中国科学院大学
University of Chinese Academy of Sciences
物理科学学院
School of Physical Sciences

Topological modes in relativistic hydrodynamics

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Ya-Wen Sun, UCAS

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KITS Kavli Institute for Theoretical Sciences
University of Chinese Academy of Sciences (UCAS)

Relativistic hydrodynamics

- Hydrodynamics: small perturbations close to thermal equilibrium, long wave length and long time limit;

- Dynamics determined by conservation equations

$$\partial_\mu T^{\mu\nu} = 0$$

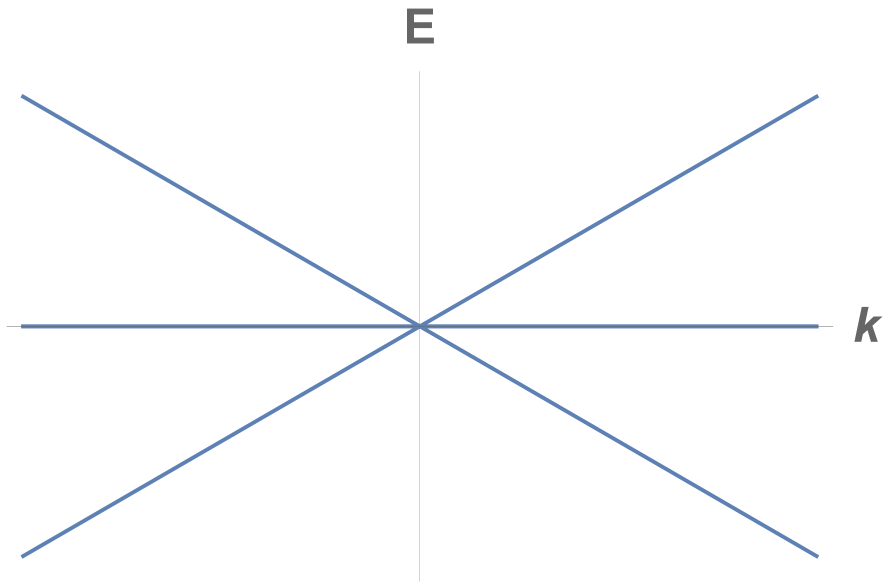
- Constitutive equations:

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p\Delta^{\mu\nu} - \eta\,\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d}\eta_{\alpha\beta}\partial_\mu u^\mu\right) - \zeta\Delta^{\mu\nu}\partial_\lambda u^\lambda$$

Hydrodynamic modes

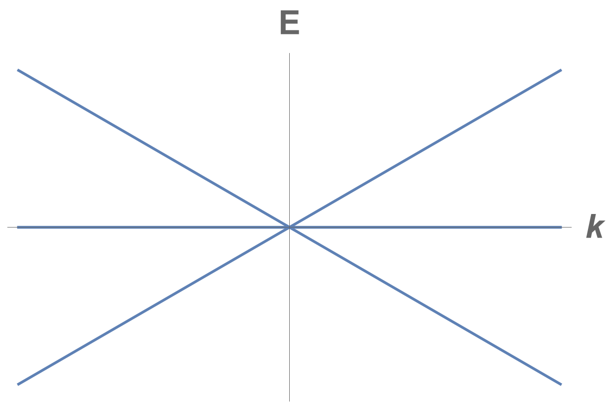
- Poles at w=0 due to conservation of energy momentum
- Propagating modes: $\omega = \pm|\mathbf{k}|v_s - i\gamma_s\mathbf{k}^2/2$
- Diffusive modes: $\omega = -iD\mathbf{k}^2$

