# Hard- & Soft- deconfinement

## from nuclear to quark matter

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Ref) K. Fukushima, T.K., and W. Weise, 2008.08436 [hep-ph]

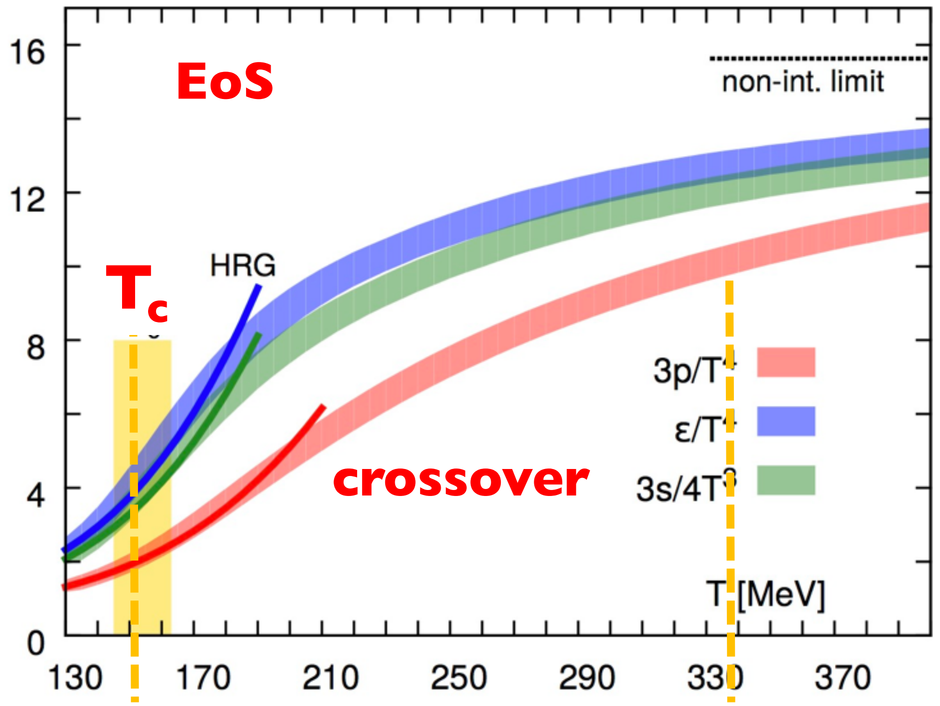
# *The purpose of this work*

- propose notions of Soft- & Hard- Deconfinement
- offer a concrete description of quark-hadron continuity
- suggest new schemes of computations for dense QCD

# Confinement-deconfinement in hot QCD

No order parameters, but the notion of conf-deconf trans. still makes sense :

Hot QCD case, lattice (e.g., Ding+, review '15)




Hadron resonance gas

semi-QGP

pQGP

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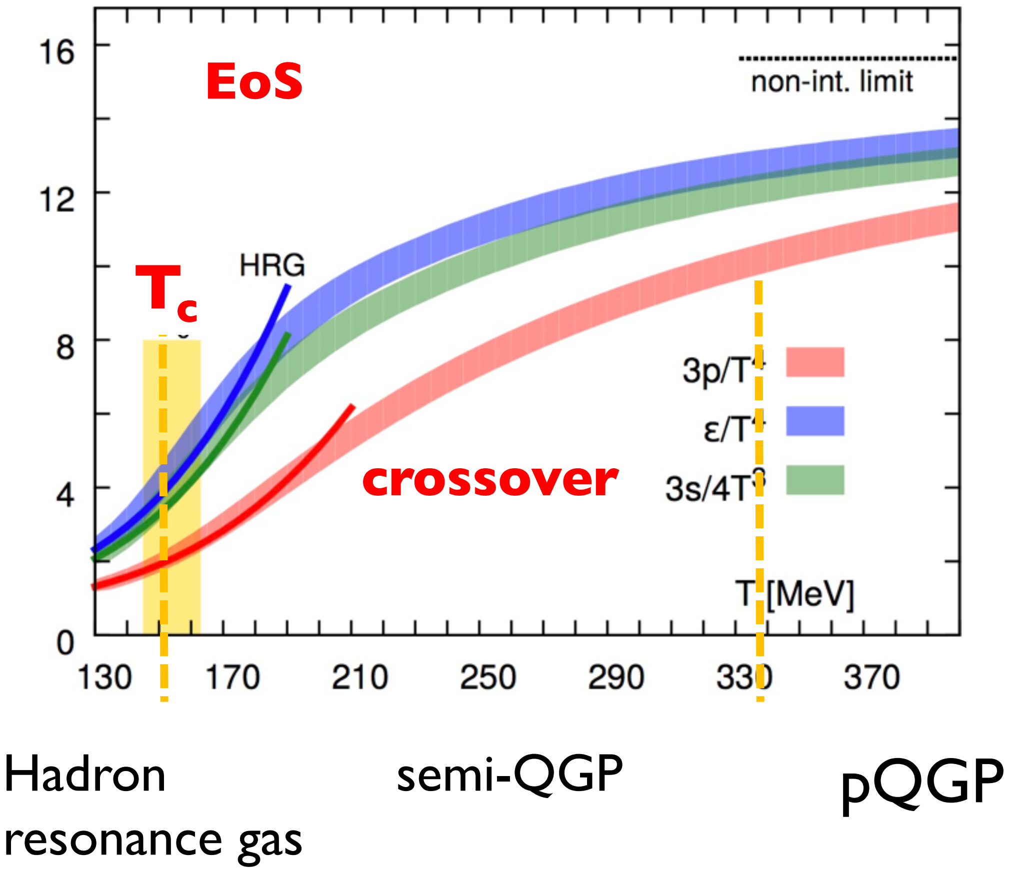
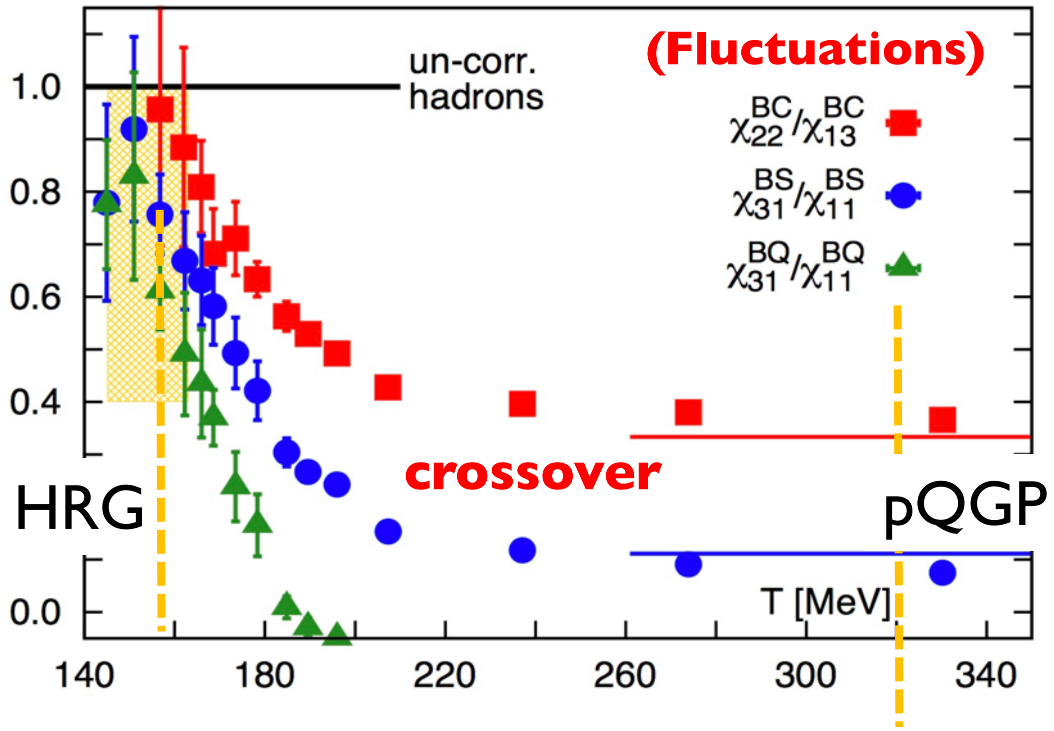
(e.g., Ding+, review '15)




**derivatives**

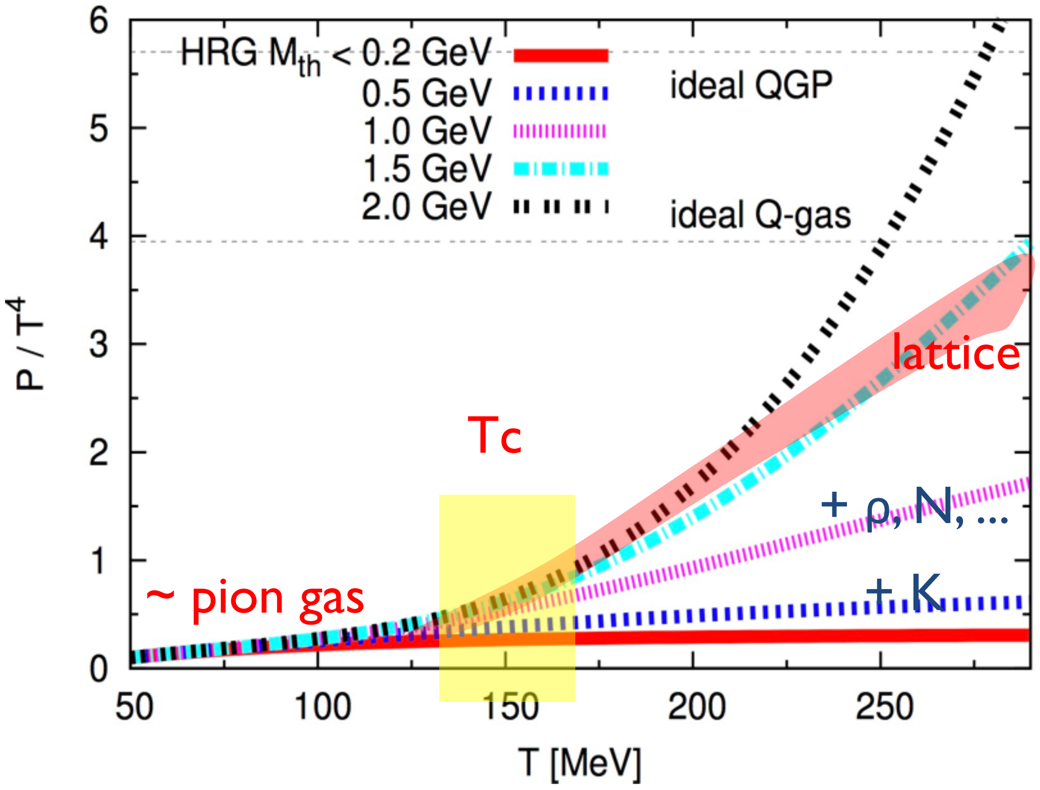
→ more info

$\mu_B$, $\mu_Q$, $\mu_S$, $\mu_C$




→ "$T_c$": universal for different flavors

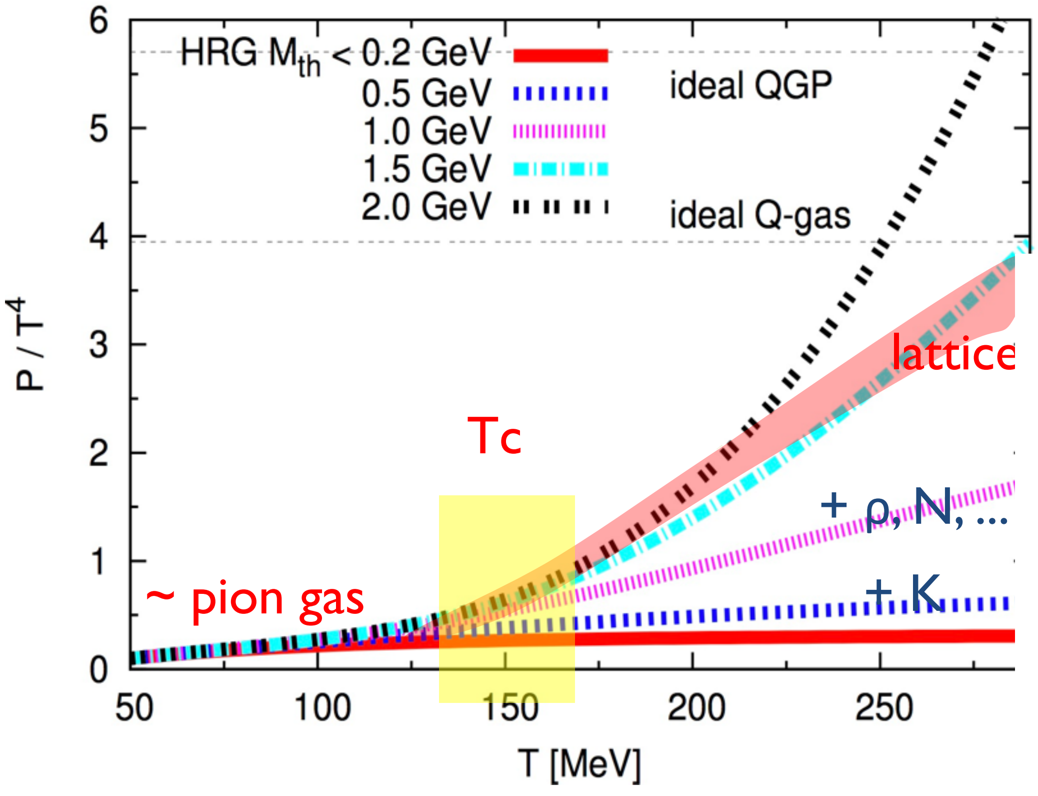
# An intuitive picture




- At ~ $T_c$, many hadrons with $E >> T$

  (**entropic** effects > **energy cost**; Hagedorn)

