Magneto-vortical effect in strong magnetic field

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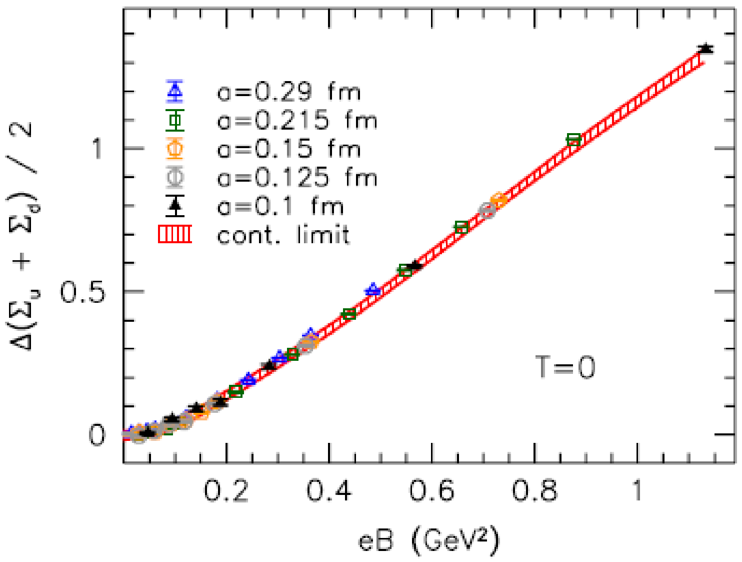
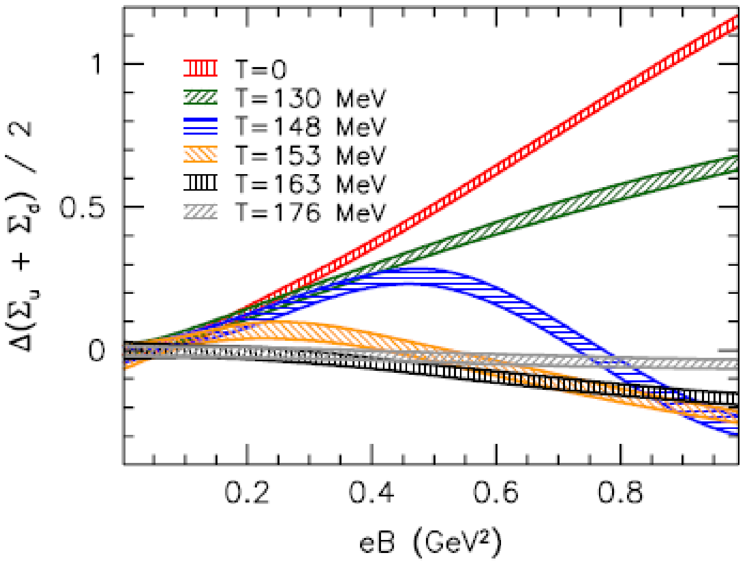
QCD theory seminar, 2021.4.13

