Transport coefficients of resonating fermions in the quantum virial expansion

KF & Y. Nishida, PRA **102**, 023310 (2020); [arXiv:2004.12154].
KF & Y. Nishida, PRA **103**, 053320 (2021); [arXiv:2103.10123].

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QCD theory seminar

5 October 2021

Plan of this talk

1. Introduction

- Resonating fermions
- Quantum virial expansion

2. Transport coefficients in the quantum virial expansion

- Review: previous results
- Shear viscosity & Thermal conductivity KF & Y. Nishida, PRA (2021).
- Bulk viscosity KF & Y. Nishida, PRA (2020).

3. Summary

Ultracold atoms

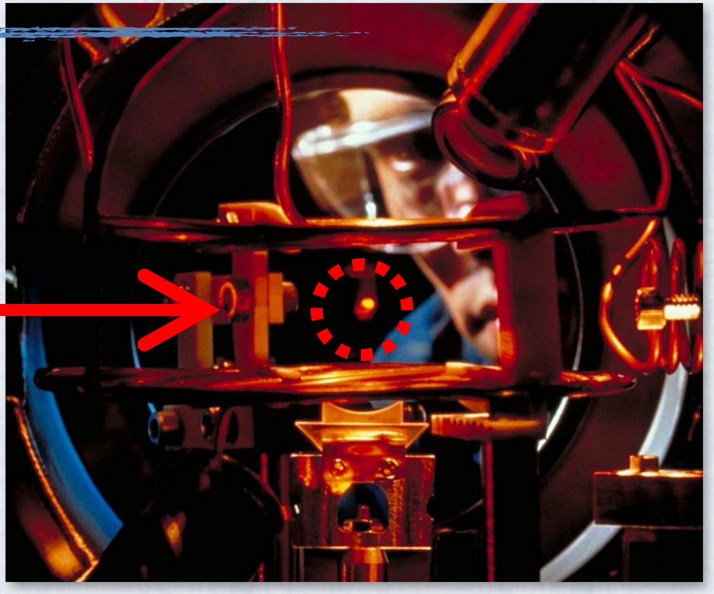
Atomic cluster :

trapped and cooled down

$T \sim 50$ nK $(T/T_F \sim 0.05)$

$n \sim 5\times10^{12}$ cm^{-3}

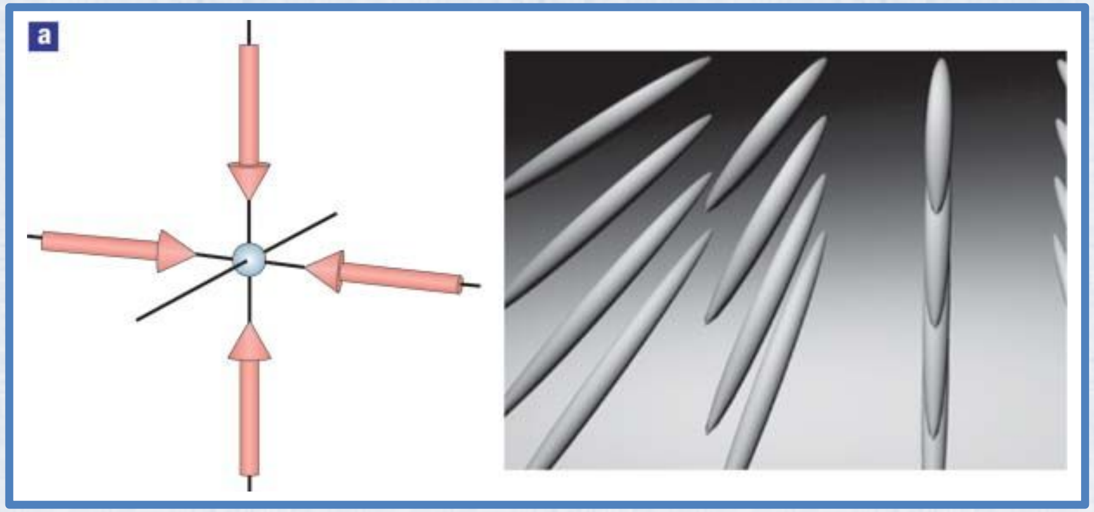
▸ Cold & dilute



https://physics.aps.org/story/v21/st11

✓ High tunability

- Spatial dimensions



I. Bloch, Nat. Phys. 1, 23-30 (2005)

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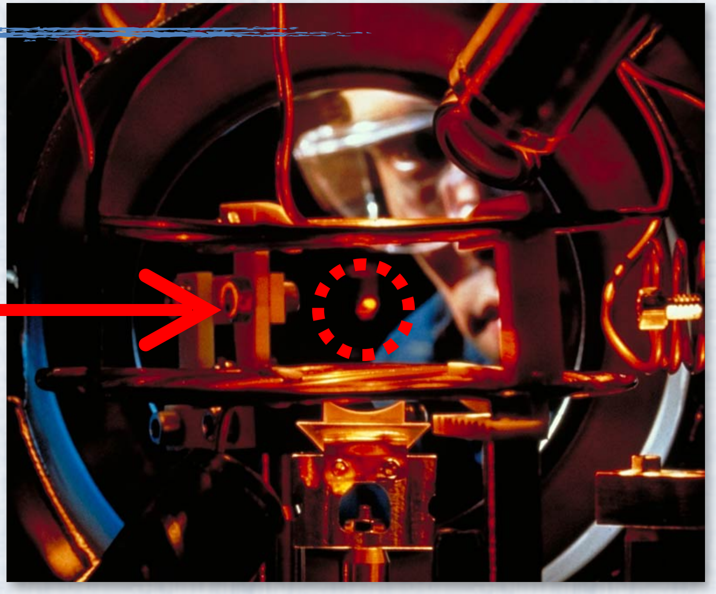


https://physics.aps.org/story/v21/st11

✓ High tunability

- Spatial dimensions
- Quantum statistics & internal degrees of freedom

Bosons : ^{7}Li, ^{23}Na, ...
Fermions : ^{6}Li, ^{40}K, ...

Ultracold atoms

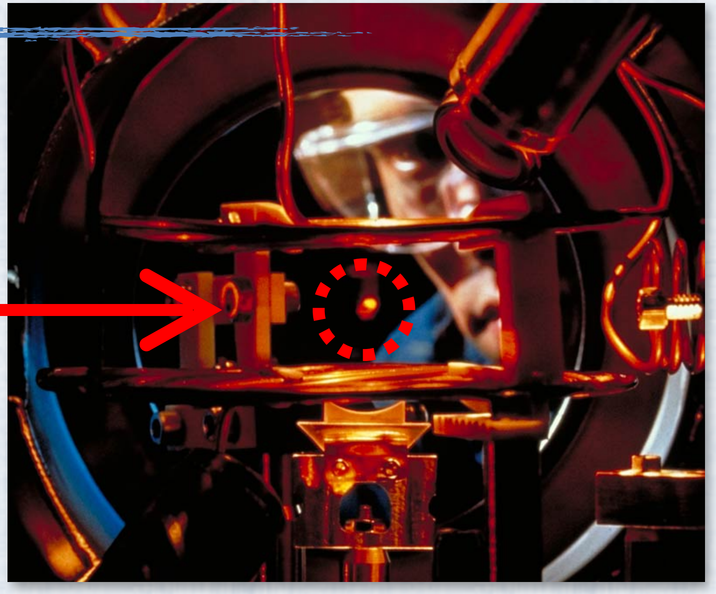
Atomic cluster :

