Updates on the QCD Phase Diagram in a Magnetic Field

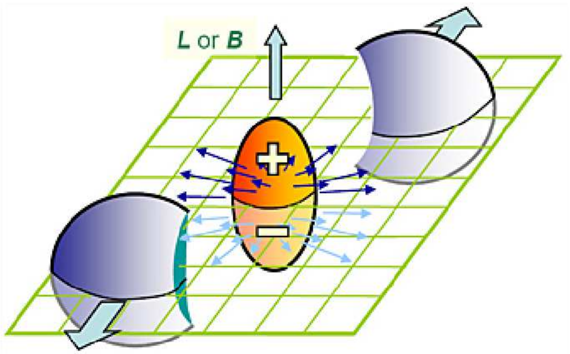
Massimo D'Elia - University of Pisa & INFN

QCD theory Seminars Series, January 11, 2022

Mostly based on work done in collaboration with **Lorenzo Maio, Francesco Sanfilippo and Alfredo Stanzione**

arXiv:2109.07456 and arXiv:2111.11237

1 – Introduction

- Strong interactions are described by QCD, the theory of quarks and gluons.
- Quarks are also subject to electroweak interactions, which in general induce small corrections to strong interaction dynamics, but exceptions are expected in presence of strong e.m. backgrounds, a situations which is relevant to many contexts:
  - Large magnetic fields ($B \sim 10^{10}$ Tesla on the surface) are expected in a class of neutron stars known as magnetars (Duncan-Thompson, 1992).
  - Large magnetic fields ($B \sim 10^{16}$ Tesla, $\sqrt{|e|B} \sim 1.5$ GeV), may have been produced at the cosmological electroweak phase transition (Vachaspati, 1991).
  - 

in non-central heavy ion collisions, largest magnetic fields ever created in a laboratory (B up to 10^{15} Tesla at LHC) with a possible rich associated phenomenology: chiral magnetic effect (Vilenkin, 1980; Kharzeev, Fukushima, McLerran and Warringa, 2008).

E.m. fields affect quarks directly and gluons only at the 1-loop level.

However non-perturbative effects can be non-trivial in the gluon sector as well.

Issues relevant this seminar:

- **Effects on the QCD vacuum structure:**
  - chiral symmetry breaking? Quite natural (Magnetic catalysis of χSB)
  - confinement? Less obvious (see later)
- Effects on the QCD phase diagram? $T_c(B)$?

LQCD is the ideal tool for a non-perturbative investigation of such issues. QCD+QED studies of the e.m. properties of hadrons go back to the early days of LQCD

Recent years have seen an increasing activity on the subject.

Lattice QCD in electromagnetic background fields

An e.m. background field a_μ modifies the continuum covariant derivative as follows:

$$D_\mu = \partial_\mu + i\,g A_\mu^a T^a \quad \to \quad \partial_\mu + i\,g A_\mu^a T^a + i\,q a_\mu$$

in the lattice formulation

$$D_\mu \psi \to \frac{1}{2a}\big(U_\mu(n) u_\mu(n)\psi(n+\hat{\mu}) - U_\mu^\dagger(n-\hat{\mu}) u_\mu^*(n-\hat{\mu})\psi(n-\hat{\mu})\big)$$

$U_\mu \in SU(3)$

$\mathbf{u}_\mu \simeq \exp(\mathbf{i}\,\mathbf{q}\,\mathbf{a}_\mu(\mathbf{n})) \in \mathbf{U(1)}$ **depends on the quark charge q.**

The discretized version of the fermion action is still a bilinear form in the fermion fields, $\bar{\psi}(i) M_{i,j} \psi(j)$ however the elements of M now belong to $U(3)$

Path integral measure:

$$\int \mathcal{D}U e^{-S_G[U]} \det M[U] \quad \rightarrow \quad \int \mathcal{D}U e^{-S_G[U]} \det M[U, u]$$

- u **fields here are considered as not dynamical (fixed): quenched QED approach. They affect the gluon field distribution through the quark determinant;**

- $\det(\not{D} + m) > 0$**: standard Monte-Carlo simulations are feasible**

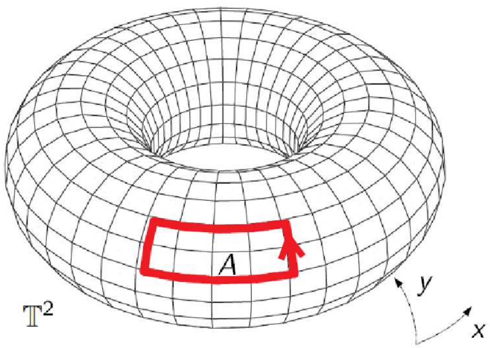
Some limitations and constraints

- **IR limitations in presence of periodic b.c.** ('t Hooft, 1979; Al-Hashimi and Wiese, arXiv:0807.0630)

  total flux through torus surface must be quantized (like Dirac quantization for monopoles), e.g. for $\vec{B} = B\hat{z}$:

$$qB = \frac{2\pi b}{L_x L_y a^2}$$

  where b is integer



- **A possible choice for gauge links, corresponding to continuum $a_y = Bx$, is:**

$$u_y(B,q)(n) = e^{i\,a^2 qB\,n_x}\,;\;\; u_\mu(B,q)(n) = 1 \text{ for } \mu = x,z,t\,;\;\; u_x(B,q)(n)|_{n_x = L_x} = e^{-i\,a^2 qL_x B\,n_y}$$

  corresponds to a uniform field plus a Dirac string, which is invisible for integer b

- **UV limitations from discretization: the plaquette sets the minimum explorable flux on the lattice, defined up to a 2π phase, thus fixing a sort of first Brillouin zone:**

$$-\frac{\pi}{a^2} < qB < \frac{\pi}{a^2}$$

OUTLINE

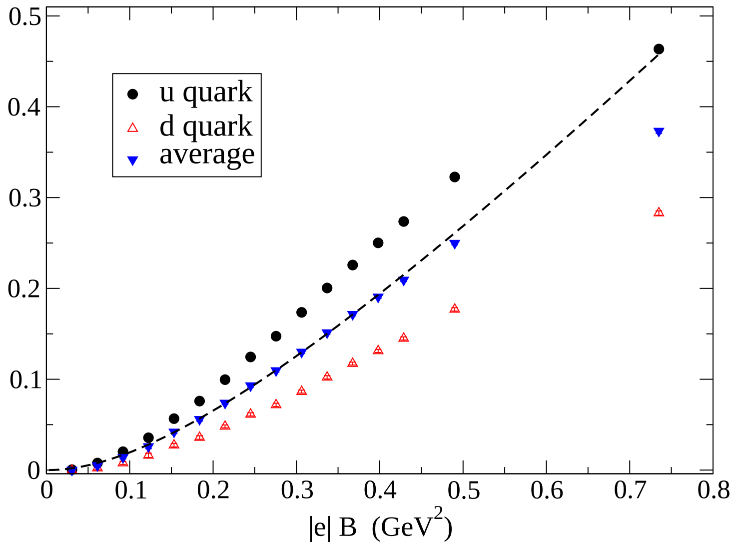
- **Known facts about the QCD in magnetic background: chiral and confining properties, phase diagram**