based on SL, Lixin Yang 2103.11577

QCD at finite magnetic field

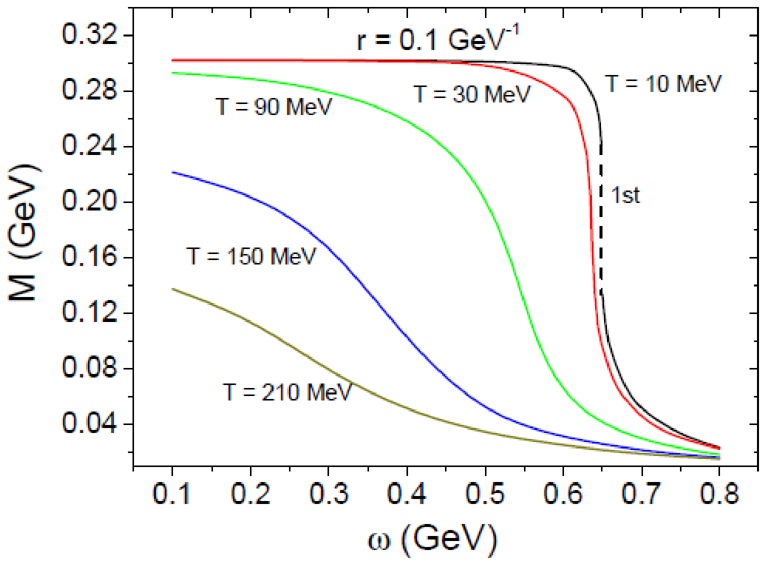




Dependent on quark mass

Gusynin, Miransky, Shovkovy, PRL 1994
Bali et al, PRD 2012, JHEP 2012
D'Elia et al, PRD 2018
Ding et al, PRD 2020

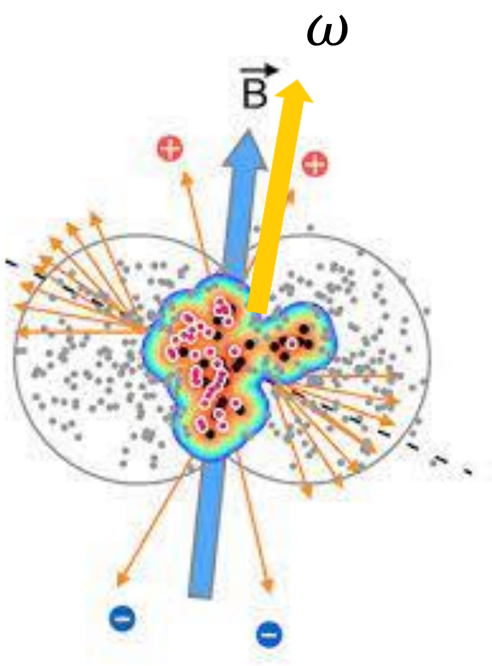
QCD with finite rotation

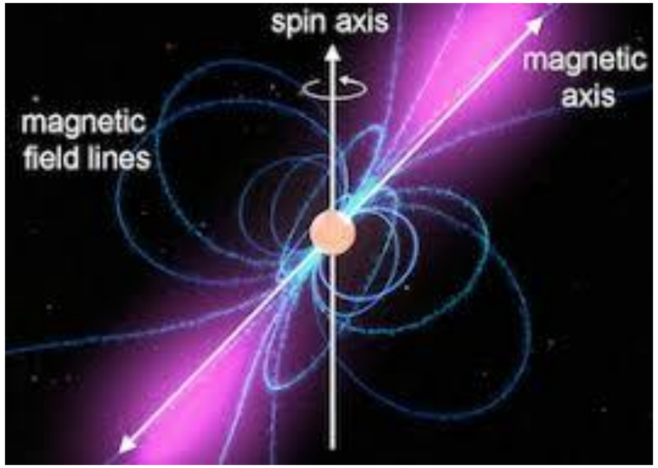


boundary needed due to causality, but not important at $T \gg 1/R$ or $\sqrt{eB} \gg 1/R$

Jiang, Liao, PRL 2016
Ebihara, Fukushima, Mameda, PLB 2017
Chernodub, Gongyo, JHEP 2017

Interplay of magnetic and vortical fields





Magneto-vortical effects?

Hattori, Yin, PRL 2017
Liu, Zahed, PRL 2017
Chen, Fukushima, Huang, Mameda, PRD 2016
Yang, Gao, Liang, Wang PRD 2020
Bu, SL, EPJC 2020

Magneto-vortical effects: vacuum part

lowest Landau level, $q_f \boldsymbol{S}$ align with B for particles

$$\Delta E = -\boldsymbol{S}\cdot\boldsymbol{\omega} = -\frac{1}{2} sgn(q_f)\omega$$

charge dependent energy shift due to vorticity,
mimicked by $\Delta\mu = \omega$

Hattori, Yin, PRL 2017

$$\Rightarrow \quad J^0 = q_f \frac{C_A}{2}\boldsymbol{B}\cdot\boldsymbol{\omega} \qquad \boldsymbol{J}_5 = |q_f|\frac{C_A}{2}\boldsymbol{B}(\boldsymbol{b}\cdot\boldsymbol{\omega})$$

