Dynamic critical behavior in the Z_2 and O(4) universality classes

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J. Berges, SS, D. Sexty, Nucl. Phys. B832 (2010) 228-240
SS, D. Smith, L. von Smekal Nucl.Phys.B 950 (2020)
D. Schweitzer, SS, L.von Smekal Nucl.Phys.B 960 (2020)
D. Schweitzer, SS, L.von Smekal arXiv:2110.01696

QCD Theory Seminar,
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Strong-interaction matter under extreme conditions

UNIVERSITÄT BIELEFELD

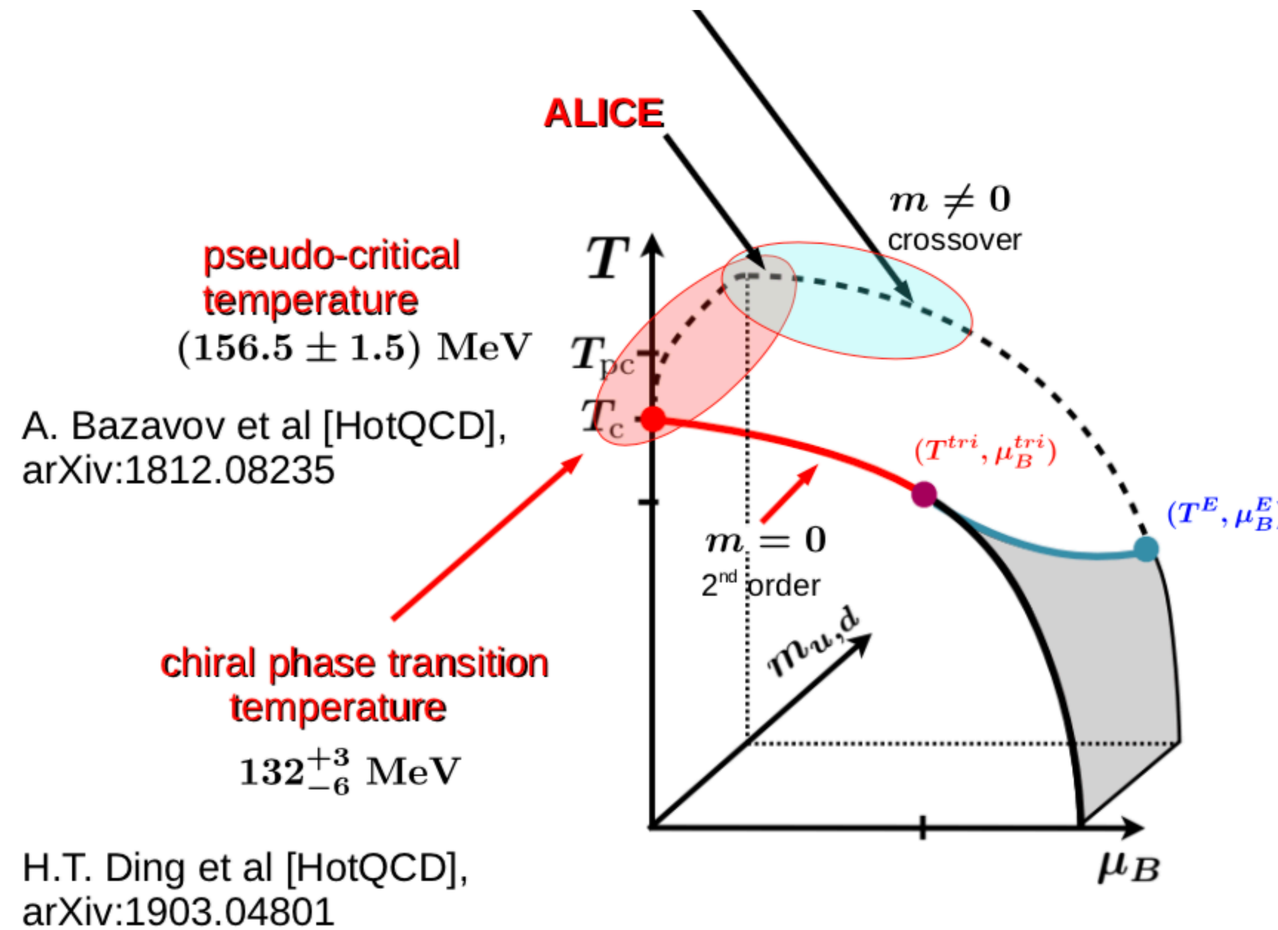
Outline

- Introduction & Motivation
- Critical dynamics & Spectral functions
  of relativistic scalar fields in (2+1)D and (3+1)D
- Critical dynamics & Spectral functions
  of relativistic diffusion in (2+1)D and (3+1)D
- Critical scaling in non-equilibrium phase-transitions
- Critical dynamics of (3+1)D O(4) model
- Summary & Conclusions

Motivation

QCD is expected to exhibit second order (Z_2) phase-transition at finite density and second order (O(4)) transition in the chiral limit

Major experimental efforts (RHIC,GSI/Fair, NICA) to unravel the QCD phase-structure



How to theoretically describe the passage of a system near the QCD critical point?

What are observable signatures of the critical point in heavy-ion experiments (RHIC BES, FAIR)?

Critical phenomena

Static critical phenomena

- Divergence of the correlation length ξ_s near the critical point of a second order phase transition

- Long distance properties near the critical point are insensitive to the microscopic physics

- Characterized in terms of critical scaling exponents

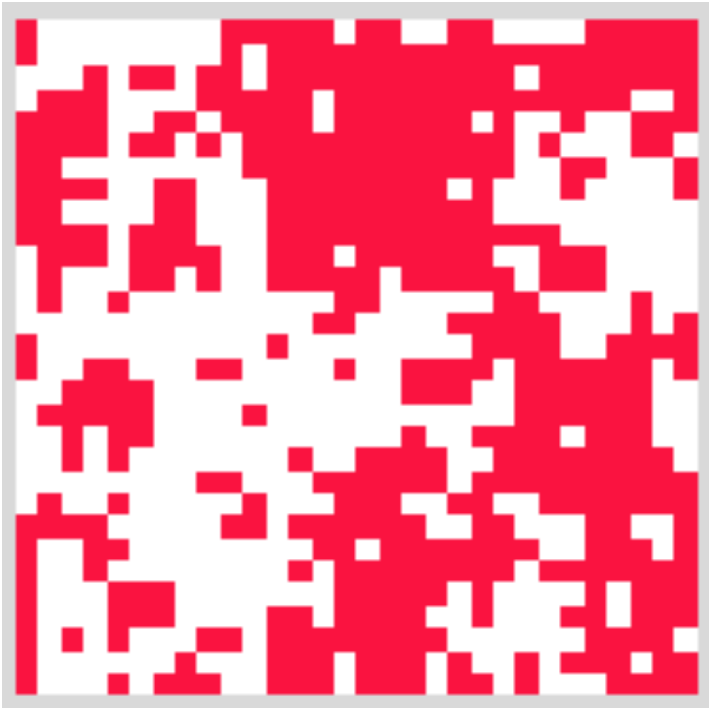
  $\alpha, \beta, \gamma, \delta, \nu, \eta$ $\qquad\qquad$ $\xi_s \sim |T-Tc|^{-\nu}$

- Universality quantities only depend on

  dimensionality, symmetry breaking pattern

Critical dynamics

Dynamic critical phenomena



- Dynamics near the critical point subject to critical slowing down *-> Divergence of the temporal correlation length ξ_t*

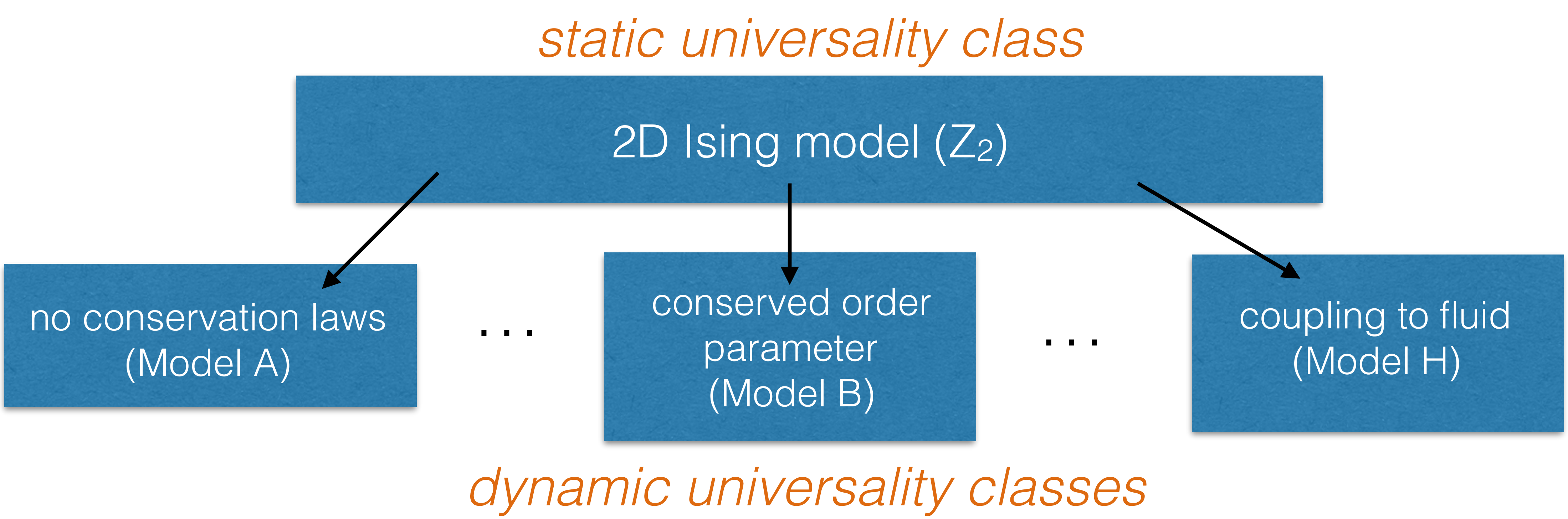
- Characterized in terms of dynamical critical exponent *z*

$$\xi_t \sim \xi^z \sim |T\text{-}Tc|^{-\nu z}$$

Critical dynamics

Dynamic critical phenomena

- Dynamical constraints (e.g conservational laws) affect the long time dynamics of the system
- Classification by Halperin & Hohenberg ’77



Critical dynamics

Dynamic critical phenomena

- Dynamical constraints (e.g conservational laws) affect the long time dynamics of the system
- C

How does a given relativistic field theory fit into this classification scheme?

What are the relevant degrees of freedom near the critical point?

no c

(Model B)

dynamic universality classes

Critical dynamics & Spectral functions of relativistic scalar fields in (2+1)D and (3+1)D

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D. Schweitzer, SS, L.von Smekal Nucl.Phys.B 960 (2020)

Basic idea of the method

- Generally not possible in the *quantum field theory*, since real-time *sign problem* (~e^{iS}) prevents use of importance sampling techniques

- However, the critical dynamics of a second order phase transition (T_c>0) is classical-statistical in nature

  -> Quantum and classical theory are in the same (static and dynamic) universality class

- No sign problem in classical-statistical field theory.

  -> Dynamic critical behavior can be studied using real-time classical lattice simulations

Scalar field theory

- Consider single component scalar field theory in 2+1 D and 3+1D

$$H = \int \mathrm{d}^2x \left(\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4!}\varphi^4 \right)$$

- Second order phase transition at $T_C>0$ for $m^2 <0$ with order parameter $\langle\phi(t,x)\rangle$

- Static universality class 2D/3D Ising (Z_2)

->Static critical properties known exactly in 2D (Onsager solution) and to high precision in 3D (conformal bootstrap)

Calculation of spectral function in real-time

Computation in *classical-statistical field theory*

1) Generate ensemble of thermal field configurations using standard importance sampling techniques

2) Solve classical-equations of motion in real-time

Model A: $$\Box\phi + \gamma\partial_t\phi + m^2\phi + \frac{\lambda}{6}\phi^3 = \sqrt{2\gamma T}\eta$$

Model C: $$\Box\phi + m^2\phi + \frac{\lambda}{6}\phi^3 = 0$$

3) Compute spectral function from unequal time correlation function $\rho_{cl}(t-t',x-x',T) = \langle\{\phi(t,x),\phi(t',x')\}_{PB}\rangle$

classical KMS $\rho_{cl}(t-t',x-x',T) = -1/T\, \partial_{t-t'} \langle\phi(t,x),\phi(t',x')\rangle$

Critical dynamics of relativistic scalar theory

- Effective degrees of freedom away from T_c are massive quasi-particles (with finite life-time)
- Spectral function $\rho(t-t',x-x',T) = i\langle[\phi(t,x),\phi(t',x')]\rangle_T$
- Mean-field approximation

$$\rho_0(\omega,p,T) = 2\pi i \,\mathrm{sgn}(\omega)\, \delta(\omega^2 - p^2 - M^2(T))$$

- Critical behavior $\rho(\omega,p,T_r) = s^{(2-\eta)}\rho(s^z\omega,sp,s^{1/\nu}T_r)$