# An intuitive picture

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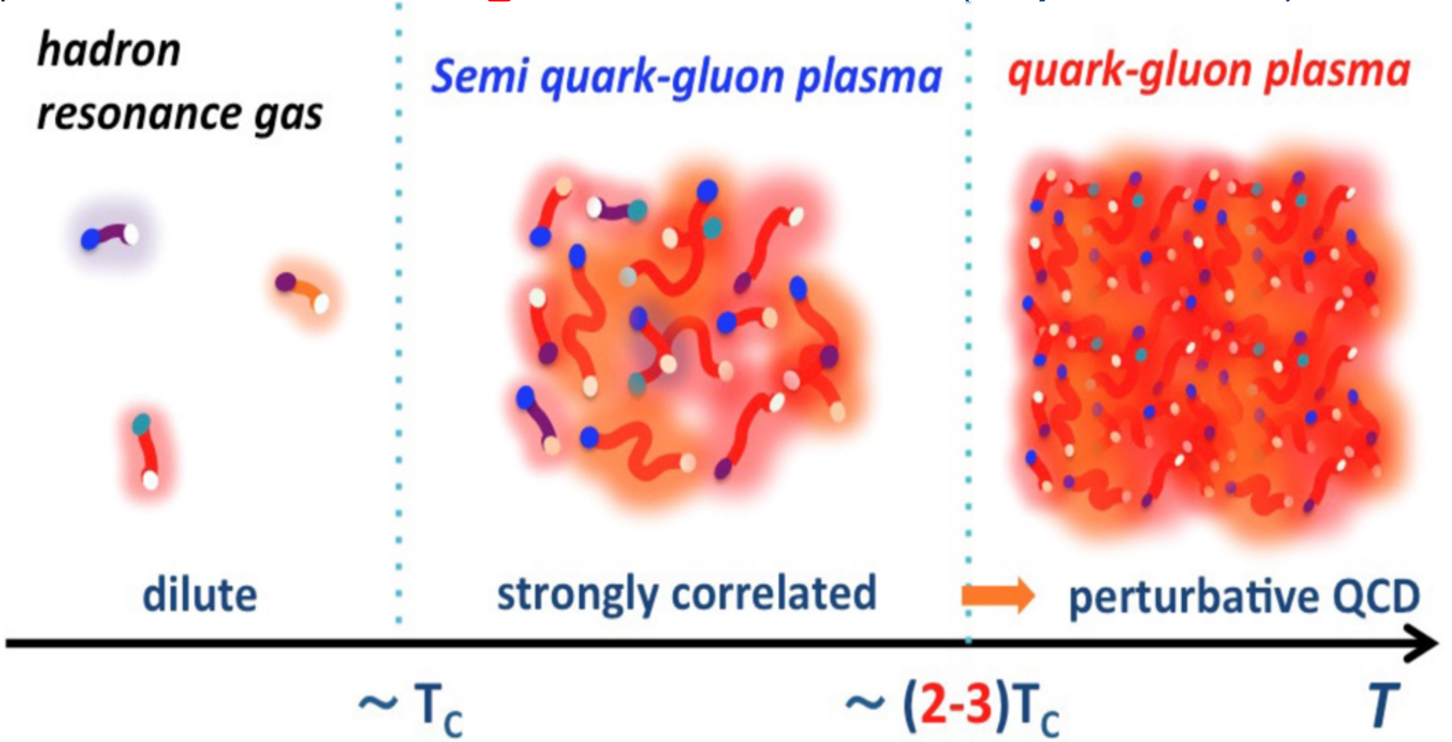
(**entropic** effects > **energy cost**; Hagedorn)

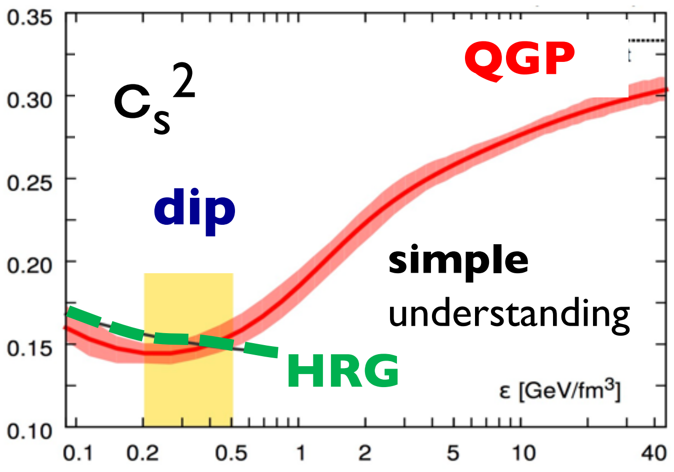
***"string condensations"*** (Polyakov '77,...)




*Can we draw this sort of cartoon for dense QCD ?*

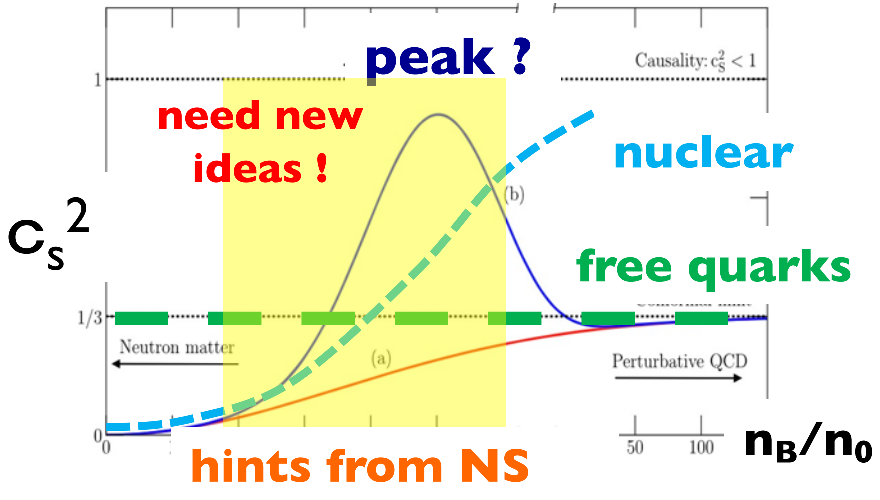
# hot QCD vs cold, dense QCD

| hot QCD | | | cold, dense QCD |
|---|---|---|---|
| · **Heavy Ion** experiments | laboratory | | · **Neutron Stars** |
| · **lattice** simulations | 1st principle | | · **no** lattice results (sign problem) |
| · pQCD (**close** to exp. domain) | dense | | · pQCD (**far** from exp. domain) |
| · HRG (**simple** calculations) | dilute | | · nuclear matter (**complicated !**) |
| · crossover (HRG→QGP) | type of trans. | | · not known (nuclear→quark matter) |

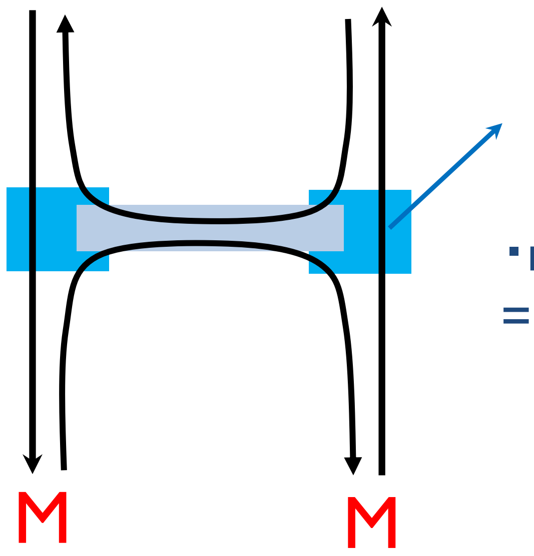



qualitative understanding

e.g.) speed of sound, d.o.f., ....



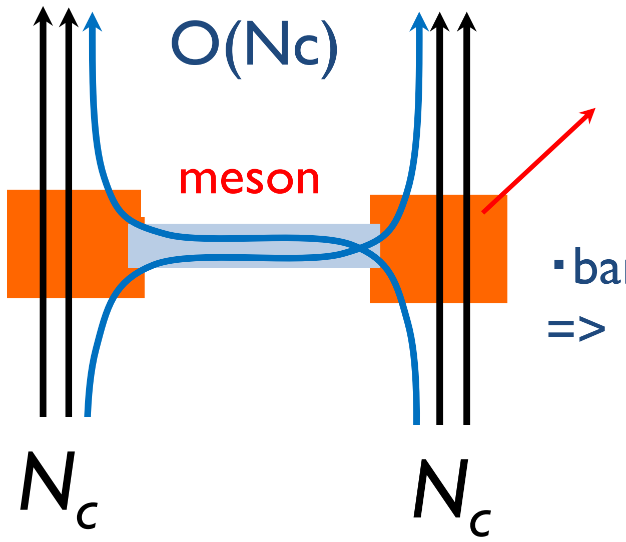
# $N_c$ counting : HRG vs Nuclear Matter




$g_{MM} \sim N_c^{-1/2}$

- mesons

=> weakly coupled

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- baryons

=> strong sources of mesons

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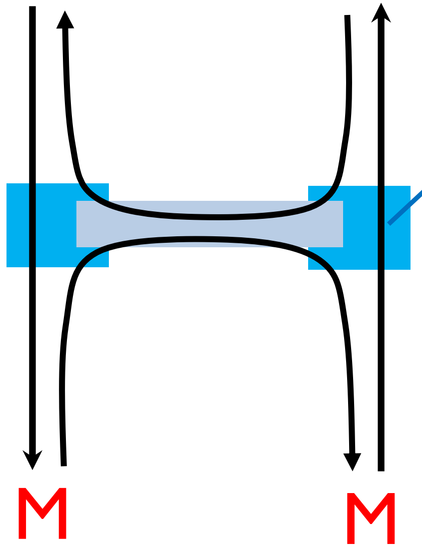


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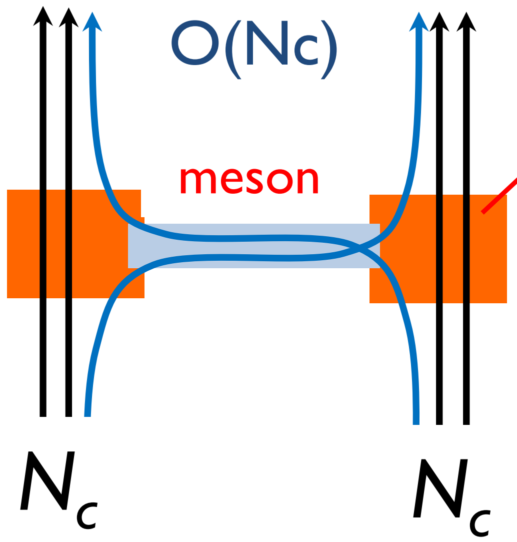