Magneto-vortical effects: expected medium part

$$\Delta J_V^0 = -\boldsymbol{\nabla}\cdot\mathbf{P} - 2\mathbf{M}\cdot\boldsymbol{\omega} \qquad \sim u_\mu \partial_\nu M^{\mu\nu}$$

$$M^{\mu\nu} = P^\nu u^\mu - P^\mu u^\nu + \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\left(M_\beta u_\alpha - M_\alpha u_\beta\right)$$

Kovtun, JHEP 2016

P: electric polarization
M: magnetization

$$\Delta\mathbf{J}_A = \left(\frac{\mu^2+\mu_5^2}{2\pi^2} + \frac{T^2}{6}\right)\boldsymbol{\omega}$$

Vilenken, PRD 1979
Landsteiner et al, PRL 2011

Outline

- Magneto-vortical effect in plasma magnetized by large B
- Chiral kinetic theory with Landau level basis
- Drift state & vortical state
- Matching with magneto-hydrostatics
- Vacuum shift
- Conclusion & Outlook

Chiral kinetic theory with Landau level basis

$$\Pi_\mu j^\mu = 0,$$

$$\Delta_\mu j^\mu = 0,$$

$$\Pi^\mu j^\nu - \Pi^\nu j^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\Delta_\rho j_\sigma,$$

Collisionless CKE for RH fermions

SL, Yang, PRD 2020

$$\Delta_\mu = \partial_\mu - \frac{\partial}{\partial p_\nu}F_{\mu\nu},\ \Pi_\mu = p_\mu - \frac{1}{12}\frac{\partial^2}{\partial p_\nu \partial p_\lambda}\frac{\partial}{\partial x^\lambda}F_{\mu\nu}$$

$$F_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}u^\rho B^\sigma + E_\mu u_\nu - E_\nu u_\mu$$

$$B^\mu \sim O(\partial^0) \quad E^\mu \sim O(\partial)$$

regime of MHD (EM field external)

$$\partial \ll p \sim T$$

Covariant background solution

$\sqrt{eB} \gg T$, lowest Landau level approximation

$$j^{\mu}_{(0)} = (u+b)^{\mu}\delta(p\cdot(u+b))f(p\cdot u)e^{\frac{p_T^2}{B}}$$

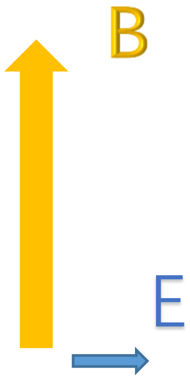
Gao, SL, Mo, PRD 2020

$$B^{\mu} = Bb^{\mu}$$ constant

$$f(p\cdot u) = \frac{2}{(2\pi)^3}\sum_{r=\pm}\frac{r\theta(rp\cdot u)}{e^{r(p\cdot u-\mu_R)/T}+1}$$

boundary set by size of fluid cell

I. Drift state



$$u^{\mu}_{(1)} \equiv \frac{1}{2B}\epsilon^{\mu\nu\rho\sigma} f_{\nu\rho} b_{\sigma}$$

drift velocity

$$u^{\mu} \to u^{\mu}_{\mathcal{D}} \equiv (u + u_{(1)})^{\mu}$$

$$j^{\mu}_{(0)} + j^{\mu}_{(1)\mathcal{D}} = (u_{\mathcal{D}} + b)^{\mu} \delta(p \cdot (u_{\mathcal{D}} + b)) f(p \cdot u_{\mathcal{D}}) e^{(p^2 - (p \cdot u_{\mathcal{D}})^2 + (p \cdot b)^2)/B}$$

Ambiguity in choosing fluid velocity:
u^{μ} thermodynamic frame
$u^{\mu}_{\mathcal{D}}$ Landau frame

Matching drift with magnetohydrostatics

with RH/LH fermions

$O(\partial^0)$

$$J^\mu_{(0)} = \frac{\mu B}{2\pi^2} u^\mu + \frac{\mu_5 B}{2\pi^2} b^\mu ,$$

$$T^{\mu\nu}_{(0)} = \frac{\chi_V B}{2\pi^2}(u^\mu u^\nu + b^\mu b^\nu) + \frac{\chi_A}{2\pi^2} u^{\{\mu} b^{\nu\}} ,$$