trapped and cooled down

$T \sim 50$ nK $\quad (T/T_F \sim 0.05)$

$n \sim 5\times10^{12}$ cm^{-3}

▸ Cold & dilute

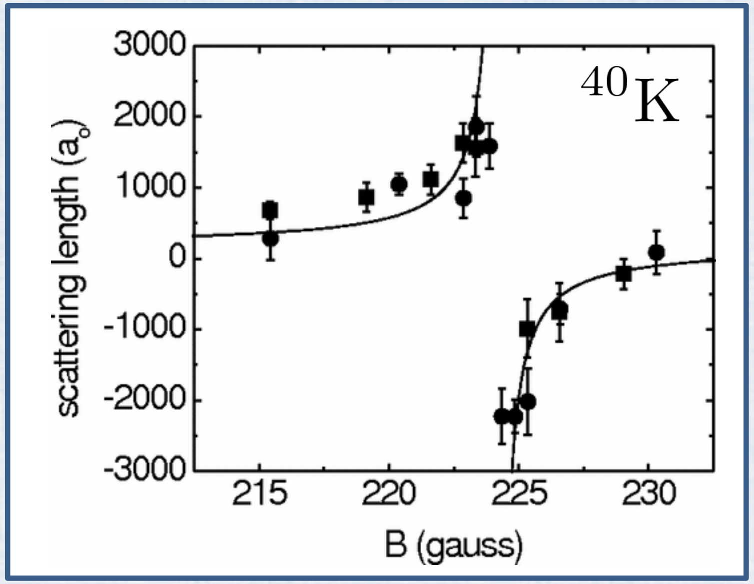


https://physics.aps.org/story/v21/st11

✓ **High tunability**

- Spatial dimensions
- Quantum statistics & internal d.o.f.
- s-wave scattering length

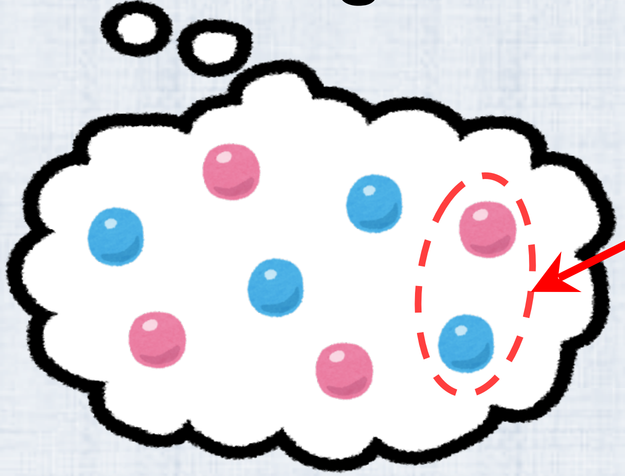
via Feshbach resonance



C.A. Regal & D.S. Jin PRL90 (2003)

Resonating fermions

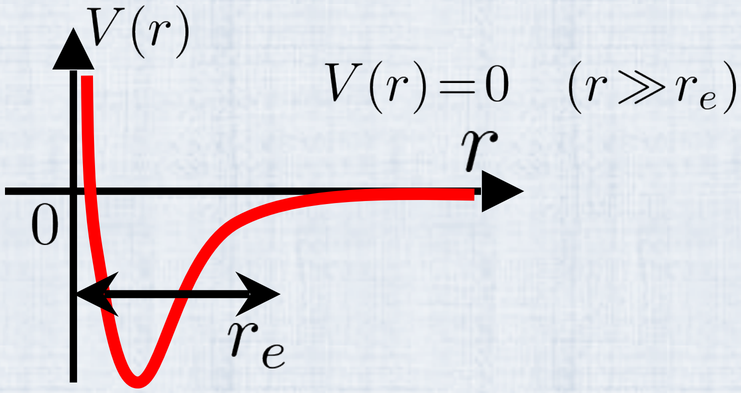
Interacting Fermionic system (spin-1/2)



with large scattering length

Interaction potential : $V(r)$

- Scattering length : a
- ~~Potential radius : r_e~~



▸ Resonance regime : $r_e \ll |a|,\ n^{-3},\ \lambda_T$

thermal de Broglie wavelength : $\lambda_T \sim T^{-1/2}$

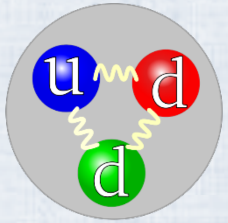
➡ **The interaction is characterized only by the scattering length.**

$r_e \to 0$

Universal !! : **Independent of the details of** $V(r)$

E.g.

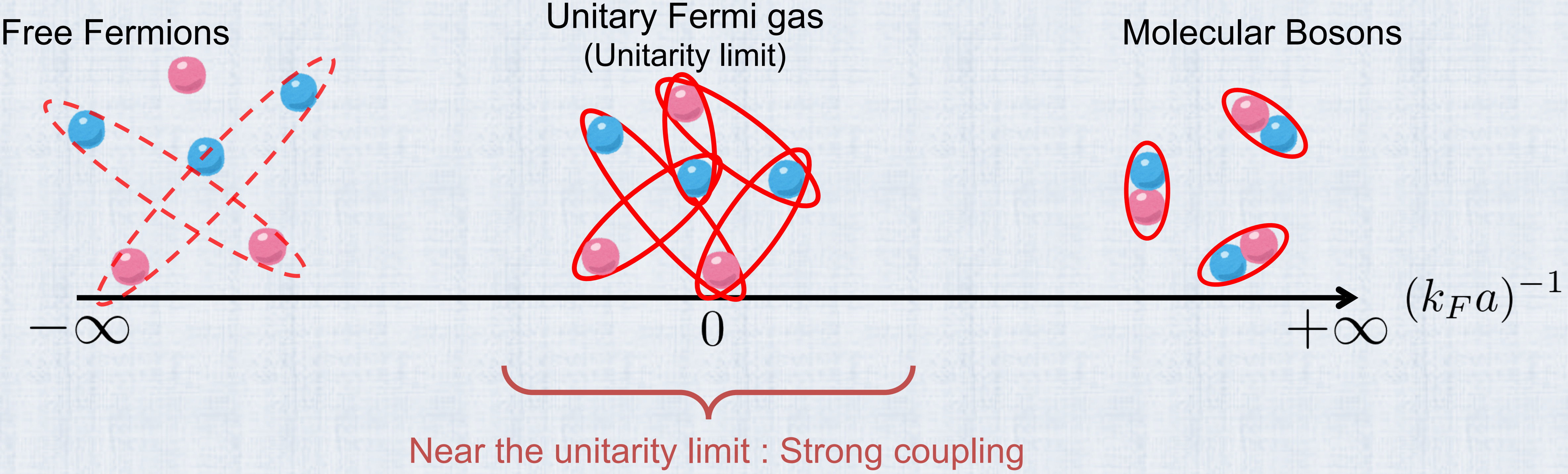
- Neutrons : $|a|/r_e \sim 18$



- Ultracold atoms : $|a|/r_e$ **tunable**

Properties of resonating fermions

- **Resonating fermions in 3D**

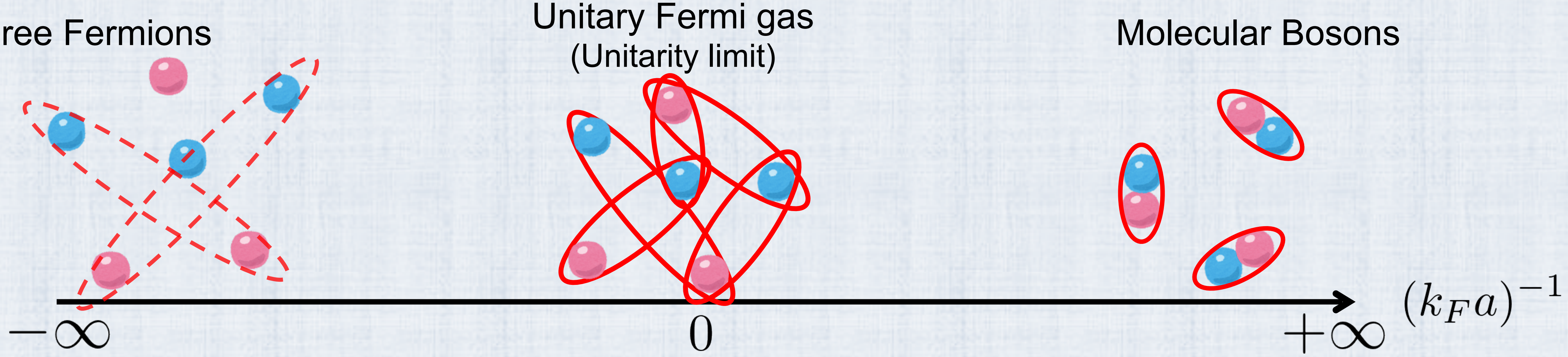


(The system shows superfluidity and BCS-BEC crossover below the critical temperature.)

