

How time-dependent electric fields affect the Schwinger mechanism?

Hidetoshi TAYA

Keio U. → RIKEN (from this April)

What is the Schwinger mechanism?

[Sauter (1932)] [Heisenberg, Euler (1936)] [Schwinger (1951)]

✓ Vacuum pair production occurs in the presence of strong fields

Intuitive picture for a slow electric field = **quantum tunneling**



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Intuitive picture for a slow electric field = **quantum tunneling**



✓ For **constant strong E-field** $E(t, x) = E$, it's understood well (at least, theoretically)

$$N_{e^\pm} = \frac{(eE)^2 VT}{(2\pi)^3} \times \exp\left[-\pi \frac{m^2}{eE}\right] \sim \exp[-\# \times (\text{gap height}) \times (\text{gap length})]$$

- **Non-perturbative** \Rightarrow Interesting, since it is the unexplored region of QED (or QFT)
- **Vacuum process** \Rightarrow Fundamental, since all the physical processes occur on top of vacuum
- However, **exponentially suppressed** i.t.o. mass \Rightarrow Not confirmed by experiments yet...

cf.) Guinness world record:

[Yanovsky et al (2008)]

$$\text{HERCULES laser } eE \sim \sqrt{10^{22} \text{ W/cm}^2} \sim (0.01 \text{ MeV})^2 \ll m_e^2 \sim \sqrt{10^{29} \text{ W/cm}^2} \sim (0.5 \text{ MeV})^2$$

Timeliness

✓ **Now is the best time to study the Schwinger mechanism !**

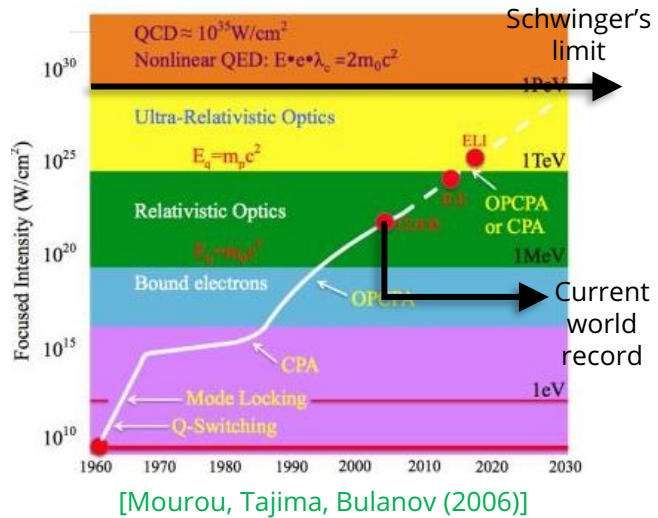
Developments in experimental technologies ⇒ **Novel strong-field sources**

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Developments in experimental technologies ⇒ **Novel strong-field sources**

ex.1) Intense lasers



ex.2) Heavy-ion collisions

- (ultra)peripheral collisions

$$eE, eB \sim \frac{\alpha Z v \gamma}{r^2} \sim \alpha Z \gamma \times m_\pi^2$$

[Skokov, Illarionov, Toneev (2009)]

[Deng, Huang (2012)]

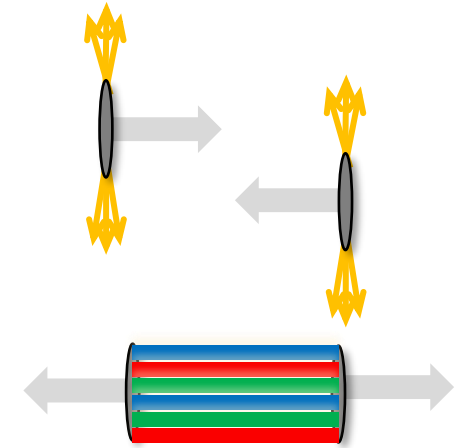
- glasma (color flux tube)

$$gE_{\text{color}}, gB_{\text{color}} \sim Q_s^2 \sim O(1) \text{ GeV}^2$$

- Also in other geometries:

- asymmetric coll. [Hirono, Hongo, Hirano (2014)] [Voronyuk, et al (2014)]

- low-energy coll. [Review: Rafelski, et al (2014)] [Maltsev et al (2019)]



[Lappi, McLerran (2006)]

[Allor, Cohen, McGady (2008)]

[Solinas, Amoretti, Giazotto (2021)]

ex.3) Cond-mat analogues: Graphene, Cold atom, Superconductor, Semiconductor, ...

[Szapak, Shutzhold (2012)]

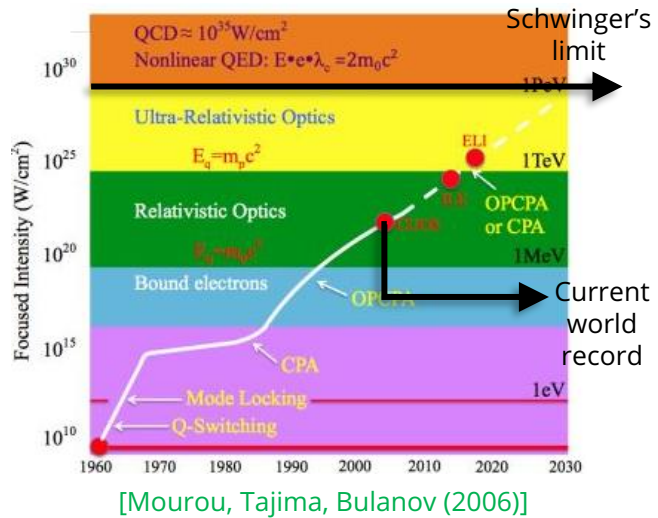
[Thesis by Linder; 1807.08050]

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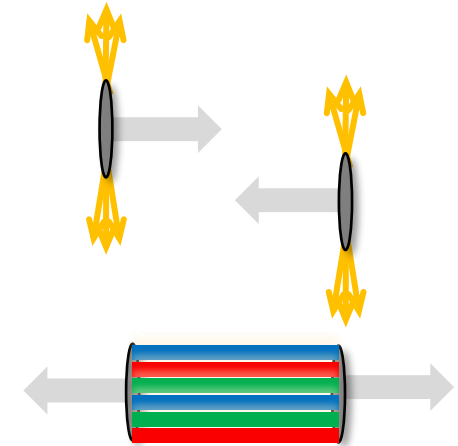
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✓ Schwinger's result is insufficient for actual problems \Rightarrow **Beyond Schwinger !**

- Inhomogeneous field (e.g., time- & space-dependence, more efficient field config., ...)
- Realtime dynamics (e.g., backreaction, intermediate particle number, ...)
- Higher loop effects (e.g., radiation, mass shift, ...)
- Other observables (e.g., spin, chirality, high-harmonic generation...)
- ...

Today's talk

Discuss the Schwinger mech. under **time-dependent** E-field

Part I: Interplay b/w non-pert. & pert. production mechanisms

[[HI](#), Fujii, Itakura, PRD (2014)]

[[HI](#) et al., JHEP (2021)]

Part II: Dynamically assisted Schwinger mechanism

[[HI](#), PRD (2019)]

[Huang, [HI](#), PRD (2019)]

[Huang, Matsuo, [HI](#), PTEP (2019)]

Part III: Dynamical assistance to chirality production [[HI](#), PRR (2020)]

Introduction



Part I:

Interplay b/w non-pert. & pert. production mechanisms



Part II:

Dynamically assisted Schwinger mechanism



Part III:

Dynamical assistance to chirality production



Summary

Background (1/2): Interplay b/w non-pert. & pert. mech.

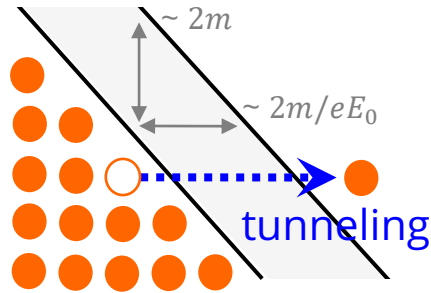
- ✓ Consider time-dependent E-field, having strength eE_0 and frequency Ω

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✓ Consider time-dependent E-field, having strength eE_0 and frequency Ω

- The tunneling picture should break down if the frequency Ω is large

Non-pert. tunneling $N \sim \exp[\# / eE_0]$



$$\text{Tunneling time } \Delta t \sim \frac{2m}{eE_0}$$

\Rightarrow E-field should be slower than Δt

$$\Rightarrow \Omega^{-1} \gtrsim \Delta t$$

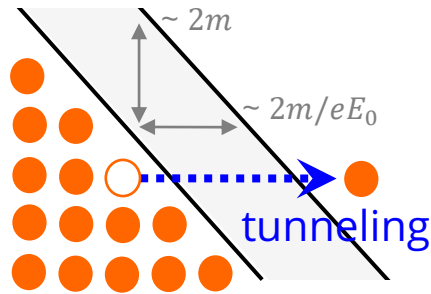
$$\Rightarrow 1 \gtrsim \frac{\Delta t}{\Omega^{-1}} = \frac{\Omega m}{eE_0} \equiv \gamma \text{ (Keldysh parameter)}$$

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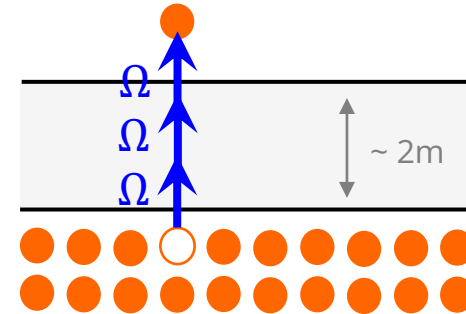
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$\Rightarrow \Omega^{-1} \gtrsim \Delta t$

$\Rightarrow 1 \gtrsim \frac{\Delta t}{\Omega^{-1}} = \frac{\Omega m}{eE_0} \equiv \gamma$ (Keldysh parameter)

Pert. multi-photon $N \sim eE_0^{2n}$



Pair production when $n\Omega > 2m$

- For large Ω , E-field may behave like a photon and interact incoherently (perturbatively)

Background (2/2): Semi-classical analysis

✓ The interplay has been “confirmed” by (but only by) semi-classical analysis

- Trans-series expansion in terms of \hbar

$$N = \sum_{n,m} N_{n,m} \hbar^n e^{-mS/\hbar} = (N_{0,1} + O(\hbar)) e^{-S/\hbar} + O(e^{-2S/\hbar})$$

- Steepest descent analysis [Brezin, Itzykson (1970)]
- Imaginary-time method [Popov (1972)]
- Divergent asymptotic series method [Berry (1989)]
- Worldline instanton method [Dunne, Schubert (2005)]
- (exact) WKB [HT, Fujimori, Misumi, Nitta, Sakai (2020)]
- ...