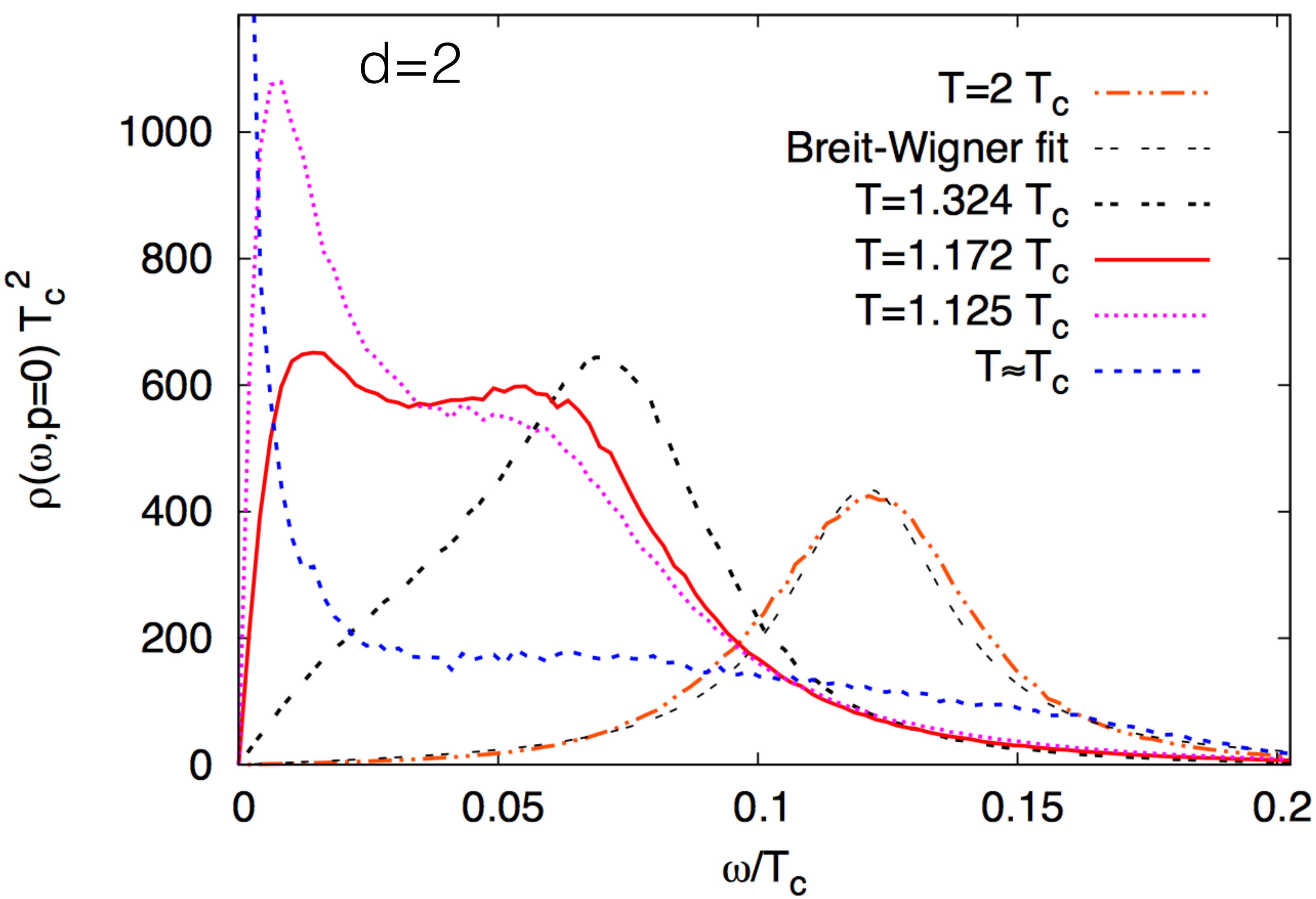
- Classification in Halperin-Hohenberg schemes

Langevin dynamics: (Model A) $z= 2.17$ (d=2) || $z=2.02$ (d=3)

Hamiltonian dynamics: (Model C) $z=2+\alpha/\nu$ — $z=2$ (d=2) || $z=2.17$ (d=3)

Critical dynamics of relativistic scalar theory

Spectral function ρ(ω,p=0,T) at zero spatial momentum at finite temperature from real-time lattice simulation



-> Change from relativistic quasi-particle to relaxation dynamics

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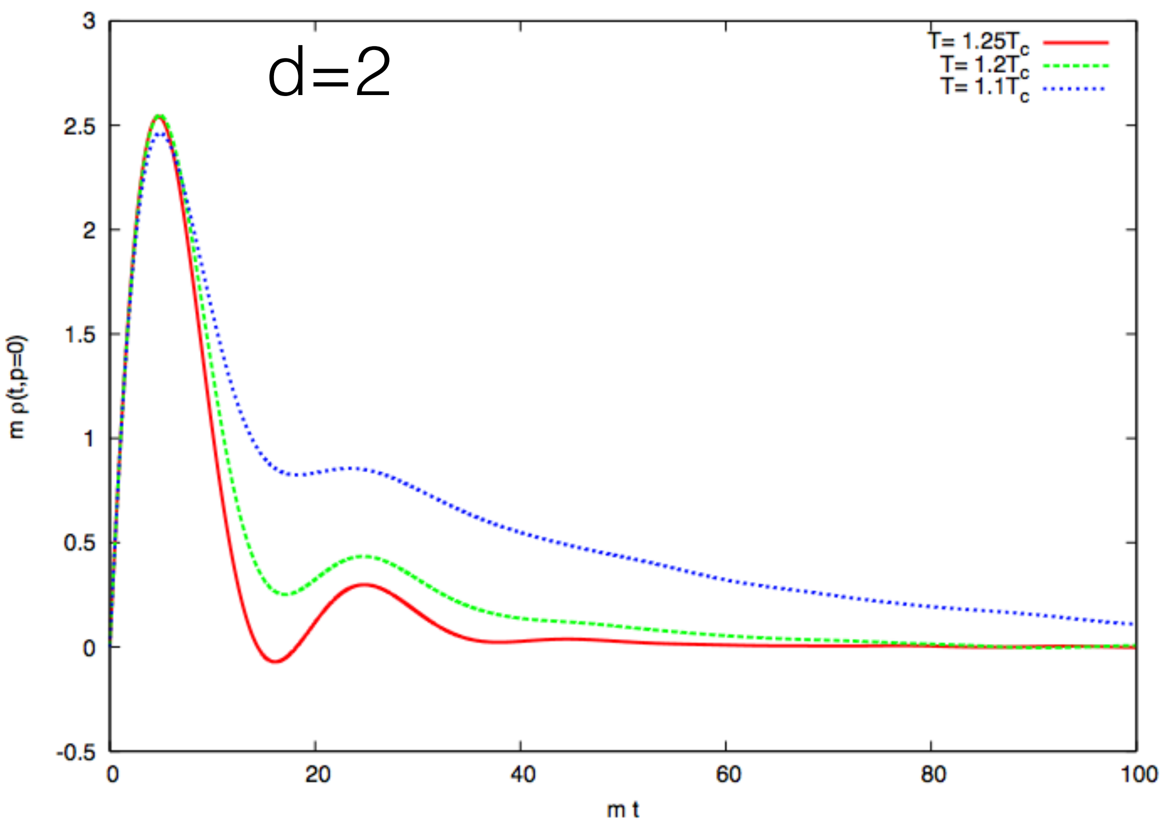
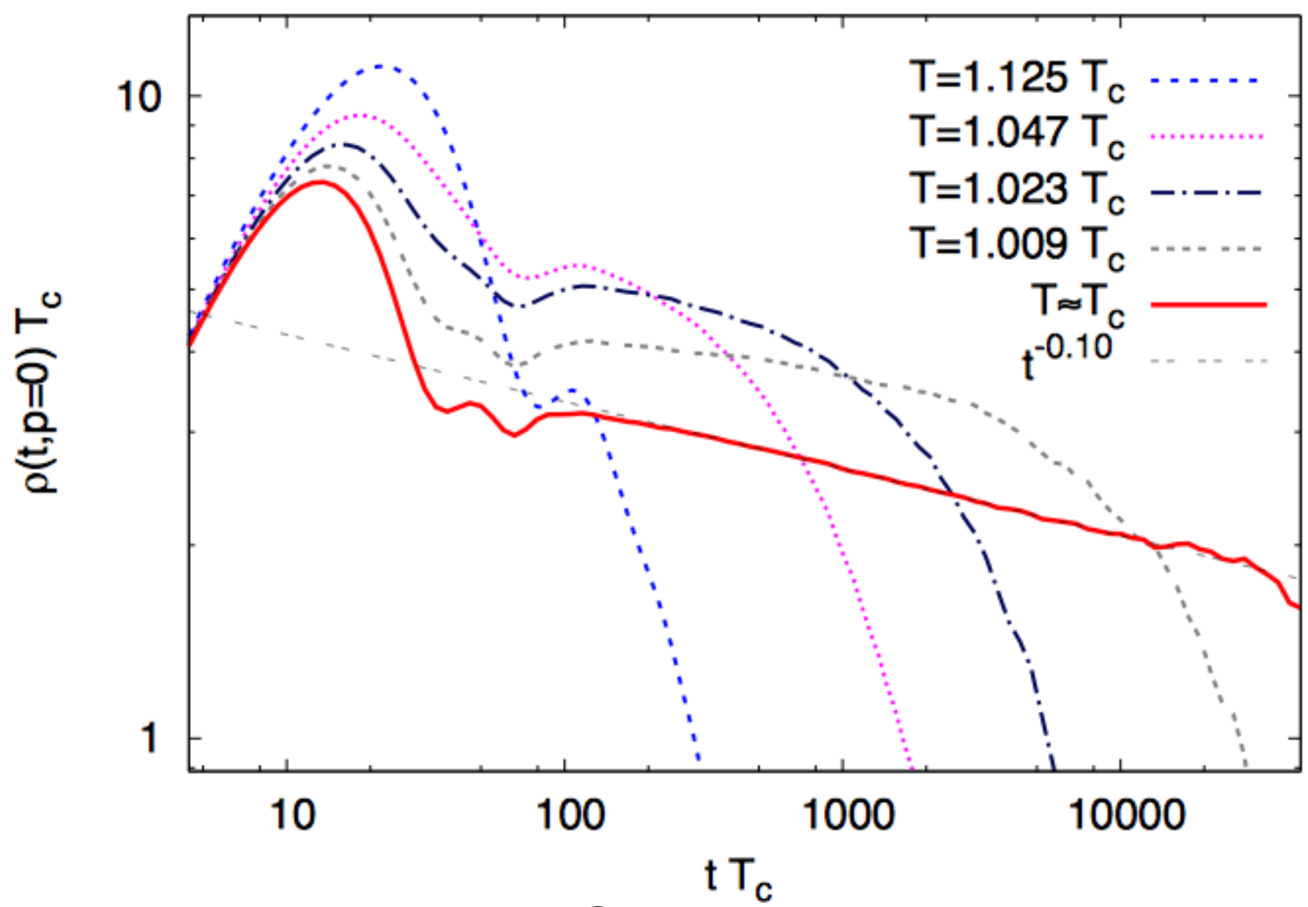
Critical dynamics of relativistic scalar theory

Spectral function ρ(ω,p=0,T) at zero spatial momentum at finite temperature from real-time lattice simulation



High temperature



Critical region

-> Change from relativistic quasi-particle to relaxation dynamics with a divergent (temporal) correlation length

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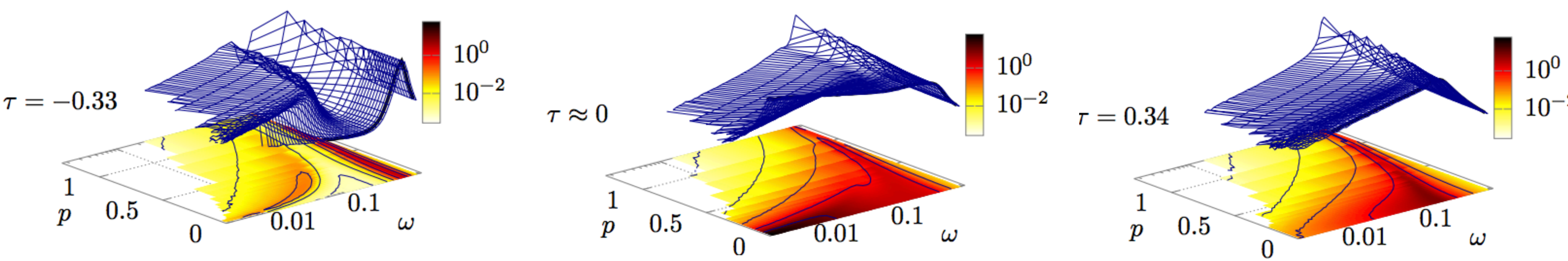
Critical dynamics of relativistic scalar theory

Spectral function ρ(t,p,T) at finite temperature T and finite momentum p

d=2



Quasi particles persist at high momentum. Critical infrared enhancement near Tc. Emergence of soft collective excitation below Tc

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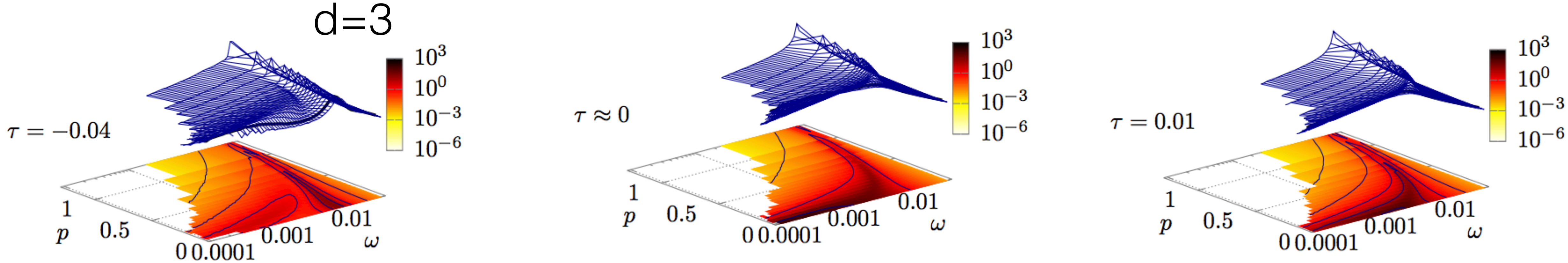
Critical dynamics of relativistic scalar theory

Spectral function ρ(t,p,T) at finite temperature T and momentum p

d=3



Quasi particles persist at high momentum. Critical infrared enhancement near Tc. Emergence of soft collective excitation below Tc

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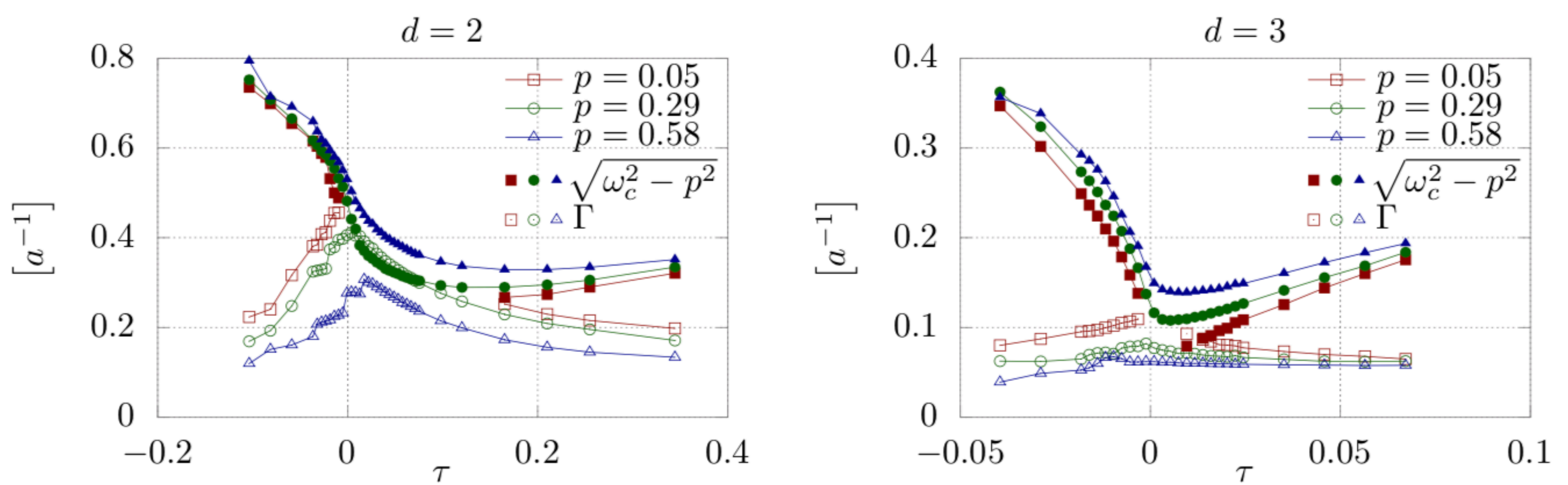
Critical dynamics of relativistic scalar theory

Breit-Wigner fits of the spectral functions



High momentum modes behave continuously across the transition.
Effective mass/damping rates show minima/maxima around Tc.