- In this talk, we work only above the critical temperature, especially at high temperatures.

Properties of resonating fermions

- **Resonating fermions in 3D**



- Shear & bulk viscosity are anomalously small.

  - Shear viscosity : close to a lower limit conjectured by AdS/CFT

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

P. K. Kovtun, D. T. Son, & A. O. Starinets (2005)

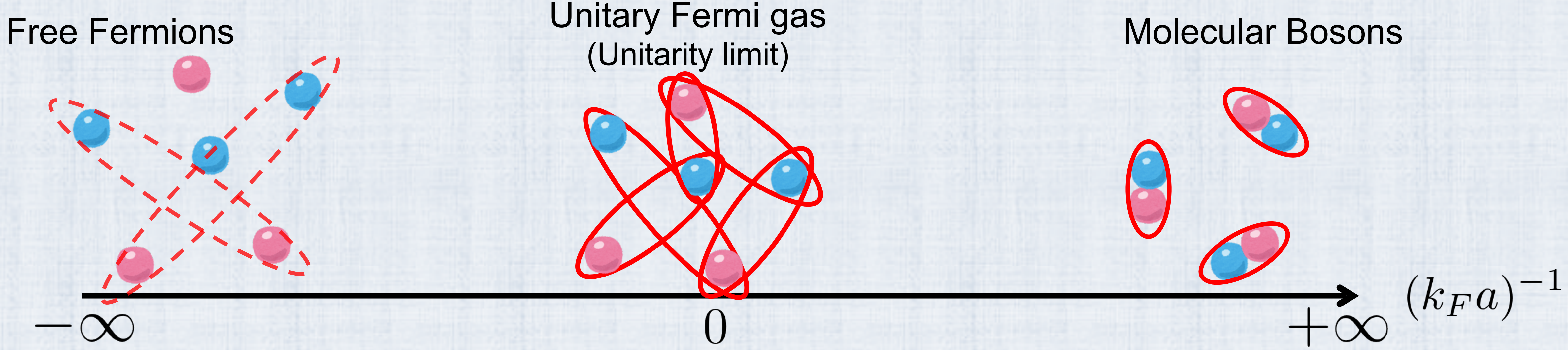
  - Bulk viscosity : vanishing at unitarity because of conformality

$$\zeta = 0 \quad \text{at} \quad a^{-1} = 0$$

D. T. Son (2007)

Properties of resonating fermions

- **Resonating fermions in 3D**



- Shear & bulk viscosity are anomalously small.

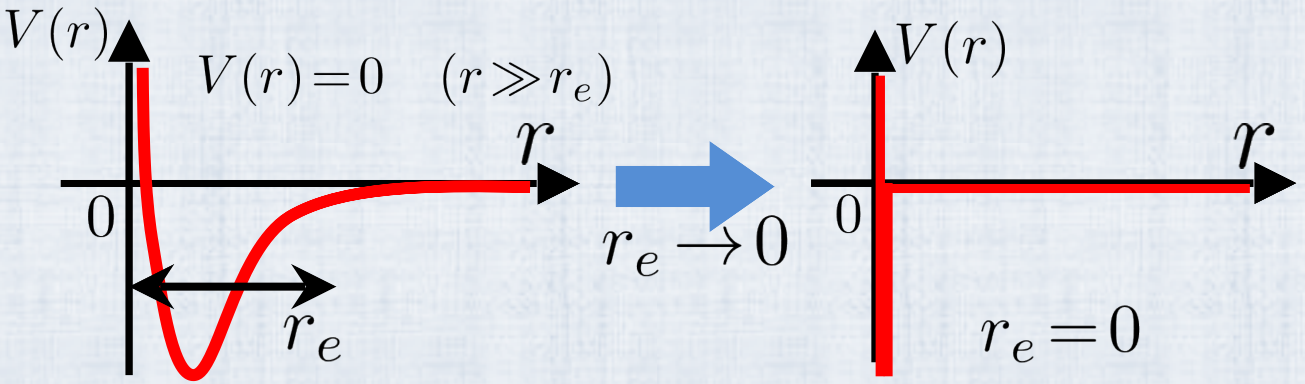
Our Research

Investigation of transport coefficients of the resonating fermions for an arbitrary scattering length

Model Hamiltonian

Interaction potential : $V(r)$

- Scattering length : a
- ~~Potential radius~~ : r_e sufficiently small



"Contact Interaction"

Fermions with contact interaction

$$\hat{H}=\sum_{\sigma=\uparrow,\downarrow}\int \mathrm{d}\boldsymbol{x}\,\hat{\psi}_\sigma^\dagger(\boldsymbol{x})\frac{-\nabla^2}{2m}\hat{\psi}_\sigma(\boldsymbol{x})+\frac{g}{2}\sum_{\sigma,\rho}\int \mathrm{d}\boldsymbol{x}\,\hat{\psi}_\sigma^\dagger(\boldsymbol{x})\hat{\psi}_\rho^\dagger(\boldsymbol{x})\hat{\psi}_\rho(\boldsymbol{x})\hat{\psi}_\sigma(\boldsymbol{x})$$

✓ Scattering amplitude

$$f(k)=\begin{cases}-\dfrac{1}{a^{-1}+ik} & \text{(3D)}\\[2ex] -\dfrac{2\pi}{\ln(ka)-i\frac{\pi}{2}} & \text{(2D)}\end{cases}$$

Cutoff regularization

$$g=\frac{\Omega_{d-1}}{m}\frac{d-2}{a^{2-d}-\frac{\Lambda^{d-2}}{\Gamma(\frac{d}{2})\Gamma(2-\frac{d}{2})}}$$

$$\Omega_{d-1}\equiv\frac{(4\pi)^{d/2}}{2\Gamma(2-d/2)}$$

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    - Resonating fermions
    - Quantum virial expansion

2. **Transport coefficients in the quantum virial expansion**
    - Review: previous results
    - Shear viscosity & Thermal conductivity
    - Bulk viscosity

3. **Summary**

Quantum Virial Expansion

Resonating fermions : Strongly correlated

➡ need a non-perturbative approach

✓ **Systematic expansion in terms of fugacity :** $z = e^{\beta\mu}$

At high temperatures with fixed density, $z \sim n\lambda_T^3 \sim nT^{-3/2}$

$$\left[n \sim \int \frac{d^3p}{(2\pi)^3} e^{-\beta(\frac{\boldsymbol{p}^2}{2m}-\mu)} = \frac{z}{\lambda_T^3} \right.$$