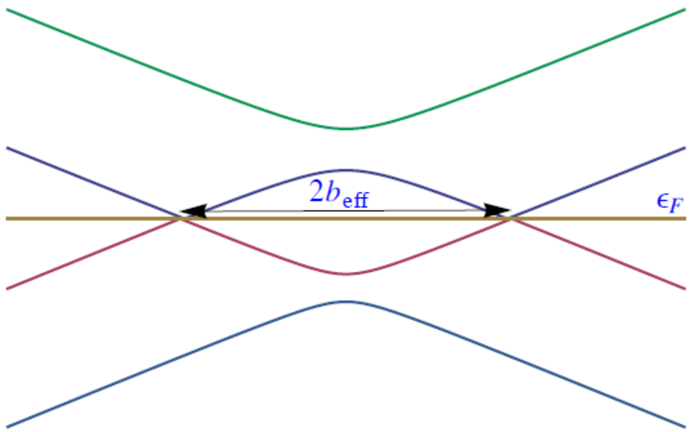
- Crossing point in the real part of the eigenvalues at E=k=0
- To first order in k: →



Looking for topological modes in hydrodynamics: motivation

- Quantum gapless topological states: Weyl semimetal; topological nodes





- Classical topological states: sounds/optics
- Possible experimental observational effects?
- Implications to other topological systems

Outline

- Effective Hamiltonian
- Engineering the Hamiltonian: making it gapped, separating the nodes
- Non-conservations: gravitons; non-inertial reference frames, symmetry;
- Transports, second order effects;
- Topological invariants: orthogonal adjacent states;
- Holography
- More general systems with two sectors of energy momentum;
- Summary and open questions

I: Looking for topological modes in hydrodynamics: effective Hamiltonian

Hydrodynamic modes, dynamics determined by the conservation equation

$$\partial_\mu \delta T^{\mu\nu} = 0$$

$$\partial_t \, \delta\epsilon + ik_x \pi_x = 0 \, ,$$
$$\partial_t \pi^{\parallel} + ik_x v_s^2 \, \delta\epsilon + \gamma_s \mathbf{k}^2 \pi_x = 0$$
$$\partial_t \pi_i^{\perp} + \gamma_\eta \mathbf{k}^2 \pi_i^{\perp} = 0 \, .$$

Define $\Psi = \left(\delta\epsilon, \delta\pi^x, \delta\pi^y, \delta\pi^z\right)^T$

$$i\partial_t \Psi = H\Psi$$

$$H = \begin{pmatrix} 0 & k_x & k_y & k_z \\ k_x v_s^2 & 0 & 0 & 0 \\ k_y v_s^2 & 0 & 0 & 0 \\ k_z v_s^2 & 0 & 0 & 0 \end{pmatrix}$$

leading order in k

H is similar to a Hermitian matrix by $\delta\epsilon \to \frac{1}{v_s}\delta\epsilon$

I: Looking for topological modes in hydrodynamics: effective Hamiltonian

- Resemblance to the equation of motion for fermions: Dirac Hamiltonian

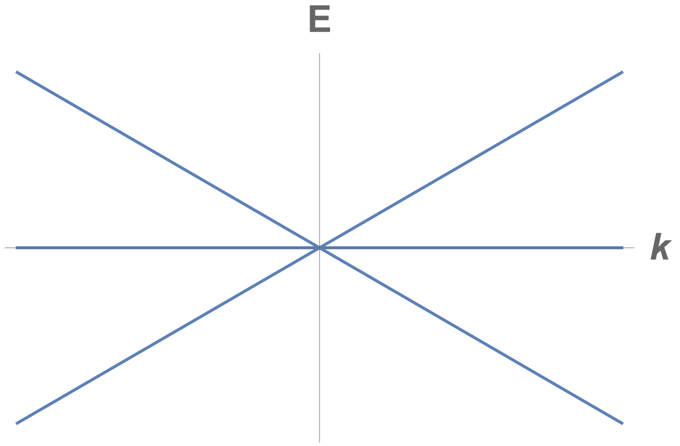
$$i\frac{\partial \psi}{\partial t} = -i\vec{\alpha}\cdot\vec{\nabla}\psi + m\beta\psi \equiv \hat{H}\psi$$

- An effective Hamiltonian in hydrodynamics, whose eigenvalues give the spectrum

$$\omega = \pm v_s\sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$\omega = 0$$

