Non-Abelian Alice strings in two-flavor dense QCD

Yuki Fujimoto

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References:

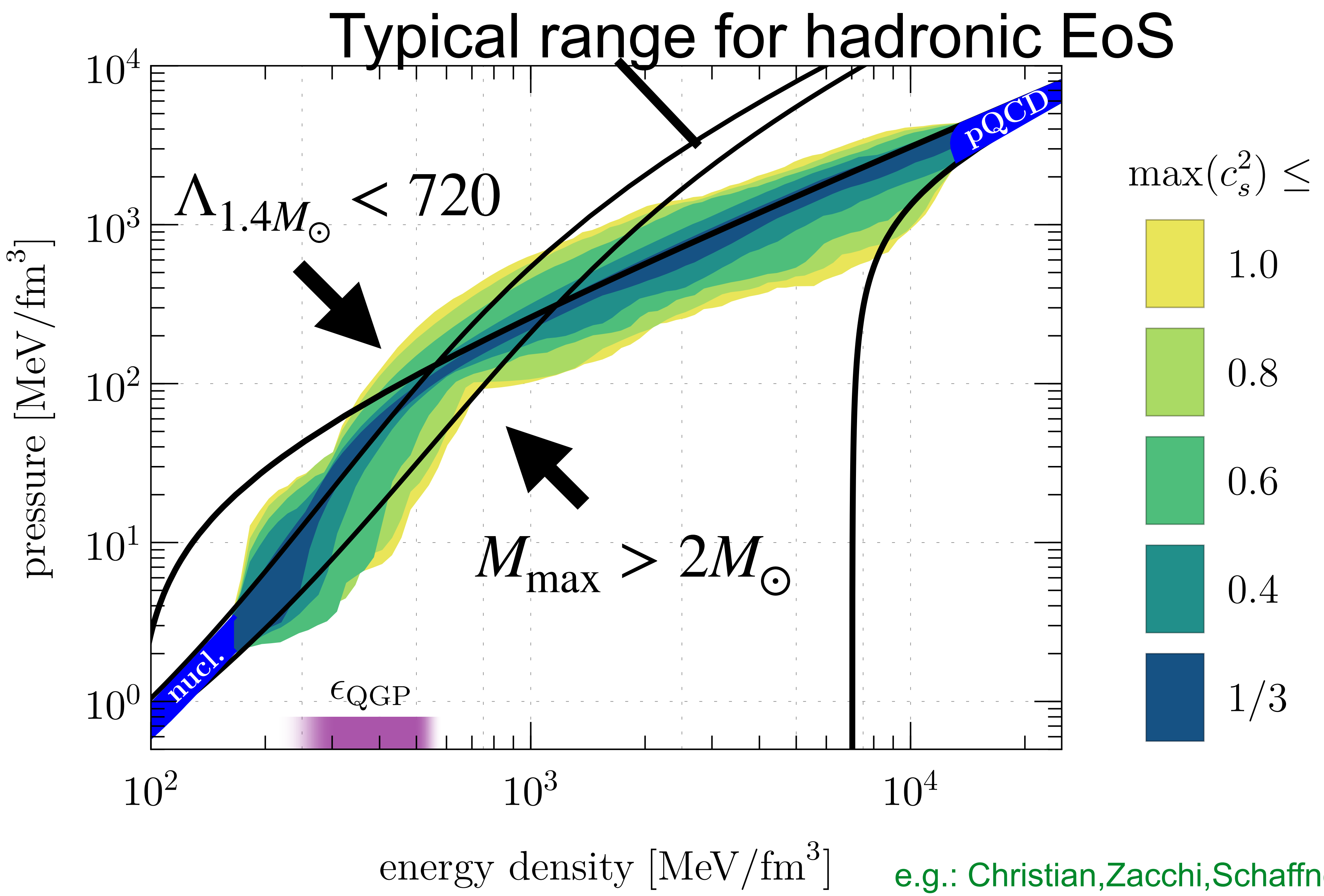
Y. Fujimoto, M. Nitta, PRD 103 (2021) 054002, [arXiv:2011.09947].
Y. Fujimoto, M. Nitta, JHEP 09 (2021) 192, [arXiv:2103.15185].
Y. Fujimoto, M. Nitta, PRD 103 (2021) 114003, [arXiv:2102.12928].
Y. Fujimoto, K. Fukushima, W. Weise, PRD 101 (2020) 094009, [arXiv:1908.09360]

19 October 2021, QCD theory seminar

Prelude: dense matter EoS

Annala,Gorda,Kurkela,Nattila,Vuorinen (2019)

Observational constraint on the dense matter EoS:



e.g.: Christian,Zacchi,Schaffner-Bieliech (2018); Han,Mamun,Constantinou,Prakash (2019), …

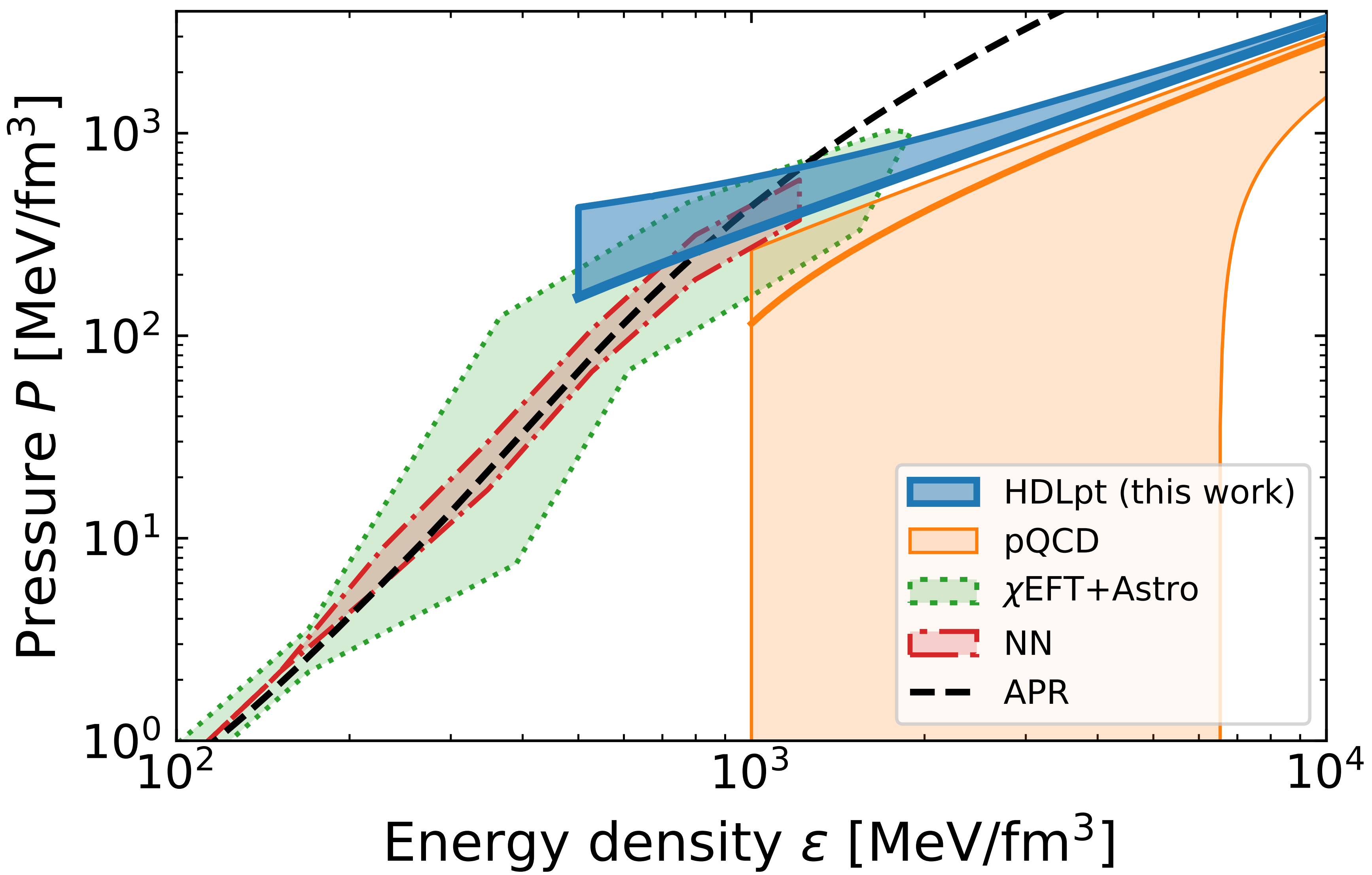
Likely to be **no strong first-order transition** in the EoS

→ **Hadron-to-quark crossover** scenario might be favored

Prelude: dense matter EoS

Fujimoto,Fukushima (2020)

Similar behavior seen also in the extended QCD calculation:



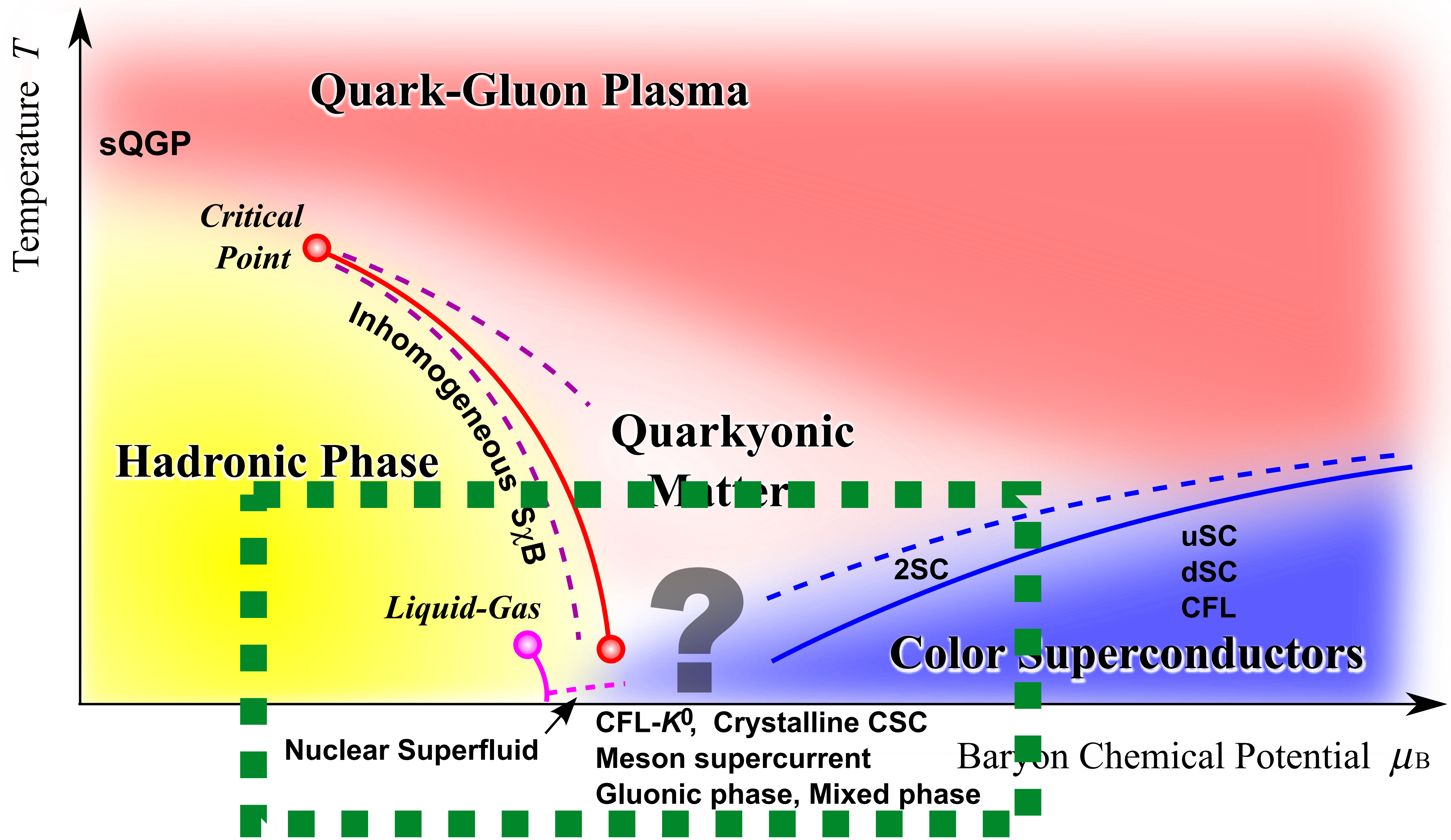
e.g.: Christian,Zacchi,Schaffner-Bieliech (2018);
Han,Mamun,Constantinou,Prakash (2019), …