Thermal de Broglie wavelength : $\lambda_T = \sqrt{2\pi\beta/m}$

$z \ll 1$ ⟺ **High-temperature & Low-density regime**

▸ Suitable for dilute gases

How to expand w.r.t fugacity

✓ **Systematic expansion in terms of fugacity :** $z = e^{\beta\mu}$

e.g. thermal expectation value

c-number

$$\mathrm{Tr}[e^{-\beta(\hat{H}-\mu\hat{N})}\hat{\mathcal{O}}] = \sum_{N=0}^{+\infty} \underline{\mathrm{Tr}_N[e^{-\beta(\hat{H}-\mu N)}\hat{\mathcal{O}}]}$$

restricted to N-body sector

$$= \sum_{N=0}^{+\infty} e^{\beta\mu N}\mathrm{Tr}_N[e^{-\beta\hat{H}}\hat{\mathcal{O}}]$$

$$= \underline{\mathrm{Tr}_0[e^{-\beta\hat{H}}\hat{\mathcal{O}}]} + z\underline{\mathrm{Tr}_1[e^{-\beta\hat{H}}\hat{\mathcal{O}}]} + z^2\underline{\mathrm{Tr}_2[e^{-\beta\hat{H}}\hat{\mathcal{O}}]} + \cdots$$

Zero-body sector — One-body sector — Two-body sector

- Lower order terms can be computed as **few-body problems**

Well studied **thermodynamic quantities** theoretically and experimentally in ultracold atoms

Xia-Ji Liu, Phys. Rep. **524**, 37-83 (2013)

Field theoretic method 1

Propagator & Vertex in Matsubara frequency representation

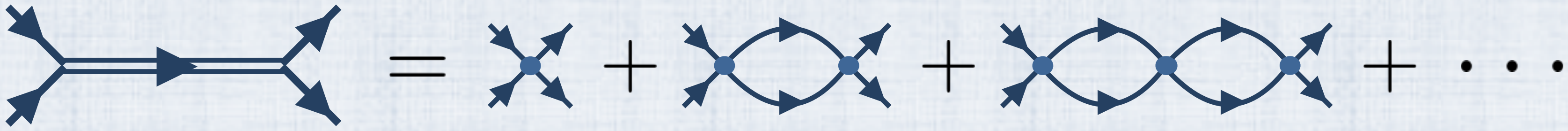
- Free fermion propagator

$$= G(i\omega^F, \boldsymbol{p}) = \frac{1}{i\omega^F - \frac{\boldsymbol{p}^2}{2m} + \mu}$$

Matsubara frequency

$$\omega^F = 2\pi(m + 1/2)/\beta$$
$$\omega^B = 2\pi m/\beta$$

- Pair propagator in the vacuum



$$= D(i\omega^B, \boldsymbol{p}) = \frac{\Omega_{d-1}}{m} \frac{d-2}{a^{d-2} - [-m(i\omega^B - \frac{\boldsymbol{p}^2}{4m} + 2\mu)]^{d/2-1}}$$

On-shell pair propagator in center-of-mass frame

$$D(\frac{\boldsymbol{k}^2}{m} - 2\mu + i0^+, \boldsymbol{0}) \propto f(k)$$

✓ Scattering amplitude

$$f(k) = \begin{cases} -\dfrac{1}{a^{-1} + ik} & (3\mathrm{D}) \\ -\dfrac{2\pi}{\ln(ka) - i\frac{\pi}{2}} & (2\mathrm{D}) \end{cases}$$

Field theoretic method 1

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- Free fermion propagator

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Matsubara frequency

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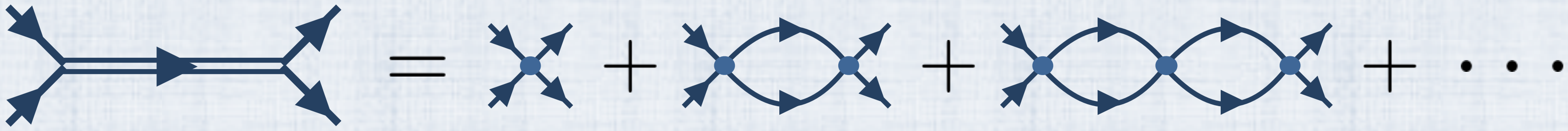


$$= D(i\omega^B, \boldsymbol{p}) = \frac{\Omega_{d-1}}{m} \frac{d-2}{a^{d-2} - [-m(i\omega^B - \frac{\boldsymbol{p}^2}{4m} + 2\mu)]^{d/2-1}}$$

- **Good point : Clear correspondence with the real frequency** $i\omega^{B/F} \to \omega + i0^+$
- **Bad point : No explicit dependence on fugacity**

The fugacity emerges from the Matsubara frequency summation

e.g. $$\frac{1}{\beta}\sum_m h(i\omega_m^F) = \sum_{w_i} f_F(w_i)\mathrm{Res}_{w=w_i} h(w_i)$$ with $$f_F(\varepsilon - \mu) = \frac{1}{e^{\beta(\varepsilon-\mu)} + 1} = ze^{-\beta\varepsilon} + O(z^2)$$

Field theoretic method 2

Propagator & Vertex in imaginary time representation

X. Leyronas, PRA **84**, 053633 (2011).

- Free fermion propagator

$$= G(\tau,\boldsymbol{p}) = -e^{-\tau(\frac{\boldsymbol{p}^2}{2m}-\mu)}[\theta(\tau) - f_F(\frac{\boldsymbol{p}^2}{2m}-\mu)]$$

Imaginary time: $-\beta < \tau < \beta$

- Good point : Explicit dependence on fugacity $f_F(\varepsilon-\mu) = \dfrac{1}{e^{\beta(\varepsilon-\mu)}+1} = ze^{-\beta\varepsilon} + O(z^2)$

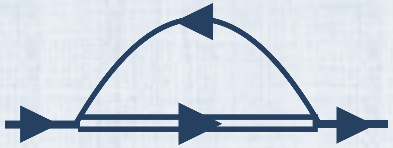
- Bad point : No clear correspondence with the real frequency

Zeroth-order part of $G(\tau,\boldsymbol{p}) \propto \theta(\tau)$: Runs only forward in imaginary time

Order of the fugacity for a given diagram

$\geq$ A number of propagators running backwards in imaginary time

Example: Fermion Self-energy



$$= \Sigma(i\omega^F,\boldsymbol{p}) = z\int_{\boldsymbol{q}} e^{-\beta\frac{\boldsymbol{q}^2}{2m}} D(i\omega^F + \frac{\boldsymbol{q}^2}{2m} + \mu, \boldsymbol{p}+\boldsymbol{q}) + O(z^2)$$

$$\int_{\boldsymbol{p}} \equiv \int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^3}$$

$$\mathrm{Re}\Sigma = \mathrm{Im}\Sigma = O(z)$$

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   - Shear viscosity & Thermal conductivity
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Microscopic theory & kinetic theory

Resonant Fermi gas in 3D $\tilde{a} = \sqrt{2\pi}a/\lambda_T$	**Microscopic theory in quantum virial expansion**	**Kinetic theory**
Shear viscosity	$\eta \overset{!}{=} \frac{1}{\lambda_T^3}\frac{15\pi}{4\sqrt{2}}\left[\int_0^\infty \mathrm{d}x \frac{e^{-x}x^3}{x+\tilde{a}^{-2}}\right]^{-1}$ T. Enss, *et al.* (2011); Y. Nishida (2019); J. Hofmann (2020).	**The same result in relaxation-time approx.** P. Massignan, *et al.* (2005); G. M. Bruun & H. Smith (2005)
	Approximate resummation	
Thermal conductivity at unitarity	$T\kappa \overset{!}{=} \frac{T}{m\lambda_T^3}\frac{225\pi}{32\sqrt{2}}$ B. Frank, *et al.* (2020)	**The same result in relaxation-time approx.** M. Braby, *et al.* (2010)
Bulk viscosity near unitarity	$\zeta = \frac{z^2}{\lambda_T^3}\frac{2\sqrt{2}}{9\pi}\frac{\ln(\tilde{a}^2 e^{-1-\gamma})}{\tilde{a}^2}$ Y. Nishida (2019); T. Enss (2019); J. Hofmann (2020).	$\zeta = \frac{z^2}{\lambda_T^3}\frac{1}{12\sqrt{2}}\frac{1}{\tilde{a}^2}$ K. Dusling & T. Schäfer (2013)

- **Shear viscosity & Thermal conductivity :** **agreement** between microscopic and kinetic results under approx.
- **Bulk viscosity : disagreement** between microscopic and kinetic results

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   - Bulk viscosity KF & Y. Nishida, PRA (2020).