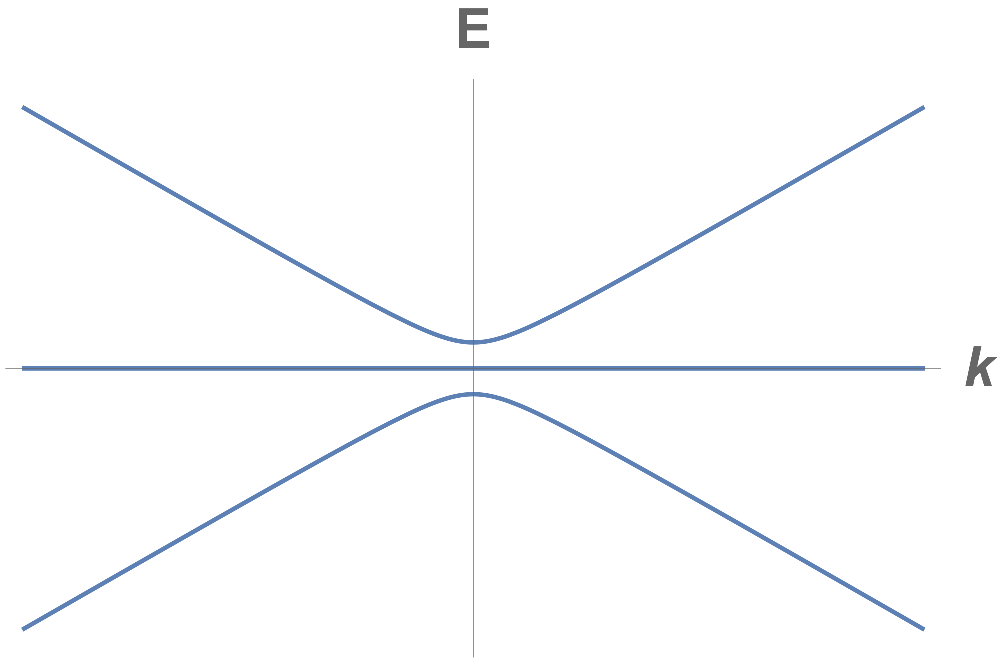
leading order in k



II: Engineering the effective Hamiltonian: gapping the hydrodynamic modes

- Gapless topological modes: stable under perturbations that usually gap the system
- First step: gap the hydrodynamic modes
- **Non-conservation of energy momentum**

$$H = \begin{pmatrix} 0 & k_x + im & k_y & k_z \\ v_s^2(k_x - im) & 0 & 0 & 0 \\ v_s^2 k_y & 0 & 0 & 0 \\ v_s^2 k_z & 0 & 0 & 0 \end{pmatrix}$$



- Order: $O(m) \sim O(k)$
- y, z direction m terms would have the same effect

II: Engineering the effective Hamiltonian: gapping the hydrodynamic modes

- Note: different from the usual momentum dissipation considered in holography
- What we have here: $\partial_\mu \delta T^{\mu t} = m \delta T^{tx}$
- Gap: $\partial_\mu \delta T^{\mu x} = -m v_s^2 \delta T^{tt}$

- What they have there: $\partial_\mu \delta T^{\mu t} = 0\,,$
- Momentum dissipation $\partial_\mu \delta T^{\mu i} = \Gamma \delta T^{ti}\,,$

II: Engineering the effective Hamiltonian: gapping the hydrodynamic modes

- Compare the effective Hamiltonians
- Momentum dissipation vs gap

$$H = \begin{pmatrix} 0 & k_x & k_y & k_z \\ v_s^2 k_x & -i\Gamma & 0 & 0 \\ v_s^2 k_y & 0 & -i\Gamma & 0 \\ v_s^2 k_z & 0 & 0 & -i\Gamma \end{pmatrix} \qquad H = \begin{pmatrix} 0 & k_x + im & k_y & k_z \\ v_s^2(k_x - im) & 0 & 0 & 0 \\ v_s^2 k_y & 0 & 0 & 0 \\ v_s^2 k_z & 0 & 0 & 0 \end{pmatrix}$$