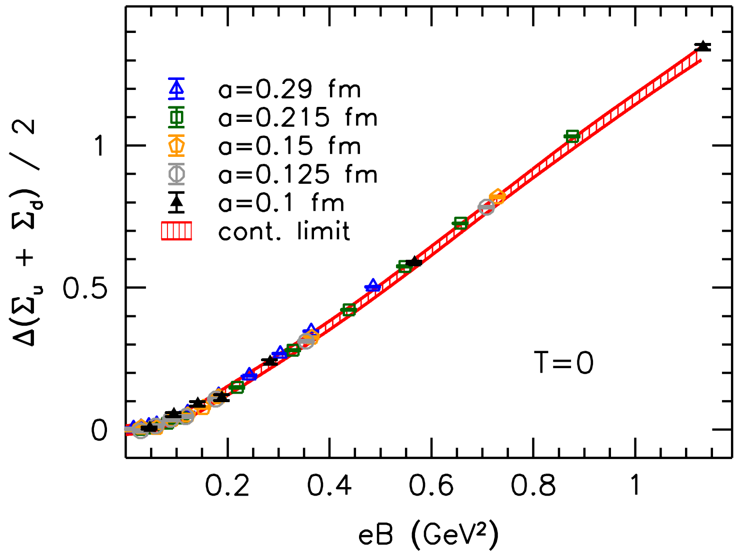
- **Updates on the properties at $T = 0$ and in the large B limit**

- **New results at $T \neq 0$ in the large B limit: updated phase diagram**

Known facts at $T = 0$ from lattice QCD simulations

Magnetic catalysis has been checked up to moderate values of eB





LEFT: **increase in the light quark condensates, $N_f = 2$ QCD, $m_\pi \simeq 200$ MeV, unimproved staggered fermions** (M. D'E and Francesco Negro, arXiv:1103.2080)

RIGHT: **$N_f = 2+1$ QCD, improved staggered fermions, physical quark masses** (G. Bali et al., arXiv:1206.4205)

early studies in pure gauge theories (Buividovich, Chernodub, Luschevskaya and Polikarpov, Phys. Lett. B 682, 484 (2010) and arXiv:1011.3795)

recently confirmed up to $eB \sim 3$ GeV^2 with HISQ staggered fermions

H. T. Ding, S. T. Li, A. Tomiya, X. D. Wang and Y. Zhang, arXiv:2008.00493

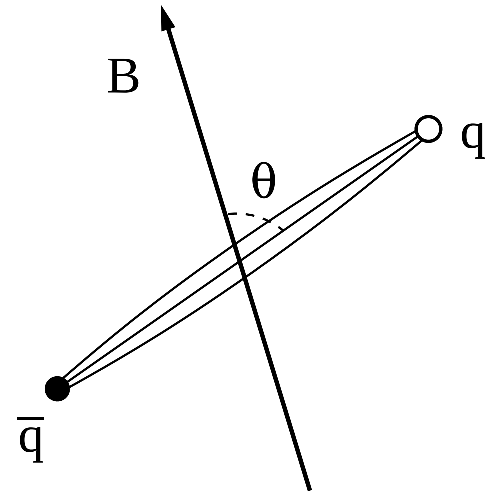
The magnetic field has a significant effect also on purely gluonic quantities

A brief review about the effect of B on confinement

The effects of a magnetic background on the static quark-antiquark potential have been studied in a couple of recent lattice studies

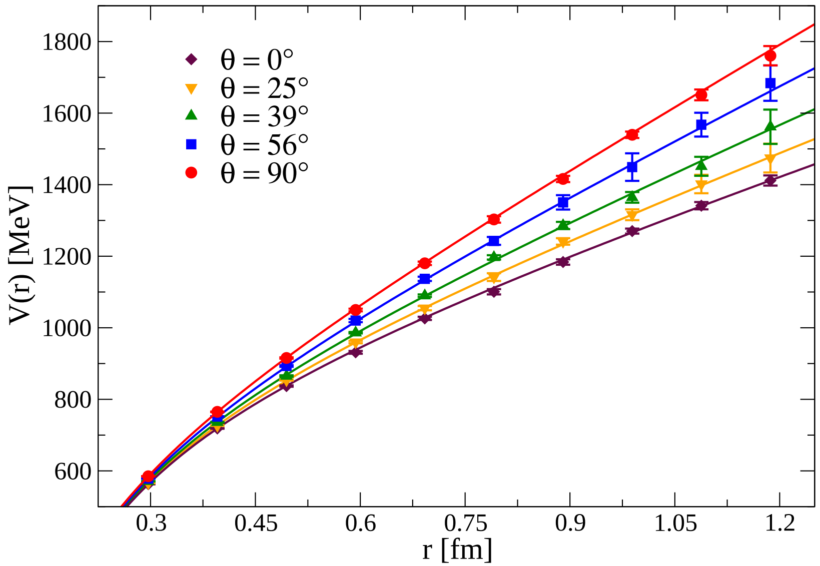
C. Bonati, MD, M. Mariti, M. Mesiti, F. Negro, A. Rucci and F. Sanfilippo, PRD 94, no.9, 094007 (2016), arXiv:1607.08160

C. Bonati, MD, M. Mariti, M. Mesiti, F. Negro and F. Sanfilippo, PRD 89, no.11, 114502 (2014), arXiv:1403.6094



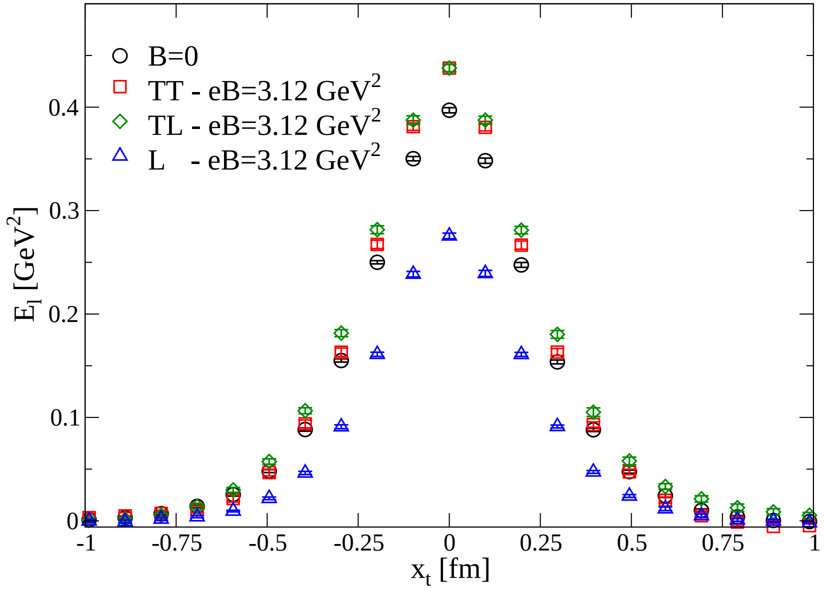
The potential becomes anisotropic, with a reduction of the string tension in the direction parallel to B, and an increase in the direction orthogonal to it

$eB \simeq 1$ **GeV**^{2} $\Longrightarrow$

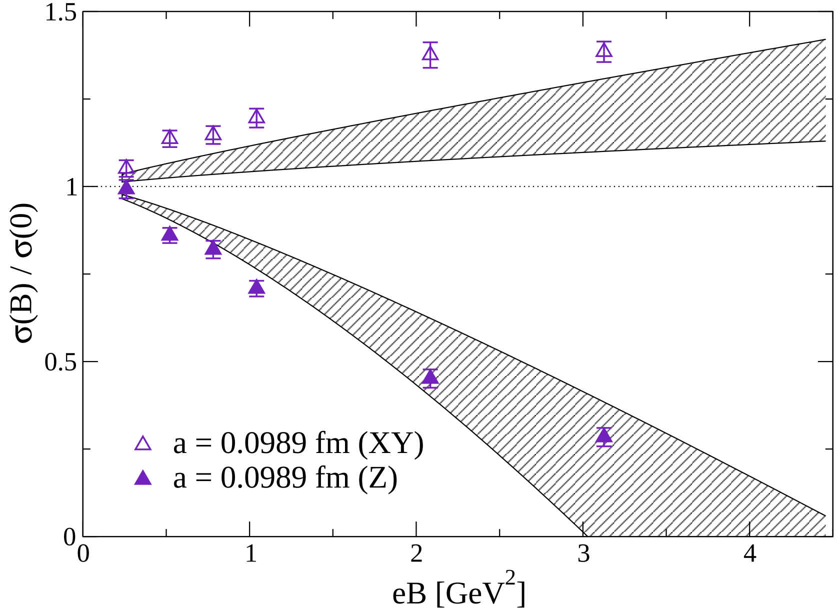


The modifications are clearly visible also at the level of the flux tube profile

C. Bonati, S. Calì, M. D'Elia, M. Mesiti, F. Negro, A. Rucci and F. Sanfilippo, PRD 98, no.5, 054501 (2018), arXiv:1807.01673