Likely to be **no strong first-order transition** in the EoS

→ **Hadron-to-quark crossover** scenario might be favored

Prelude: QCD phase diagram

Fukushima,Hatsuda (2010)

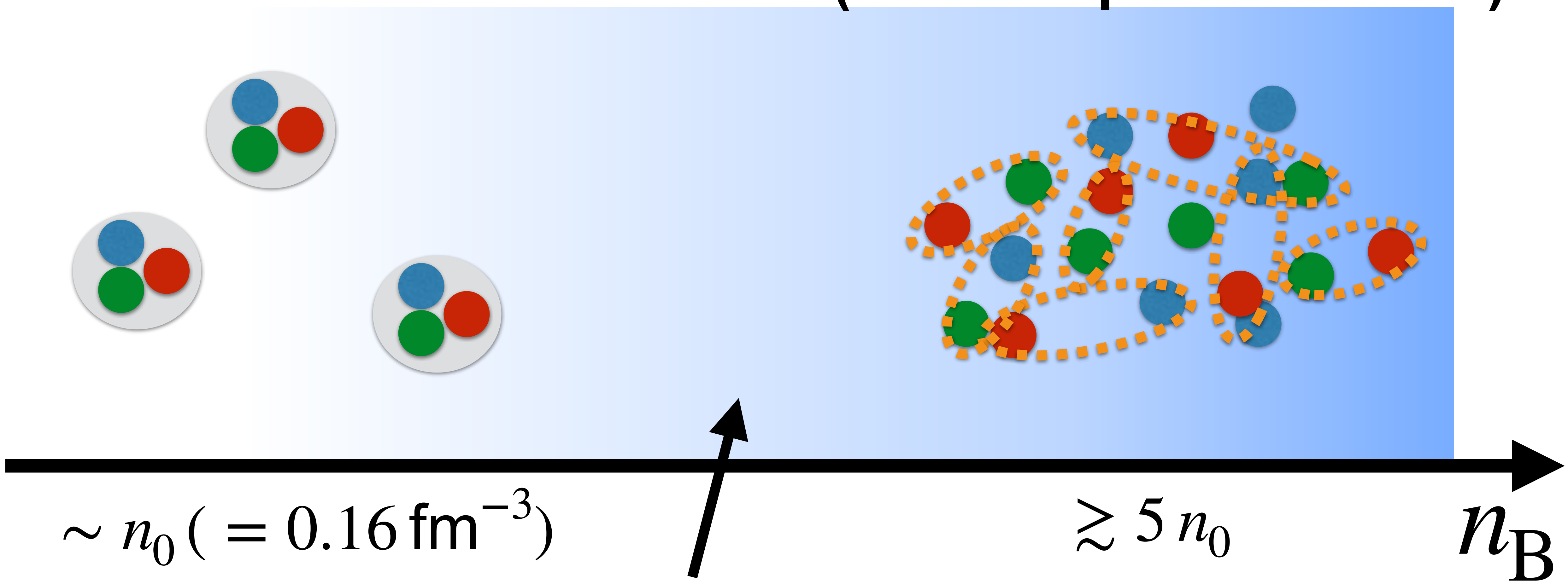


Prelude: crossover scenario

Crossover scenario of cold dense matter

Hadronic matter

Quark matter (Color superconductor)



smoothly connected

"Quark-hadron continuity"

Schafer,Wilczek (1998); Hatsuda,Tachibana,Yamamoto,Baym (2006); Fujimoto,Fukushima,Weise (2019)

Prelude: 3-flavor crossover scenario

Schafer,Wilczek (1998)

When $m_u = m_d = m_s$:

Hyperonic superfluid

Color-flavor locked (CFL) quark matter

$\mathrm{SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_B}$
$\to \mathrm{SU(3)_V}$

Identical symmetry breaking pattern = continuity

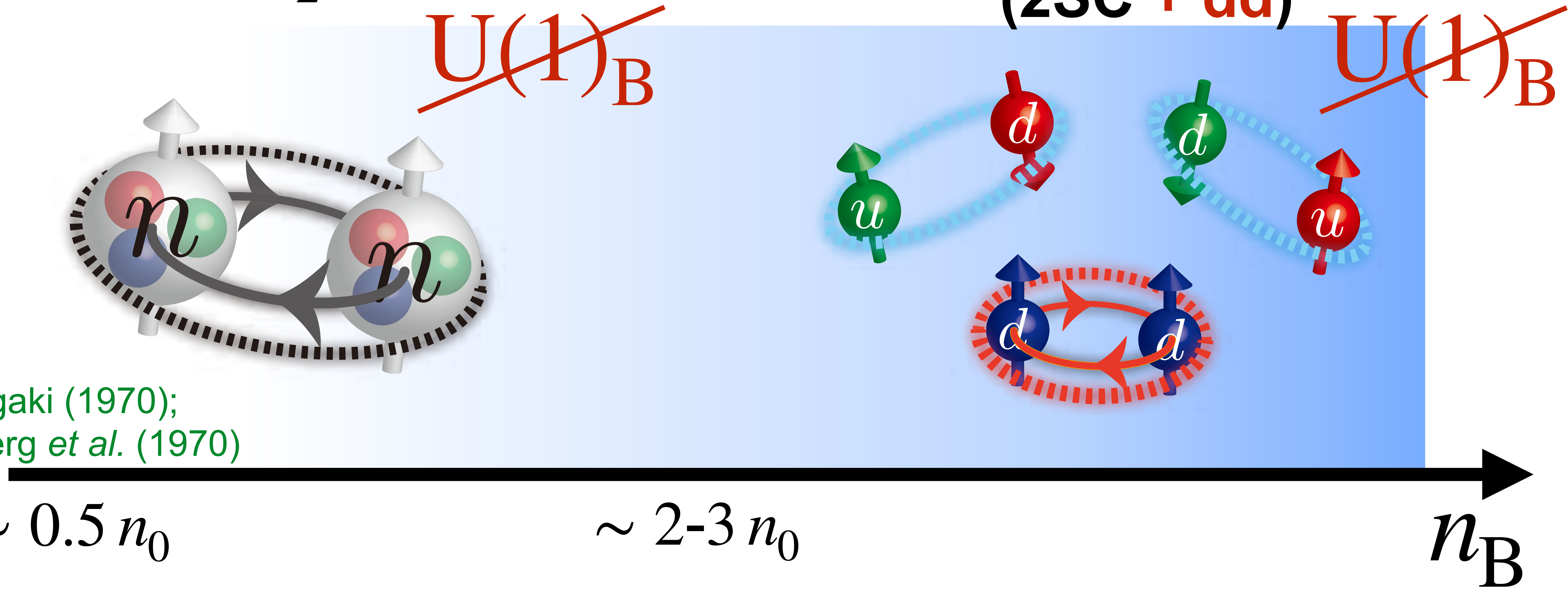
2-flavor crossover scenario

Fujimoto,Fukushima,Weise(2019)

When $m_u = m_d \ll m_s$:

Neutron $^3P_2$ superfluid

Quark matter (2SC + dd)

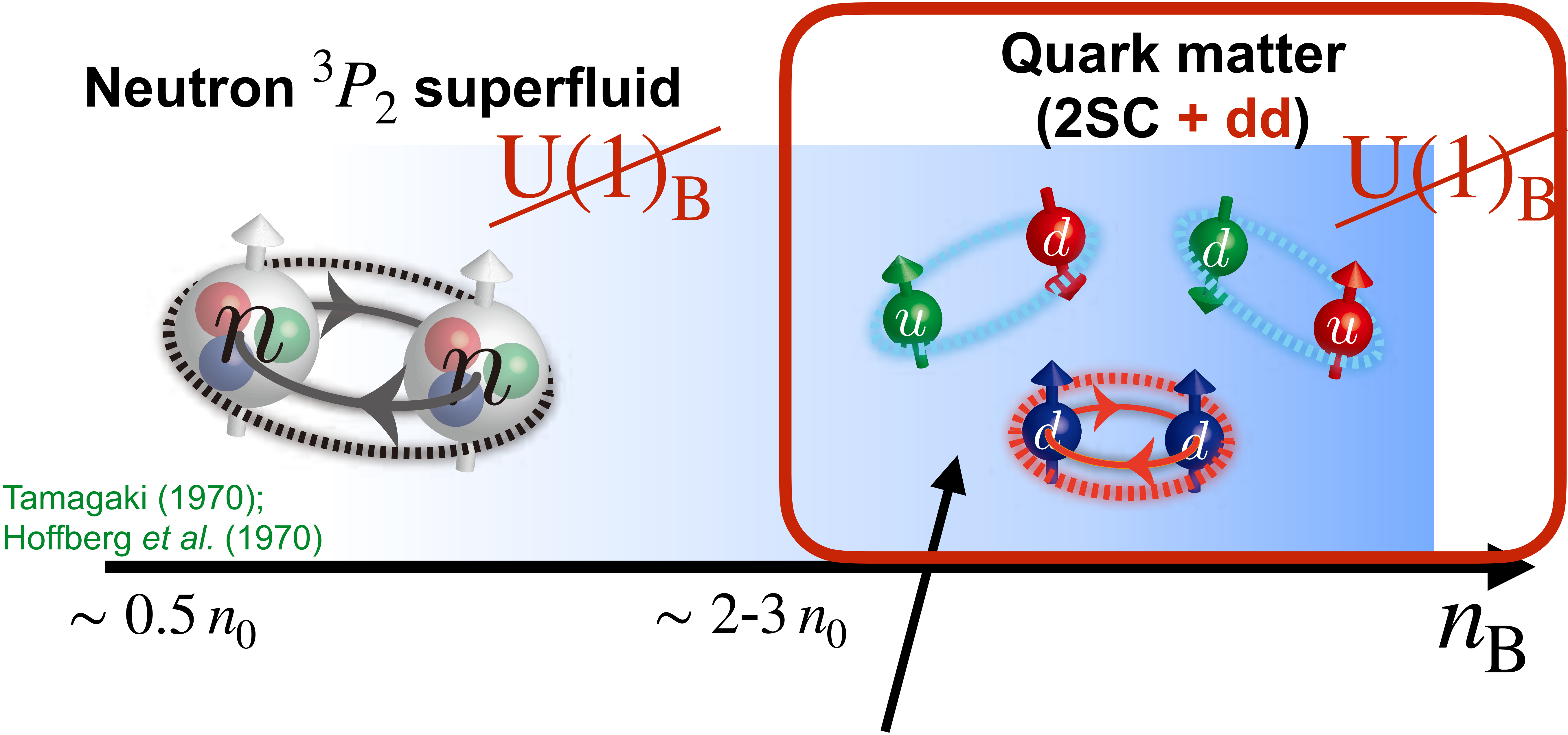


Tamagaki (1970);
Hoffberg *et al.* (1970)

2-flavor crossover scenario

Fujimoto,Fukushima,Weise(2019)

When $m_u = m_d \ll m_s$:



Central topic of this talk:

What are the vortices in this phase?

