Nuclear Science Computing Center at CCNU

QCD phase structure in strong magnetic fields

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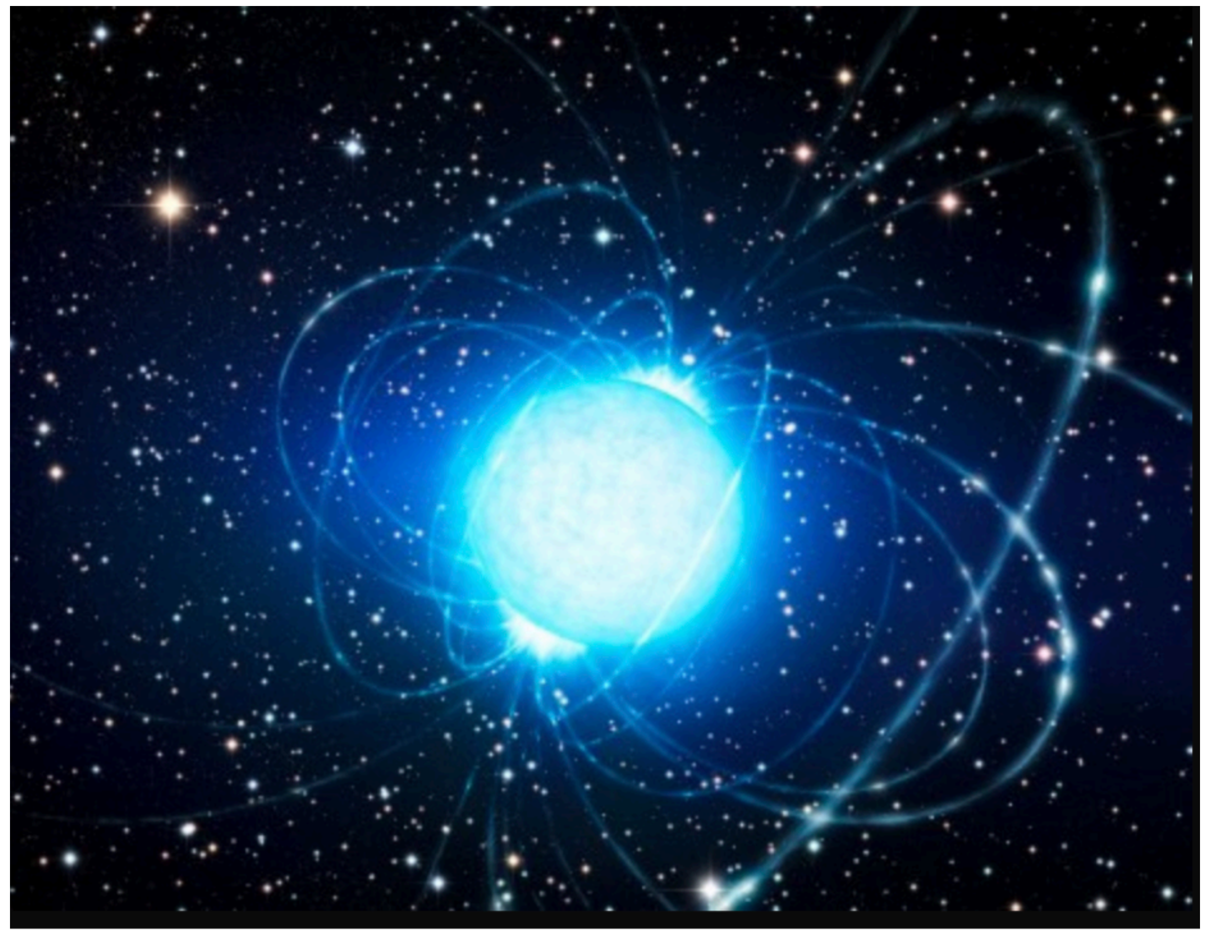
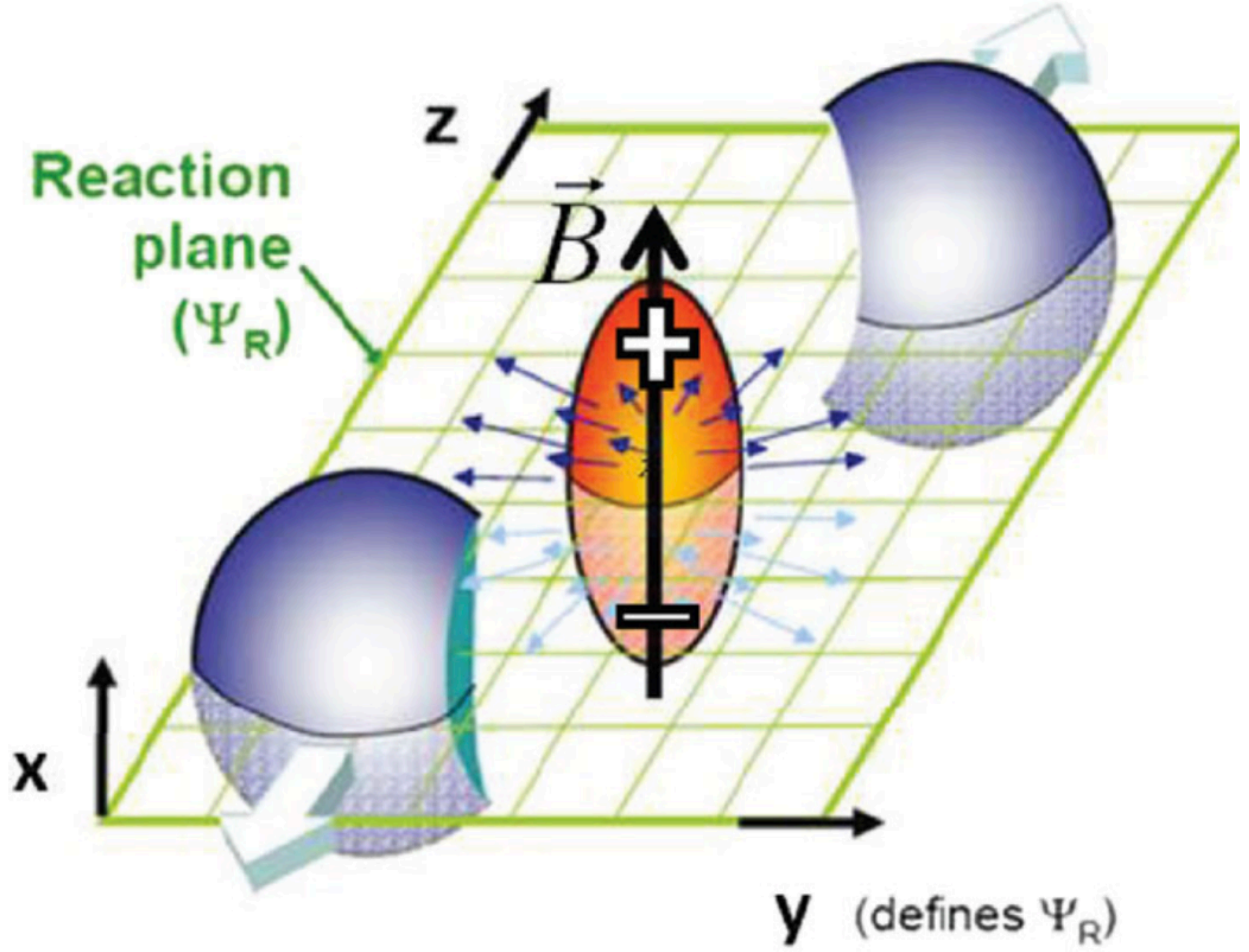
Central China Normal University (華中師範大學)

HTD, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, arXiv:2008.00493
HTD, S.-T. Li, Q. Shi and X.-D. Wang, arXiv:2104.06843

online QCD theory seminars

July 20, 2021

Strong magnetic fields

Earth	A common, hand-held magnet	Magnetar	Heavy-Ion collision
			
0.6 Gauss	100 Gauss	10^{15} Gauss	$10^{17\text{-}18}$ Gauss

$$\Lambda_{QCD}^2 \sim 10^4 \text{ MeV}^2 \sim 10^{17} \text{ Gauss}$$

$$1 \text{ Gauss} = 1.95 \times 10^{-14} \text{ MeV}^2$$

Chiral properties of (2+1)-flavor QCD in strong magnetic fields at zero temperature

HTD, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, arXiv:2008.00493

- Gell-Mann-Oakes-Renner relation
- qB scaling
- Masses of pseudo-scalar mesons

ChPT: T=0: $eB \uparrow \ \langle\bar{\psi}\psi\rangle \ \uparrow \Rightarrow \mathrm{T_{pc}} \uparrow$

ELSEVIER

12 June 1997

PHYSICS LETTERS B

Physics Letters B 402 (1997) 351–358

Quark condensate in a magnetic field

I.A. Shushpanov [a], A.V. Smilga [a,b]

[a] *Institute for Theoretical and Experimental Physics, B. Cheremushkinskaya 25, Moscow 117259, Russia*

[b] *TPI, 116 Church St. S.E., University of Minnesota, MN 55455, USA*

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Editor: M. Dine

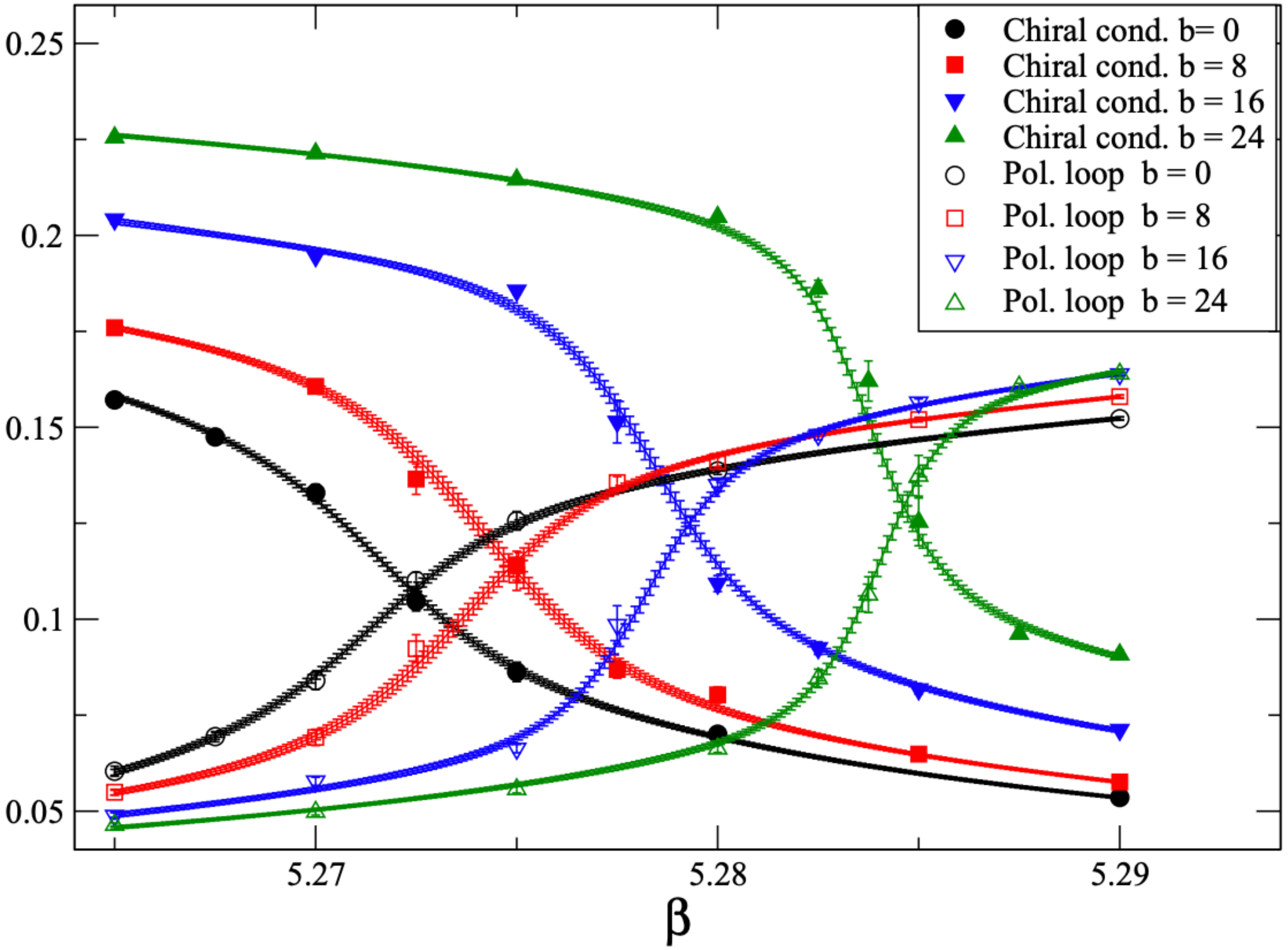
Abstract

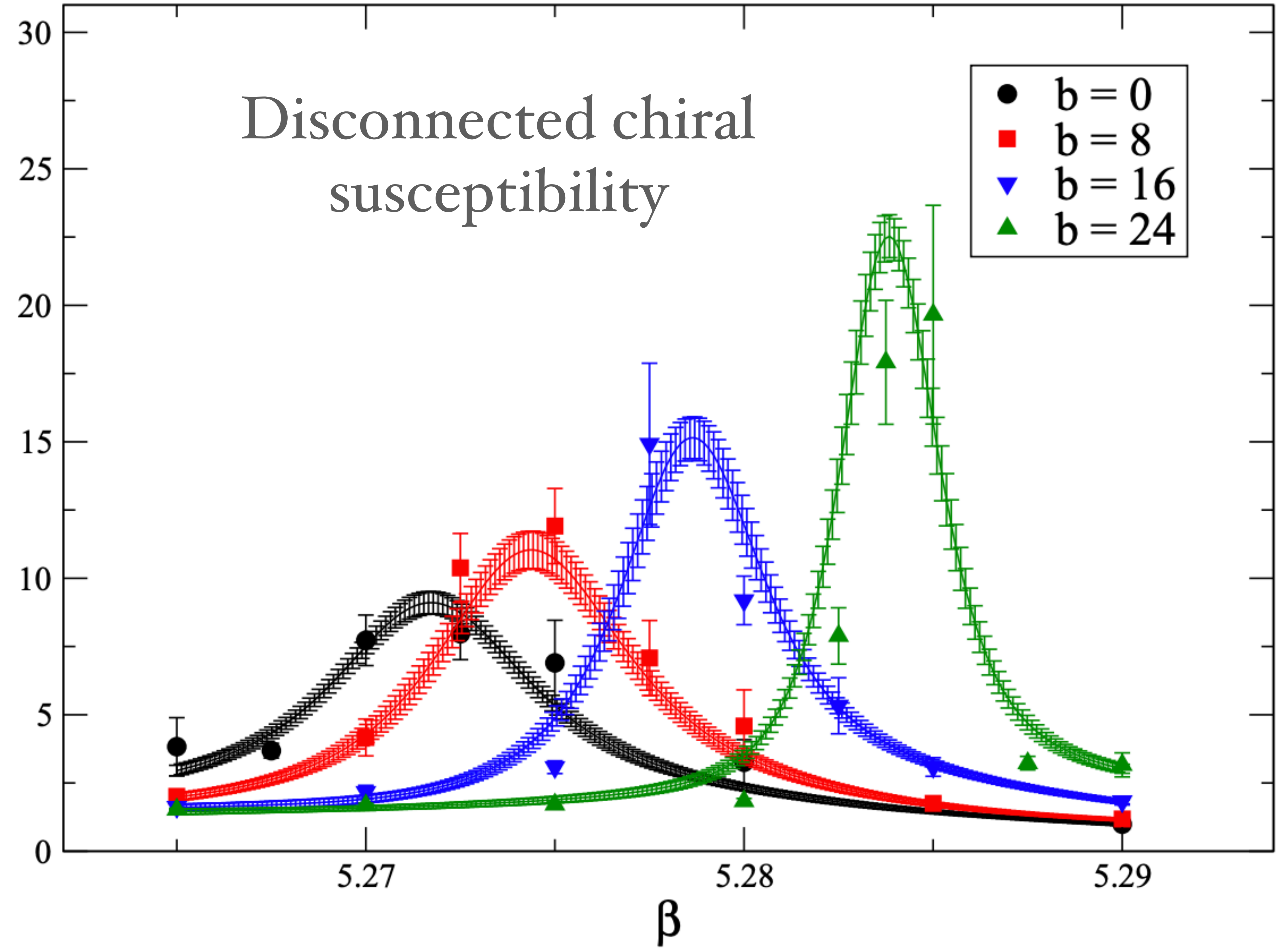
We study the dependence of quark condensate Σ on an external magnetic field. For weak fields, it rises linearly: $\Sigma(H) = \Sigma(0)[1 + eH \ln 2/(16\pi^2 F_\pi^2) + \ldots]$. M_π and F_π are also shifted so that the Gell-Mann–Oakes–Renner relation is satisfied. In the strong field region, $\Sigma(H) \propto (eH)^{3/2}$.

An external magnetic field increases the condensat
which means that it should make the chiral restora
tion phase transition in temperature and/or in baryo
chemical potential more difficult. That means, in par
ticular, that the critical temperature T_c (at $H = 0$, i
is estimated to be of order 200 MeV [1]) should in
crease with H. According to recent work [18], fo
strong fields, T_c is of order of the dynamically gener
ated mass (23) and grows with H, indeed. The esti
mate $T_c \sim \alpha_s \sqrt{|e_q H|}$ obtained earlier in [19] is prob
ably too rough and can be treated as an upper limit fo

Early lattice results: Magnetic catalysis & $T_{pc}\uparrow$

naive staggered fermion, larger-than-physical pion mass
not-continuum-extrapolated





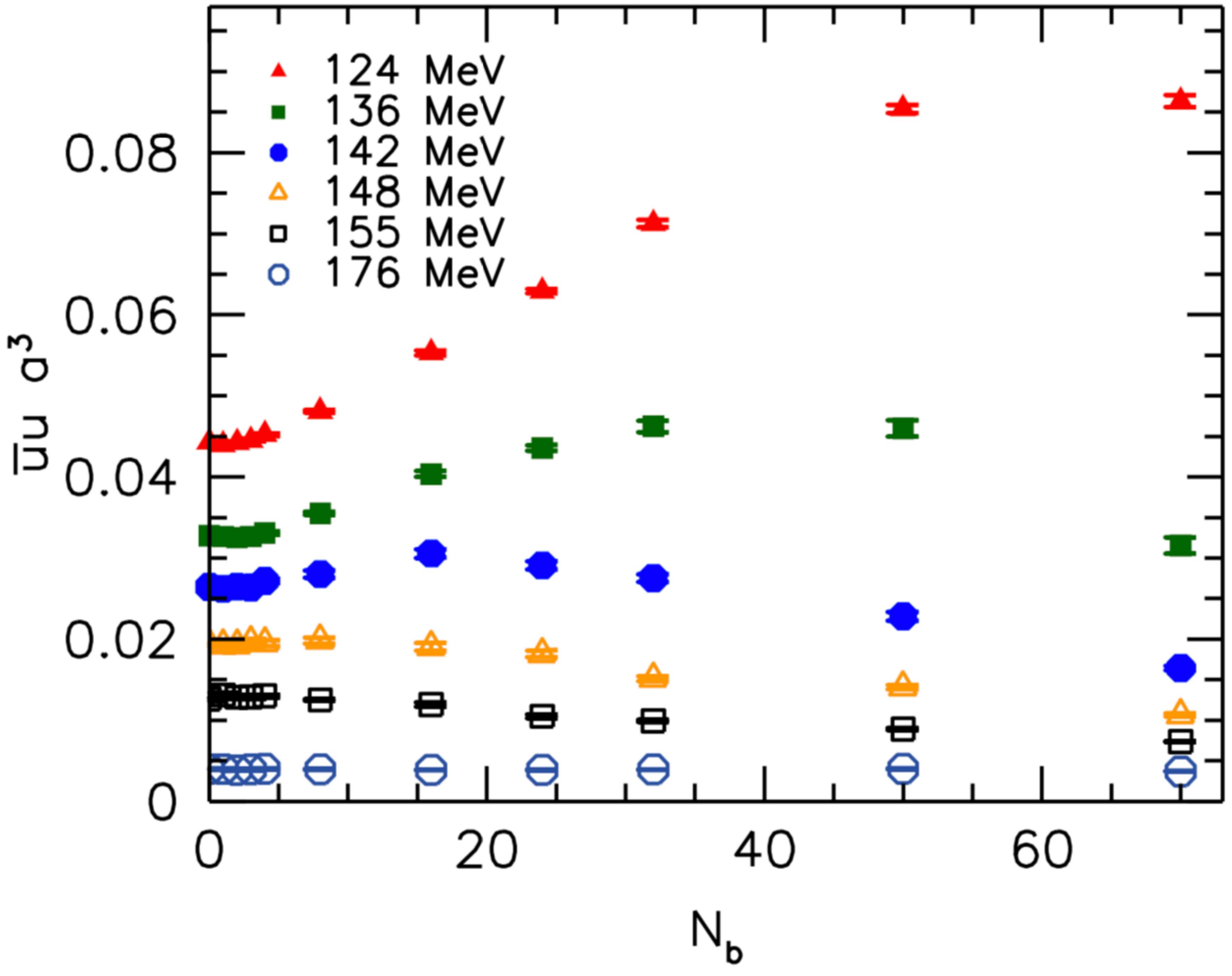
M. D'Elia, S. Mukherjee, F. Sanfilippo, *Phys.Rev.D* 82 (2010) 051501

Surprise came later…

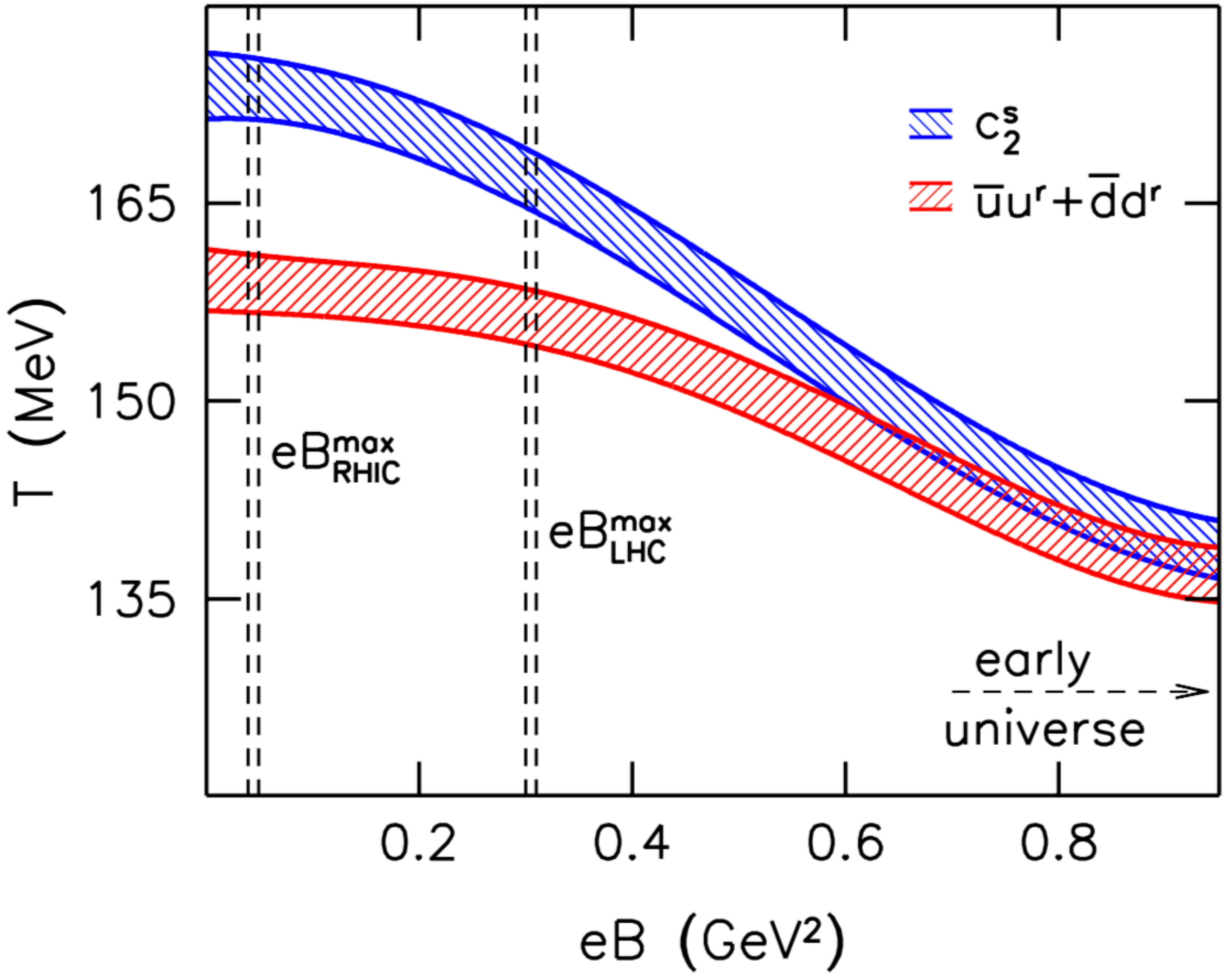
Continuum extrapolated lattice QCD results with physical pion mass

Bali et al., JHEP02(2012)044

Inverse magnetic catalysis



$eB\uparrow\ T_{pc}\downarrow$



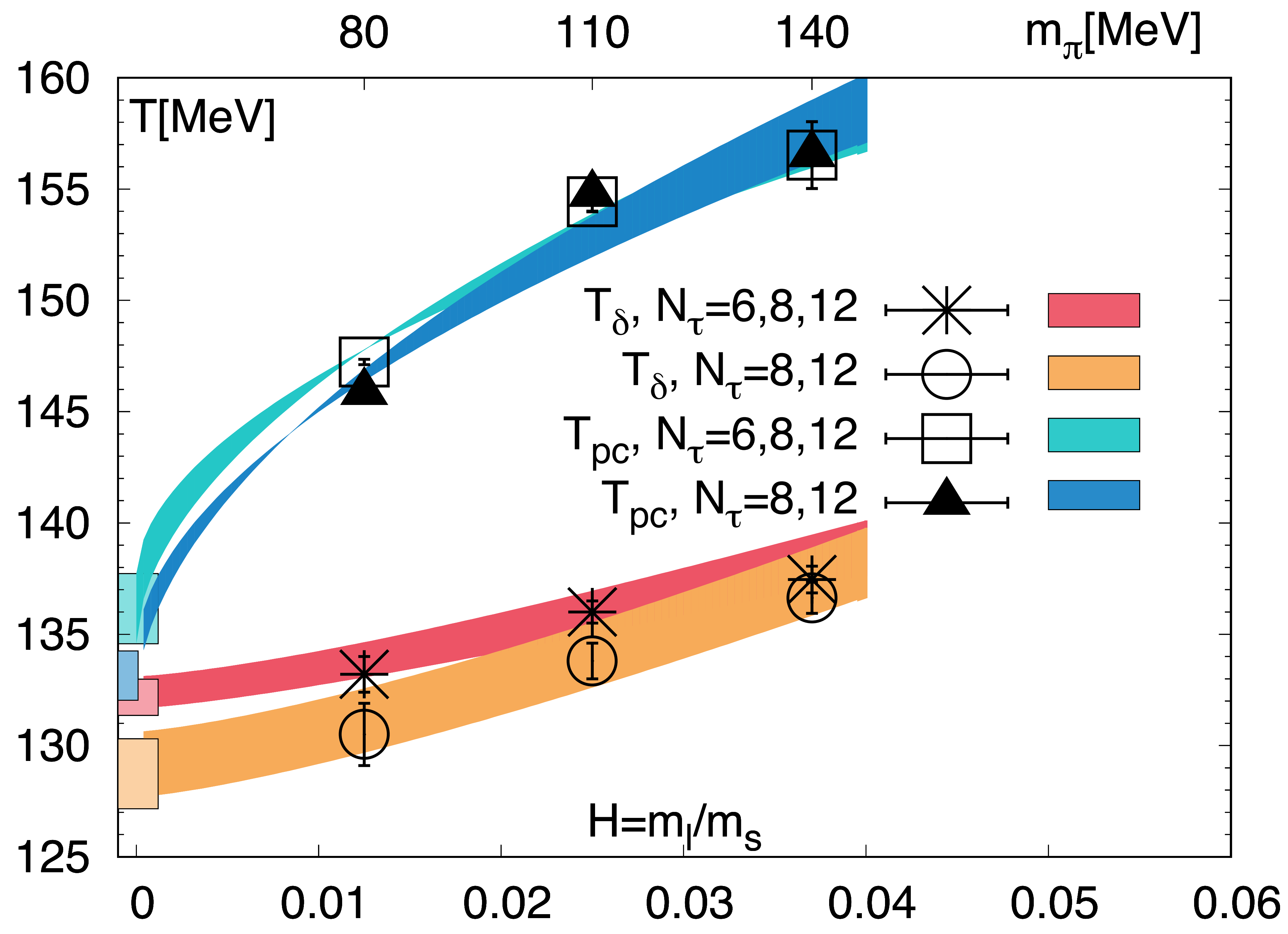
See recent reviews e.g.
G. Cao, arXiv:2103.00456
Andersen et al., Rev. Mod. Phys. 88(2016)02001

Chiral condensate always increases as eB at $T<<T_{pc}$

reduction of T_{pc} associated with inverse magnetic catalysis?