- Valid in the slow limit $\hbar \ll 1 \Leftrightarrow \Omega \ll 1$: $\because i\hbar\partial_t\psi = H(\Omega t)\psi \xrightarrow{\tau \equiv t/\hbar} i\partial_\tau\psi = H(\hbar\Omega\tau)\psi$

- Production number N is controlled solely by the Keldysh parameter $\gamma \equiv \frac{m\Omega}{eE_0}$ [Keldysh (1965)]

$$S = \pi \frac{m^2}{eE_0} g(\gamma) \xrightarrow{\text{Example: } eE(t) = eE_0 \cos(\Omega t)} \begin{cases} \pi \frac{m^2}{eE_0} + O(\gamma^1) & \text{(slow limit } \gamma \ll 1) \\ \frac{2m}{\Omega} \ln \gamma^2 + \text{const.} + O(\gamma^{-1}) & \text{(fast limit } \gamma \gg 1) \end{cases}$$

See, e.g., [Dunne, Gies, Schubert, Wang (2006)]
for an explicit expression for $g(\gamma)$

of photons pert. power-dep.
after exponentiation

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✓ However, you **should not** be convinced with the semi-classical argument

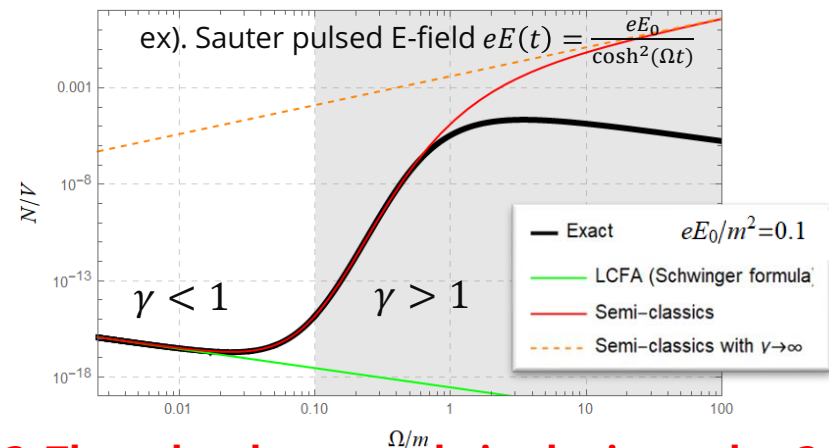
- Fast limit $\gamma \gg 1$ is dangerous

⇒ Q1: When are semi-classical methods really OK i.t.o. Ω ?

- Another dimensionless parameter should exist

\therefore 3 dimensionful parameters m, eE_0, Ω

⇒ 2 dimensionless parameters ⇒ Q2: Why only γ ? The other has no role in the interplay?



Idea: One-photon process

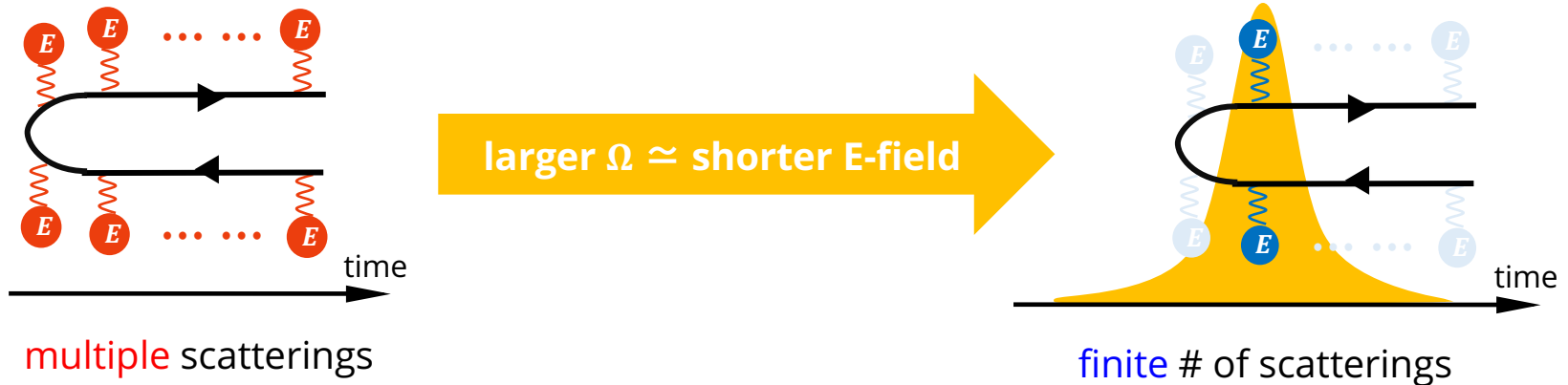
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Idea: One-photon process

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✓ Larger Ω , more perturbative \Rightarrow one-photon dominates eventually

- because there is **no time to interact many times**, no matter how strong eE_0 is



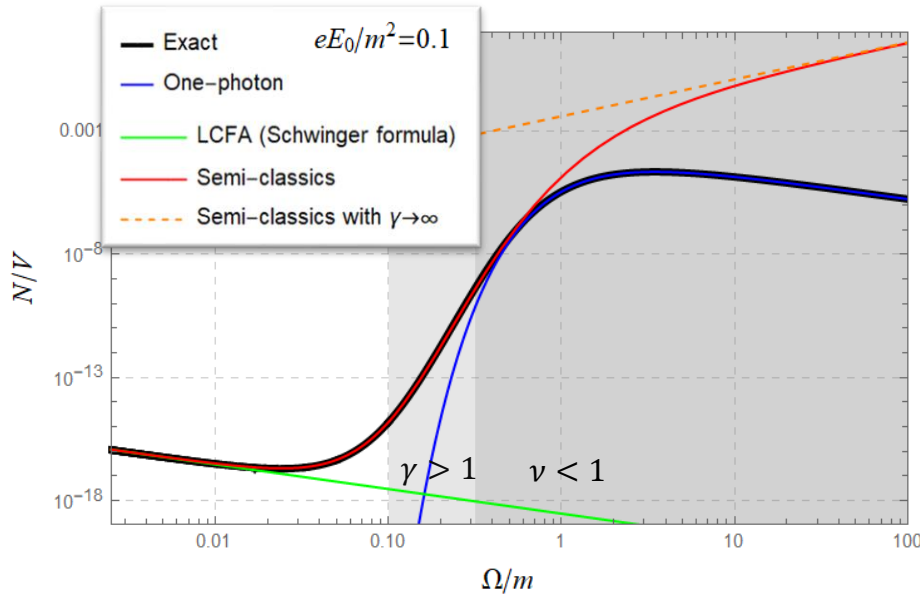
- should cover different region, as photon # should be huge $\frac{2m}{\Omega} \rightarrow \infty$ in the semi-classics
- analytical formula is available (for any field config. !)

$$N_{1\text{-photon}} = \sum_{s,s'} \int d^3\mathbf{p} d^3\mathbf{p}' \left| \begin{array}{c} \text{E} \text{ wavy line} \\ \text{particle path} \end{array} \right|^2 = \frac{V}{(4\pi)^2} \int_{2m}^{\infty} d\omega \sqrt{1 - \frac{4m^2}{\omega^2}} \frac{1}{3} \left(2 + \frac{4m^2}{\omega^2} \right) |e\tilde{E}(\omega)|^2$$

[HI, Fujii, Itakura (2014)] [QFT textbook by Itzykson, Zuber]

Result: Interplay b/w tunneling, multi-, one-photon

✓ An explicit demonstration for Sauter pulsed E-field $eE(t) = \frac{eE_0}{\cosh^2(\Omega t)}$



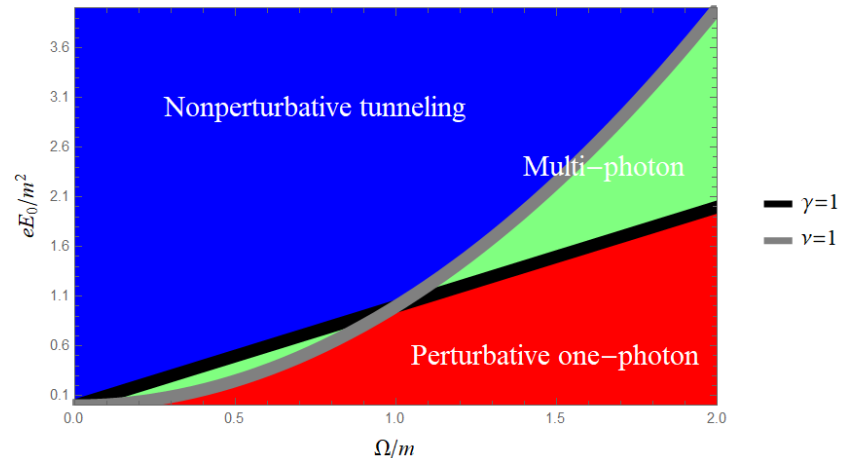
- **One-photon dominates for large Ω** , where semi-classical methods fail,
- **One-photon is more efficient** than tunneling ; N is the largest at $\Omega \sim 2m$

✓ **Two dimensionless parameters $\gamma = \frac{m\Omega}{eE_0}$, $\nu \equiv \frac{eE_0}{\Omega^2}$ control the interplay**

- Sauter field is solvable, which can be compared w/ Schwinger & one-photon

$\gamma \gg 1, \nu \ll 1 \Rightarrow$ **pert. one-photon**
 $\gamma \ll 1, \nu \gg 1 \Rightarrow$ **non-pert. tunneling**

• $\nu = \frac{eE_0/\Omega}{\Omega} = \frac{\text{(work done by E-field)}}{\text{(photon energy)}} = \text{(# of photons involved)}$



Message of Part I

- (1) The interplay b/w non-pert. & pert. production mechanisms is controlled by $\gamma = \frac{m\Omega}{eE}$ (Keldysh parameter) and also by $\nu = \frac{eE}{\Omega^2}$ (\sim # of photons involved)
- (2) Semi-classical methods (e.g., worldline) are dangerous for $\gamma \gg 1$;
It breaks down for $\nu \gtrsim 1$, where one-photon process dominates
- (3) One-photon production is very efficient, compared to non-pert. tunneling

[[HT](#), Fujii, Itakura, PRD (2014)]

[[HT](#), Fujimori, Misumi, Nitta, Sakai, JHEP (2021)]

Introduction



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Dynamically assisted Schwinger mechanism



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Summary

Background (1/2): Dynamically assisted Schwinger mechanism

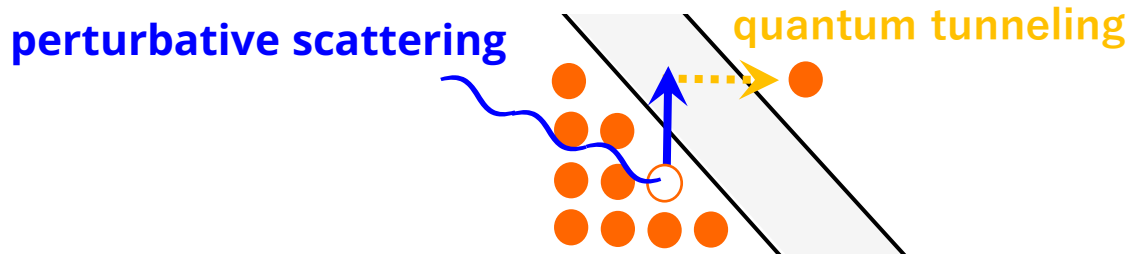
- ✓ In Part I, I discussed E-field having a single frequency mode Ω
- ✓ What if E-field is bi-frequent; superposition of slow strong + weak fast E-fields ?