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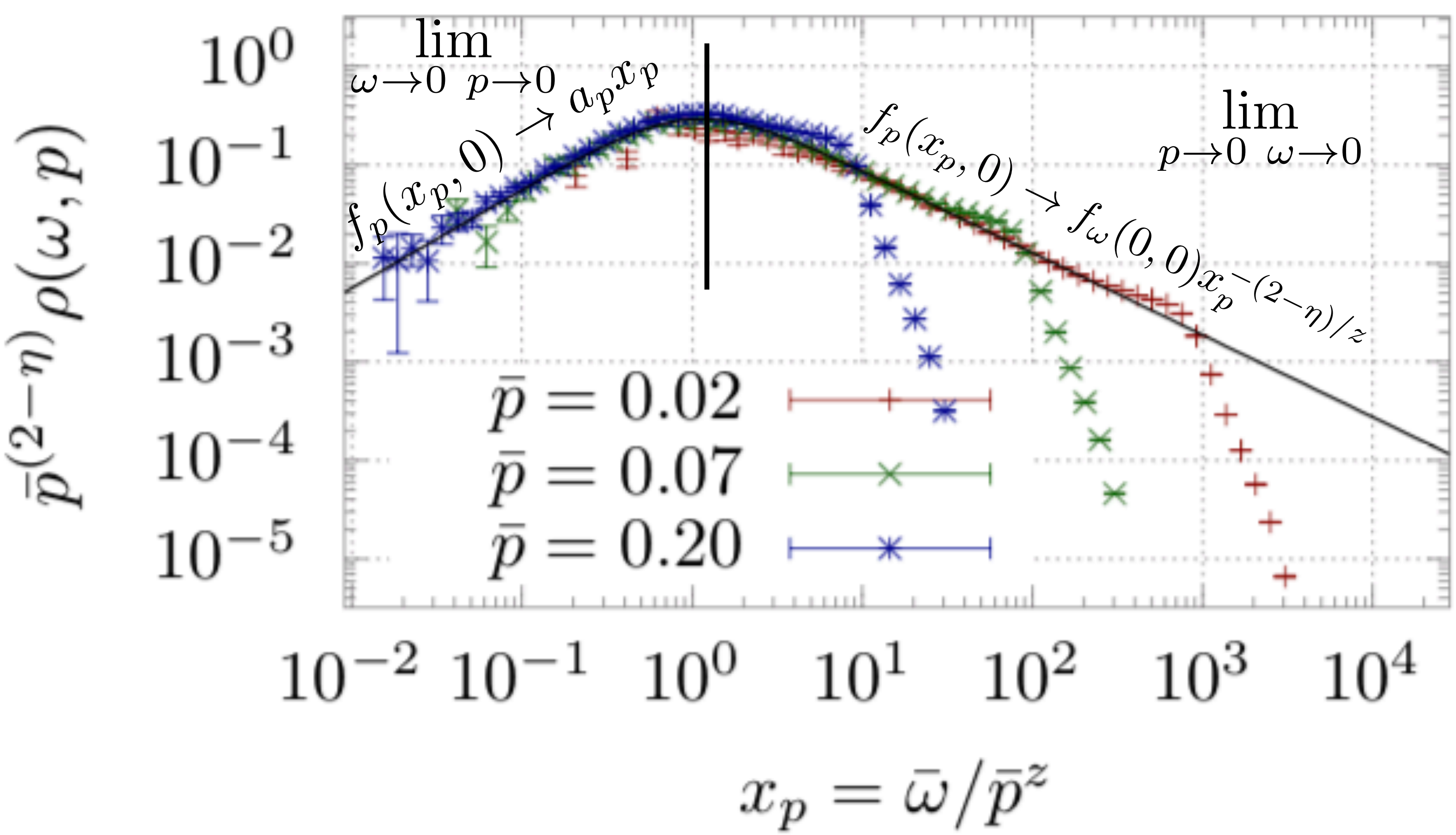
Near Tc spectral function exhibits universal scaling behavior in low frequency/low momentum regime

$$\rho(\omega,p,T_r)=s^{(2-\eta)}\,\rho(s^z\omega,sp,s^{1/\nu}T_r\,)$$

Spectral function described by critical scaling functions

$$\rho\,(\omega,p,\tau)=\bar{\omega}^{-(2-\eta)/z}\,f_\omega\Big(\bar{p}^z/\bar{\omega},\tau/\bar{\omega}^{1/\nu z}\Big),\qquad \rho\,(\omega,p,\tau)=\bar{p}^{-(2-\eta)}\,f_p\Big(\bar{\omega}/\bar{p}^z,\tau/\bar{p}^{1/\nu}\Big),$$



Scaling functions interpolate between critical frequency scaling (p->0 ω->0)

$$\rho(\bar{\omega},0,0)=f_\omega(0,0)\;\bar{\omega}^{-(2-\eta)/z}$$

and regular behavior on time scales larger than auto-correlation time (ω->0 p->0)

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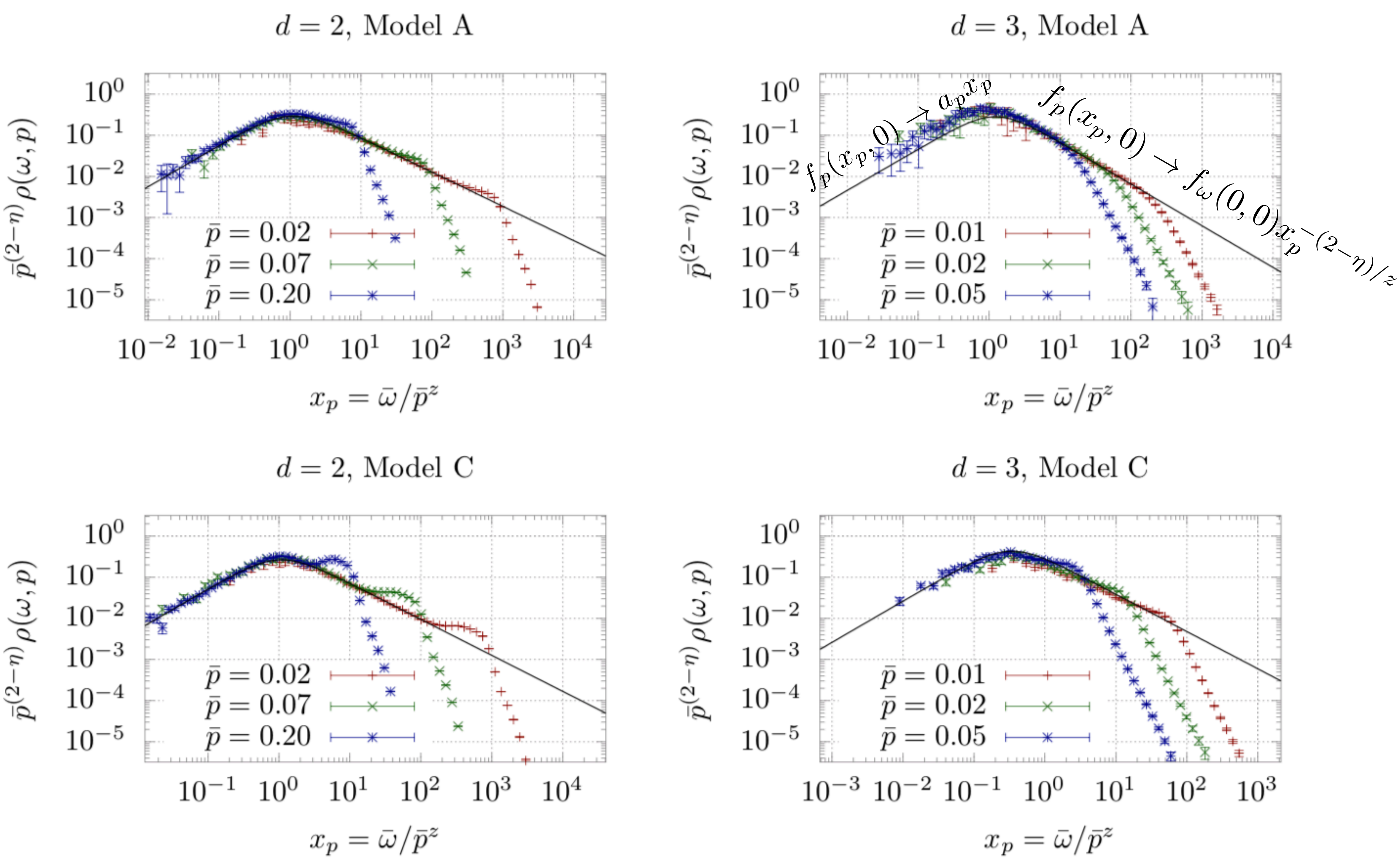


Explicit verification of dynamical scaling hypothesis and first determination of dynamical scaling functions

D. Schweitzer, SS, L.von Smekal Nucl.Phys.B 960 (2020)

Critical dynamics & Spectral functions of relativistic diffusion in (2+1)D and (3+1)D

D. Schweitzer, SS, L.von Smekal arXiv:2110.01696

So far considered non-conserved order parameter, governed by Hamiltonian/Langevin dynamics; when order parameter corresponds to conserved quantity (e.g. n_B) need to consider modified EOMs

Non-relativistic literature considers standard diffusion equation, classifies as Models B (or D when coupled to conserved density e)

$$\dot{\phi}(\boldsymbol{x},t) = \mu\nabla^2 \frac{\delta\mathfrak{H}'[\phi,\rho]}{\delta\phi(\boldsymbol{x},t)} + \xi(\boldsymbol{x},t),$$

$$\dot{\rho}(\boldsymbol{x},t) = \lambda\nabla^2 \frac{\delta\mathfrak{H}'[\phi,\rho]}{\delta\rho(\boldsymbol{x},t)} + \zeta(\boldsymbol{x},t),$$

with stochastic white noise

$$\langle\xi(\boldsymbol{x},t)\xi(\boldsymbol{x}',t')\rangle = -2T\mu\nabla^2\delta(\boldsymbol{x}-\boldsymbol{x}')\delta(t-t'),$$

$$\langle\zeta(\boldsymbol{x},t)\zeta(\boldsymbol{x}',t')\rangle = -2T\lambda\nabla^2\delta(\boldsymbol{x}-\boldsymbol{x}')\delta(t-t'),$$

$$\langle\xi(\boldsymbol{x},t)\zeta(\boldsymbol{x}',t')\rangle = 0.$$

Diffusion dynamics

We consider Israel-Stuart type diffusion equation, where spatial currents relax to diffusion currents on time scale $1/\gamma$

$$\partial_\mu J^\mu = 0 \,, \qquad J^\mu = \phi u^\mu + \nu^\mu \qquad u^\mu = (1,0,0,0)$$

$$\partial_t \nu^\mu = -\gamma\Big(\nu^\mu - \frac{\mu}{\gamma}\nu^\mu_{\mathrm{NS}}\Big) - \sqrt{2\gamma\mu T}\,\zeta^\mu_\perp$$

$$\nu^\mu_{\mathrm{NS}} = -\nabla^\mu \frac{\partial H}{\partial \phi} = -\nabla^\mu\Big(m^2\phi - \triangle\phi + \frac{\lambda}{6}\phi^3\Big)$$

with spatial white noise $\langle \zeta^\mu_\perp(x)\zeta^\nu_\perp(y)\rangle = \Delta^{\mu\nu}\delta(x-y)$

Guarantees stationarity of equilibrium probability distribution; static critical behavior remains unchanged

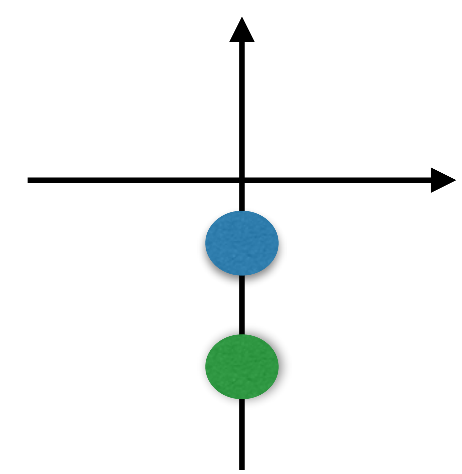
Will consider generic case $\gamma > 0$ (rel. Model B) as well as $\gamma = 0$ where noise is absent, and system features a conserved energy density

Hydro & Non-Hydro Excitations

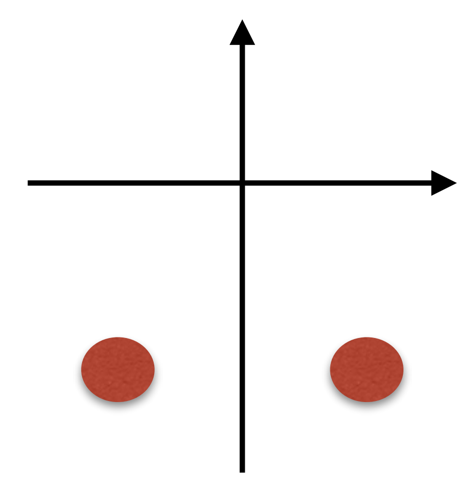
Second order/Israel-Stuart type diffusion equation features hydrodynamic (diffusive) excitations and (decaying) non-hydrodynamic

$$G(z,\boldsymbol{p}) = \frac{(1 - \mathrm{i}z\tau_R)\chi(\boldsymbol{p})}{D_{\mathrm{diff}}(\boldsymbol{p})\,\boldsymbol{p}^2 - \tau_R z^2 - \mathrm{i}z}, \qquad D_{\mathrm{diff}}(\boldsymbol{p}) \equiv \frac{\mu}{\gamma}(\bar{m}^2 + \boldsymbol{p}^2)$$

$$\mu\mathbf{p}^2 \ll \frac{\gamma^2}{m^2}: \quad z_{\mathrm{hydro}} = -iD_{\mathrm{diff}}(\mathbf{p})\mathbf{p}^2\ , \qquad z_{\mathrm{non-hydro}} = -i\gamma$$