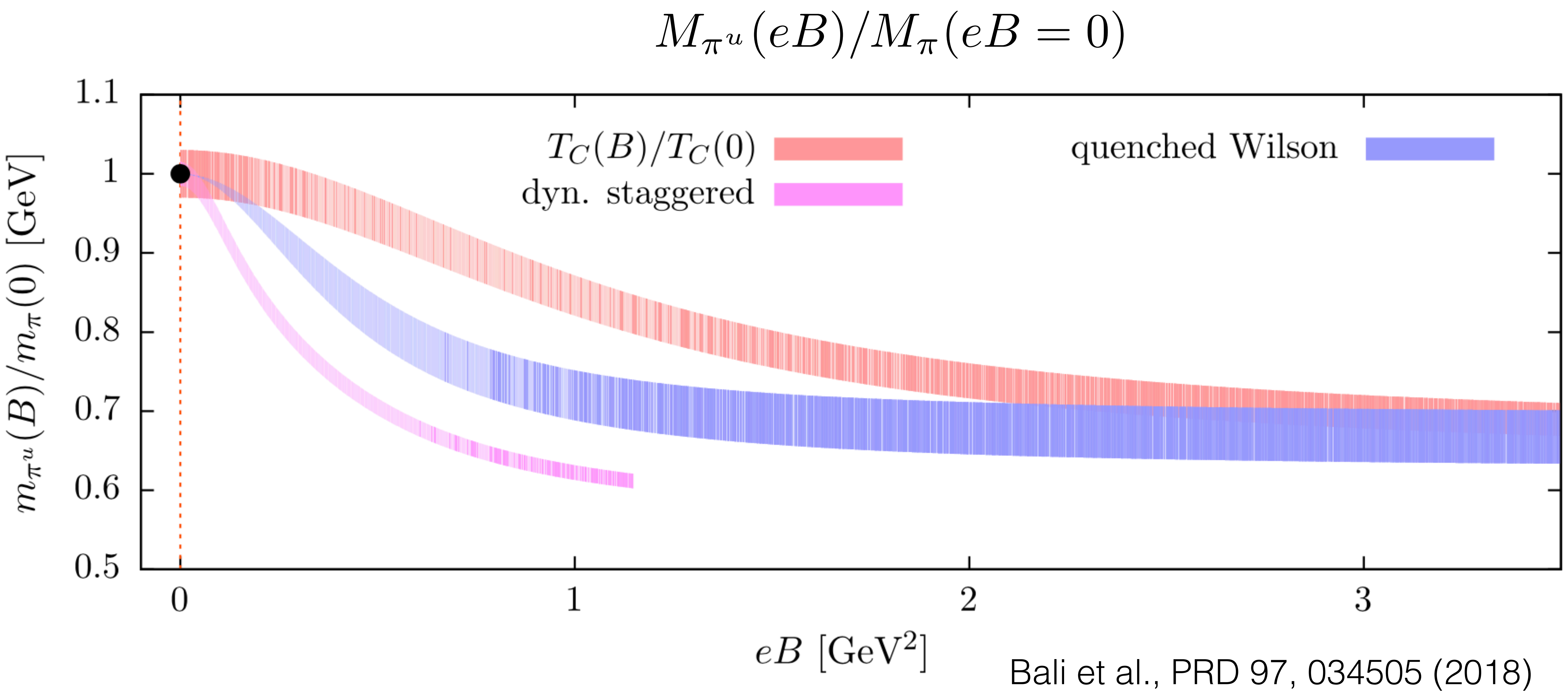
Reduction of T_{pc} v.s. lighter pion

eB=0, full QCD



HTD, P. Hegde, O. Kaczmarek et al.[HotQCD],
Phys. Rev. Lett. 123 (2019) 062002
HTD, arXiv:2002.11957

eB=/=0, quenched QCD



Bali et al., PRD 97, 034505 (2018)

Is (neutral) pion still a Goldstone boson at eB=/=0 ?

Gell-Mann-Oakes-Renner relation

M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968), J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985)

$$(m_u + m_d)\ \left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) = 2 f_\pi^2 M_\pi^2$$

Gell-Mann-Oakes-Renner relation

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Explicit chiral symmetry breaking

$$(m_u + m_d)\left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) = 2f_\pi^2 M_\pi^2$$

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$$(m_u + m_d)\left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) = 2f_\pi^2 M_\pi^2$$

Spontaneous chiral symmetry breaking

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M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968), J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985)

Explicit chiral symmetry breaking

$$(m_u + m_d)\left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) = 2f_\pi^2 M_\pi^2\,(1-\delta_\pi)$$

Spontaneous chiral symmetry breaking

At physical pion mass δ_π~6%

Gell-Mann-Oakes-Renner relation

M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968), J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985)

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Spontaneous chiral symmetry breaking

At physical pion mass $\delta_\pi \sim 6\%$

- At T=0, in the weak magnetic field the 2-flavor GMOR relation holds true for chiral (& point-like) pions from LO ChPT Shushpanov and Smilga, PLB402(1997)351

Gell-Mann-Oakes-Renner relation

M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968), J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985)

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- At T=0, in the weak magnetic field the 2-flavor GMOR relation holds true for chiral (& point-like) pions from LO ChPT Shushpanov and Smilga, PLB402(1997)351

- At eB=/=0, additional pion decay constants appear due to a nonzero pion-to-vacuum transition via the vector electroweak current Fayazbakhsh & Sadooghi, PRD 88(2013)065030

Bali et al., PRD121(2018)072001

Coppola et al., PRD.99 (2019)0540312

Lattice QCD in a background magnetic field

No sign problem: detM(eB) >0

- B pointing to the z direction & Gauge link multiplied by a U(1) factor

$$u_x(n_x,n_y,n_z,n_\tau) = \begin{cases} \exp[-iqa^2BN_xn_y] & (n_x = N_x - 1) \\ 1 & (\text{otherwise}) \end{cases}$$

$$u_y(n_x,n_y,n_z,n_\tau) = \exp[iqa^2Bn_x],$$

$$u_z(n_x,n_y,n_z,n_\tau) = u_t(n_x,n_y,n_z,n_\tau) = 1.$$

- Quantization of the magnetic field

$$qB = \frac{2\pi N_b}{N_xN_y}a^{-2}$$

q_u=2/3e, q_d=-1/3e, q_s=-1/3e →

$$eB = \frac{6\pi N_b}{N_xN_y}a^{-2}$$

Lattice setup

HTD et al., arXiv:2008.00493,2104.06843

- Symanzik-improved gauge action with HISQ fermions
- 32^3x96 lattices, with a=0.117 fm (a^{-1}=0.17 GeV), m_l/m_s =1/10 (M_π =220 MeV)
- In our setup f_π= 96.93(2) MeV, f_K=112.50(2) MeV, f_K/f_π=1.1606(3)

FLAG 2019: At physical mass point f_π= 92.1(6) MeV, f_K=110.1(5) MeV, f_K/f_π=1.1917(37)

- ✦ Magnetic field is quantized as $eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$
- ✦ Magnetic flux: N_b=0,1,2,3,4,6,8,10,12,16,20,24,32,48 & 64
- ✦ 0 ≤ eB ≤ 3.35 GeV^2 (~70 M_π^2)

✦ Fixed scale approach to nonzero T up to 281 MeV

UV divergence of chiral condensate

$$(m_u + m_d)\ \left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) = 2f_\pi^2 M_\pi^2\,(1-\delta_\pi)$$

- UV-divergence term dominates by the linear-in-quark-mass term

$$\langle\bar{\psi}\psi\rangle_{q,UV-div} = \frac{\nu_f}{2}\left(\frac{\pi}{a}\right)^2\frac{1}{(2\pi)^2}m_q + \frac{\nu_f}{2}ln\left(\frac{am_q}{2\pi}\right)\frac{1}{(2\pi)^2}m_q^3.$$

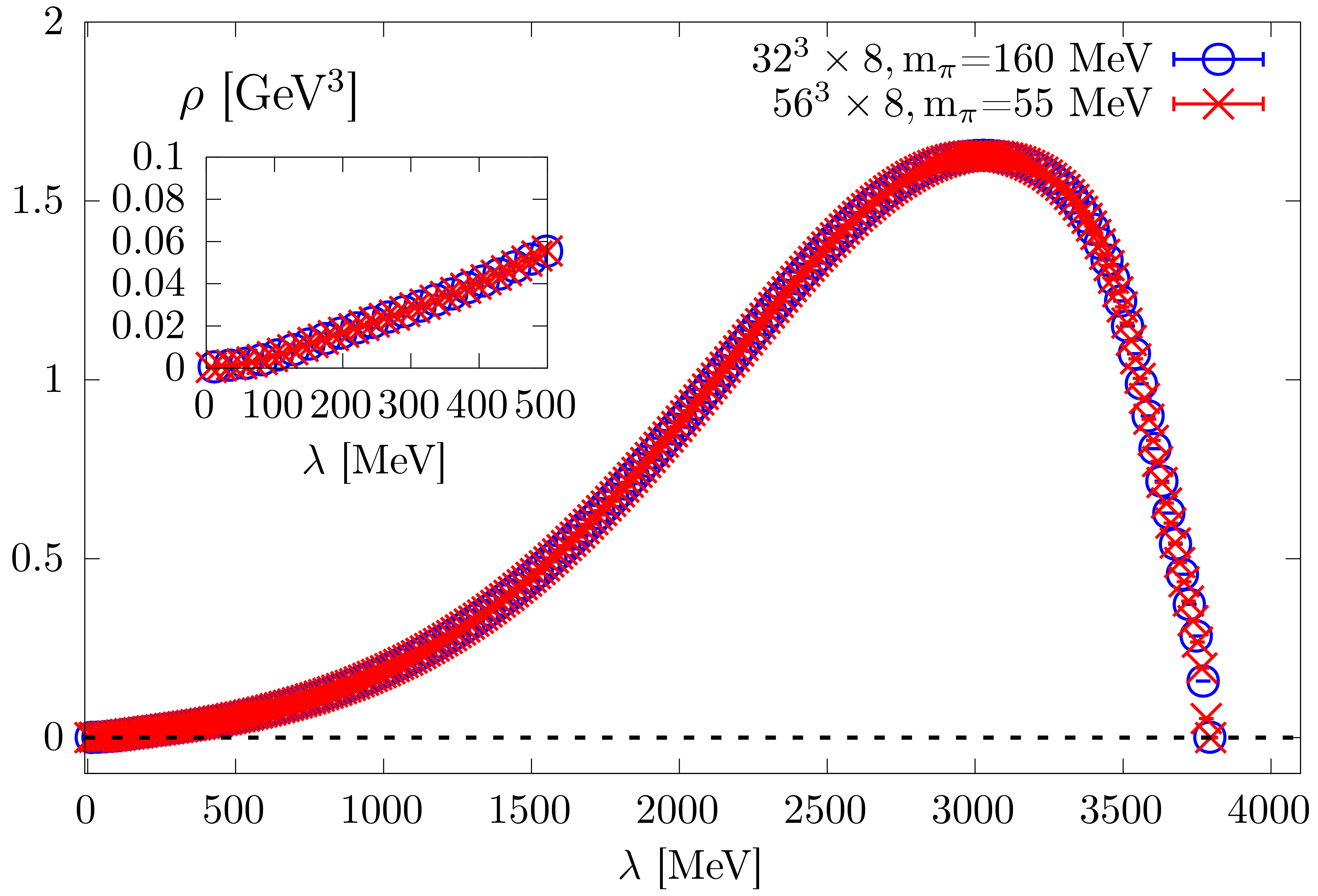
- Commonly used methods to get rid of the UV-divergence part

Subtracted chiral condensate: $\langle\bar{\psi}\psi\rangle_{sub} = \langle\bar{\psi}\psi\rangle_l - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_s$ ✗

Zero T/eB subtraction: $\langle\bar{\psi}\psi\rangle_{UV\,free} = \langle\bar{\psi}\psi\rangle_l(eB\neq 0) - \langle\bar{\psi}\psi\rangle_l(eB=0)$ ✗

A complete Eigenvalue spectrum

$$\langle\bar{\psi}\psi\rangle = \int_0^\infty \frac{4m_l\,\rho}{\lambda^2 + m_l^2}\,\mathrm{d}\lambda$$



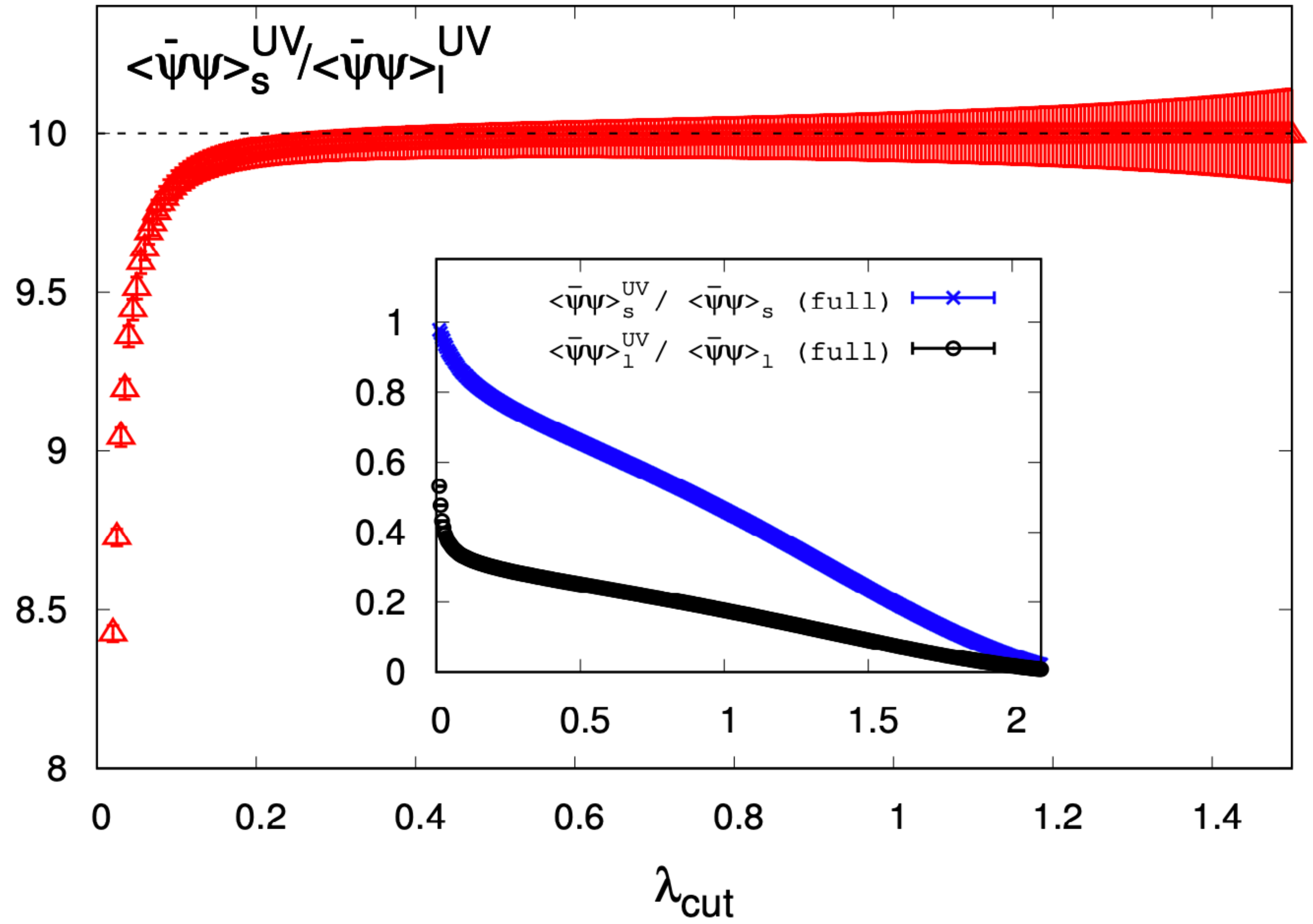
via Chebyshev Polynomial filtering technique

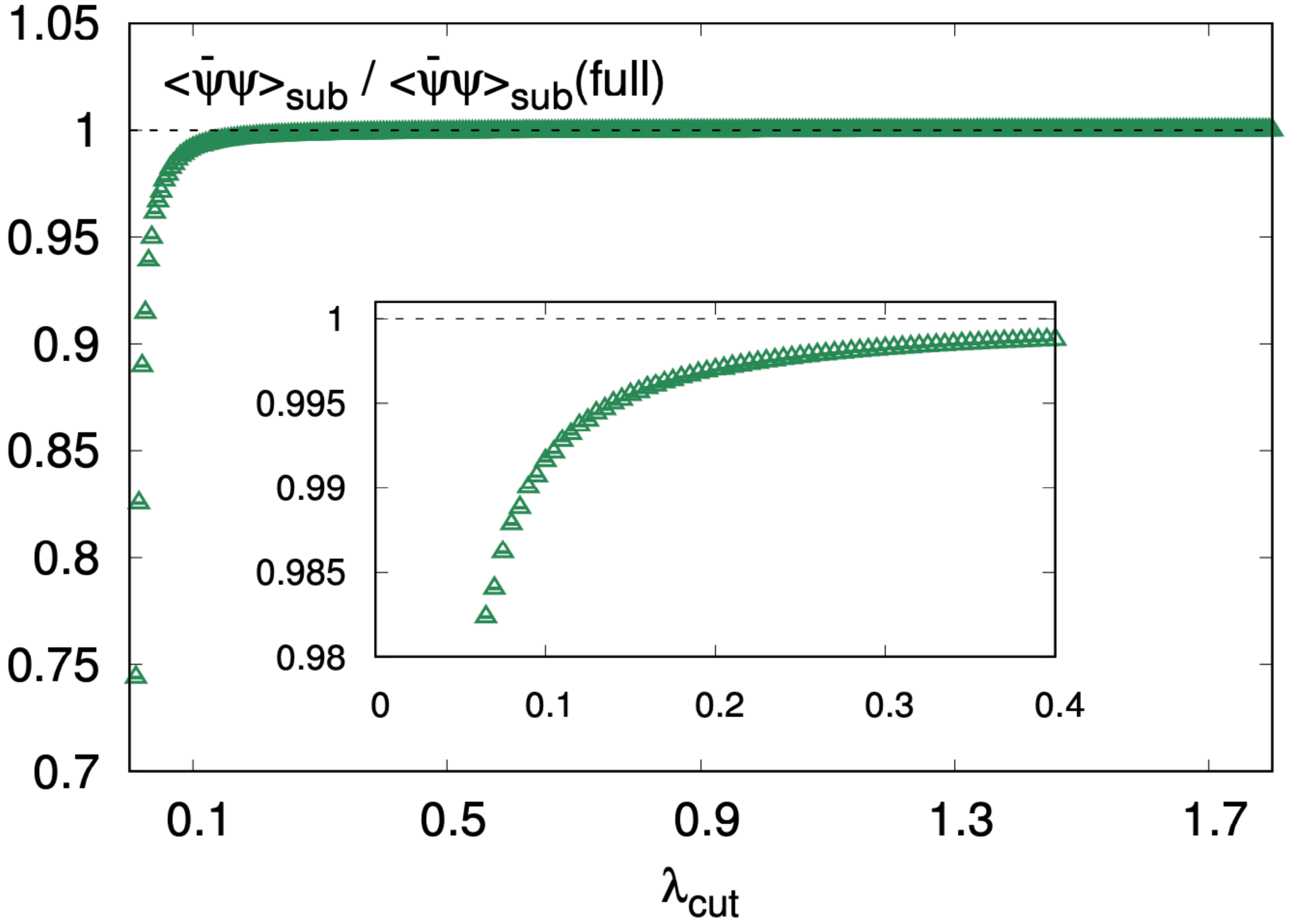
HTD, S.-T. Li, S. Mukherjee, A. Tomiya, X.-D. Wang, Y. Zhang, Phys. Rev. Lett. 126 (2021) 082001

UV-free chiral condensate

$$\langle\bar{\psi}\psi\rangle^{\mathrm{UV}}_{l,s} = \int_{\lambda^{\mathrm{UV}}_{cut}}^{\infty} \frac{2\,m_{l,s}\,\rho(\lambda)}{\lambda^2 + m^2_{l,s}}\,\mathrm{d}\lambda.$$

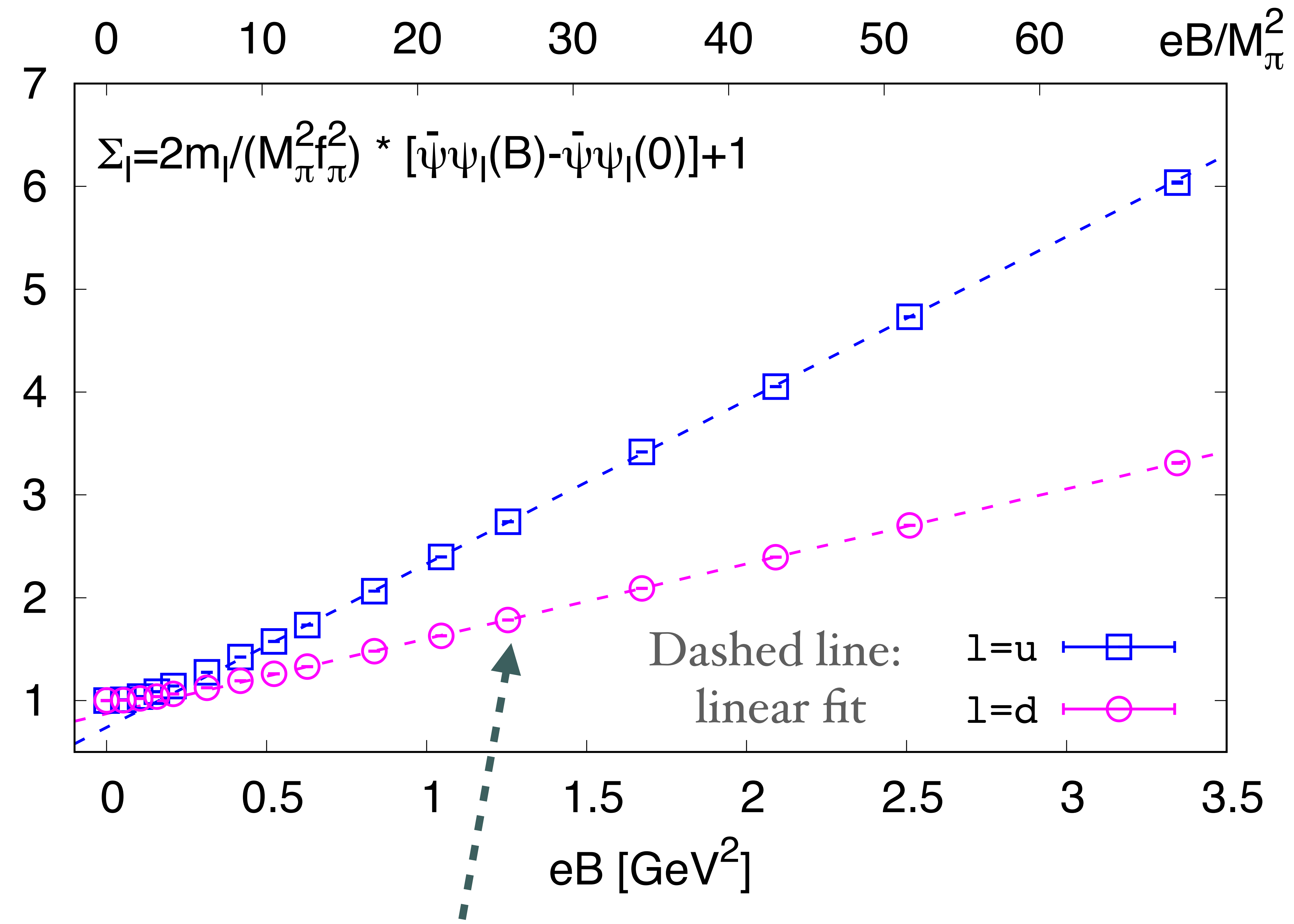
$$\langle\bar{\psi}\psi\rangle_{sub} \equiv \langle\bar{\psi}\psi\rangle_l - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_s = \int_0^{\infty} \frac{2m_l\,(m_s^2 - m_l^2)\rho(\lambda)}{(\lambda^2+m_l^2)(\lambda^2+m_s^2)}\,\mathrm{d}\lambda$$