Background (1/2): Dynamically assisted Schwinger mechanism

- ✓ In Part I, I discussed E-field having a single frequency mode Ω
- ✓ What if E-field is bi-frequent; superposition of slow strong + weak fast E-fields ?

- Combinatorial effects of non-pert. & pert. mechs. \Rightarrow **dynamically assisted Schwinger mech.**

[Dunne, Gies, Schutzhold (2008), (2009)]
[Piazza et al (2009)] [Monin, Voloshin (2010)]



$$N \sim \exp[-\# \times (\text{gap height}) \times (\text{gap length})] \Rightarrow \text{Enhancement in production}$$

(even though the fast field is very weak)

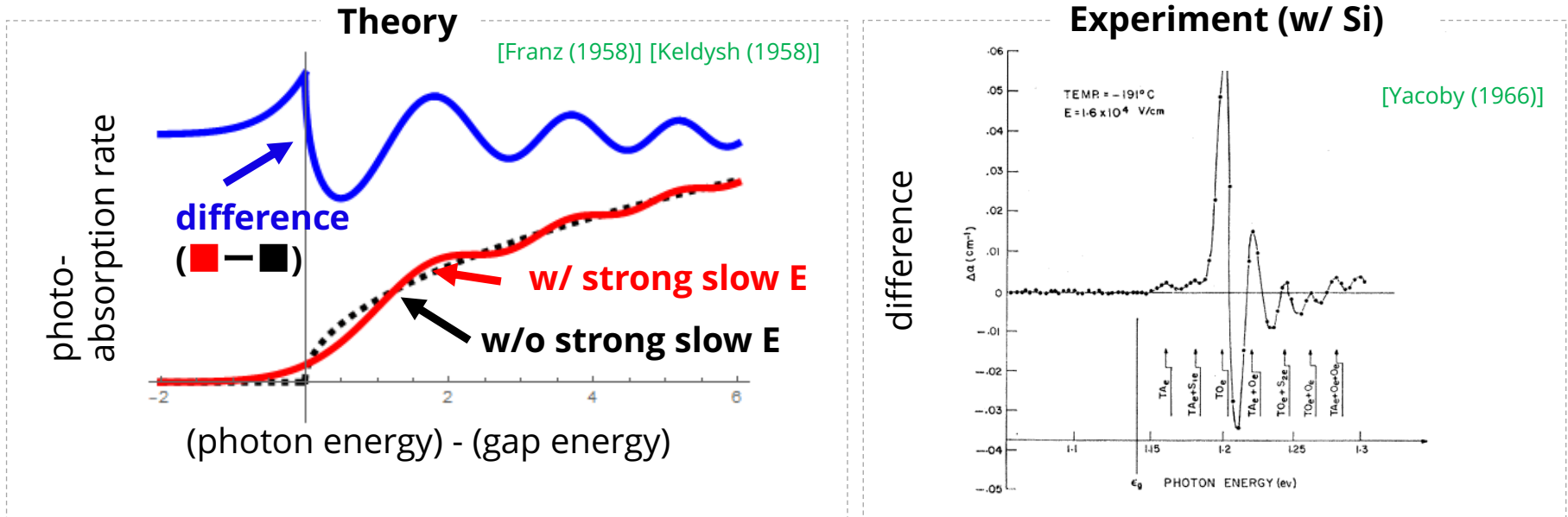
reduced by pert.

- Typically, analyzed within semi-classical methods [Dunne, Gies, Schutzhold (2008), (2009)]
- Phenomenological importance:
 - ex1) laser:** available E-field is still weak \Rightarrow needs enhancement to observe the Schwinger mech
 - ex2) heavy-ion collisions:** (Mini-)jets on top of glasma, Event generators (e.g., PYTHIA)

Background (2/2): Franz-Keldysh effect in cond-mat

✓ Apply strong slow E-field & a photon (~ weak fast E-field) onto a semi-conductor, and measure photo-absorption rate

- photo-absorption rate $\sim \text{Im}[1\text{-loop action}] \sim$ particle production rate



- Enhancement below the mass gap
⇒ Looks very similar to the dynamically assisted Schwinger mechanism (?)
- Oscillation above the mass gap (Franz-Keldysh oscillation)
- Enhancement is maximized around the mass gap

What I am going to do

✓ Get a better understanding of the dynamical assistance by ...

- Clarifying **the relation to the Franz-Keldysh effect** in cond-mat
- Establishing **a novel analytical method**, applicable for very fast E-field

⇐ Why such method needed ?

- (1) Conventional semi-classical approaches are invalid for very fast E-field (result of Part I)
- (2) Enhancement by pert. one-photon around $\Omega \sim 2m$ may be important (result of Part I)
- (3) Such an enhancement is observed in Franz-Keldysh effect
- (4) The Franz-Keldysh oscillation occurs for very fast E-field

- Revealing **novel features**, e.g., behavior at large Ω , spin generation, effective mass concept, ...

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✓ Use “the perturbation theory in the Furry picture”

Goal: Evaluate $\frac{d^3 N_s}{dp^3} = \langle \text{vac} | \hat{a}_{\mathbf{p},s}^\dagger \hat{a}_{\mathbf{p},s} | \text{vac} \rangle$ in the presence of strong slow E_s & weak fast \mathcal{E}_f

Idea: Perturbative expansion $\hat{a}_{\mathbf{p},s}$ i.t.o. \mathcal{E}_f , but no expansion i.t.o. E_s

[Furry (1951)] [Fradkin, Gitman, Shvartsman (1991)] [Torgrimsson, Schneider, Shutzhold (2017)]

Perturbation theory in Furry picture (1/3)

STEP 1



Separate the total E into **strong slow** E_s & **weak fast** \mathcal{E}_f

$$E = E_s + \mathcal{E}_f$$

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STEP 2

Solve Dirac eq. non-pert. w.r.t. E_s , but just pert. w.r.t. \mathcal{E}_f

$$[i\partial - e\mathcal{A}_s - m]\hat{\psi} = e\mathcal{A}_f\hat{\psi}$$

$$\Rightarrow \hat{\psi}(x) = \hat{\psi}^{(0)}(x) + \int_{-\infty}^{\infty} dy^4 S_R(x, y) e\mathcal{A}_f(y) \hat{\psi}^{(0)}(y) + O(|e\mathcal{A}_f|^2)$$

Here, $\hat{\psi}^{(0)}$ and S_R are **non-perturbatively dressed by** E_s as

$$[i\partial - e\mathcal{A}_s - m]\hat{\psi}^{(0)} = 0$$

$$[i\partial - e\mathcal{A}_s - m]S_R(x, y) = \delta^4(x - y)$$

Perturbation theory in Furry picture (2/3)

STEP 3

Compute in/out annihilation operators $\hat{a}_{p,s}^{\text{in/out}}$, $\hat{b}_{p,s}^{\text{in/out}}$ from $\hat{\psi}$

$$\begin{pmatrix} \hat{a}_{p,s}^{\text{in/out}} \\ \hat{b}_{-p,s}^{\text{in/out}\dagger} \end{pmatrix} \equiv \lim_{t \rightarrow -\infty / +\infty} \int d^3x \begin{pmatrix} (u_{p,s} e^{-i\omega_p t} e^{i\mathbf{p}\cdot\mathbf{x}})^\dagger \\ (v_{p,s} e^{+i\omega_p t} e^{i\mathbf{p}\cdot\mathbf{x}})^\dagger \end{pmatrix} \hat{\psi}(x)$$

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$\Rightarrow \hat{a}_{p,s}^{\text{in}}, \hat{a}_{p,s}^{\text{out}}$ are inequivalent $\hat{a}_{p,s}^{\text{in}} \neq \hat{a}_{p,s}^{\text{out}}$ and related with each other by a Bogoliubov transformation

$$\begin{pmatrix} \hat{a}_{p,s}^{\text{out}} \\ \hat{b}_{-p,s}^{\text{out}\dagger} \end{pmatrix} = \sum_{s'} \int d^3 \mathbf{p}' \begin{pmatrix} \alpha_{p,s;p',s'} & \beta_{p,s;p',s'} \\ -\beta_{p,s;p',s'}^* & \alpha_{p,s;p',s'}^* \end{pmatrix} \begin{pmatrix} \hat{a}_{p',s'}^{\text{in}} \\ \hat{b}_{-p',s'}^{\text{in}\dagger} \end{pmatrix}$$

where, up to 1st order in $e\mathcal{A}_f$,

$$\alpha_{p,s;p',s'} = \int d^3 \mathbf{x} \psi_{p,s}^{(0)\text{out}\dagger} + \psi_{p',s'}^{(0)\text{in}} - i \int d^4 x \bar{\psi}_{p,s}^{(0)\text{out}} e\mathcal{A}_f + \psi_{p',s'}^{(0)\text{in}} + O(|e\mathcal{A}_f|^2)$$

$$\beta_{p,s;p',s'} = \int d^3 \mathbf{x} -\psi_{p,s}^{(0)\text{out}\dagger} + \psi_{p',s'}^{(0)\text{in}} - i \int d^4 x -\bar{\psi}_{p,s}^{(0)\text{out}} e\mathcal{A}_f + \psi_{p',s'}^{(0)\text{in}} + O(|e\mathcal{A}_f|^2)$$