3.

- ▸ **Shear viscosity & Thermal conductivity :**
  agreement between microscopic and kinetic results under approx.
- ▸ **Bulk viscosity : disagreement** between microscopic and kinetic results

For shear viscosity & thermal conductivity

- **Shear viscosity & Thermal conductivity :**
  agreement between microscopic and kinetic results under approx.
- **Bulk viscosity : disagreement** between microscopic and kinetic results

The **approximated** results at leading order in the quantum virial expansion
= the results from the kinetic theory in **the relaxation-time approximation**

Q. Do they agree more exactly ?

The **exact** results at leading order in the quantum virial expansion
?↗ = the **exact** results from the kinetic theory

For shear viscosity & thermal conductivity

- **Shear viscosity & Thermal conductivity :**
  agreement between microscopic and kinetic results under approx.
- **Bulk viscosity : disagreement** between microscopic and kinetic results

The **approximated** results at leading order in the quantum virial expansion
= the results from the kinetic theory in **the relaxation-time approximation**

Q. Do they agree more exactly ? A. Yes !! KF & Y. Nishida, PRA **103**, 053320 (2021).

The **exact** results at leading order in the quantum virial expansion
= the **exact** results from the kinetic theory

Yes ↗

We give an exact microscopic computation by taking into account a singularity.

Our Result

Self-consistent equation for resummation = **Linearized Boltzmann equation**

⇒ "Microscopic results = kinetic results" without any approximation

Pinch singularity

Product of single-particle Green functions — Eliashberg (1962); Jeon (1995); Jeon & Yaffe (1996)

$$\mathcal{G}^R(\varepsilon,\boldsymbol{p})\mathcal{G}^A(\varepsilon,\boldsymbol{p}) = \frac{\mathrm{Im}[\mathcal{G}^R(\varepsilon,\boldsymbol{p})]}{\mathrm{Im}[\Sigma(\varepsilon+i0^+,\boldsymbol{p})]} \sim O(z^{-1})$$

Requiring resummation

$$\mathcal{G}^R(\varepsilon,\boldsymbol{p}) = \frac{1}{\varepsilon-\frac{\boldsymbol{p}^2}{2m}+\mu-\Sigma(\varepsilon+i0^+,\boldsymbol{p})}$$

$$\mathrm{Im}\Sigma(\omega,\boldsymbol{k}) \sim O(z^1)$$

- Appear in the static limit $\omega \to 0$ of response function

Kubo formula

$$\sigma_{\mathcal{O}} = \lim_{\omega\to 0}\frac{\mathrm{Im}[\chi_{\mathcal{O}}(\omega+i0^+)]}{\omega}$$

with

$$\chi_{\mathcal{O}}(i\omega^B) = \int_0^\beta d\tau \int d^d\boldsymbol{x}\, e^{i\omega^B\tau}\langle \mathcal{T}_\tau \hat{\mathcal{O}}(\tau,\boldsymbol{x})\hat{\mathcal{O}}(0,\boldsymbol{0})\rangle$$

$$\hat{\mathcal{O}}(\boldsymbol{p}=\boldsymbol{0}) = \sum_{\sigma=\uparrow,\downarrow}\sum_{\boldsymbol{k}} \hat{\psi}^\dagger_\sigma(\boldsymbol{k})\mathcal{Q}(\boldsymbol{k})\hat{\psi}_\sigma(\boldsymbol{k})$$

: One-body operator

$$\eta = \lim_{\omega\to 0}\frac{\mathrm{Im}[\chi_{\Pi_{xy}}(\omega+i0^+)]}{\omega}$$

$$T\kappa = \lim_{\omega\to 0}\frac{\mathrm{Im}[\chi_{\mathcal{J}^q_x}(\omega+i0^+)]}{\omega}$$

$$\hat{\Pi}_{xy}(\boldsymbol{p}=\boldsymbol{0}) = \sum_{\sigma=\uparrow,\downarrow}\sum_{\boldsymbol{k}} \hat{\psi}^\dagger_\sigma(\boldsymbol{k})\frac{k_x k_y}{m}\hat{\psi}_\sigma(\boldsymbol{k})$$

$$\hat{\mathcal{J}}^q_x(\boldsymbol{p}=\boldsymbol{0}) = \sum_{\sigma=\uparrow,\downarrow}\sum_{\boldsymbol{k}} \hat{\psi}^\dagger_\sigma(\boldsymbol{k})\frac{k_x}{m}\left(\frac{\boldsymbol{k}^2}{2m}-\frac{\mathcal{E}+p}{n}\right)\hat{\psi}_\sigma(\boldsymbol{k}) + \text{Two-body operator}$$

Negligible
at high temperatures

Pinch singularity

Product of single-particle Green functions Eliashberg (1962); Jeon (1995); Jeon & Yaffe (1996)

$$\mathcal{G}^R(\varepsilon,\boldsymbol{p})\mathcal{G}^A(\varepsilon,\boldsymbol{p}) = \frac{\mathrm{Im}[\mathcal{G}^R(\varepsilon,\boldsymbol{p})]}{\mathrm{Im}[\Sigma(\varepsilon+i0^+,\boldsymbol{p})]} \sim O(z^{-1})$$

$$\mathcal{G}^R(\varepsilon,\boldsymbol{p}) = \frac{1}{\varepsilon-\frac{\boldsymbol{p}^2}{2m}+\mu-\Sigma(\varepsilon+i0^+,\boldsymbol{p})}$$

Requiring resummation $\mathrm{Im}\Sigma(\omega,\boldsymbol{k}) \sim O(z^1)$

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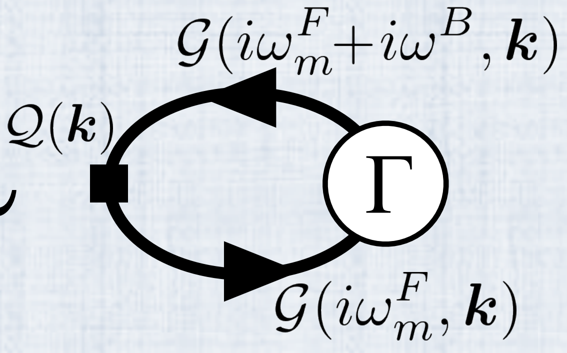
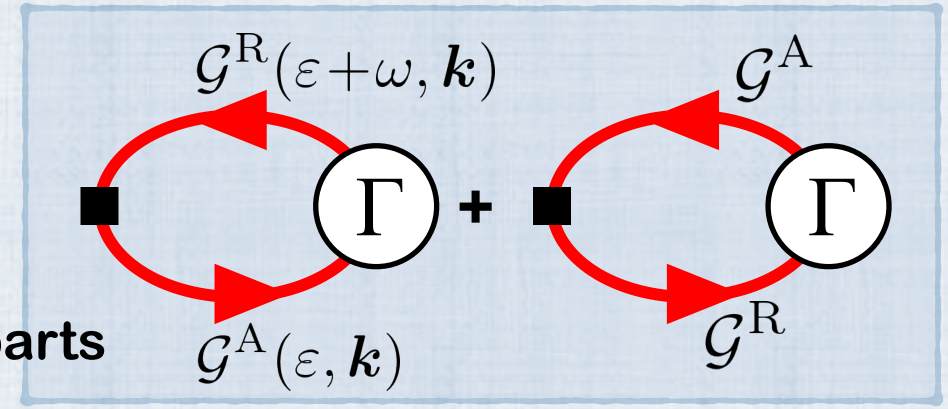
$$\hat{\mathcal{O}}(\boldsymbol{p}=\boldsymbol{0}) = \sum_{\sigma=\uparrow,\downarrow}\sum_{\boldsymbol{k}} \hat{\psi}^\dagger_\sigma(\boldsymbol{k})\mathcal{Q}(\boldsymbol{k})\hat{\psi}_\sigma(\boldsymbol{k})$$ **: One-body operator**