$$\lambda^{\mathrm{UV}}_{cut} \in [0.12, 0.36]$$

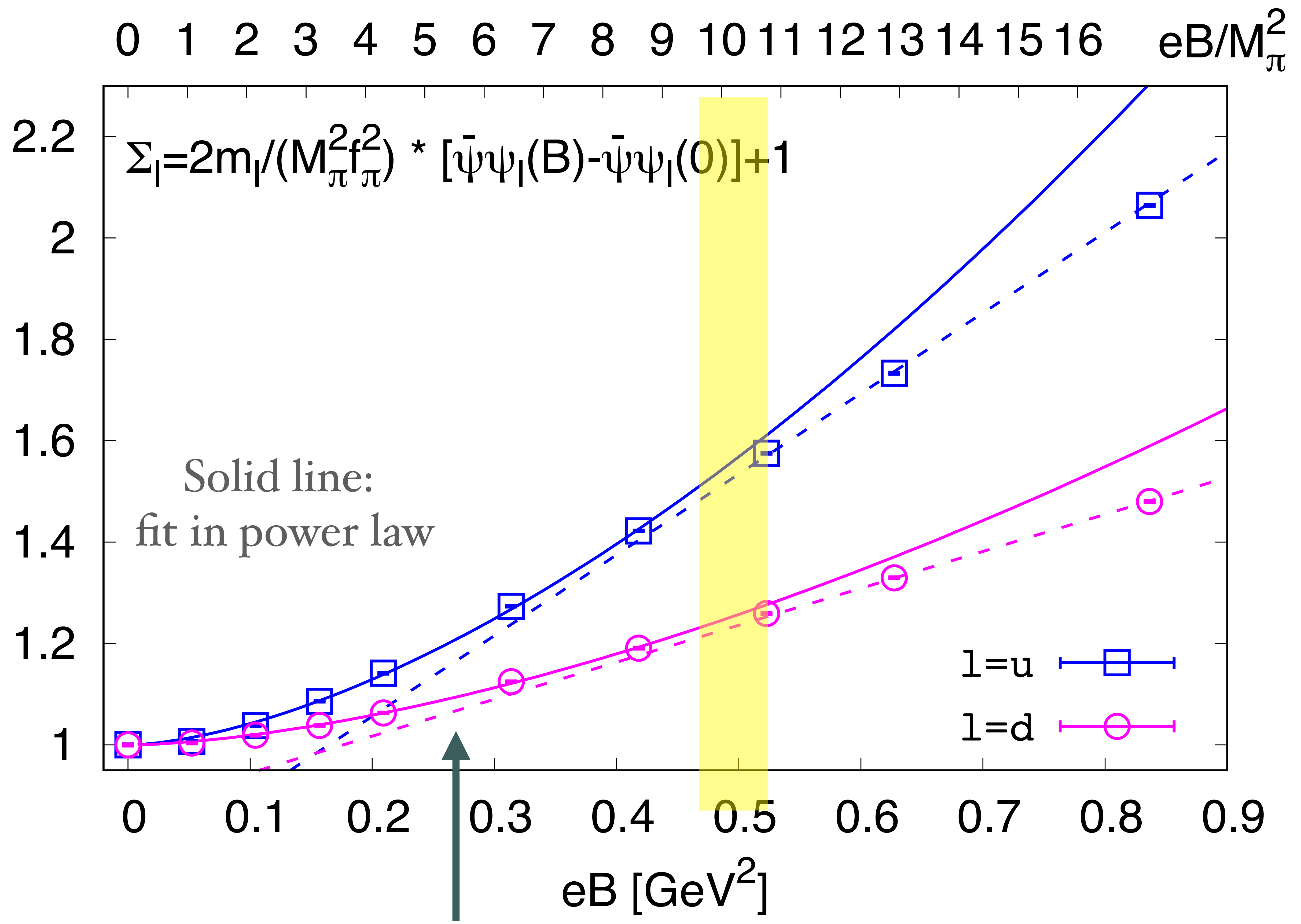
Magnetic catalysis at T=0



Linear in eB at large eB ≳ 0.5 GeV²

Dimensional reduction & Quark mass gap

T. Kojo and N. Su, *Phys.Lett.B* 720 (2013) 192



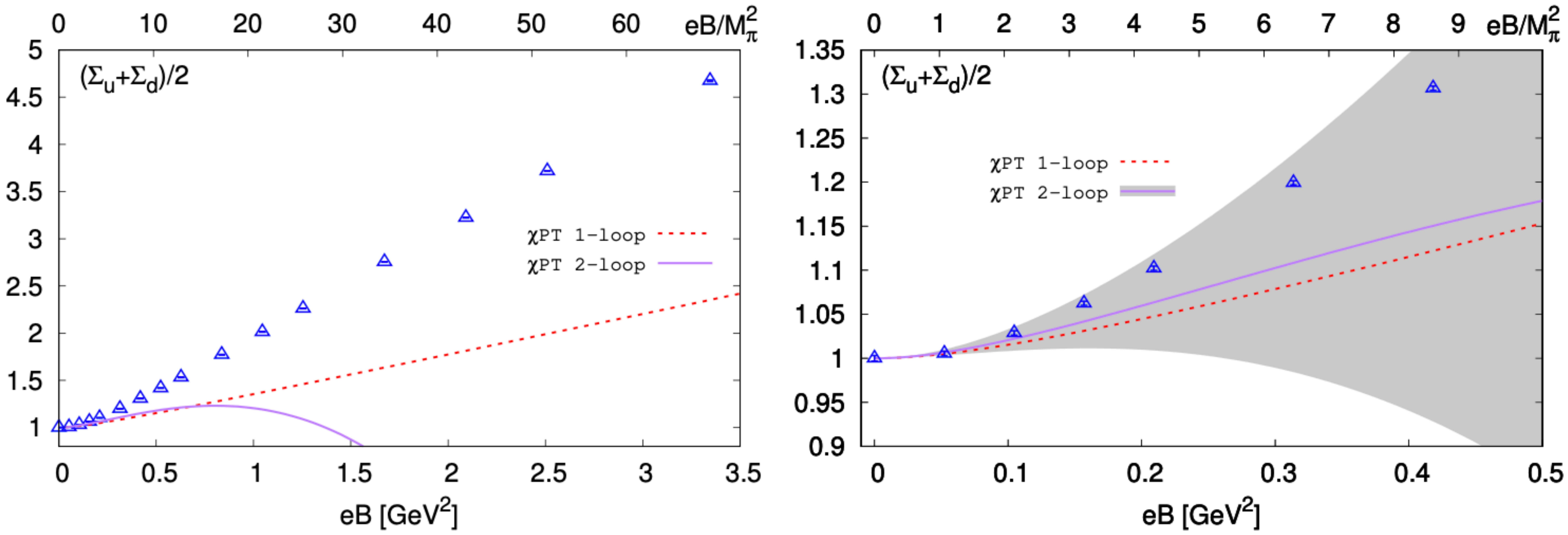
Power law in eB at small eB ≲ 0.5 GeV²

In contrast to 1-loop ChPT in chiral limit:

$$\Sigma(H) = \Sigma(0)\,[1 + eH \ln 2/(16\pi^2 F_\pi^2) + \ldots]$$

Shushpanov and Smilga, PLB402(1997)351

Comparison to ChPT



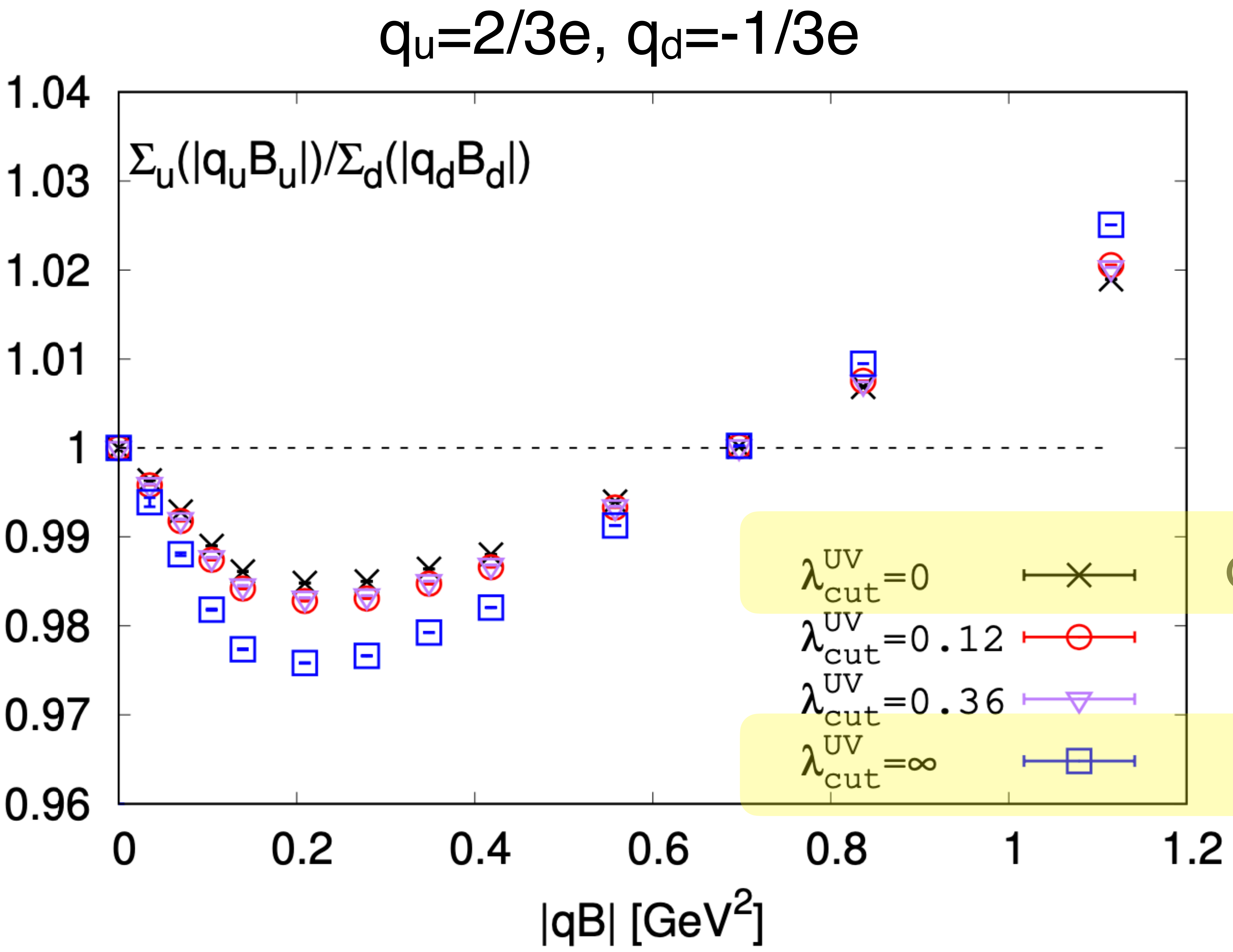
ChPT: Extended to nonzero values of pion mass

all consider degenerate u and d chiral condensates

1-loop: T. D. Cohen, D. A. McGady, and E. S. Werbos, Phys. Rev. C76, 055201 (2007)

2-loop: E. S. Werbos, Phys. Rev. C77, 065202 (2008)

qB scaling of up and down quark chiral condensates

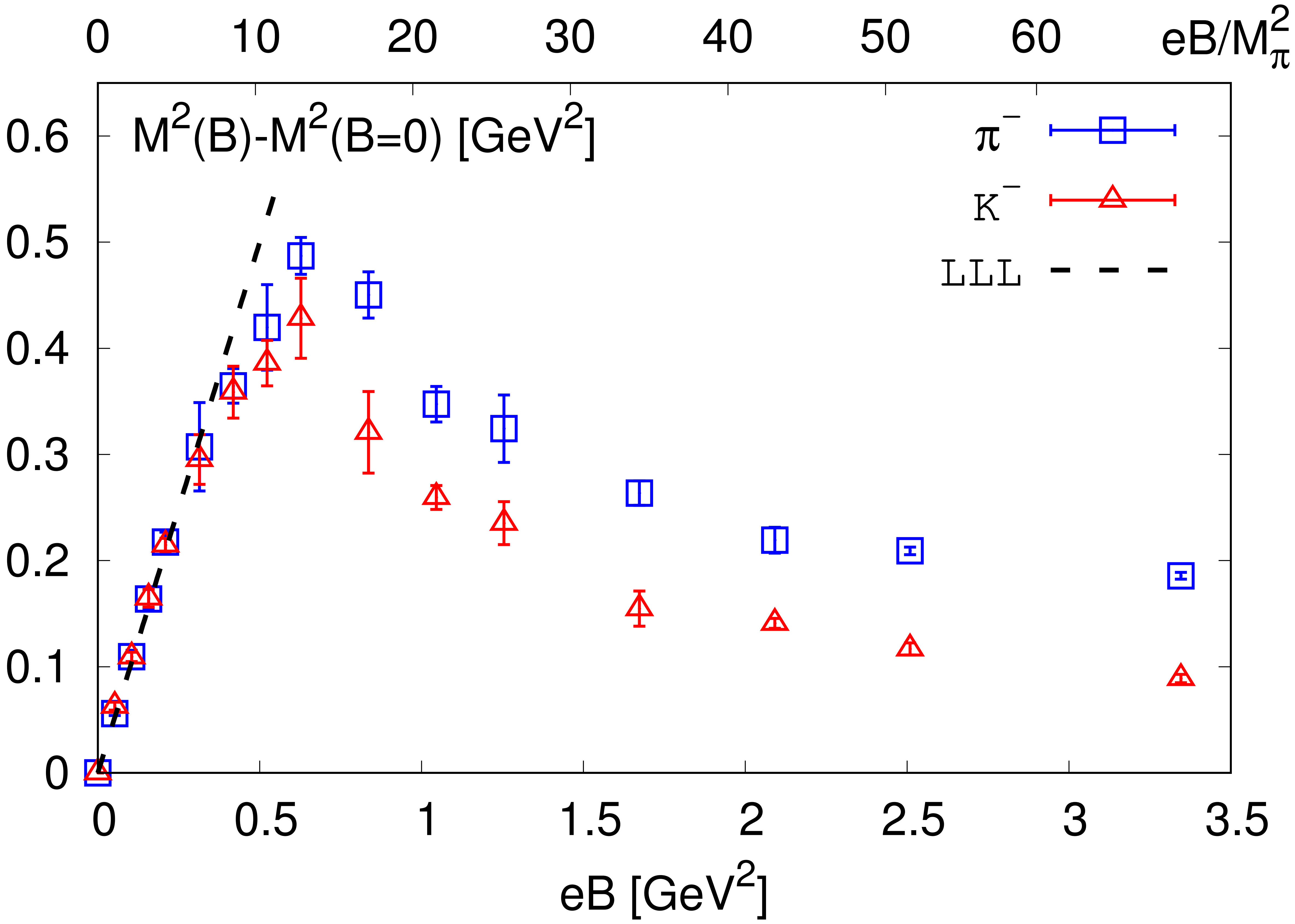


$\lambda_{\rm cut}^{\rm UV}$: different estimates of UV divergence in the chiral condensate are removed

Chiral condensate at T=0&B=0 is subtracted

No subtraction

Masses of charged pseudo scalar mesons



Lowest Landau-Level (LLL):

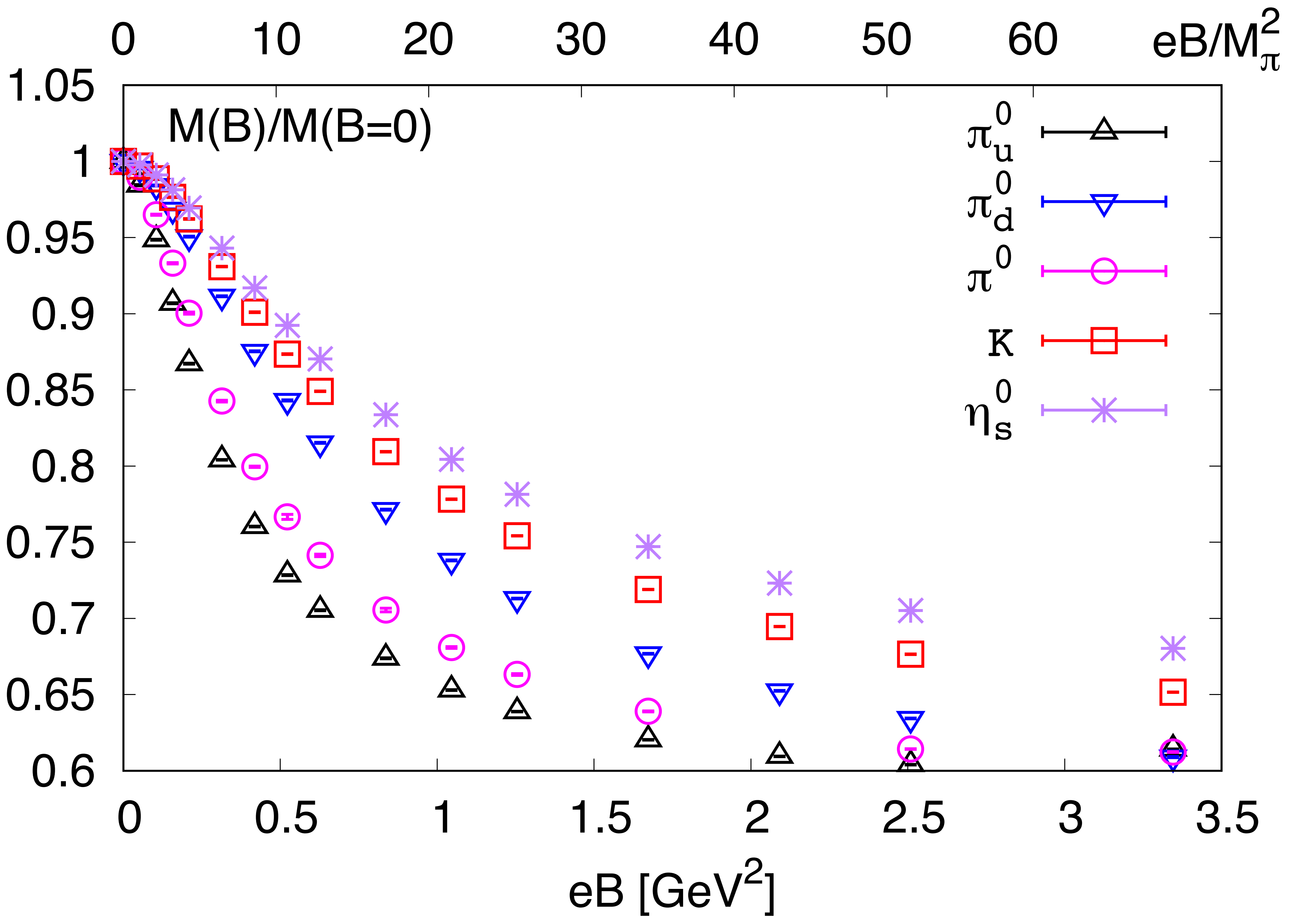
$$M^{\pm}_{\rm ps}(B) = \sqrt{\left(M^{\pm}_{\rm ps}(B=0)\right)^2 + |eB|}.$$

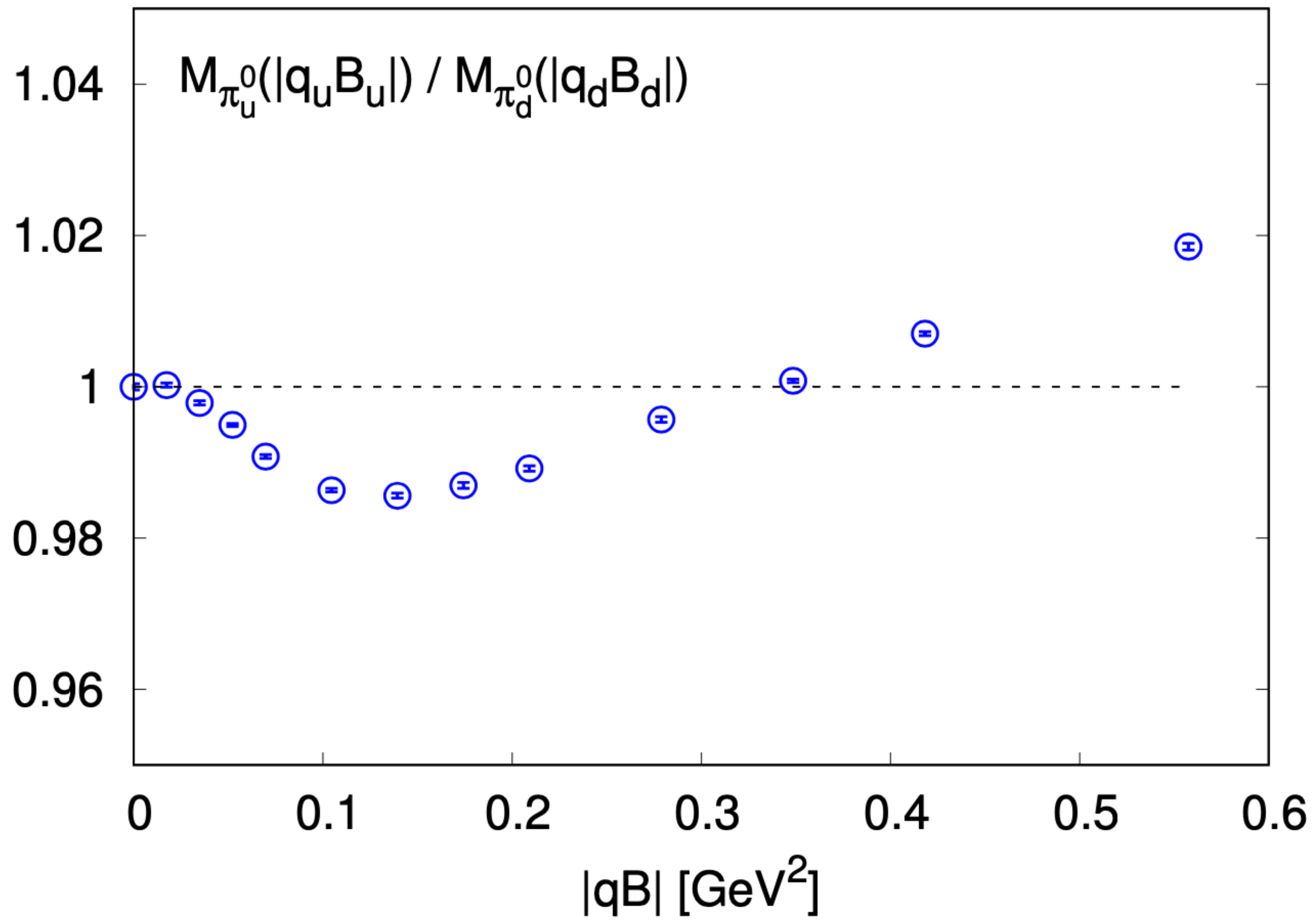
In contrast to Quenched QCD results where M increases monotonously with *eB*

Bali et al., PRD 97, 034505 (2018) Luschevskaya et al, PLB 761 (2016) 393

Not point particles anymore? Effects from dynamic quarks?

Masses of neutral pseudo scalar mesons



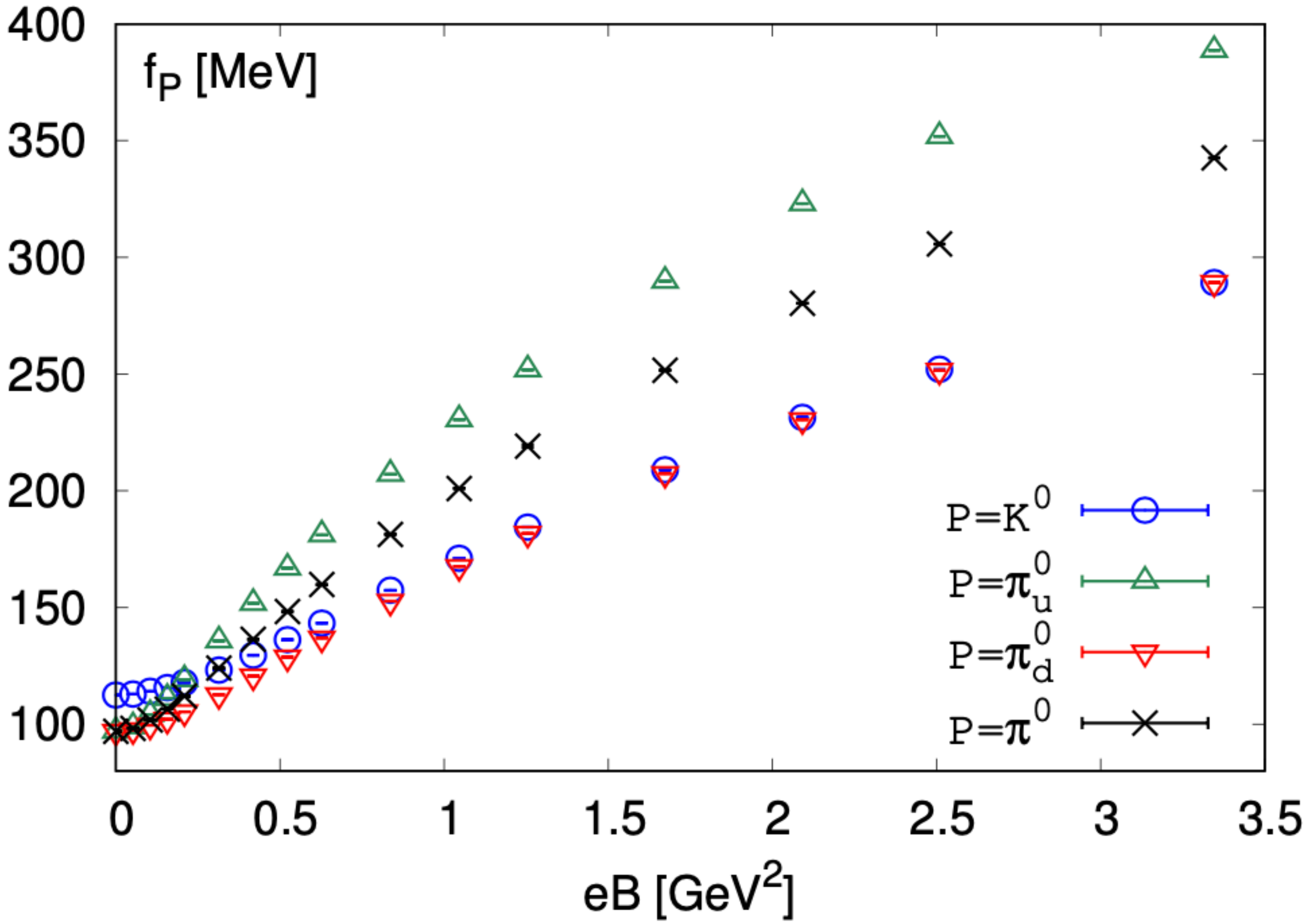


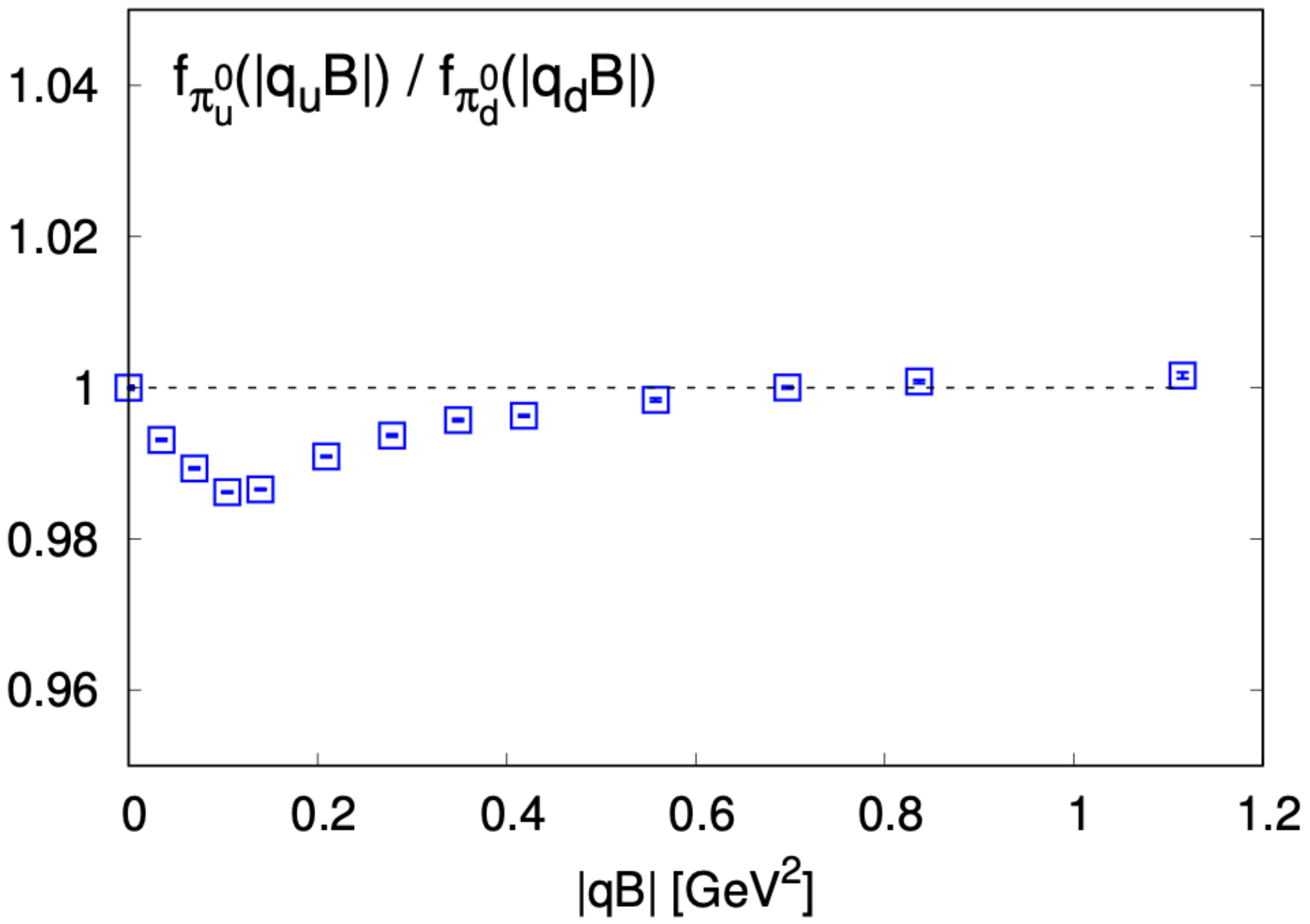
Mass of neutral pseudo scalar meson decreases with eB

$$|\pi^0\rangle = \alpha|u\gamma_5\bar{u}\rangle - \beta|d\gamma_5\bar{d}\rangle$$

qB scaling observed in the up and down quark flavor components of neutral pion mass