The continuum extrapolated results for σ predicted a vanishing longitudinal string tension for $eB \sim 4$ GeV^2

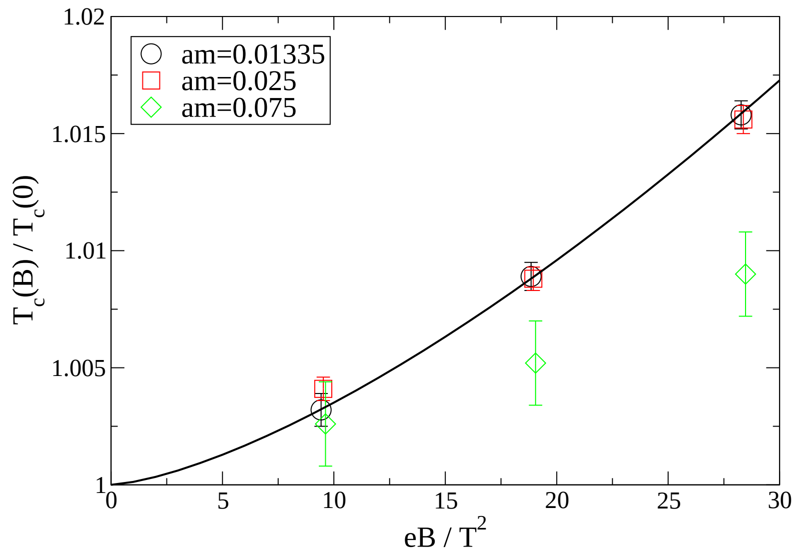


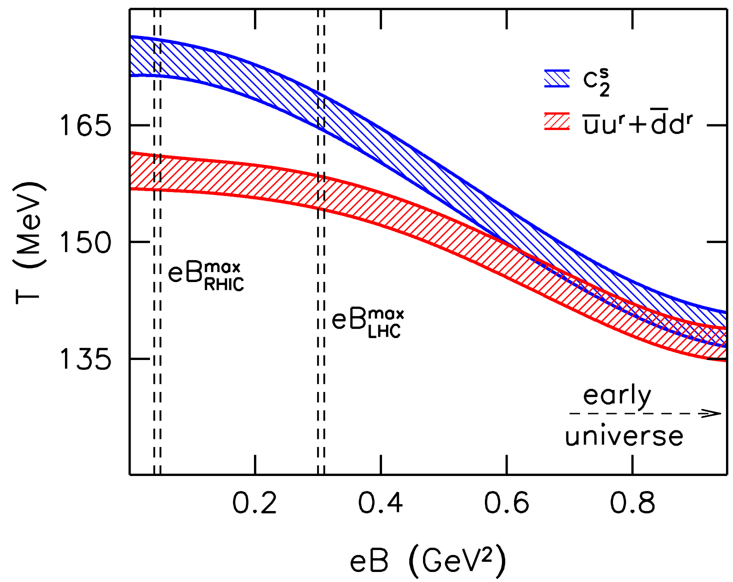
This is however, outside the range explored for the continuum extrapolation, $eB \lesssim 1$ GeV^2

Is that really true?

Known facts at $T \neq 0$ from lattice QCD simulations

Early lattice results on the QCD phase diagram in a magnetic background produced contrasting results: $T_c(B)$ increasing vs decreasing





$N_f = 2$ **standard staggered fermions, plaquette gauge action,** $m_\pi \simeq 200$ **MeV,** $a \simeq 0.3$ **fm**

MD, S. Mukherjee and F. Sanfilippo, PRD 82, 051501 (2010), arXiv:1005.5365

$N_f = 2+1$ **stout improved staggered fermions, Symanzik improved gauge action, physical quark masses, continuum extrap.**

G. S. Bali *et al*, JHEP 02, 044 (2012), arXiv:1111.4956

Decreasing behaviour confirmed by later studies, together with a substantial strengthening of the transition

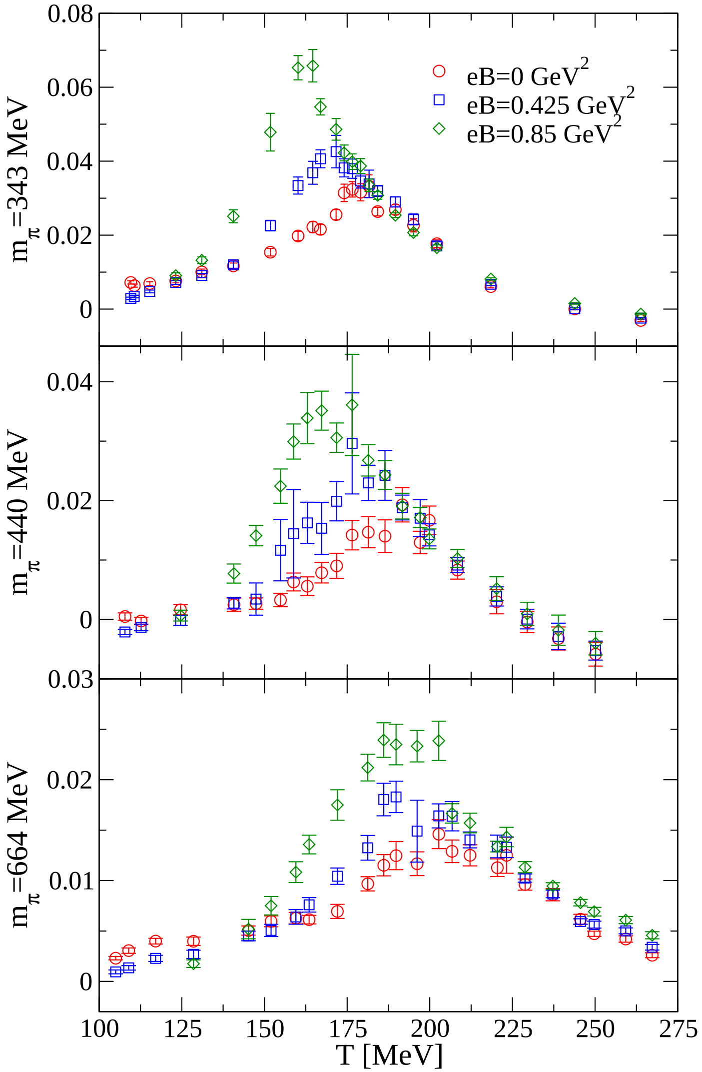
Early results affected by lattice artefacts, the decrease of T_c confirmed also for heavier pion masses

(MD, F. Manigrasso, F. Negro and F. Sanfilippo, PRD 98, no.5, 054509 (2018), arXiv:1808.07008)

(G. Endrodi, M. Giordano, S. D. Katz, T. G. Kovács and F. Pittler, JHEP 07, 007 (2019), arXiv:1904.10296)