Diagrammatic representation

$\chi_{\mathcal{O}}(i\omega^B) \sim$



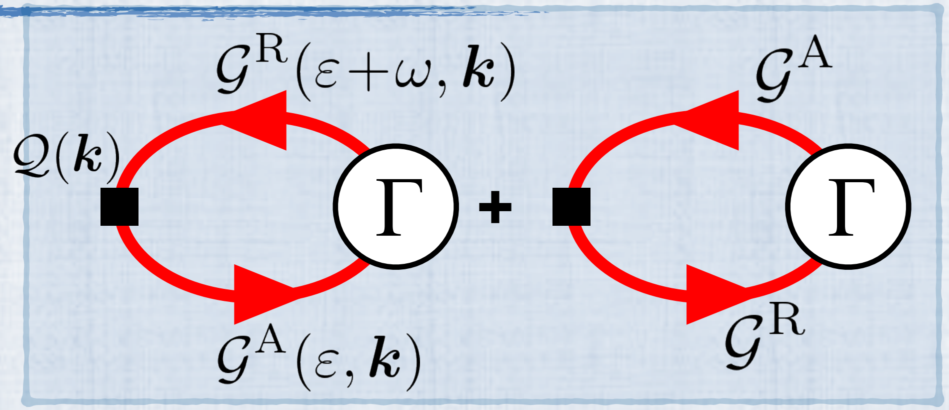
keep only **pinch-singular** **parts**



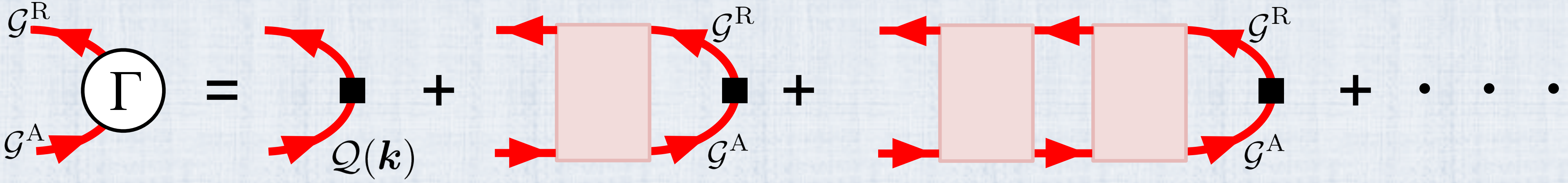
Self-consistent eq. for vertex function

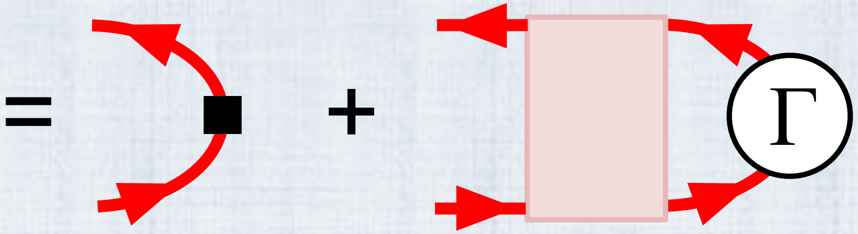
Kubo formula

$$\sigma_{\mathcal{O}} = \lim_{\omega \to 0} \frac{\mathrm{Im}[\chi_{\mathcal{O}}(\omega + i0^+)]}{\omega} \quad \text{with} \quad \chi_{\mathcal{O}}(i\omega^B) \sim$$

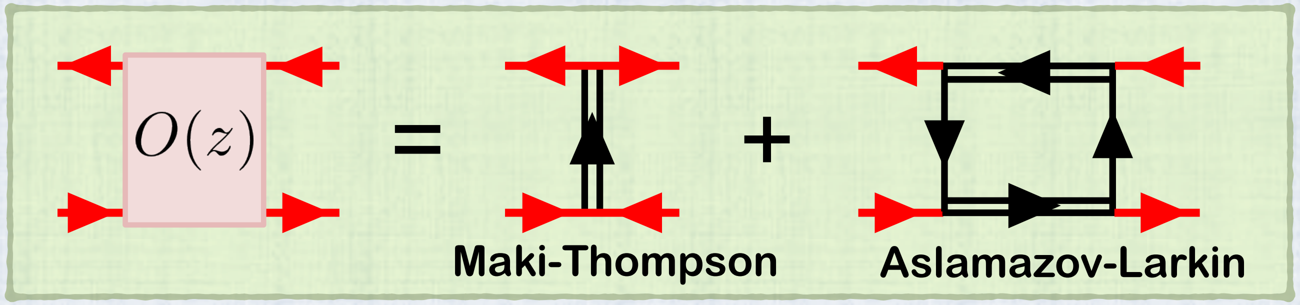


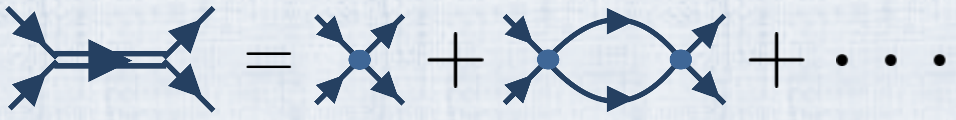
Only need Γ connected to $\mathcal{G}^R$ & $\mathcal{G}^A$





Self-consistent equation

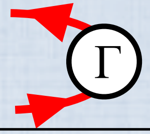




Linearized Boltzmann equation

Self-consistent equation

$$\varphi(\boldsymbol{k}) \equiv \left.\frac{z \times [\Gamma]}{-2\mathrm{Im}\Sigma(\varepsilon+i0^+, \boldsymbol{k})}\right|_{\varepsilon=\frac{\boldsymbol{k}^2}{2m}-\mu}$$



Jeon (1995); Jeon & Yaffe (1996); Hidaka & Kunihiro (2011).