Here, $\pm\psi_{p,s}^{(0)\text{in/out}}$ are sol. of Dirac eq. **dressed by $e\mathcal{A}_s$** w/ different B.C.

$$[i\partial - e\mathcal{A}_s - m] \pm\psi_{p,s}^{(0)\text{in/out}} = 0 \quad \text{w/} \quad \lim_{t \rightarrow -\infty / +\infty} \begin{pmatrix} +\psi_{p,s}^{(0)\text{in/out}} \\ -\psi_{p,s}^{(0)\text{in/out}} \end{pmatrix} = \begin{pmatrix} u_{p,s} e^{-i\omega_p t} e^{i\mathbf{p}\cdot\mathbf{x}} \\ v_{p,s} e^{-i\omega_p t} e^{i\mathbf{p}\cdot\mathbf{x}} \end{pmatrix}$$

Perturbation theory in Furry picture (3/3)

STEP 4

Evaluate the in-vacuum expectation value of # operator

$$\frac{d^3 N_e}{d\mathbf{p}^3} \equiv \langle \text{vac}; \text{in} | a_{\mathbf{p},s}^{\text{out}\dagger} a_{\mathbf{p},s}^{\text{out}} | \text{vac}; \text{in} \rangle = \sum_{s'} \int d^3 \mathbf{p}' |\beta_{\mathbf{p},s;\mathbf{p}',s'}|^2$$

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Assume \mathbf{E}_s is so slow that it can be approximated as a static E-field

⇒ analytical sol. of Dirac eq. $\pm \psi_{\mathbf{p},s}^{(0)\text{in/out}}$ is known

⇒ one can evaluate $\beta_{\mathbf{p},s;\mathbf{p}',s'}$ **exactly!**

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✓ Remarks

- **Directly computing VEV of # operator**

⇒ **inclusive quantity** that includes all the processes up to 1st order in \mathcal{E}_f

$$\langle \text{vac}; \text{in} | a_{\mathbf{p},s}^{\text{out}\dagger} a_{\mathbf{p},s}^{\text{out}} | \text{vac}; \text{in} \rangle = \sum_X |\langle e_{\mathbf{p},s} X; \text{out} | \text{vac}; \text{in} \rangle|^2 \quad [\text{Baltz, McLerran (2001)}]$$

$$= \sum_X \left| \left\{ \begin{array}{c} \text{Furry picture diagram} \\ \text{X;out} \end{array} \right\} \right|^2 = \left| \begin{array}{c} \text{Schwinger} \\ \text{diagram} \end{array} \right|^2 + \left| \begin{array}{c} \text{multiple Schwinger} \\ \text{diagram} \end{array} \right|^2 + \left| \begin{array}{c} \text{Breit-Wheeler} \\ \text{diagram} \end{array} \right|^2 + \left| \begin{array}{c} \text{Schwinger} \\ \text{+ absorption} \\ \text{diagram} \end{array} \right|^2 + \left| \begin{array}{c} \text{Schwinger} \\ \text{+ annihilation} \\ \text{diagram} \end{array} \right|^2 + \dots$$

- **Production number \neq Vacuum decay rate**

[Cohen McGady (2008)]

- **No approximation in evaluating $\beta_{\mathbf{p},s;\mathbf{p}',s'}$**

within 0-th order WKB [Torgrimsson et al (2017)]

Result (1/6): Analytical formula for parallel $E_s \parallel \mathcal{E}_f$

✓ Analytical formula for $E = (0,0, E_s + \mathcal{E}_f(t))$, with **arbitrary** time-dep.

- is applicable even for very fast \mathcal{E}_f and **reproduces numerics very well** (show later)

$$\frac{d^3 N_e}{d\mathbf{p}^3} = \frac{V}{(2\pi)^3} \exp\left[-\frac{\pi(m^2 + \mathbf{p}_\perp^2)}{eE_s}\right] \\ \times \left| 1 + \frac{1}{2} \frac{m^2 + \mathbf{p}_\perp^2}{eE_s} \int_0^\infty d\omega \frac{\tilde{\mathcal{E}}_f(\omega)}{E_s} \exp\left[-\frac{i}{4} \frac{\omega^2 + 4\omega p_\parallel}{eE_s}\right] {}_1F_1\left(1 - \frac{i}{2} \frac{m^2 + \mathbf{p}_\perp^2}{eE_s}; 2; \frac{i}{2} \frac{\omega^2}{eE_s}\right) \right|^2$$

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Schwinger mech. by slow E_s

Dynamical assistance by fast \mathcal{E}_f

- describes interplay b/w Schwinger & one-photon process smoothly:

- Slow limit $\omega/\sqrt{eE_s} \ll 1$: ■ dominates \Rightarrow **Schwinger formula**

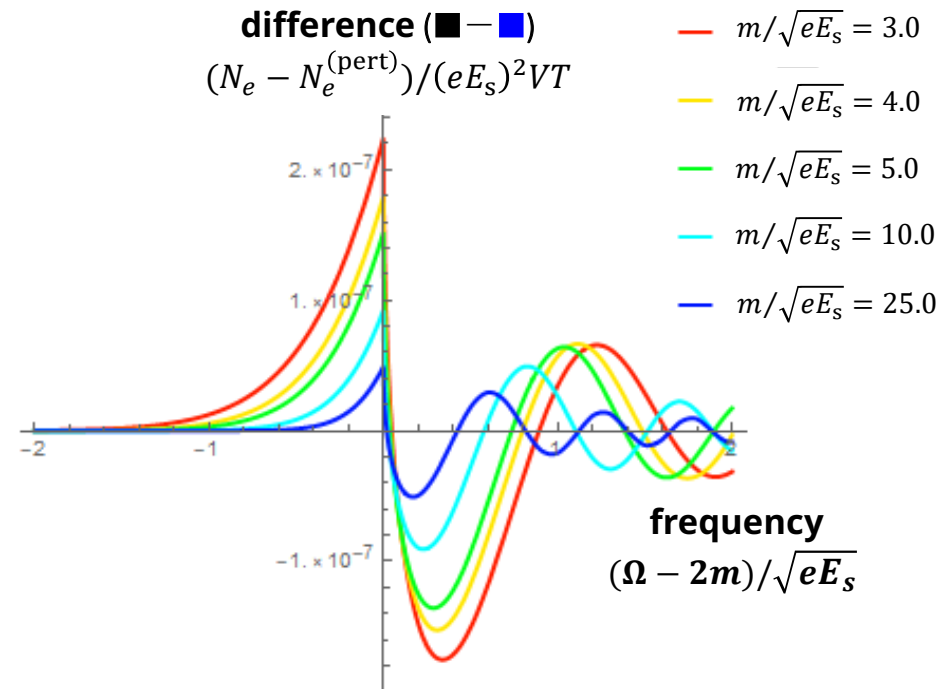
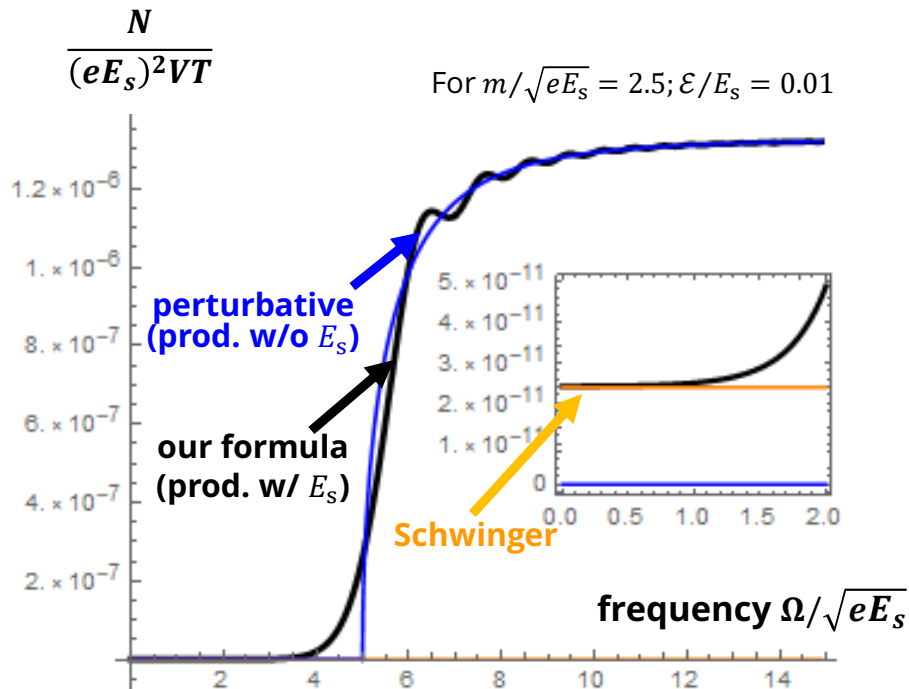
$$\frac{d^3 N_e}{d\mathbf{p}^3} \sim \frac{V}{(2\pi)^3} \exp \left[-\frac{\pi(m^2 + \mathbf{p}_\perp^2)}{eE_s} \right] \left| 1 + \frac{\pi}{2} \frac{m^2 + \mathbf{p}_\perp^2}{eE_s} \frac{\mathcal{E}_f}{E_s} \right|^2 \sim \frac{V}{(2\pi)^3} \exp \left[-\frac{\pi(m^2 + \mathbf{p}_\perp^2)}{e(E_s + \mathcal{E}_f)} \right]$$

- Fast limit $\omega/\sqrt{eE_s} \gg 1$: ■ dominates \Rightarrow **one-photon process**

$$\frac{d^3 N_e}{d\mathbf{p}^3} \sim \frac{V}{(2\pi)^3} \frac{1}{4} \frac{m^2 + \mathbf{p}_\perp^2}{\omega_p^2} \frac{|e\tilde{\mathcal{E}}_f(2\omega_p)|^2}{\omega_p^2}$$

Result (2/6): Total N & Relation to Franz-Keldysh effect

✓ For an oscillating perturbation $\mathcal{E}_f(t) = \mathcal{E} \cos(\Omega t)$ as a demonstration