↓

$$\omega = \left(-i\Gamma, \ -i\Gamma, \ \frac{1}{2}(-i\Gamma - \sqrt{4v_s^2k^2 - \Gamma^2}), \ \frac{1}{2}(-i\Gamma + \sqrt{4v_s^2k^2 - \Gamma^2}) \right)$$

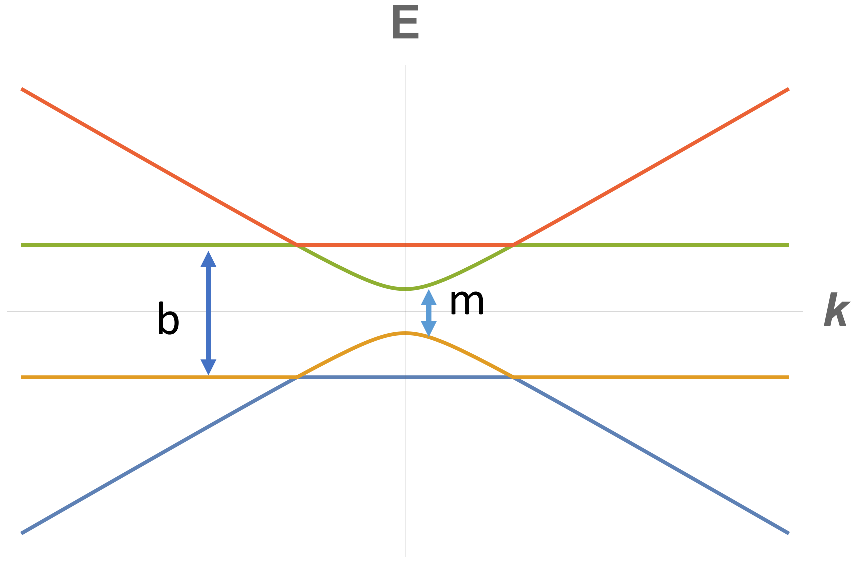
II: Engineering the effective Hamiltonian: separating the hydrodynamic modes

- More non-conservation terms

$$\partial_\mu \delta T^{\mu t} = m \delta T^{tx}\,, \quad \partial_\mu \delta T^{\mu x} = -m v_s^2 \delta T^{tt}$$

$$\partial_\mu \delta T^{\mu y} = b v_s \delta T^{tz}\,, \quad \partial_\mu \delta T^{\mu z} = -b v_s \delta T^{ty}$$

$$H = \begin{pmatrix} 0 & k_x + im & k_y & k_z \\ v_s^2(k_x - im) & 0 & 0 & 0 \\ v_s^2 k_y & 0 & 0 & ib \\ v_s^2 k_z & 0 & -ib & 0 \end{pmatrix}$$