✓ Linearized Boltzmann equation

$$\mathcal{Q}(\boldsymbol{k}) = \int_{\boldsymbol{k}_2,\boldsymbol{p}_1,\boldsymbol{p}_2} e^{-\beta\frac{\boldsymbol{k}_2^2}{2m}} \mathcal{W}(\boldsymbol{p}_1,\boldsymbol{p}_2|\boldsymbol{k},\boldsymbol{k}_2)\Big[\varphi(\boldsymbol{k})+\varphi(\boldsymbol{k}_2)-\varphi(\boldsymbol{p}_1)-\varphi(\boldsymbol{p}_2)\Big] \qquad \int_{\boldsymbol{p}} \equiv \int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^3}$$

Transition rate :

$$\mathcal{W}(\boldsymbol{p}_1,\boldsymbol{p}_2|\boldsymbol{k}_1,\boldsymbol{k}_2) = \left|D\left(\frac{\boldsymbol{p}_1^2}{2m}+\frac{\boldsymbol{p}_2^2}{2m}-2\mu+i0^+, \boldsymbol{p}_1+\boldsymbol{p}_2\right)\right|^2 (2\pi)^{d+1}\delta\left(\frac{\boldsymbol{p}_1^2}{2m}+\frac{\boldsymbol{p}_2^2}{2m}-\frac{\boldsymbol{k}_1^2}{2m}-\frac{\boldsymbol{k}_2^2}{2m}\right)\delta^d(\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{k}_1-\boldsymbol{k}_2)$$

Boltzmann equation at high temperatures

✓ Linearized Boltzmann equation

$$\mathcal{Q}(\boldsymbol{k}) = \int_{\boldsymbol{k}_2,\boldsymbol{p}_1,\boldsymbol{p}_2} e^{-\beta\frac{\boldsymbol{k}_2^2}{2m}} \mathcal{W}(\boldsymbol{p}_1,\boldsymbol{p}_2|\boldsymbol{k},\boldsymbol{k}_2)\Big[\varphi(\boldsymbol{k})+\varphi(\boldsymbol{k}_2)-\varphi(\boldsymbol{p}_1)-\varphi(\boldsymbol{p}_2)\Big]$$

Transition rate :

$$\mathcal{W}(\boldsymbol{p}_1,\boldsymbol{p}_2|\boldsymbol{k}_1,\boldsymbol{k}_2) = \underline{\left|D\left(\frac{\boldsymbol{p}_1^2}{2m}+\frac{\boldsymbol{p}_2^2}{2m}-2\mu+i0^+,\boldsymbol{p}_1+\boldsymbol{p}_2\right)\right|^2}(2\pi)^{d+1}\delta\left(\frac{\boldsymbol{p}_1^2}{2m}+\frac{\boldsymbol{p}_2^2}{2m}-\frac{\boldsymbol{k}_1^2}{2m}-\frac{\boldsymbol{k}_2^2}{2m}\right)\delta^d(\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{k}_1-\boldsymbol{k}_2)$$

Transport coefficient : $\sigma_{\mathcal{O}} = 2\beta\int_{\boldsymbol{k}} e^{-\beta\frac{\boldsymbol{k}^2}{2m}}\mathcal{Q}(\boldsymbol{k})\varphi(\boldsymbol{k})$

In 3D $|D|^2 = \left(\frac{4\pi}{m}\right)^2 \frac{1}{a^{-2}+\frac{(\boldsymbol{p}_1-\boldsymbol{p}_2)^2}{4}}$

Boltzmann equation

For shear viscosity : P. Massignan, G. M. Bruun, & H. Smith, (2005); G. M. Bruun & H. Smith (2005)
For thermal conductivity : M. Braby, J. Chao & T. Schäfer (2010)

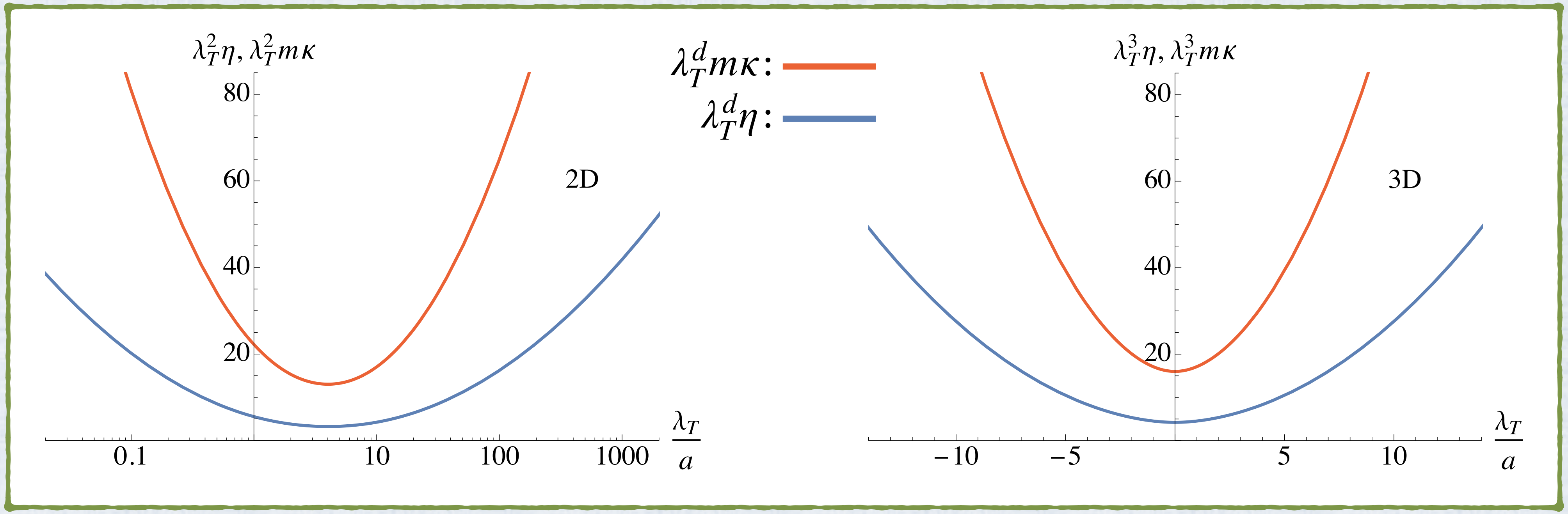
$$\left[\frac{\partial}{\partial t}+\frac{\boldsymbol{k}}{m}\cdot\frac{\partial}{\partial \boldsymbol{x}}\right]f(\boldsymbol{k}) = \left(\frac{\partial f_{\boldsymbol{k}}}{\partial t}\right)_{\text{coll.}}$$

$f(\boldsymbol{k}) = f(t,\boldsymbol{x},\boldsymbol{k})$: distribution function

Collision term : $\left(\frac{\partial f_{\boldsymbol{k}}}{\partial t}\right)_{\text{coll.}} = \int_{\boldsymbol{k}_2,\boldsymbol{p}_1,\boldsymbol{p}_2} \mathcal{W}(\boldsymbol{p}_1,\boldsymbol{p}_2|\boldsymbol{k},\boldsymbol{k}_2)\Big[f(\boldsymbol{p}_1)f(\boldsymbol{p}_2)-f(\boldsymbol{k})f(\boldsymbol{k}_2)\Big]$

Result: Shear viscosity & Thermal conductivity

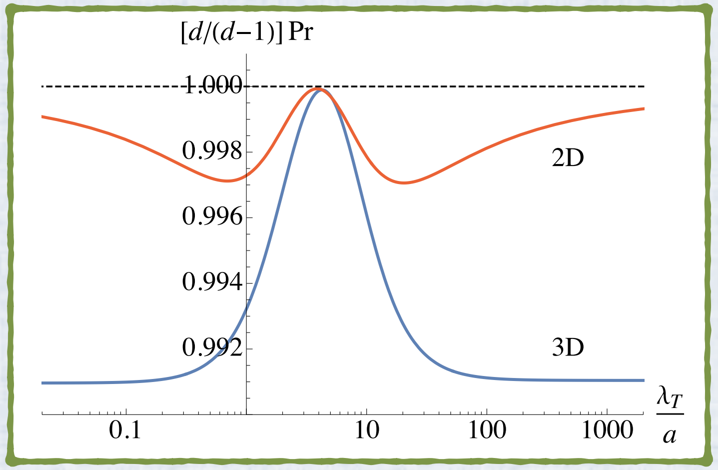
In the high-temperature limit for an arbitrary scattering length



- Prandtl number at high temperatures

$$\mathrm{Pr} \equiv \frac{\eta}{m\kappa} c_{\mathcal{P}} \simeq \frac{d-1}{d} \qquad c_{\mathcal{P}} = \frac{d+2}{2} + O(z)$$

Equal within the relaxation time approximation.
Slightly dependent on the scattering length



Plan of this talk

1. Introduction
   - Resonating fermions
   - Quantum virial expansion

2. Transport coefficients in the quantum virial expansion
   - Review: previous results
   - Shear viscosity & Thermal conductivity KF & Y. Nishida, PRA (2021).
   - Bulk viscosity KF & Y. Nishida, PRA (2020).