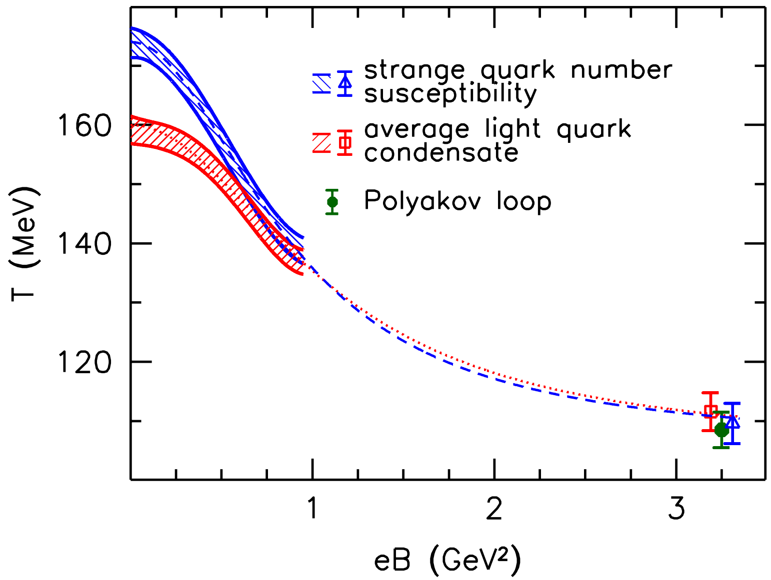
Renormalized chiral susceptibility for different temperatures, magnetic fields and pion masses

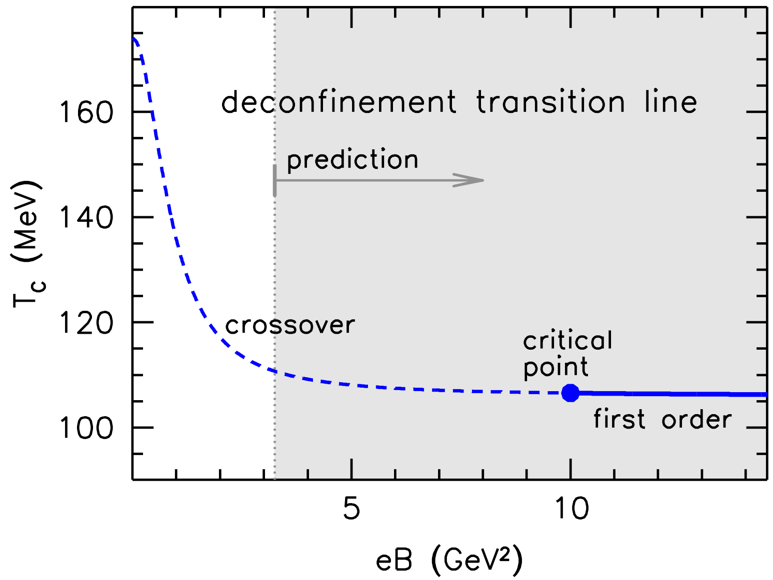
T_c **decreases with** B **for all pion masses, likely up to the quenched limit, and the transition strengthens**



Later studies have extended lattice simulations of the phase diagram up to $eB \simeq 3$ GeV^2 and speculated about the possible presence of a critical endpoint for $eB \simeq 10$ GeV^2, where the transition would turn into first order

likely relevant for the Early Universe





from G. Endrodi, JHEP 07, 173 (2015) [arXiv:1504.08280 [hep-lat]]

Direct confirmations of first order only obtained with unimproved staggered fermions

H. T. Ding, C. Schmidt, A. Tomiya and X. D. Wang, PRD 102, 054505 (2020) [arXiv:2006.13422 [hep-lat]].

Looking for updates

- is there a critical magnetic field B_c, at $T = 0$, where confinining properties of QCD get disrupted? (anisotropic deconfinement?)
- What is fate of $T_c(B)$ for large magnetic fields? And what the fate of the order of the phase transition?

Recently, we started some efforts in this direction:

MD, L. Maio, F. Sanfilippo, A. Stanzione, arXiv:2109.07456 and arXiv:2111.11237

- $N_f = 2 + 1$ QCD with physical quark masses and two large magnetic fields, $eB = 4$ and 9 GeV^2
- 2-level stout improved stag. fermions, Symanzik tree level improved gauge action
- three different lattice spacings, $a = 0.057, 0.086, 0.114$ fm, spatial size mostly kept fixed around 3 fm
- additional UV effects expected from large B: maximum flux across a plaquette sets $eB \lesssim 2\pi/a^2 \sim 18$ GeV^2 for $a = 0.114$ fm, so 9 GeV^2 is borderline ...

Technical Details

We consider a discretization of $N_f = 2+1$ QCD based on the tree-level improved Symanzik pure gauge action and on stout rooted staggered fermions

$$Z = \int [DU]\, e^{-S_{YM}} \prod_{f=u,d,s} \det\,(M^f_{st})^{\frac{1}{4}},$$

$$M^f_{st\ ij} = \hat{m}_f \delta_{ij} + \sum_{\nu=1}^{4} \frac{\eta_{i;\nu}}{2} \left(U^{(2)}_{i;\nu} \delta_{i\,j-\hat{\nu}} - U^{(2)\dagger}_{i-\hat{\nu};\nu} \delta_{i\,j+\hat{\nu}} \right) \;;\; S_{YM} = -\frac{\beta}{3} \sum_{\substack{i \\ \mu\neq\nu}} \left(\frac{5}{6} W^{1\times1}_{i,\mu\nu} - \frac{1}{12} W^{1\times2}_{i,\mu\nu} \right)$$

Bare quark masses and β tuned so as to move on a line of constant physics

Renormalized chiral condensate and susceptibility:

$$\langle \bar{\psi}\psi \rangle_f = \frac{\partial}{\partial m_f} \left(\frac{T}{V_s} \log Z \right) = \frac{1}{4 a^3 L_s^3 N_t} \left\langle (M^f_{st})^{-1} \right\rangle$$

$$\langle \bar{\psi}\psi \rangle^r_f(B,T) = \frac{m_f}{m_\pi^2 F_\pi^2} \left(\langle \bar{\psi}\psi \rangle_f(B,T) - \langle \bar{\psi}\psi \rangle_f(0,0) \right)$$

The zero-T subtraction, performed at fixed UV cut-off, eliminates additive divergences, while multiplication by the bare quark mass m_f takes care of multiplicative ones.

Just for the purpose of a finite size scaling analysis around the transition, we will consider also the unrenormalized disconnected chiral susceptibility

$$\chi^{disc}_{\bar\psi\psi,f} \equiv \frac{1}{16 L_s^3 N_t} \left[\langle (M_f^{-1})^2 \rangle - \langle M_f^{-1} \rangle^2 \right] \; .$$

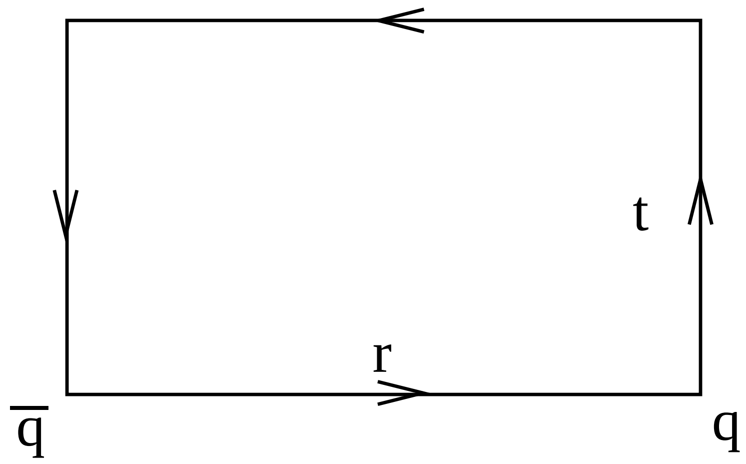
The dimensionless susceptibility of the strange quark number is instead defined as follows (with $f = s$):

$$\chi_f \equiv \frac{1}{T^2} \frac{\partial}{\partial \mu_f^2} \left(\frac{T}{V_s} \log Z \right) = \frac{N_t}{4 L_s^3} \left\langle \left[M_f^{-1} \partial^2_{a\mu_f} M_f - \left(M_f^{-1} \partial_{a\mu_f} M_f \right)^2 \right] \right\rangle$$

where μ_f is the quark chemical potential and, in the last line, only terms not vanishing at $\mu_f = 0$ have been left