Brief summary

	Vortex	$U(1)_B$ winding	Color flux	Moduli	Aharonov-Bohm phase
2SC+<dd> (2-flavor)	Non-Abelian **Alice** string	1/3	**1/6**	$\mathbb{R}P^2 \cong S^2/\mathbb{Z}_2$	Color non-singlet
CFL (3-flavor)	Non-Abelian string	1/3	1/3	$\mathbb{C}P^2$	Color singlet

Outline of the talk

- Introduction
- Setup of our study: two-flavor dense quark matter and symmetry breaking patterns
- Classification of vortices: CFL strings and non-Abelian Alice strings
- Aharonov-Bohm defects
  and the topological confinement of Alice strings
- Summary

Outline of the talk

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2-flavor dense quark matter

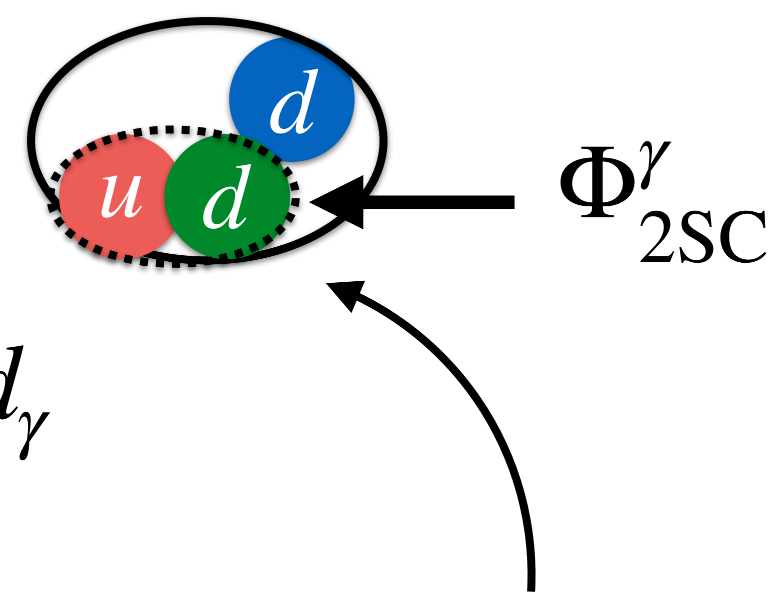
Fujimoto,Fukushima,Weise (2019)

- **Order parameter of $^3P_2$ neutron superfluid:**

$$\Upsilon_{nn} \equiv n^{\top} C \gamma^{i} \nabla^{j} n$$

spin   angular momentum

$$n \equiv \epsilon^{\alpha\beta\gamma}(u_{\alpha}^{\top} C\gamma^{5} d_{\beta}) d_{\gamma} = \Phi^{\gamma}_{2\mathrm{SC}}\, d_{\gamma}$$

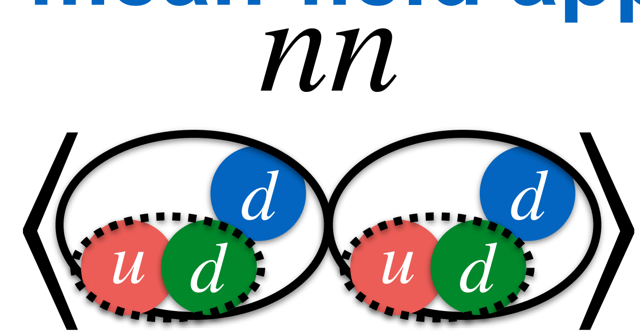


(this schematic figure assumes (unitary) gauge fixing)

2-flavor dense quark matter

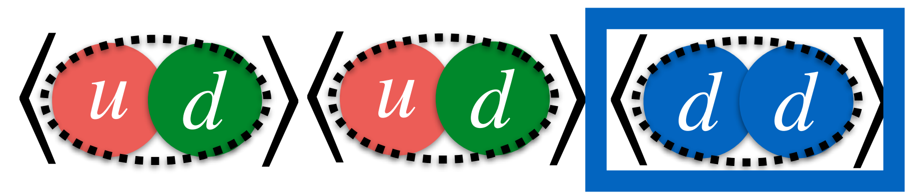
Fujimoto, Fukushima, Weise (2019)

- Υ_{nn} (OP of superfluid) in the mean-field approx:

in hadron phase $\langle \Upsilon_{nn} \rangle \approx$ 

$^3P_2$ pairing

in quark phase $\langle \Upsilon_{nn} \rangle \approx \Phi^{\alpha}_{2\mathrm{SC}} \Phi^{\alpha'}_{2\mathrm{SC}} \langle d^{\top}_{\alpha} C \gamma^i \nabla^j d_{\alpha'} \rangle$

$\approx$ 

New

$2\mathrm{SC} + \langle dd \rangle$

… in the quark phase $U(1)_B$ symmetry broken

→ leads to the topological **superfluid vortex**

From this argument based on the OP, we find the new condensate <dd> in the quark phase

How does <dd> develop expectation value?

Fujimoto,Fukushima,Weise (2019)

- This <dd> should have non-negligible value due to the **coupling to the energy momentum tensor**:

- Four-fermion coupling term:

$$\hat{\mathcal{J}} = (\bar{\psi}_d \gamma^i \nabla^j C \bar{\psi}_d^\top)(\psi_d^\top C \gamma_i \nabla_j \psi_d)$$

$$\begin{cases} \text{EMT:} & T^{\mu\nu} = \bar{\psi}_d i\gamma^\mu \partial^\nu \psi_d \\ \text{In equilibrium:} & T^{\mu\nu} = \mathrm{diag}[\varepsilon, P, P, P] \end{cases}$$

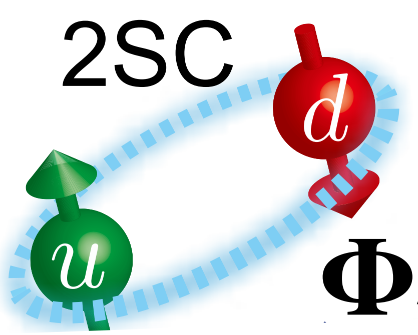
Fierz transformation (neglecting some terms)

$$\langle \hat{\mathcal{J}} \rangle \approx \frac{3}{4} P^2$$

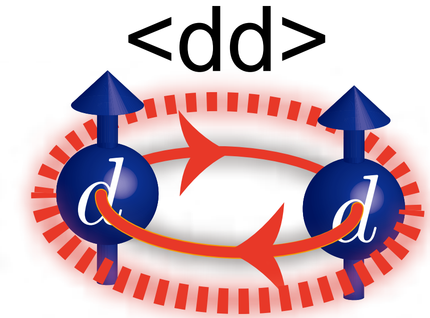
Macroscopic value related to EoS

Symmetry breaking pattern

$$G_{\mathrm{QCD}} = \mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{U}(1)_{\mathrm{B}}$$



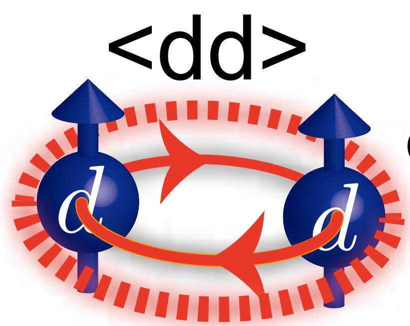
$\Phi_{2\mathrm{SC}} \neq 0$



$\Phi_{dd} \neq 0$

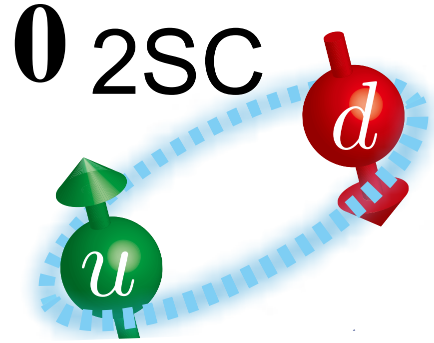
$$\tilde{H}_{2\mathrm{SC}} = \mathrm{SU}(2)_{\mathrm{C}} \times \mathrm{U}(1)_{\mathrm{C+B}}$$

$$H_{dd} = \mathrm{SO}(3)_{\mathrm{C}} \rtimes (\mathbb{Z}_6)_{\mathrm{C+B}}$$



$\Phi_{dd} \neq 0$

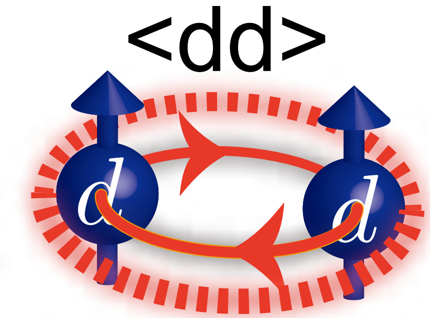
$\Phi_{2\mathrm{SC}} \neq 0$



$$K_{2\mathrm{SC}+dd} = \mathrm{SO}(2)_{\mathrm{C}} \times (\mathbb{Z}_6)_{\mathrm{C+B}}$$

Symmetry breaking pattern

$$G_{\mathrm{QCD}} = \mathbf{SU(3)_C \times U(1)_B}$$



Ansatze:

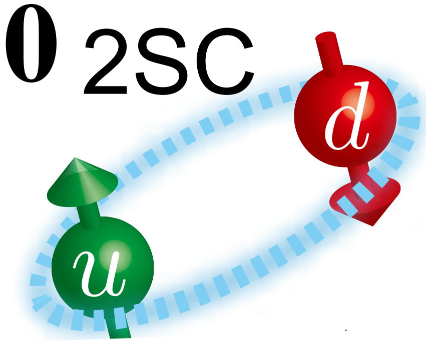
$$(\Phi_{dd})_{\alpha\beta} = \begin{pmatrix} m & & \\ & m & \\ & & m \end{pmatrix}$$

$$(\Phi_{2\mathrm{SC}})^{\alpha} = \begin{pmatrix} \Delta_r \\ \Delta_g \\ \Delta_b \end{pmatrix}$$

$$\Phi_{dd} \neq 0$$

$$H_{dd} = \mathbf{SO(3)_C \rtimes (\mathbb{Z}_6)_{C+B}}$$

$$\Phi_{2\mathrm{SC}} \neq 0$$ 2SC



$$K_{2\mathrm{SC}+dd} = \mathbf{SO(2)_C \times (\mathbb{Z}_6)_{C+B}}$$

Symmetry breaking pattern

$$G_{\mathrm{QCD}} = \mathrm{SU(3)_C} \times \mathrm{U(1)_B}$$

Vortices develop associated with this condensate

$$\Phi_{dd} \neq 0$$

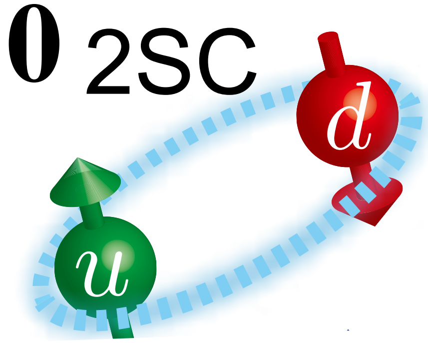
Ansatze:

$$(\Phi_{dd})_{\alpha\beta} = \begin{pmatrix} m & & \\ & m & \\ & & m \end{pmatrix}$$

$$(\Phi_{2\mathrm{SC}})^{\alpha} = \begin{pmatrix} \Delta_r \\ \Delta_g \\ \Delta_b \end{pmatrix}$$

$$H_{dd} = \mathrm{SO(3)_C} \rtimes (\mathbb{Z}_6)_{\mathrm{C+B}}$$

$$\Phi_{2\mathrm{SC}} \neq 0$$ 2SC



$$K_{2\mathrm{SC}+dd} = \mathrm{SO(2)_C} \times (\mathbb{Z}_6)_{\mathrm{C+B}}$$

Outline of the talk

- Introduction
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- **Classification of vortices: CFL strings and non-Abelian Alice strings**
- Aharonov-Bohm defects and the topological confinement of Alice strings
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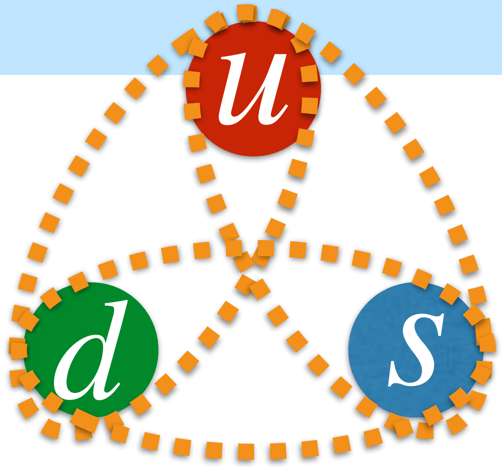
Vortices in CFL phase (3-flavor)