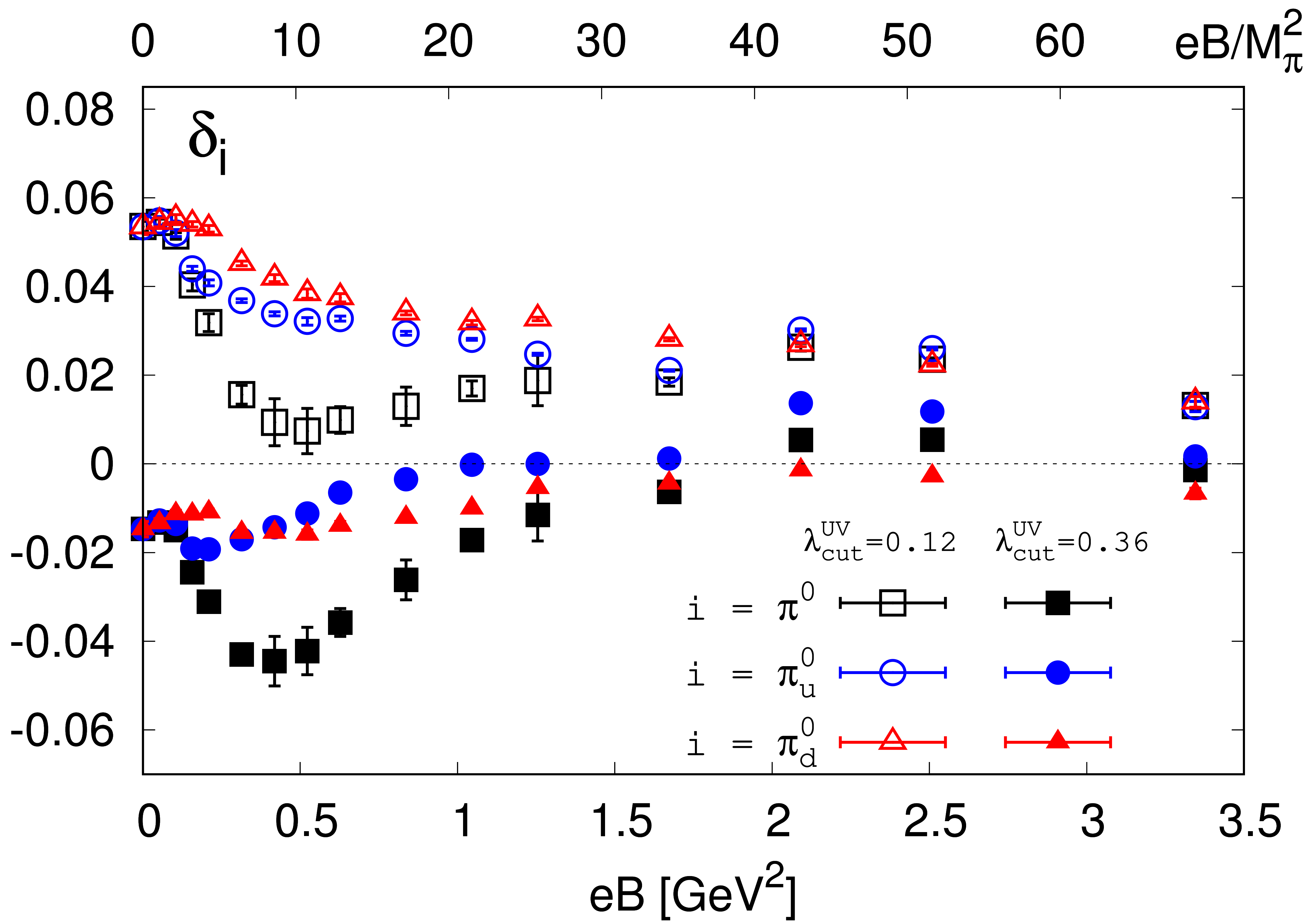
Decay constants of neutral pion and kaon





- All the decay constants increase with eB
- qB scaling observed in u and d quark flavor components of f_π

Gell-Mann-Oakes-Renner relation

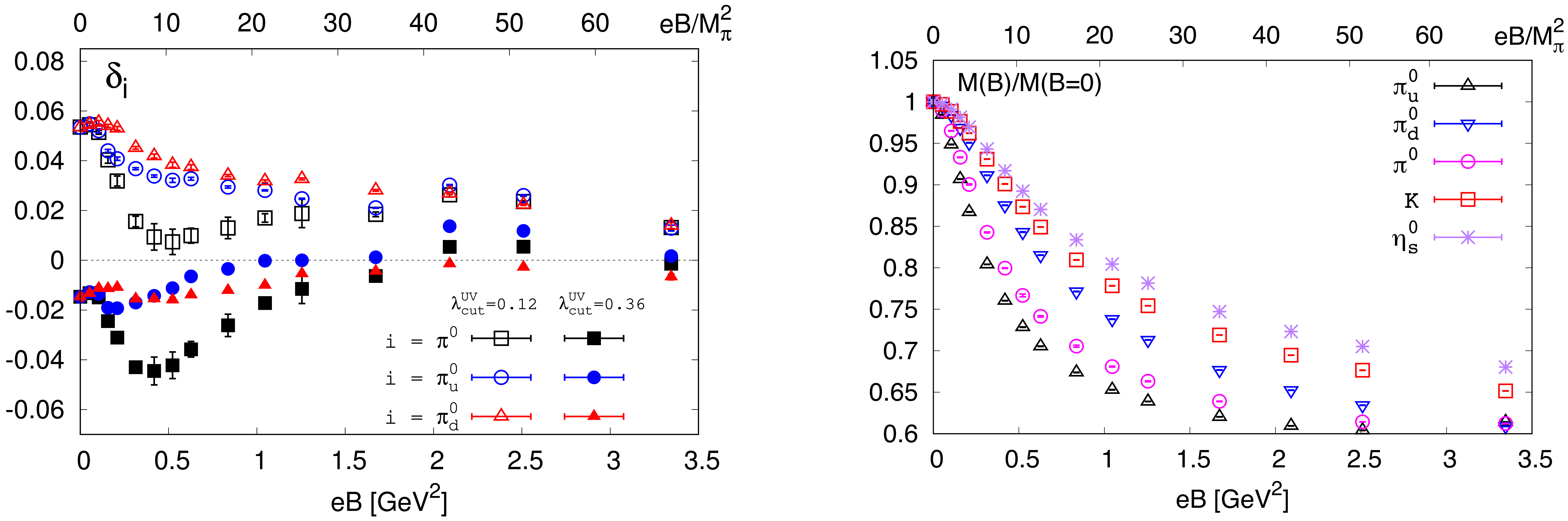


$$4m_u \,\langle\bar{\psi}\psi\rangle_u = 2f^2_{\pi^0_u} M^2_{\pi^0_u}\,(1-\delta_{\pi^0_u})$$
$$4m_d \,\langle\bar{\psi}\psi\rangle_d = 2f^2_{\pi^0_d} M^2_{\pi^0_d}\,(1-\delta_{\pi^0_d})\,.$$

$$(m_u+m_d)\,\left(\langle\bar{\psi}\psi\rangle_u+\langle\bar{\psi}\psi\rangle_d\right) = 2f_\pi^2 M_\pi^2\,(1-\delta_\pi)$$

neutral pion remains as a Goldstone boson with eB up to ~3.5 GeV^2

Gell-Mann-Oakes-Renner relation



T_{pc} decreases with eB regardless of (inverse) magnetic catalysis

Low T: Hadron resonance gas model

Non-interacting hadron resonance gas

Dashen, Ma & Bernstein,
Phys. Rev. 187 (1969) 345.

With eB=/=0 pressure: $p = p_c^{\mathrm{M/B}} + p_n^{\mathrm{M/B}}$

Charged Meson/Baryons: $p_c^{\mathrm{M/B}} = \mp \frac{|q_i|BT}{2\pi^2} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \int_0^\infty \mathrm{d}p_z \ln\left[1 \mp e^{-(E_c-\mu_i)/T}\right]$

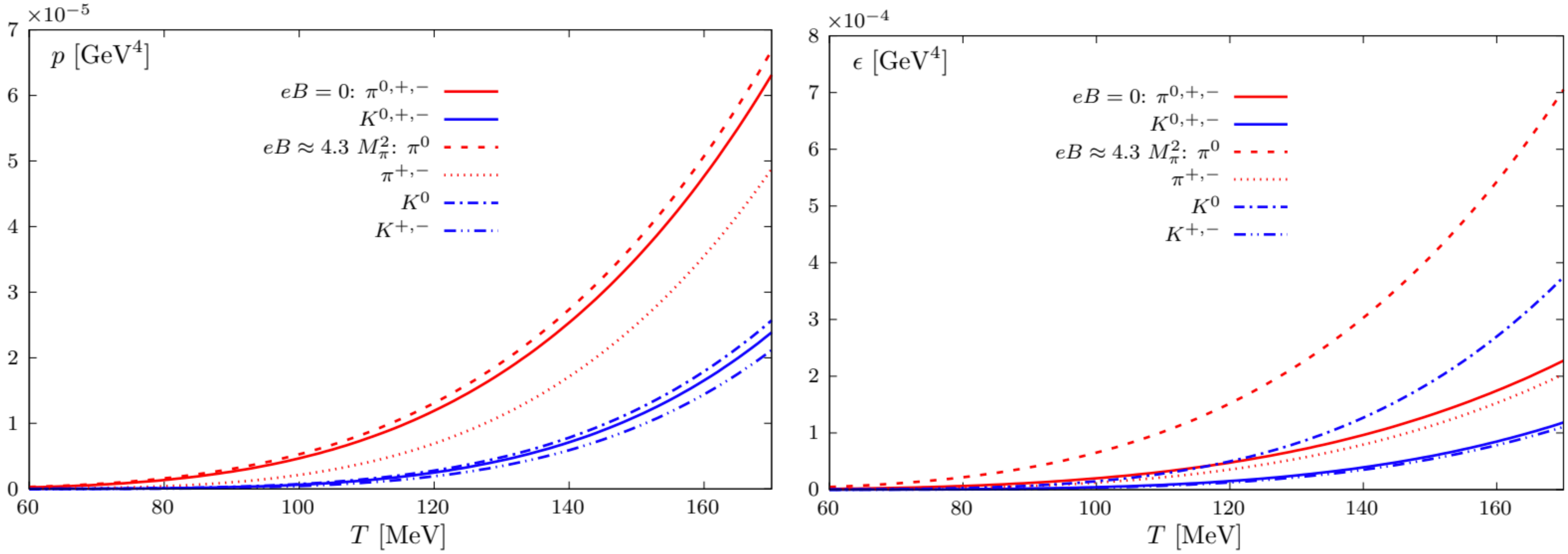
Neutral Meson/Baryons: $p_n^{\mathrm{M/B}} = \mp \frac{d_i T}{2\pi^2} \int_0^\infty \mathrm{d}p |\vec{p}|^2 \ln\left[1 \mp e^{-(E_n-\mu_i)/T}\right]$

Bhattacharyya et al., EPL 115 (2016) 62003

Fukushima and Hidaka, Phys.Rev. Lett. 117 (2016)102301

HTD, S.-T. Li, Q. Shi and X.-D. Wang, arXiv:2104.06843

Contributions to pressure and energy density from individual hadrons in HRG



HTD, S.-T. Li, Q. Shi, A. Tomiya, X.-D. Wang, Y. Zhang, arXiv: 2011.04870

Ward Identities

$$\langle\bar{\psi}\psi\rangle_f = m_f \chi_{ps_f^0}$$

At *eB*=0: G. W. Kilcup and S. R. Sharpe, Nucl. Phys. B283, 493 (1987)

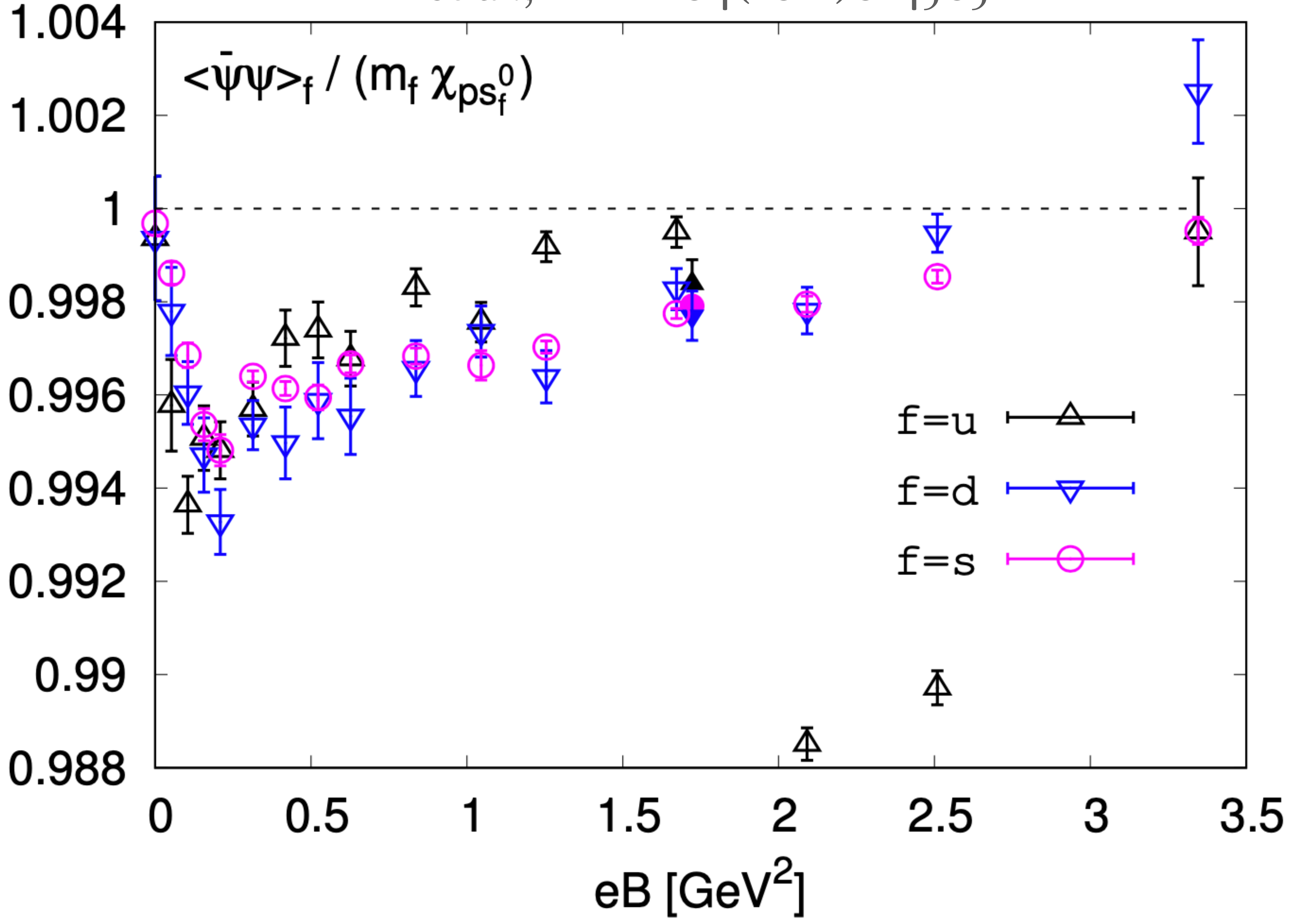
At *eB*=/=0: HTD et al., PRD 104(2021)014505

$$\chi_{\mathrm{ps_f^0}} = \sum_{\tau=0}^{N\tau-1} G_{\mathrm{ps}_f^0}(\tau)$$

$$\lim_{\tau\to\infty} G_{ps_f^0}(\tau) \sim e^{-M_{ps_f^0}\tau}$$

Ward identities hold true at eB=/=0!

HTD et al., PRD 104(2021)014505



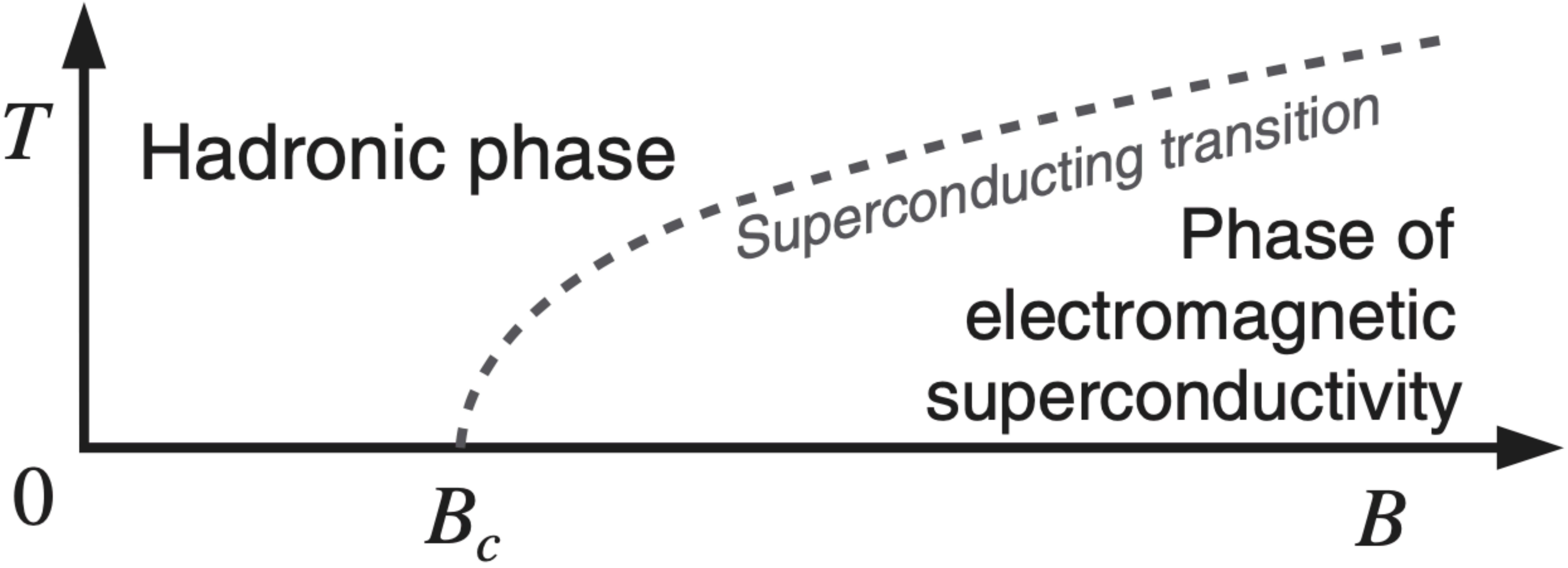
$eB \uparrow$ $\langle\bar{\psi}\psi\rangle_l \uparrow$ $M_{\pi^0} \downarrow$ Consistent within the WI

Fluctuations and correlations of net baryon number, electric charge and strangeness in a background magnetic field

HTD, S.-T. Li, Q. Shi and X.-D. Wang, arXiv:2104.06843

- Possibilities to detect the existence of a magnetic field in heavy-ion collisions

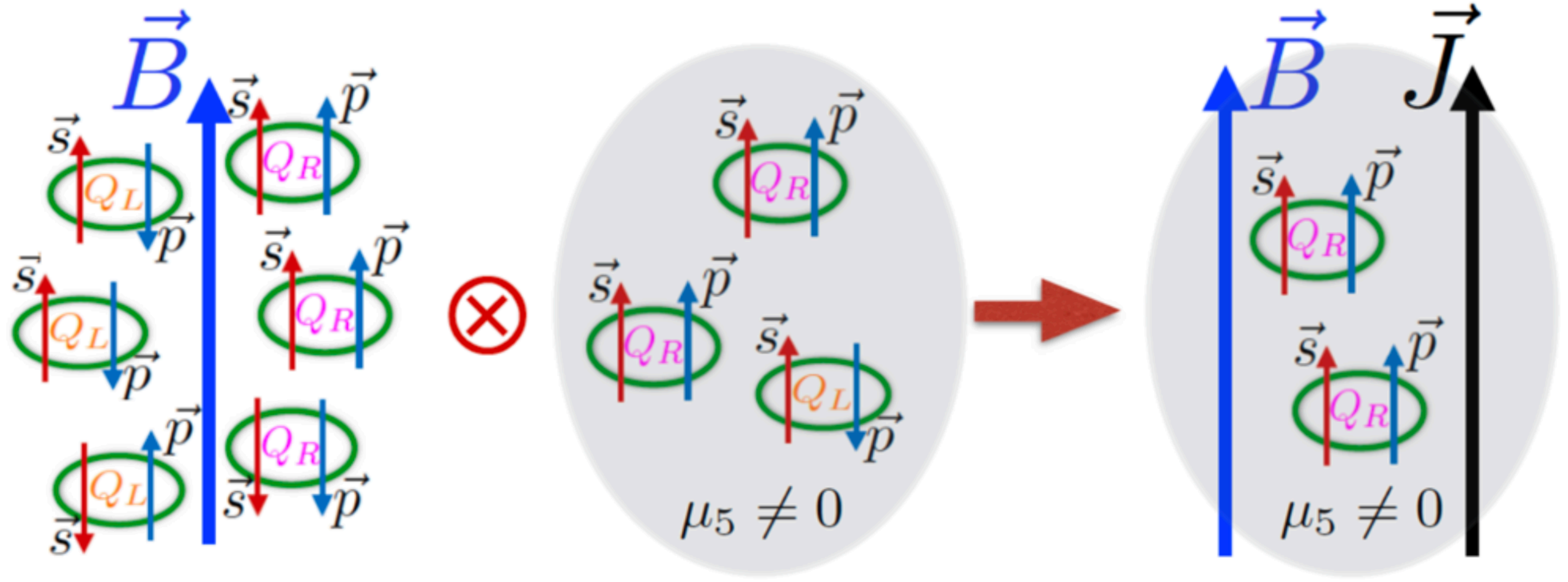
Superconducting phase at T=0



Signaled by the condensation of rho

M. N. Chernodub, Phys. Rev. Lett. 106 (2011) 142003

Chiral magnetic effects



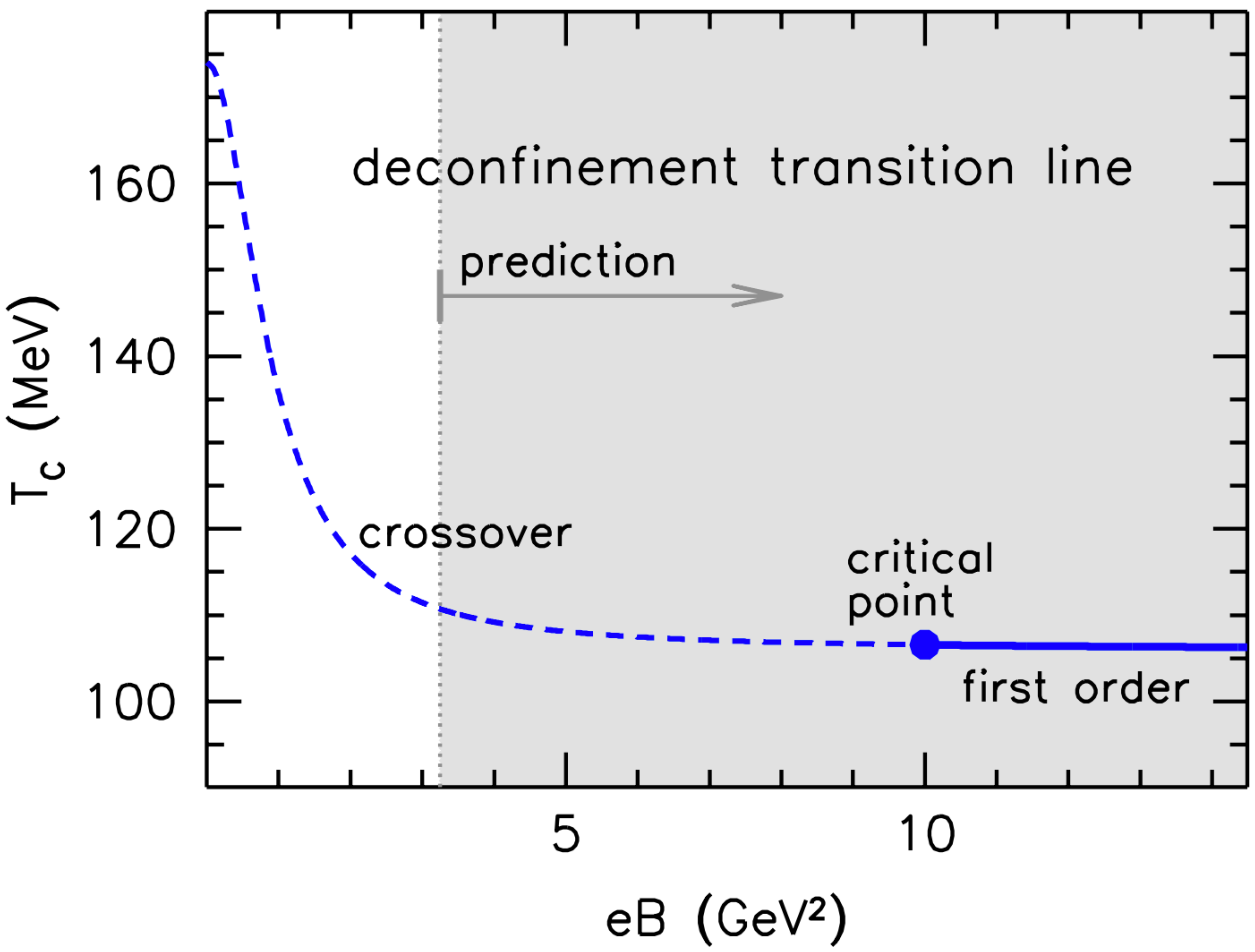
$$\vec{J} = \frac{Q^2}{2\pi^2}\mu_5 \vec{B}$$