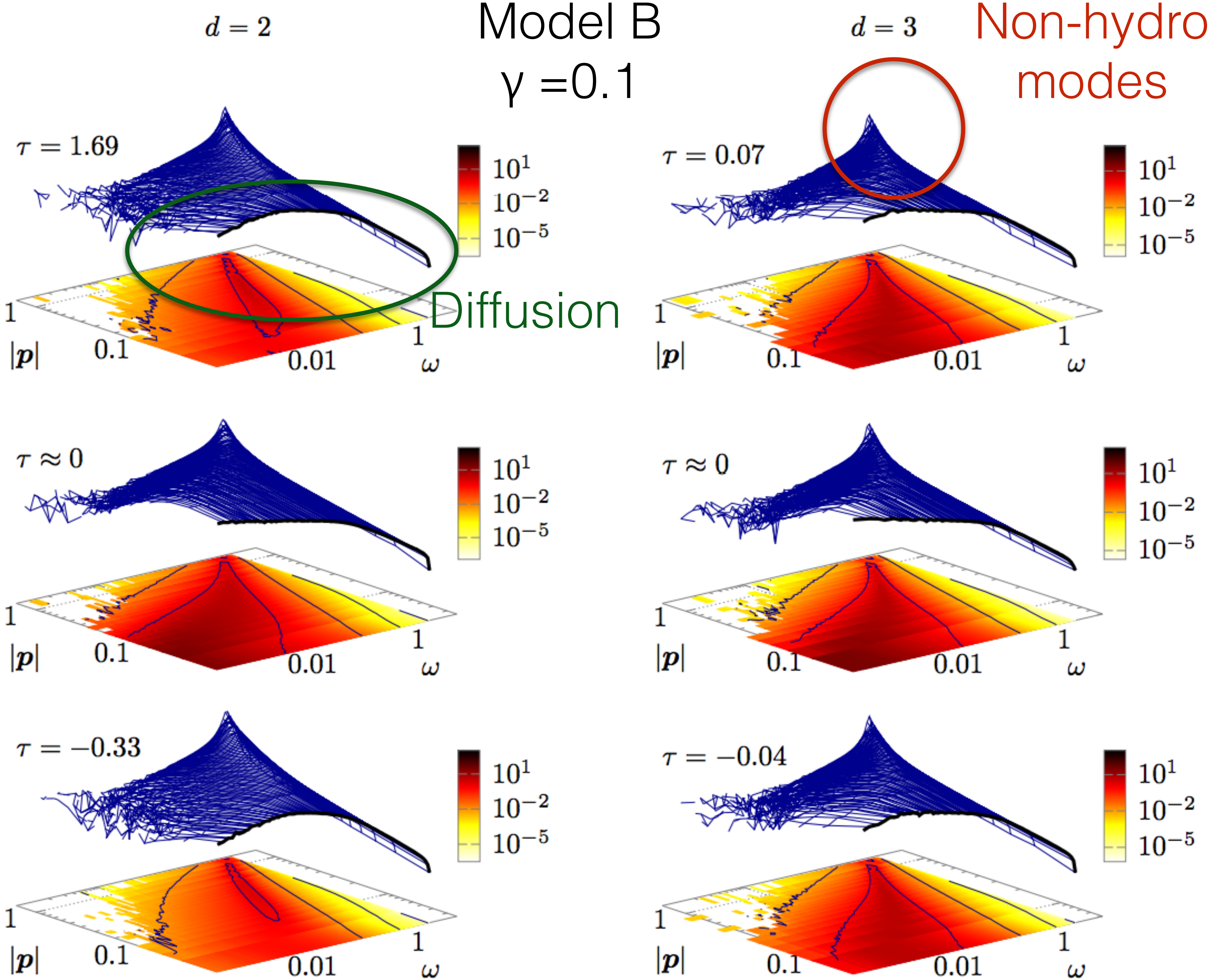


$$\mu\mathbf{p}^2 \gg \frac{\gamma^2}{m^2}: \quad z_{\mathrm{prop}} = -\frac{i\gamma}{2} \pm \sqrt{\mu(\bar{m}^2 + \mathbf{p}^2)}|\mathbf{p}|$$



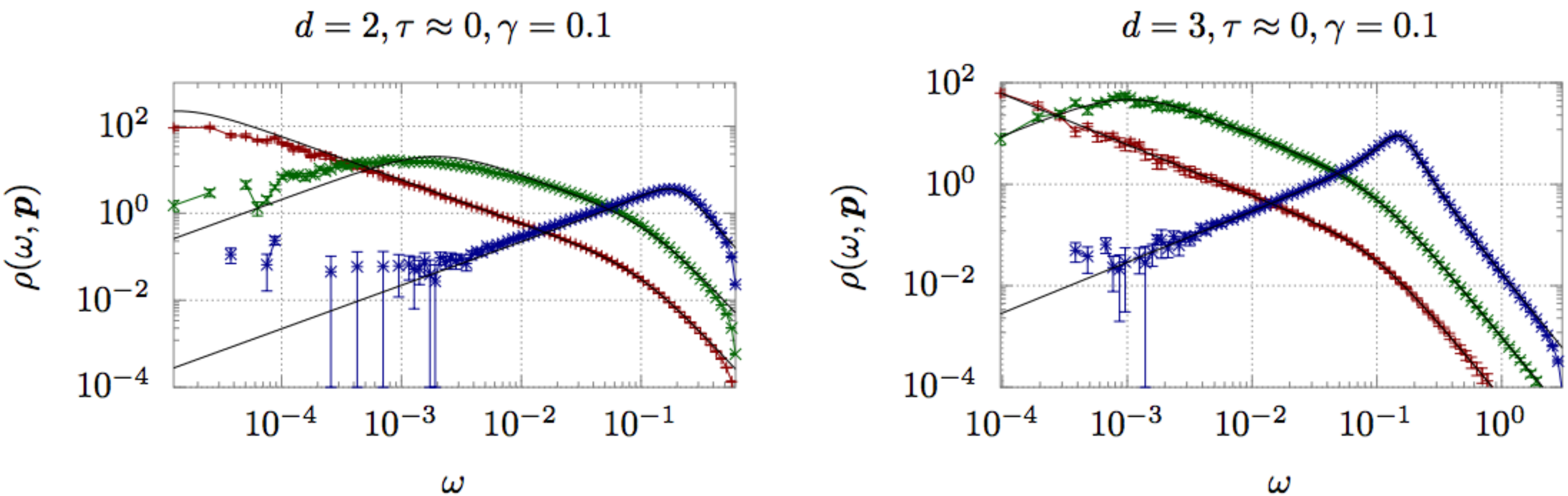
in the limit γ ->0 only non-hydro modes survive at mean-field level yielding conserved order parameter dynamics without actual diffusion

Model B
γ =0.1



$d = 2$
$d = 3$
Non-hydro modes
Diffusion
$\tau = 1.69$
$\tau = 0.07$
$\tau \approx 0$
$\tau \approx 0$
$\tau = -0.33$
$\tau = -0.04$
10^{1}
10^{-2}
10^{-5}
$|\boldsymbol{p}|$
1
0.1
0.01
1
ω

Spectral functions



Spectral functions surprisingly well described by mean-field form even in the vicinity of the critical point.

$$\rho_{\mathrm{BW}}(\omega, \boldsymbol{p}) = \frac{\mu \boldsymbol{p}^2 \Gamma_p \, \omega}{\left(\omega^2 - \omega_p^2\right)^2 + \Gamma_p^2 \omega^2}$$

However, near Tc the dispersion relations are strongly modified

$$\omega_p^2 = \mu \mathbf{p}^2 (m^2 + \mathbf{p}^2)$$
$$\Gamma_p = \gamma$$

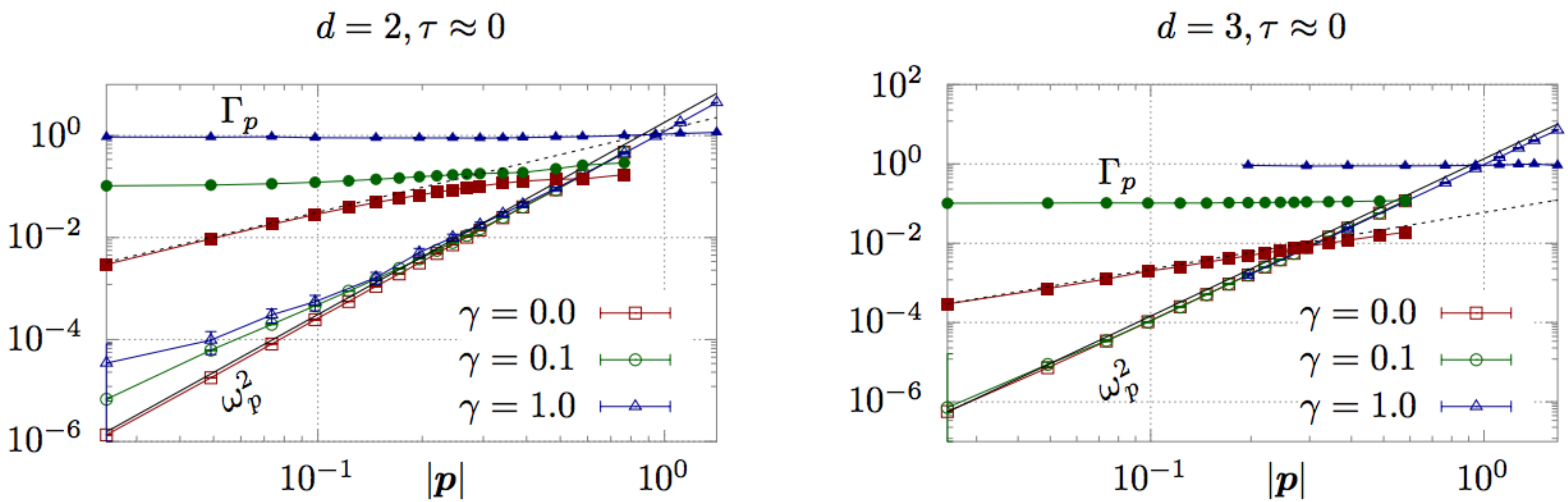
mean-field

$$\longrightarrow$$

$$\omega_p^2 = \omega_0^2 \, \bar{p}^{z_\omega}$$
$$\Gamma_p = \Gamma_0 \, \bar{p}^{z_\Gamma}$$

critical scaling

Brett-Wigner fits at T$_c$



Central frequencies exhibit power-law dependence

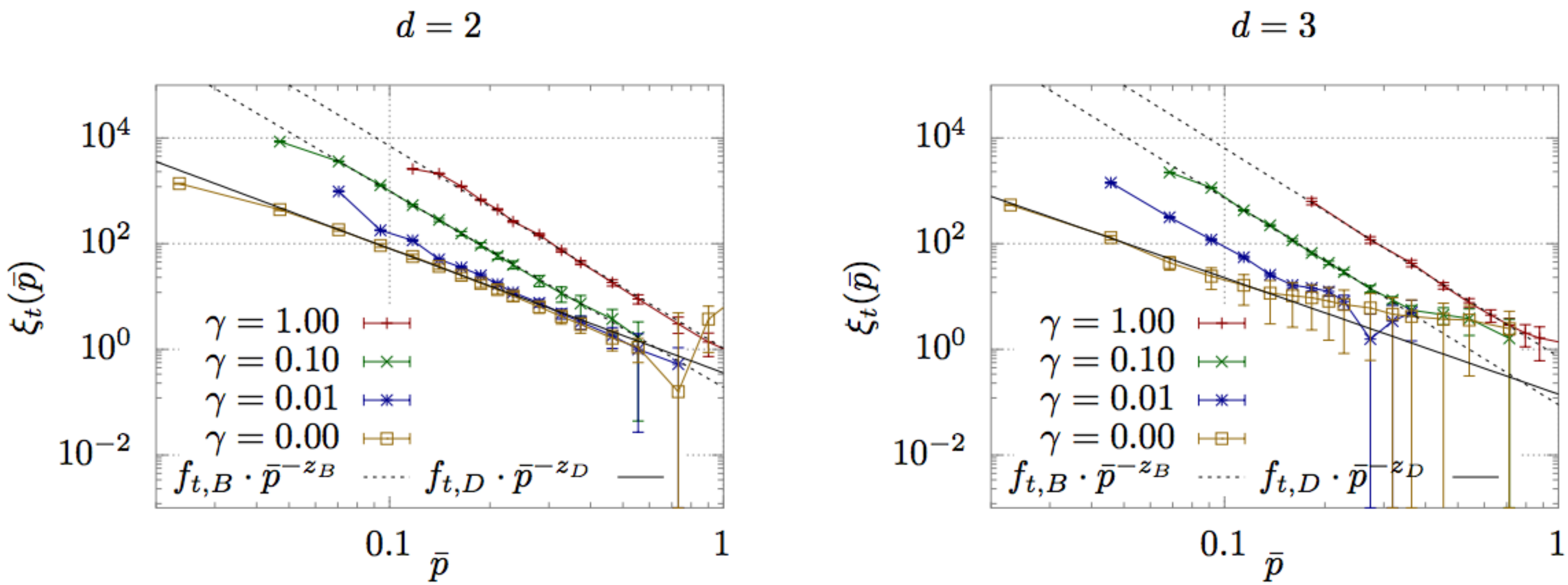
$$\omega_p^2 = \omega_0^2 \, \bar{p}^{z_\omega} \qquad z_\omega = 4 - \eta$$

Decay width Γ bounded by dissipative coupling γ

$$\Gamma_p = \Gamma_0 \, \bar{p}^{z_\Gamma}$$

γ>0: $z_\Gamma = 0$ γ=0: $z_\Gamma \approx \begin{cases} 1.6 & d = 2 \\ 1.4 & d = 3 \end{cases}$