- **Gap matrix in the color-flavor space:**

$$\Phi_{\alpha i} = \epsilon_{\alpha\beta\gamma}\epsilon_{ijk}q_j^{\beta}q_k^{\gamma}$$

$$(\alpha = r, g, b; \quad i = u, d, s)$$

- **Ground state (Color-flavor locking):**

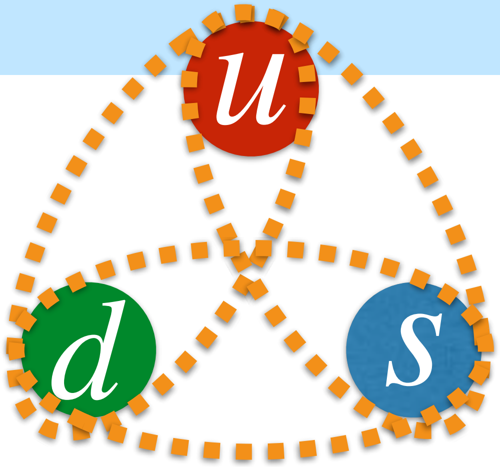
$$\langle \Phi_{\mathrm{CFL}} \rangle_{\alpha i} = \begin{pmatrix} \langle d_{[g} s_{b]} \rangle & 0 & 0 \\ 0 & \langle s_{[b} u_{r]} \rangle & 0 \\ 0 & 0 & \langle u_{[r} d_{g]} \rangle \end{pmatrix}$$

$$G = SU(3)_C \times SU(3)_F \times U(1)_B$$
$$\to H_{\mathrm{CFL}} = SU(3)_{C+F}$$

$$\Phi_{\alpha i} \to e^{i\theta_B} g_{\mathrm{color}} \, \Phi_{\alpha i} \, g_{\mathrm{flavor}}$$
$$g_{\mathrm{color}} = g_{\mathrm{flavor}}^{-1}$$

Alford,Rajagopal,Wilczek (1998)

Vortices in CFL phase (3-flavor)



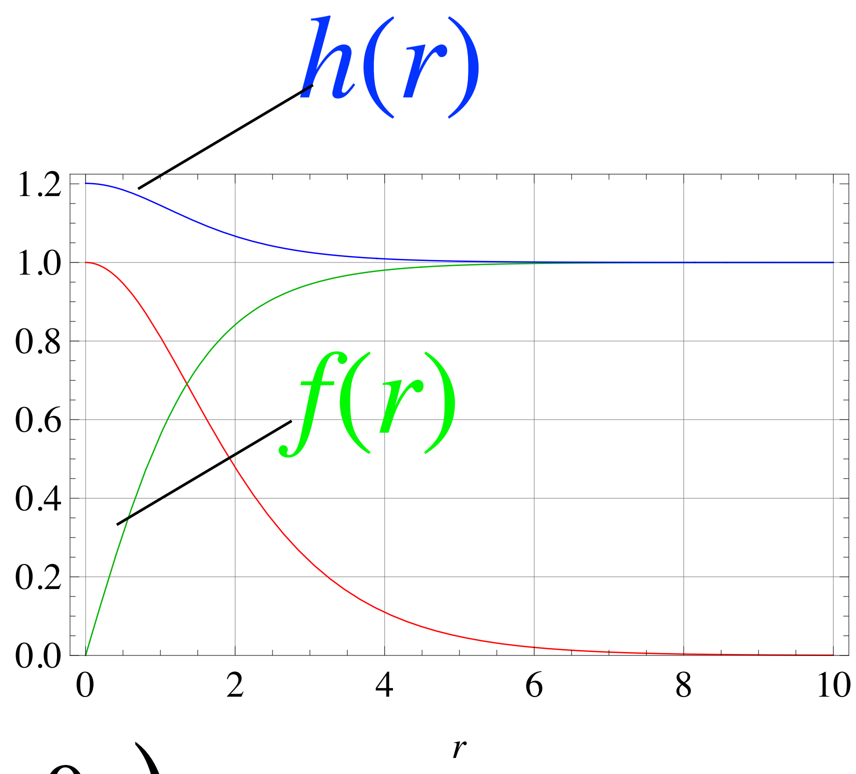
- **Abelian vortices:**

$$\Phi_{\rm CFL}(\varphi) = \begin{pmatrix} f(r)e^{i\varphi} & 0 & 0 \\ 0 & f(r)e^{i\varphi} & 0 \\ 0 & 0 & f(r)e^{i\varphi} \end{pmatrix}$$

- **Non-Abelian vortices**

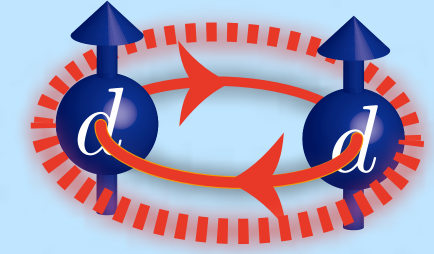
$$\Phi_{\rm CFL}(\varphi) = \begin{pmatrix} h(r) & 0 & 0 \\ 0 & h(r) & 0 \\ 0 & 0 & f(r)e^{i\varphi} \end{pmatrix}$$

$$= e^{\frac{i\varphi}{3}} e^{\frac{i\varphi}{3}{\rm diag}(-1,-1,2)} \begin{pmatrix} h(r) & 0 & 0 \\ 0 & h(r) & 0 \\ 0 & 0 & f(r) \end{pmatrix}$$



Balachandran,Digal,Matsuura (2005); Nakano,Nitta,Matsuura (2007)

Vortices with Φ_{dd} (2-flavor)



- **Ground state (in color symmetric rep.):**

$$(\Phi_{dd})_{\alpha\beta} = \begin{pmatrix} \Delta_{dd} & & \\ & \Delta_{dd} & \\ & & \Delta_{dd} \end{pmatrix}$$

$$G = SU(3)_C \times U(1)_B$$

$$\rightarrow H_{dd} = SO(3)_C \rtimes (\mathbb{Z}_6)_{C+B}$$

$$(\Phi_{dd})_{\alpha\beta} \rightarrow e^{i\theta_B} g_{\text{color}} (\Phi_{dd})_{\alpha\beta}\, g_{\text{color}}^T$$

$$e^{i\theta_B} = 1, \quad g_{\text{color}} g_{\text{color}}^T = 1$$

$$e^{i\theta_B} = \omega^{-2k}, \quad g_{\text{color}} = [\text{diag}(\omega, \omega, \omega^{-2})]^k \leftarrow T^8$$

$$(\omega \equiv e^{i\pi/3},\ k = 0,1,2,3,4,5)$$

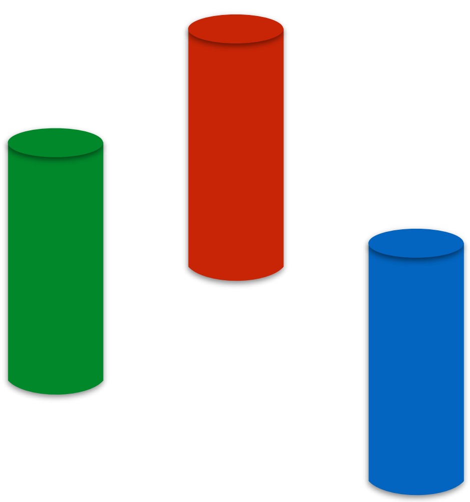
Vortices with Φ_{dd} (2-flavor)

Abelian vortex

$$\Phi_{dd}(\varphi) = \boxed{e^{i\varphi}}\,\Phi_{dd}(\varphi=0) = mf(r)\begin{pmatrix} e^{i\varphi} & & \\ & e^{i\varphi} & \\ & & e^{i\varphi} \end{pmatrix}$$

unit $\mathrm{U}(1)_\mathrm{B}$ winding

Non-Abelian vortex (Alice string)



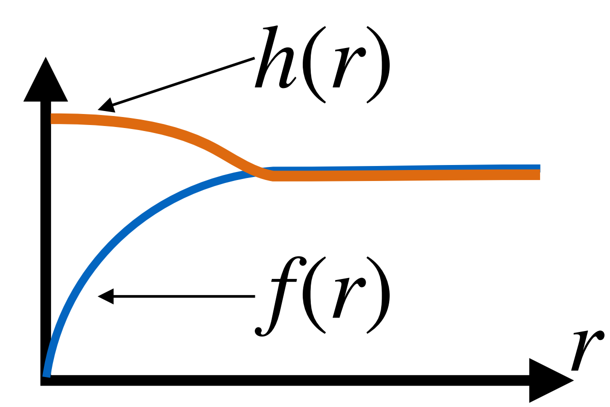
$$\Phi_{dd}(\varphi) = \boxed{e^{i\varphi/3}}\,g_c(\varphi)\Phi_{dd}(\varphi=0)g_c^{\top}(\varphi) = \Delta_{dd}\begin{pmatrix} h(r) & & \\ & h(r) & \\ & & f(r)\,e^{i\varphi} \end{pmatrix}$$

1/3 $\mathrm{U}(1)_\mathrm{B}$ winding

gauge transform: $g_c(\varphi) = e^{i(\varphi/6)\mathrm{diag}(-1,-1,2)}$

profile functions: $f(r), h(r)$

$$A_i(r) = -\frac{1}{6g}\frac{\epsilon_{ij}x^j}{r^2}a(r)\begin{pmatrix} -1 & & \\ & -1 & \\ & & 2 \end{pmatrix}$$



Vortices with Φ_{dd} (2-flavor)

Abelian vortex

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unit $\mathrm{U}(1)_{\mathrm{B}}$ winding

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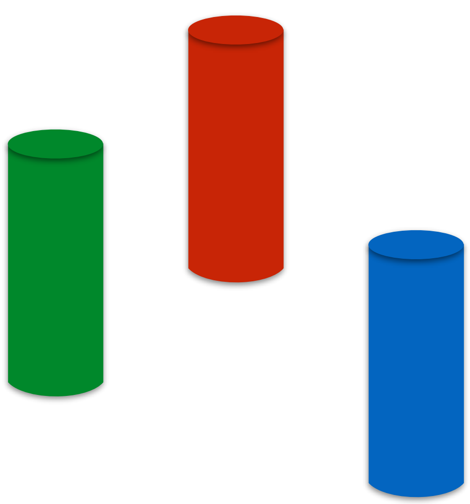

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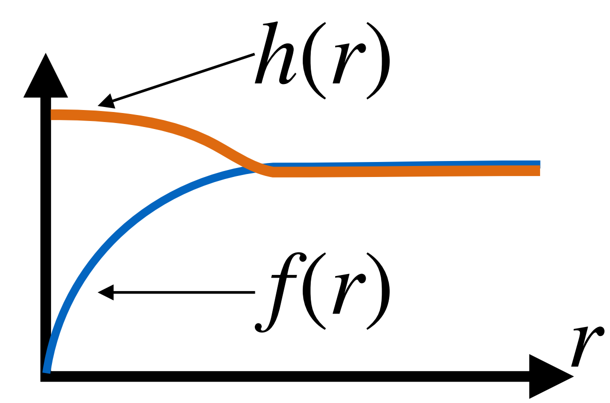
1/3 $\mathrm{U}(1)_{\mathrm{B}}$ winding



Tension of vortices: $E(n) = n^2 \log \Lambda$

(n: $\mathrm{U}(1)_{\mathrm{B}}$ winding, Λ: system size)

Vortices with Φ_{dd} (2-flavor)