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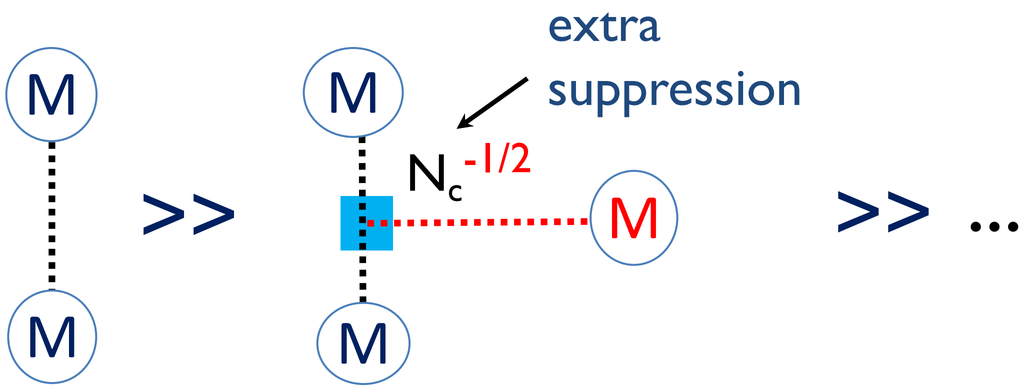
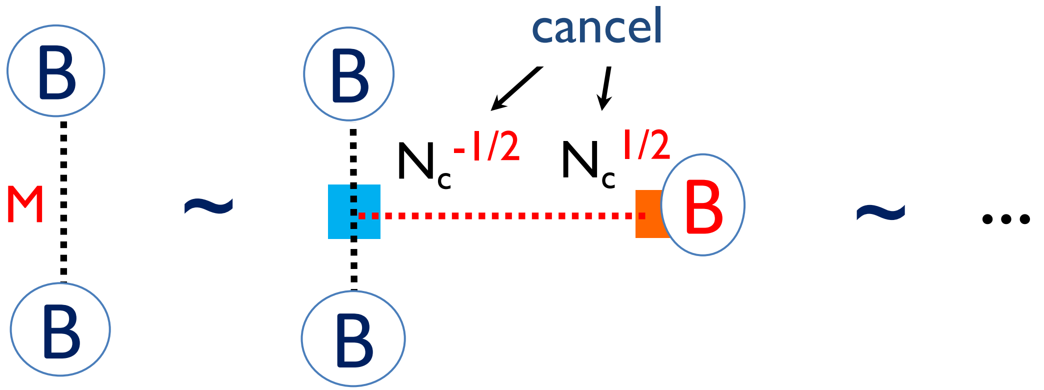
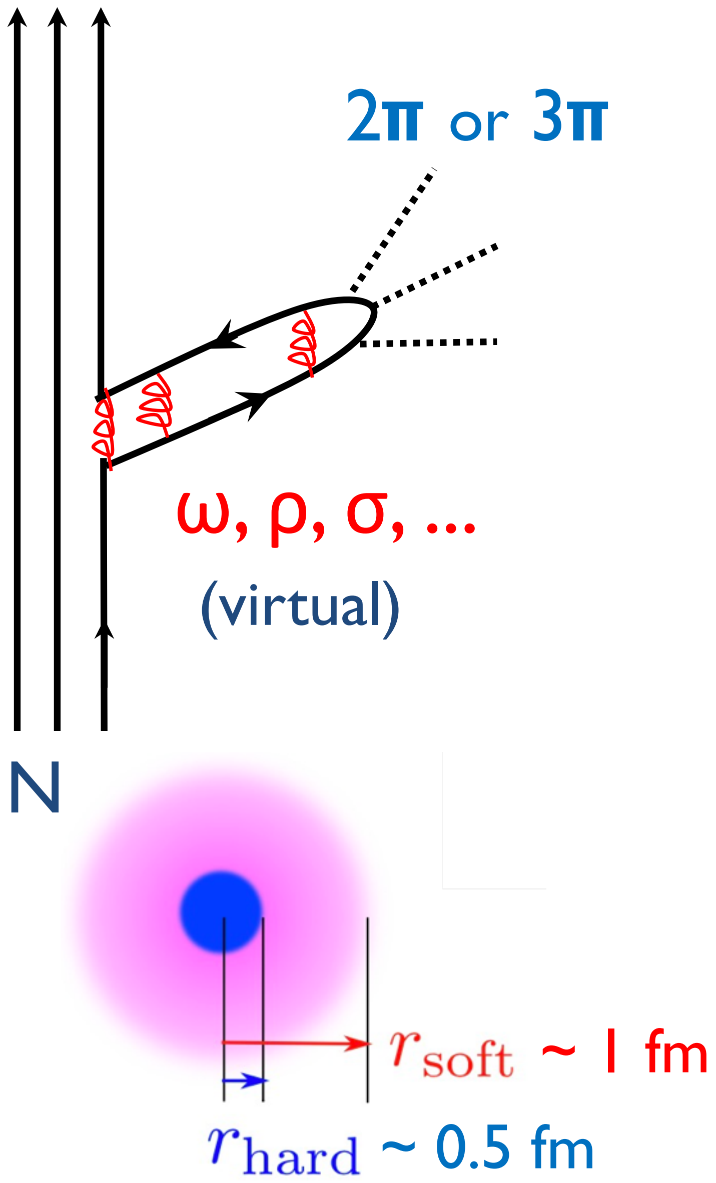



$\langle V_{2\text{-body}} \rangle \gg \langle V_{3\text{-body}} \rangle \gg \langle V_{4\text{-body}} \rangle \gg \dots$

$\langle V_{2\text{-body}} \rangle \sim \langle V_{3\text{-body}} \rangle \sim \langle V_{4\text{-body}} \rangle \sim \dots \sim O(\mathbf{Nc})$ !

# "Soft" & "Hard" scales in a nucleon



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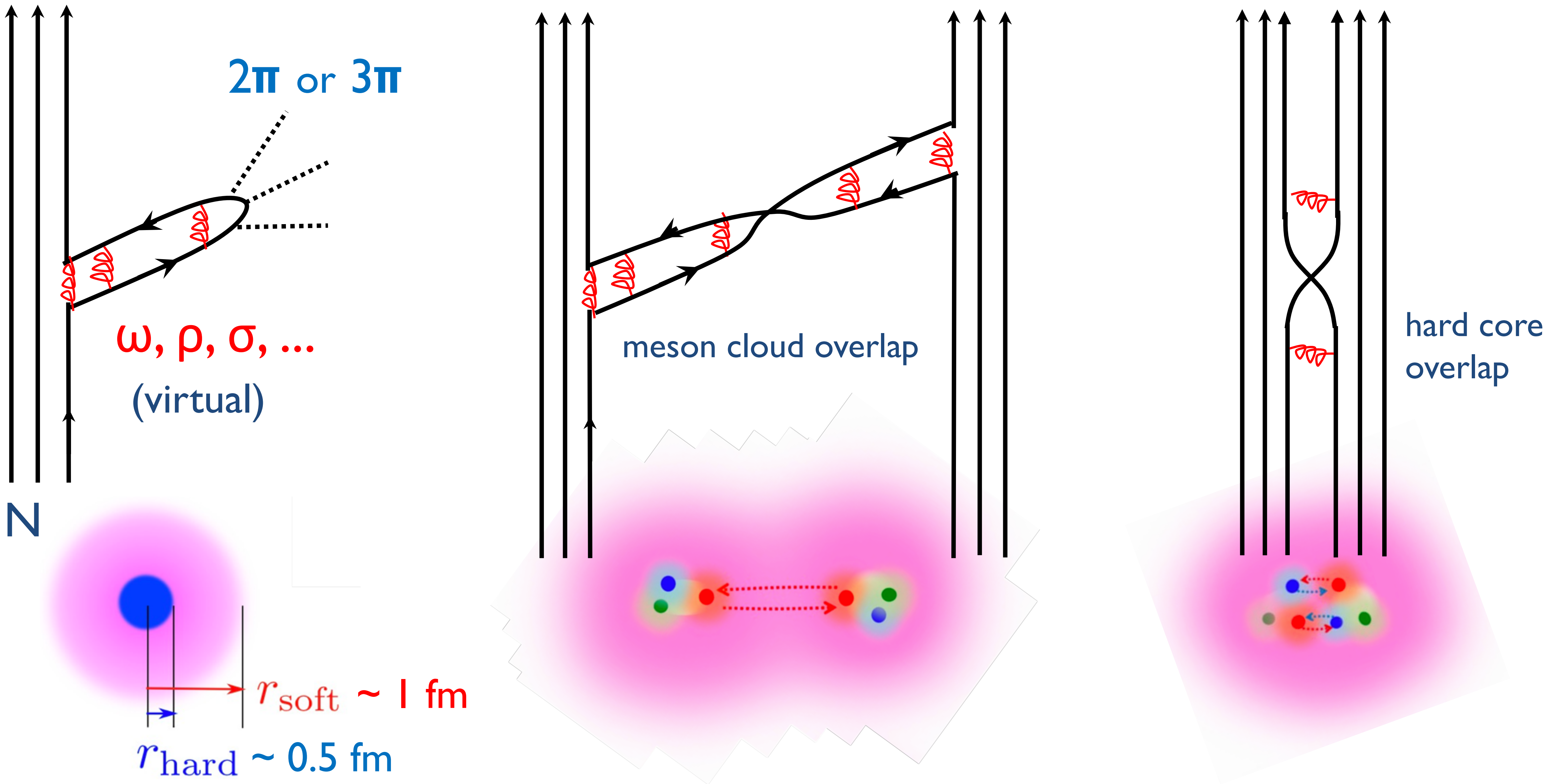
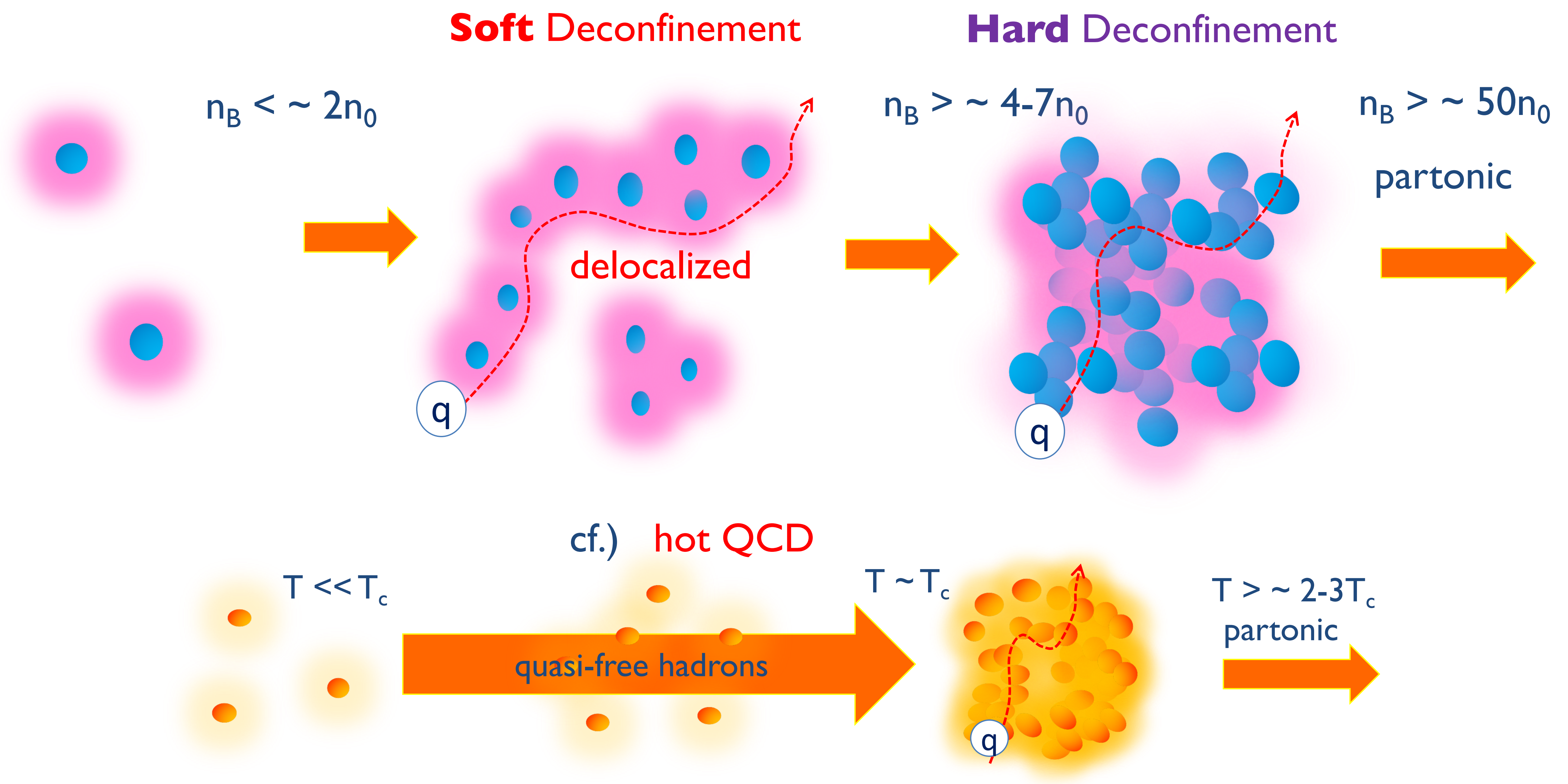


