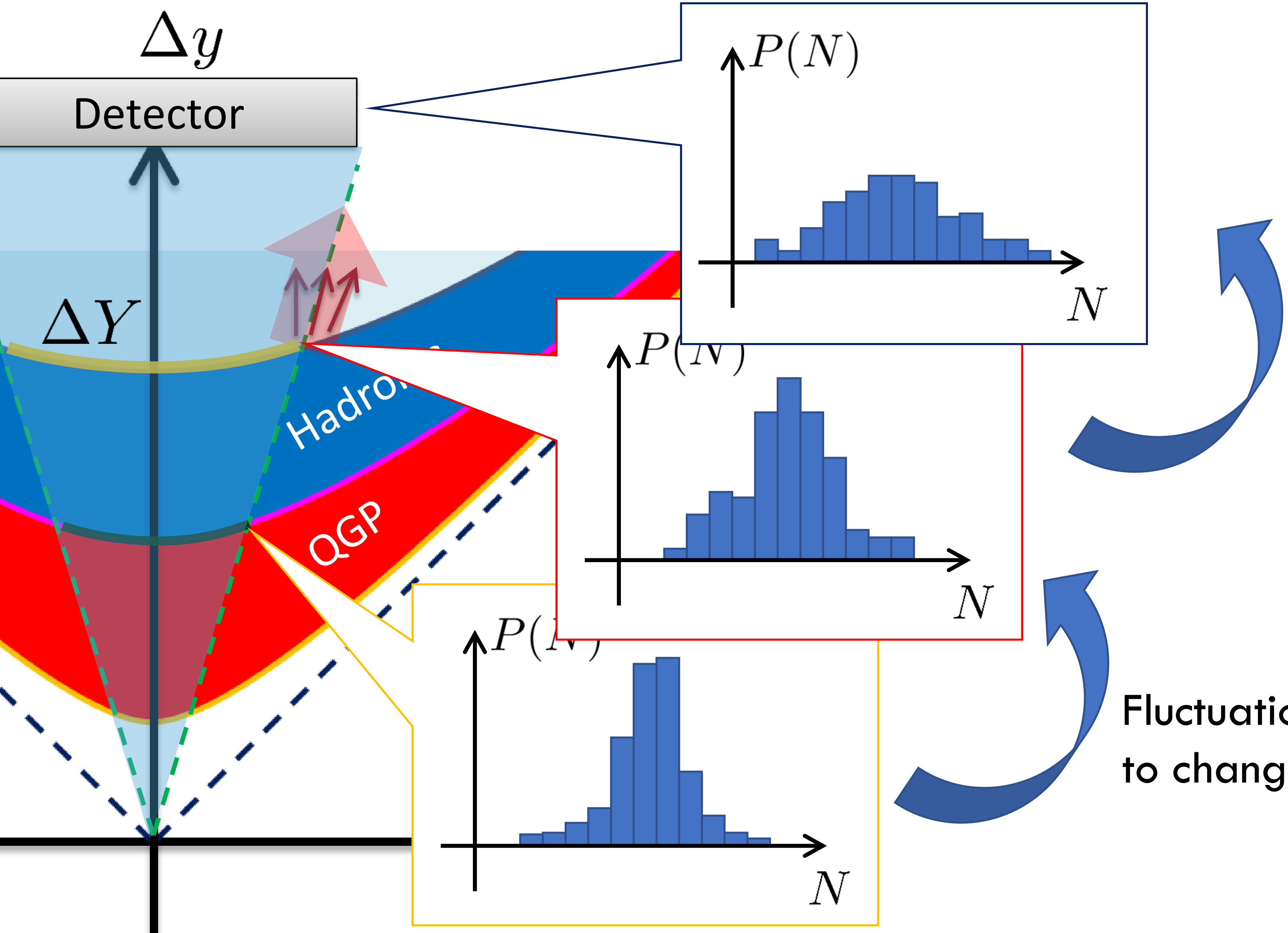
measurement with 50% efficiency loss

- Clear difference b/w these cumulants.
- Isospin randomization justifies the reconstruction of $\langle N_B^n \rangle_c$ via the binomial model.
- Similar problem on the **momentum cut…**

Only 54% of charged particles are observed in the momentum bin



Time Evolution of Fluctuations

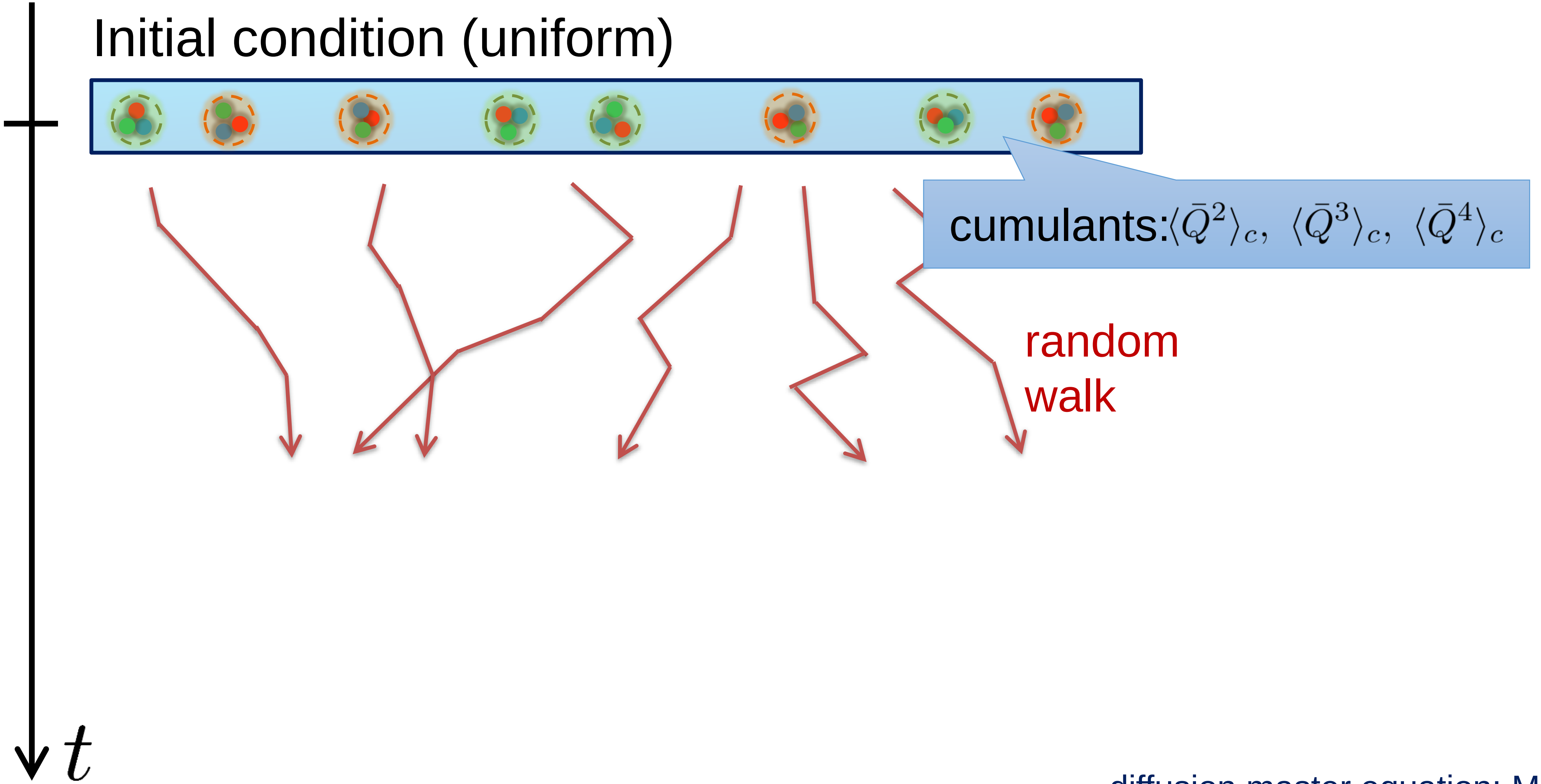


Distributions in ΔY and Δy are different due to "thermal blurring".

Ohnishi, MK, Asakawa, PRC(2016)

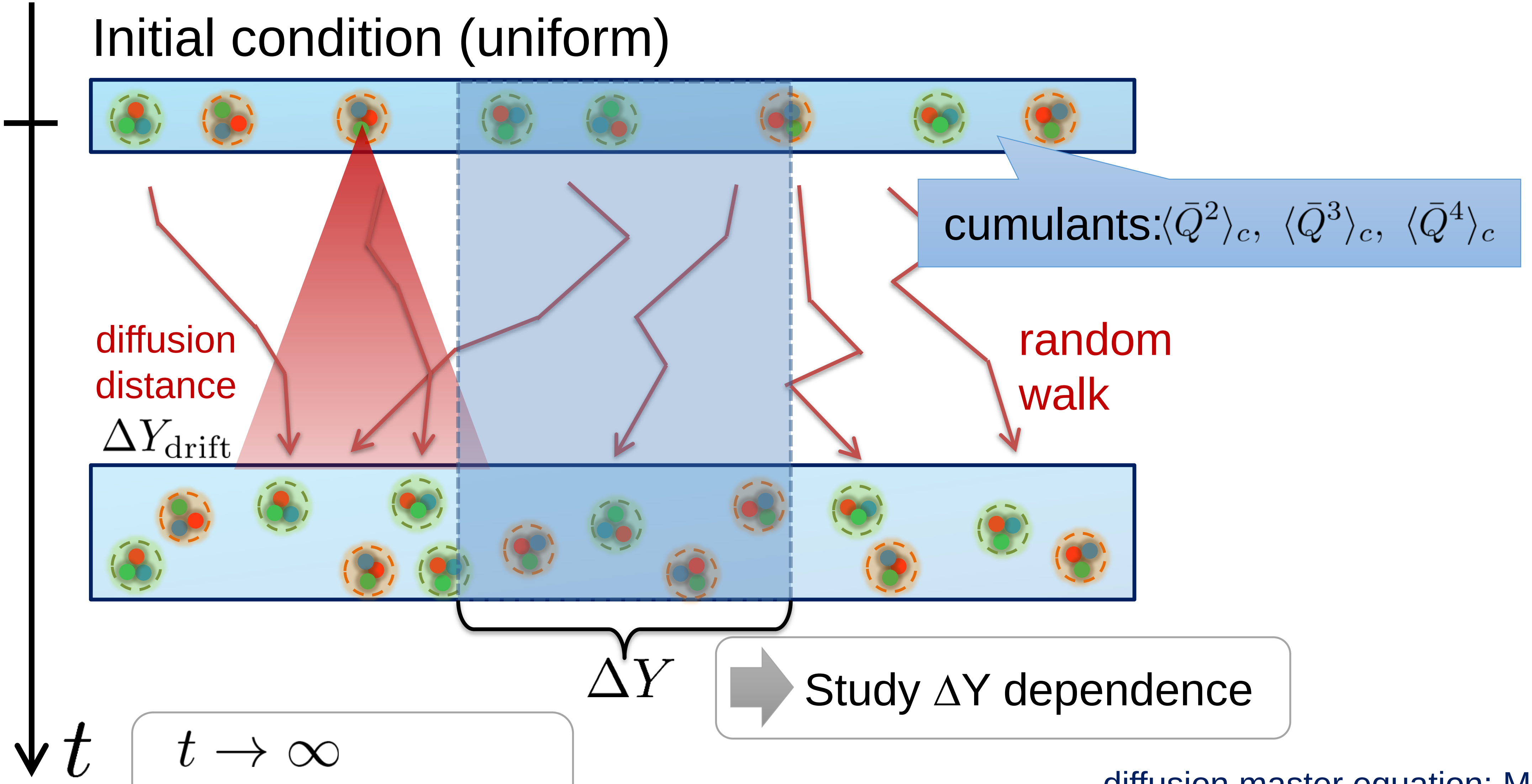
Fluctuations in ΔY continue to change until kinetic f.o.