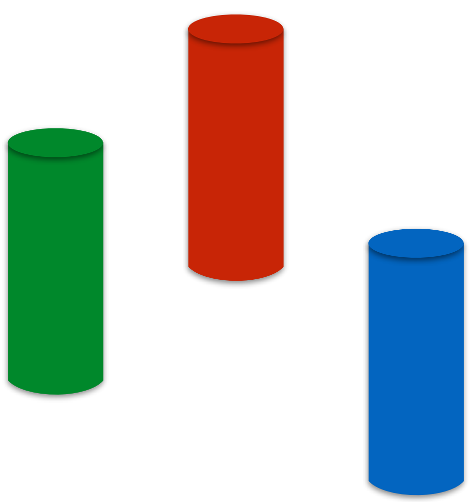
Abelian vortex

$$\Phi_{dd}(\varphi) = \boxed{e^{i\varphi}}\Phi_{dd}(\varphi=0) = mf(r)\begin{pmatrix} e^{i\varphi} & & \\ & e^{i\varphi} & \\ & & e^{i\varphi} \end{pmatrix}$$

unit $U(1)_B$ winding

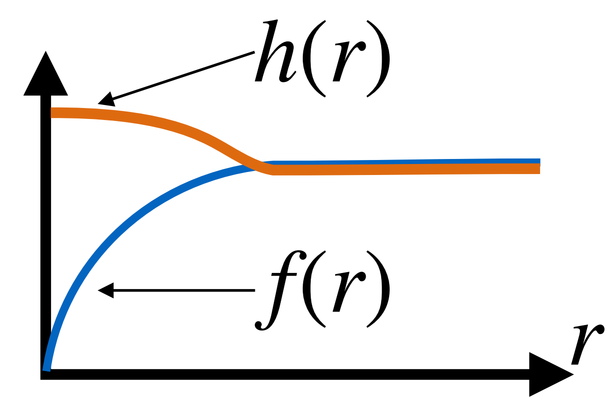
$$E(n=1) = 9E(n=1/3)$$

Non-Abelian vortex (Alice string) Energetically favorable



$$\Phi_{dd}(\varphi) = \boxed{e^{i\varphi/3}} g_c(\varphi)\Phi_{dd}(\varphi=0)g_c^{\top}(\varphi) = \Delta_{dd}\begin{pmatrix} h(r) & & \\ & h(r) & \\ & & f(r)\,e^{i\varphi} \end{pmatrix}$$

1/3 $U(1)_B$ winding



$$3E(n=1/3)$$

Tension of vortices: $E(n) = n^2 \log \Lambda$

(n: $U(1)_B$ winding, Λ: system size)

Orientational moduli in CFL phase

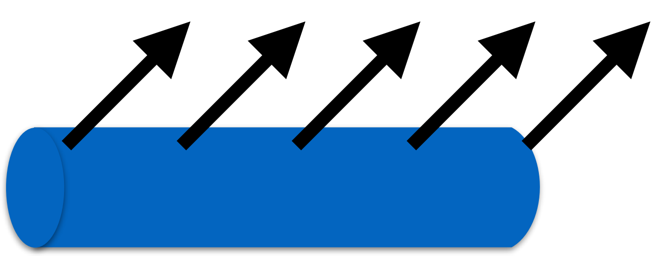
$$\Phi_{\mathrm{CFL}}(\varphi) = \left(\begin{array}{cc|c} h(r) & 0 & 0 \\ 0 & h(r) & 0 \\ \hline 0 & 0 & f(r)e^{i\varphi} \end{array}\right)$$

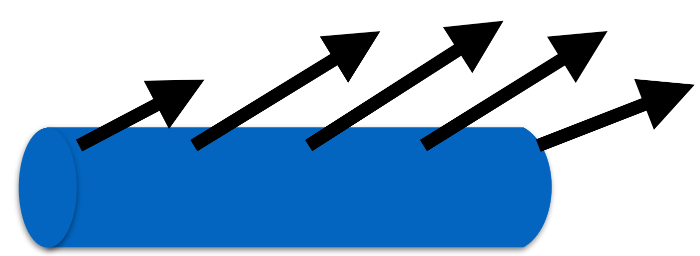
$$H_{\mathrm{CFL}} = SU(3)_{C+F}$$
$$\to K_{\mathrm{vortex}} = [SU(2) \times U(1)]_{C+F}$$ **@ vortex core**

NG modes in the vicinity of vortices (orientational moduli):

$$\frac{H_{\mathrm{CFL}}}{K_{\mathrm{vortex}}} = \frac{SU(3)_{C+F}}{SU(2) \times U(1)} \cong \mathbb{C}P^2$$

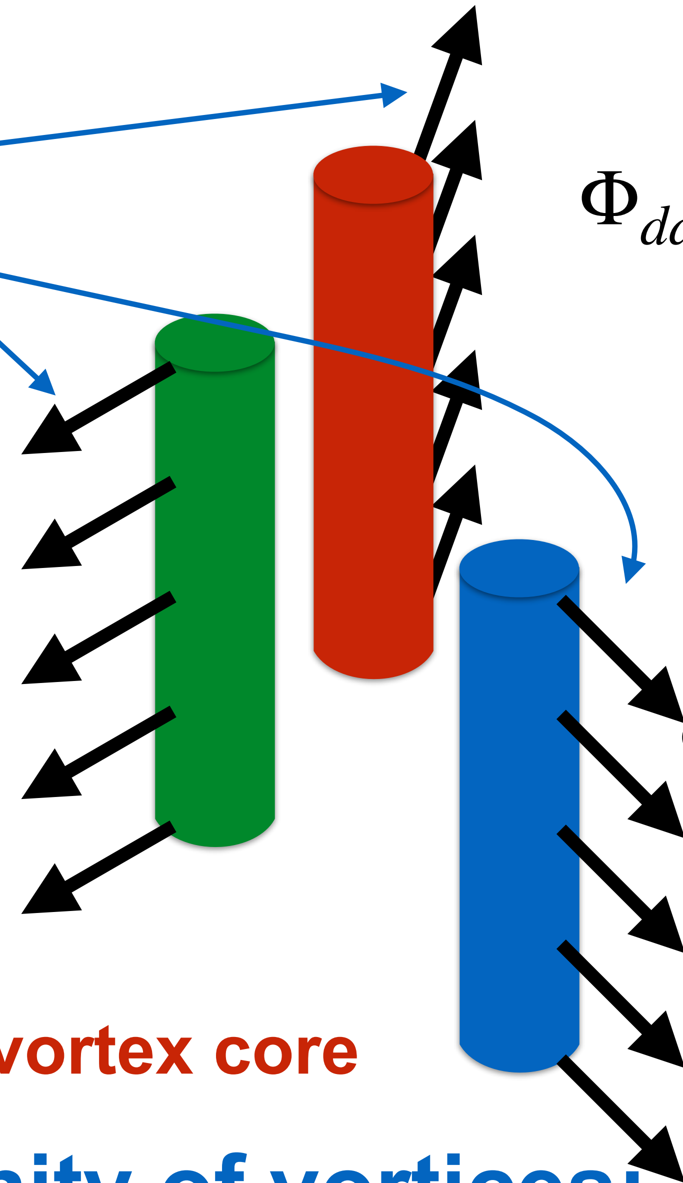
… gapless modes propagating along the vortex





Orientational moduli in 2-flavor

Orientational moduli



$$\Phi_{dd}(\varphi) = \Delta_{dd} \begin{pmatrix} f(r)\,e^{i\varphi} & & \\ & h(r) & \\ & & h(r) \end{pmatrix}$$

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$$\Phi_{dd}(\varphi) = \Delta_{dd} \begin{pmatrix} h(r) & & \\ & h(r) & \\ & & f(r)\,e^{i\varphi} \end{pmatrix}$$

$H_{dd} = SO(3)_C \rtimes (\mathbb{Z}_6)_{C+B}$

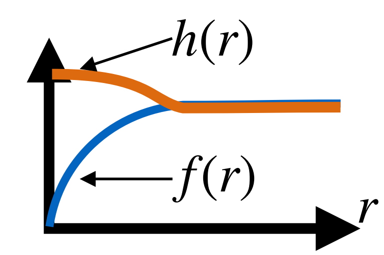
$\to K_{\text{vortex}} = O(2)_C \times \mathbb{Z}_6$ **@ vortex core**

cf: $\Phi_{dd}(\varphi = 0) \propto \text{diag}(1,1,1)$ @ $r = \infty$

NG modes in the vicinity of vortices:

$$\frac{H_{dd}}{K_{\text{vortex}}} = \frac{\text{SO}(3) \rtimes \mathbb{Z}_6}{\text{O}(2) \times \mathbb{Z}_6} \simeq S^2/\mathbb{Z}_2 \simeq \mathbb{R}P^2$$

… orientational moduli



Each vortices: **Non-Abelian Alice string**

Non-Abelian Alice phenomenon

Fujimoto,Nitta (2020)

- In a certain gauge theory, "charge conjugation" can be **local** symmetry due to topological obstruction

- $G \to H_\varphi$; H_φ is position (azimuthal angle φ) dependent and $H_\varphi = g_{\rm color}(\varphi)\, H_{\varphi=0}\, g^{-1}_{\rm color}(\varphi)$

- For the whole group $H_{\varphi=2\pi} \cong H_{\varphi=0} = SO(3) \rtimes \mathbb{Z}_6$.
  But, not true for an individual generator of H_φ because embedding of H_φ in G is position dependent
  $\to$ **No global & continuous definition** of generators of H_φ

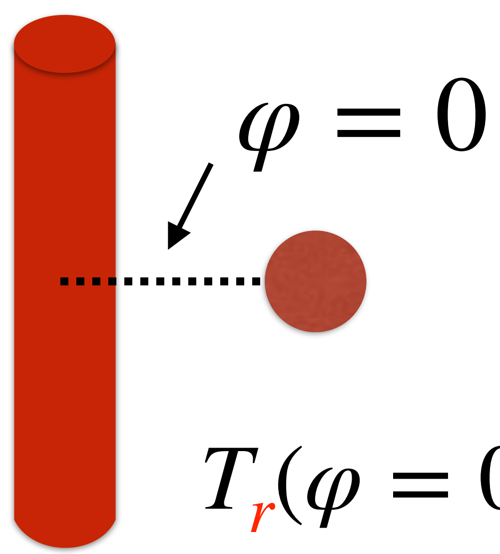
Non-Abelian Alice phenomenon

Fujimoto,Nitta (2020)

- Generators $T_{r,g,b} \in SO(3)$:

$$T_{r,g,b}(\varphi) = g_{\rm color}(\varphi)\, T_{r,g,b}\, g_{\rm color}^{-1}(\varphi)$$

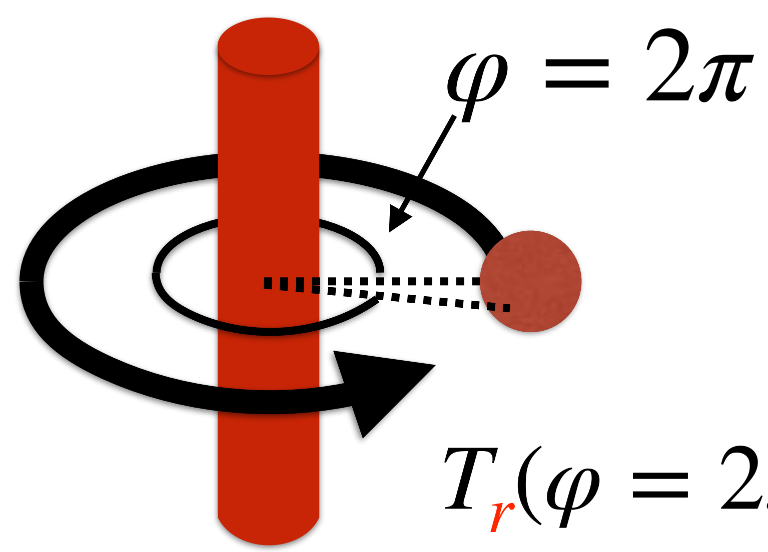
Around non-Abelian Alice strings:



$T_r(\varphi = 0) = T_r$

$T_g(\varphi = 0) = T_g$

$T_b(\varphi = 0) = T_b$



$T_r(\varphi = 2\pi) = T_r$

$T_g(\varphi = 2\pi) = -T_g$

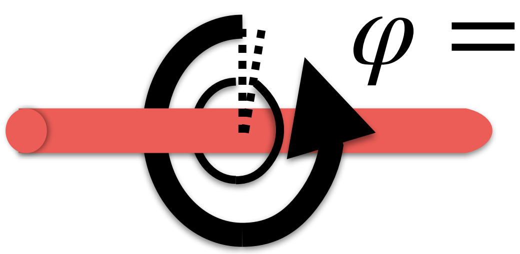
$T_b(\varphi = 2\pi) = -T_b$

Non-Abelian generalization of Alice property

Aharonov-Bohm phase

- Aharonov-Bohm phase of q (light quarks u, d):

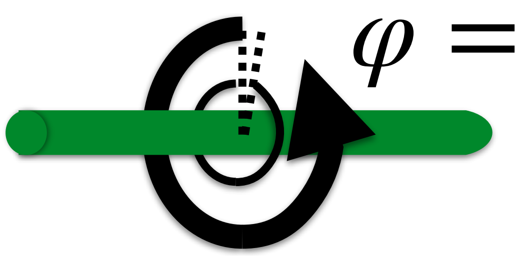
$$q \to e^{i\varphi/6}\,\mathrm{P}\exp\left(-ig\int_0^{\varphi} \boldsymbol{A}\cdot d\boldsymbol{\ell}\right) q; \quad q = \begin{pmatrix} q_r \\ q_g \\ q_b \end{pmatrix}$$



$\varphi = 2\pi$

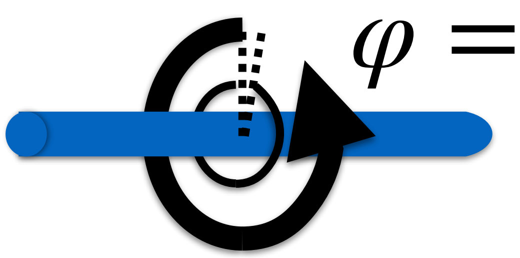
$$q \to (2\pi \text{ rotation}) \to \mathrm{diag}(\underline{-1}, 1, 1)\, q$$

color non-singlet



$\varphi = 2\pi$

$$q \to (2\pi \text{ rotation}) \to \mathrm{diag}(1, \underline{-1}, 1)\, q$$

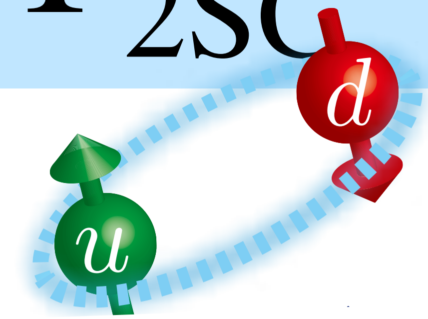


$\varphi = 2\pi$

$$q \to (2\pi \text{ rotation}) \to \mathrm{diag}(1, 1, \underline{-1})\, q$$

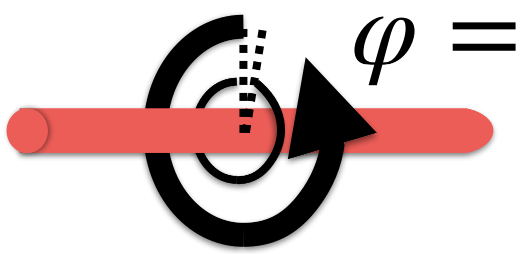
Quarks can detect color flux at infinity

Consistency with 2SC condensate Φ_{2SC}



- Aharonov-Bohm phase of Φ_{2SC} consistent with gauge-transformation of Φ_{dd} :

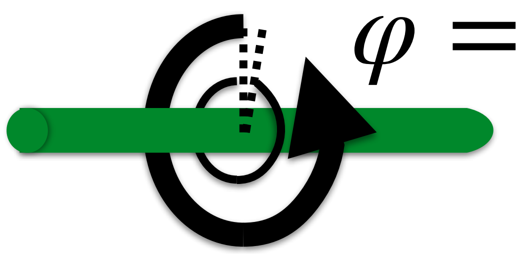
$$\Phi_{2SC} \to e^{i\varphi/3}\,\mathrm{P}\exp\left(-ig\int_0^{\varphi} \boldsymbol{A}\cdot d\boldsymbol{\ell}\right)\Phi_{2SC}; \qquad \Phi_{2SC} = \begin{pmatrix} \Delta_r \\ \Delta_g \\ \Delta_b \end{pmatrix}$$



$\varphi = 2\pi$

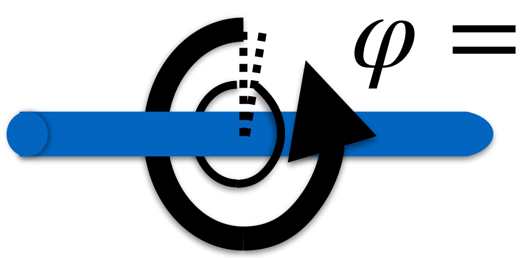
$$\Phi_{2SC} \to (2\pi \text{ rotation}) \to \mathrm{diag}(1, -1, -1)\,\Phi_{2SC}$$

inconsistent



$\varphi = 2\pi$

$$\Phi_{2SC} \to (2\pi \text{ rotation}) \to \mathrm{diag}(-1, 1, -1)\,\Phi_{2SC}$$

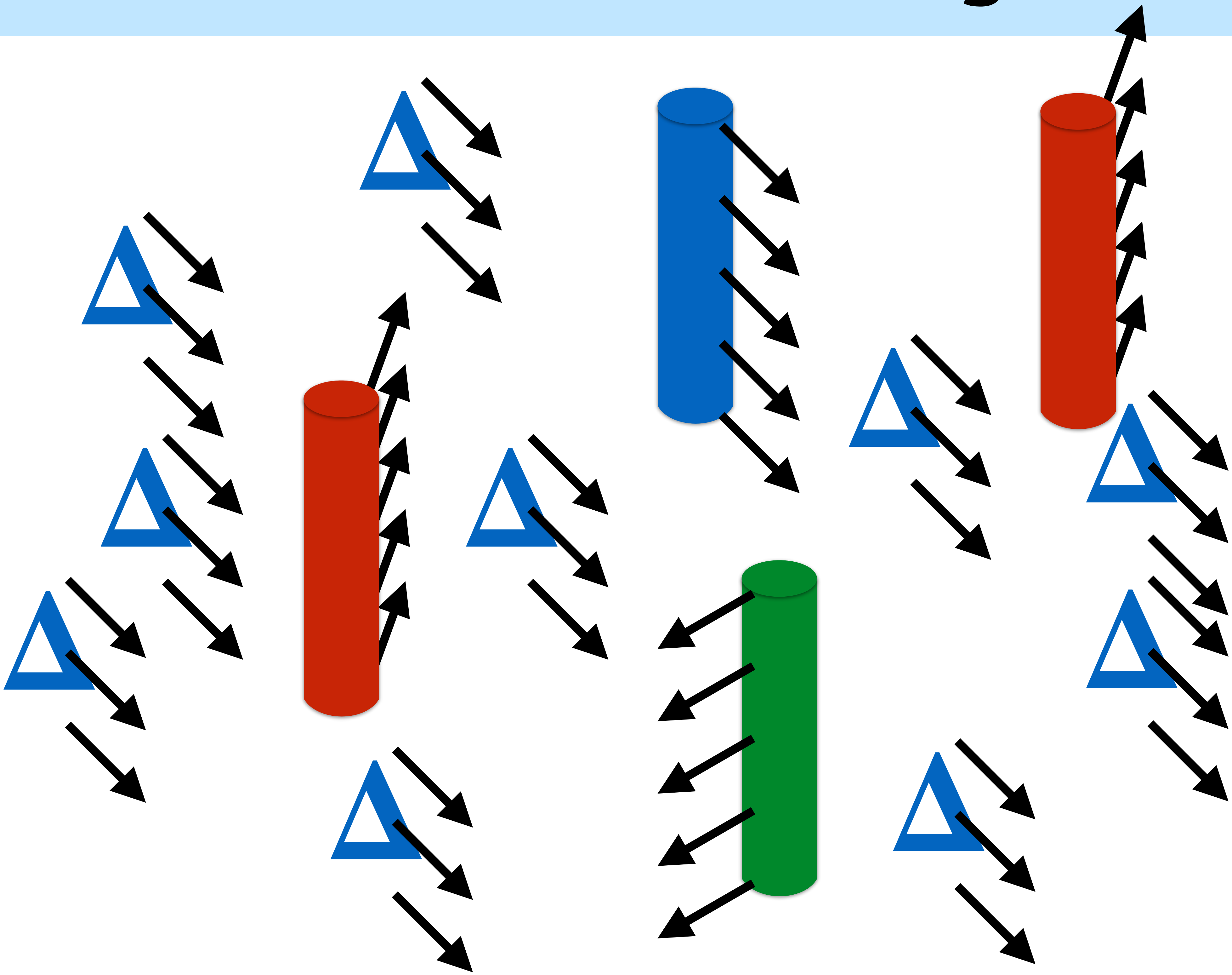


$\varphi = 2\pi$

$$\Phi_{2SC} \to (2\pi \text{ rotation}) \to \mathrm{diag}(-1, -1, 1)\,\Phi_{2SC}$$

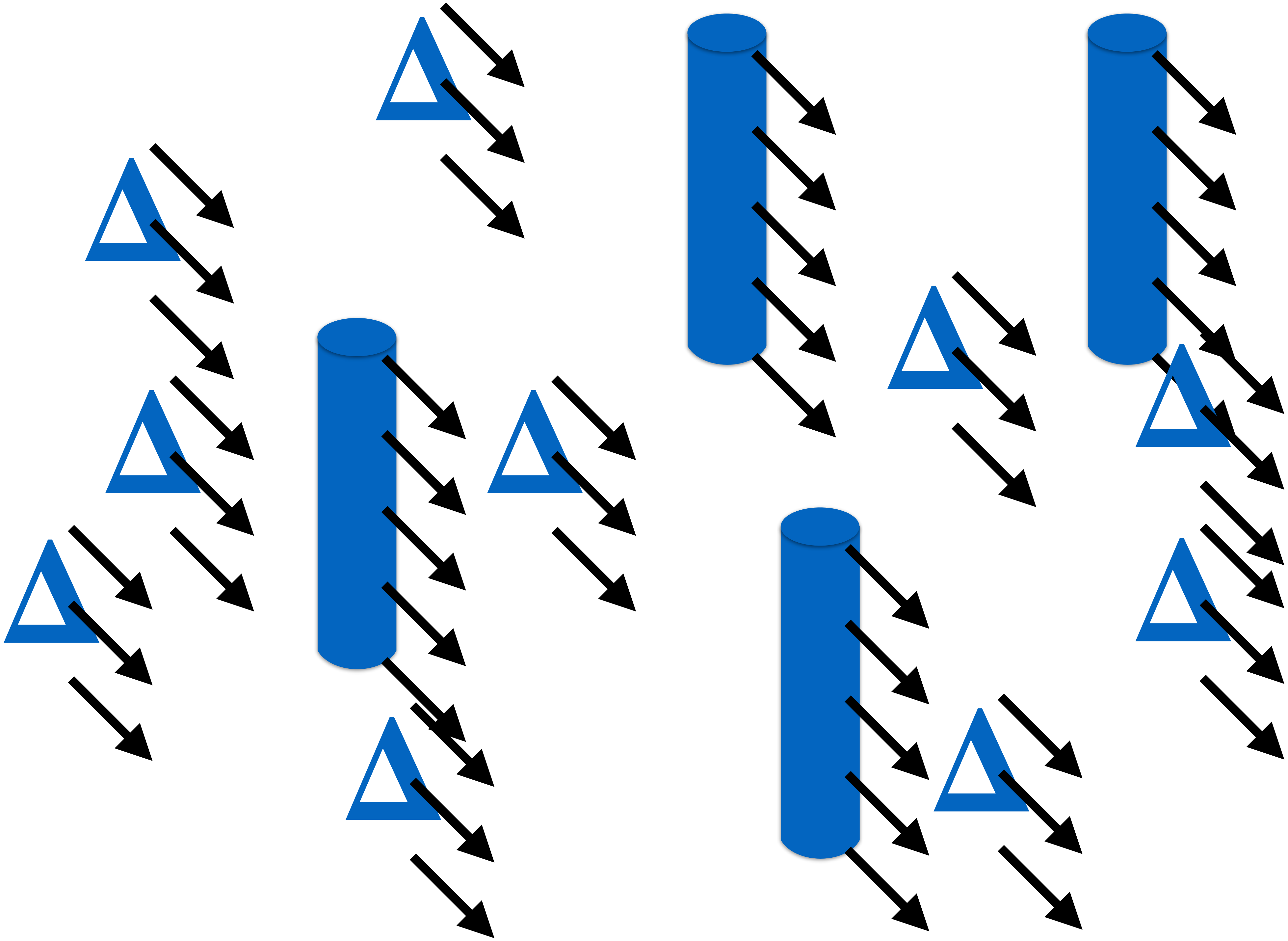
Φ_{2SC} (bulk quantity) should be aligned with soliton moduli!