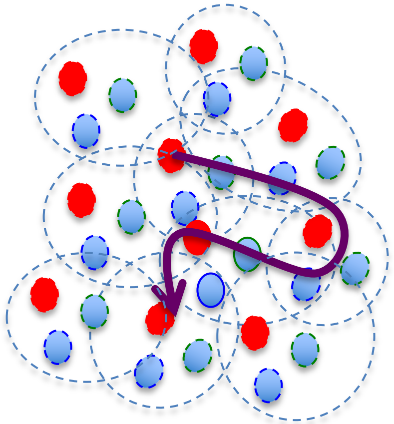
# Soft & Hard Deconfinement



# Hard Deconfinement

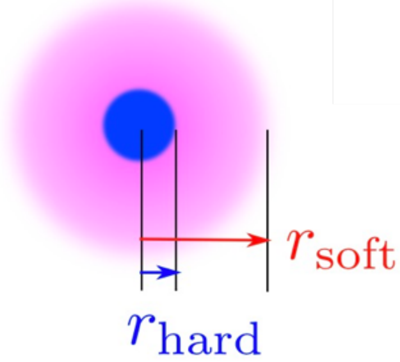
**$4\text{-}7\ n_0 \quad < \quad n_B \quad < \quad \sim 50\ n_0$**

hard core overlap



pQCD valid

# Hard Deconfinement

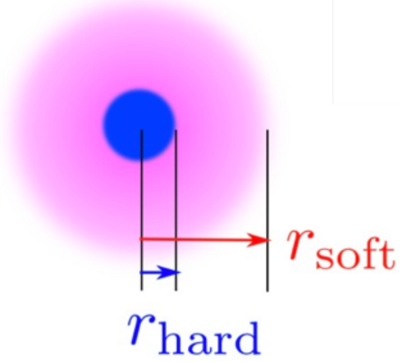



hard core overlap $\neq$ perturbative regime

core properties of a nucleon $\rightarrow$ alternative baselines ?

e.g. ***mechanical*** **p** & **ε** in a nucleon

# Hard Deconfinement




hard core overlap ≠ perturbative regime

core properties of a nucleon → alternative baselines ?

e.g. ***mechanical*** **p** & **ε** in a nucleon

**gravitational** form factors [Kobzarev-Okun('63), Pagels('66)]

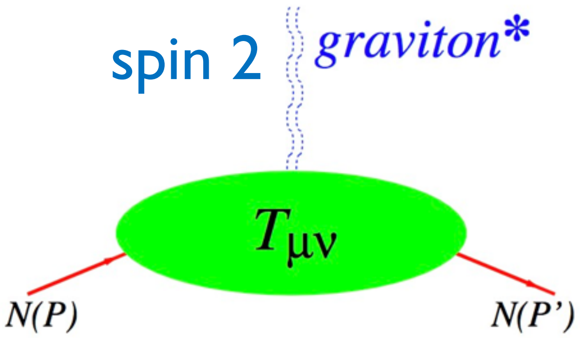
proton state

$$\langle p_2|\hat{T}^q_{\mu\nu}|p_1\rangle = \bar{U}(p_2)\left[M_2^q(t)\frac{P_\mu P_\nu}{M} + J^q(t)\frac{\imath(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + d_1^q(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M}\right]U(p_1)$$

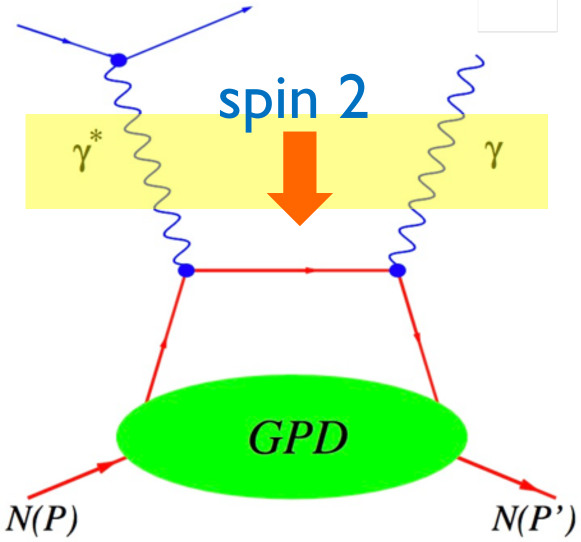
energy mom. tensor — energy density — angular mom — pressure & shear forces

$P = (p_1+p_2)/2$ ; $\Delta = p_1 - p_2$

# On-going programs



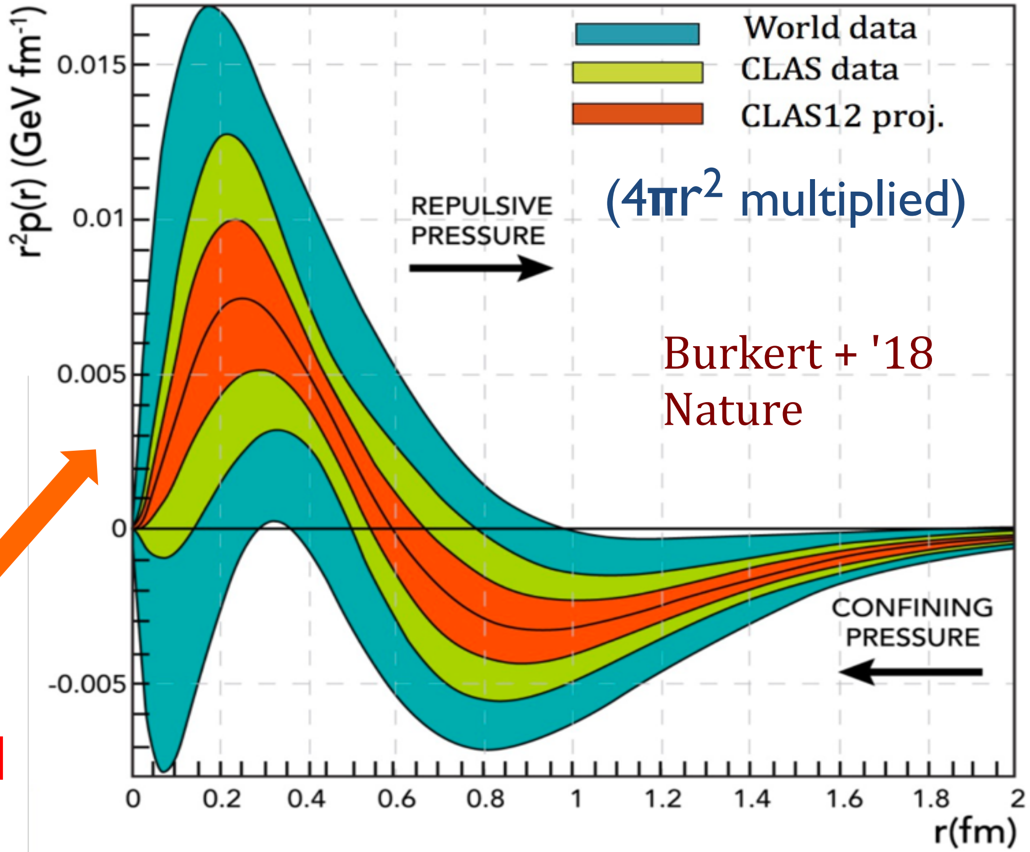

not practical




a target in **EIC**

**errors to be reduced**

mechanical pressure distribution

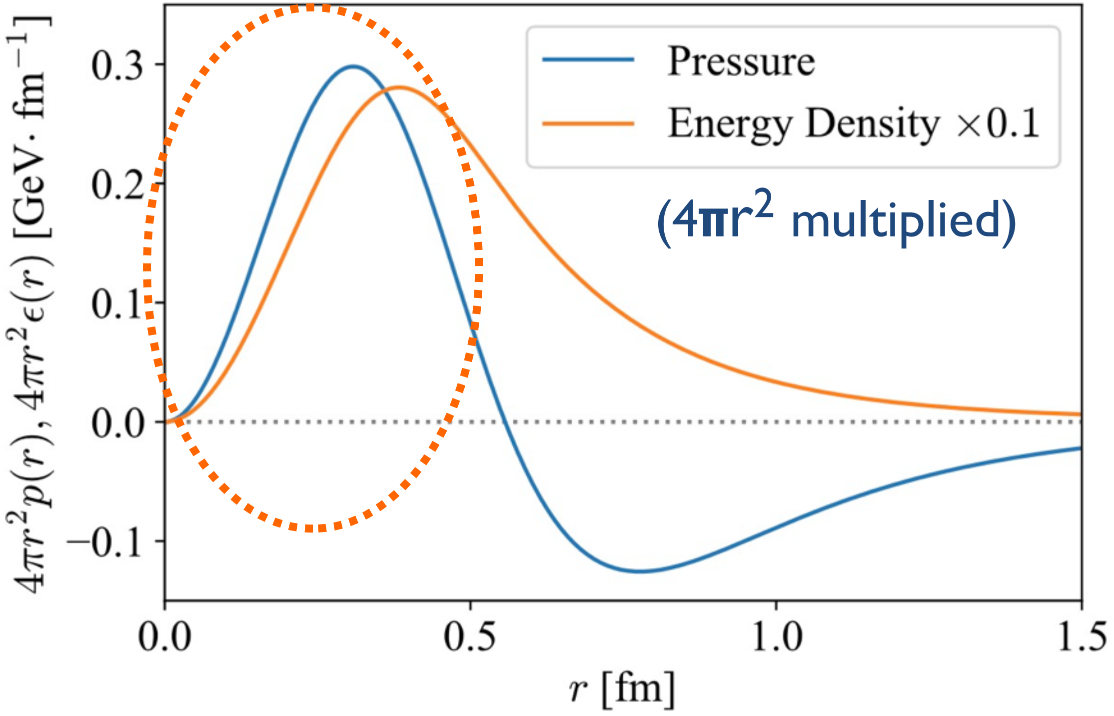


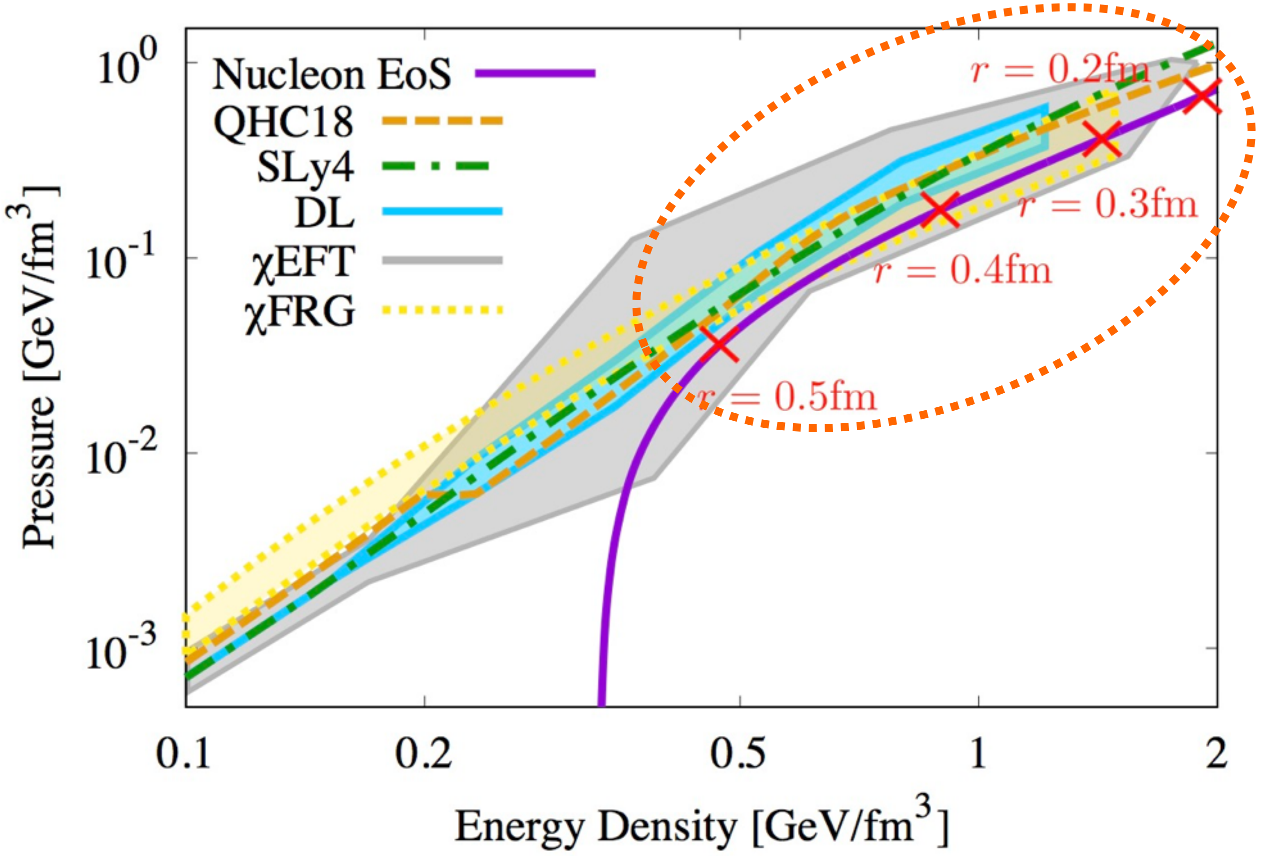

models on the market: bag model, chiral quark soliton, Skyme model,... (see also Rahan+ 2018)

For tentative estimates, we use a chiral soliton model + ω, ρ mesons (next slide)

# "Nucleon" EoS vs neutron star EoS

Distributions in a nucleon



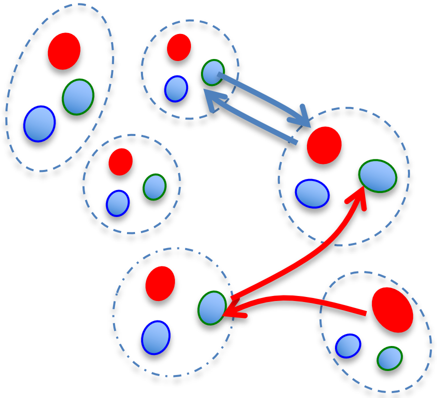

close to realistic **NS EoS** : a useful **baseline ?** (like $R_{AA}$ in HIC)

With this, can we quantify medium correlations such as BCS pairing?