Evolution in Brownian Particle Model



diffusion master equation: MK+, PLB('14)
probabilistic argument: Ohnishi+, PRC('16)

Evolution in Brownian Particle Model



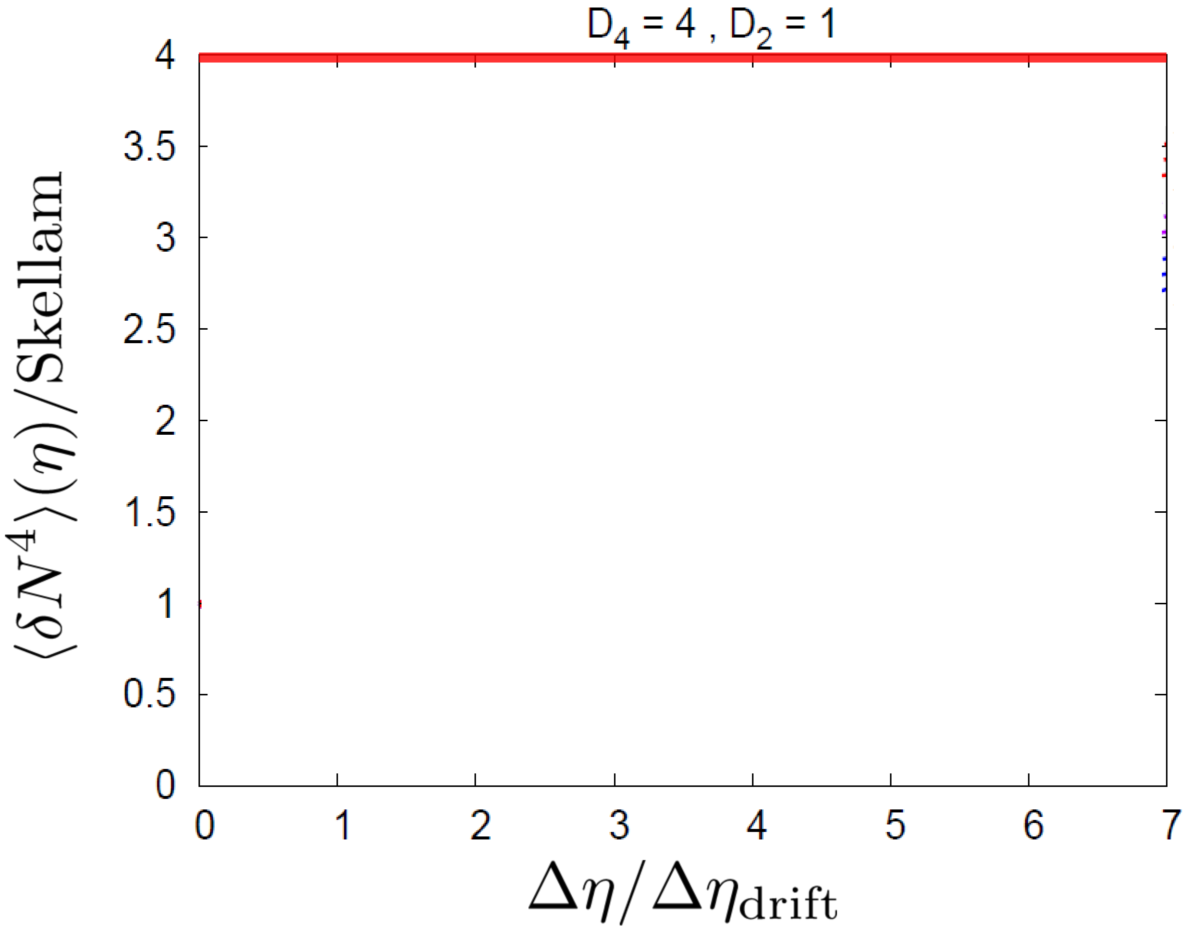
Study ΔY dependence

$t \to \infty$
Poisson distribution

diffusion master equation: MK+, PLB('14)
probabilistic argument: Ohnishi+, PRC('16)

4th Order Cumulant

Before the diffusion



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

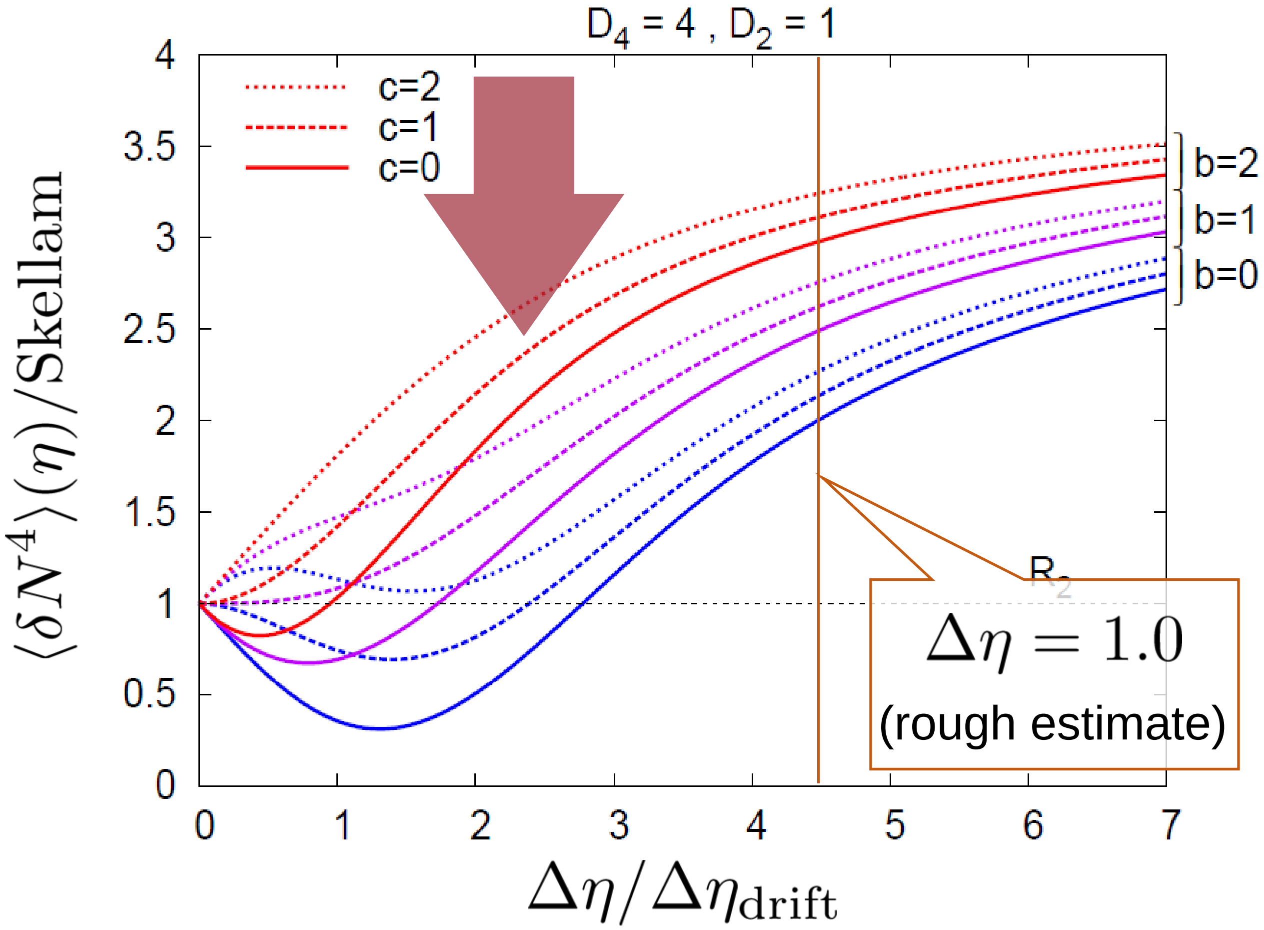
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

MK+ (2014)
MK (2015)

4th Order Cumulant

After the diffusion



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$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

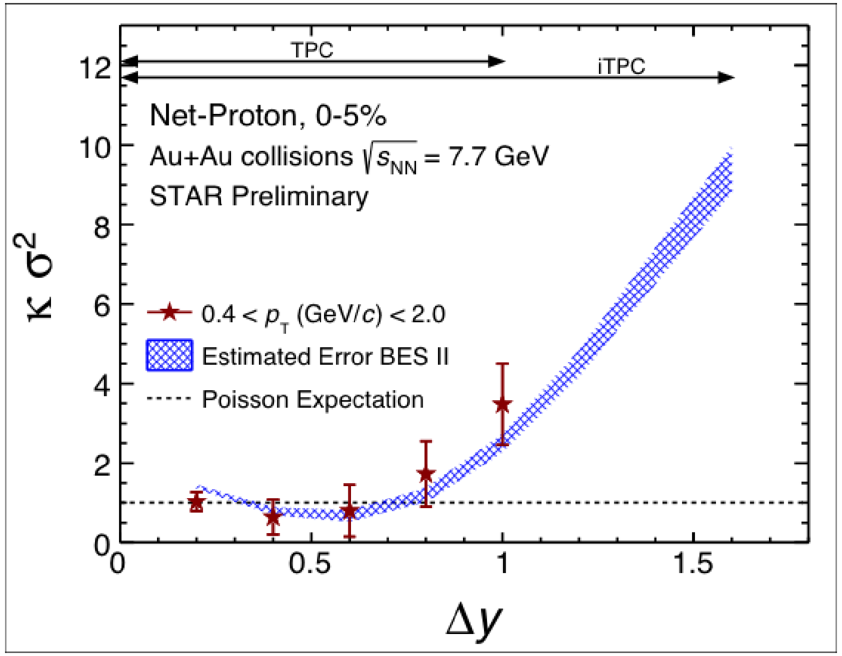
MK+ (2014)
MK (2015)

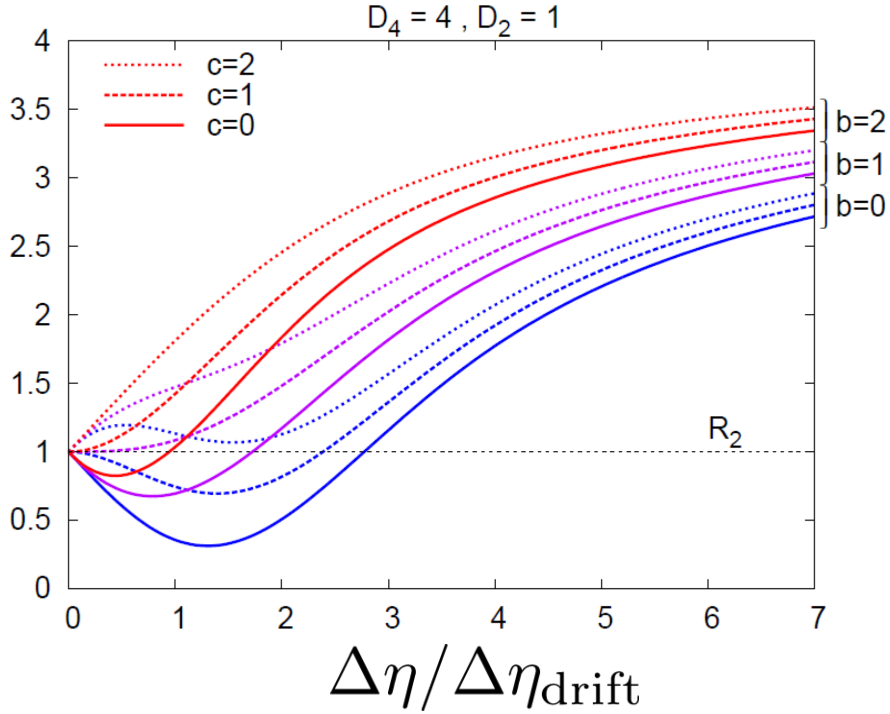
- Cumulant at small Δη is modified toward a Poisson value.
- Non-monotonic behavior can appear.