Axial U(1) anomaly & deconfined phase & magnetic field

See recent reviews e.g. D.E. Kharzeev and J. Liao, Nature Rev. Phys. 3(2021)55

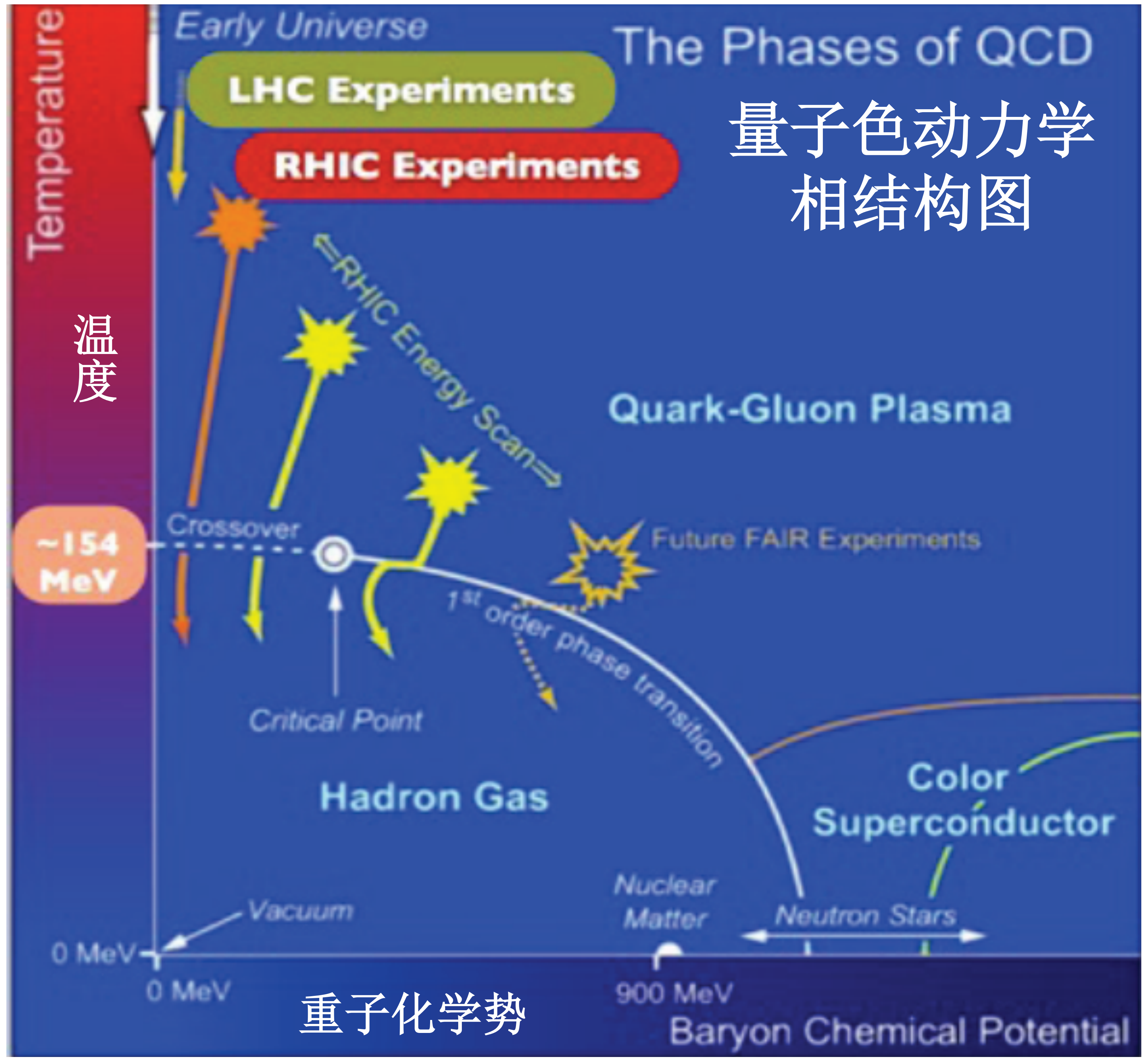
QCD critical end point

T-*eB* plane

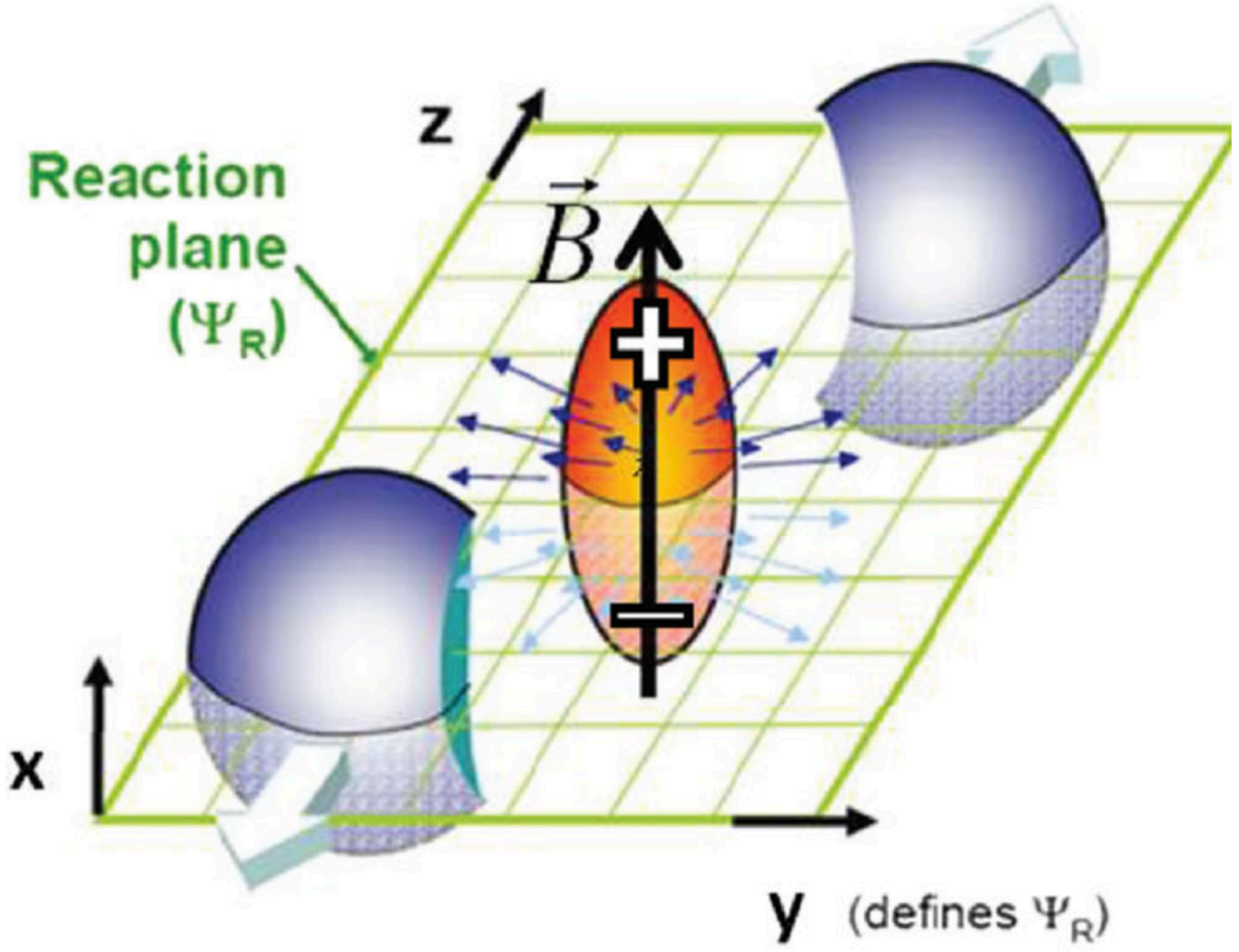


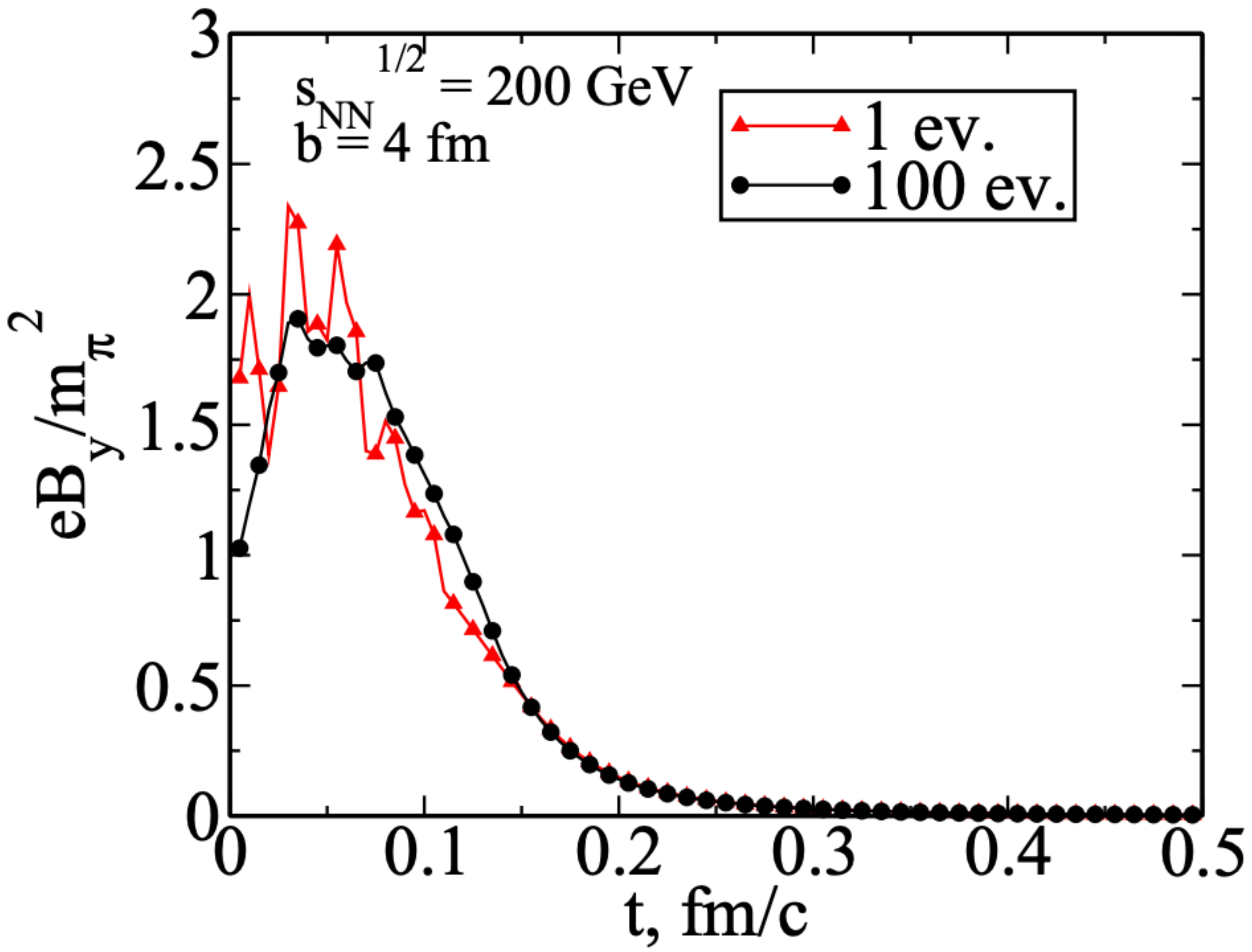
G. Endrodi, JHEP 1507(2015) 173

T-μ_B plane



Lifetime of the magnetic field created in the early stage of HIC



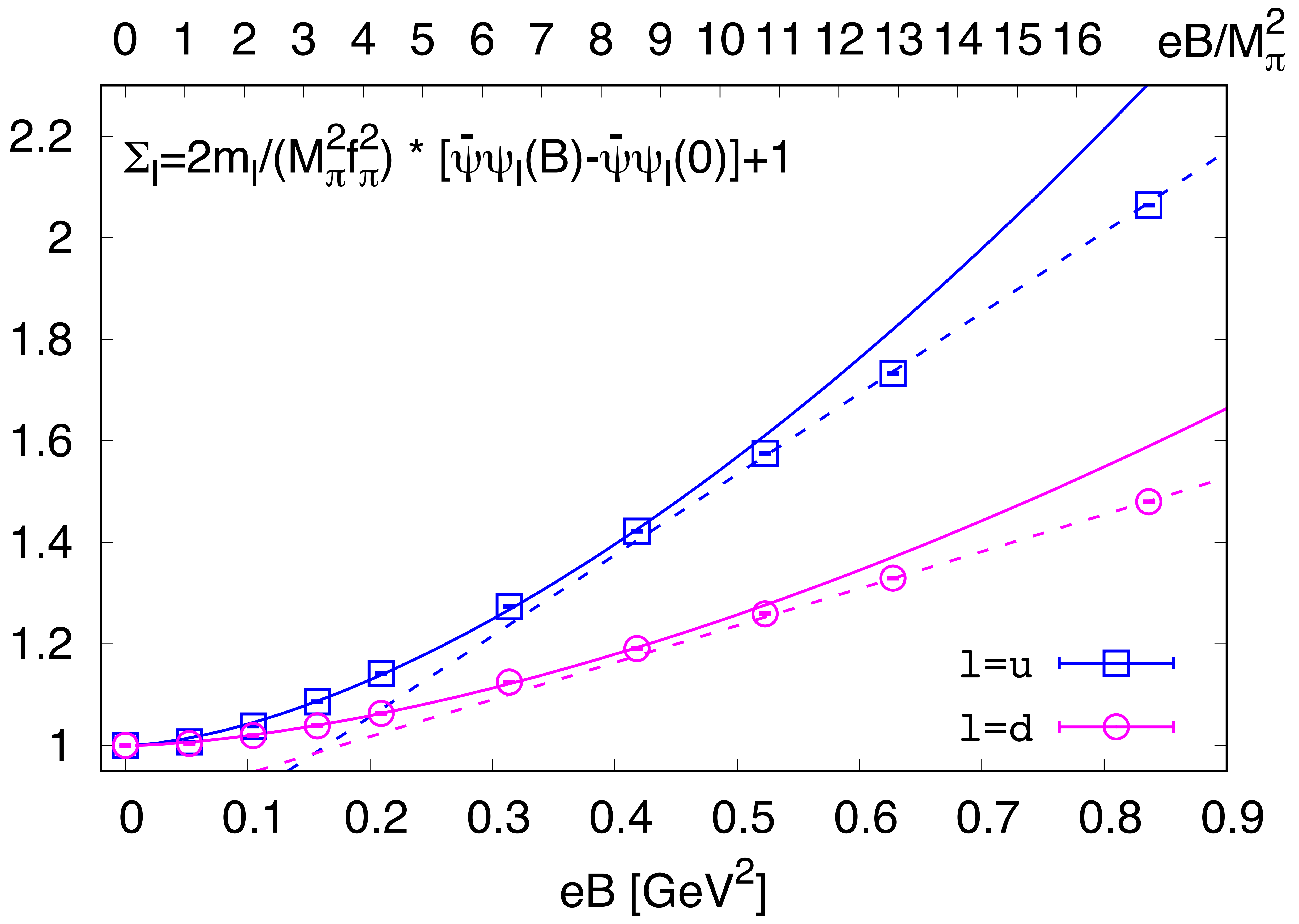


Skokov, Illarionov and V.Toneev, IJMPA 24 (2009) 5925

t=0: RHIC: $eB \sim m_\pi^2$

LHC: $eB \sim 15 m_\pi^2$

Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates



Not accessible in experiments

HTD, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, arXiv:2008.00493

See also in Bali et al., Phys.Rev.D86(2012)071502

Explore the QCD phase diagram through fluctuations of conserved charges x=B,Q,S

HIC

mean: M_x

variance: σ_x^2

skewness: S_x

kurtosis: κ_x

hyper-skewness: S_x^h

hyper-kurtosis: κ_x^h

Proxies:
proton, charge particles, kaons

$$\frac{M_x(\sqrt{s})}{\sigma_x^2(\sqrt{s})} = \frac{\langle N_x \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi_1^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{12}^x(T,\mu_B)$$

$$\frac{S_x(\sqrt{s})\,\sigma_x^3(\sqrt{s})}{M_x(\sqrt{s})} = \frac{\langle (\delta N_x)^3 \rangle}{\langle N_x \rangle} = \frac{\chi_3^x(T,\mu_B)}{\chi_1^x(T,\mu_B)} = R_{31}^x(T,\mu_B)$$

$$\kappa_x(\sqrt{s})\,\sigma_x^2(\sqrt{s}) = \frac{\langle (\delta N_x)^4 \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi_4^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{42}^x(T,\mu_B)$$

$$\frac{S_x^h(\sqrt{s})\,\sigma_x^5(\sqrt{s})}{M_x(\sqrt{s})} = \frac{\langle (\delta N_x)^5 \rangle}{\langle N_x \rangle} = \frac{\chi_5^x(T,\mu_B)}{\chi_1^x(T,\mu_B)} = R_{51}^x(T,\mu_B)$$

$$\kappa_x^h(\sqrt{s})\,\sigma_x^4(\sqrt{s}) = \frac{\langle (\delta N_x)^6 \rangle}{\langle (\delta N_x)^2 \rangle} = \frac{\chi_6^x(T,\mu_B)}{\chi_2^x(T,\mu_B)} = R_{62}^x(T,\mu_B)$$

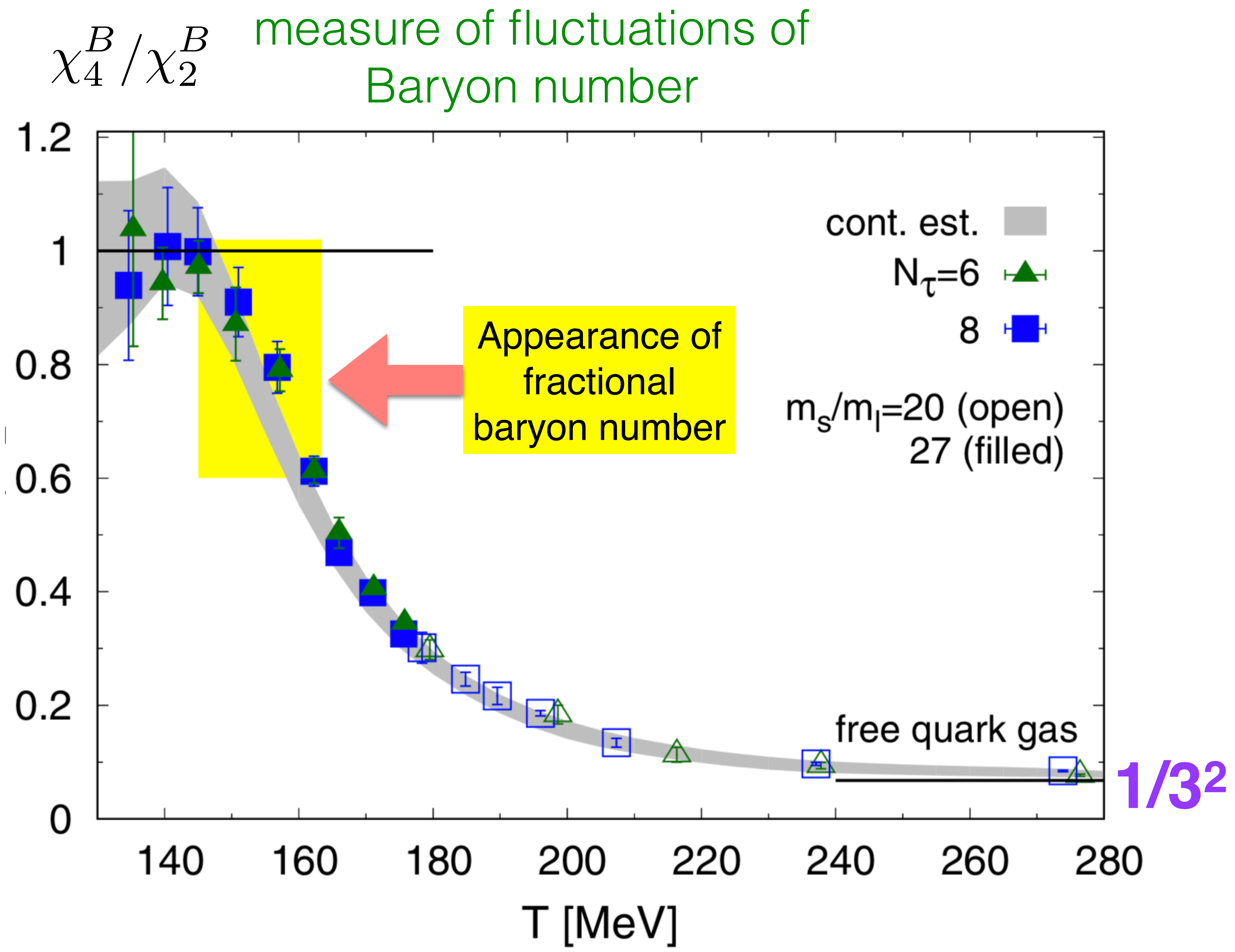
LQCD

generalized susceptibilities

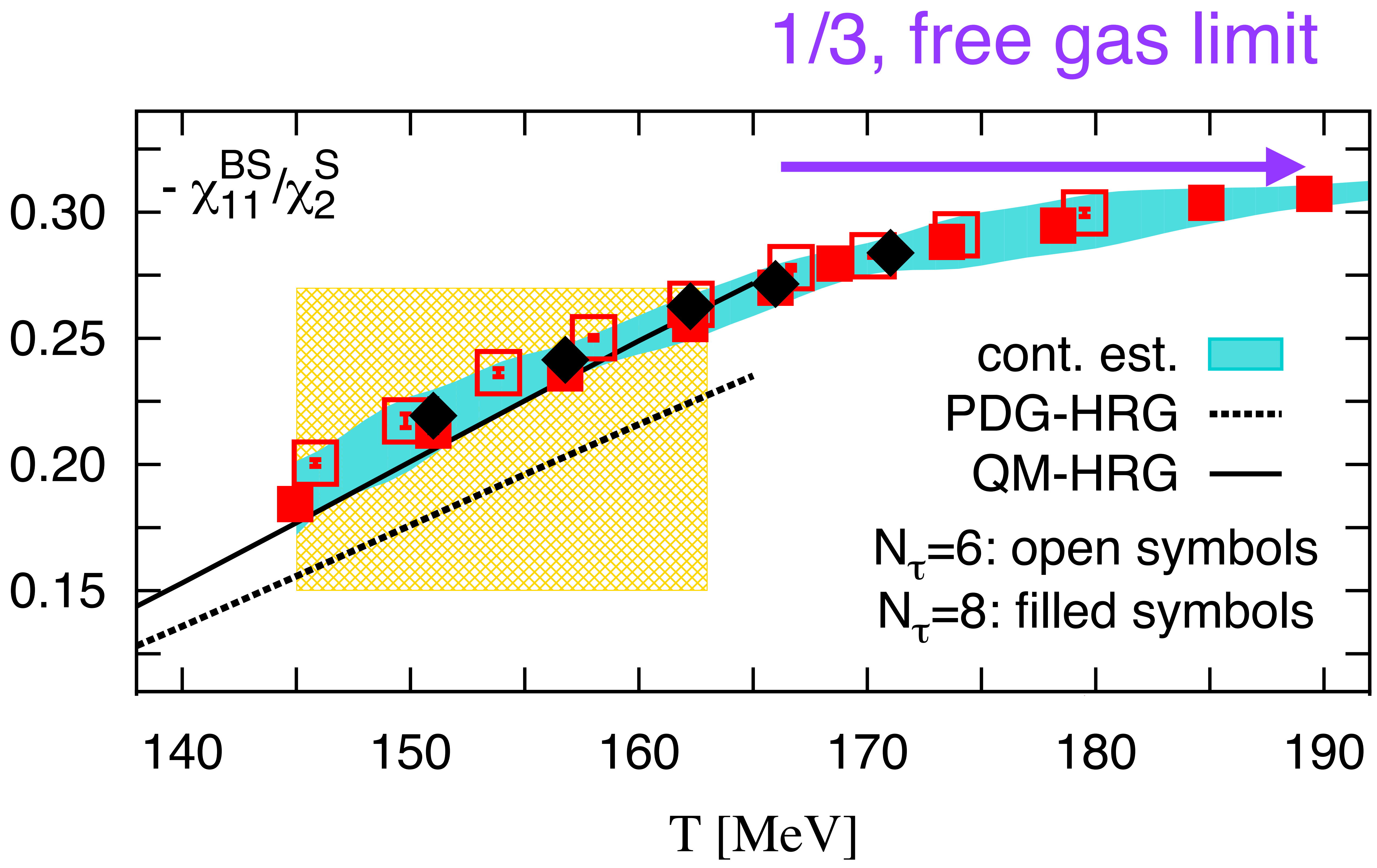
$$\chi_n^x(T,\mu_B) = \frac{1}{VT^3}\frac{\partial^n \ln Z(T,\vec{\mu})}{\partial(\mu_x/T)^n}$$

See recent reviews e.g. X.F. Luo and N. Xu, Nucl. Sci. Tech. 28 (2017) 112,
HTD, S. Mukherjee and F. Karsch, Int. J. Mod. Phys. E24 (2015) 1530007

Changes of degrees of freedom in thermal QCD



HotQCD: PRL 111(2013) 082301,
HTD, F. Karsch, S. Mukherjee, arXiv: 1504.05274



HotQCD, PRL 113 (2014) 072001

V. Koch, A. Majumder, and J. Randrup , PRL95 (2005) 182301

Fluctuations of net baryon number, electric charge and strangeness

- Taylor expansion of the QCD pressure:

Allton et al., Phys.Rev. D66 (2002) 074507
Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln \mathcal{Z}(T,V,\hat{\mu}_u,\hat{\mu}_d,\hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!}\left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- Taylor expansion coefficients at μ=0 are computable in LQCD

$$\hat{\chi}_{ijk}^{uds} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial(\mu_u/T)^i\,\partial(\mu_d/T)^j\,\partial(\mu_s/T)^k}\right|_{\mu_{u,d,s}=0}$$

$$\hat{\chi}_{ijk}^{\mathrm{BQS}} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial(\mu_\mathrm{B}/T)^i\,\partial(\mu_\mathrm{Q}/T)^j\,\partial(\mu_\mathrm{S}/T)^k}\right|_{\mu_{\mathrm{B,Q,S}}=0}$$

$$\mu_u = \frac{1}{3}\mu_\mathrm{B} + \frac{2}{3}\mu_\mathrm{Q}\,, \quad \mu_d = \frac{1}{3}\mu_\mathrm{B} - \frac{1}{3}\mu_\mathrm{Q}\,, \quad \mu_s = \frac{1}{3}\mu_\mathrm{B} - \frac{1}{3}\mu_\mathrm{Q} - \mu_\mathrm{S}\,.$$

- At eB=/=0 a lot more need to be explored

HRG: G. Kadam et al., JPG 47 (2020) 125106, Ferreira et al., PRD 98(2018)034003, Fukushima and Hidaka, PRL117 (2016)102301 Bhattacharyya et al., EPL115(2016)62003

PNJL: W.-J. Fu, Phys. Rev. D 88 (2013) 014009

2nd order fluctuations and correlations

B,Q & S $\Longleftrightarrow$ u, d & s

$$\chi_2^B = \frac{1}{9}\left(\chi_2^u + \chi_2^d + \chi_2^s + 2\chi_{11}^{us} + 2\chi_{11}^{ds} + 2\chi_{11}^{ud}\right),$$

$$\chi_2^Q = \frac{1}{9}\left(4\chi_2^u + \chi_2^d + \chi_2^s - 4\chi_{11}^{us} + 2\chi_{11}^{ds} - 4\chi_{11}^{ud}\right),$$

$$\chi_2^S = \chi_2^s,$$

$$\chi_{11}^{BQ} = \frac{1}{9}\left(2\chi_2^u - \chi_2^d - \chi_2^s + \chi_{11}^{us} - 2\chi_{11}^{ds} + \chi_{11}^{ud}\right),$$

$$\chi_{11}^{BS} = -\frac{1}{3}\left(\chi_2^s + \chi_{11}^{us} + \chi_{11}^{ds}\right),$$

$$\chi_{11}^{QS} = \frac{1}{3}\left(\chi_2^s - 2\chi_{11}^{us} + \chi_{11}^{ds}\right).$$