# Soft Deconfinement

**$\sim 2\ n_0 \quad < \quad n_B \quad < \quad 4\text{-}7\ n_0$**

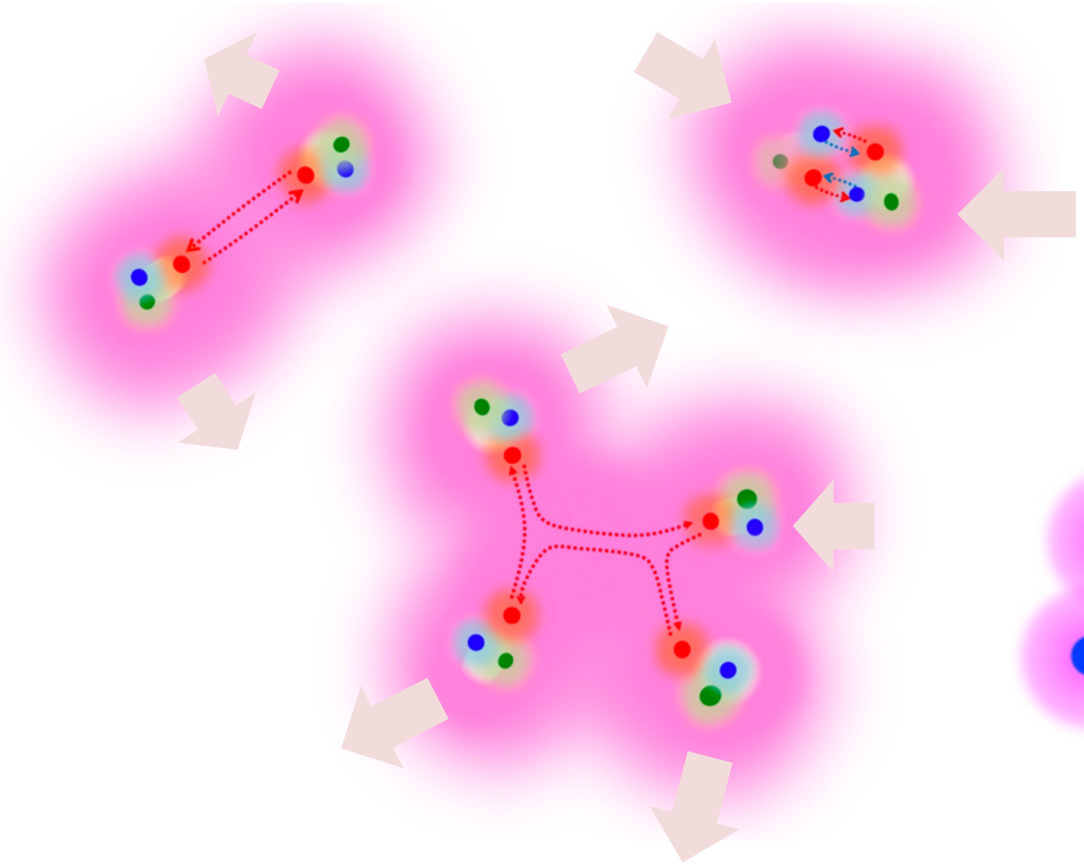
nuclear

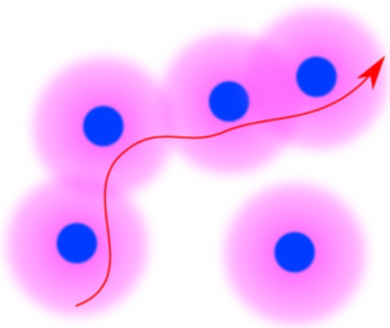


hard core overlap

# Soft Deconfinement

relating "*multi-quark exchanges*" to "*delocalization of quark w.f.*"





**Need**

to solve
the dynamics of quarks
being exchanged
among moving baryons

looks very complicated...

# strategy

Separate **fast** quark dynamics from **slow** baryon dynamics

=> *Born-Oppenheimer* descriptions

1, The velocity : $k_B/E_B \sim 1/Nc \ll k_q/E_q \sim 1$ $(k_B \sim k_q \sim n_B^{1/3})$

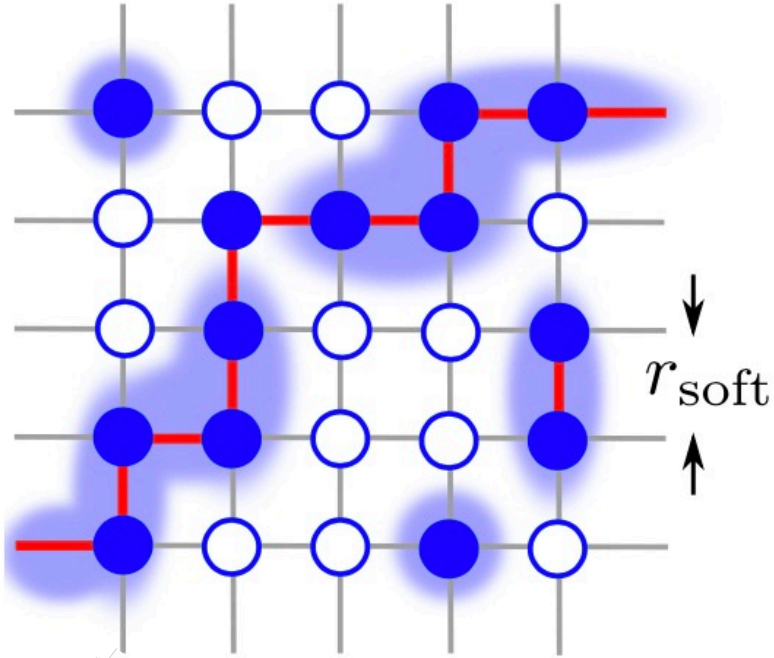
$n_B = n_q^R = n_q^G = n_q^B$

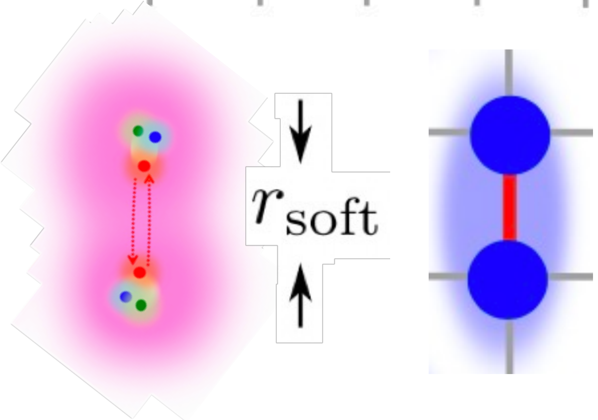
2, Find quark eigenstates for **a given** baryon configuration

3, Take the "**time** average" $\rightarrow$ "**ensemble** average" of baryons

# A model of **classical** percolation

3D cubic lattice




probability

a site is occupied by a baryon : $p$

unoccupied : $1 - p$

[one baryon per site]

$$\langle N_B \rangle = p \times \text{sites} = \langle n_B \rangle \times \text{volume}$$

$$\rightarrow \quad p \sim \langle n_B \rangle \times r_{\mathbf{soft}}^3$$

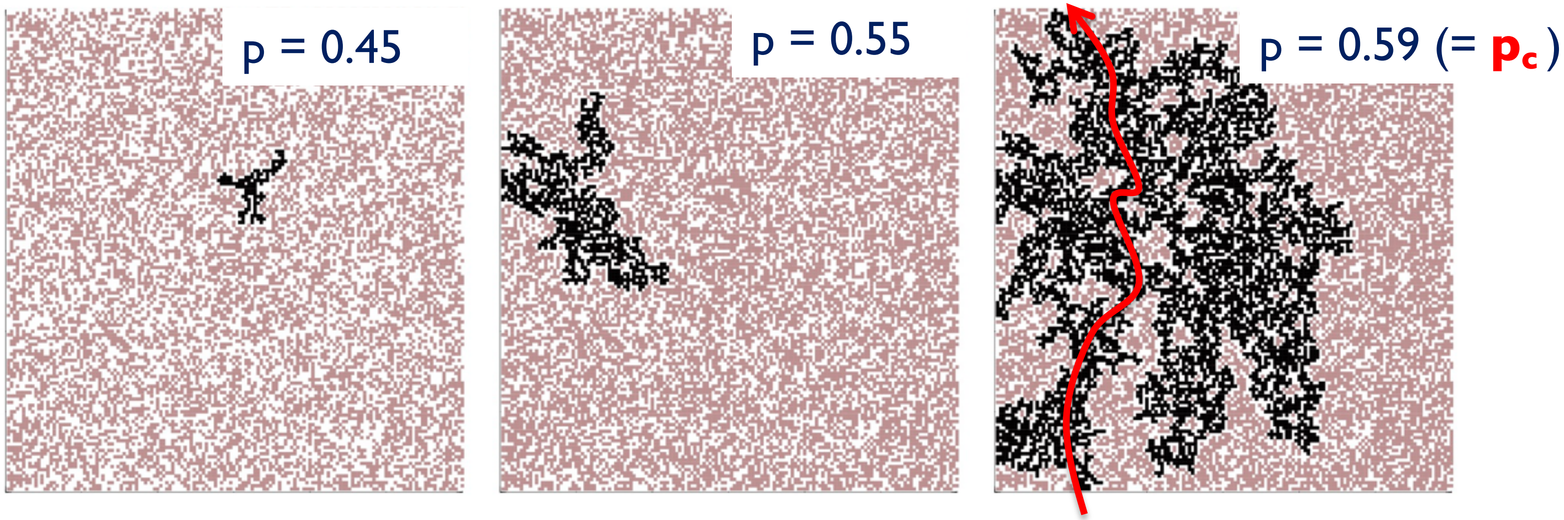
**bonds** for nearest neighbor hopping of quarks

# Classical Percolation (no quark dynamics)

When do baryon clusters connect two opposite boundaries?

( for $V \to \infty$ )

e.g. 2D lattice




$\mathbf{p_c}$ : critical probability (concentration)

def) at $p = p_c$, a cluster reaches opposite boundaries for the first time

# Classical Percolation (no quark dynamics)

For 3D cubic lattice : $p_c = 0.34...$

A rough estimate of the critical density

Assuming $r_{soft} \sim 0.7$ fm

$$n_B^c \sim \mathbf{0.34} \times (4\pi\, r_{soft}^3/3)^{-1} \sim 0.24\ \mathrm{fm}^{-3}$$

$$\sim \mathbf{1.4}\, n_0 \;!$$

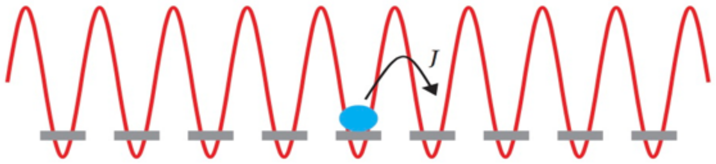
This may happen within the nuclear territory

But the CL percolation tells us only about the availability of paths for quarks...