Rapidity-Window Dependence

4^{th}-order cumulant

STAR Collab. (X. Luo, CPOD2014)





Initial Condition

$$D_4 = \frac{\langle Q^4_{\rm (net)}\rangle_c}{\langle Q_{\rm (tot)}\rangle} = 4$$

$$b = \frac{\langle Q^2_{\rm (net)} Q_{\rm (tot)}\rangle_c}{\langle Q_{\rm (net)}\rangle}$$

$$c = \frac{\langle Q^2_{\rm (tot)}\rangle_c}{\langle Q_{\rm (tot)}\rangle}$$

$$D_2 = \frac{\langle Q^2_{\rm (net)}\rangle_c}{\langle Q_{\rm (tot)}\rangle} = 1$$

MK+, 2014
MK, 2015

- Is non-monotonic $\Delta\eta$ dependence already observed?
- Different initial conditions give rise to different characteristic $\Delta\eta$ dependence. → Study initial condition

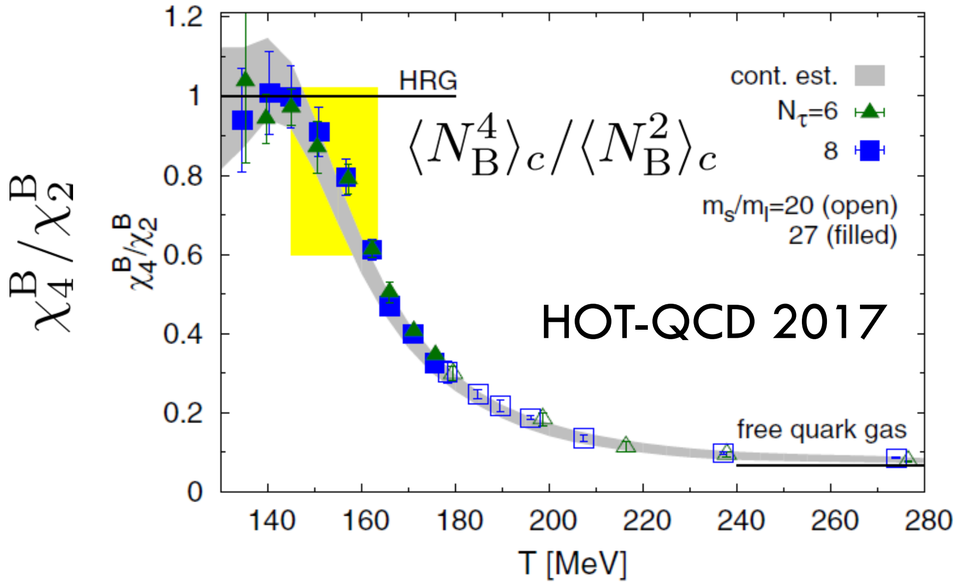
Finite volume effects: Sakaida+, PRC90 (2015)

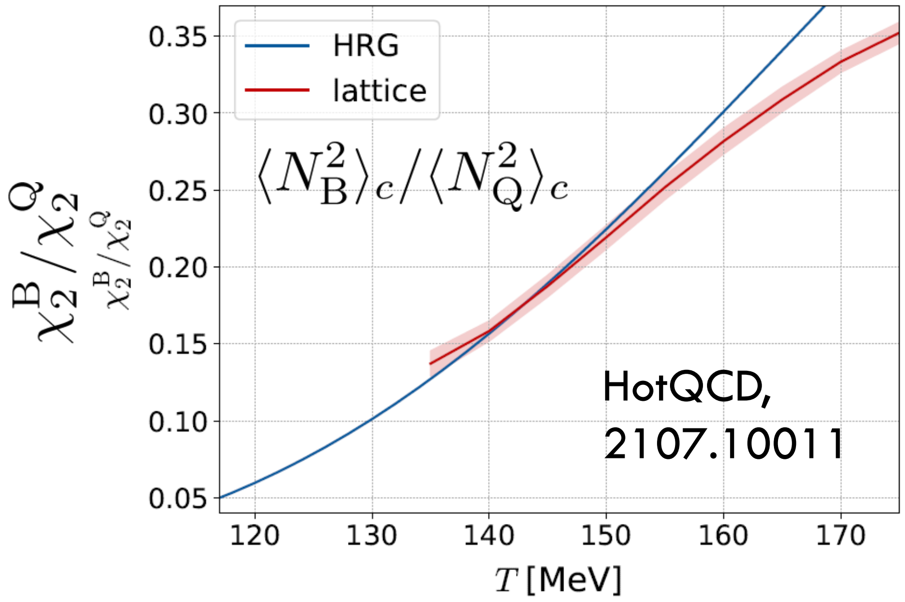
$\langle N_B^2\rangle_c/\langle N_Q^2\rangle_c$

$$\langle N_{\rm B}^2\rangle_c/\langle N_{\rm Q}^2\rangle_c \simeq \chi_2^{\rm B}/\chi_2^{\rm Q}$$

- ☐ Combination of lower-order cumulants.
- ☐ χ_2^B/χ_2^Q has a linear T dependence.
- ☐ Lattice data are available.

MK, Esumi, Nonaka, in prep.





Experimental Data

$\langle N_p^2 \rangle_c$

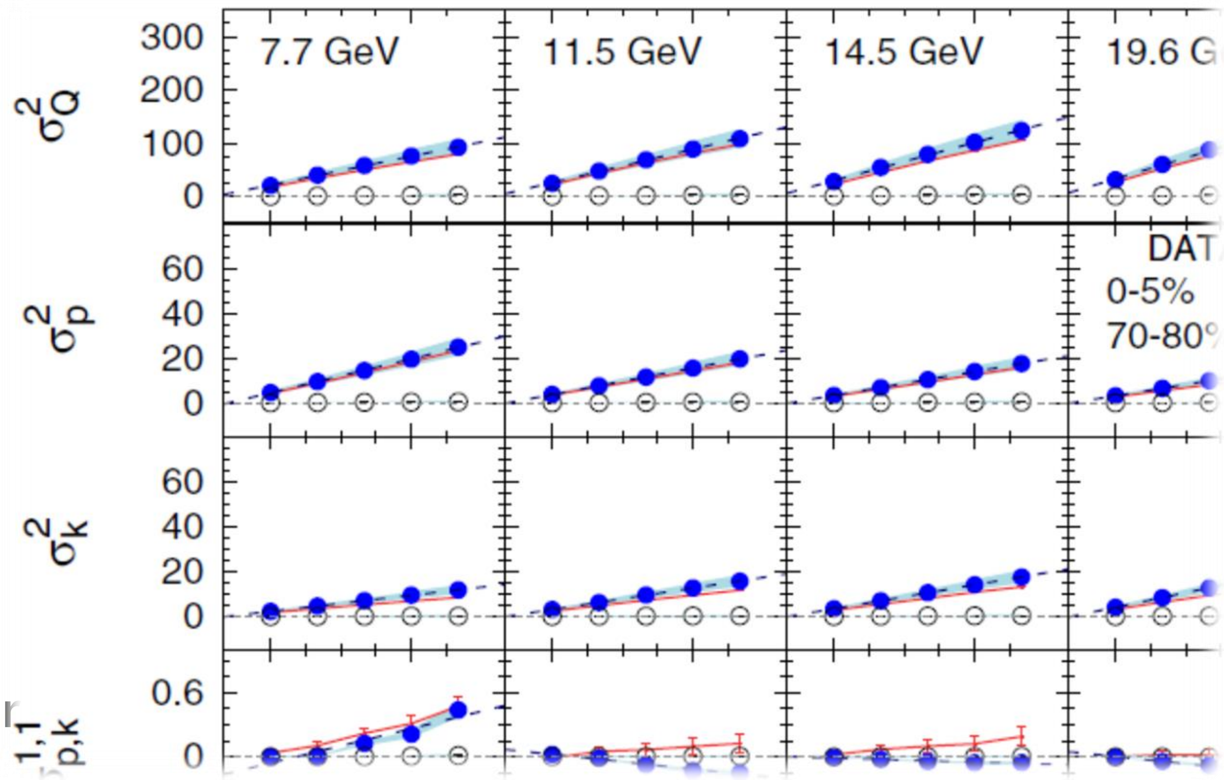
STAR, PRC104,024902 (2021)

- Proton cumulants up to 4th order
- Δy dependence
- $0.4 < p_T < 2.0$GeV/c

$\langle N_Q^2 \rangle_c$

STAR, PRC100,014902 (2019)

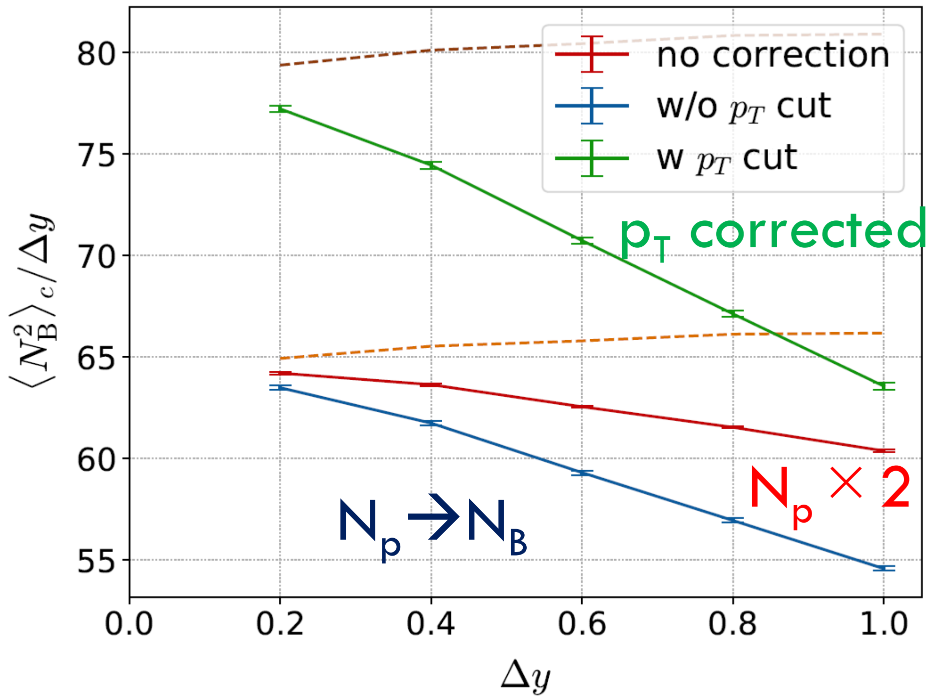
- Mixed cumulants of p, Q, S at 2nd order
- **Pseudo rapidity** $\Delta\eta$
- $0.4 < p_T < 1.6$GeV/c
- Total charge: private comm. A. Chattergee