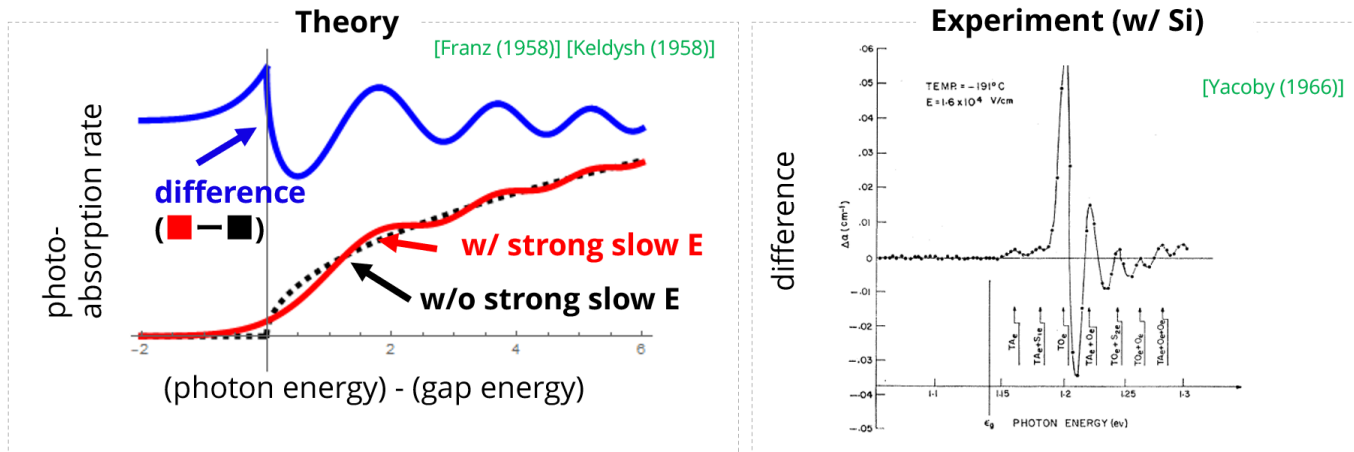
Result (2/6): Total N & Relation to Franz-Keldysh effect

✓ For

Background (2/2): Franz-Keldysh effect in cond-mat

✓ Apply strong slow E-field & a photon (~ weak fast E-field) onto a semi-conductor, and measure photo-absorption rate

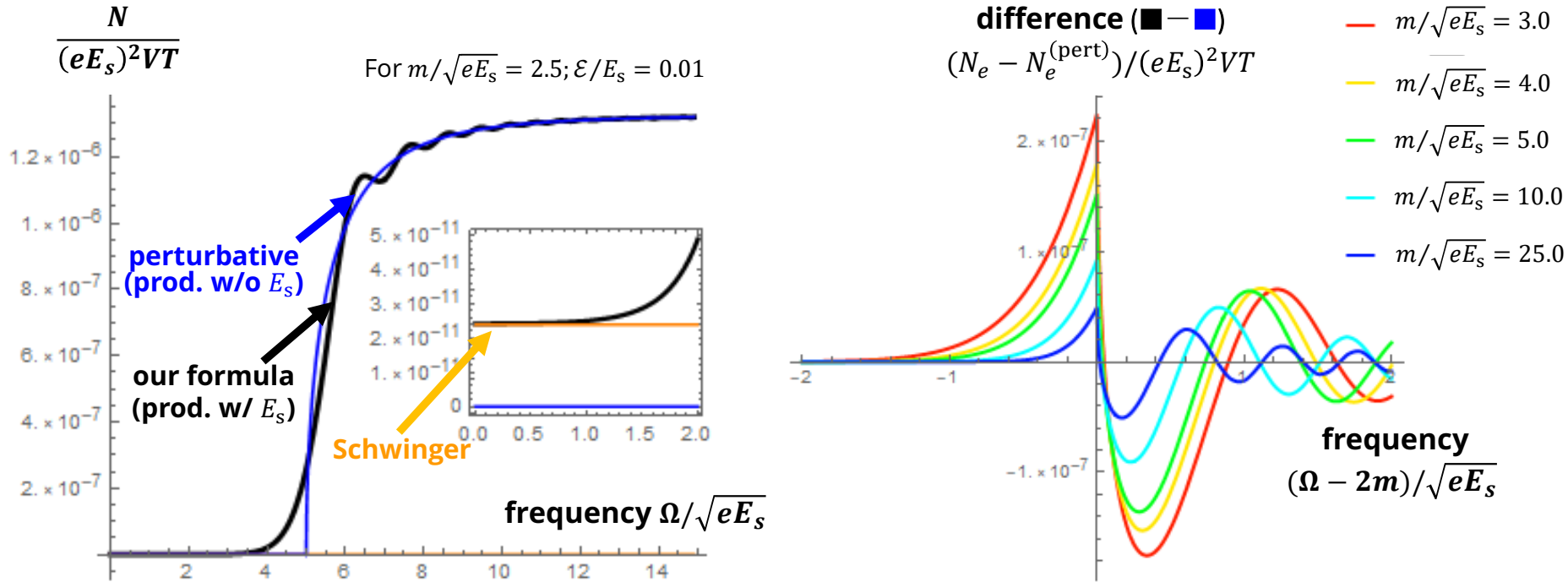
- photo-absorption rate $\sim \text{Im}[1\text{-loop action}] \sim$ particle production rate



- Enhancement below the mass gap
⇒ **Looks very similar to the dynamically assisted Schwinger mechanism (?)**
- Oscillation above the mass gap (Franz-Keldysh oscillation)
- Enhancement is maximized around the mass gap

Result (2/6): Total N & Relation to Franz-Keldysh effect

✓ For an oscillating perturbation $\mathcal{E}_f(t) = \mathcal{E} \cos(\Omega t)$ as a demonstration



✓ Completely the same as the Franz-Keldysh effect !

- enhancement below the mass gap
- oscillation above the mass gap
- enhancement becomes the maximum at around the mass gap

⇒ **dynamically assisted Schwinger mechanism = Franz-Keldysh effect**

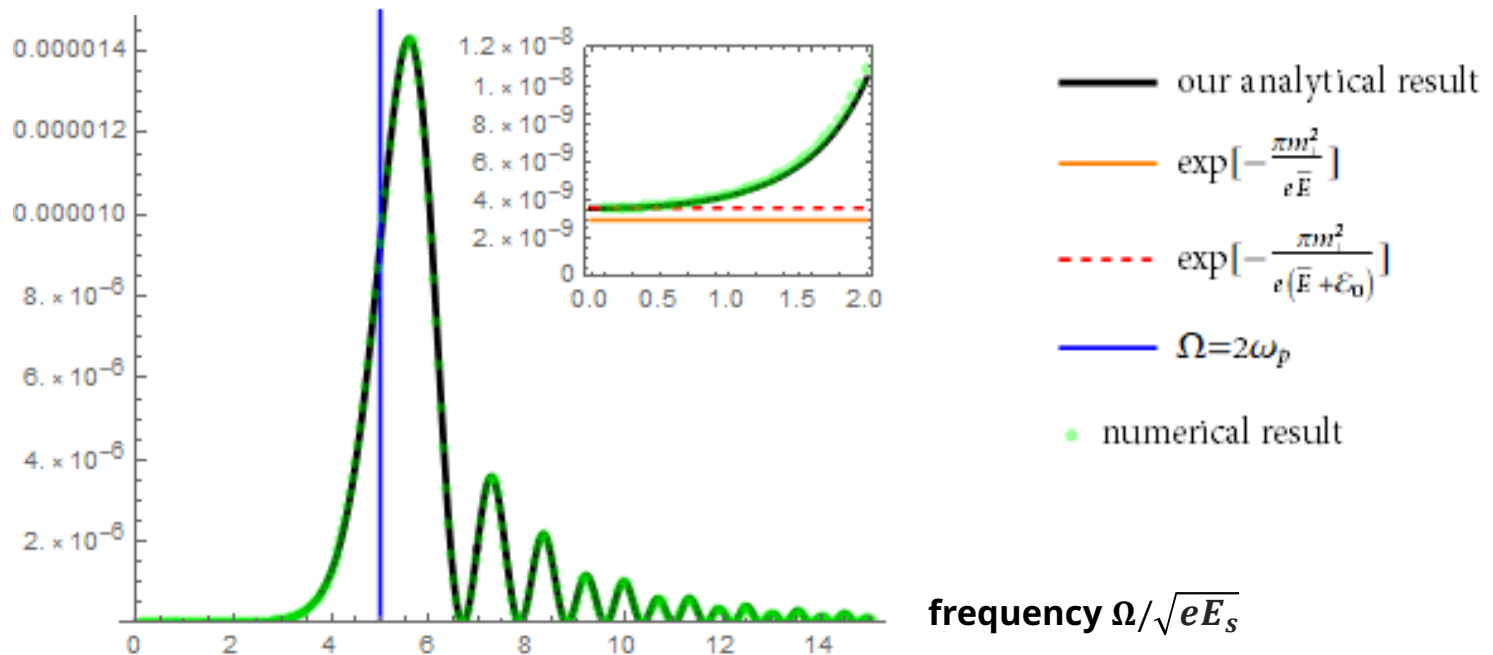
Result (3/6): Momentum dist. & comp. w/ numerics

✓ Similar enhancement/oscillating behavior for the momentum dist.

momentum dist.

$$\frac{(2\pi)^3 d^3 N_e}{V dp^3}$$

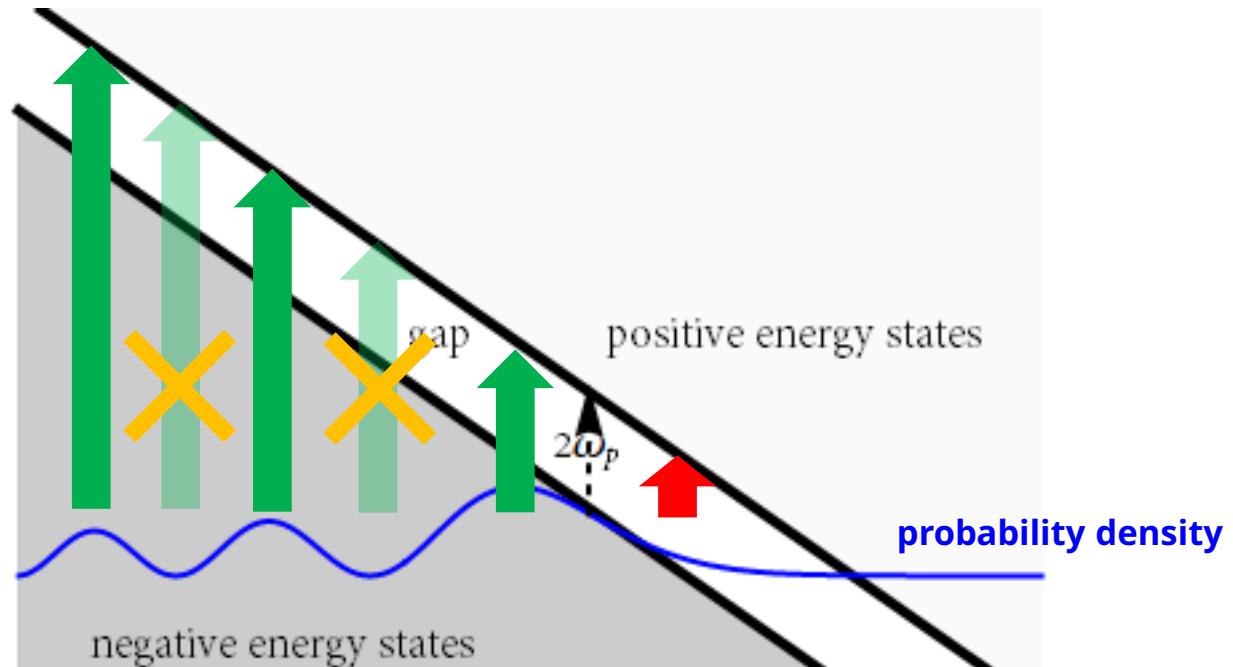
For $m/\sqrt{eE_s} = 2.5$; $\mathcal{E}/E_s = 0.01$