Extraction of the static quark-antiquark potential

At $T = 0$, the potential is determined through Wilson loop expectation values

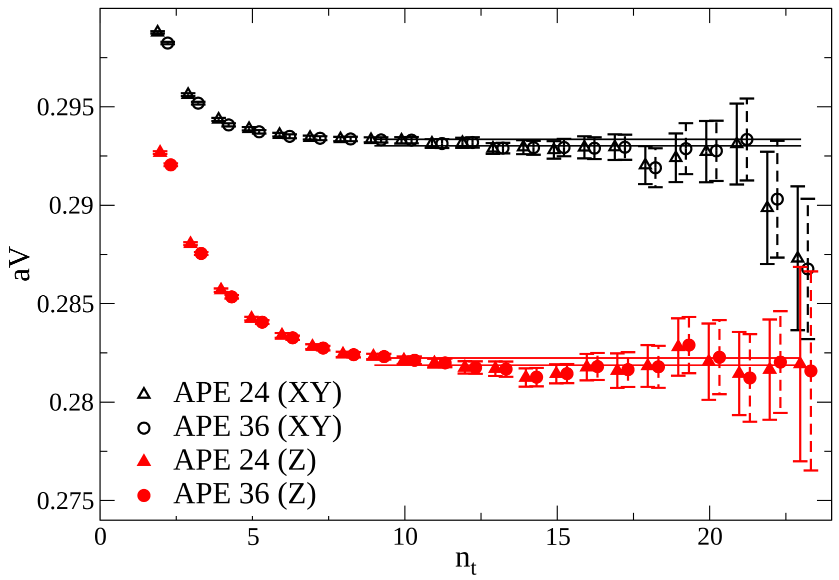


1 HYP smearing for temporal links and various APE smearings for spatial links to reduce UV fluctuations

As usual

$$aV(a\vec{n}) = \lim_{n_t \to \infty} \log \left(\frac{\langle W(\vec{n}, n_t) \rangle}{\langle W(\vec{n}, n_t + 1) \rangle} \right)$$

results in the figure refer to two different orientations with respect to $\vec{B} = B\hat{z}$, and for simulations performed at $a \simeq 0.0989$ fm with $|e|B \simeq 1$ GeV^2.



Lattice determinations of color flux tubes make use of correlation between Wilson loops and plaquette operators.

Connected correlators allow the determination of the field strength itself

[Di Giacomo, Maggiore, Olejnik, 1990] [Cea, Cosmai, Cuteri, Papa, 2017]

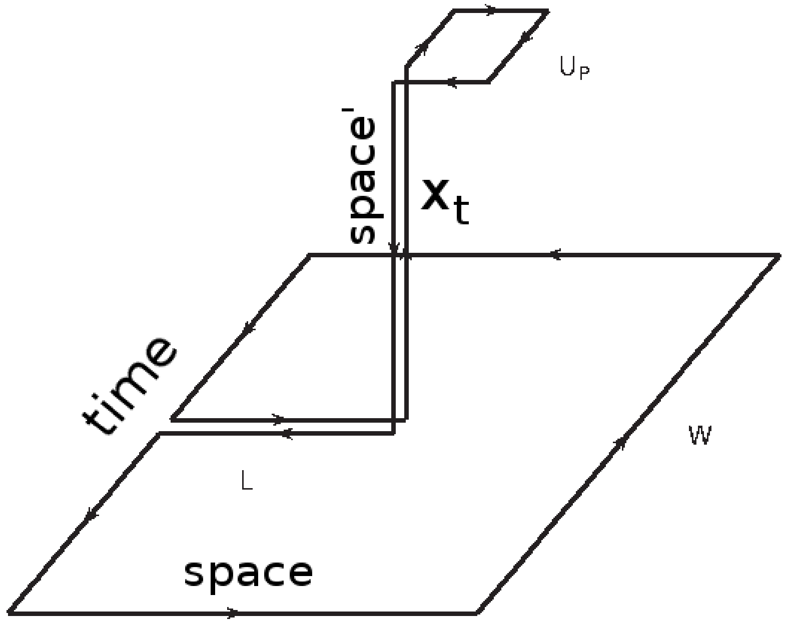
$$E_l^{chromo} = \lim_{a\to 0} \frac{1}{a^2 g}\left[\frac{\langle Tr(WLU_PL^\dagger)\rangle}{\langle Tr(W)\rangle} - \frac{\langle Tr(W)Tr(U_P)\rangle}{\langle Tr(W)\rangle}\right]$$

W is the open Wilson loop operator

U_P is the open plaquette operator

L is the adjoint parallel transport

A smearing procedure is adopted (1 HYP for temporal links, several APE for spatial links) as a noise reduction technique

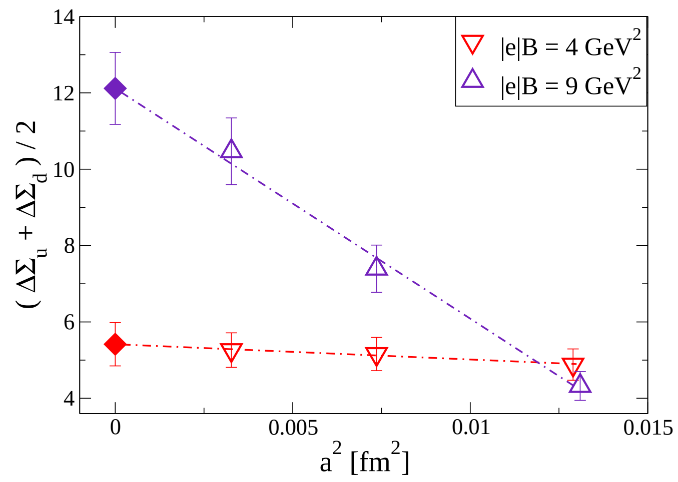


Updates on $T = 0$ results

MD, L. Maio, F. Sanfilippo, A. Stanzione, arXiv:2109.07456

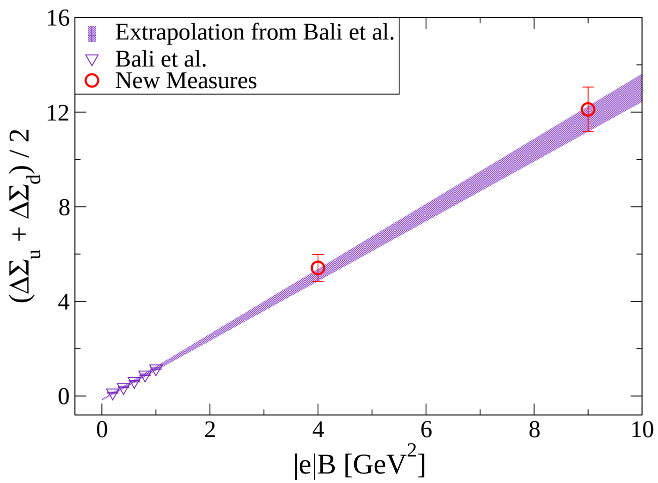
increase of the renormalized light chiral condensate, simulations on $24^3 \times 48$, $32^3 \times 64$, $48^3 \times 96$ lattices

lattice artefacts significant for $eB = 9$ GeV^2, minimum phase around a plaquette is $\simeq 2\pi/3$ for the up quark on the coarsest lattice