$O(\partial)$

$$J^\mu_{(1)\mathcal{D}} = -\frac{\mu}{2\pi^2}\epsilon^{\mu\nu\rho\sigma} u_\nu E_\rho b_\sigma ,$$

$$T^{\mu\nu}_{(1)\mathcal{D}} = -\frac{1}{2\pi^2}\left(\frac{B}{4} + \chi\right) u^{\{\mu}\epsilon^{\nu\}\lambda\rho\sigma} u_\lambda E_\rho b_\sigma ,$$

$$\chi_V \equiv \frac{\mu^2+\mu_5^2}{2} + \frac{\pi^2 T^2}{6} \qquad \chi_A \equiv \mu\mu_5$$

set $\mu_5 = 0$

MHS in thermodynamic frame

$$J^\mu = \mathcal{N} u^\mu + \mathcal{J}^\mu$$

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

Kovtun, Hernandez, JHEP 2017
Grozdanov et al, PRD 2017
Hongo, Hattori, JHEP 2021

matching at $O(\partial^0)$ determined pressure

$$\mathcal{J}^\mu_{(1)} = -\alpha_{BB,\mu}\epsilon^{\mu\nu\rho\sigma} u_\nu E_\rho B_\sigma ,$$

$$\mathcal{Q}^\mu_{(1)} = \left(M_{\omega,\mu} + 2p_{,B^2}\right)\epsilon^{\mu\nu\rho\sigma} u_\nu E_\rho B_\sigma$$

matching at $O(\partial)$ determined susceptibility

$$M^{\mathcal{D}}_\omega = -\frac{\mu}{8\pi^2} - \frac{\xi}{2\pi^2 B} \qquad \xi \equiv \frac{1}{3}\mu\left(\mu^2 + \pi^2 T^2\right)$$

II. Vortical state

$$\Pi_\mu j^\mu = 0,$$
$$\Delta_\mu j^\mu = 0,$$
$$\Pi^\mu j^\nu - \Pi^\nu j^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\Delta_\rho j_\sigma,$$

$$\omega^\mu = \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu\partial_\rho u_\sigma = \omega b^\mu$$
$$F_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}u^\rho B^\sigma$$

$$\Delta_\mu = \partial_\mu - \frac{\partial}{\partial p_\nu}F_{\mu\nu},\ \Pi_\mu = p_\mu - \frac{1}{12}\frac{\partial^2}{\partial p_\nu \partial p_\lambda}\frac{\partial}{\partial x^\lambda}F_{\mu\nu}$$

gradient on $f(p\cdot u)$ gradient on field strength

$$j^\mu_{(1)\mathcal{V}} = (u+b)^\mu\left[-\frac{\omega}{3}\left(\frac{p_T^2}{B}+1\right)\delta'(p\cdot(u+b))\,f(p\cdot u) + \frac{2\omega p_T^2}{3B}\delta(p\cdot(u+b))\,f'(p\cdot u)\right.$$
$$\left. - \frac{2\omega p_T^2}{B^2}p\cdot u\,\delta(p\cdot(u+b))\,f(p\cdot u)\right]e^{\frac{p_T^2}{B}} + \frac{\omega p_T^\mu}{B}\delta(p\cdot(u+b))\,f(p\cdot u)e^{\frac{p_T^2}{B}}.$$

upto homogeneous solutions: $O(\partial)$ change of $f(p\cdot u)$ in the background solution

Matching without knowing homogeneous solution

matching without knowing homogeneous solution:
assuming $O(\partial)$ change of $f(p \cdot u)$ through $O(\partial)$ change of $T\&\mu$

$$
\begin{aligned}
T(x) &\rightarrow T'(x) = T(x) + \delta T(x)\,, \\
\mu(x) &\rightarrow \mu'(x) = \mu(x) + \delta\mu(x)\,, \\
u^\mu(x) &\rightarrow u'^\mu(x) = u^\mu(x) + \delta u^\mu(x)\,,
\end{aligned}
$$