Dynamic critical exponent z



Dynamic critical exponent z determined from the auto-correlation time as

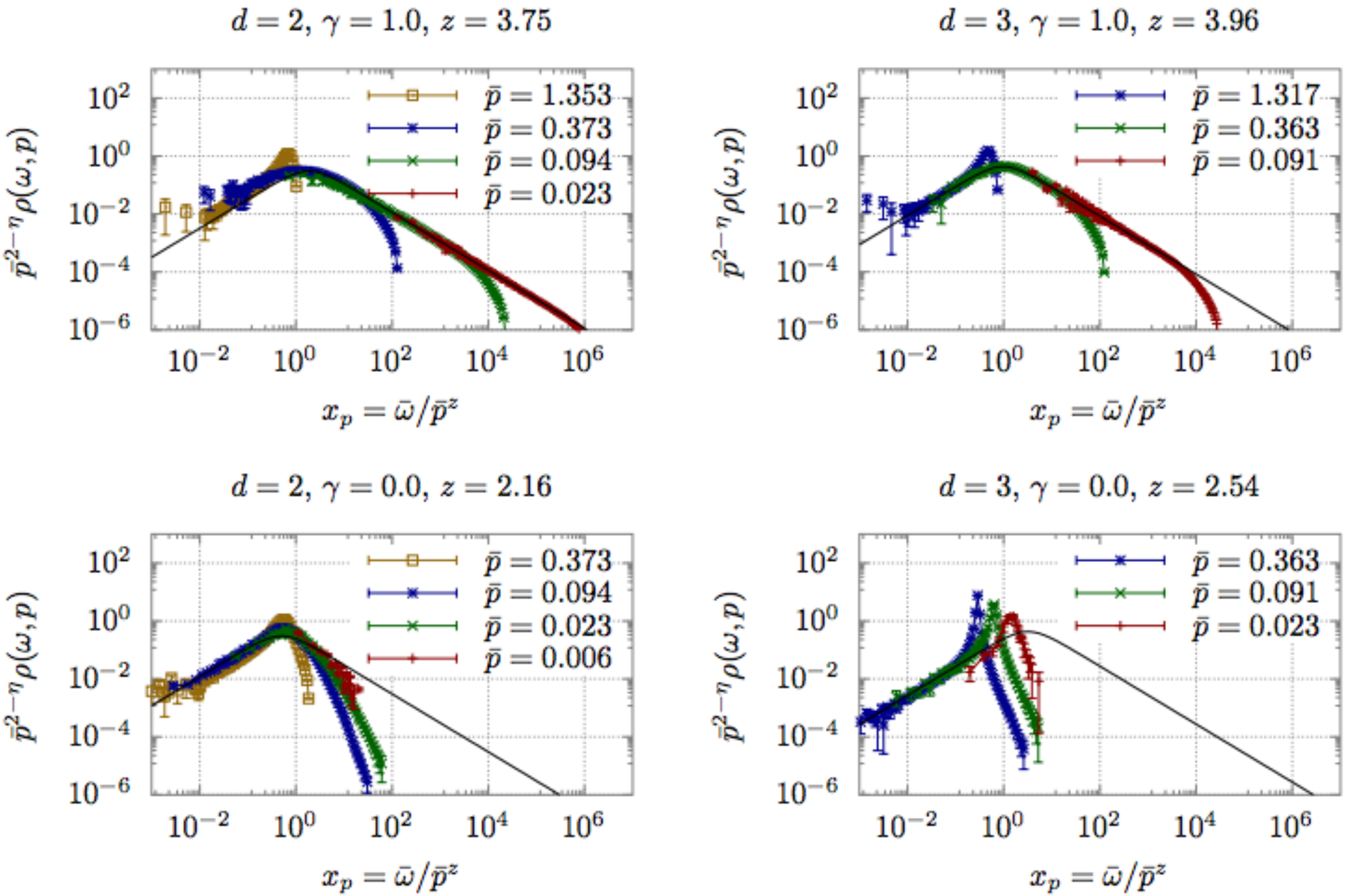
$$\xi_t(\bar{p}) = f_t \bar{p}^{-z}, \qquad \xi_t(p) \simeq \Gamma_p/\omega_p^2 \qquad z = z_\omega - z_\Gamma$$

Scaling arguments for the spectral function give the same result and also fix $z_\omega = 4 - \eta$ as for standard diffusion dynamics of Model B

d	$z(\gamma = 1.0)$	$f_t(\gamma = 1.0)$	$z(\gamma = 0.1)$	$f_t(\gamma = 0.1)$	$z(\gamma = 0.0)$	$f_t(\gamma = 0.0)$
2	3.83(10)	1.04(14)	3.716(17)	0.190(8)	2.354(23)	0.358(19)
3	3.95(8)	0.73(6)	3.91(6)	0.090(10)	2.20(13)	0.14(5)

Deduce critical scaling functions
from Breit-Wigner form
with scaling forms of ω,Γ

$$f_p(x_p,0) = \frac{(f_\xi^+)^{-2}}{\omega_0^4\left(\frac{\Gamma_0}{f_t}x_p\right)^{-1} + \frac{\Gamma_0}{f_t}x_p}.$$

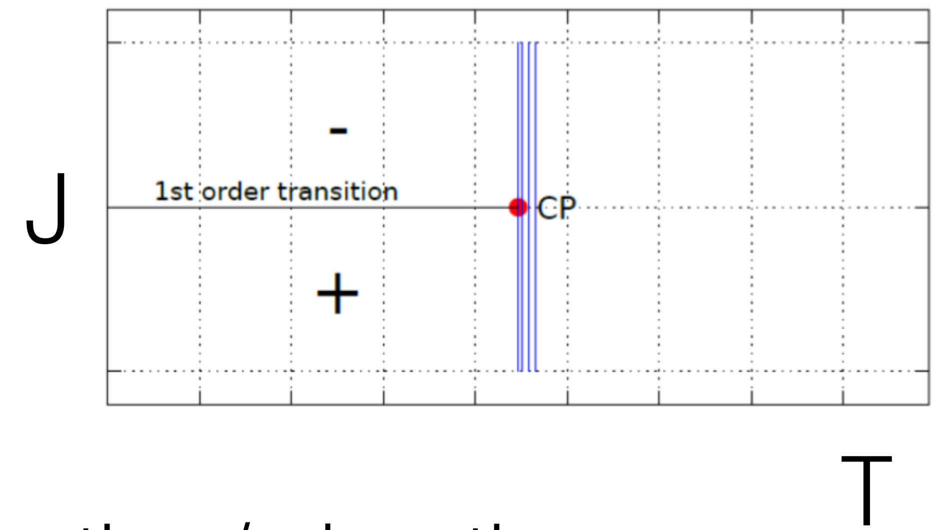


Critical scaling in non-equilibrium phase-transitions

M.Harhoff, SS work in progress

Non-equilibrium Phase-transitions

So far considered dynamic critical behavior in equilibrium; however in the real-world e.g. heavy-ion collisions system dynamically transits critical point



Trans-Critical Protocol: Since auto-correlation time/relaxation time diverges as system approaches critical point system falls out of equilibrium at finite time t_{KZ}, with $T_{KZ}(t)$,$J_{KZ}(t)$

$$\frac{\partial_t \phi(T(t),J(t))}{\phi(T(t),J(t))} \gtrsim \xi_t^{-1}(T(t),J(t))$$

Kibble-Zurek (KZ) scaling: Non-equilibrium near critical point again governed by universal scaling exponents (related to scaling with $T_{KZ}(t)$,$J_{KZ}(t)$) and scaling functions

We consider variation of the explicit symmetry breaking J(t)= Γt and monitor the evolution of cumulants of the order parameter

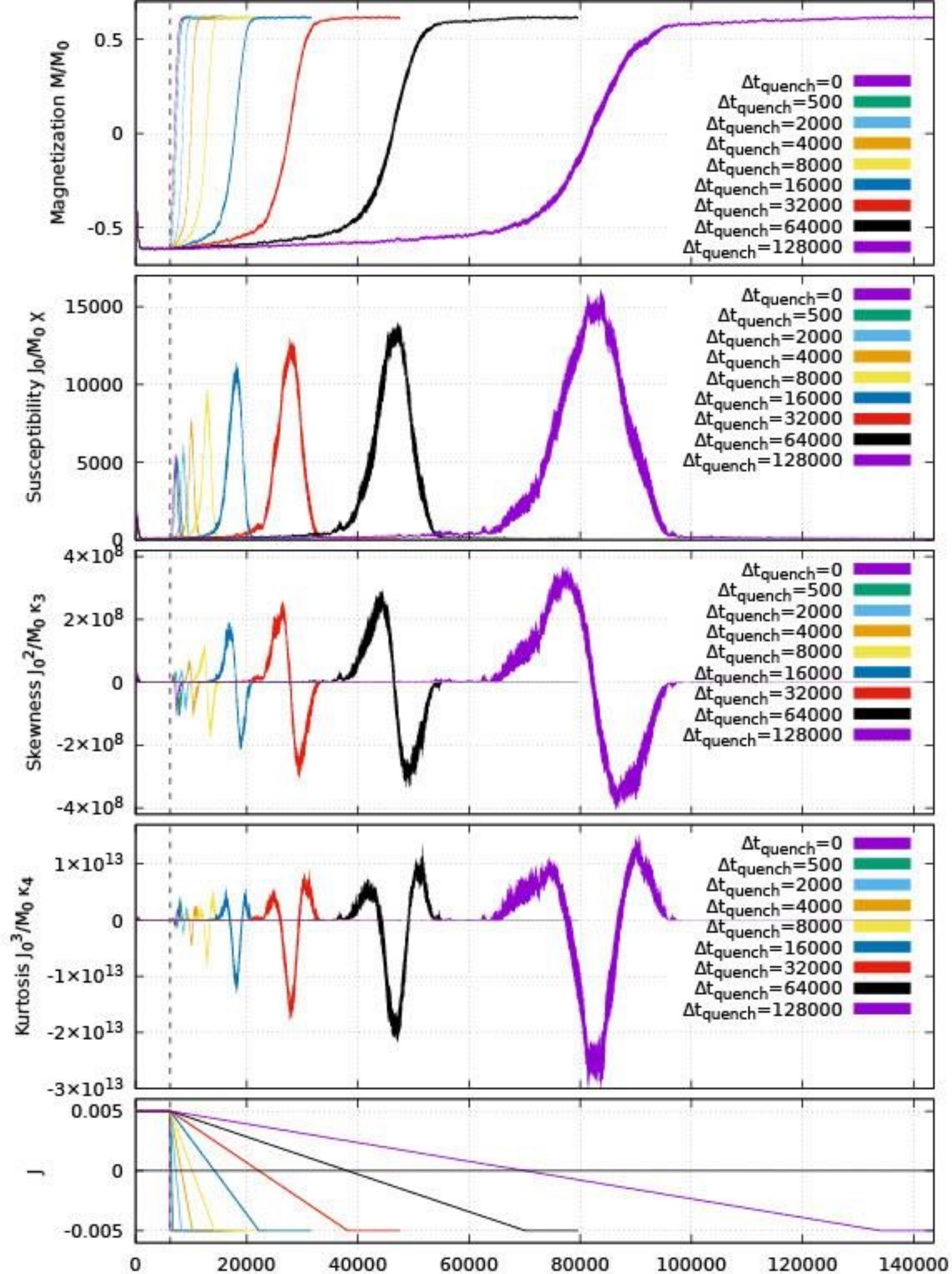
$$\langle \phi(t) \rangle$$

$$\langle \phi^2(t) \rangle - \langle \phi(t) \rangle^2$$

for Model A dynamics in 2+1D

Clearly observe delay in evolution of Φ for slow quenches

Higher-cumulants follow order parameter



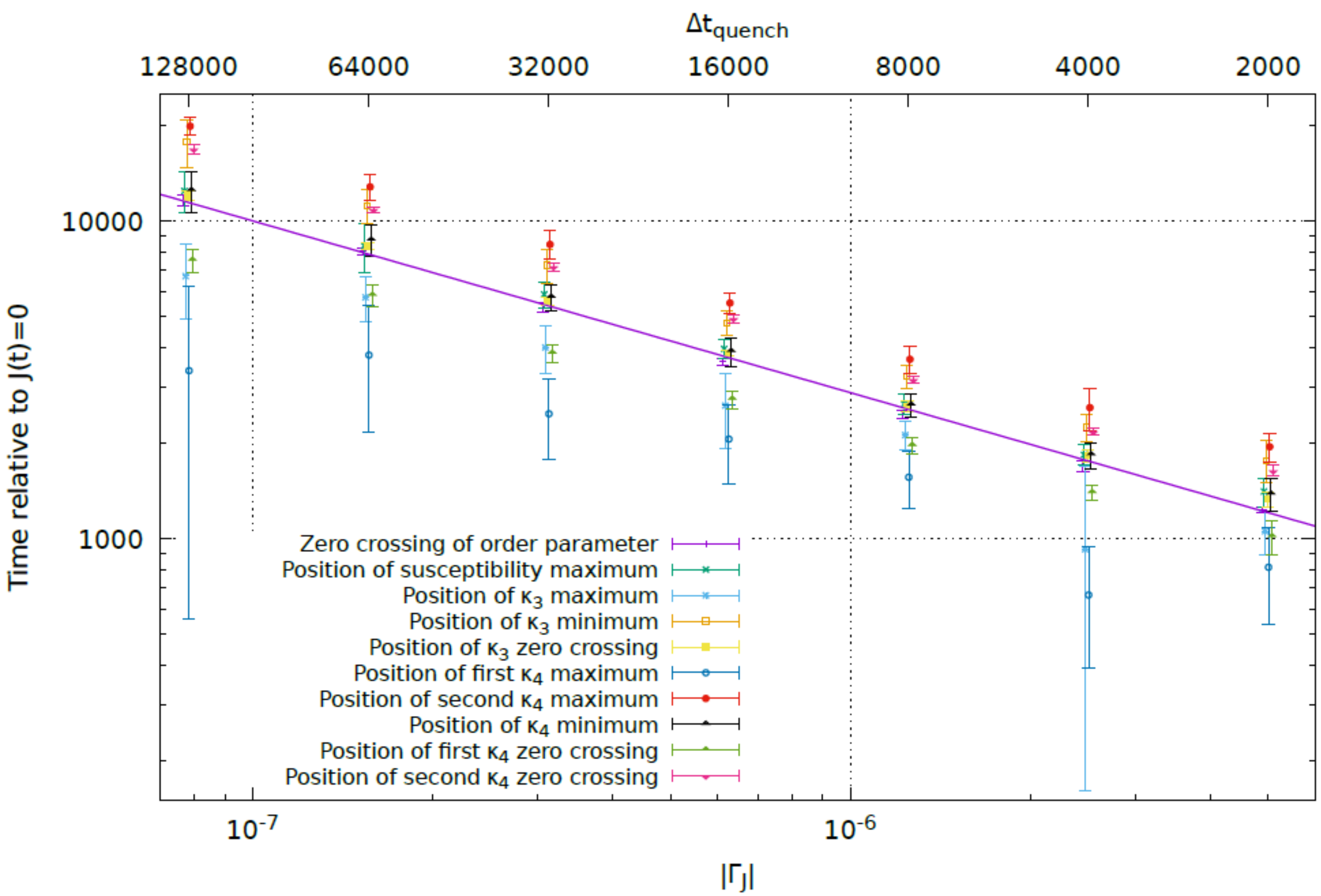
Kibble-Zurek scaling

Based on KZ formalism expect time scale to behave as

$$\frac{\partial_t \phi(T(t), J(t))}{\phi(T(t), J(t))} \gtrsim \xi_t^{-1}(T(t), J(t))$$

$$t_{KZ} \sim \Gamma^{\frac{1}{1+\frac{\nu z}{\beta\delta}} - 1} \approx -0.533$$

$$j_{KZ} \sim \Gamma t_{KZ} \sim \Gamma^{\frac{1}{1+\frac{\nu z}{\beta\delta}}}$$