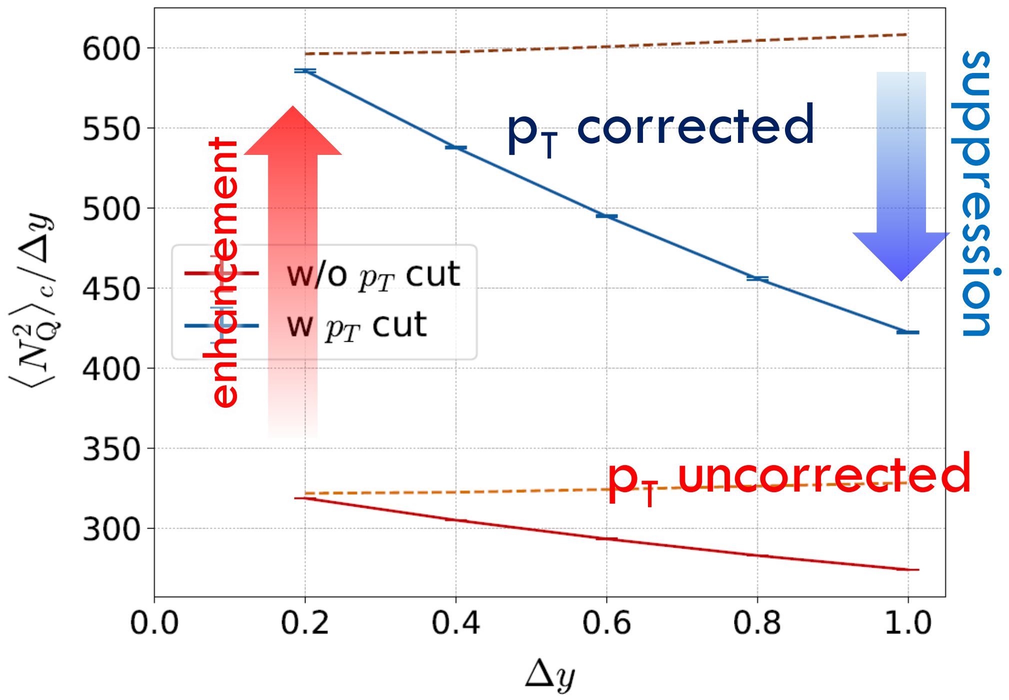
p_T correction with binomial model

Baryon Number Cumulants w/ Acceptance Correction

$\langle N_B^2 \rangle_c / \Delta y$



$\langle N_Q^2 \rangle_c / \Delta y$

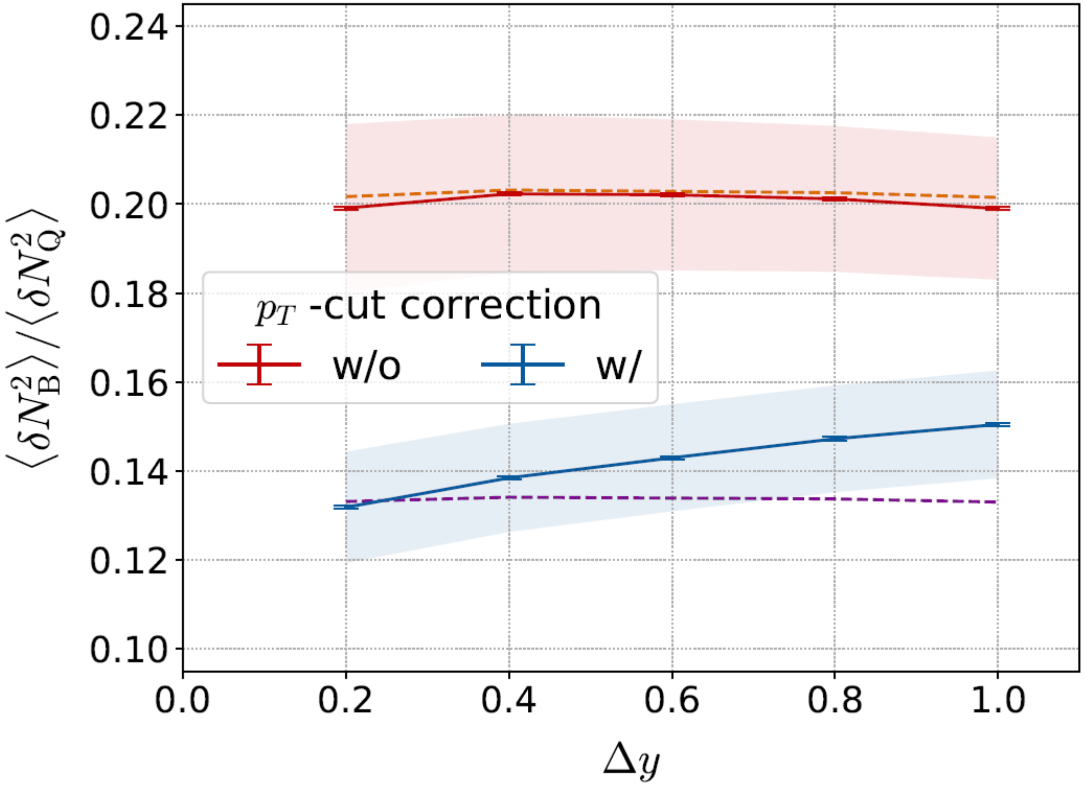


dashed:
$\langle N_B^{tot} \rangle$~
Poisson値

p_T correction with binomial model

$$\langle N_{\text{net}}^2 \rangle_c^{\text{corrected}} = \frac{1}{p^2}\Big(\langle n_{\text{net}}^2 \rangle_c - (1-p)\langle n_{\text{tot}} \rangle_c \Big)$$

$\langle N_B^2\rangle_c/\langle N_Q^2\rangle_c$



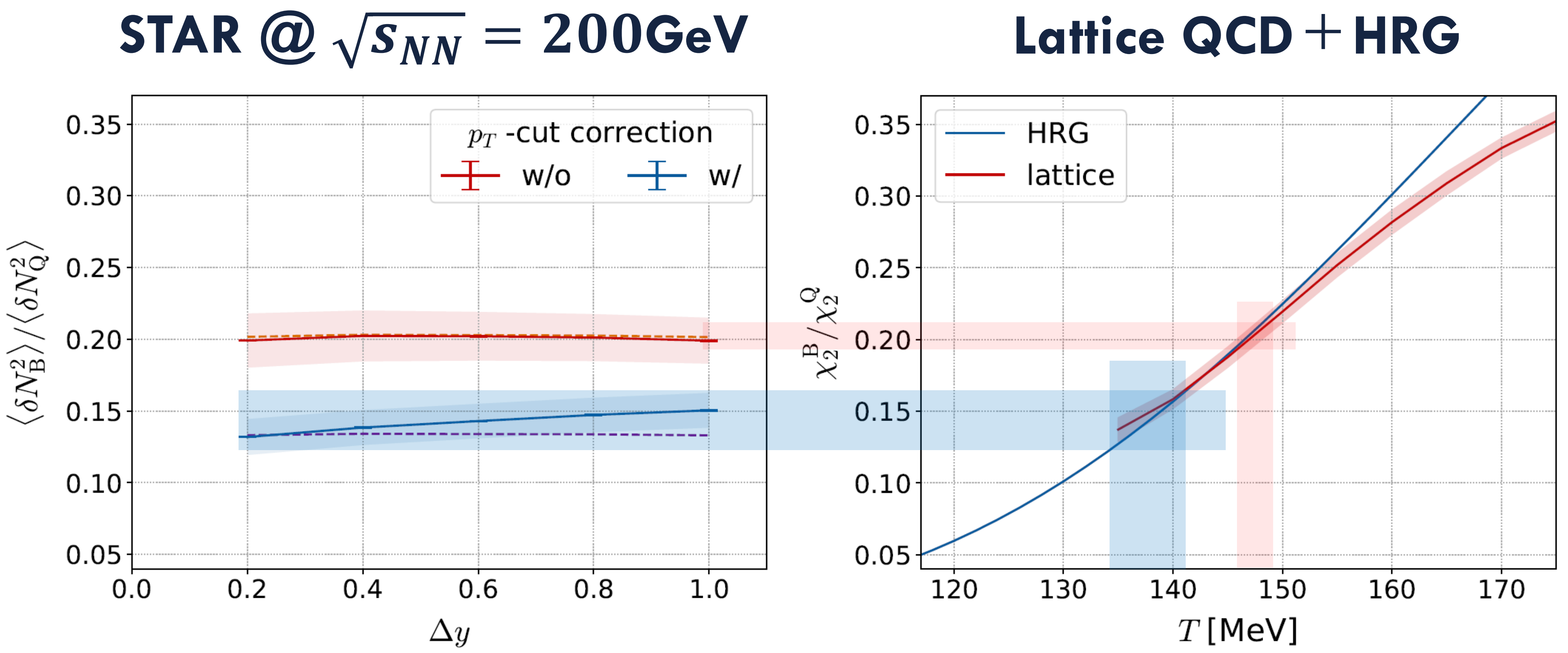
p_T uncorrected

p_T corrected

dashed: Poisson

- p_T cut correction reduces the value of $\langle N_B^2\rangle_c/\langle N_Q^2\rangle_c$.
- Δy dependence in $\langle N_B^2\rangle_c/\langle N_Q^2\rangle_c$ ➡ Dynamical effect

HIC vs LAT

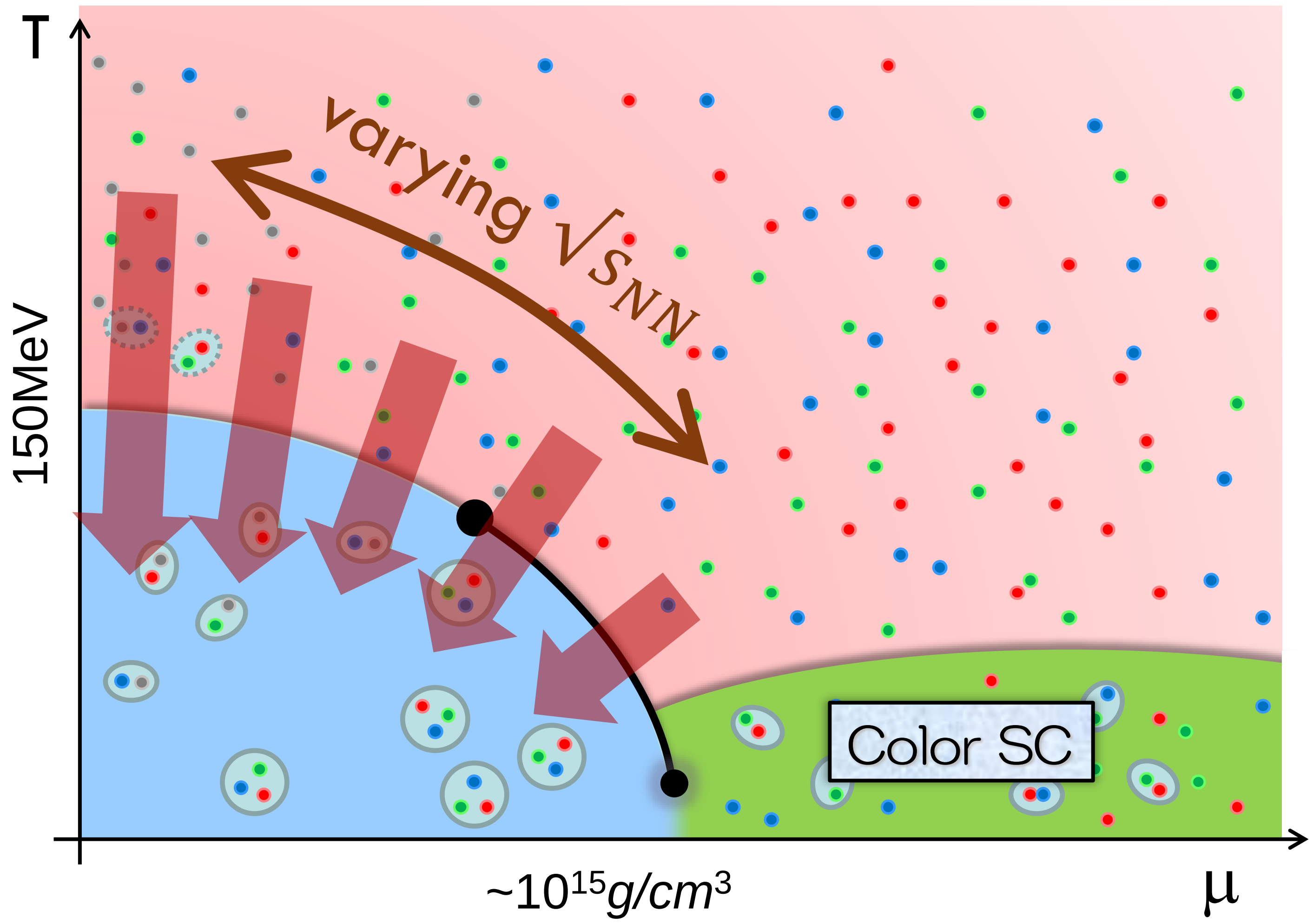


- ☐ ***T*=135~140MeV** if exp. measures thermal fluctuations.
- ☐ Lower than chemical freezeout *T*
- ☐ Exp. results are not those produced at chemical freezeout!

Anomalous Dilepton Production as a Precursor of CSC

Nishimura, MK, Kunihiro, arXiv:2201.01963

Color SuperConductiviting Phases (CSC)

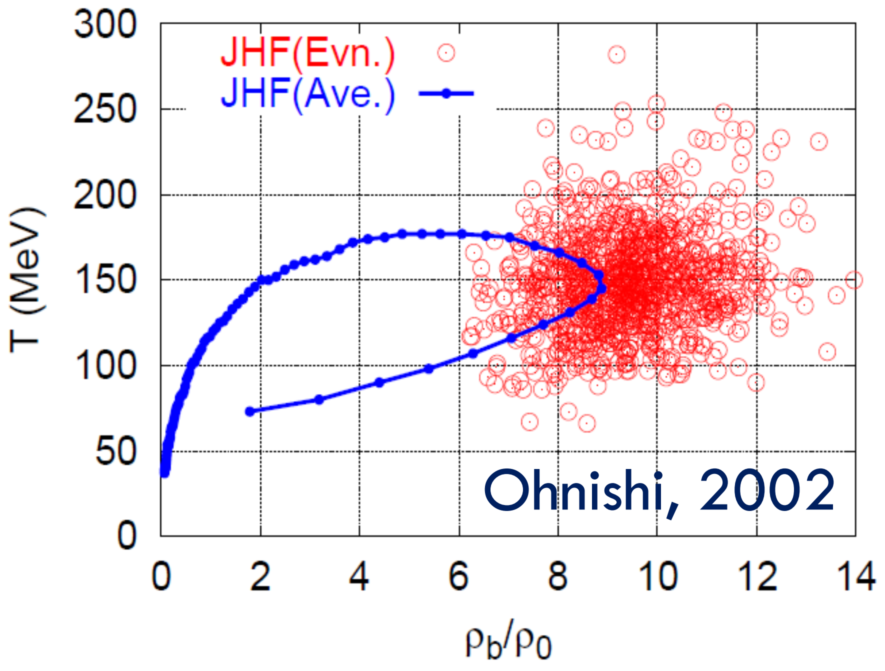


- Attractive qq interaction in $\bar{3}$ channel in one-gluon exchange
- Cooper instability at sufficiently low T
- Various phases due to color/flavor d.o.f.
  - CFL, 2SC, …
- SC in a strongly coupled system
  - BCS-BEC crossover Abuki, Hatsuda, Itakura ('02) MK, Rischeke, Shovkovy ('08)
  - "pseudogap" region MK, Koide, Kunihiro, Nemoto ('03)

Observing CSC in HIC

- **Difficulties**
  - CSC would not be created if Tc is not high enough.
  - Even if created, its lifetime would be short.
  - Since CSC is created in the early stage, its signal would be blurred during the evolution in later stage.