- **excellent agreement** b/w our analytical formula and the numerics
 \Rightarrow One-photon process is the essence of the dynamical assistance
- Another notable feature: the biggest peak (\sim 1-photon peak) is above the gap $\Omega > 2\omega_p$

cf) effective mass shift in the multi-photon regime
[Kohlfurst, Gies, Alkofer (2014)]

Result (4/6): Physical interpretation



- quantum tunneling \Rightarrow **enhancement**
- quantum reflection \Rightarrow **oscillation**
 \Rightarrow production occurs most efficiently at the maxima

Result (5/6): Analytical formula for general $E_s \not\parallel \mathcal{E}_f$

✓ Generalization of the analytical formula for $E_s \parallel \mathcal{E}_f \rightarrow E_s \not\parallel \mathcal{E}_f$

$$\frac{d^3 N_e}{d\mathbf{p}^3} = \frac{V}{(2\pi)^3} \exp\left[-\frac{\pi(m^2 + \mathbf{p}_\perp^2)}{eE_s}\right] \times \left[\begin{aligned} & 1 + \int_0^\infty d\omega e^{-i\frac{\omega p_\parallel}{eE_s}} \frac{1}{2} \frac{m^2 + \mathbf{p}_\perp^2}{eE_s} \frac{\tilde{\mathcal{E}}_f(\omega) \cdot \mathbf{E}_s}{E_s^2} e^{-i\frac{\omega^2}{4eE_s}} {}_1F_1\left(1 - \frac{i}{2} \frac{m^2 + \mathbf{p}_\perp^2}{eE_s}; 2; \frac{i}{2} \frac{\omega^2}{eE_s}\right) \\ & + i \int_0^\infty d\omega e^{-i\frac{\omega p_\parallel}{eE_s}} \frac{\tilde{\mathcal{E}}_f(\omega) \cdot \mathbf{p}_\perp}{E_s \omega} \operatorname{Re} \left[e^{-i\frac{\omega^2}{4eE_s}} {}_1F_1\left(1 - \frac{i}{2} \frac{m^2 + \mathbf{p}_\perp^2}{eE_s}; 1; \frac{i}{2} \frac{\omega^2}{eE_s}\right) \right] \\ & + s \times i \int_0^\infty d\omega e^{-i\frac{\omega p_\parallel}{eE_s}} \frac{(\tilde{\mathcal{E}}_f(\omega) \times \mathbf{p}_\perp) \cdot \mathbf{E}_s}{E_s^2 \omega} \operatorname{Im} \left[e^{-i\frac{\omega^2}{4eE_s}} {}_1F_1\left(1 - \frac{i}{2} \frac{m^2 + \mathbf{p}_\perp^2}{eE_s}; 1; \frac{i}{2} \frac{\omega^2}{eE_s}\right) \right] \Bigg|^2 \\ & + \left[\int_0^\infty d\omega e^{-i\frac{\omega p_\parallel}{eE_s}} \frac{m}{\omega} \frac{\tilde{\mathcal{E}}_f^x(\omega) + is\tilde{\mathcal{E}}_f^y(\omega)}{E_s} \operatorname{Im} \left[e^{-i\frac{\omega^2}{4eE_s}} {}_1F_1\left(1 - \frac{i}{2} \frac{m^2 + \mathbf{p}_\perp^2}{eE_s}; 1; \frac{i}{2} \frac{\omega^2}{eE_s}\right) \right] \right]^2 \end{aligned} \right]$$

• becomes complicated (green = new terms), but the basic structure is the same

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$$+ \left[\int_0^\infty d\omega e^{-i\frac{\omega p_\parallel}{eE_s}} \frac{m \tilde{\mathcal{E}}_f^x(\omega) + is \tilde{\mathcal{E}}_f^y(\omega)}{\omega E_s} \operatorname{Im} \left[e^{-i\frac{\omega^2}{4eE_s}} {}_1F_1\left(1 - \frac{i}{2} \frac{m^2 + \mathbf{p}_\perp^2}{eE_s}; 1; \frac{i}{2} \frac{\omega^2}{eE_s}\right) \right] \right]^2$$

Schwinger mechanism by slow E_s

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Schwinger mechanism by slow E_s

- becomes complicated (green = new terms), but the basic structure is the same
- **spin-dependence appears even without magnetic fields** [Takayoshi, Wu, Oka (2020)]

∴ Dirac particle has a spin-orbit coupling $\mathbf{s} \cdot (\mathbf{p} \times \mathcal{E})$

[Foldy, Wouthuysen (1950)] [Tani (1951)]

- can be applied to rotating E-fields: $E = E_0(\cos(\Omega t), \sin(\Omega t), 0) \sim (E_0, E_0 \Omega t, 0)$

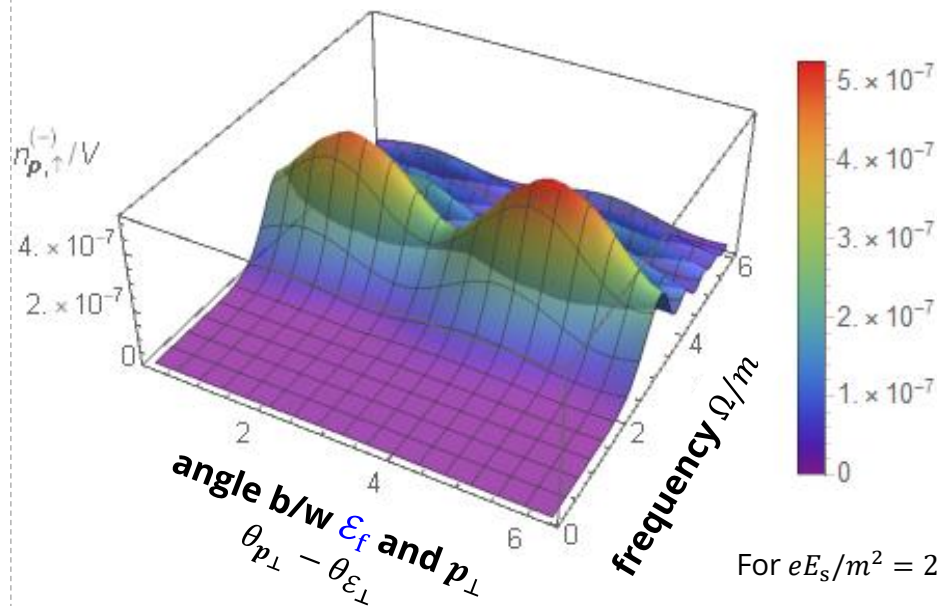
Numerical studies: [Blinne, Strobel (2015)] [Strobel, Xue (2015)] [Woller, Bauke, Keitel (2015)] [Kohlfurst (2019)]



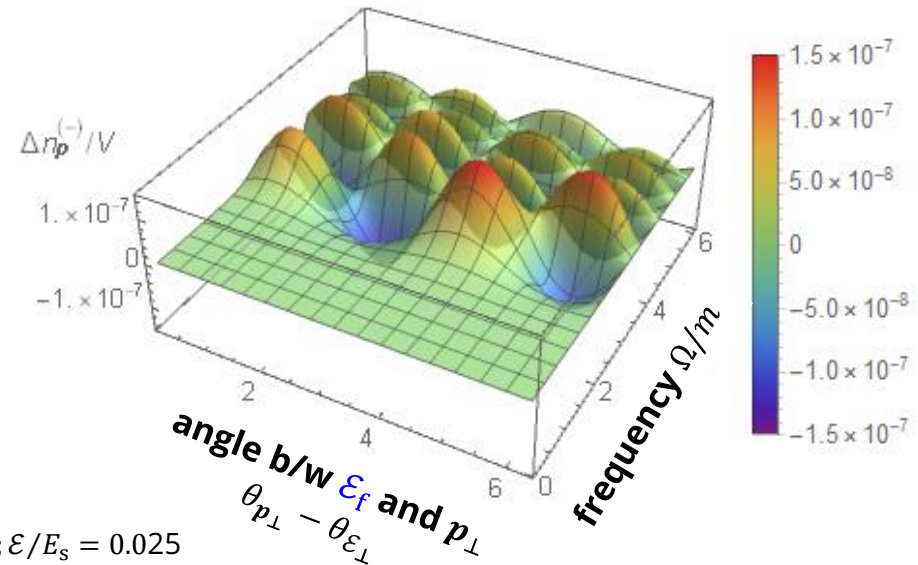
Result (6/6): Spin-dependent production

✓ Non-parallel superposition : $E = \begin{pmatrix} \mathcal{E} \cos \theta_{\varepsilon_{\perp}} \times \cos \Omega t \\ \mathcal{E} \sin \theta_{\varepsilon_{\perp}} \times \cos \Omega t \\ E_s \end{pmatrix}$

momentum dist. of spin up $\frac{1}{V} \frac{d^3 N_{\uparrow}}{dp^3}$



diff. b/w spin up and down $\frac{1}{V} \frac{d^3 N_{\uparrow}}{dp^3} - \frac{1}{V} \frac{d^3 N_{\downarrow}}{dp^3}$



For $eE_s/m^2 = 2.5$; $\mathcal{E}/E_s = 0.025$

- The dynamical assistance to N is basically the same as the parallel case
- Spin-dependence appears \Rightarrow O(10%) effect \Rightarrow **non-negligible**
- $\theta_{p_{\perp}}$ -dependent because of the spin-orbit interaction $\mathbf{s} \cdot (\mathbf{p} \times \mathcal{E})$