Bulk-soliton moduli locking



Soliton moduli is aligned with Δ (bulk quantity)!

Bulk-soliton moduli locking



Soliton moduli is aligned with Δ (bulk quantity)!

Brief summary

	Vortex	$U(1)_B$ winding	Color flux	Moduli	Aharonov-Bohm phase
2SC+<dd> (2-flavor)	Non-Abelian **Alice** string	1/3	**1/6**	$\mathbb{R}P^2 \cong S^2/\mathbb{Z}_2$	Color non-singlet
CFL (3-flavor)	Non-Abelian string	1/3	1/3	$\mathbb{C}P^2$	Color singlet

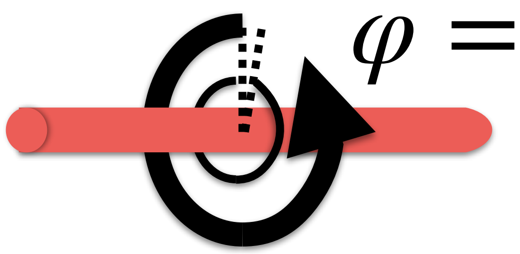
Outline of the talk

- Introduction
- Setup of our study: two-flavor dense quark matter and symmetry breaking patterns
- Classification of vortices: CFL strings and non-Abelian Alice strings
- **Aharonov-Bohm defects and the topological confinement of Alice strings**
- Summary

Aharonov-Bohm defects in $\Phi_{2\mathrm{SC}}$

- AB phase consistent with gauge-transformation of Φ_{dd} :

$$\Phi_{2\mathrm{SC}} \to e^{i\varphi/3}\,\mathrm{P}\exp\left(-ig\int_0^{\varphi} \boldsymbol{A}\cdot d\boldsymbol{\ell}\right)\Phi_{2\mathrm{SC}}; \quad \Phi_{2\mathrm{SC}} = \begin{pmatrix} \Delta_r \\ \Delta_g \\ \Delta_b \end{pmatrix}$$



$\varphi = 2\pi$

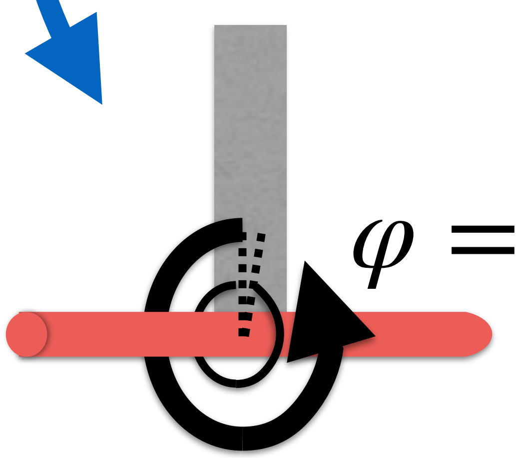
$$\Phi_{2\mathrm{SC}} \to (2\pi \text{ rotation}) \to \mathrm{diag}(1, -1, -1)\ \Phi_{2\mathrm{SC}}$$

inconsistency

Singlevaluedness of $\Phi_{2\mathrm{SC}}$ can be maintained by inserting kink profile $w(\varphi)$ (AB defect)

$$\Phi_{2\mathrm{SC}} = \begin{pmatrix} \Delta_r \\ \Delta_g w(\varphi) \\ \Delta_b w(\varphi) \end{pmatrix}$$

$$w(\varphi = 0) = 1,\ w(\varphi = 2\pi) = -1$$



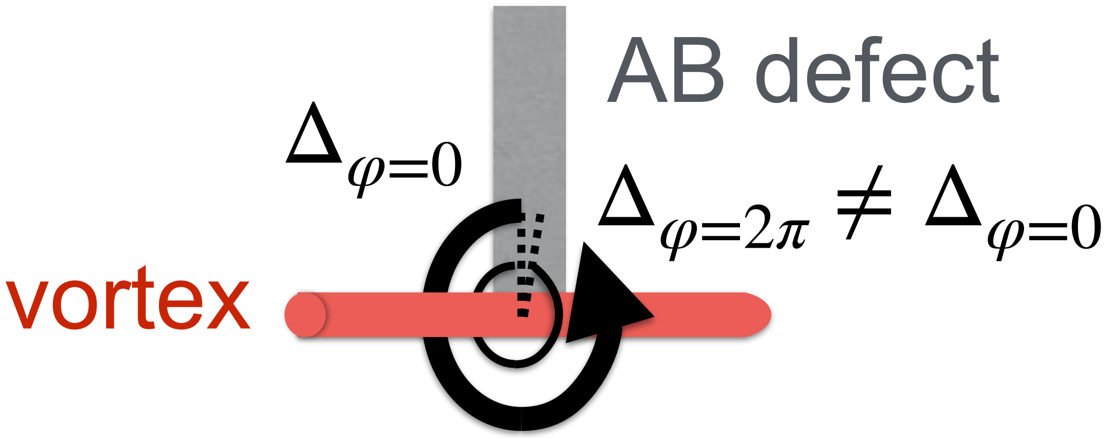
$\varphi = 2\pi$

$$\Phi_{2\mathrm{SC}} \to (2\pi \text{ rotation}) \to \mathrm{diag}(1, -w(\varphi), -w(\varphi))\ \Phi_{2\mathrm{SC}}$$

consistent

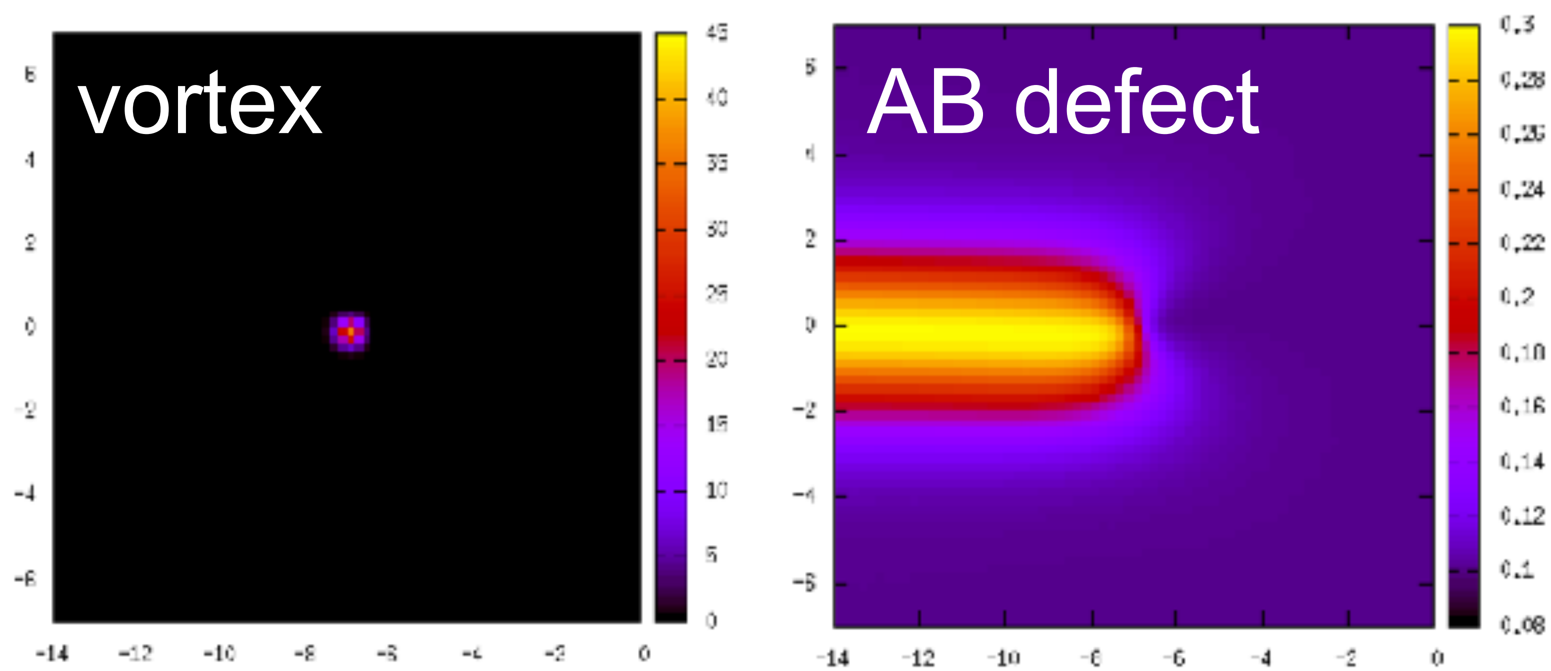
Aharonov-Bohm defects

- When there is a nontrivial AB phase around a vortex, the AB defect (kink) attached to the vortex appears to maintain the single-valuedness

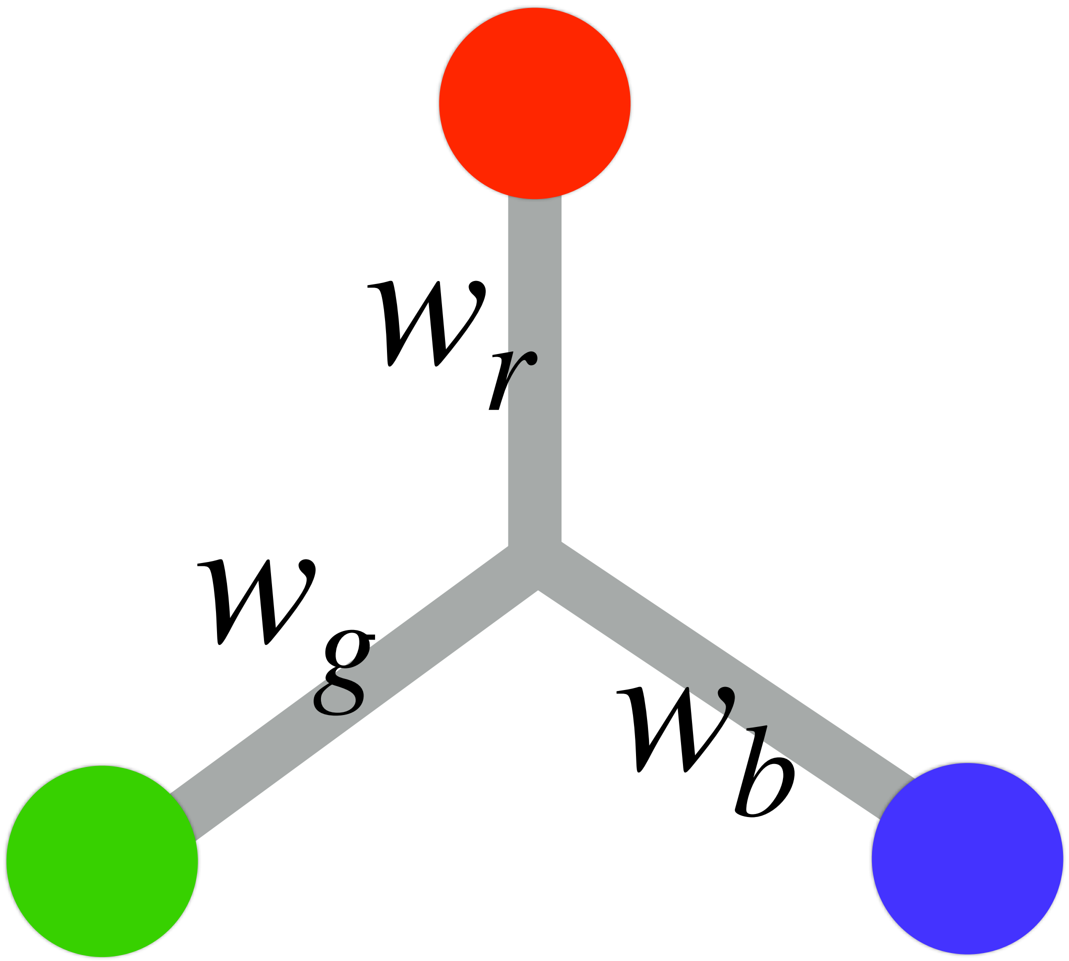


Chatterjee,Nitta (2019)