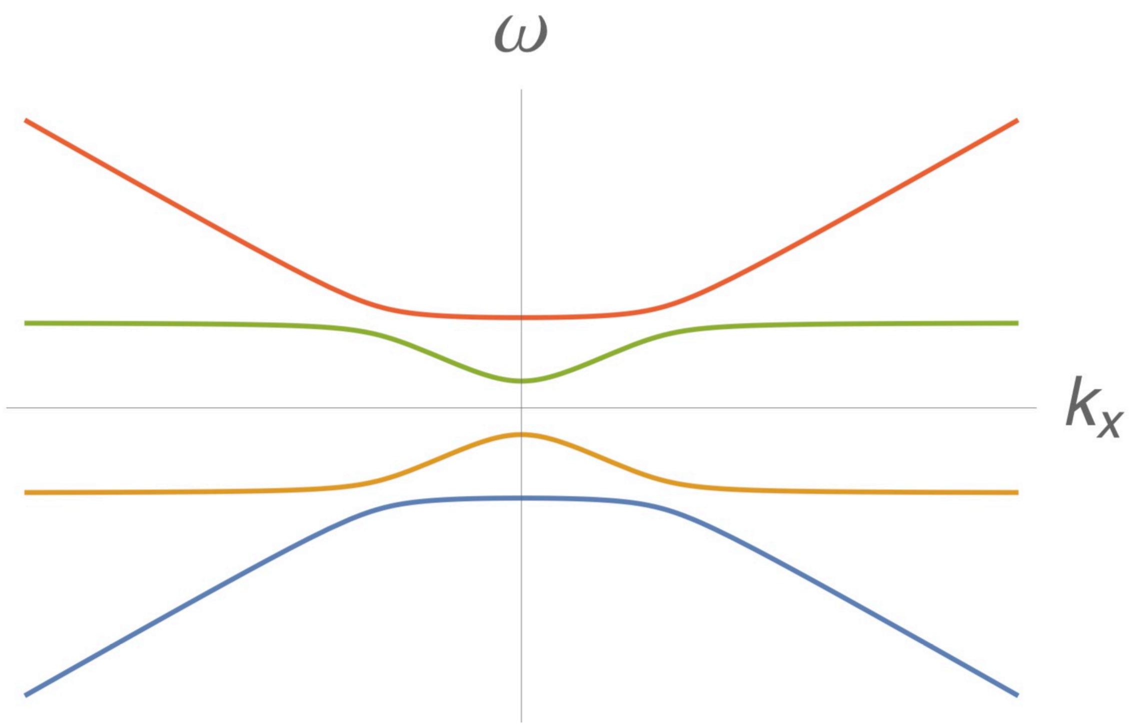


- H is similar to a Hermitian matrix by $\delta\epsilon \to \frac{1}{v_s}\delta\epsilon$

$$\omega = \pm\frac{1}{\sqrt{2}}\sqrt{b^2 + k^2 + m^2 \pm \sqrt{(k_x^2 + m^2 - b^2)^2 + (k_y^2 + k_z^2)^2 + 2(k_y^2 + k_z^2)(k_x^2 + m^2 + b^2)}}$$

II: Engineering the effective Hamiltonian: separating the hydrodynamic modes

- Note that the y or z direction mass terms could still gap the system

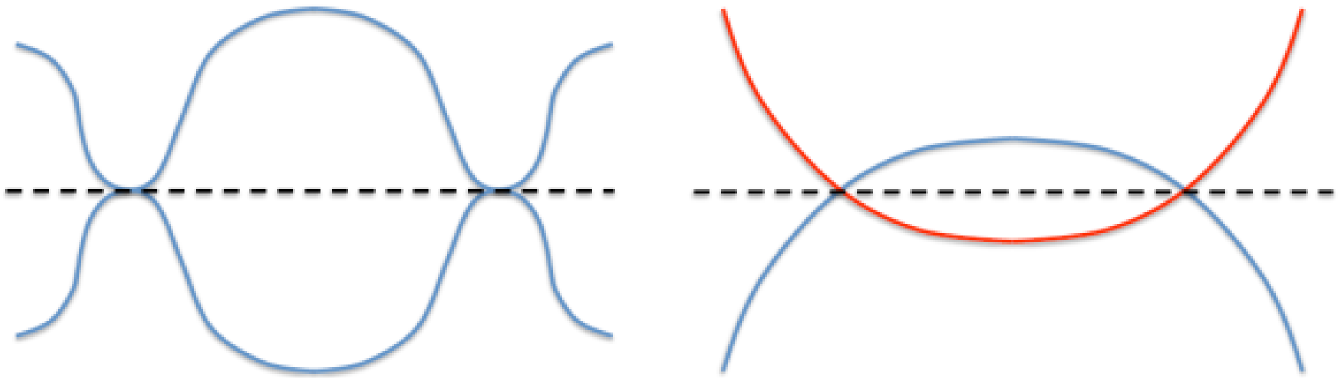


The spectrum with y or z mass terms

Topologically nontrivial protected by certain symmetries: not gapped by the x direction m terms.