Feature	Exponent from fit
Zero crossing of order parameter	-0.5418 ± 0.0082
Position of susceptibility maximum	-0.534 ± 0.013
Position of κ_3 maximum	-0.467 ± 0.029
Position of κ_3 minimum	-0.566 ± 0.019
Position of κ_3 zero crossing	-0.5426 ± 0.0066
Position of κ_4 minimum	-0.541 ± 0.012
Position of first κ_4 zero crossing	-0.4936 ± 0.0085
Position of second κ_4 zero crossing	-0.576 ± 0.012
Position of first κ_4 maximum	-0.432 ± 0.084
Position of second κ_4 maximum	-0.577 ± 0.017

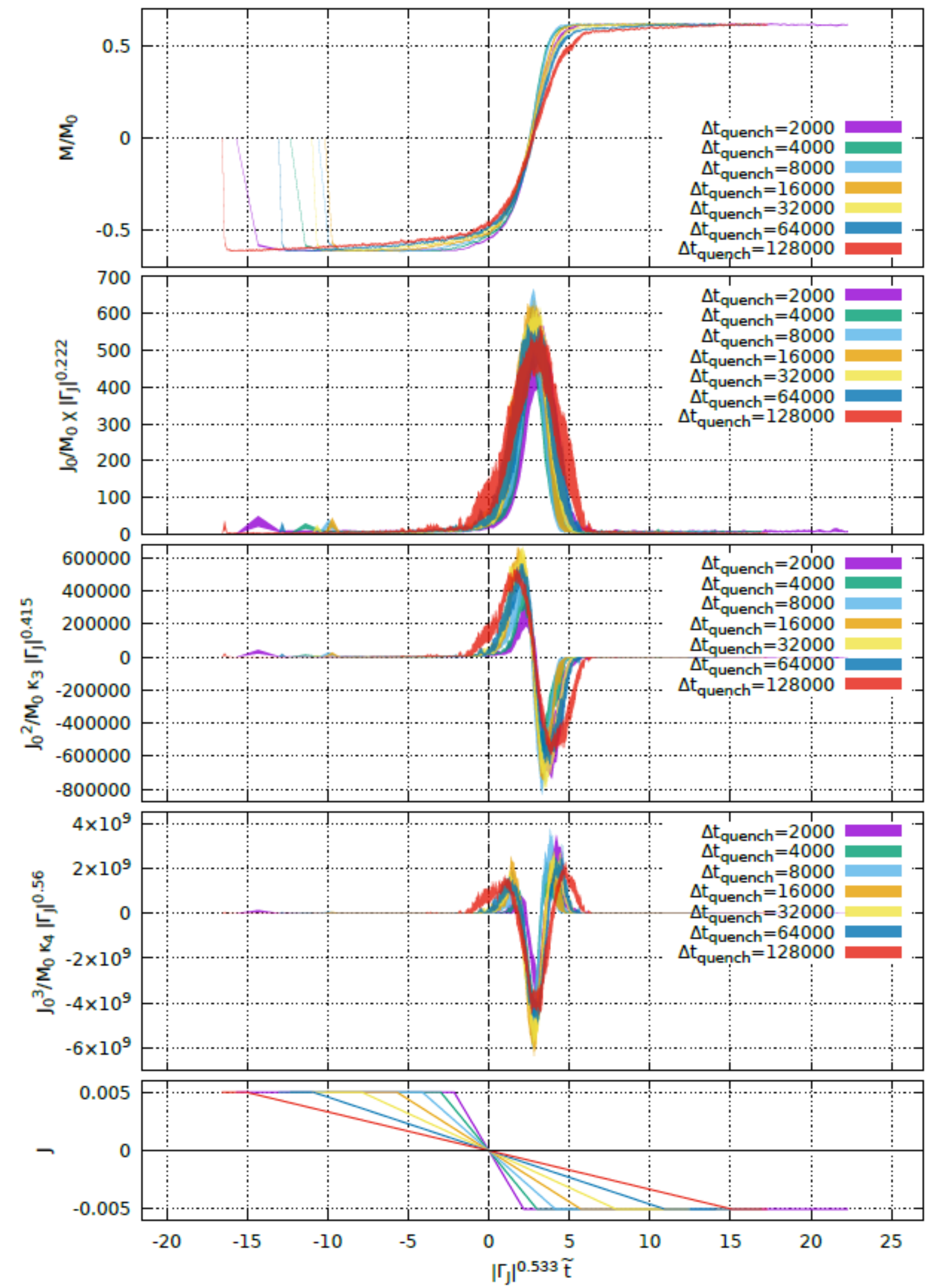
Non-equilibrium scaling functions

Based on KZ formalism expect normalization of the scaling functions to scale with j_{KZ}

$$\kappa_n(T=T_c) \sim j_{KZ}^{1/\delta-(n-1)} \sim \Gamma^{\frac{1-(n-1)\delta}{\delta+\nu z/\beta}}$$

in infinite volume limit

Evidence of KZ scaling in simulation; residual finite size effects hinder extraction of scaling exponents and scaling functions



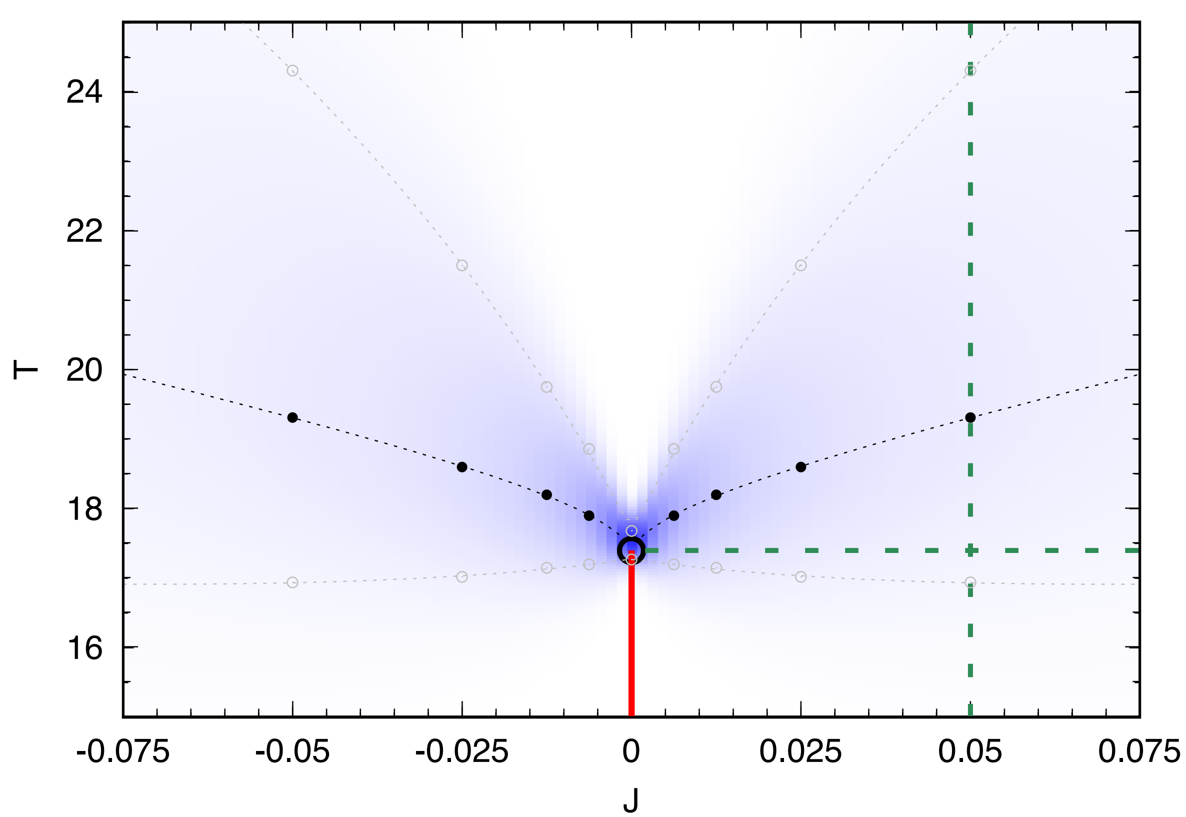
Critical dynamics of 3+1D O(4) model

SS, D. Smith, L. von Smekal Nucl.Phys.B 950 (2020)

Spectral functions and critical dynamics in 3D O(4) model

Study dynamic critical behavior in O(4) chiral transition ($m_{u/d}$->0) based on O(4) scalar field theory as low-energy EFT

Distinction between Pion and Sigma modes difficult in finite volume
-> only possible with explicit symmetry breaking



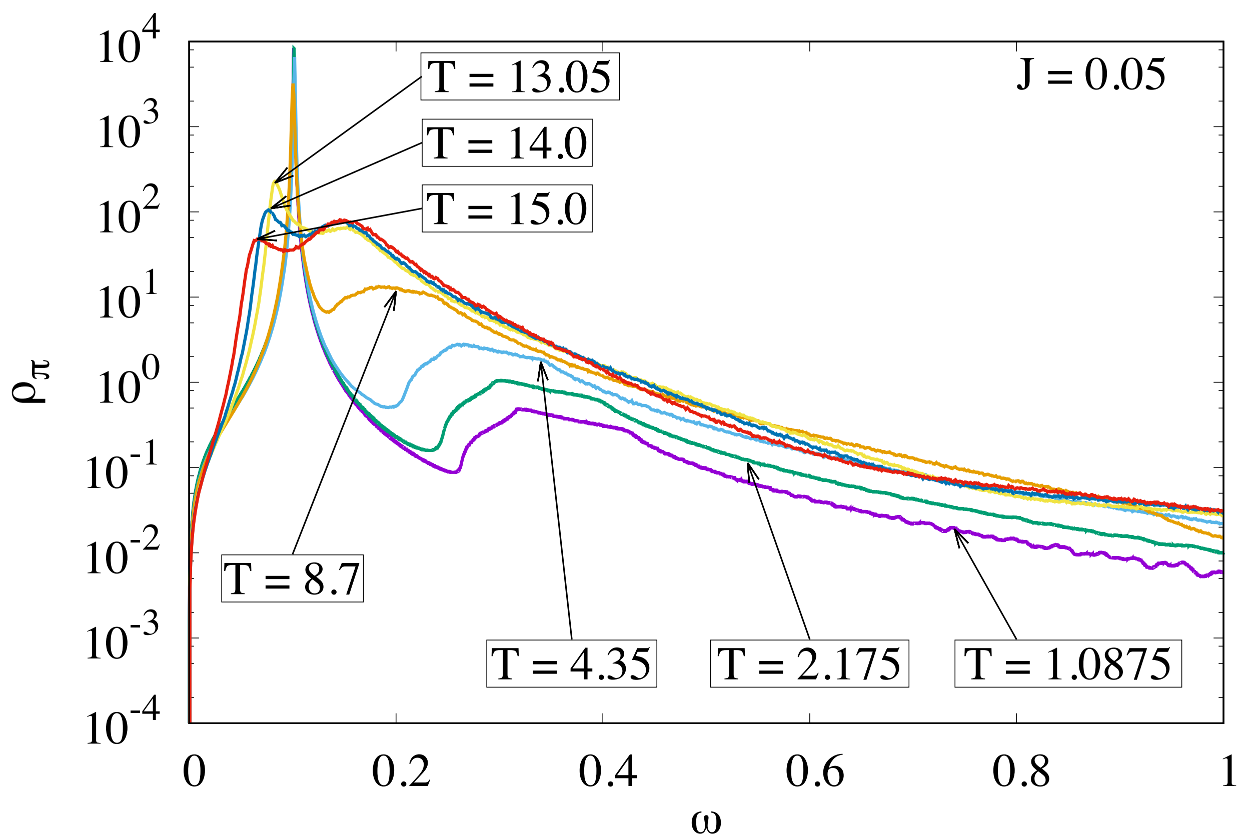
1) Scan temperature axis for pseudo-critical behavior at finite symmetry breaking

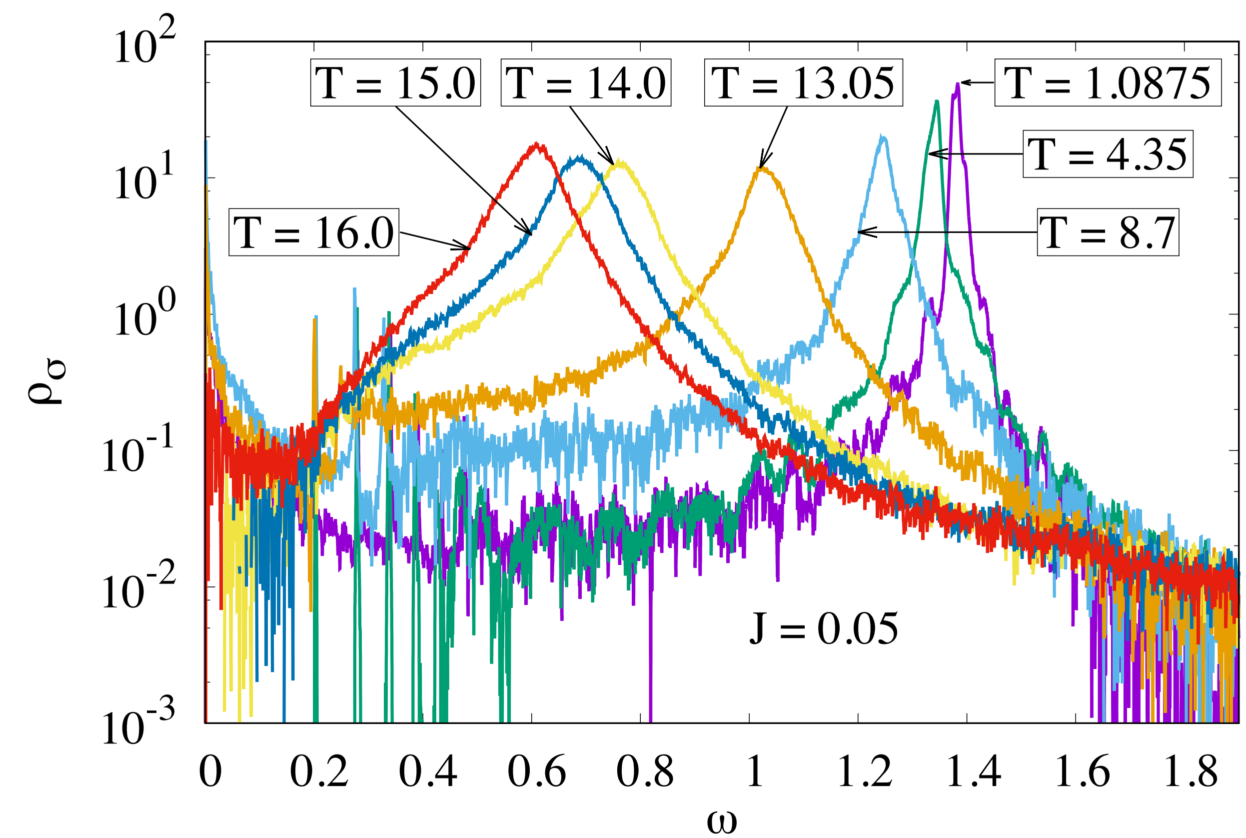
2) Scan critical behavior by tuning explicit symmetry breaking to zero at T=Tc