# Quantum Percolation

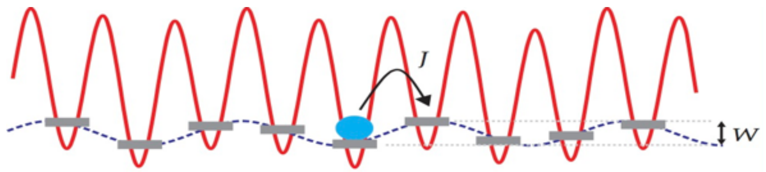
Quantum amplitudes from various paths may cancel: (destructive) **interference**

e.g. *Anderson localization* ('57)




**clean**

**dirty**



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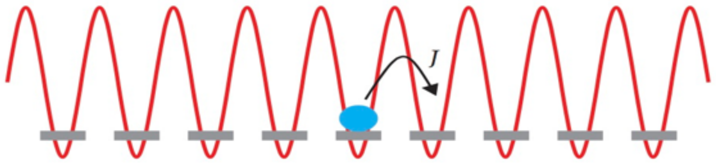


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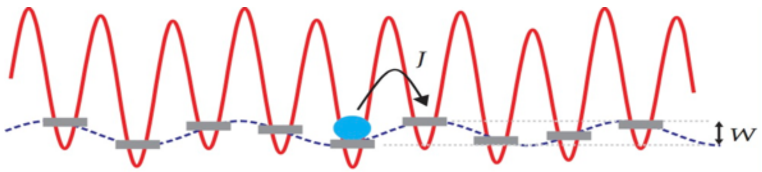
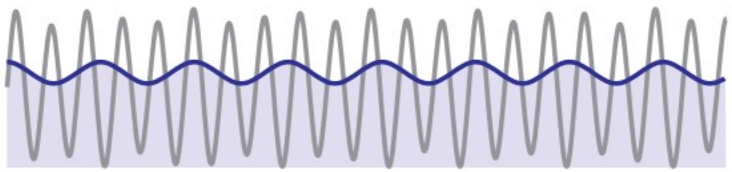
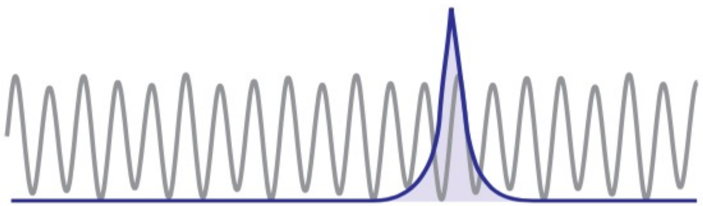


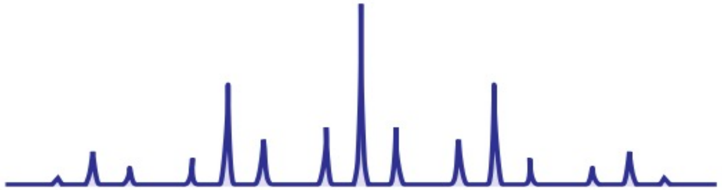

→ **Bloch waves** (phase coherence)

→ **Localized states** (random phase)



coordinate space



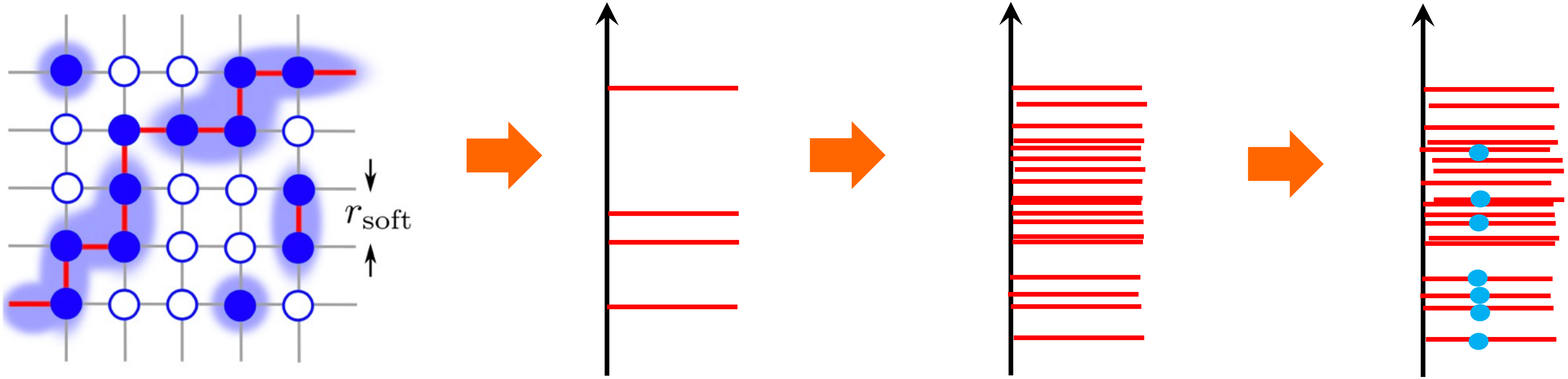


momentum space

# Delineating quark wavefunctions

## procedures

solve a **single** particle problem for **a given geometry** → prepare single particle levels → sum geometries (& normalize) → fill levels by quarks




=> we diagnose the **quark contents** of given baryon configurations

# A model of **quantum** percolation

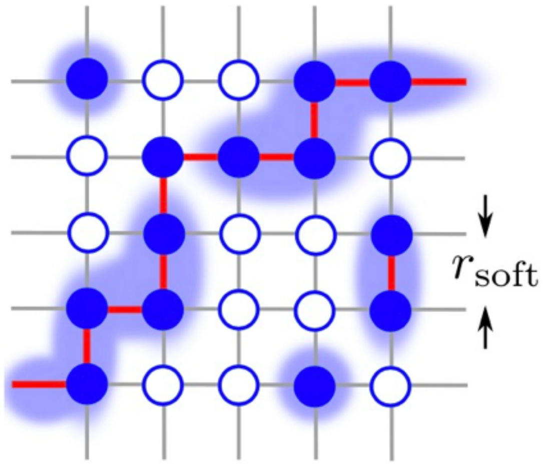
[ Kirkpatrick-Eggarter '72,... ]

tight-binding Hamiltonian

on-site energy, hopping

$$H = \sum_n |n\rangle \varepsilon_n \langle n| + \sum_{n \neq m} |n\rangle V_{nm} \langle m|$$

$|n>$ : a quark state exists at a site $n$




$$V_{nm} = -V \quad (V > 0)$$

nearest-neighbor hopping

the on-site energy is generated with probabilities

$$P(\varepsilon_n) = p\delta(\varepsilon_n - \varepsilon_{\rm on}) + (1-p)\delta(\varepsilon_n - \varepsilon_{\rm off})$$

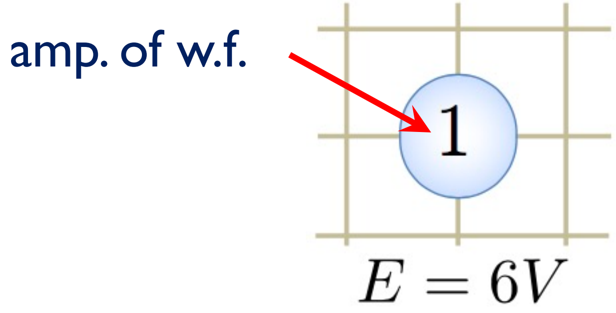
$\varepsilon_{\rm on}$ = 6V (convenient choice)

$\varepsilon_{\rm off} \to \infty$ (confinement)

quarks hop only within connected clusters

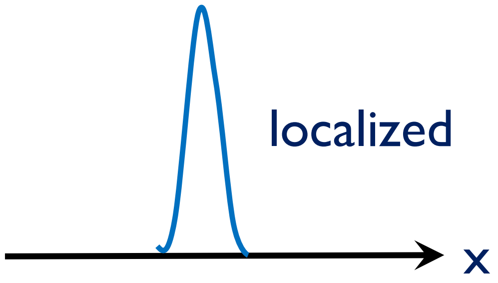
# Examples of one-particle states

1) $p \to 0$ (dilute limit) : **isolated** baryons only




localized in x, y, z-directions
(no hopping)

$E = \varepsilon_{\mathbf{on}} = 3 \times 2V$




$V \sim 1/\,(2M_q\, r_{soft}^2)$

like kin. energy (NR)

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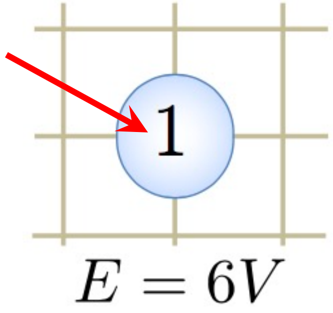
amp. of w.f.

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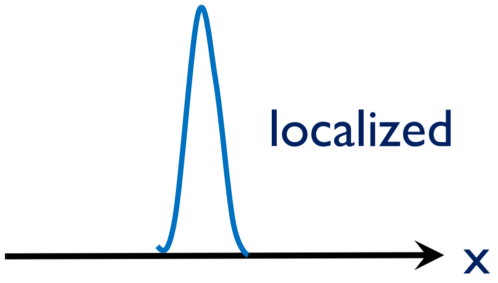



$V \sim 1/\,(2M_q\, r_{soft}^2)$

like kin. energy (NR)

2) $p \to 1$ (dense limit): **all** sites are **filled**

Eigenstates => plane waves with wavenumbers $k_i$

(Bloch type)

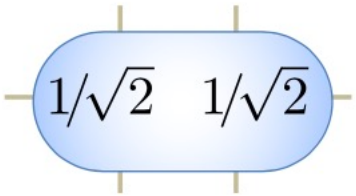
$$E(k) = 4V \sum_{i=x,y,z} \sin^2(k_i/2) \quad \longrightarrow \quad V\, k^2$$

small k limit

# Examples of one-particle states

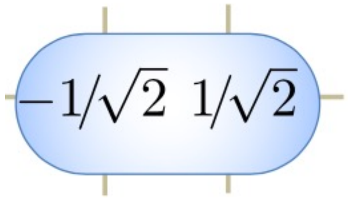
for few baryon clusters (localized in y, z-directions)

a) 2-baryons




$E = 5V$

0-node




$E = 7V$

1-node

# Examples of one-particle states

for few baryon clusters (localized in y, z-directions)

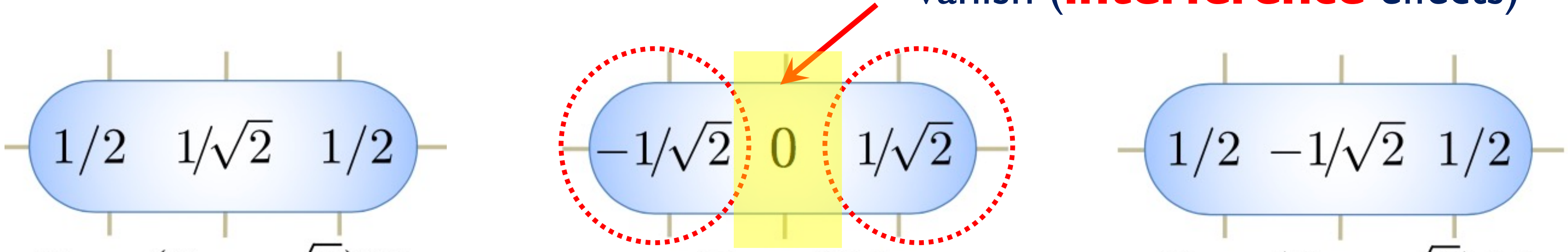
a) 2-baryons


$E = 5V$ 0-node

$E = 7V$ 1-node

vanish (**interference** effects)