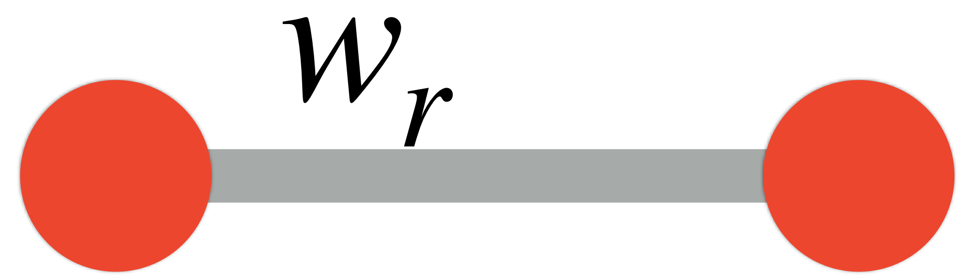
- Numerical solution has been obtained in an $SU(2) \times U(1)$ model:



Confinement owing to AB defects



Baryon-type

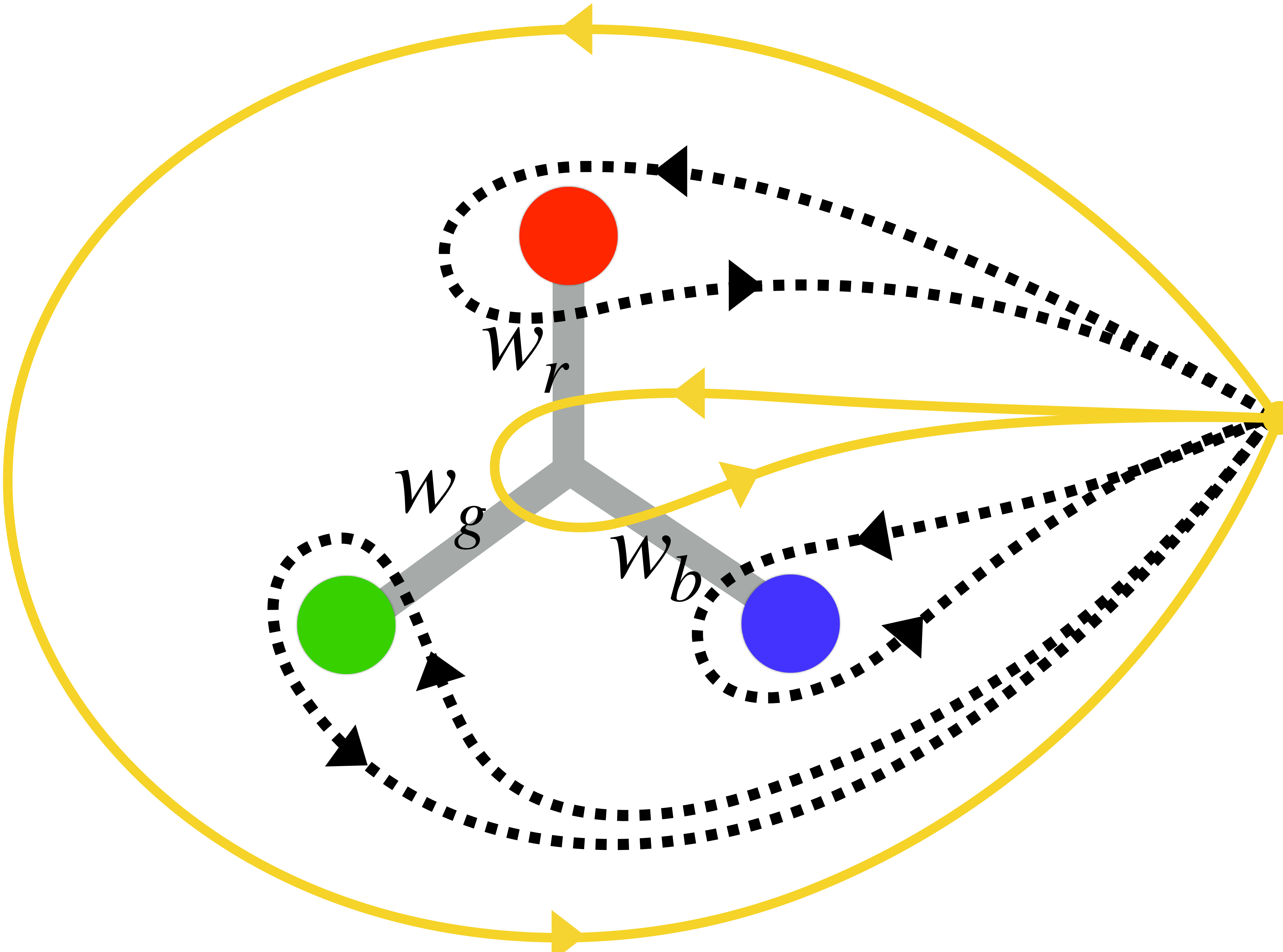


Meson-type

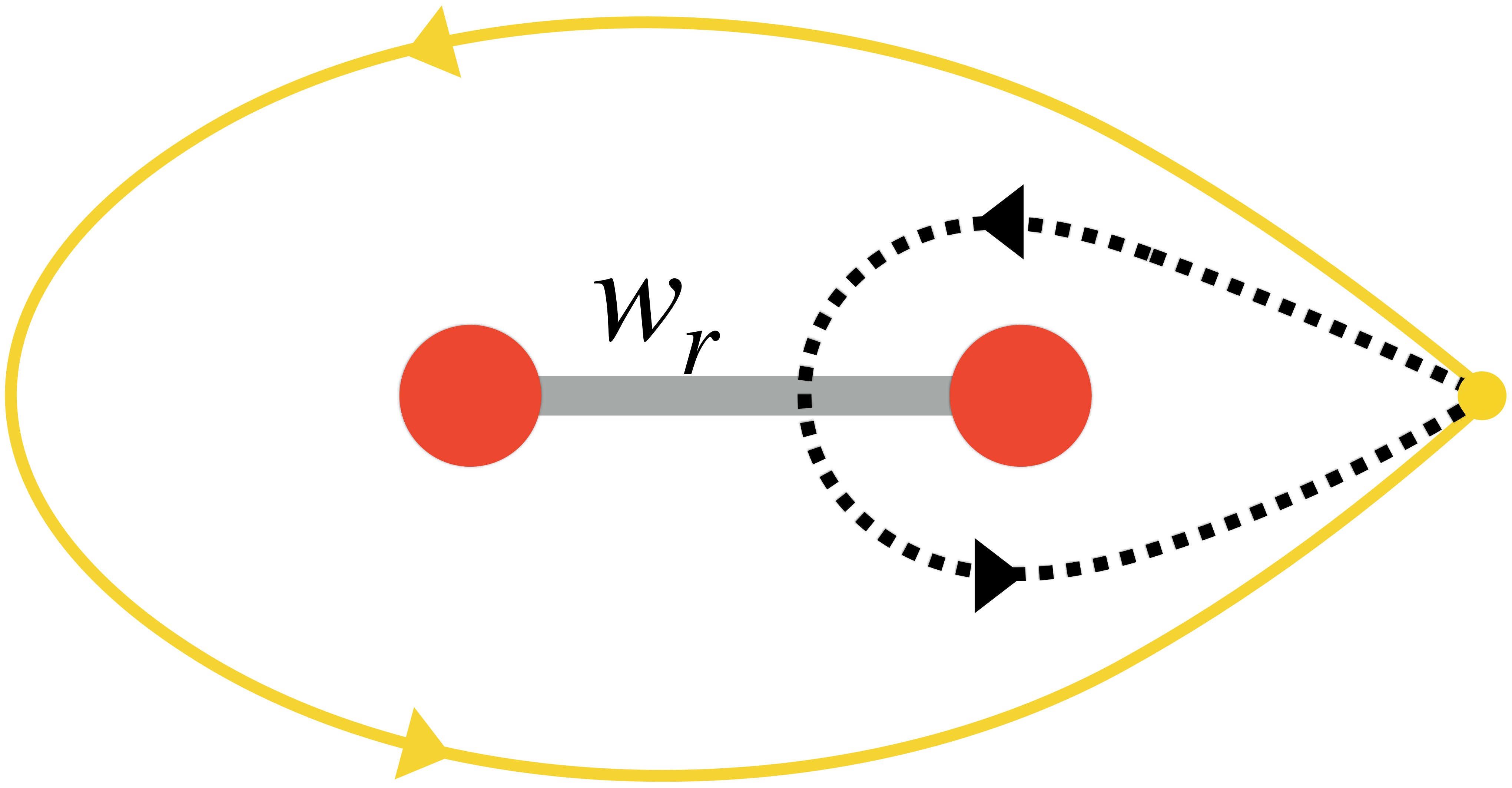
… Two types of confining vortices appear due to different pattern of the cancellation of non-trivial AB phases in AB defects

No color can be detected by AB phase → color confinement

“Baryonic” molecule



“Mesonic” molecule



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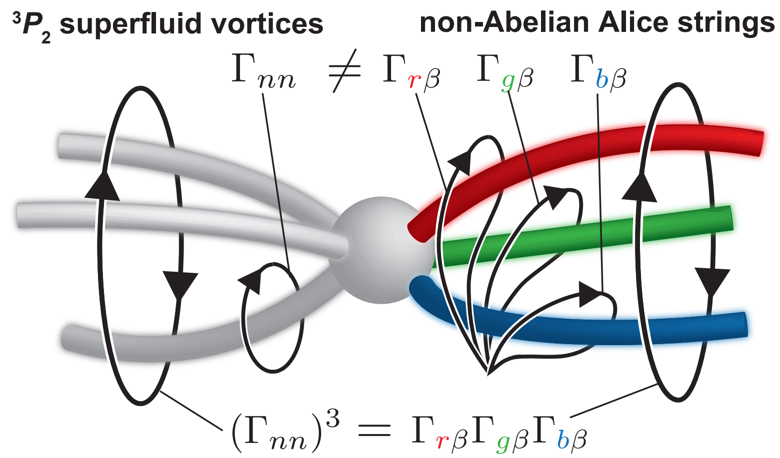
Summary

- In 2-flavor quark matter with ${}^3P_2$ superfluidity, we found non-Abelian vortices; it is the non-Abelian version of **Alice strings**. Multivalued nature of the generator is essential.

- In discussing the property of vortices, **Aharonov-Bohm (AB) phase** is a useful tool:
  - 2SC condensate (Δ) is aligned with soliton moduli of Alice string → **Bulk-soliton moduli locking**
  - Formation of AB defect leads to the confinement of vortices

- Connecting hadronic and color-superconducting vortices:



← AB phase doesn't match: boojum needed?

What can higher form symmetry tell?