- **Strategy** in the present study:
  - Focus on precursory phenomena of CSC
  - Use dilepton production as an observable

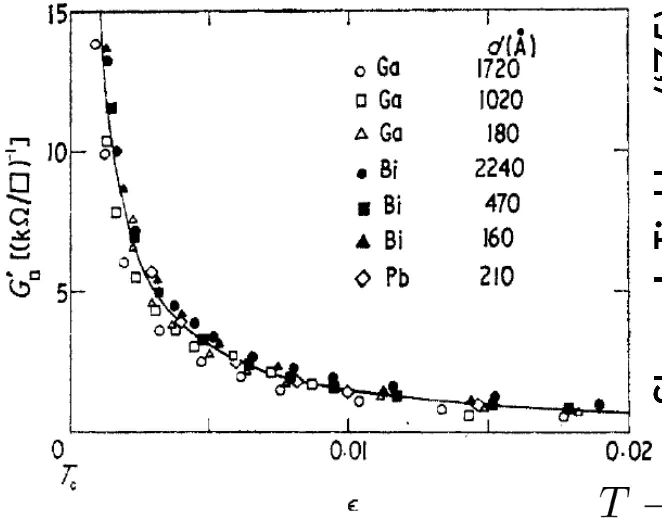


Precursor of CSC

- **Anomalous behavior of observables near but above Tc of SC**
  - electric conductivity
  - magnetic susceptibility
  - pseudogap

- Enhanced pair fluctuations is one of the origins of precursory phenomena.
- More significant phenomena in strongly-coupled systems.

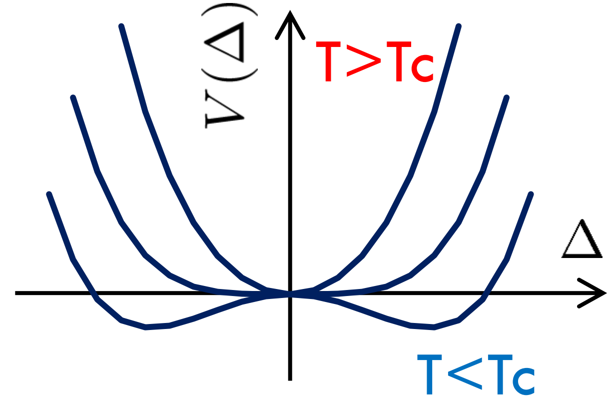
Electric conductivity



Skopal, Tinkham('75)

$$\varepsilon = \frac{T - T_c}{T_c}$$

Landau's free energy



Model

NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \mathcal{L}_S + \mathcal{L}_C$$

$$\mathcal{L}_S = G_S\left((\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \tau \psi)^2\right)$$

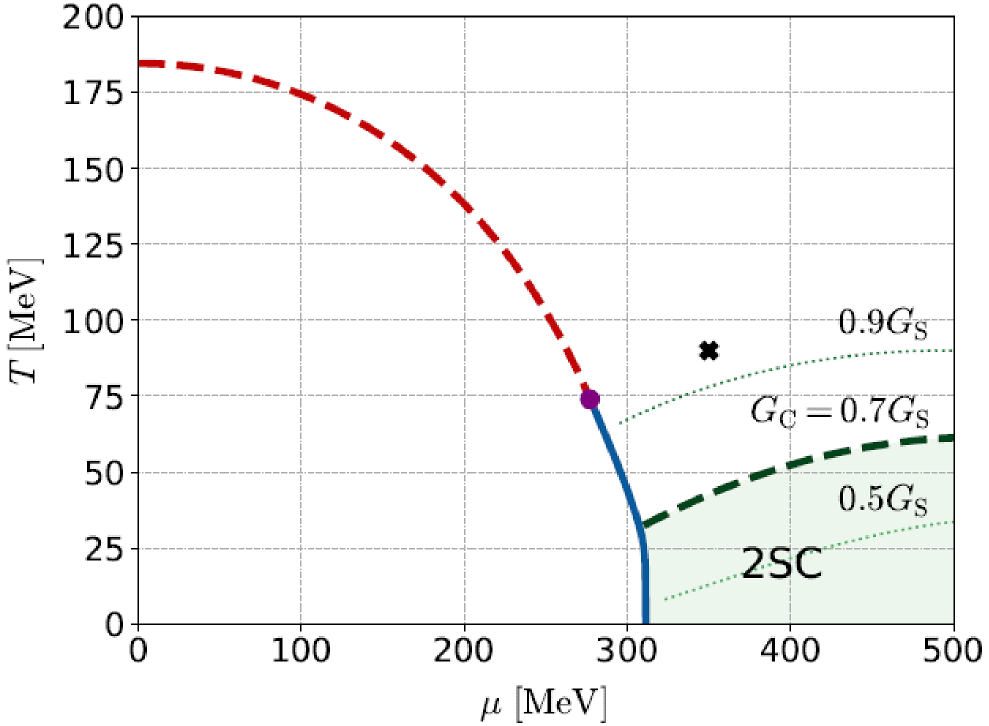
$$\mathcal{L}_C = G_C\left((\bar{\psi} i\gamma_5 \tau_A \lambda_A \psi^C\right)(\text{h.c.})$$

diquark interaction

Parameters

$$G_S = 5.01\ \text{GeV}^{-2}, \quad \Lambda = 650\text{MeV}, \quad m_q = 0$$

Phase Diagram in MFA



- Order of phase transition
  - 2nd in the MFA
  - can be 1st due to gauge fluctuation

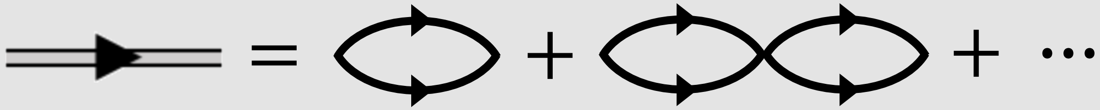
Matsuura+('04), Giannakis+('04)
Noronha+('06), Fejos, Yamamoto('19)

Di-quark Fluctuations

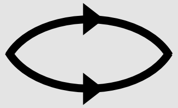
Diquark Propagator

$$D^R(x) = \langle[\Delta^\dagger(x), \Delta(0)]\rangle\theta(t) =$$ 

- Random Phase Approximaion



$$= \frac{Q^R(\boldsymbol{k},\omega)}{1 + G_C Q^R(\boldsymbol{k},\omega)}$$

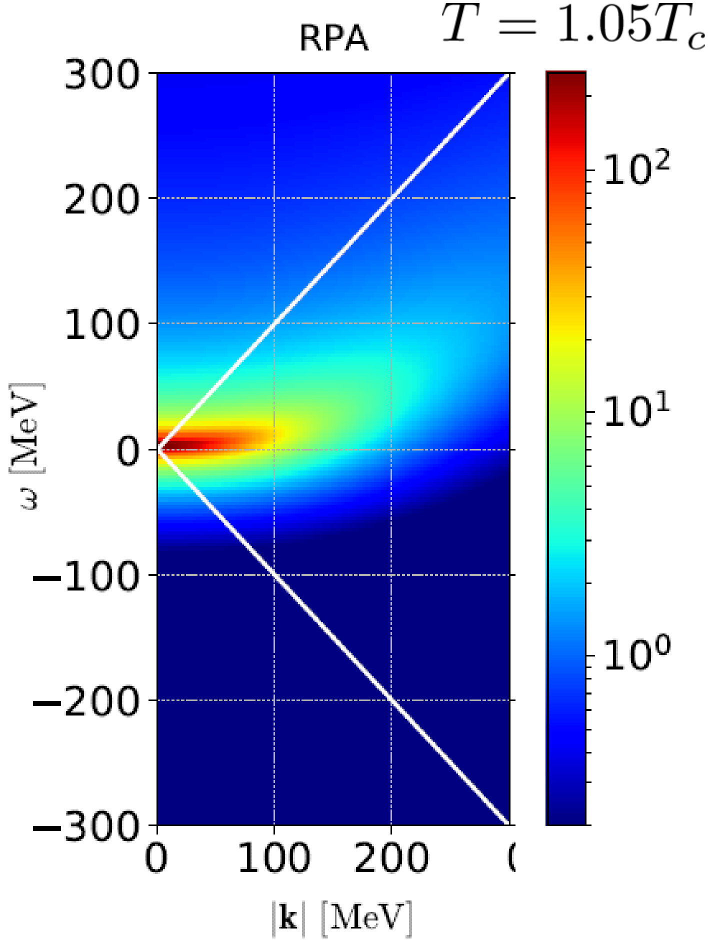
$$Q^R(\boldsymbol{k},\omega) =$$ 

- Diquark field becomes massless at T=Tc
- Soft mode of CSC transition
- Strength in the space-like region

Dynamical Structure Factor

$$S(\boldsymbol{k},\omega) = -\frac{1}{\pi}\frac{1}{1-e^{-\beta\omega}}\mathrm{Im}\mathrm{D}^{\mathrm{R}}(\mathrm{k},\omega)$$