Kovtun, Hernandez, JHEP 2017

addition of homogeneous solution mimicked by hydrodynamic frame transformation

frame invariant variables

$$
\begin{aligned}
f &\equiv f_{\mathcal{P}} - \left(\frac{\partial \Pi}{\partial \epsilon}\right)_n f_{\mathcal{E}} - \left(\frac{\partial \Pi}{\partial n}\right)_\epsilon f_{\mathcal{N}}, \\
t &\equiv f_{\mathcal{T}} - \frac{B^2}{3}\left[\left(\frac{\partial \alpha_{BB}}{\partial \epsilon}\right)_n f_{\mathcal{E}} + \left(\frac{\partial \alpha_{BB}}{\partial n}\right)_\epsilon f_{\mathcal{N}}\right]
\end{aligned}
$$

Matching vortical with magnetohydrostatics

with RH/LH fermions

$O(\partial)$

$$J^{\mu}_{(1)\mathcal{V}} = \frac{\omega}{2\pi^2}\left(2\chi_V + \frac{2}{3}B\right)u^\mu + \frac{\omega}{2\pi^2}2\chi_A b^\mu,$$

$$T^{\mu\nu}_{(1)\mathcal{V}} = \frac{\omega}{2\pi^2}\left(2\xi_V + \frac{2}{3}\mu B\right)u^\mu u^\nu + \frac{\omega}{2\pi^2}\left(2\xi_V - \frac{1}{3}\mu B\right)b^\mu b^\nu + \frac{\omega}{2\pi^2}\left(2\xi_A + \frac{1}{6}\mu_5 B\right)u^{\{\mu}b^{\nu\}} + \frac{\omega}{2\pi^2}\frac{\mu B}{2}P^{\mu\nu},$$

$$\xi_V \equiv \tfrac{1}{3}\mu\left(\mu^2 + 3\mu_5^2 + \pi^2 T^2\right)$$

$$\xi_A \equiv \tfrac{1}{3}\mu_5\left(\mu_5^2 + 3\mu^2 + \pi^2 T^2\right)$$

MHS in thermodynamic frame

$$\mathcal{N}_{(1)} = f_{\mathcal{N}} = -2\left(2p_{,B^2} + M_{\omega,\mu}\right)B\omega,$$

$$\mathcal{E}_{(1)} = f_{\mathcal{E}} = -2\left(TM_{\omega,T} + \mu M_{\omega,\mu} - 2M_\omega\right)B\omega,$$

$$\mathcal{P}_{(1)} = f_{\mathcal{P}} = \frac{2}{3}\left(M_\omega + 4M_{\omega,B^2}B^2\right)B\omega,$$

$$\mathcal{T}_{(1)} = f_{\mathcal{T}} = -\frac{4}{3}\left(M_{\omega,B^2}B^2 + M_\omega\right)B\omega,$$

$$M_\omega^{\mathcal{V}} = \frac{\mu}{8\pi^2} + \frac{\#\xi}{B}$$

to be compared with $M_\omega^{\mathcal{D}} = -\frac{\mu}{8\pi^2} - \frac{\xi}{2\pi^2 B}$

vacuum part inconsistent!

Vacuum ambiguity

Magneto-vortical susceptibility $M_\omega = -\frac{\partial \mathcal{F}}{2\partial(B\cdot\omega)}$

assumes vacuum state unchanged as vorticity is turned on adiabatically

$$M_\omega^{\mathcal{D}} = -\frac{\mu}{8\pi^2} - \frac{\xi}{2\pi^2 B}$$

$$M_\omega^{\mathcal{V}} = \frac{\mu}{8\pi^2} - \frac{\xi}{2\pi^2 B}$$

vacuum energy density lowered $\frac{\mu B\omega}{2\pi^2}$

$$\frac{\mu B\omega}{2\pi^2} = \omega \times \frac{\mu B}{2\pi^2} = \Delta\mu \times n$$