b) 3-baryons



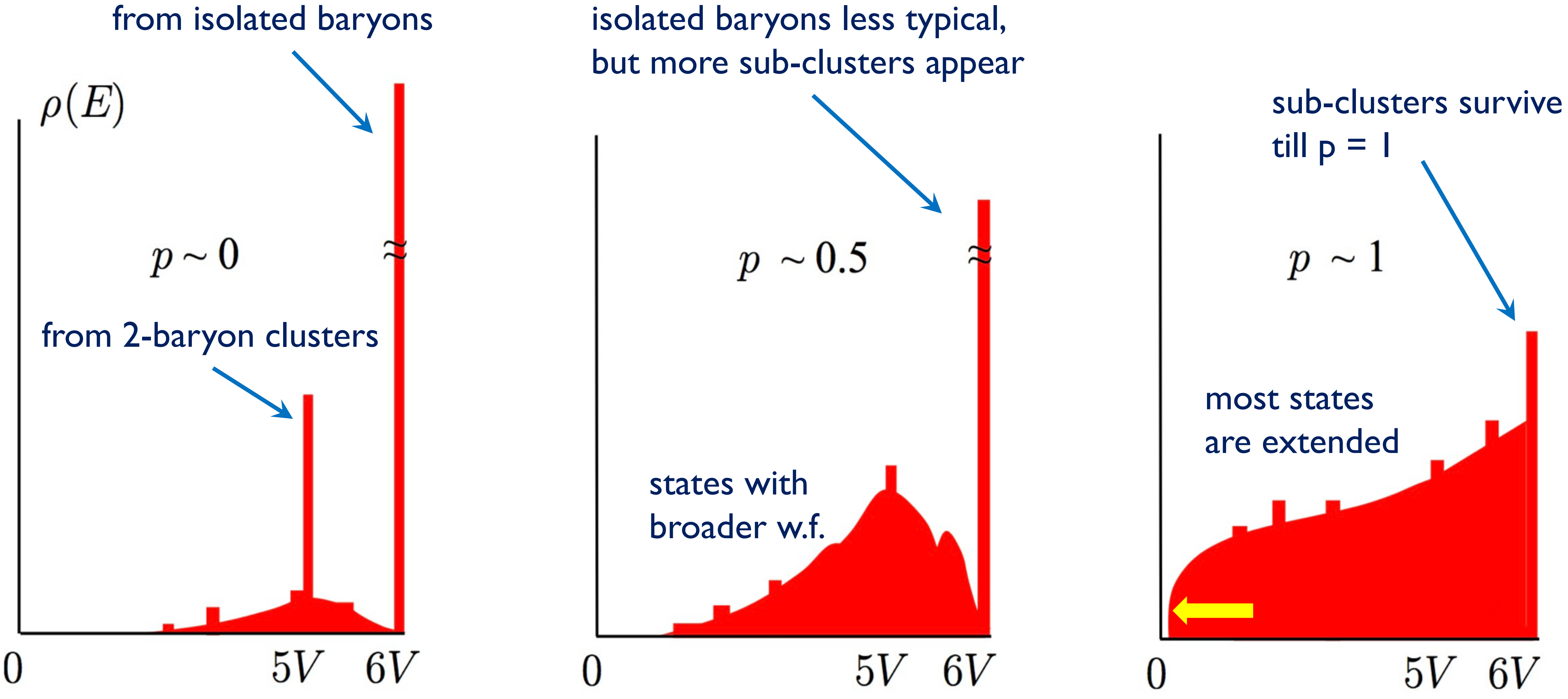

$E = (6 - \sqrt{2})V$ 0-node

$E = 6V$ **localized**

$E = (6 + \sqrt{2})V$ 2-node

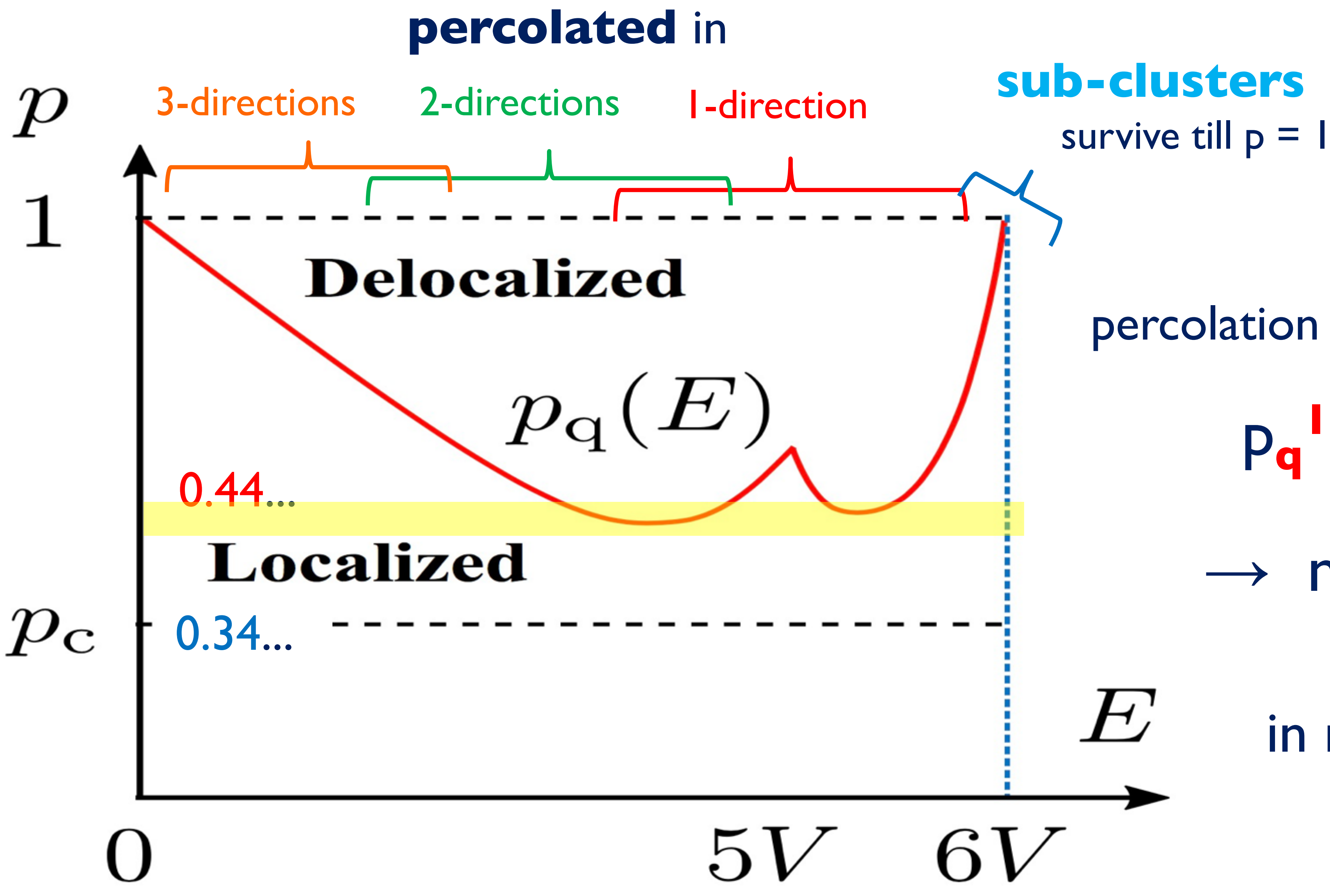
classically connected contain sub-clusters (localized states)

# Histograms of quark eigenstates $\int dE \rho(E) = 1$




Note: classical percolation is necessary condition for quantum one

# **Mode-by-mode** percolation




percolation proceeds from 1D to 3D

$p_q^{1D} \sim 0.44...$

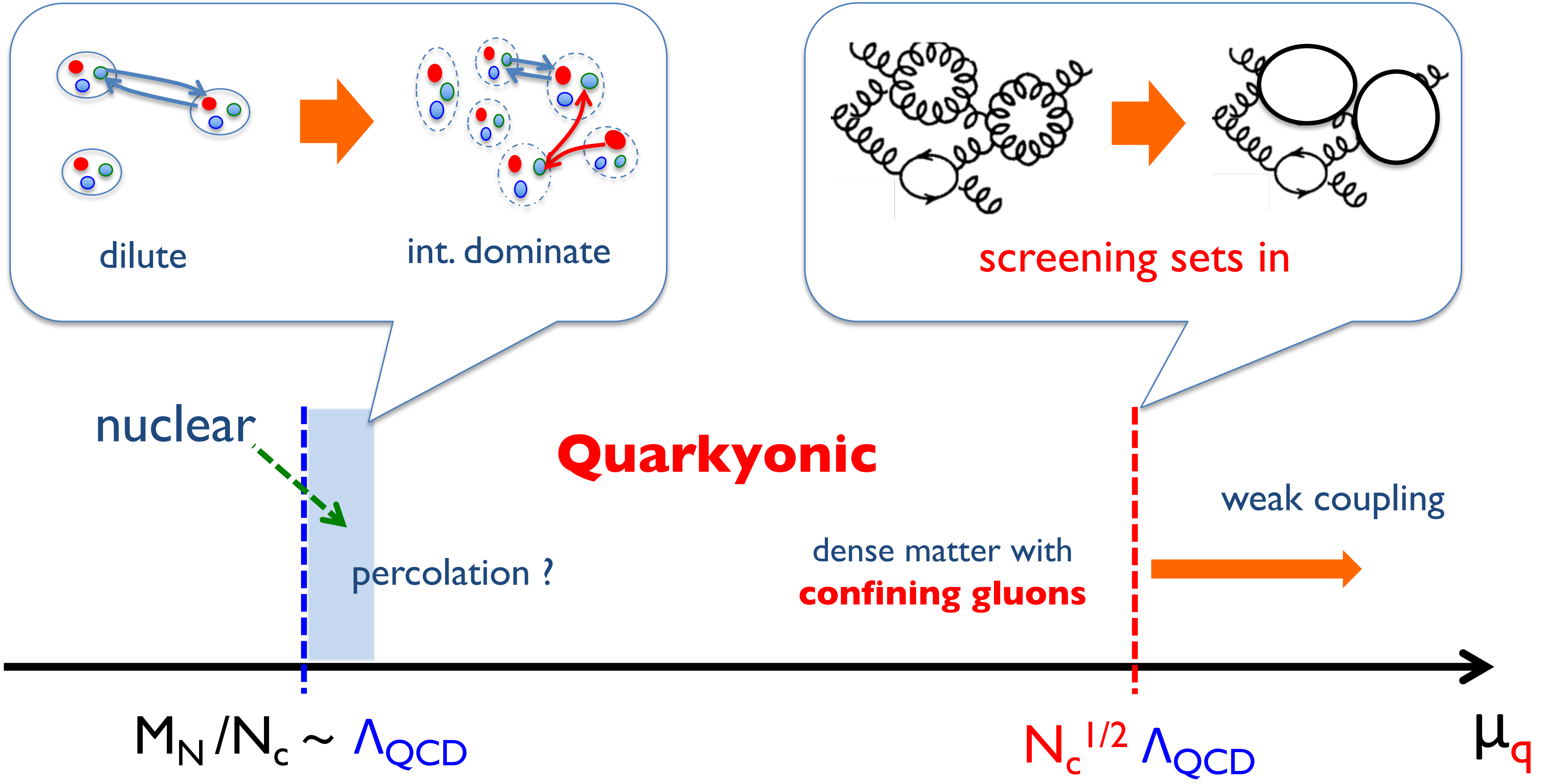
$\rightarrow n_B^{1D} \sim$ **1.8** $n_0$

in nuclear territory?

# A momentum shell in Quarkyonic Matter

# Quarkyonic Matter

[ McLerran-Pisarski '07 ]



# Momentum shell

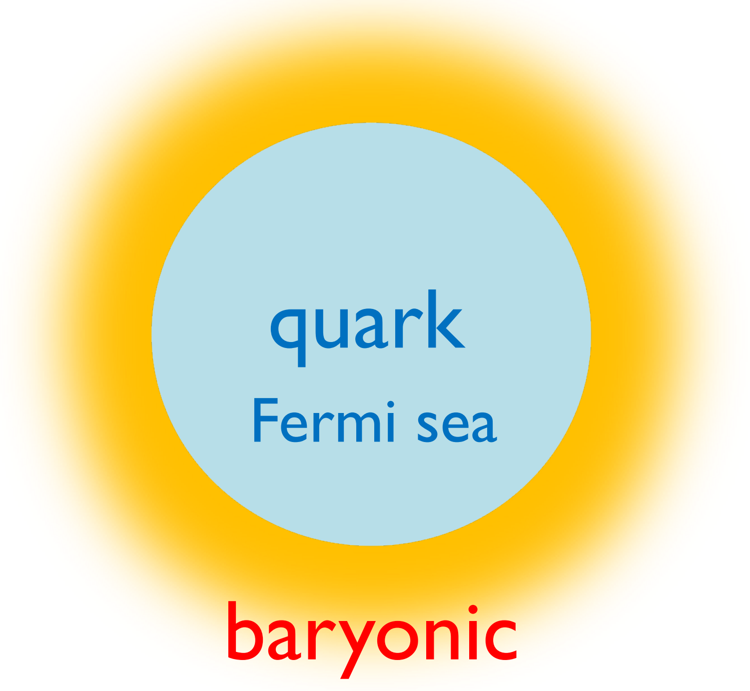
Dense matter with **confining gluons**

**effective d.o.f ?** (minimizing impacts of interactions?)

use only colorless d.o.f.