$T = 1.05T_c$

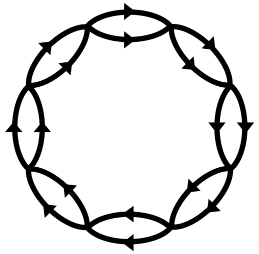


MK, Koide, Kunihiro, Nemoto, '01,'05

Effects on Observables

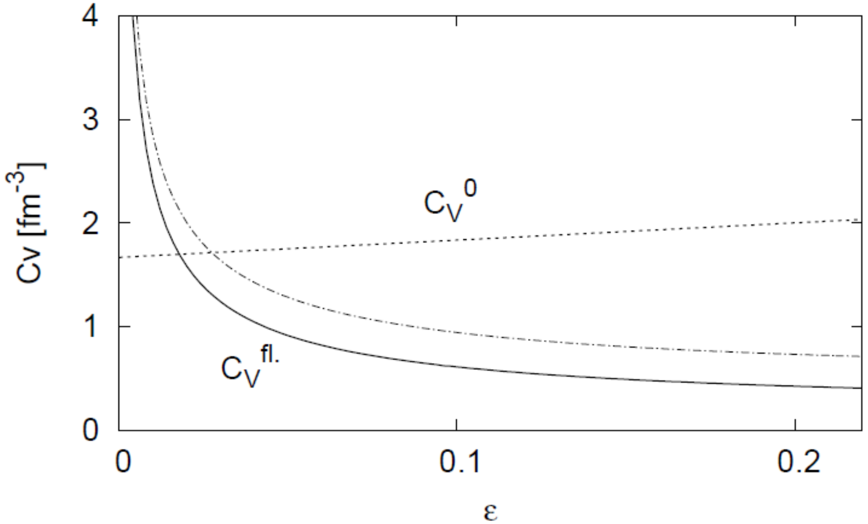
MK, Koide, Kunihiro, Nemoto, '03, '05

□Thermodynamic Potential

$\Omega =$ 

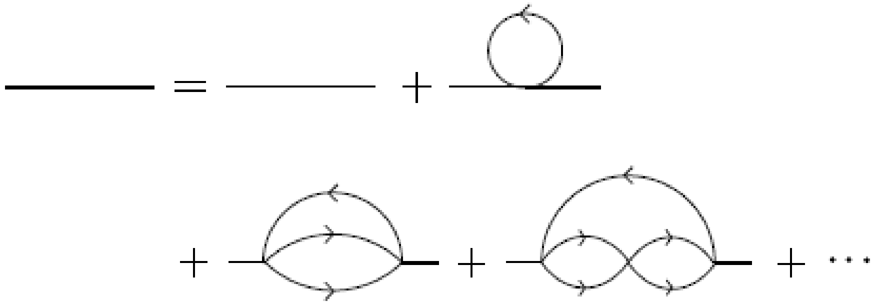
Specific heat

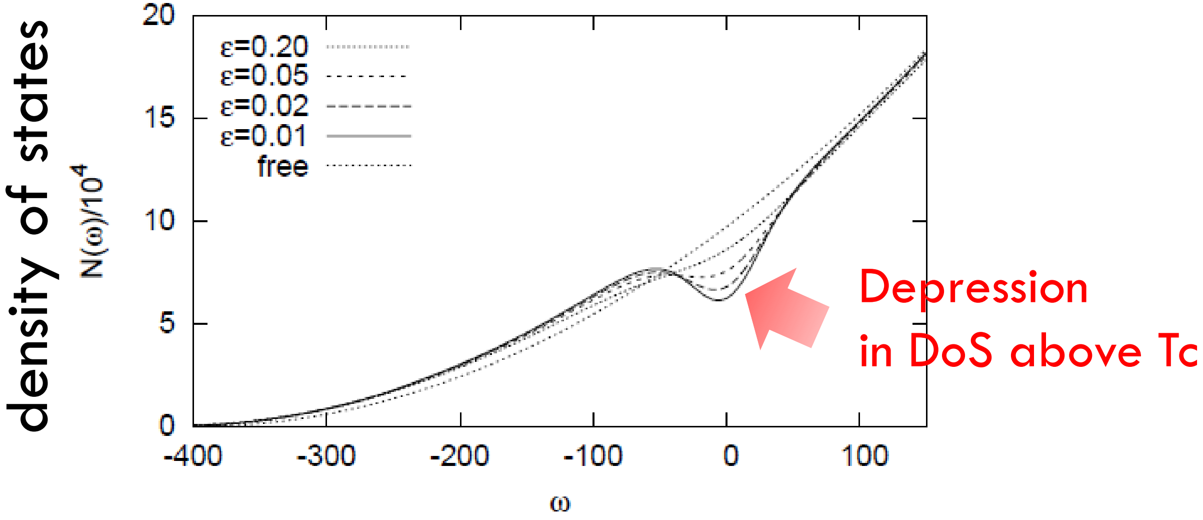
$$c = -T\frac{\partial^2\Omega}{\partial T^2}$$



$$\varepsilon = \frac{T-T_c}{T_c}$$

□Pseudogap

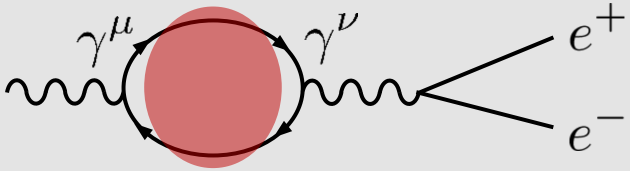




Photon Self-Energy: Precursor of CSC

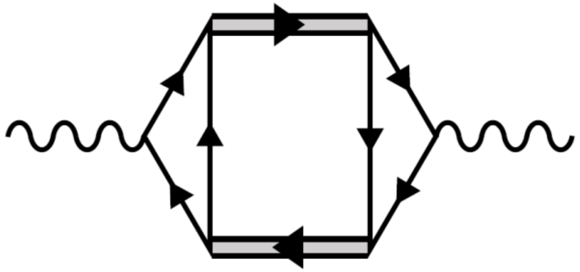
☐ Dilepton Production Rate

$$\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4}\frac{1}{k^2}\frac{1}{e^{\beta\omega-1}}\mathrm{Im}\Pi^{R\mu}_{\mu}(k)$$

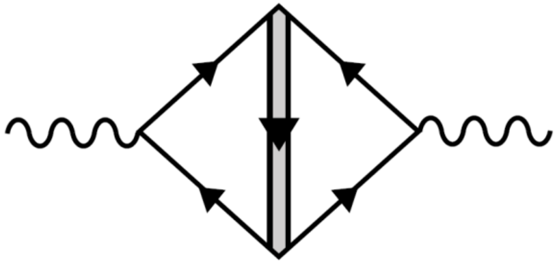


☐ Effect of Di-quarks on $\Pi^{\mu\nu}(k)$

Aslamasov-Larkin term



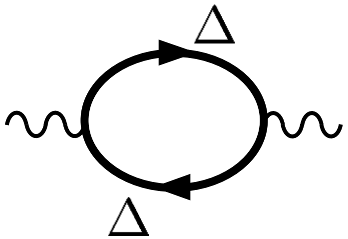
Maki-Thompson term

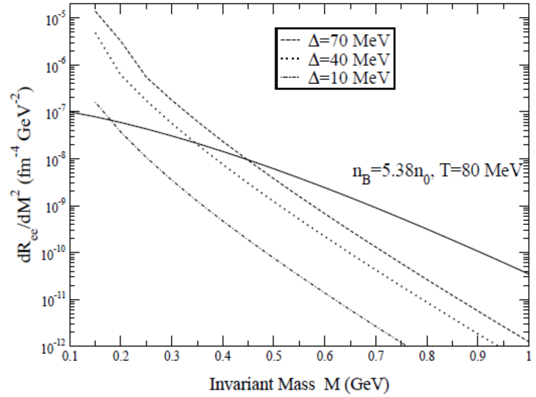


Well-known diagrams in metallic SC
for describing paraconductivity

☐ DPR from CFL phase

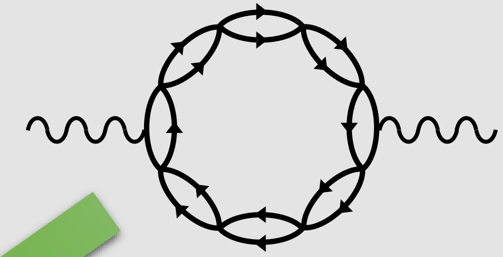
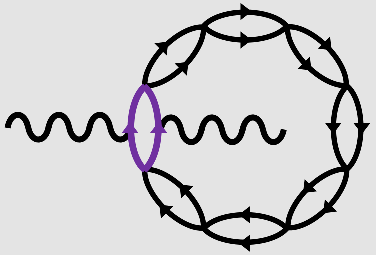
Jaikumar, Rapp, Zahed ('02)



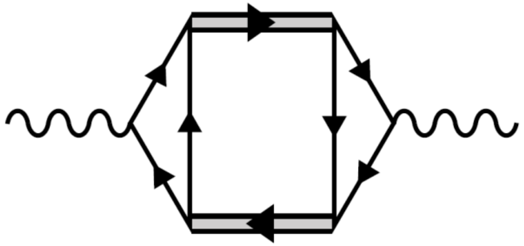


Gauge-Invariant Construction of $\Pi_{\mu\nu}(k)$

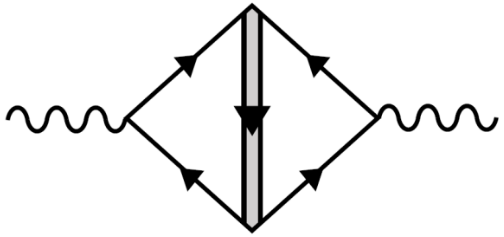
Insert two photon vertices in thermodynamic potential

$\Pi^{\mu\nu}(k) =$  

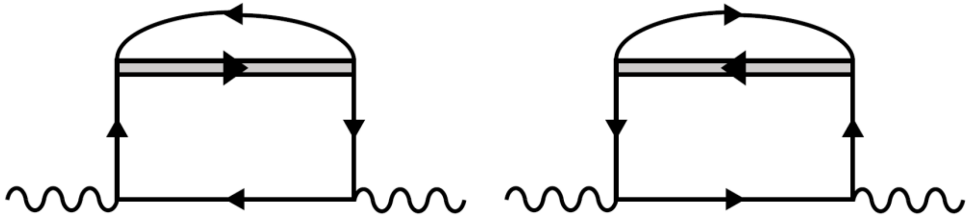
Aslamasov-Larkin (AL)



Maki-Thompson (MT)



Density of States (DoS)



- ☐ WT identity $k_\mu \Pi^{\mu\nu}(k) = 0$ is satisfied with AL, MT and DoS terms.

Time-Dependent Ginzburg-Landau Approximation

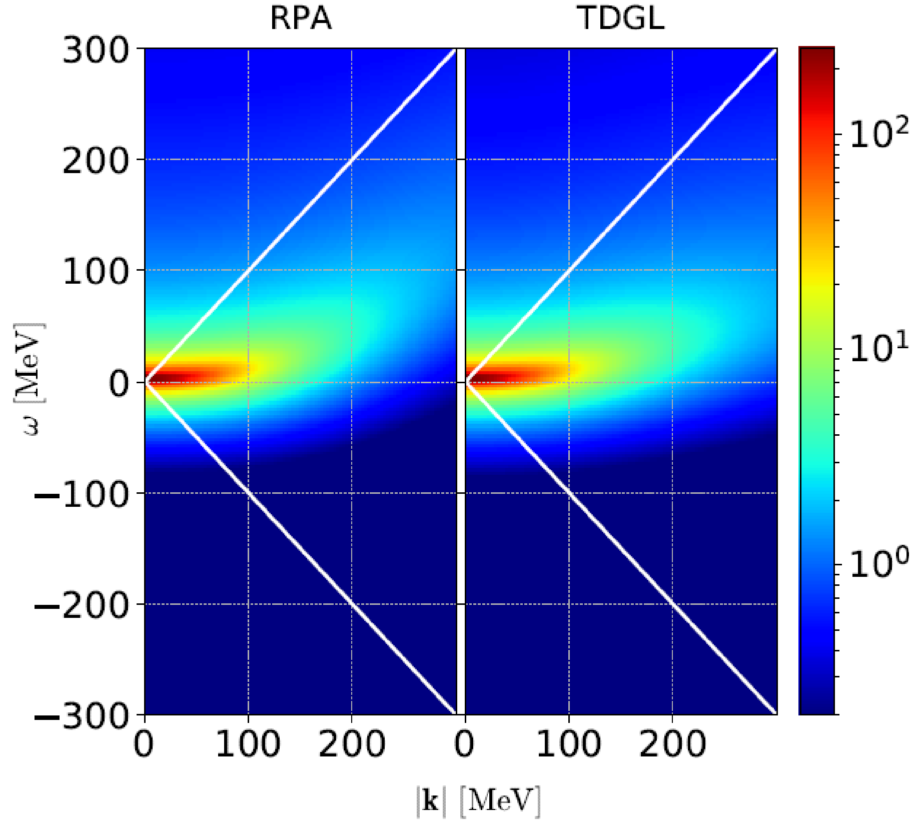
TDGL approximation for T-matrix

$$\Xi^R(\boldsymbol{k},\omega) = \frac{G_C}{1 + G_C Q^R(\boldsymbol{k},\omega)} \simeq \frac{1}{c\omega + \Xi^R(\boldsymbol{k},0)^{-1}} \qquad c = \left.\frac{\partial(\Xi^R)^{-1}}{\partial\omega}\right|_{\omega=0}$$

$$\Xi^R(\boldsymbol{k},\omega) = \frac{G_C}{Q^R(\boldsymbol{k},\omega)} D^R(\boldsymbol{k},\omega)$$

Note:

- Valid in low energy region
- $[\Xi^R(0,0)]^{-1} = 0$ at $T = T_c$
- We do not expand w.r.t. k

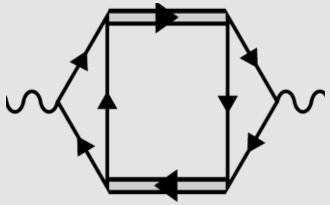


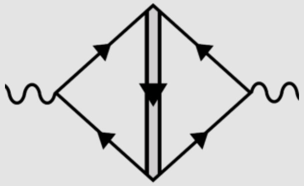
$$\Xi^R(\boldsymbol{k},\omega) \simeq \frac{1}{c\omega + \Xi^R(\boldsymbol{k},0)^{-1}} \simeq \cancel{\frac{1}{c\omega + a + b\boldsymbol{k}^2}}$$

⟺ TDGL equation: $ic\frac{\partial}{\partial t}\Delta + a\Delta - b\nabla^2\Delta = 0$

Vertices

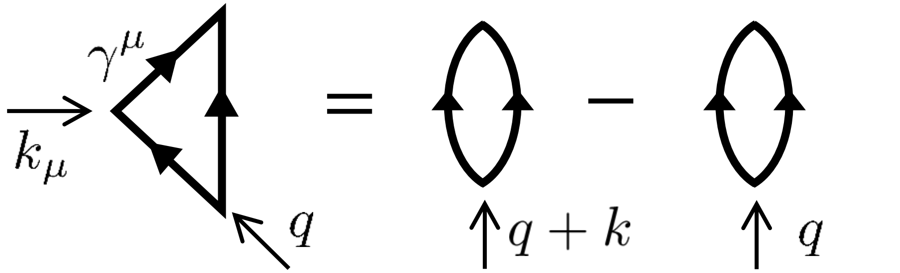
Vertices must be determined to be consistent with the TDGL approx.