At eB=0 (isospin symmetric case)

$$\chi_{11}^{us} = \chi_{11}^{ds}, \quad \chi_2^u = \chi_2^d$$

$$2\chi_{11}^{\mathrm{QS}} - \chi_{11}^{\mathrm{BS}} = \chi_2^{\mathrm{S}},$$
$$2\chi_{11}^{\mathrm{BQ}} - \chi_{11}^{\mathrm{BS}} = \chi_2^{\mathrm{B}}.$$

High T: Ideal gas limit

- **At eB=0:** $\varepsilon^2 = m^2 + |\vec{p}|^2$ Kapusta & Gale, Finite-temperature field theory: Principles and applications

$$\frac{p}{T^4} = \frac{8\pi^2}{45} + \frac{7\pi^2}{20} + \sum_{f=u,d,s}\left[\frac{1}{2}\hat{\mu}_f^2 + \frac{1}{4\pi^2}\hat{\mu}_f^4\right]$$

- **At eB=/=0:** $\varepsilon_l^2 = p_z^2 + m^2 + 2qB(l + 1/2 - s_z)$ HTD, S.-T. Li, Q. Shi and X.-D. Wang, 2104.06843

$$\frac{p}{T^4} = \frac{8\pi^2}{45} + \sum_{f=u,d,s}\frac{3|q_f|B}{\pi^2T^2}\left[\frac{\pi^2}{12} + \frac{\hat{\mu}_f^2}{4} + 2\frac{\sqrt{2|q_f|B}}{T}\sum_{l=1}^{\infty}\sqrt{l}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\cosh\left(k\hat{\mu}_f\right)\times \mathrm{K}_1\left(\frac{k\sqrt{2|q_f|Bl}}{T}\right)\right]$$

High T: Ideal gas limit

$$\frac{\chi_2^{\mathrm{B}}}{eB} = \frac{4}{9\pi^2}\left(\frac{1}{2} + \hat{b}\sum_{l=1}^{\infty}\sqrt{l}\sum_{k=1}^{\infty}(-1)^{k+1}k\times\left[\sqrt{2}\,\mathrm{K}_1\left(k\,\hat{b}\sqrt{2l}\right) + \mathrm{K}_1\left(k\,\hat{b}\sqrt{l}\right)\right]\right)$$

$$\frac{\chi_{11}^{\mathrm{BQ}}}{eB} = \frac{4}{9\pi^2}\left(\frac{1}{4} + \hat{b}\sum_{l=1}^{\infty}\sqrt{l}\sum_{k=1}^{\infty}(-1)^{k+1}k\times\left[2\sqrt{2}\,\mathrm{K}_1\left(k\,\hat{b}\sqrt{2l}\right) - \mathrm{K}_1\left(k\,\hat{b}\sqrt{l}\right)\right]\right)$$

$$\hat{b} = \sqrt{2eB/3}/T$$

$$\sqrt{eB}/T \to \infty$$

Quantity	Value
χ_2^u/eB	$1/\pi^2$
$\chi_2^{d/s/S}/eB$	$1/(2\pi^2)$
$\chi_{11}^{ud}/eB = \chi_{11}^{us}/eB = \chi_{11}^{ds}/eB{=}0$	0
χ_2^{B}/eB	$2/(9\pi^2)$
χ_2^{Q}/eB	$5/(9\pi^2)$
$\chi_{11}^{\mathrm{BQ}}/eB$	$1/(9\pi^2)$
$\chi_{11}^{\mathrm{QS}}/eB = -\chi_{11}^{\mathrm{BS}}/eB = \chi_2^{\mathrm{S}}/3eB$	$1/(6\pi^2)$

$$eB = 0$$

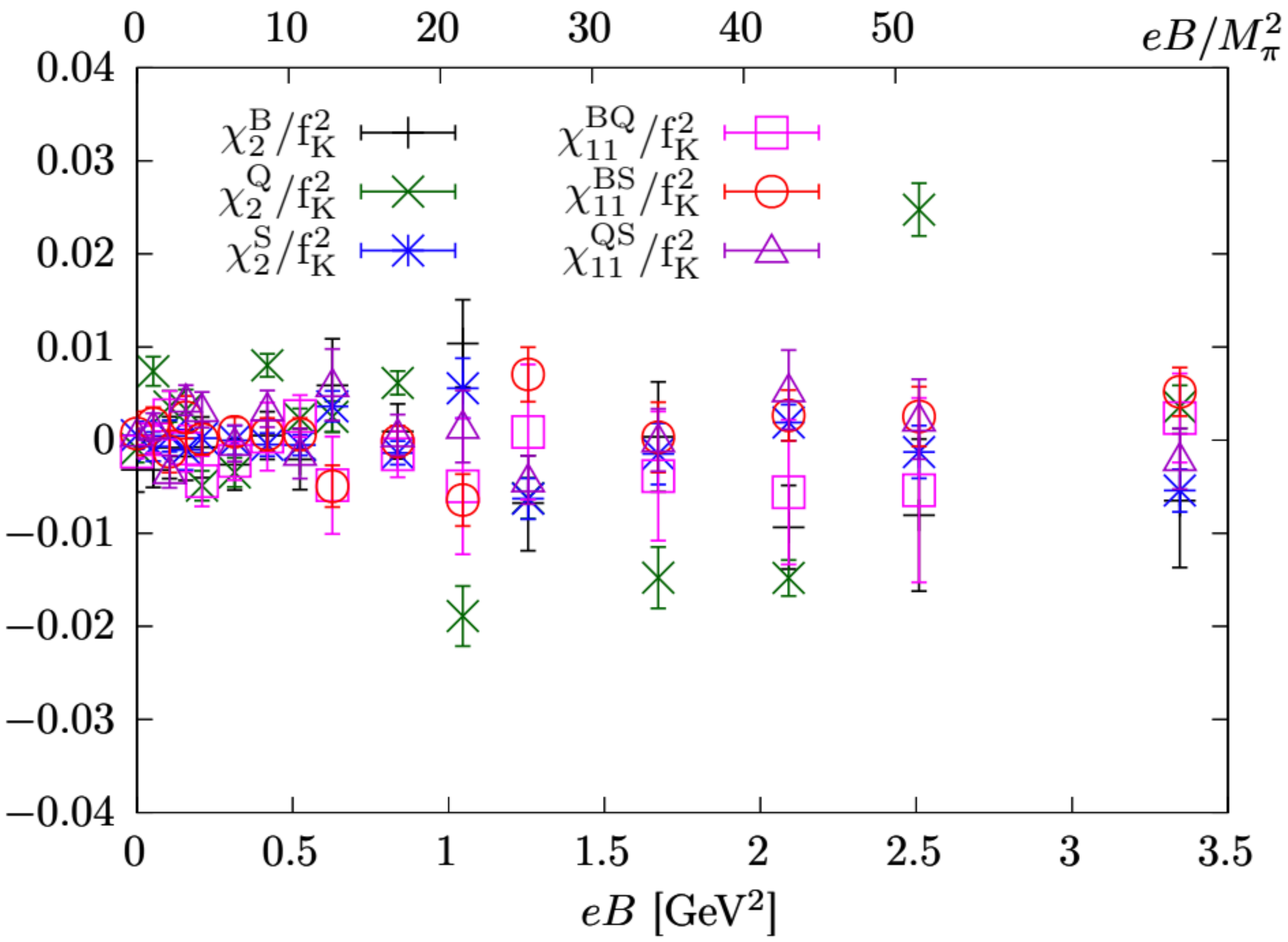
$$\chi_2^{\mathrm{B}} = \chi_{11}^{\mathrm{QS}} = -\chi_{11}^{\mathrm{BS}} = \chi_2^{\mathrm{Q}}/2 = \chi_2^{\mathrm{S}}/3 = 1/3$$

$$\chi_{11}^{\mathrm{BQ}} = 0\,.$$

Holds at both eB=0 and eB=/=0 with T→∞

$$\chi_{11}^{\mathrm{BS}}/\chi_2^{\mathrm{S}} = -\chi_{11}^{\mathrm{QS}}/\chi_2^{\mathrm{S}} = -\frac{1}{3}$$

No evidence for a Superconducting phase at T=0

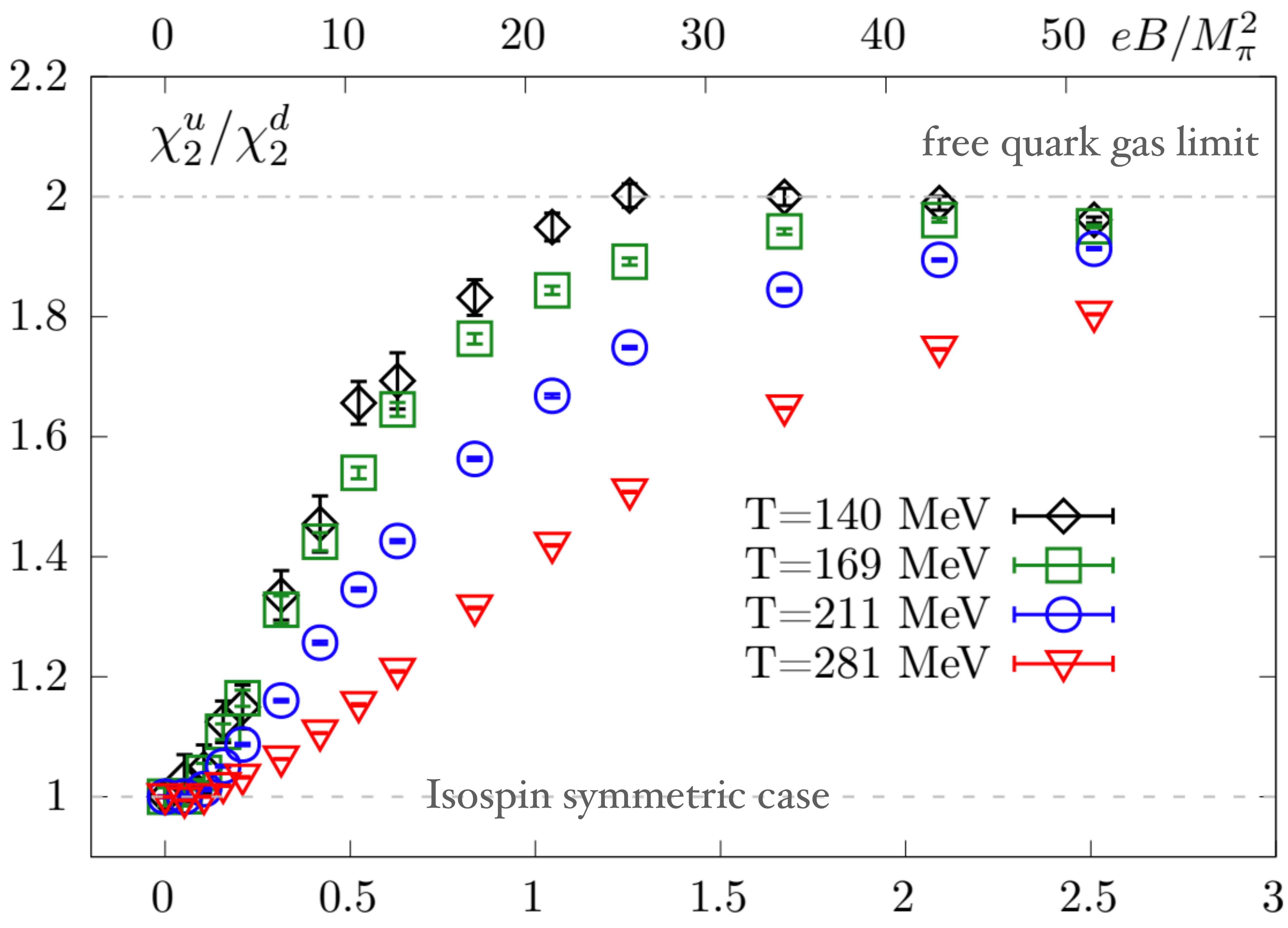


rho is a boson

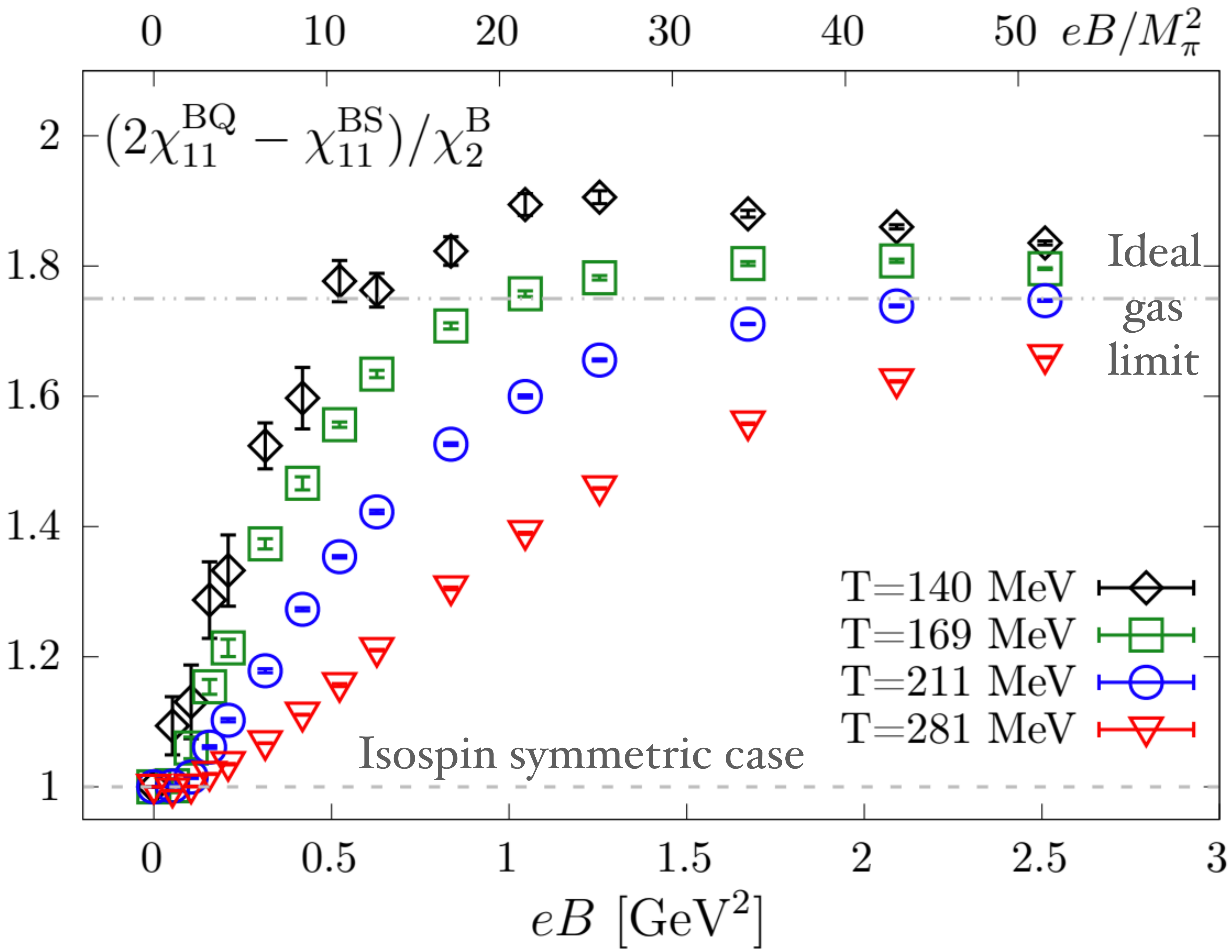
$$f(E) = \frac{1}{e^{E/kT} - 1}$$

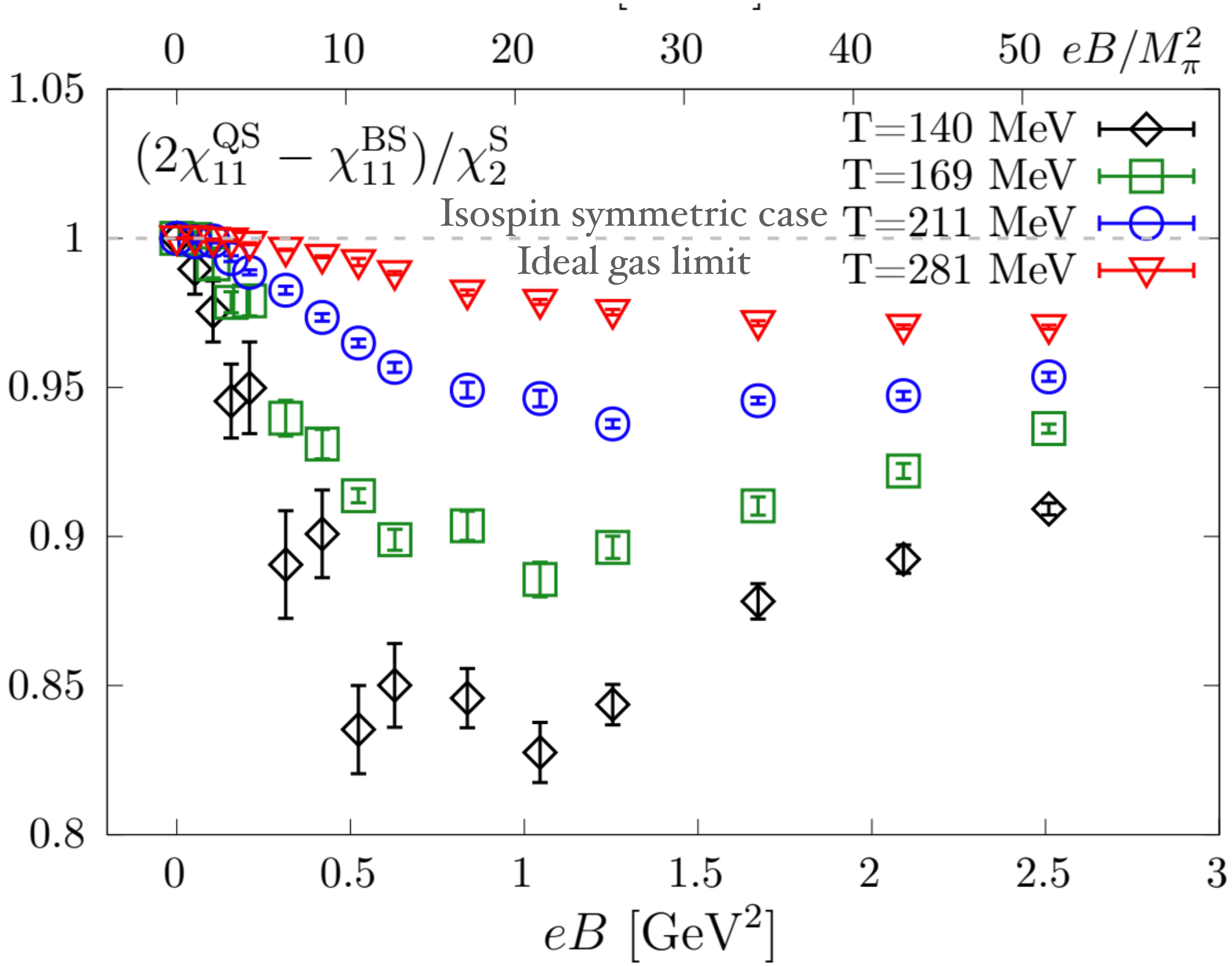
If the energy of rho becomes zero, electric charge fluctuation χ_2^Q shall be divergent

Isospin symmetry breaking at $eB\neq 0$

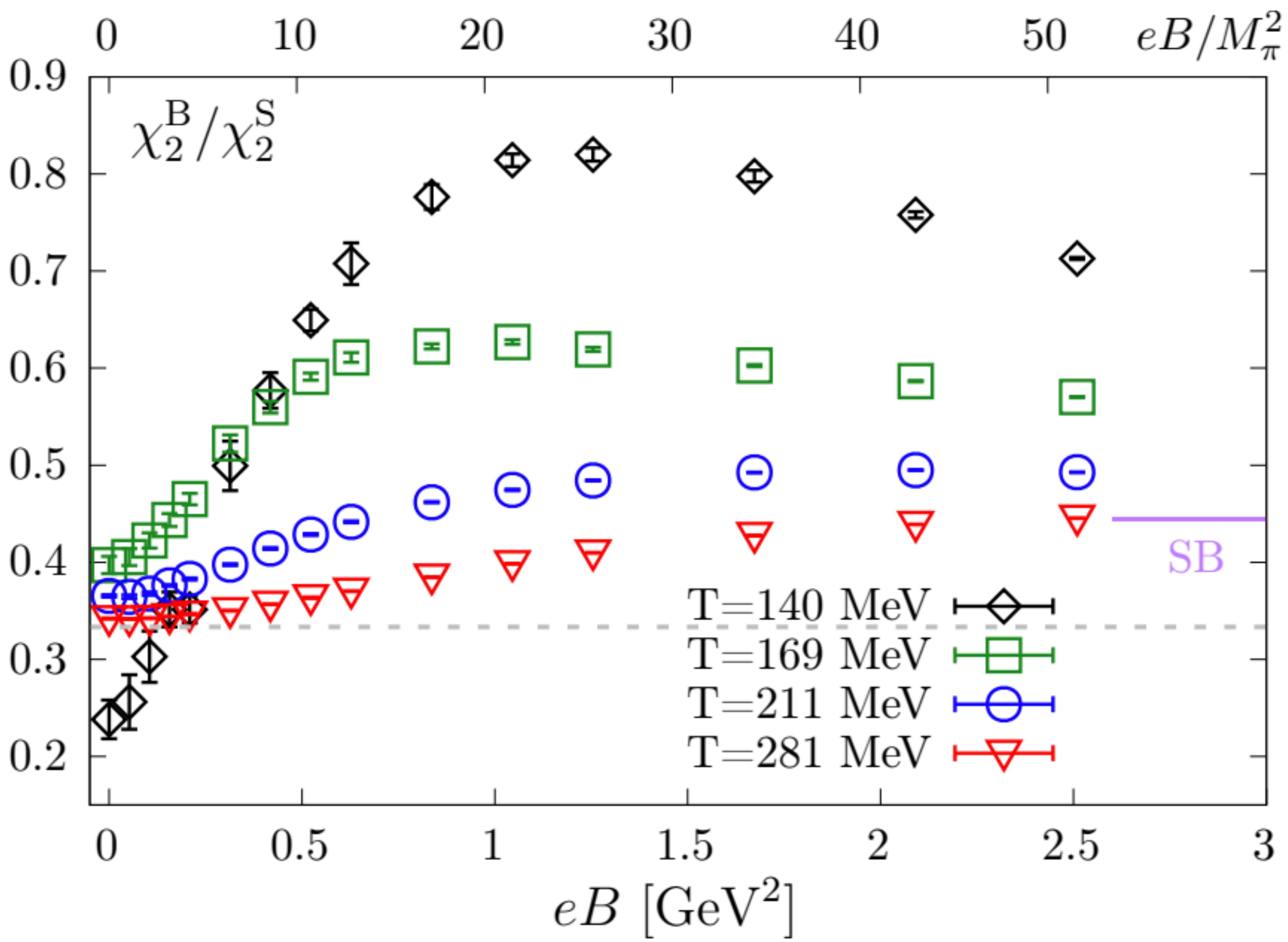


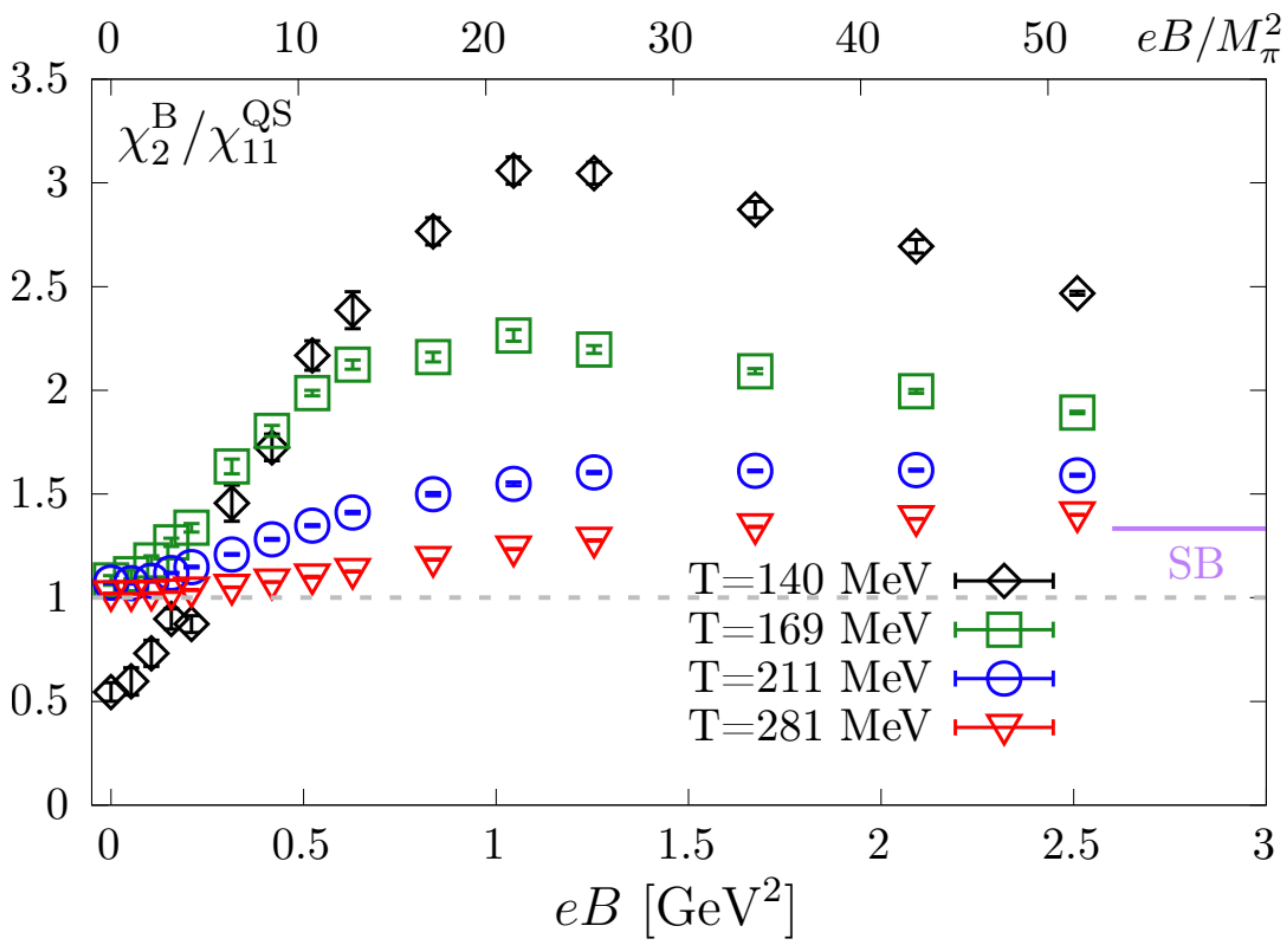
Experimentally accessible quantities for probing isospin symmetry breaking