3.

▸ **Shear viscosity & Thermal conductivity :**
agreement between microscopic and kinetic results under approx.

▸ **Bulk viscosity : disagreement** between microscopic and kinetic results

For bulk viscosity

- Shear viscosity & Thermal conductivity :
  agreement between microscopic and kinetic results under approx.
- **Bulk viscosity : disagreement** between microscopic and kinetic results

✓ Two-body operator is essential

$$\zeta = \lim_{\omega\to 0} \frac{\mathrm{Im}[\chi_{\tilde{\Pi}}(\omega+i0^+)]}{\omega}$$

$$\hat{\tilde{\Pi}} = \frac{1}{d}\sum_i \hat{\Pi}_{ii} - \left(\frac{\partial \mathcal{P}}{\partial n}\right)_{\mathcal{E}} \hat{\mathcal{N}} - \left(\frac{\partial \mathcal{P}}{\partial \mathcal{E}}\right)_n \hat{\mathcal{H}} \sim \frac{\hat{\mathcal{C}}}{d\Omega_{d-1} m a^{d-2}}$$

Contact density : $\hat{\mathcal{C}}(x) = \frac{(mg)^2}{2}\sum_{\sigma,\rho} \hat{\psi}^\dagger_\sigma(x)\hat{\psi}^\dagger_\rho(x)\hat{\psi}_\rho(x)\hat{\psi}_\sigma(x)$

- ζ cannot be calculated in the Boltzmann eq.

$\zeta = 0$ in the Boltzmann eq. : $\left[\frac{\partial}{\partial t} + \frac{\boldsymbol{k}}{m}\cdot\frac{\partial}{\partial \boldsymbol{x}}\right] f(\boldsymbol{k}) = \left(\frac{\partial f_{\boldsymbol{k}}}{\partial t}\right)_{\mathrm{coll.}}$

Need to beyond the Boltzmann equation to calculate ζ

Landau kinetic theory

- Shear viscosity & Thermal conductivity :
  agreement between microscopic and kinetic results under approx.
- **Bulk viscosity : disagreement** between microscopic and kinetic results

Landau kinetic equation for quasi-particles

$$\left[\frac{\partial}{\partial t} + \frac{\partial E}{\partial \boldsymbol{k}} \cdot \frac{\partial}{\partial \boldsymbol{x}} - \frac{\partial E}{\partial \boldsymbol{x}} \cdot \frac{\partial}{\partial \boldsymbol{k}}\right] f_{\boldsymbol{k}} = \left(\frac{\partial f_{\boldsymbol{k}}}{\partial t}\right)_{\rm coll} \quad \Rightarrow \quad \zeta = \frac{z^2}{\lambda_T^3} \frac{1}{12\sqrt{2}} \frac{1}{\tilde{a}^2}$$

K. Dusling & T. Schäfer (2013)

- Excitation energy of quasi-particles : $E = E[t, \boldsymbol{x}, \boldsymbol{k}; f] = \frac{\boldsymbol{k}^2}{2m} + \mathrm{Re}\Sigma[t, \boldsymbol{x}, \boldsymbol{k}; f]$

on-shell self-energy correction

At high temperatures, $\mathrm{Re}\Sigma \propto z$ & $\mathrm{Im}\Sigma \propto z$

Non-negligible

cf. weak coupling regime
$\mathrm{Re}\Sigma \propto g$ & $\mathrm{Im}\Sigma \propto g^2$

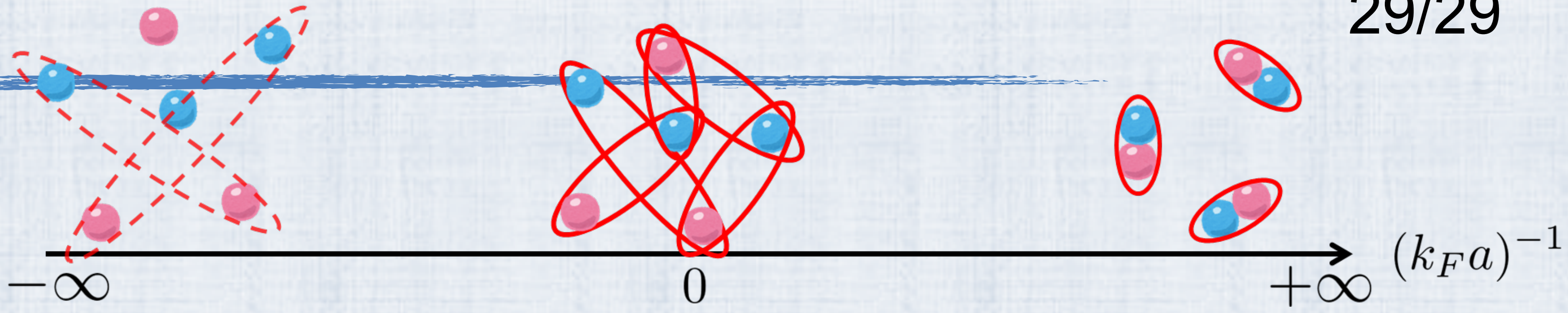
✓ **Quasiparticle approx. is broken in $O(z)$:** $A(\omega, \boldsymbol{p}) = 2\pi\delta(\omega - \frac{\boldsymbol{p}^2}{2m} + \mu) + O(z)$

non-peak function

KF & Y. Nishida, PRA **102**, 023310 (2020).

Summary

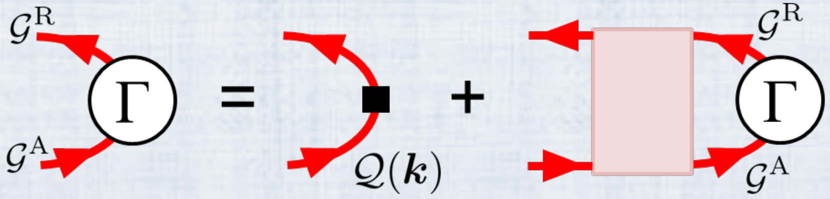
Resonating fermions



- Quantum virial expansion : expansion w.r.t $z = e^{\beta\mu}$

  valid for high-temperature & low-density regime $z \sim n\lambda_T^3$

- For shear viscosity & thermal conductivity KF & Y. Nishida, PRA(2021).
  - **Pinch singularity** $\mathcal{G}^{\mathrm{R}}(\omega,\boldsymbol{k})\mathcal{G}^{\mathrm{A}}(\omega,\boldsymbol{k}) \sim O(z^{-1})$
  - **Self-consistent eq. = Linearized Boltzmann eq.**



- For bulk viscosity
  - Kinetic theory is not capable of computing the bulk viscosity up to $O(z^2)$

    KF & Y. Nishida, PRA(2020).
  - **Pinch singularity** in the bulk viscosity calculation

    KF & T. Enss, in preparation.

- Future work : Quantum virial expansion in Keldysh formalism