$\Pi^{\mu\nu}_{\mathrm{AL}}(k) =$ 

$\Pi^{\mu\nu}_{\mathrm{MT}}(k) =$ 

☐ WT identity for AL vertex

$$k_\mu \Gamma^\mu(q, q+k) = \Xi^{-1}(q+k) - \Xi^{-1}(k)$$

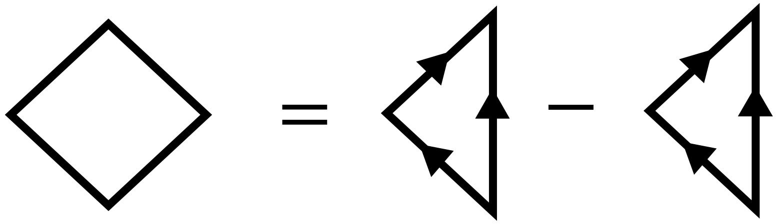


At the lowest order in k

$$\begin{cases} \Gamma^0 = e_\Delta c \\ \Gamma^i = e_\Delta \dfrac{\partial^2 \Xi(q)^{-1}}{\partial q^2}(2q^i + k^i) \end{cases}$$

e_Δ: electric charge of diquarks

☐ MT+DoS



Similar formula for MT+DoS vertex

Photon Self-Energy

□ Temporal Component

$$\Pi^{00}(k) = \frac{\boldsymbol{k}^2}{k_0^2}\Pi_{\mathrm{L}}(k)$$

➡ $\Pi^{00}(k)$ is obtained from spatial components.

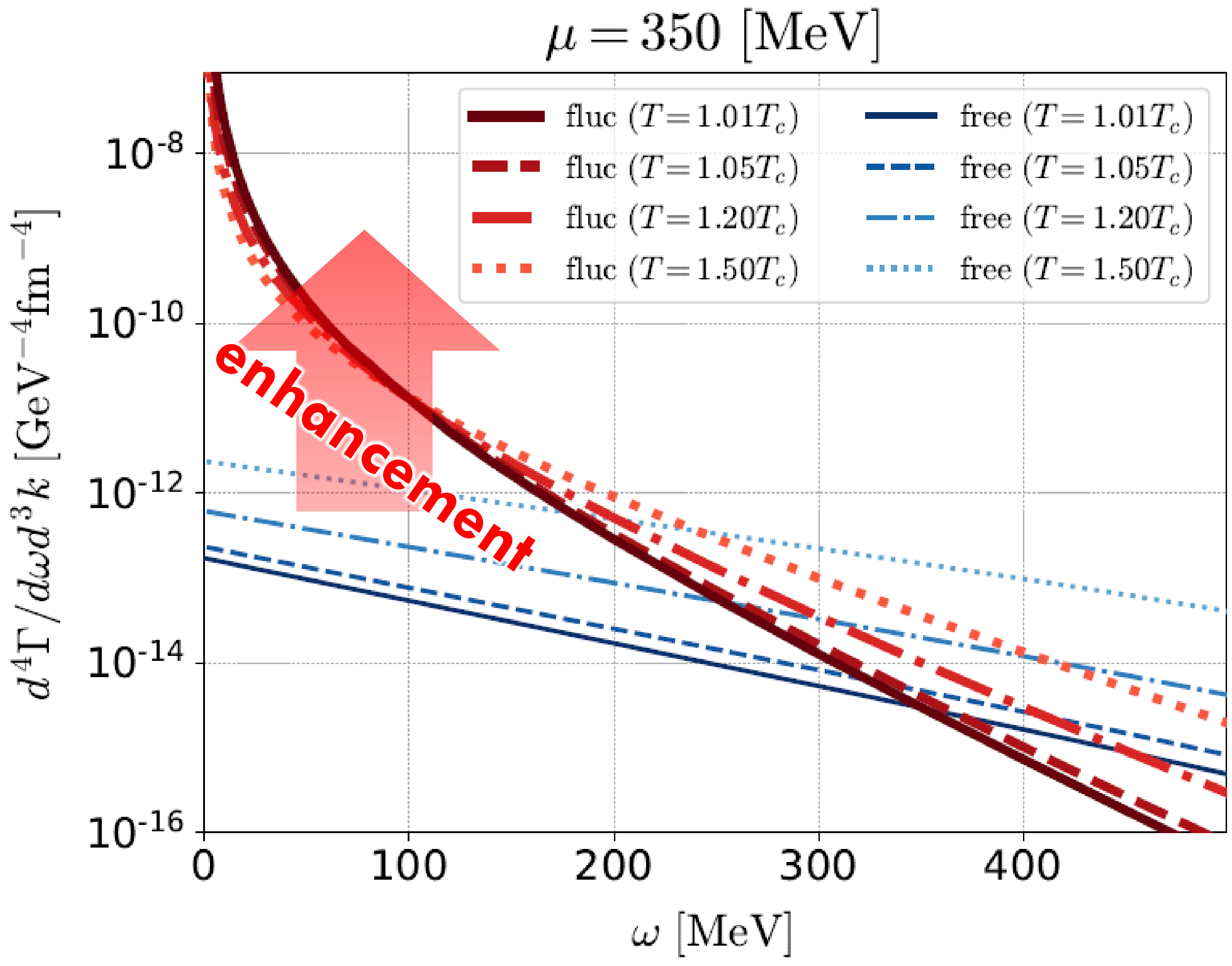
□ Cancellation of MT+DoS

$$\mathrm{Im}\Pi^{Rij}_{\mathrm{MT+DoS}}(k) = 0$$

➡ Calculation of AL term is sufficient to obtain $\mathrm{Im}\Pi^{\mu\nu}(k)$

$$\mathrm{Im}\Pi^{R\mu}_{\mu}(k) = \frac{\boldsymbol{k}^2 - k_0^2}{k_0^2}\mathrm{Im}\Pi^{R\mu}_{\mathrm{AL,L},\mu} + 2\mathrm{Im}\Pi^{R\mu}_{\mathrm{AL,T},\mu}$$

Production Rate at $\boldsymbol{k} = 0$



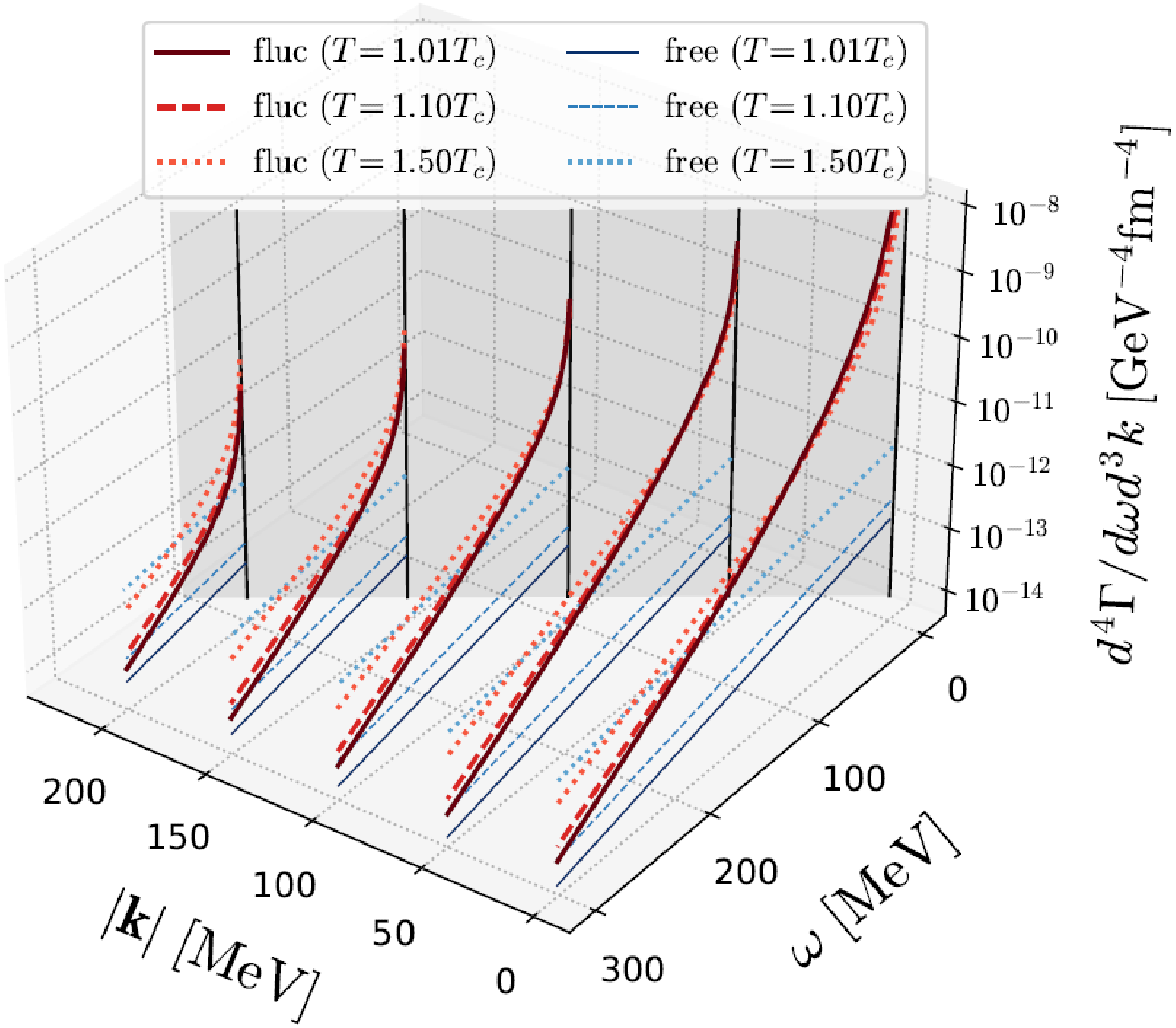
Red: fluctuation contribution

Blue: free quarks

$G_C = 0.7G_s$, $T_c \simeq 45$ MeV

- Di-quark fluctuations give rise to large enhancement in the low energy region $\omega < 200$ MeV and $T < 1.5T_c$.
- Anomalous enhancement is not sensitive to T.

Energy-Momentum Dependence



Red: fluctuation contribution

Blue: free quarks

$G_C = 0.7G_S, T_c \simeq 45$ MeV

- Enhancement due to diquark fluctuations is more suppressed for larger k.

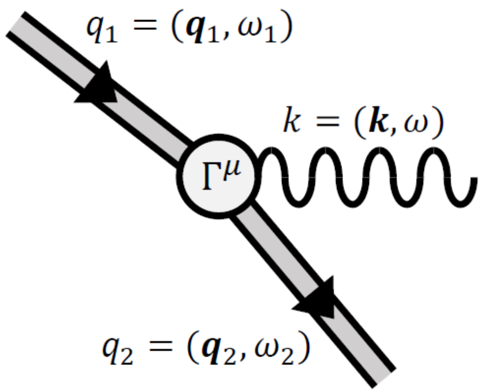
Production Mechanism of Virtual Photons

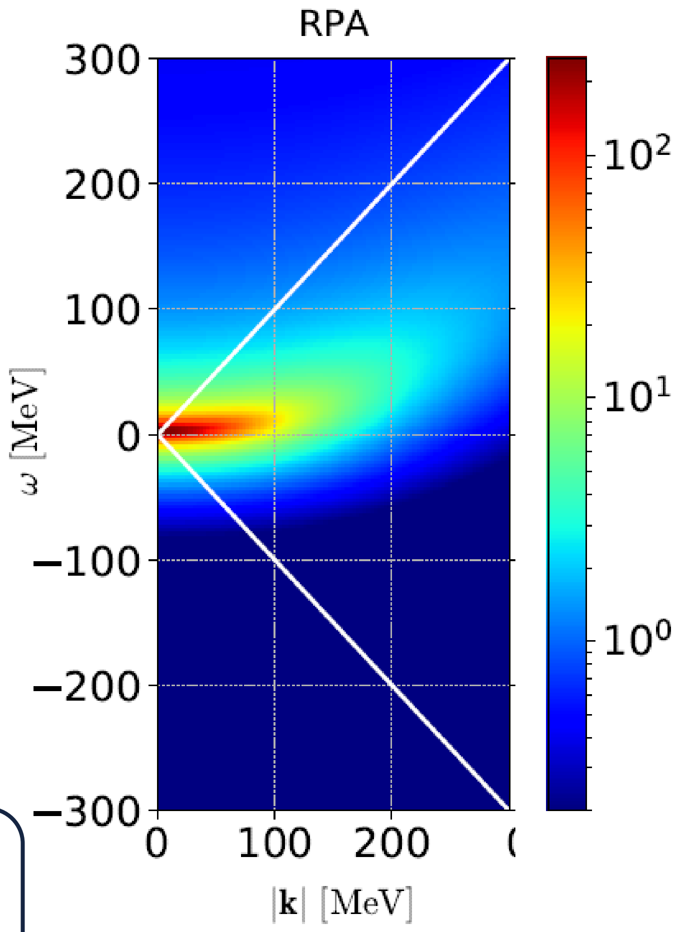
☐ Production mechanism

- ☐ scattering of diquarks
- ☐ diquarks: space-like region

$$\omega = \omega_1 - \omega_2$$

$$\boldsymbol{k} = \boldsymbol{q}_1 - \boldsymbol{q}_2$$



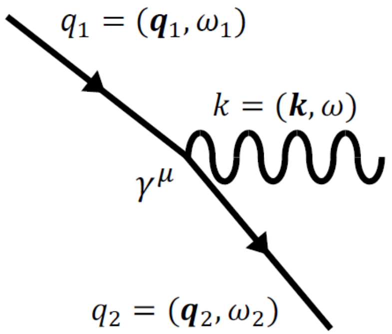


➡ Production in the time-like region is possible.