Message of Part II

- (1) The dynamically assisted Schwinger mechanism is an analogue of the Franz-Keldysh effect in cond-mat
- (2) Perturbation theory in the Furry picture provides a very powerful analytical formula
- (3) Not only the enhancement due to the quantum tunneling and one-photon assistance, but also oscillation appears due to the quantum reflection
- (4) Spin-dependent production occurs when transverse E-field is time-dependent

[[HT](#), PRD (2019)]

[Huang, [HT](#), PRD (2019)]

[Huang, Matsuo, [HT](#), PTEP (2019)]

Introduction



Part I:

Interplay b/w non-pert. & pert. production mechanisms



Part II:

Dynamically assisted Schwinger mechanism



Part III:

Dynamical assistance to chirality production



Summary

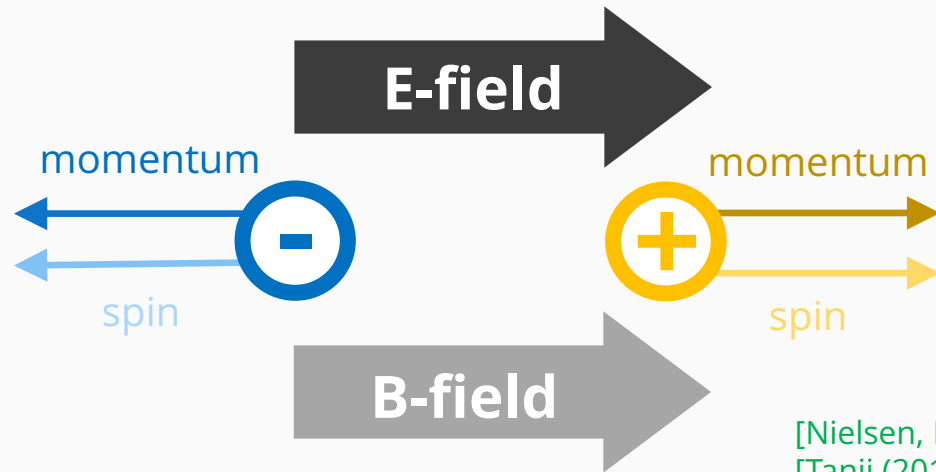
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- ✓ Chirality is produced through anomaly when $E \cdot B \neq 0$

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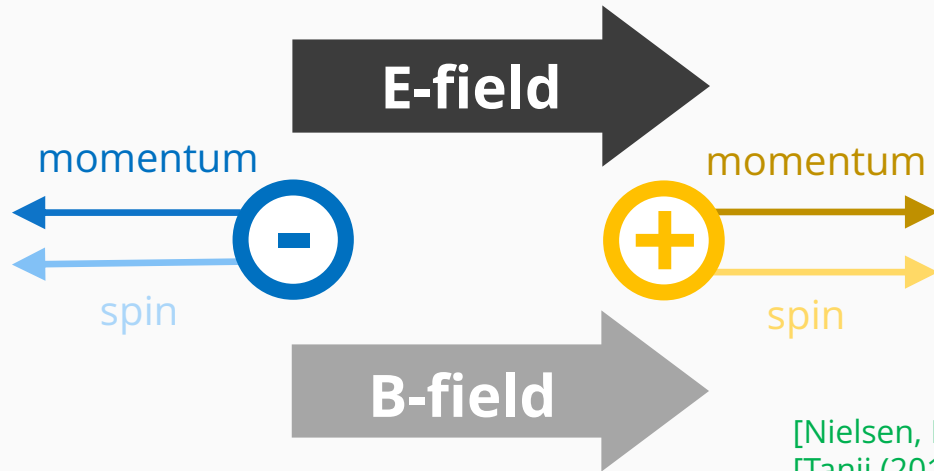
[Nielsen, Ninomiya (1983)]
[Tanji (2010)]
[Fukushima, Kharzeev, Warringa (2010)]
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etc ...

- $E \cdot B \neq 0$ may/can be realized at: heavy-ion collisions, early Universe, laser, ...

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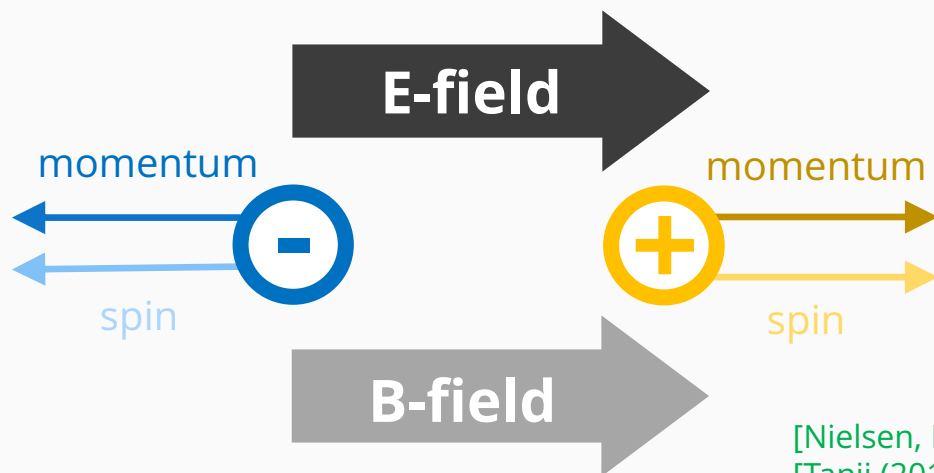
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Q: Any way to avoid the mass suppression ? ⇒ A: Dynamical assistance !

Theory: Perturbation theory in the Furry picture

- ✓ Extend the E-field case (Part II) to $E||B$ case

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✓ Extend the E-field case (Part II) to $E||B$ case

- Repeat essentially the same calculation:

SETUP: Parallel $\bar{\mathbf{E}}, \bar{\mathbf{B}}$ with perturbative weak fast E-field \mathcal{E}

$$\begin{aligned} \mathbf{E}(t) &= \bar{\mathbf{E}} + \mathcal{E}(t) \\ \mathbf{B}(t) &= \bar{\mathbf{B}} \end{aligned}$$

STEP 1: Solve Dirac eq. under $\bar{\mathbf{E}}, \bar{\mathbf{B}}$ non-perturbatively, and include effects of \mathcal{E} perturbatively

$$\begin{aligned} [i\partial - e\bar{\mathbf{A}} - m]\hat{\psi} &= e\mathcal{A}\hat{\psi} \\ \Rightarrow \hat{\psi}(x) &= \hat{\psi}^{(0)}(x) + \int_{-\infty}^{\infty} dy^4 S_R(x, y) e\mathcal{A}(y) \hat{\psi}^{(0)}(y) + O(|e\mathcal{A}|^2) \end{aligned}$$

STEP 2: Compute VEV of chirality operator

$$Q_5 \equiv \lim_{t \rightarrow \infty} \int dx^3 \left\langle \text{vac; in} \left| \hat{\psi} \gamma^5 \hat{\psi} \right| \text{vac; in} \right\rangle = O(1) + O(|e\mathcal{A}|^1) + O(|e\mathcal{A}|^2) + \dots$$

Result (1/2): Analytical formula

$$\frac{Q_5}{VT} = \frac{e\bar{E}e\bar{B}}{2\pi^2} e^{-\pi \frac{m^2}{e\bar{E}}} \times \left[1 + \frac{2\pi}{T} \left(\frac{m^2}{e\bar{E}} \right)^2 \int_0^\infty d\omega \left| \frac{\tilde{\mathcal{E}}(\omega)}{\bar{E}} {}_1\tilde{F}_1\left(1 - \frac{i m^2}{2 e\bar{E}}; 2; \frac{i \omega^2}{2 e\bar{E}}\right) \right|^2 \right]$$

Dynamical assistance by fast E-field \mathcal{E}

- Dynamical assistance

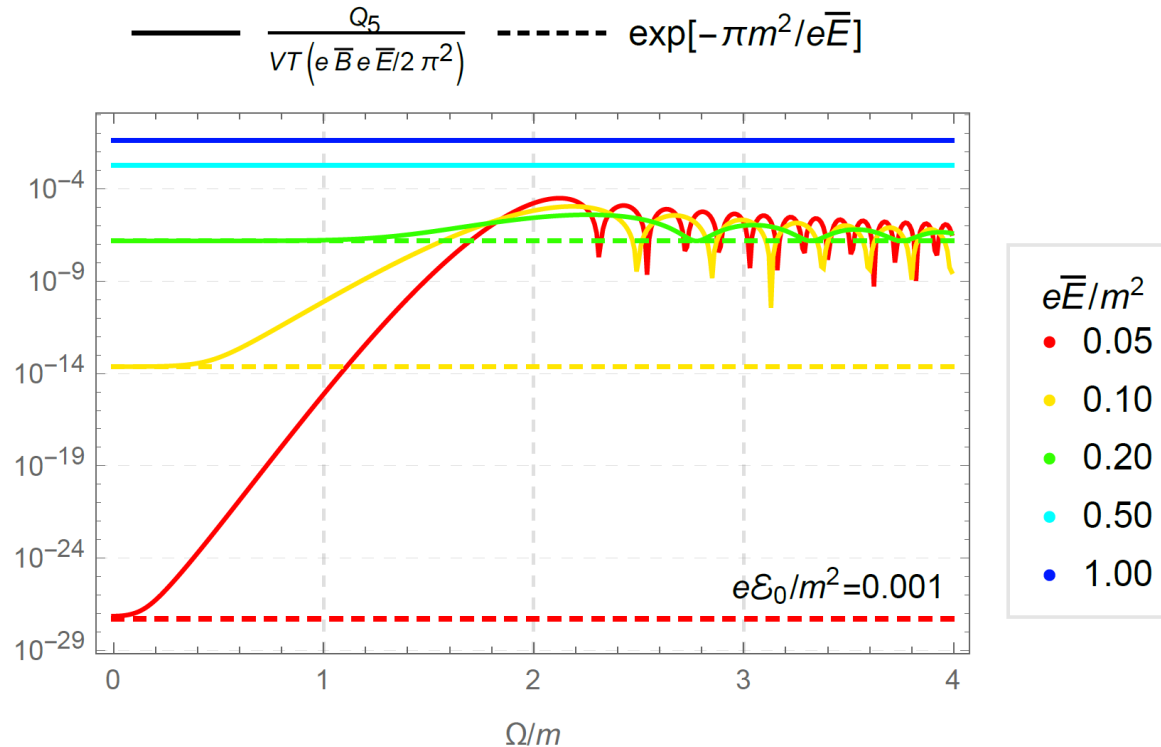
- is positive \Rightarrow **chirality is always enhanced**

- goes away when $m \rightarrow 0 \Rightarrow$ **important only for massive case**

- is independent of B-field

(\because B-field does not supply energy, i.e., does not affect the prod. mech.)