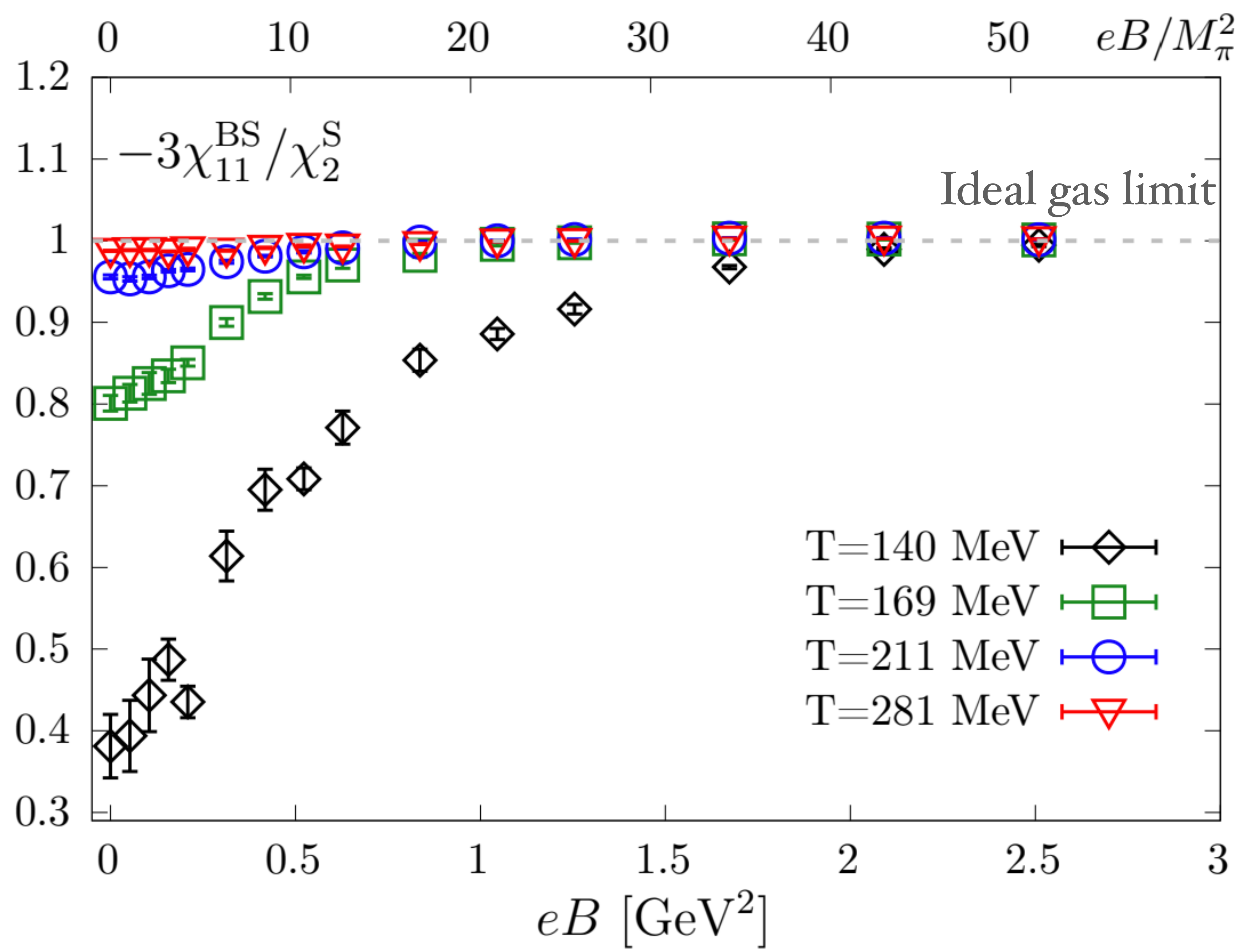


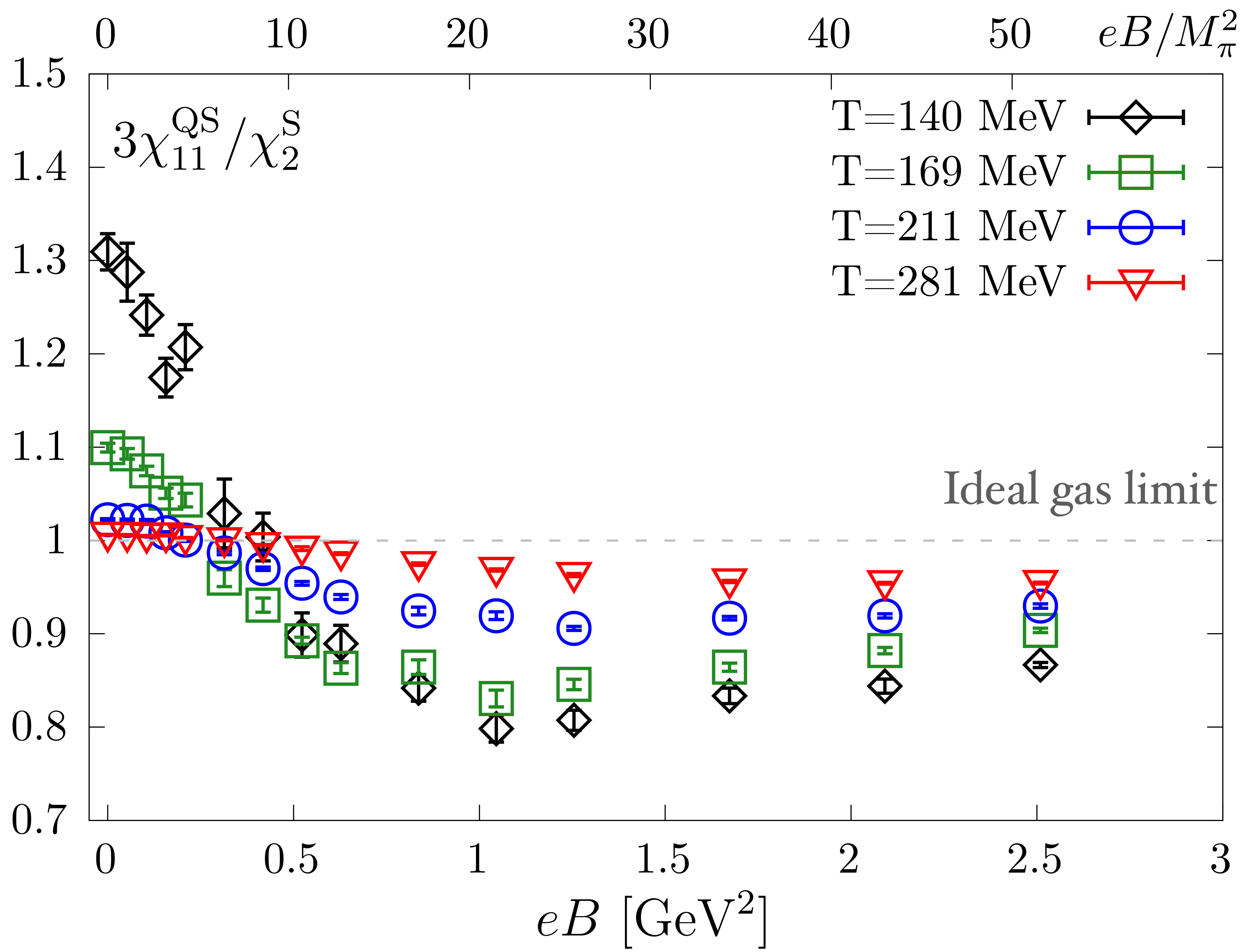
Experimentally accessible quantities for probing the (non-)existence of a magnetic field





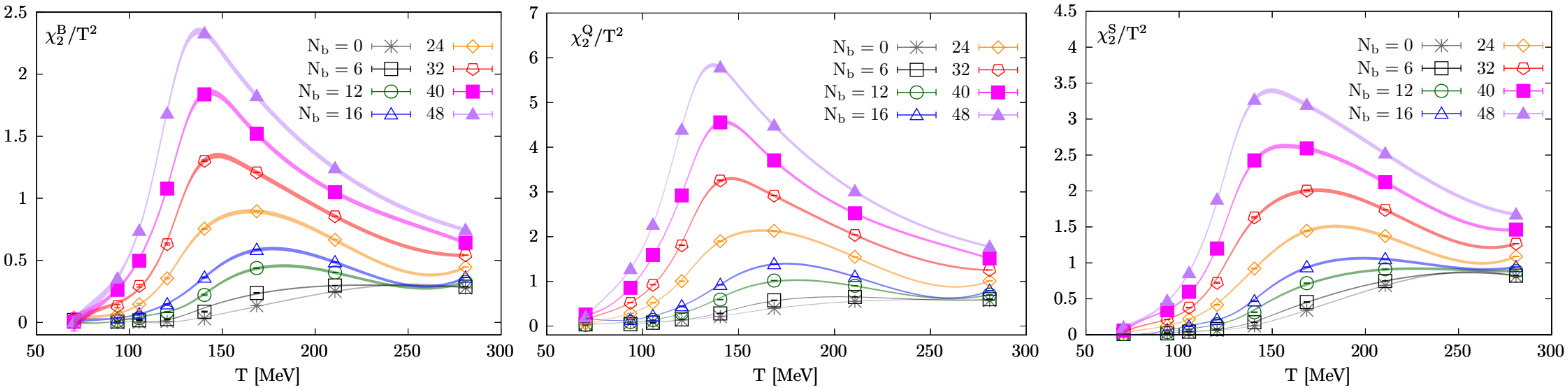
Experimentally accessible quantities for probing the (non-)existence of a magnetic field





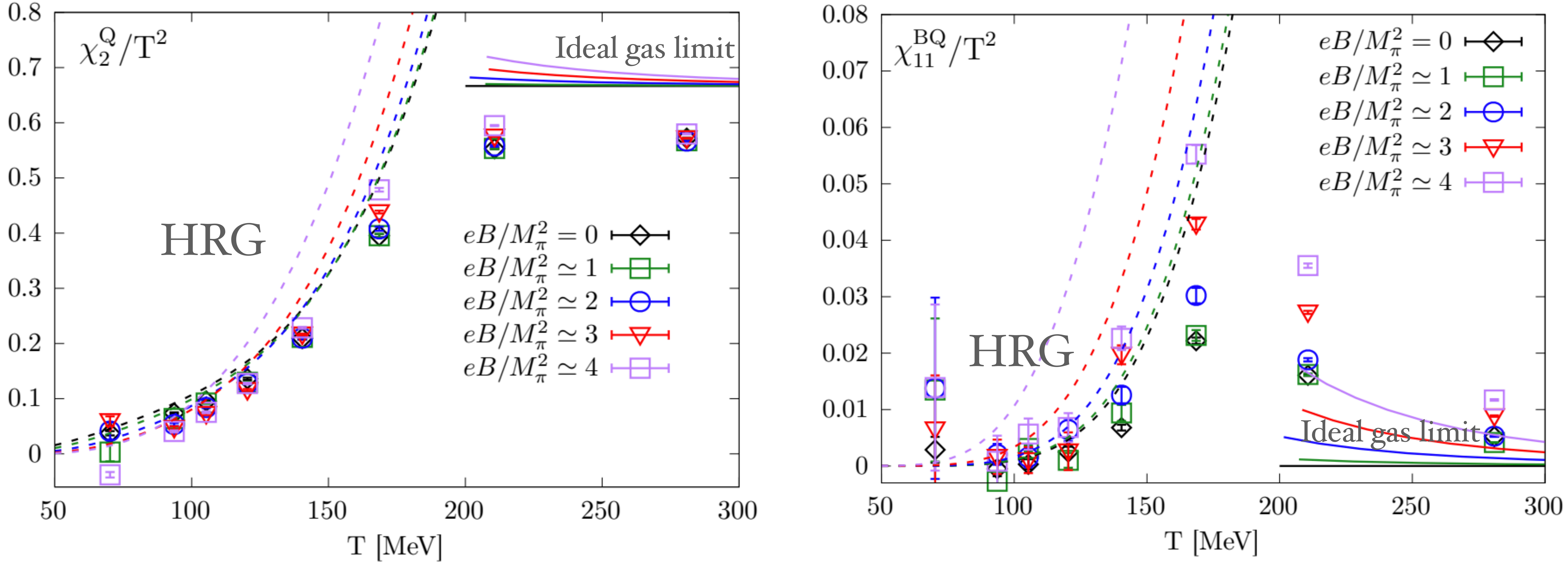
At both eB=0 and eB=/=0 with T→∞ : $-3\chi_{11}^{BS}/\chi_2^S = 3\chi_{11}^{QS}/\chi_2^S = 1$

2nd order fluctuations of net baryon number, electric charge and strangeness

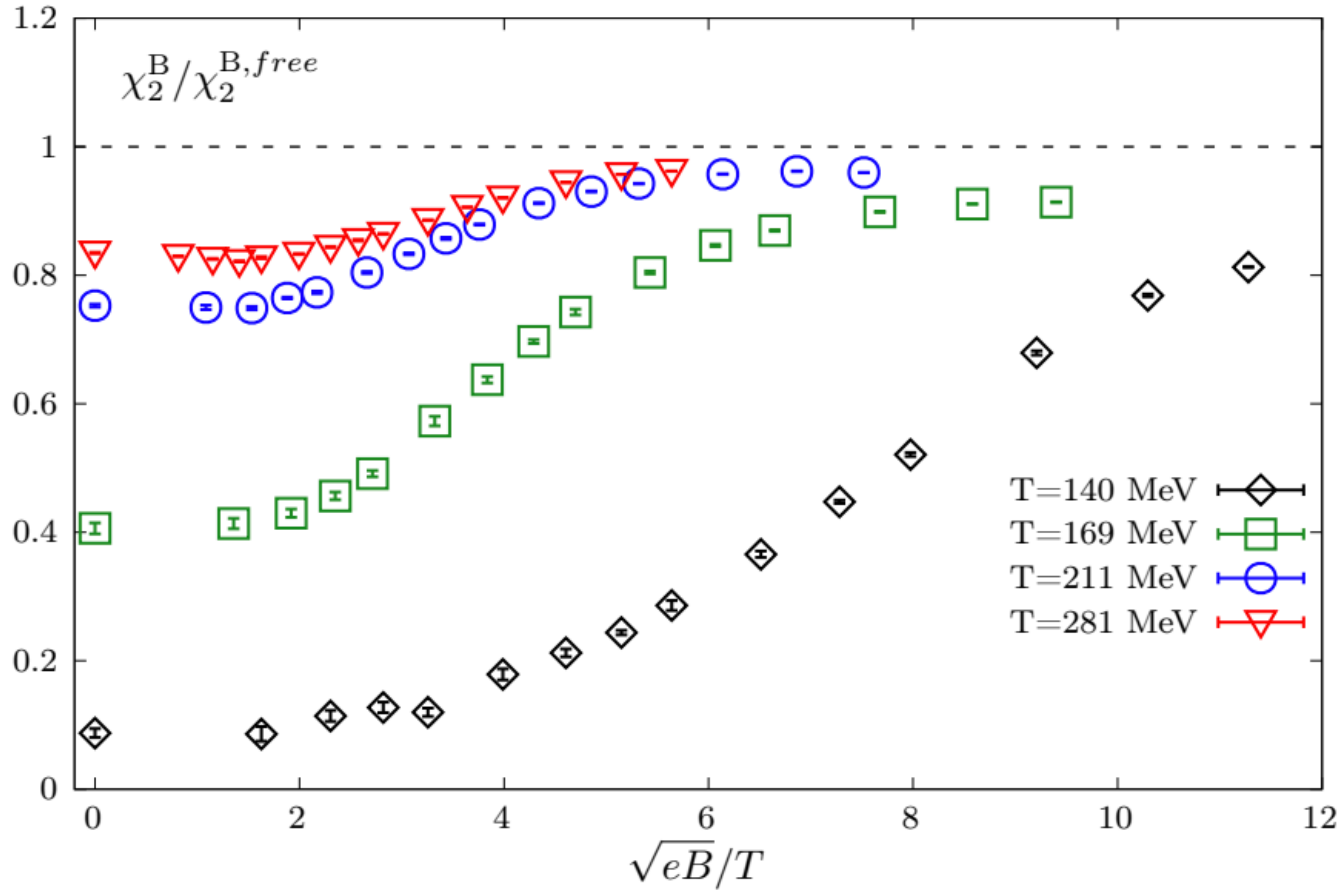


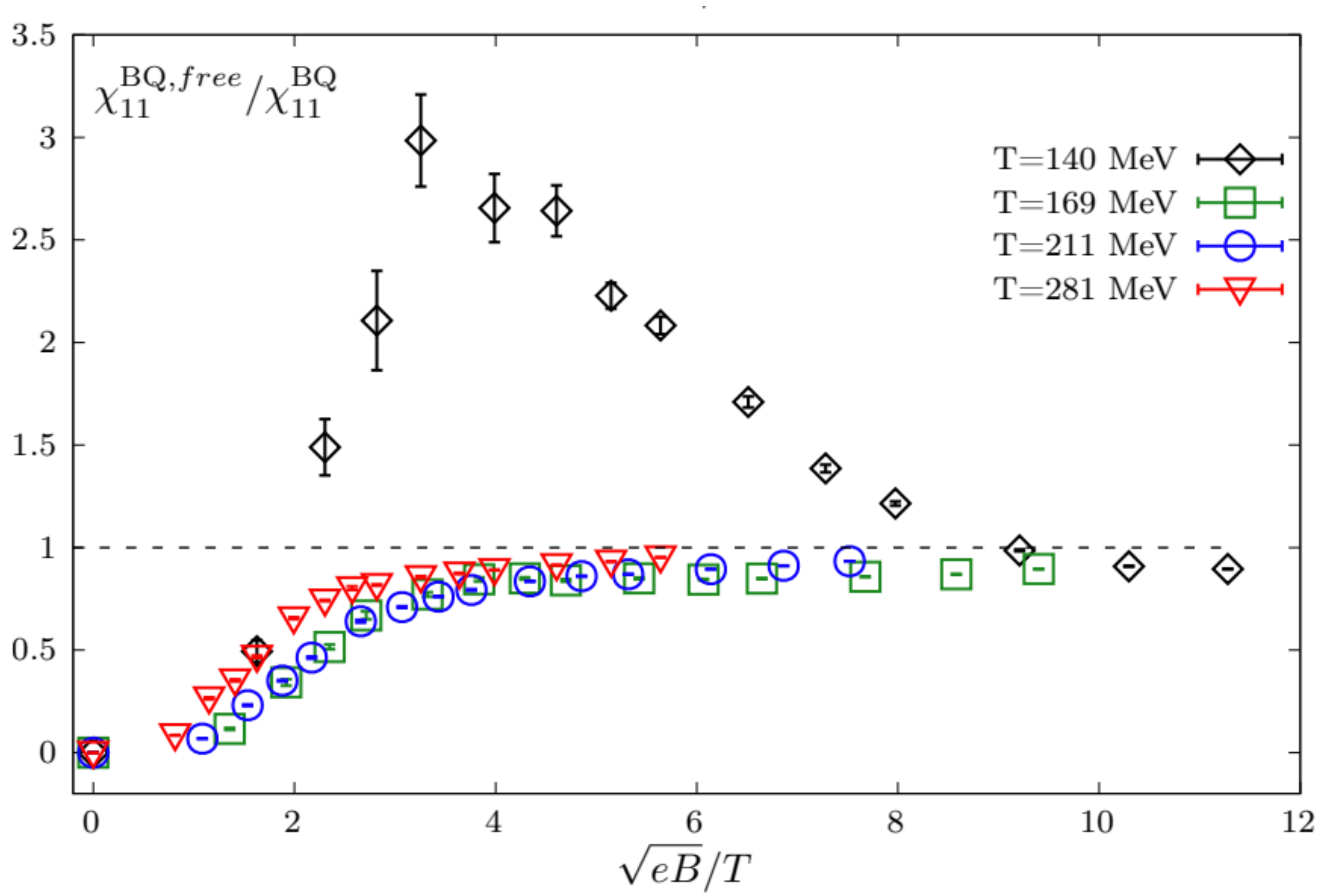
Signal for a Critical end point in the T-*eB* plane of QCD phase diagram?