⬌

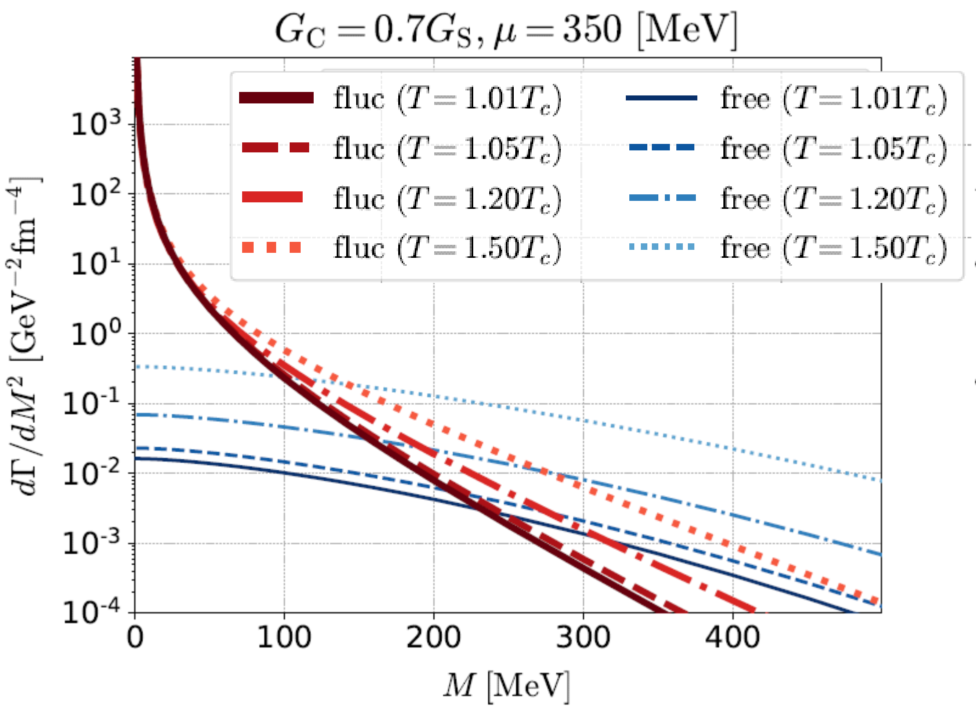
☐ C.f.) Scattering of free quarks produces virtual photons only in the space-like region.

$$|\boldsymbol{q}_1 - \boldsymbol{q}_2| \geq \omega_1 - \omega_2$$

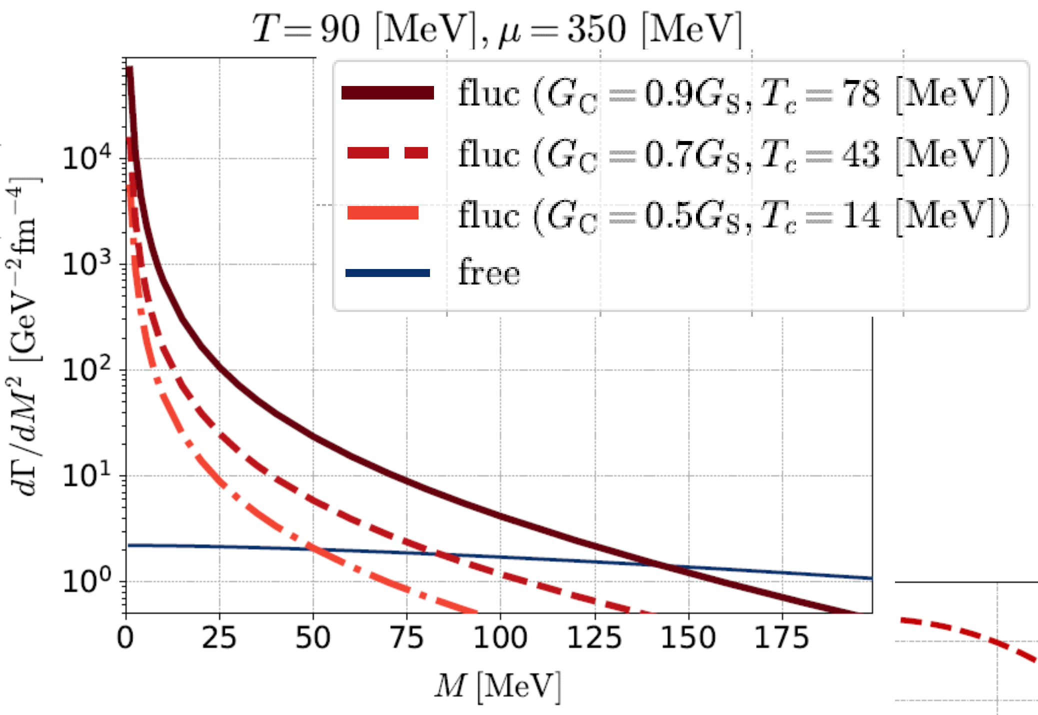


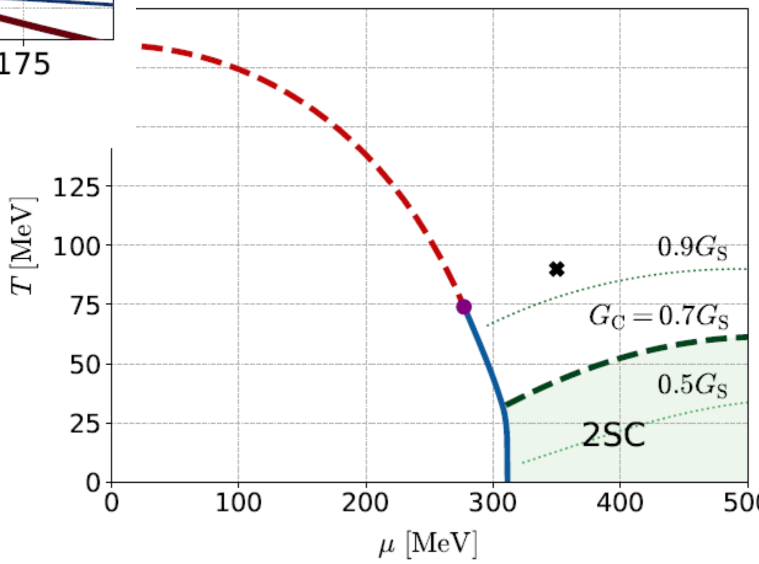
Invariant-Mass Spectrum

Fixed G_C



Fixed Temperature





- ☐ Strong enhancement at low invariant mass, though the range of M is narrower than the previous results.
- ☐ **Observable in the HIC?**

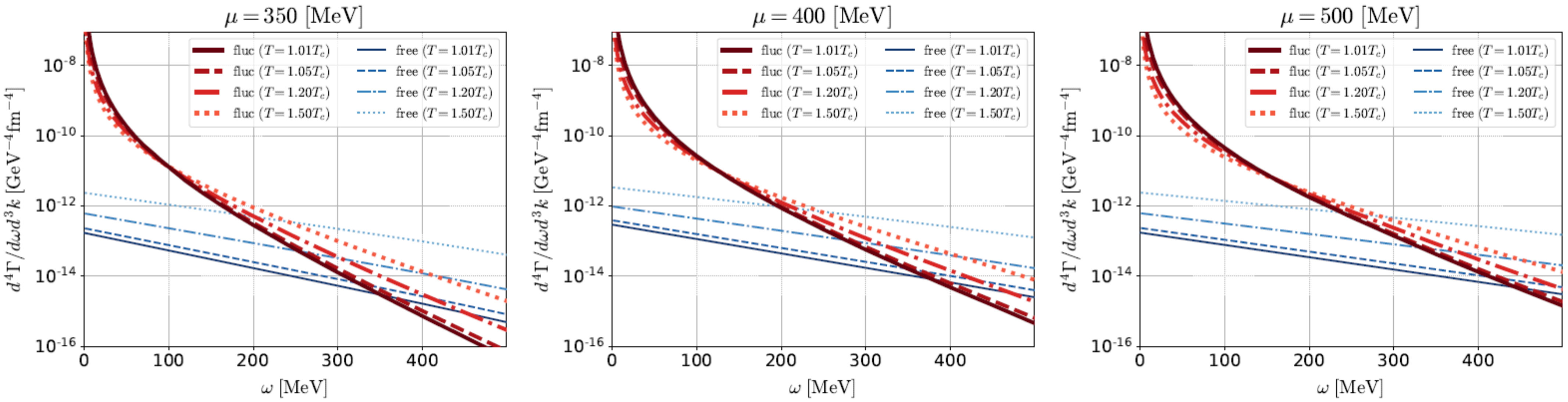
Summary

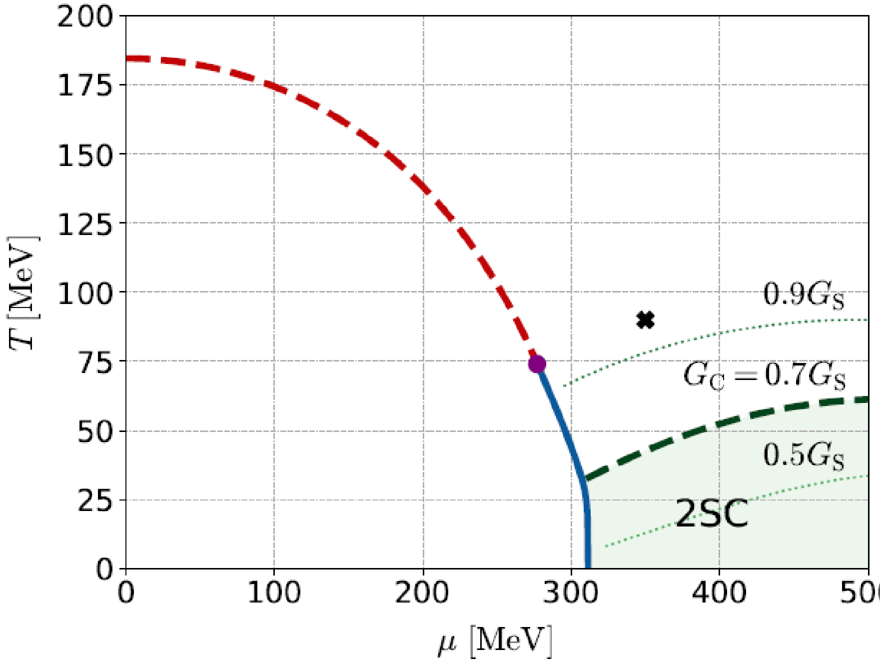
- The dilepton production rate is calculated incorporating the effects of diquark fluctuations enhanced near Tc of the 2SC.
  - AL, MT and DoS terms for the photon self-energy.
  - TDGL approximation for diquark propagator and vertices.

- Dilepton production rate is enhanced significantly near but above Tc at low invariant-mass region.
- **Signal of the onset of the CSC phase transition?**

- **Future**
  - Comparison with bremsstrahlung in QGP, hadronic effects, Dalitz decays, etc.
  - Quantitative estimate on dynamical models
  - Electric conductivity

μ Dependence





- The enhancement from di-quark fluctuations is more pronounced at the higher density region.