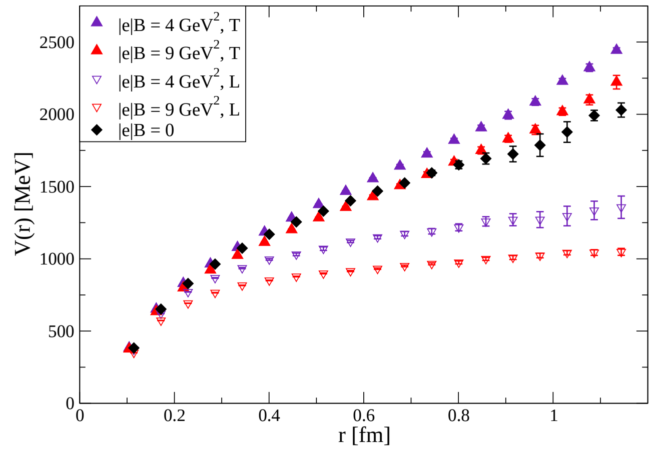
Nevertheless, after continuum extrapolation magnetic catalysis $\sim$ linear with eB confirmed up to $eB \sim 9$ GeV^2

similar results up to $eB \sim 3$ GeV^2 with HISQ staggered fermions H. T. Ding, S. T. Li, A. Tomiya, X. D. Wang and Y. Zhang, arXiv:2008.00493

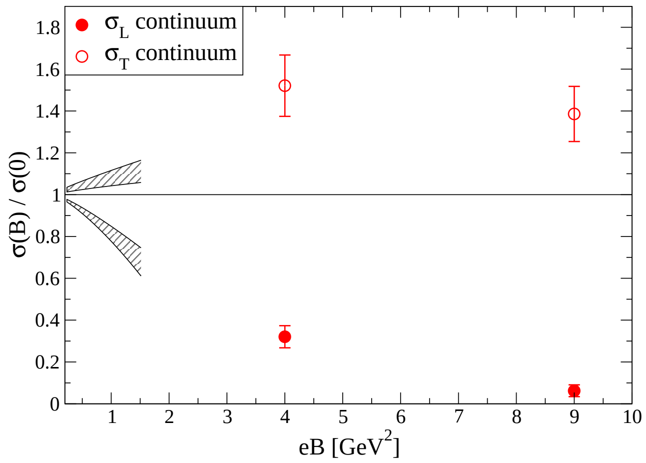


Results for the static potential (longitudinal vs transverse) on the finest lattice

Contrary to previous extrapolations, the longitudinal string tension does not seem to vanish, neither at 4 GeV^2, nor at 9 GeV^2



After proper continuum extrapolation, the transverse string tension seems to saturate, the longitudinal string tension is strongly suppressed, could vanish at some larger magnetic field



Updates on the finite B - finite T Phase Diagram

MD, L. Maio, F. Sanfilippo, A. Stanzione, in progress

Finite T simulations performed at fixed cut-off

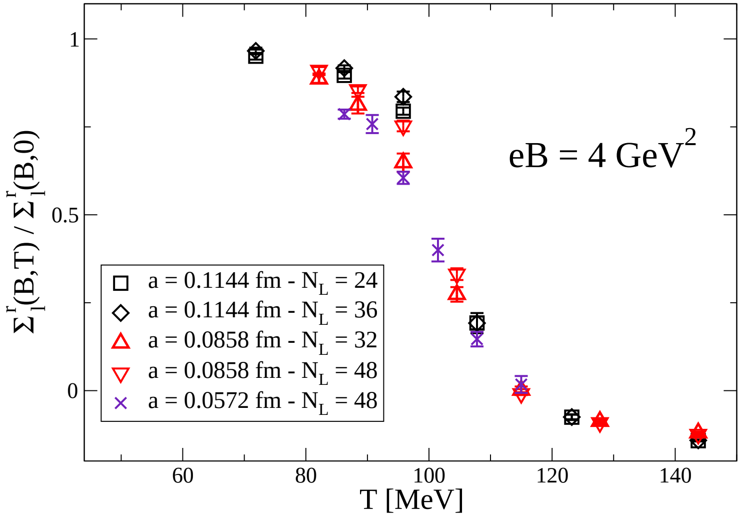
$$T = 1/(N_t a)$$

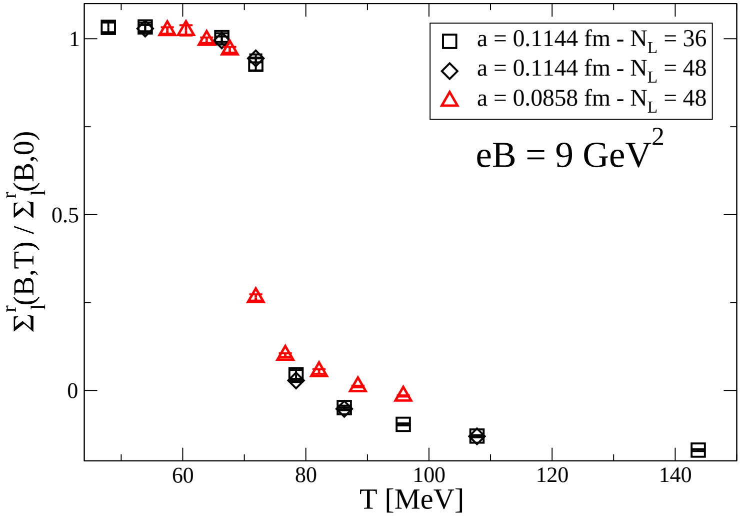
keeping a fixed and changing N_t, three sets of lattice spacings

Renormalized light chiral condensate, normalized by $T=0$ values

the inflection point moves to lower and lower temperatures as eB increases

at $eB = 9$ GeV^2 a jump seems to appear (first order transition?)

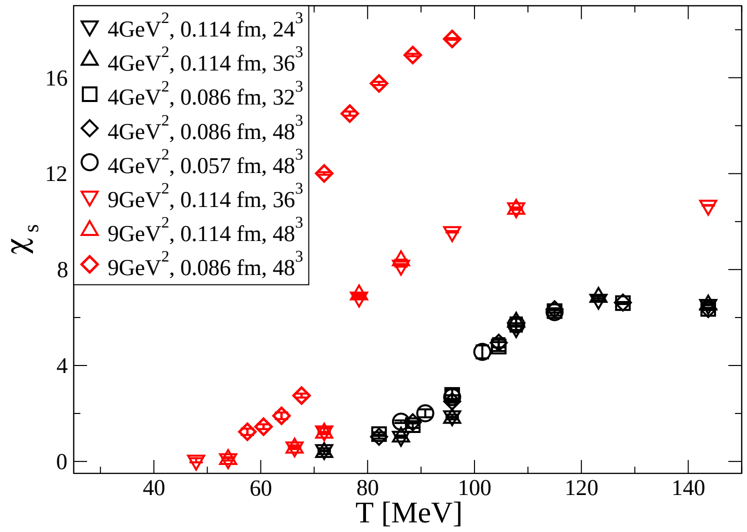




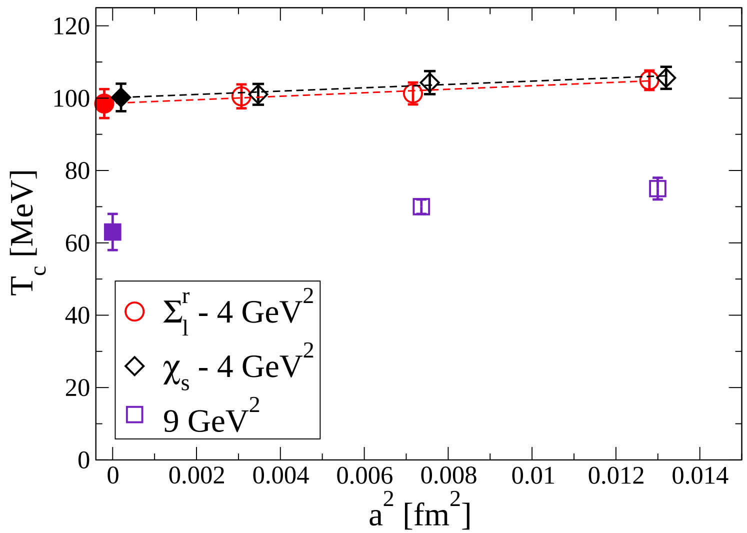
Similar results from the strange quark number susceptibility

large cutoff effects at 9 GeV^2, but the jump is stable or even stronger on the finer lattice