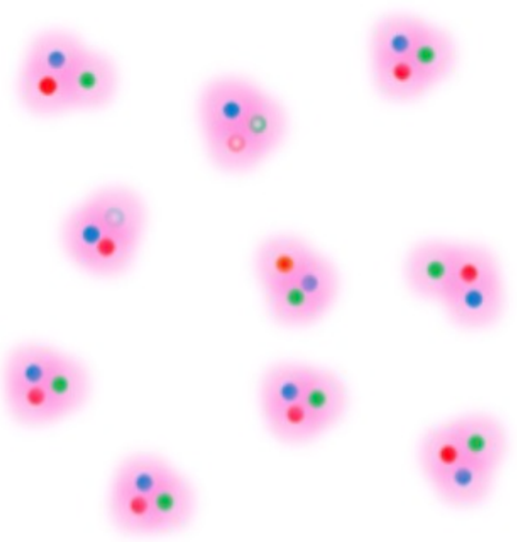

often claimed picture

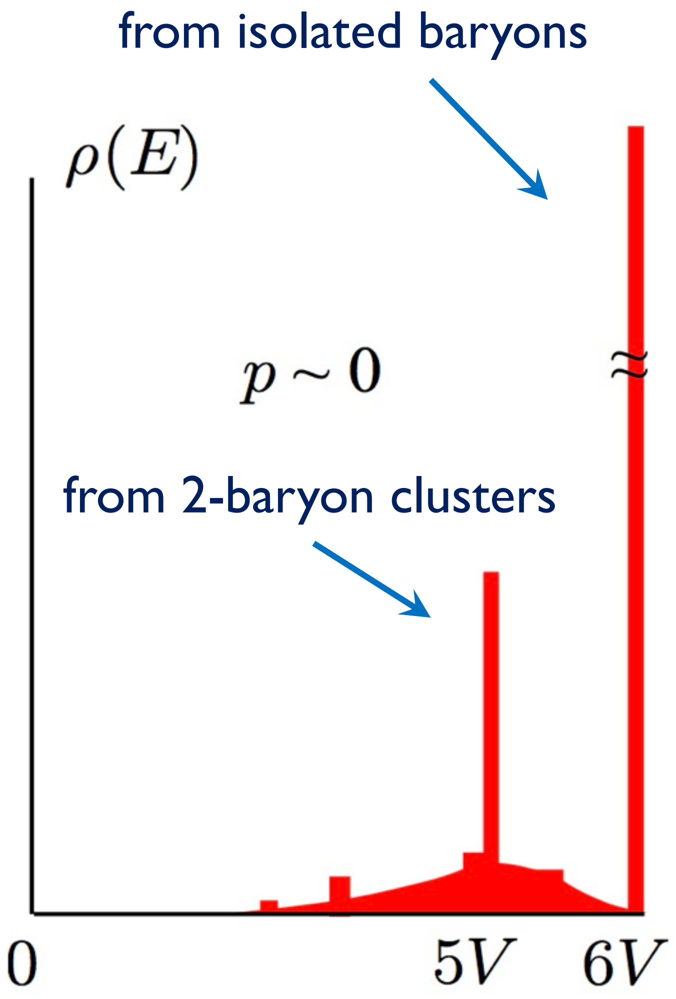

Quarkyonic



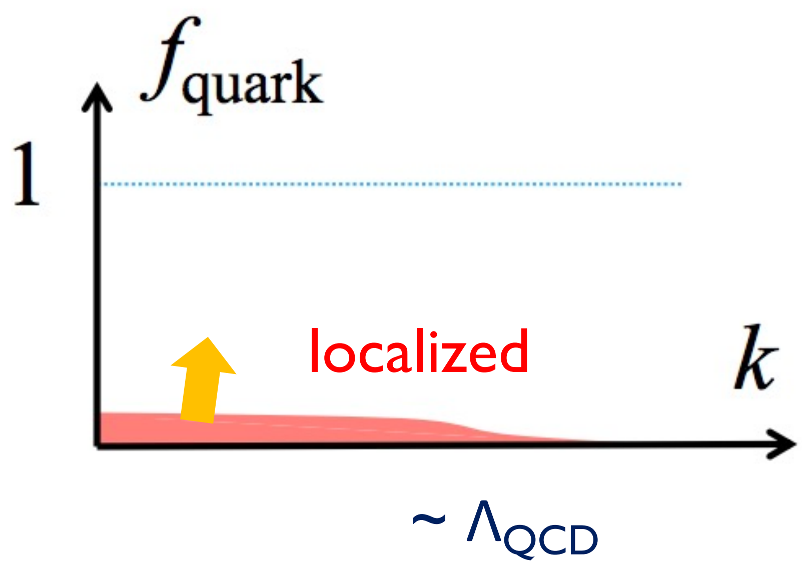
# quark Fermi sea & mode-by-mode percolation

$$\int dE \rho(E) = 1$$






occupation probability

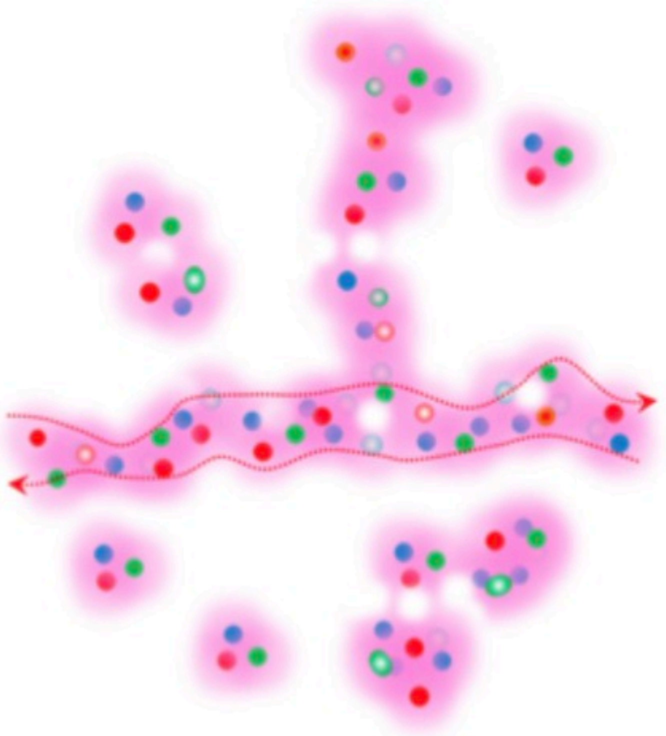


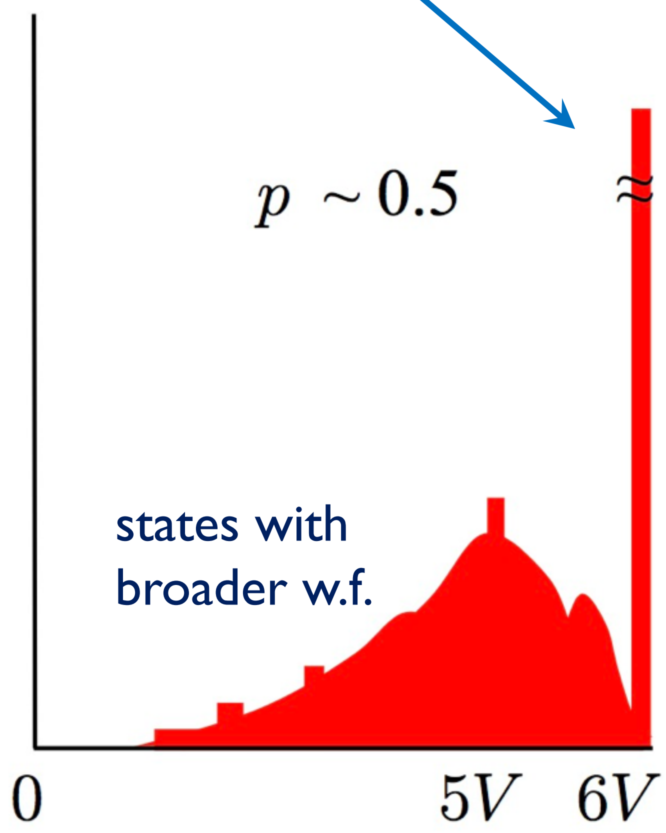

**"nuclear"**

# quark Fermi sea & mode-by-mode percolation

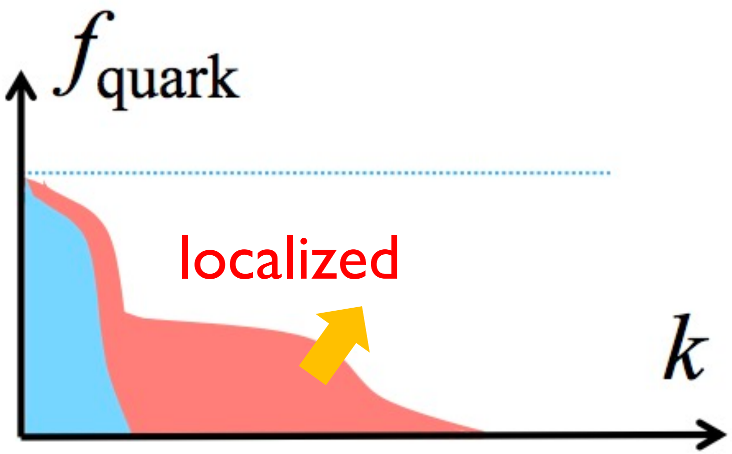
$$\int dE \rho(E) = 1$$

isolated baryons + sub-clusters






occupation probability




extended $\sim \Lambda_{QCD}$

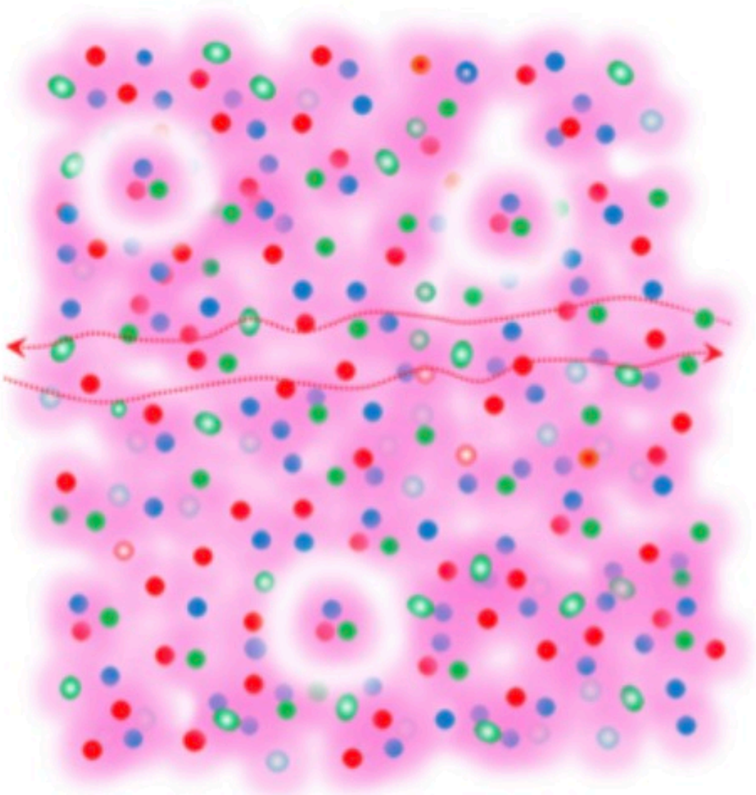
↑

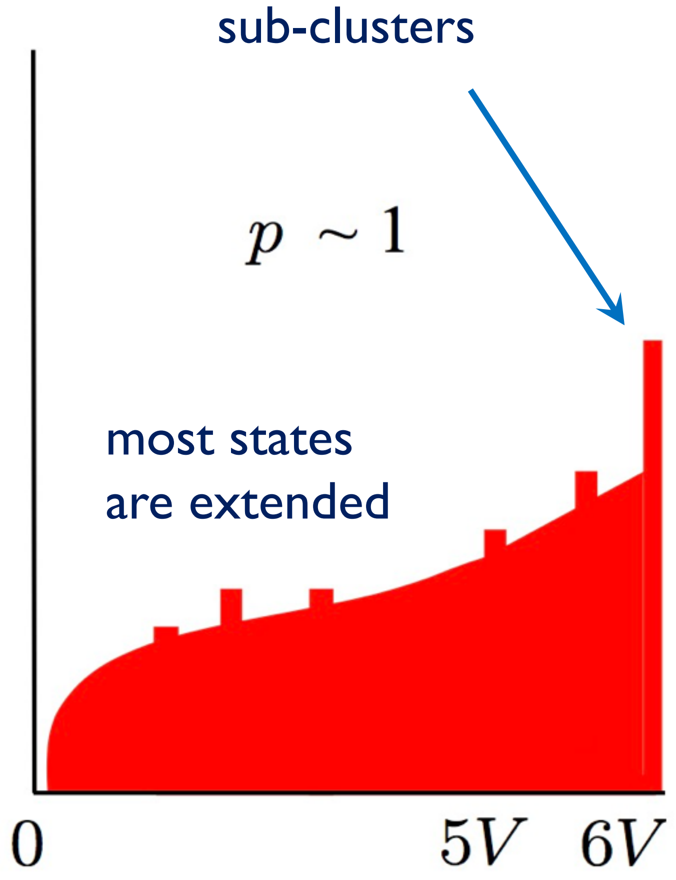
quark bases reasonable

**"quark-hadron continuity"**

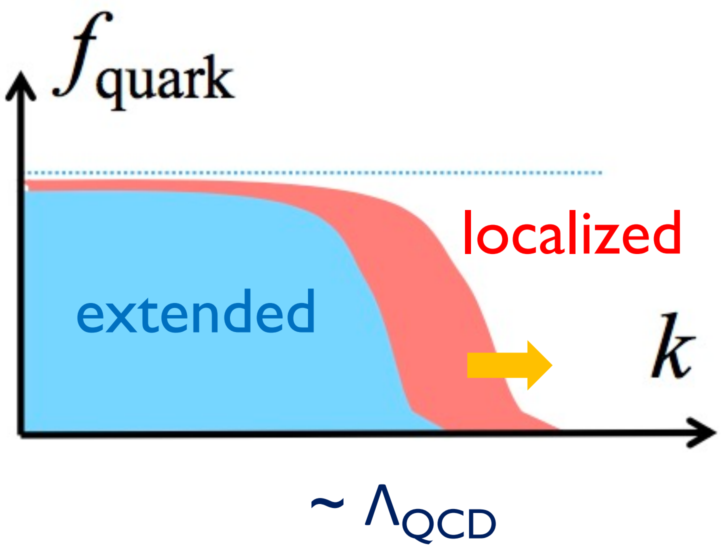
# quark Fermi sea & mode-by-mode percolation

$$\int dE \rho(E) = 1$$






occupation probability




**"quarkyonic"**

# Summary: a cartoon