Result (2/2): Plot for chirality production



✓ An oscillating pert. as demonstration: $\frac{eE(t)}{m^2} = \frac{e\bar{E}}{m^2} + 0.001 \cos \Omega t$, $\frac{eB(t)}{m^2} = \frac{e\bar{B}}{m^2}$

✓ The same behavior as the dynamically assisted Schwinger mech.

- Free from the exponential suppression due to the enhancement
- Enhancement becomes largest around the mass gap
- Oscillation above the mass gap

Message of Part III

Chirality production can be enhanced significantly via the dynamical assistance

[[HI](#), PRR (2020)]

Introduction



Part I:

Interplay b/w non-pert. & pert. production mechanisms



Part II:

Dynamically assisted Schwinger mechanism



Part III:

Spin & chirality production



Summary

Summary

I discussed the Schwinger mechanism under **time-dependent E-field**:

Part I: Interplay b/w non-pert. & pert. production mechanisms

- The interplay is controlled by $\gamma \equiv \frac{m\Omega}{eE}$ (Keldysh parameter) and also by $\nu \equiv \frac{eE}{\Omega^2}$
- Semi-classical methods are invalid when $\nu \gtrsim 1$, where one-photon process dominates
- One-photon production is very efficient, compared to non-pert. tunneling

Part II: Dynamically assisted Schwinger mechanism

- Point out relation to Franz-Keldysh effect in cond-mat.
- Analytical analysis based on perturbation theory in the Furry picture
- Not only enhancement due to quantum tunneling & one-photon assist, but also oscillation due to quantum reflection
- Spin-dependent production occurs when transverse E-field is time-dependent

Part III: Dynamical assistance to chirality production

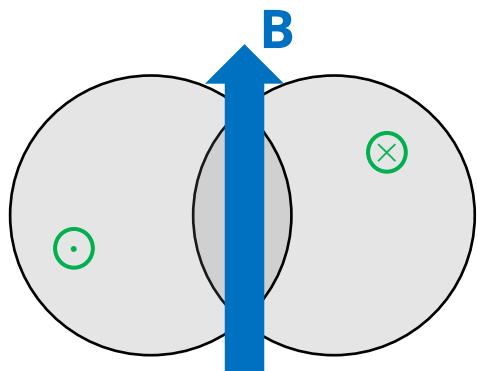
- Chirality production can be enhanced significantly via the dynamical assistance

BACKUP

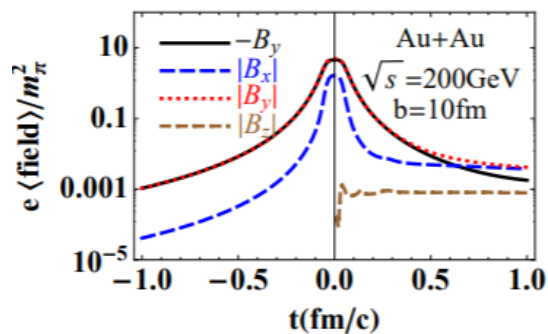
Strong EM fields in HIC

✓ 3 ways to produce (as far as I know)

① (ultra)peripheral

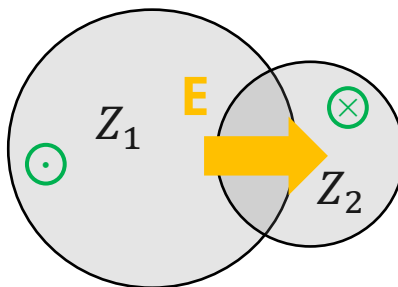


$$eB \sim \frac{\alpha Z v \gamma}{r^2} \sim \alpha Z \gamma \times m_\pi^2$$

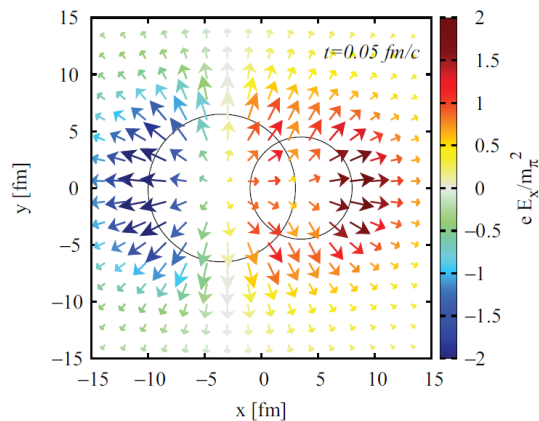


[Deng, Huang (2012)]

② asymmetric

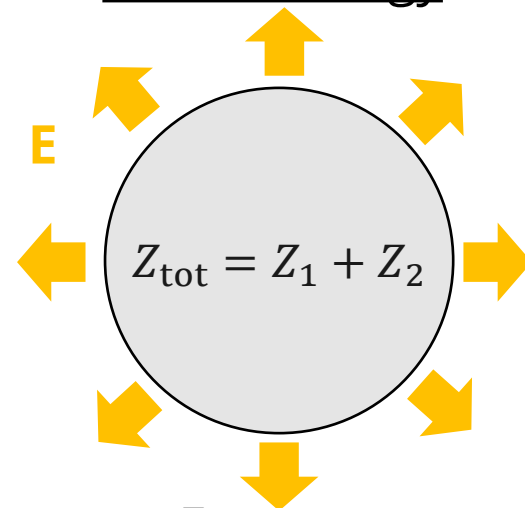


$$eE \sim \frac{\alpha(Z_1 - Z_2)\gamma}{r^2} \sim \alpha(Z_1 - Z_2)\gamma \times m_\pi^2$$

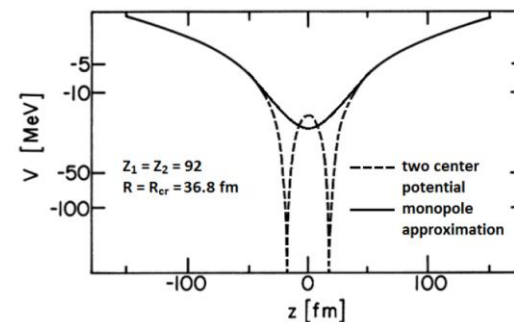


[Voronyuk, Toneev, Voloshin, Cassing (2014)]

③ low-energy



$$eE \sim \frac{\alpha Z_{\text{tot}}}{r^2} \sim \alpha Z_{\text{tot}} \times m_\pi^2$$



[Review: Rafelski, Kirsch, Muller, Reinhardt, Greiner (2014)]

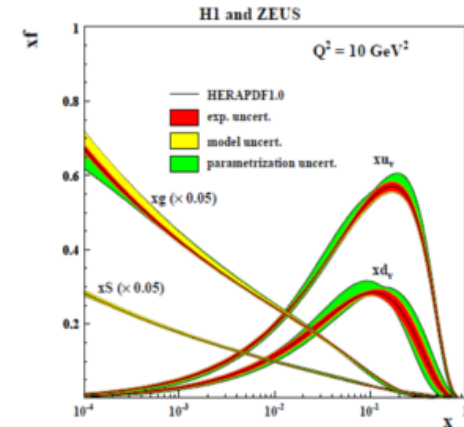
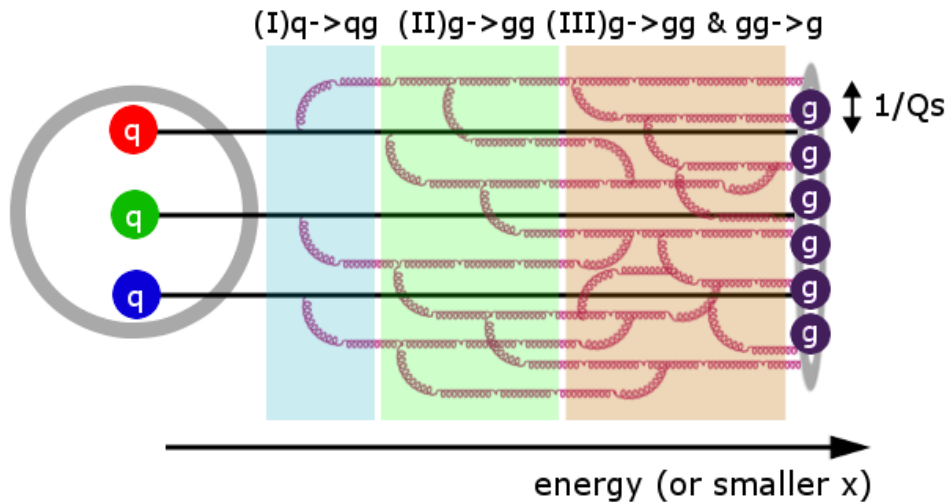
Glasma

✓ Gluon saturation of ultra-relativistic nuclei

[McLerran, Venugopalan (1994)]

⇒ something like a “color capacitor”

w/ huge color charge density = $O(Q_s) = O(\text{a few GeV})$

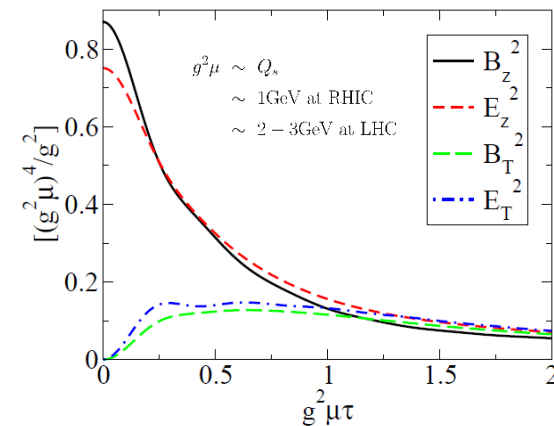
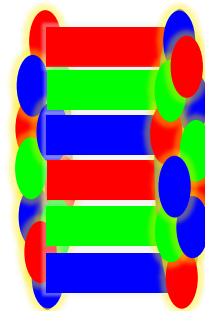


[Lappi, McLerran (2006)]

✓ High-energy heavy-ion collisions

≈ formation of “color condenser”

⇒ strong color flux tubes



Old ideas: [Low (1975)] [Nussinov (1975)] [Casher, Neuberger, Nussinov (1979)]