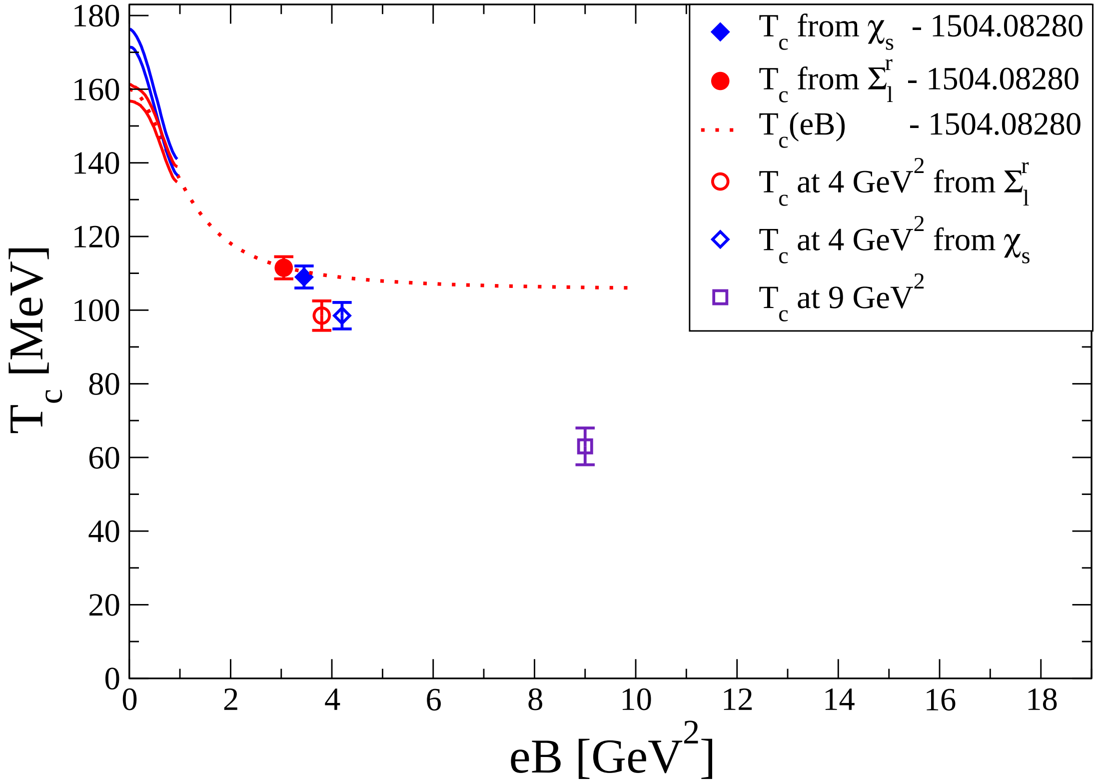
We also observe a significant increase of quark number fluctuations with eB, confirming results reported in H. T. Ding, S. T. Li, Q. Shi, A. Tomiya, X. D. Wang and Y. Zhang, arXiv:2011.04870



The critical temperature extrapolates smoothly to the continuum limit and is observable-independent



Updated phase diagram - a first sketch



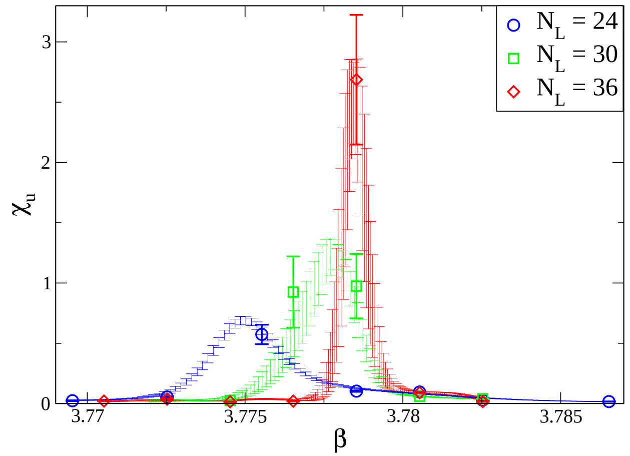
main observations:

- the steady decrease of T_c continues ... hitting the ground somewhere?
- **morover, it seems that at 9 GeV^2 the transition is first order, but we just see a large jump on discrete temperature mesh. We need to check more carefully ...**

Finite size scaling analysis around 9 GeV^2

In order to fine tune T and perform a multi-histogram analysis, we give up the fixed cut-off and the line-of-constant-physics setup, changing just the inverse gauge coupling β around there

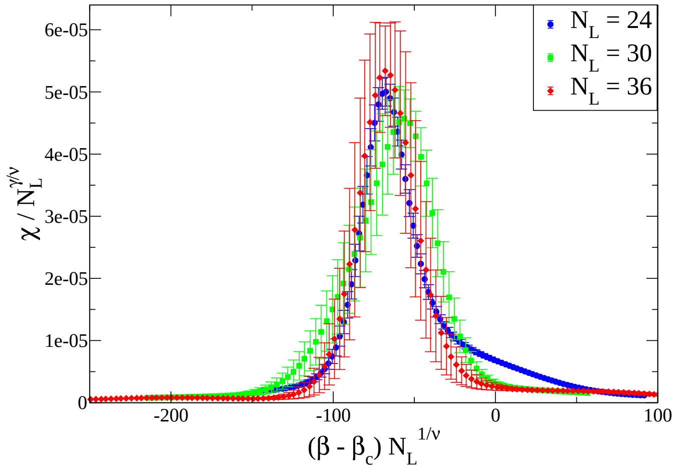
This is of course irrelevant in order to assess if there is a first order transition around there



FSS analysis for the unrenormalized, disconnected up-quark chiral susceptibility

$$\chi_u / N_L^{\gamma/\nu} = \phi\left((\beta - \beta_c) N_L^{1/\nu}\right)$$

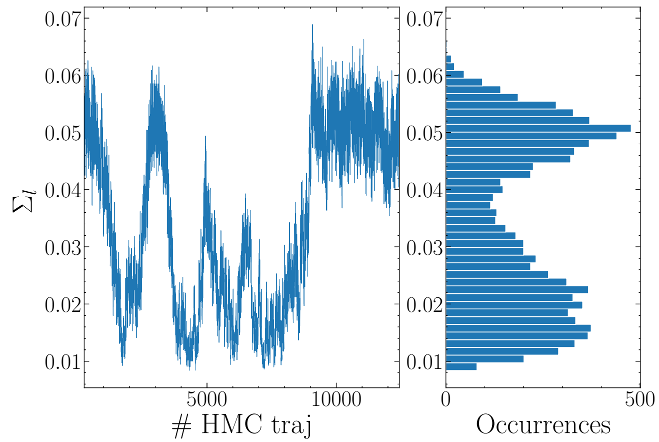
correct scaling observed with first order indexes, $\nu = 1/3$ and $\gamma = 1$