Comparisons to HRG and Ideal gas limit

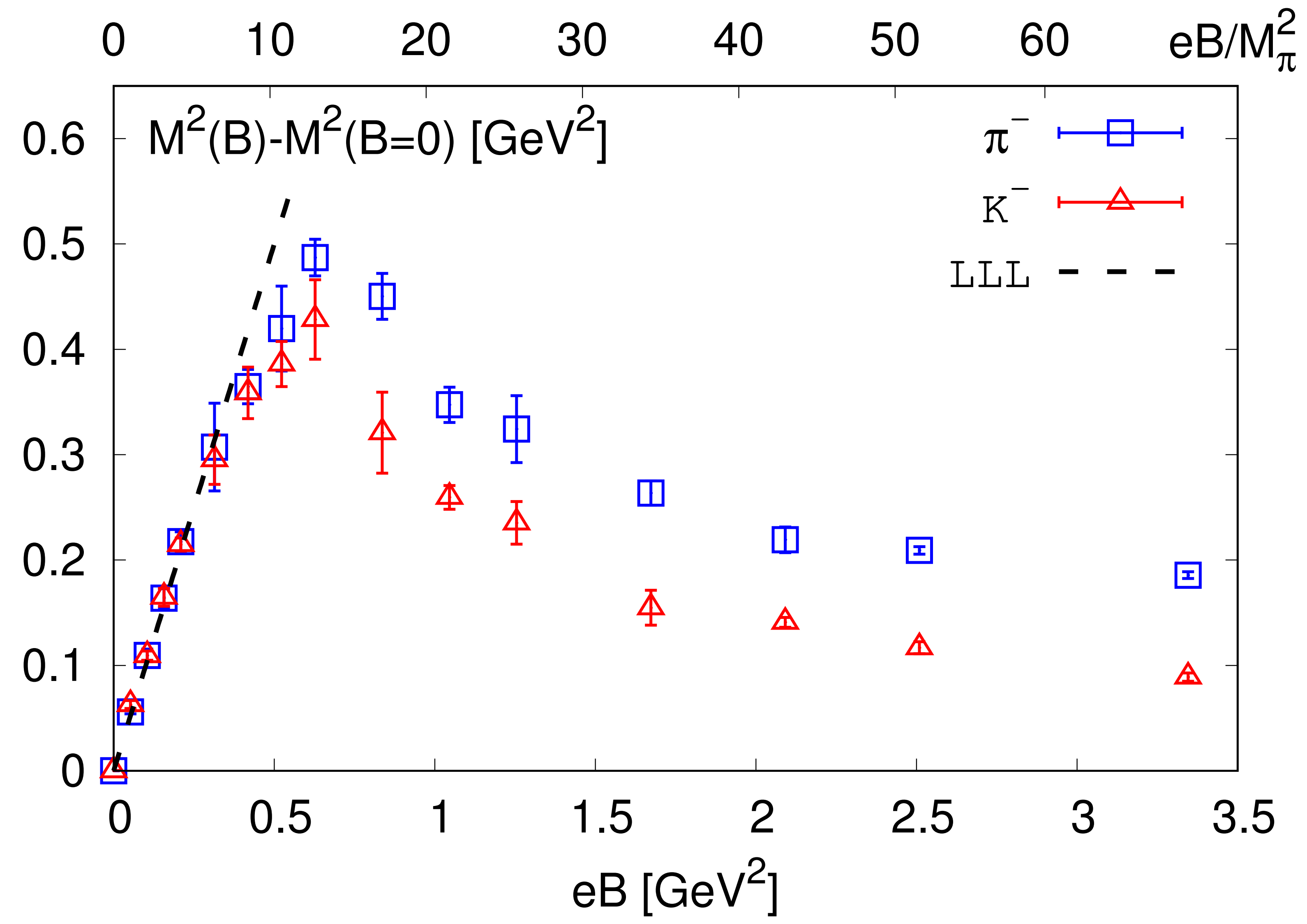


Ratio to ideal gas limits

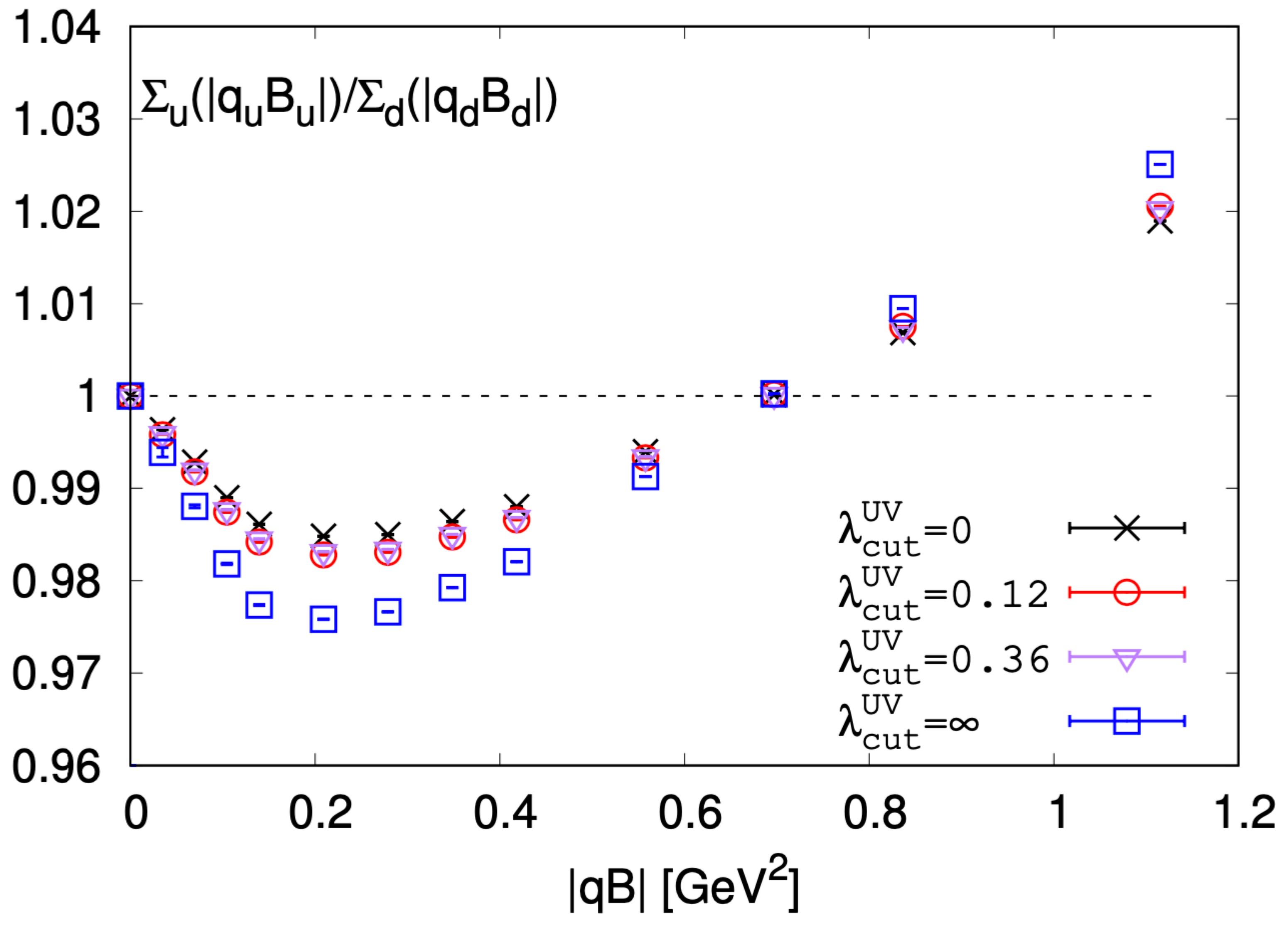




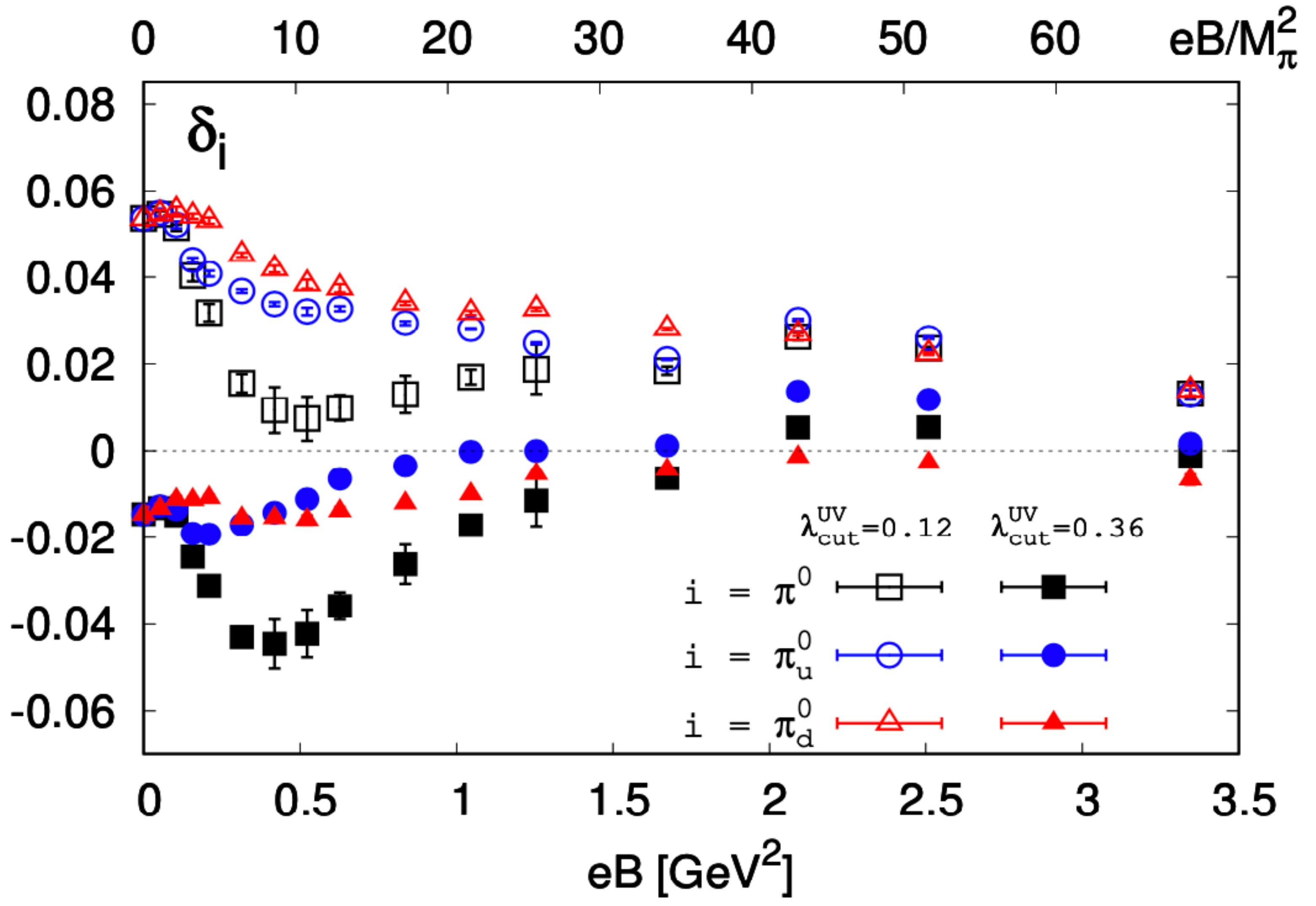
Summary



$M^2(B)-M^2(B=0)$ [GeV^2]
π^-
K^-
LLL
eB [GeV^2]
eB/M_π^2

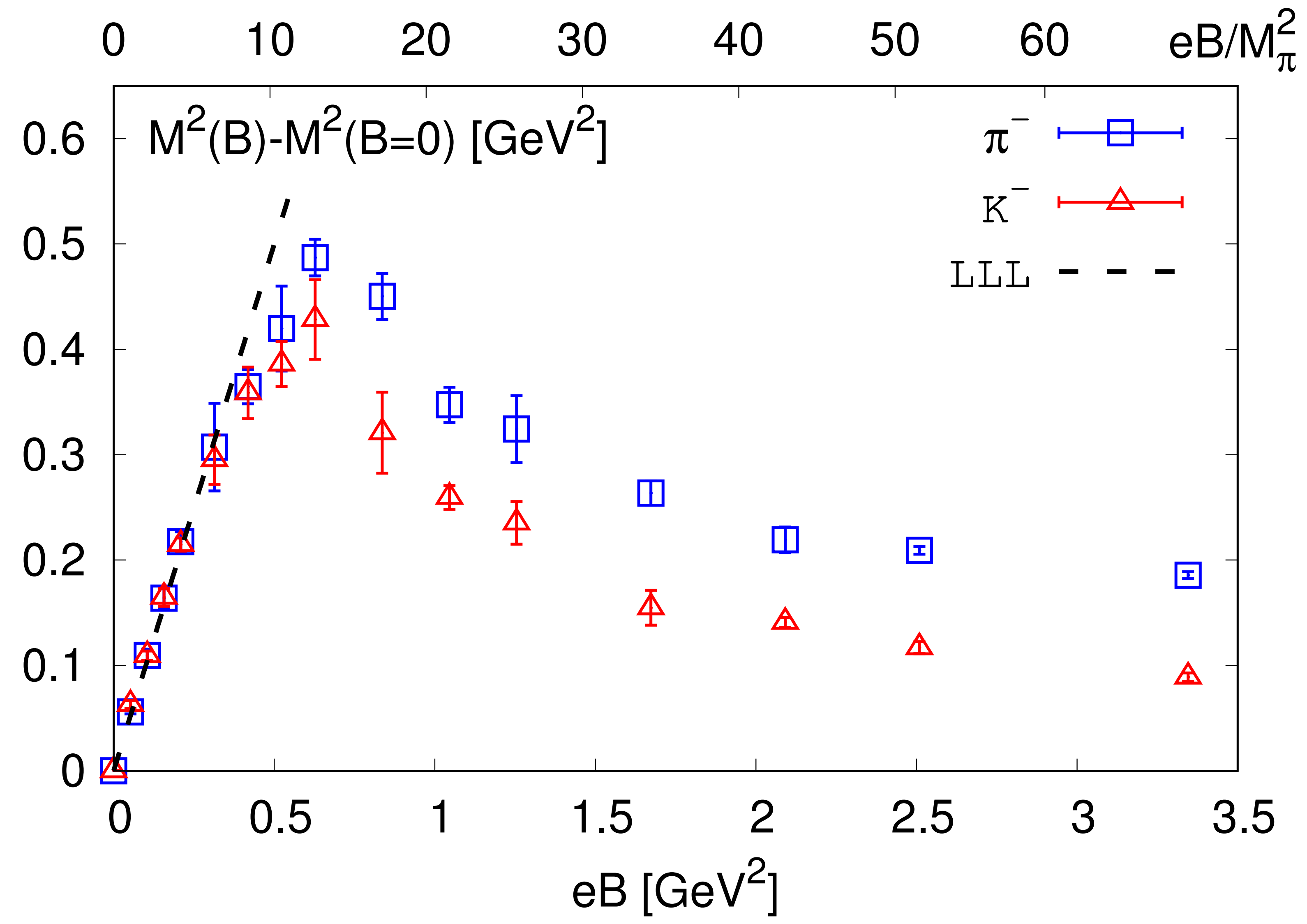


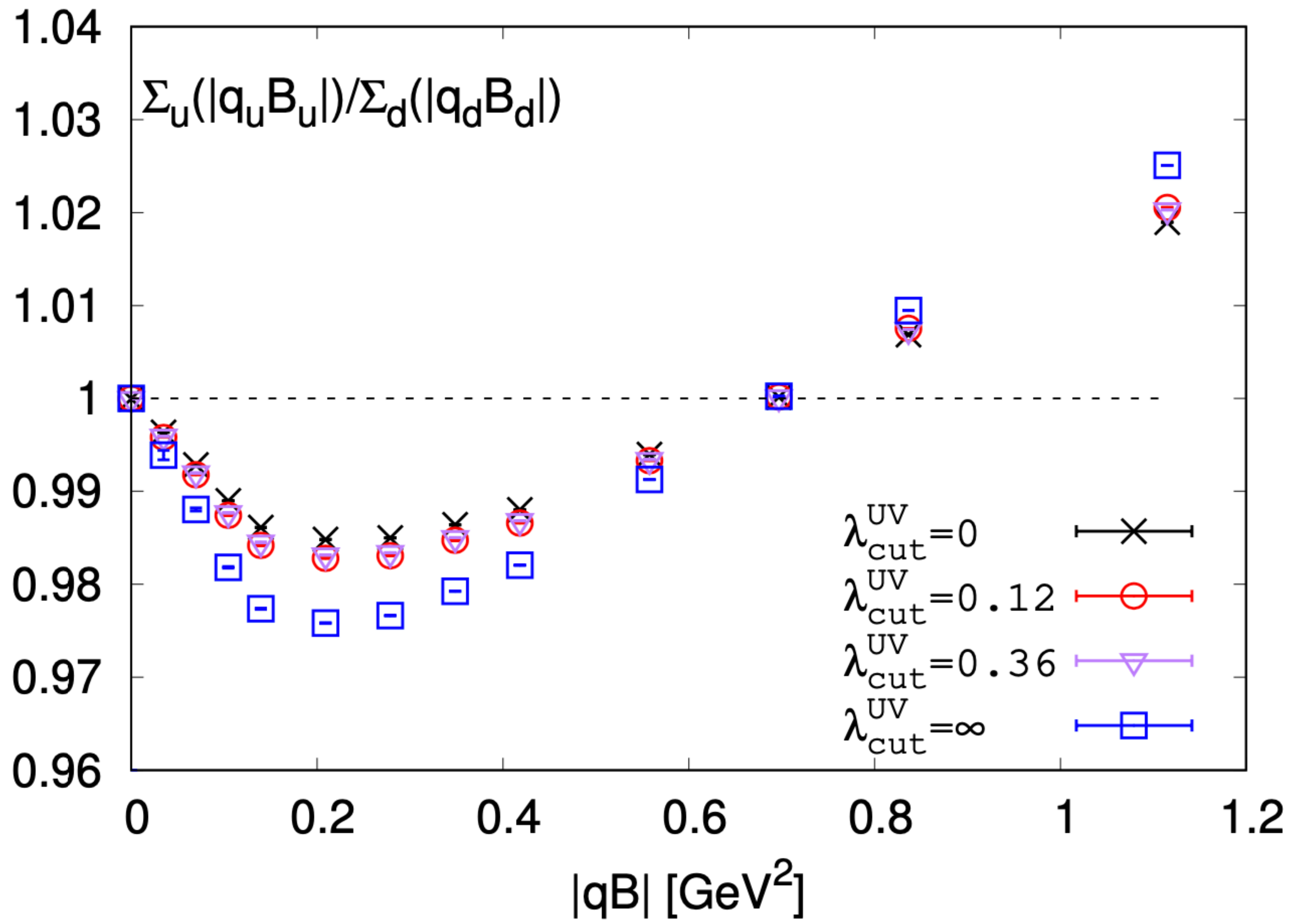
$\Sigma_u(|q_uB_u|)/\Sigma_d(|q_dB_d|)$
$\lambda_{cut}^{UV}=0$
$\lambda_{cut}^{UV}=0.12$
$\lambda_{cut}^{UV}=0.36$
$\lambda_{cut}^{UV}=\infty$
|qB| [GeV^2]

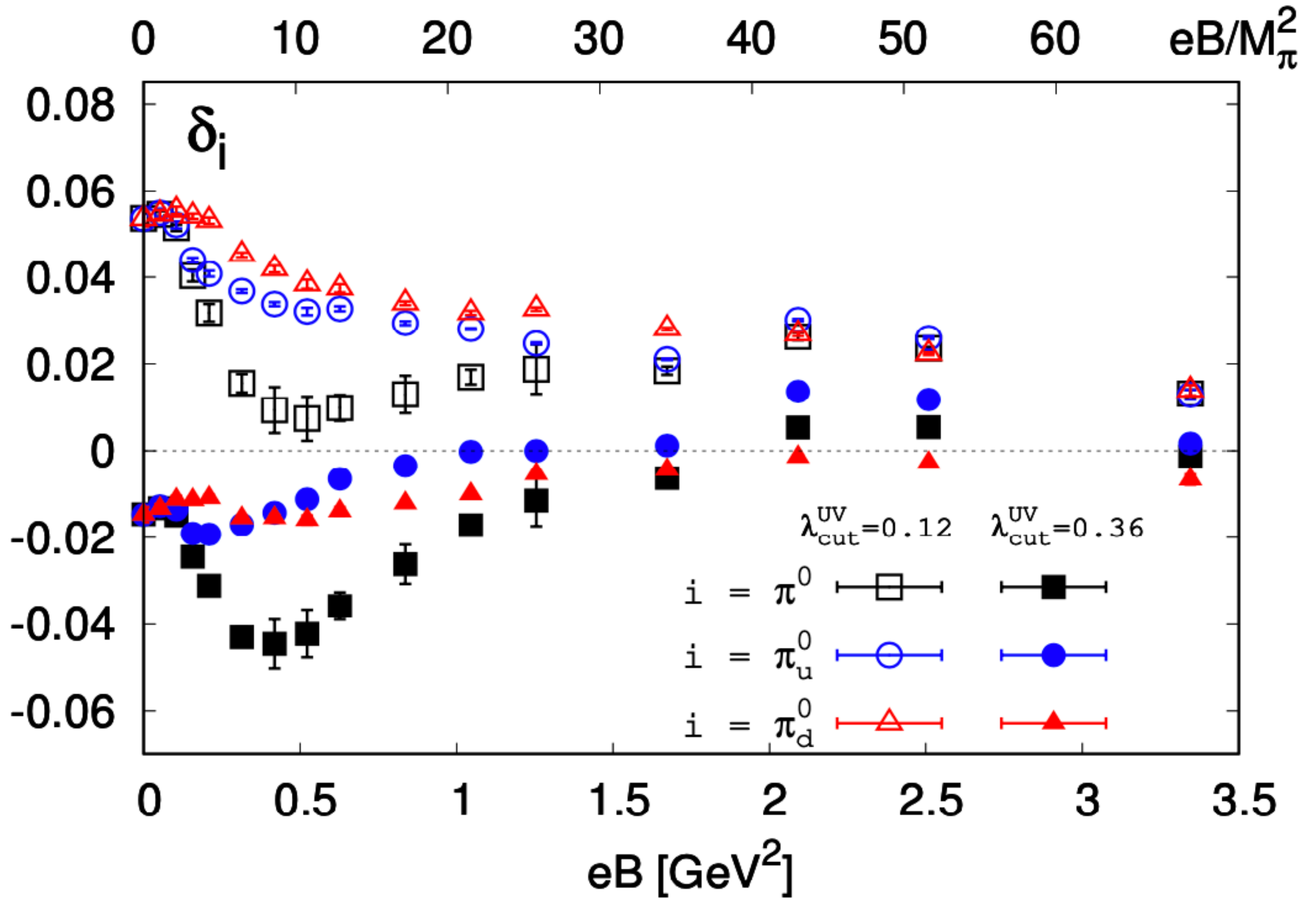


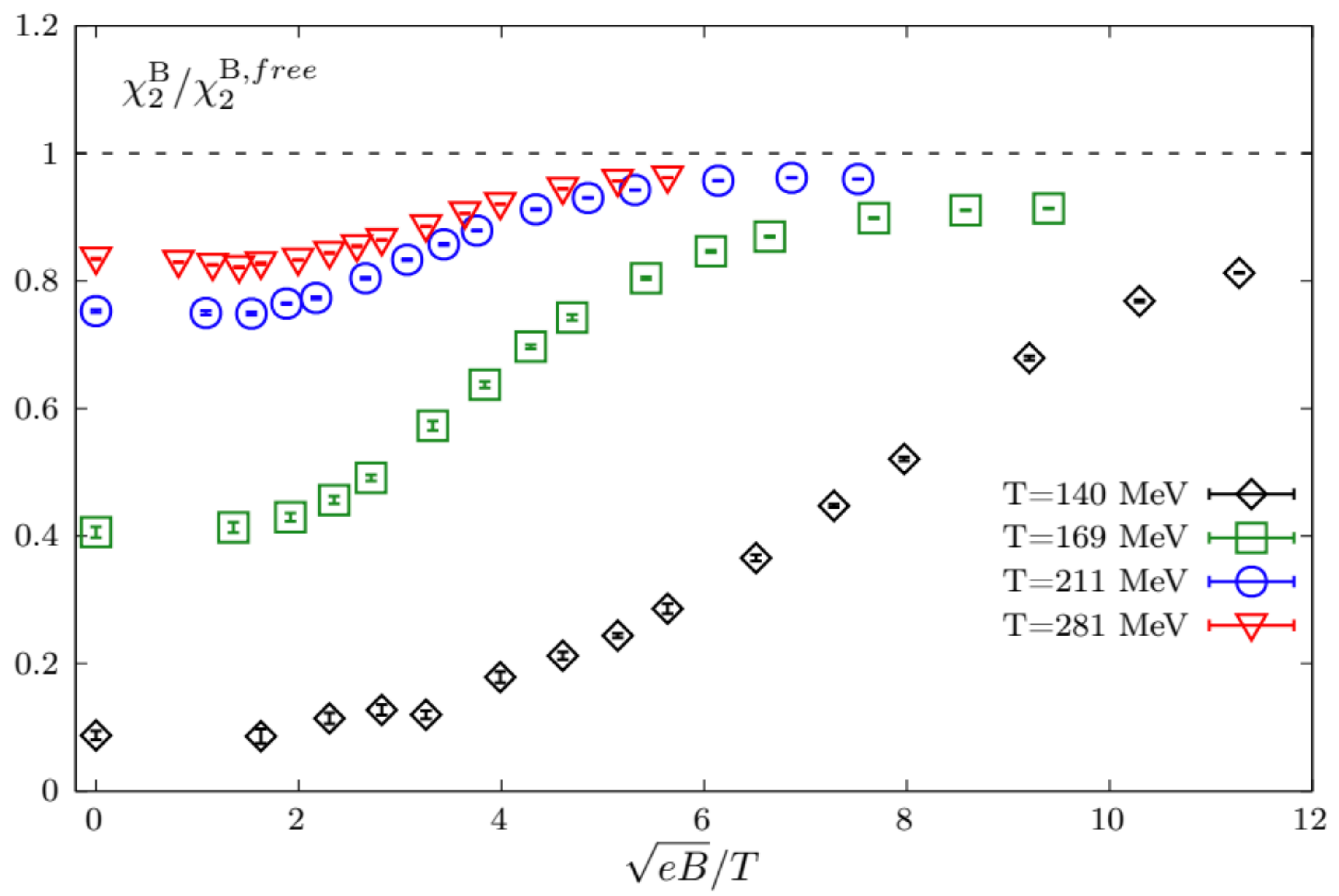
δ_i
$\lambda_{cut}^{UV}=0.12$ $\lambda_{cut}^{UV}=0.36$
i = π^0
i = π_u^0
i = π_d^0
eB [GeV^2]
eB/M_π^2

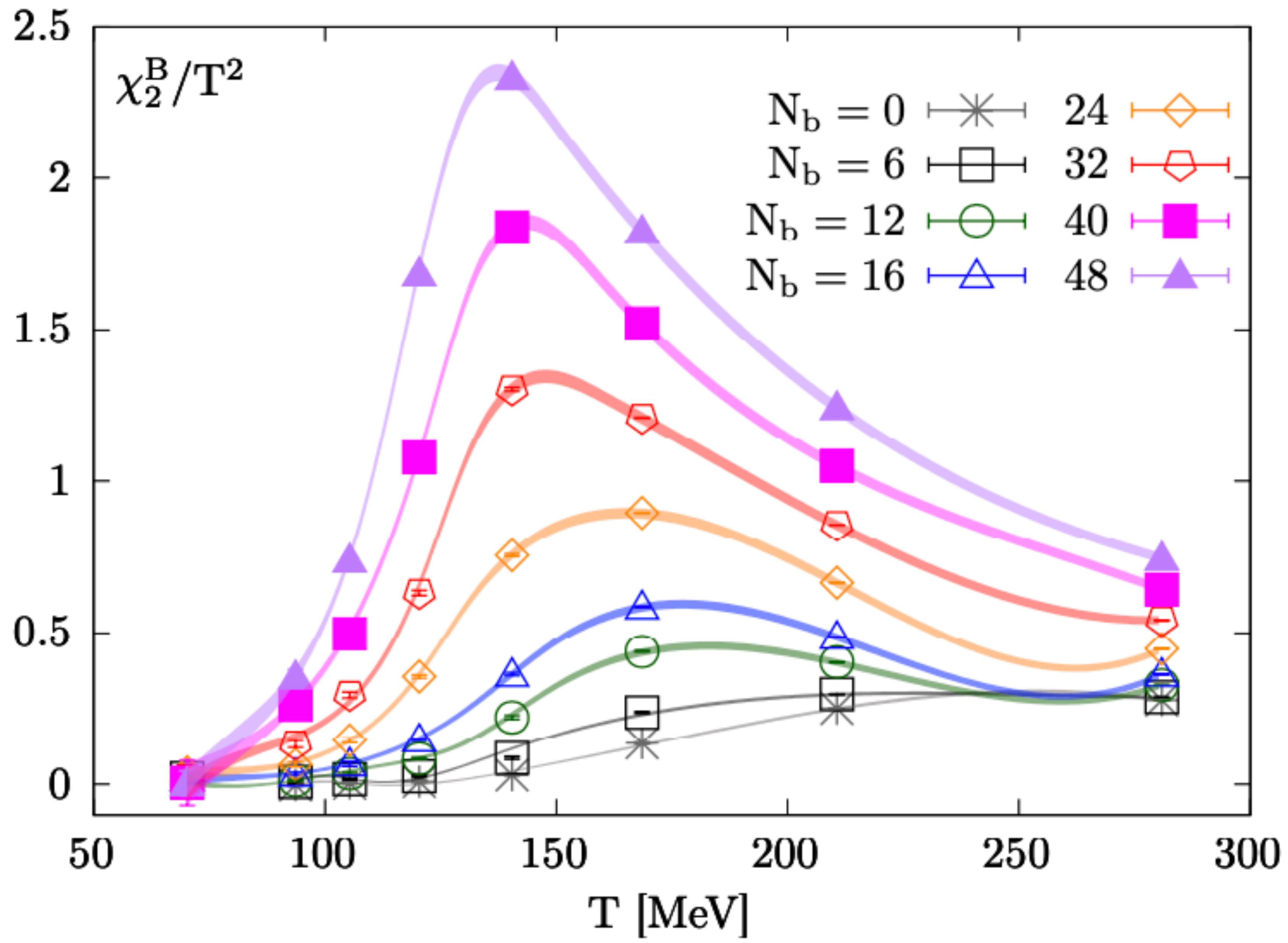
Summary

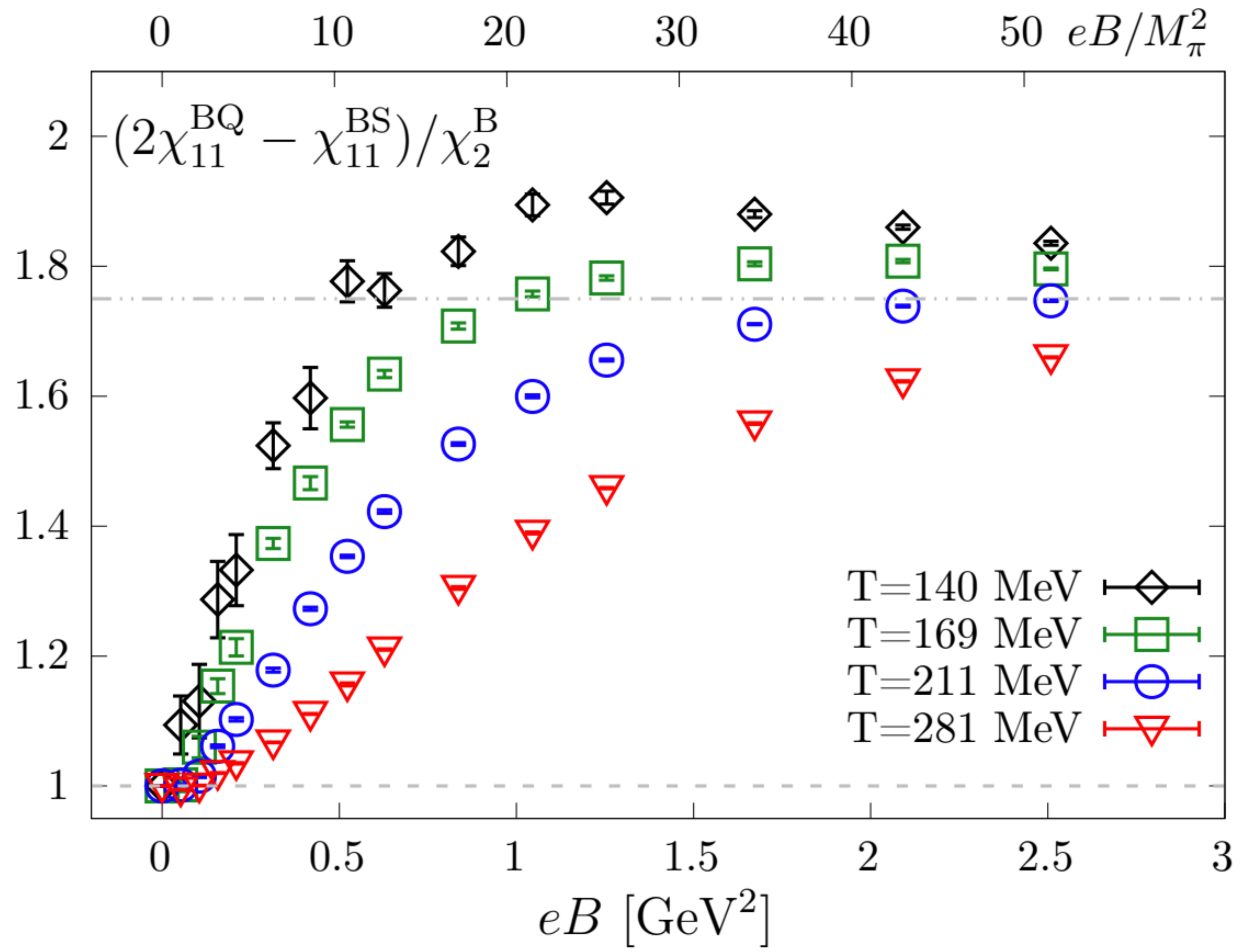

$M^2(B)-M^2(B=0)$ [GeV^2]
π^-
K^-
LLL
eB [GeV^2]
eB/M_π^2


$\Sigma_u(|q_uB_u|)/\Sigma_d(|q_dB_d|)$
λ^{UV}_{cut}=0
λ^{UV}_{cut}=0.12
λ^{UV}_{cut}=0.36
λ^{UV}_{cut}=∞
|qB| [GeV^2]


δ_i
λ^{UV}_{cut}=0.12
λ^{UV}_{cut}=0.36
i = π^0
i = π^0_u
i = π^0_d
eB [GeV^2]
eB/M_π^2


$\chi_2^B/\chi_2^{B,free}$
T=140 MeV
T=169 MeV
T=211 MeV
T=281 MeV
$\sqrt{eB}/T$


χ_2^B/T^2
$N_b = 0$
$N_b = 6$
$N_b = 12$
$N_b = 16$
24
32
40
48
T [MeV]


$(2\chi_{11}^{BQ} - \chi_{11}^{BS})/\chi_2^B$
T=140 MeV
T=169 MeV
T=211 MeV
T=281 MeV
eB [GeV^2]
eB/M_π^2