SS, D. Smith,L. von Smekal *Nucl.Phys.B* 950 (2020)

Spectral functions in 3D O(4) model

Low temperature $T<T_{pc}$





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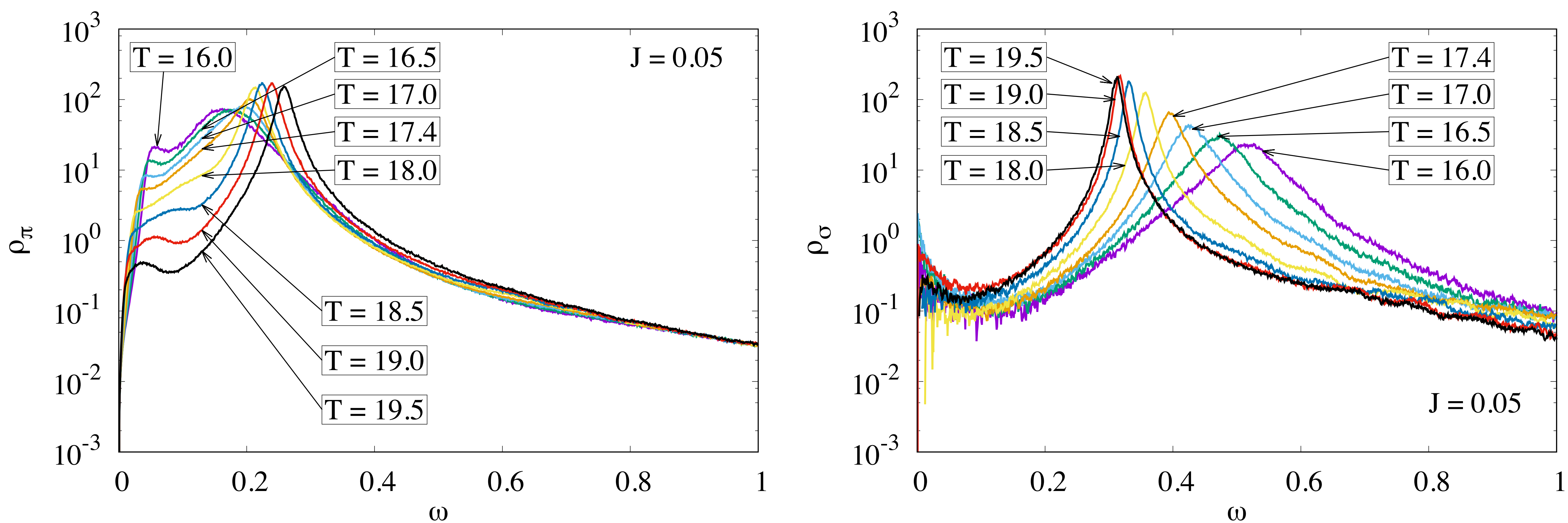
Spectral functions in 3D O(4) model

Pseudo-critical region $T \sim T_{pc}$



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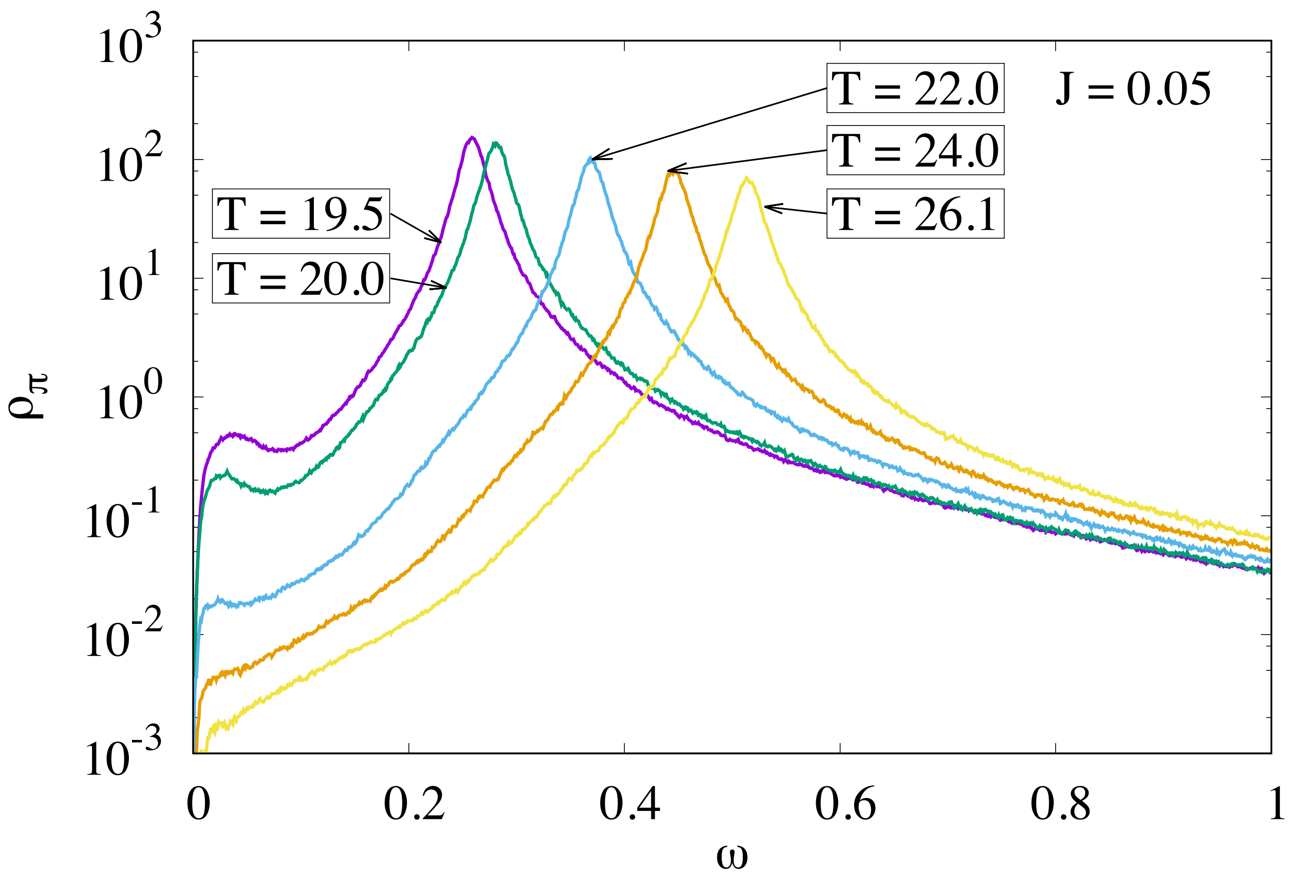
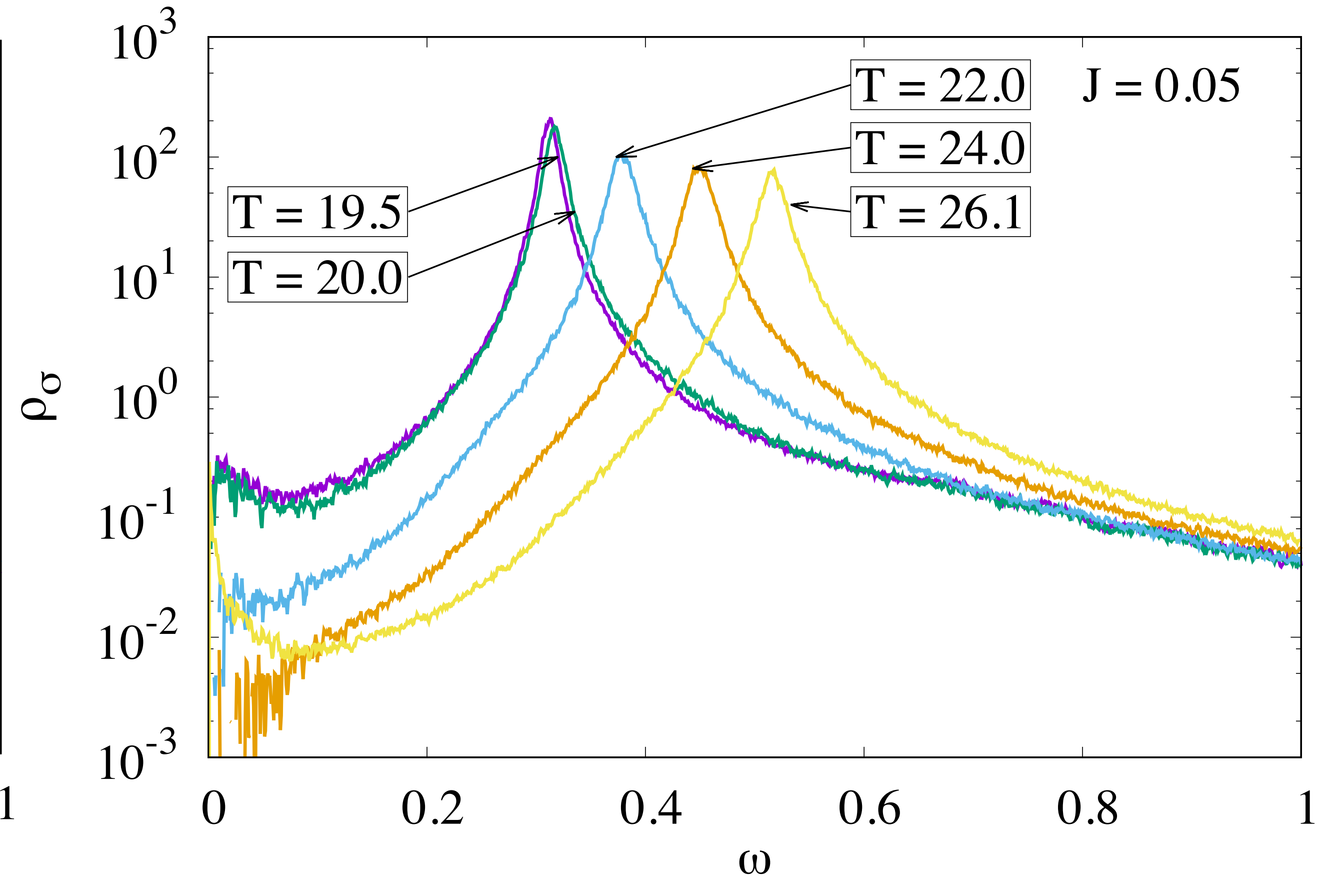
Spectral functions in 3D O(4) model

High temperature $T>T_{pc}$





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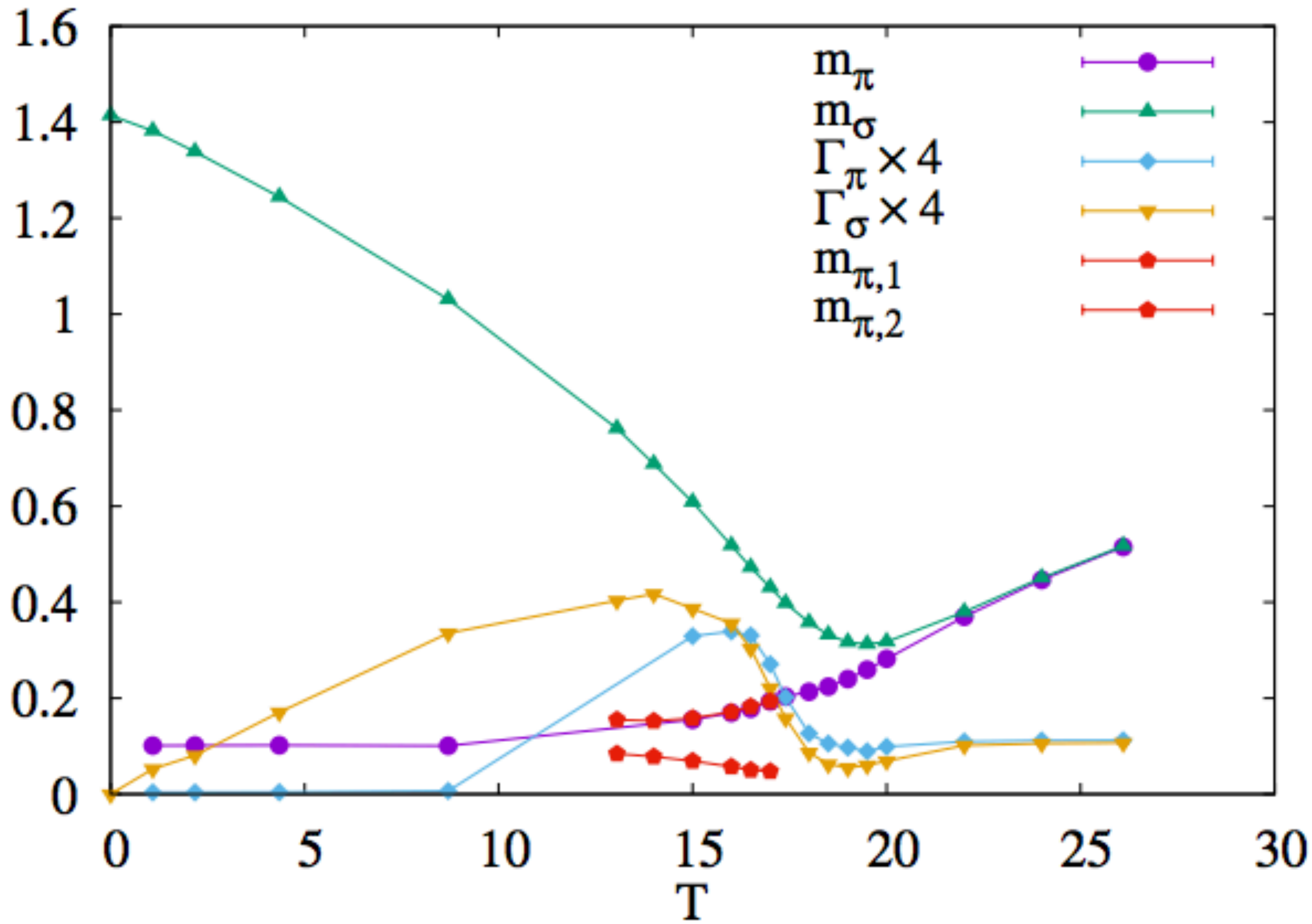
Spectral functions in 3D O(4) model