Other smoking guns for a first order transition at 9 GeV^2

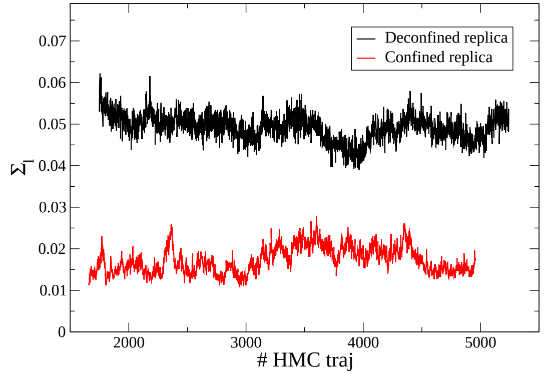
Double peak in the distribution of the light chiral condensate observed at the transition point on a $24^3 \times 20$ lattice

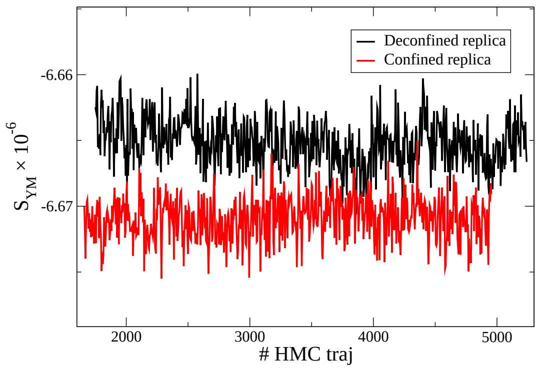
Similar bistability observed in other observables, including the pure gauge action



On a larger lattice, $36^3 \times 20$, twin pair of runs starting from different sides of the transition with identical parameters stay separated

$\Longrightarrow$ strong metastability observed both in the chiral condensate and in the action density

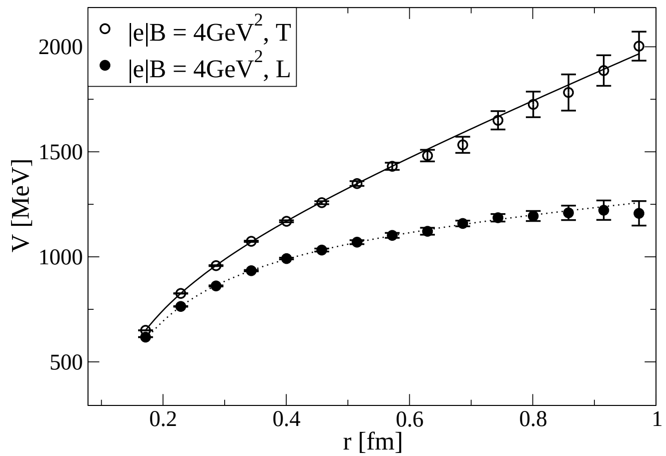




What about the confining properties on the two sides of the transition?

Static potential at $T = 86$ MeV and $eB = 4$ GeV^2, i.e. in the chirally broken phase

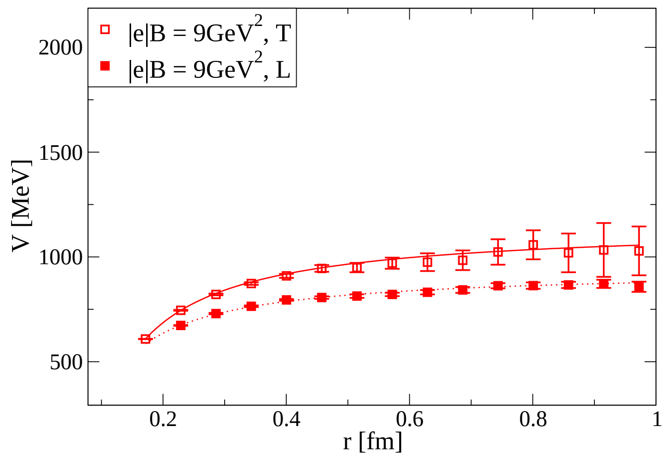
The system seems confined, the confined phase is still strongly anisotropic, $\sqrt{\sigma_T} = 475(15)$ MeV, $\sqrt{\sigma_L} = 215(15)$ MeV



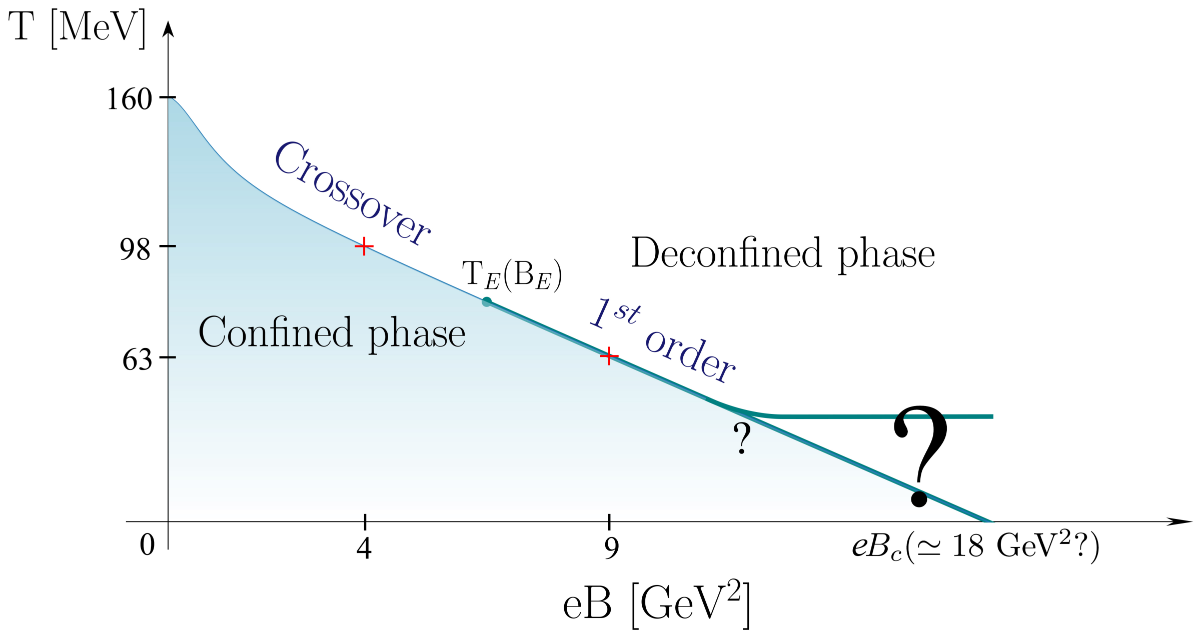
Static potential at $T = 86$ MeV and $eB = 9$ GeV^2, i.e. in the chiral restored phase

results are compatible with deconfinement

The potential is well fitted by a purely Coulombic term, even if we cannot exclude a non-zero σ within errors.



Updated phase diagram: new facts and new speculations



- T_c **decreases at least down to** 60 **MeV**
- **The transition becomes first order with a critical endpoint** $65 \text{ MeV} < T_E < 95 \text{ MeV}$**,** $4 \text{ GeV}^2 < eB_E < 9 \text{ GeV}^2$
- **The transition at large** eB **seems deconfining, with the string tension anisotropic in the confined phase and likely vanishing in the deconfined phase**
- **Does** $T_c(B)$ **hit the ground at some large** $eB_c \sim 20 \text{ GeV}^2$**, or not? Would that be a natural scale for** $N_f = 2 + 1$ **QCD ?**

Perspective

- The critical endpoint in the $B - T$ could be extremely interesting for the physics of the Early Universe

- Future simulations could locate the critical endpoint more precisely

- Properties of the two metastable phases should be better clarified by measuring other interesting observables, for instance electric conductivity (N. Astrakhantsev et al., arXiv:1910.08516) or other transport properties

- Studies at $T = 0$ and larger magnetic field could help clarifying if a finite B_c exists along that axis, and if $T_c(B)$ indeed hits the ground at B_c