$\Delta\mu = \omega$ consistent with shift of chemical potential

New vacuum

$$j^{\mu}_{\text{vac}} = \omega(u \pm b)^{\mu}\delta\left(p\cdot(u \pm b)\right) f'\left(p\cdot u\right) e^{\frac{p_T^2}{B}}$$

interpretation $\delta\mu_{\text{vac}} = -\omega$

$$T^{\mu\nu}_{\text{vac}} = \frac{1}{2}\int d^4p p^{\{\mu} j^{\nu\}}_{\text{vac}} = -\frac{\mu B\omega}{2\pi^2}\left(u^{\mu}u^{\nu} + b^{\mu}b^{\nu}\right), \quad J^{\mu}_{\text{vac}} = \int d^4p j^{\mu}_{\text{vac}} = -\frac{B\omega}{2\pi^2}u^{\mu}$$

MHD thermodynamic quantities measured with respective to this new vacuum state

Consistent calculations of J^0 and J_A

$$\Delta J^0 = -2(2p_{,B^2} + M^{\mathcal{D}}_{\omega,\mu})B\omega = \left(\frac{B}{4\pi^2} + \frac{\chi}{\pi^2}\right)\boldsymbol{\omega}\cdot\mathbf{b}$$

MHD with unshifted vacuum

$$-2M\cdot\omega$$

$$\Delta J^0 = -2(2p_{,B^2} + M^{\mathcal{V}}_{\omega,\mu})B\omega - J^0_{\mathrm{vac}} = \left(\frac{B}{4\pi^2} + \frac{\chi}{\pi^2}\right)\boldsymbol{\omega}\cdot\mathbf{b}$$

MHD with shifted vacuum

$$\Delta \mathbf{J}_A = \left(\frac{B}{4\pi^2} + \frac{\chi}{\pi^2}\right)\boldsymbol{\omega}$$

vacuum part $\propto$ B, medium part $\propto$ χ

The full vortical solution

$$f_{\mathcal{N}} = f'_{\mathcal{N}} + \delta T\, n_{,T} + \delta\mu\, n_{,\mu},$$

$$f_{\mathcal{E}} = f'_{\mathcal{E}} + \delta T\, \epsilon_{,T} + \delta\mu\, \epsilon_{,\mu},$$

$$f_{\mathcal{P}} = f'_{\mathcal{P}} + \delta T\, \Pi_{,T} + \delta\mu\, \Pi_{,\mu},$$

$$f_{\mathcal{T}} = f'_{\mathcal{T}} + \frac{B^2}{3}\left(\delta T\, \alpha_{BB,T} + \delta\mu\, \alpha_{BB,\mu}\right)$$

homogeneous solution

$$\delta T = \frac{3\omega\mu}{\pi^2 T}, \qquad \delta\mu = -\frac{7\omega}{6}$$

$$j^{\mu}_{(1)\mathcal{V}} - j^{\mu}_{\mathrm{vac}} = (u+b)^{\mu}\Bigg[-\frac{\omega}{3}\left(\frac{p_T^2}{B}+1\right)\delta'\left(p\cdot(u+b)\right) f(p\cdot u) + \left(\frac{2\omega p_T^2}{3B} + \frac{\omega}{6} - \frac{3\omega\mu}{2\pi^2 T^2}(p\cdot u-\mu)\right)\delta\left(p\cdot(u+b)\right) f'(p\cdot u) - \frac{2\omega p_T^2}{B^2} p\cdot u\,\delta\left(p\cdot(u+b)\right) f(p\cdot u)\Bigg] e^{\frac{p_T^2}{B}} + \frac{\omega p_T^{\mu}}{B}\delta\left(p\cdot(u+b)\right) f(p\cdot u) e^{\frac{p_T^2}{B}}$$

modified disperision

$\delta\mu - \delta\mu_{vac} \& \delta T$

other p_T dependent terms: excitation of LLL states

Conclusion&Outlook

- Covariant chiral kinetic theory with Landau level basis
- Find drift state with Hall current and heat current
- Find vortical state and apparent discrepancy in matching with magnetohydrodynamics
- Consistency restored after vacuum shift taken into account
- Results medium contribution in addition to vacuum contribution in magneto-vortical effect, consistent with MHD and CVE expectation
- Higher LL contributions
- Collisional effect

Thank you!