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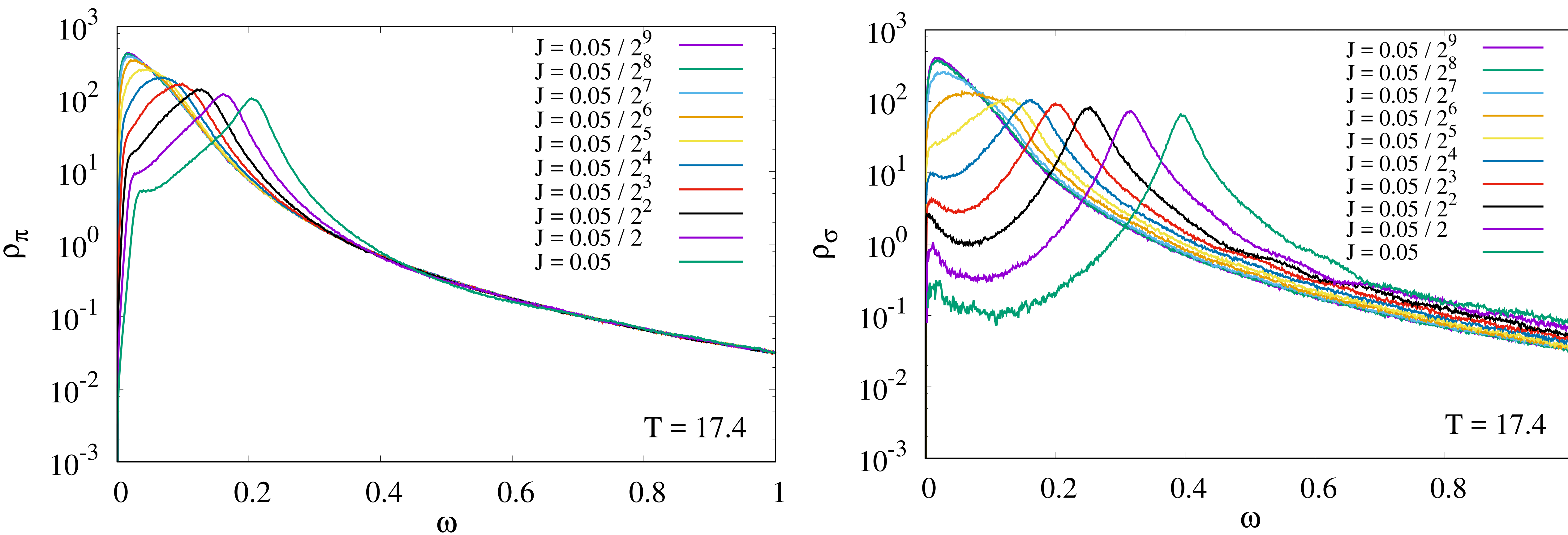
Spectral functions in 3D O(4) model

Critical behavior for Hamiltonian dynamics believed to be Model G (z=3/2)



Distinction between Pion and Sigma becomes increasingly difficult as explicit symmetry breaking is removed

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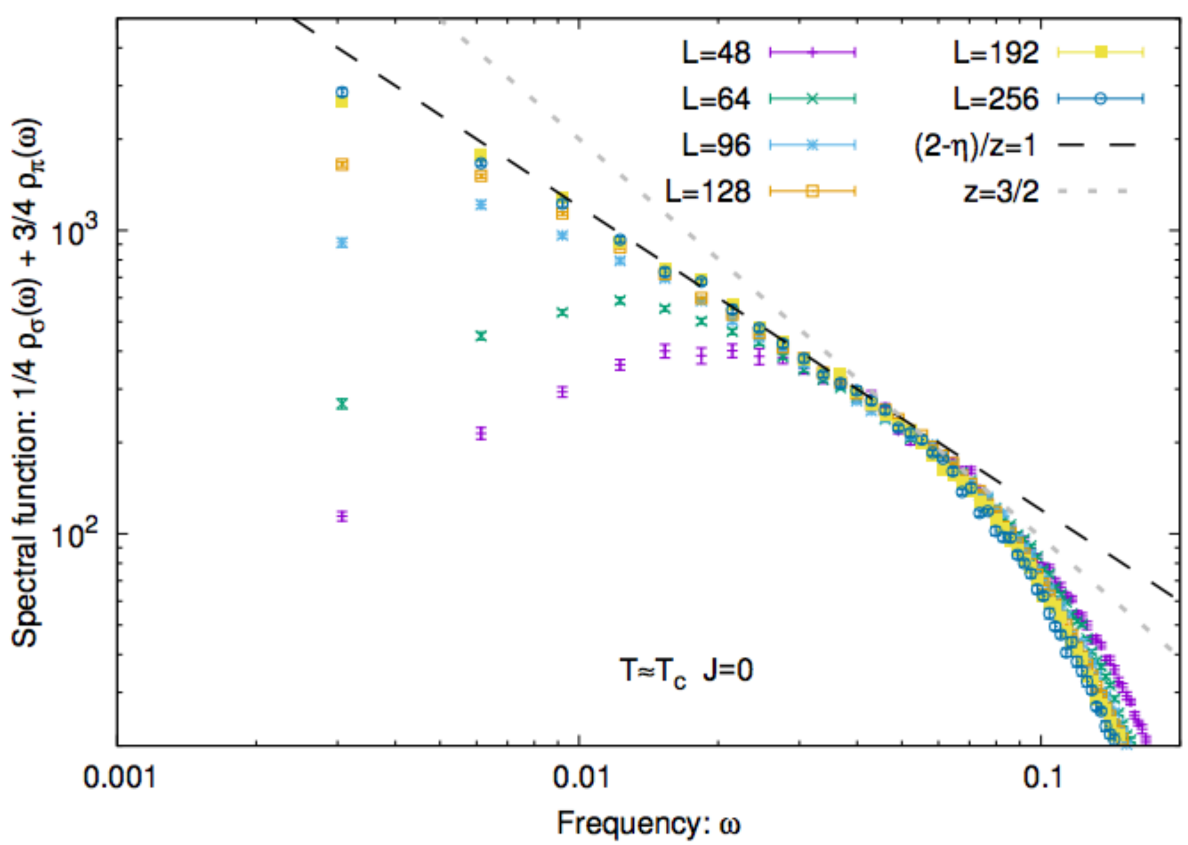
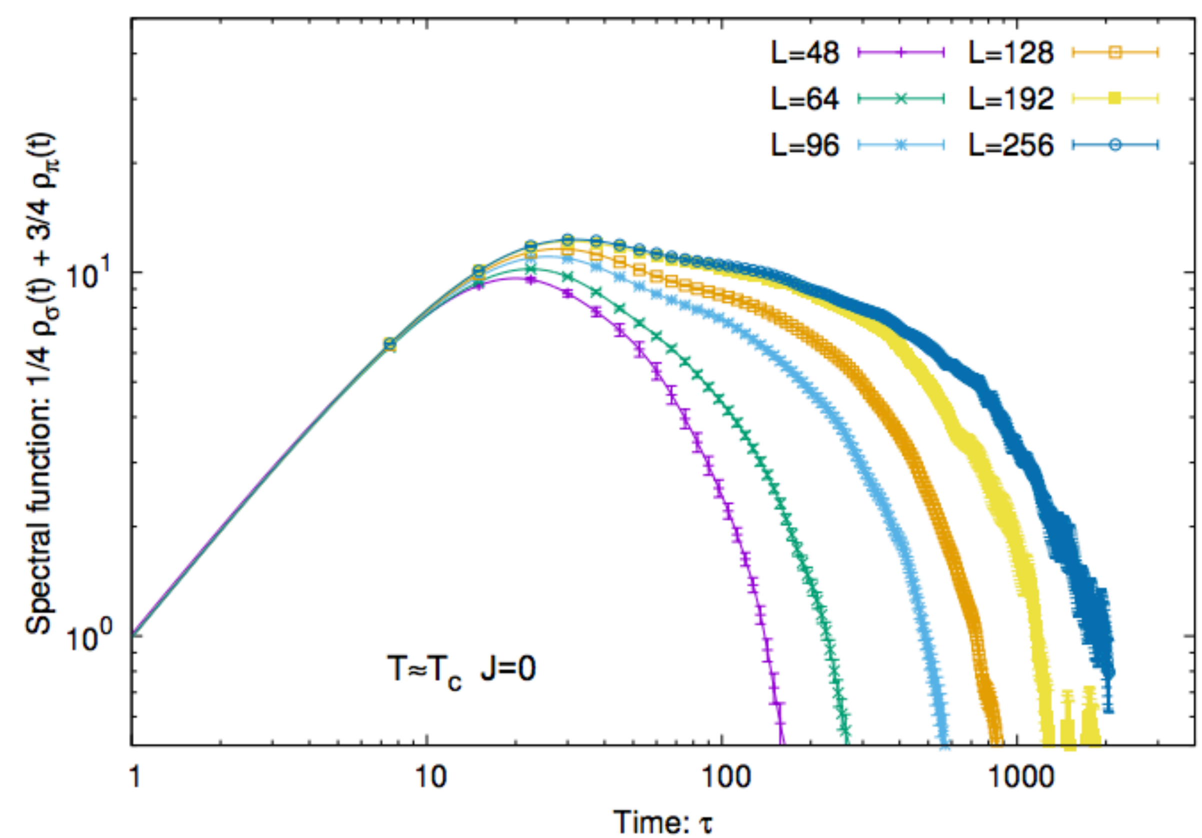
Spectral functions in 3D O(4) model





Behavior of spectral function in frequency space consistent with naive scaling (z=2-η); However finite size scaling of divergence of auto-correlation time shows hints at Model G behavior (z=3/2)

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Summary & Conclusion

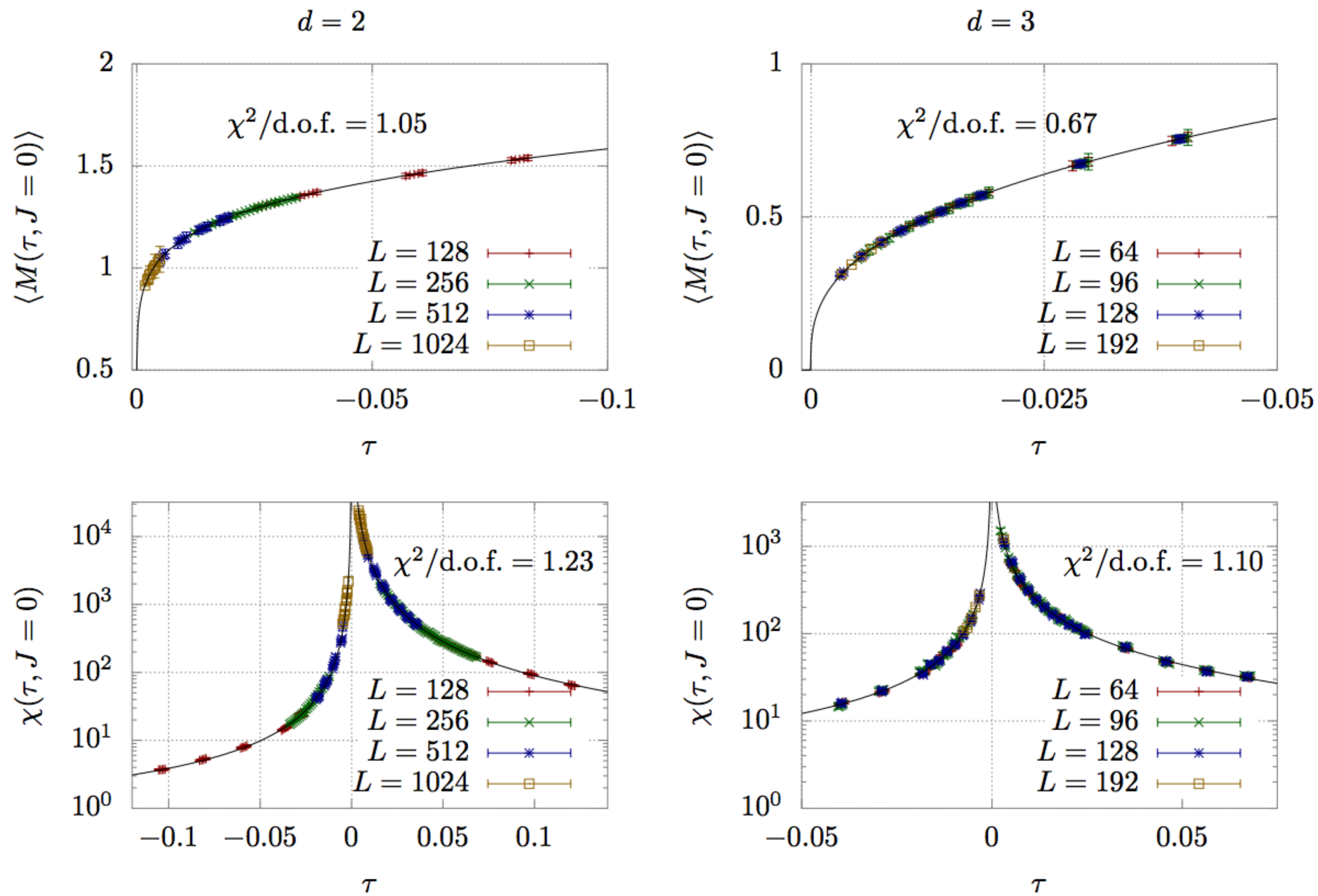
- Dynamic critical behavior governed by universal scaling functions for the spectral function
  Models A &C: governed by relaxation dynamics
  Model B & D: governed by diffusion dynamics

Equilibrium studies provide baseline for studies of non-equilibrium phase-transitions

- Evidence of Kibble-Zurek Scaling in trans-critical quenches; hope to extract non-equilibrium scaling functions for 2D/3D models

Extensions of studies in O(4) model and of Z_2 Model H dynamics in progress/planned

Critical dynamics of relativistic scalar theory



Static critical behavior well under control, and results for critical exponents and universal amplitude ratios in good agreement with literature although precision is not competitive compared to optimized spin models

D. Schweitzer, SS, L.von Smekal Nucl.Phys.B 960 (2020)