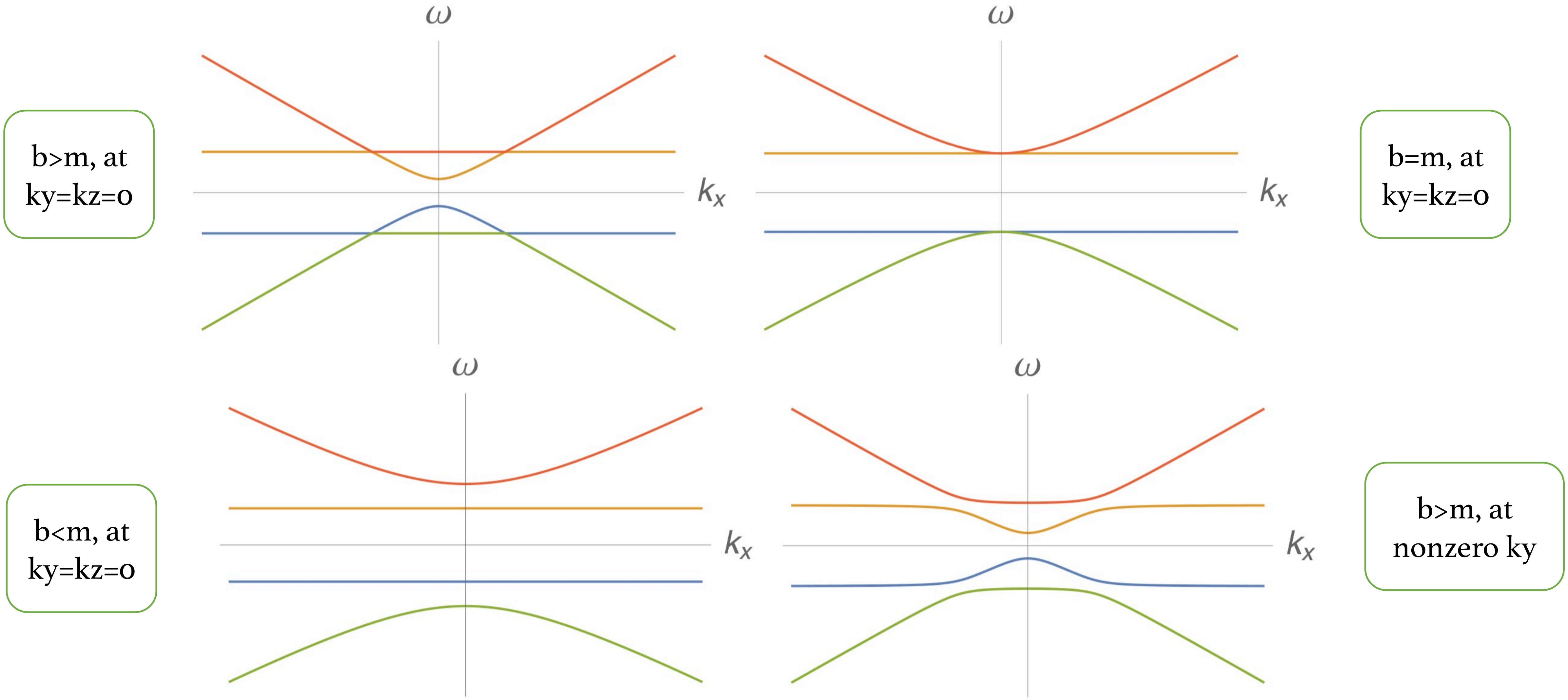
Meaning of topologically gapless states

- Accidental vs topological



- Symmetry protection

- (Topological) phase transition: tuning b from larger than m to smaller than m



III Origin for non-conservation of energy momentum

- From a symmetric tensor external field: $f_{\mu\nu}O^{\mu\nu}$
- Non-conservation equation: $\partial^\mu T_{\mu\nu} = O^{\alpha\beta}(\partial_\nu f_{\alpha\beta} - 2\partial_\alpha f_{\nu\beta})$
- Choose the operator to be $T^{\mu\nu}$; carefully choosing the nonzero components of the external field could give us the non-conservation equation that is needed.
- The symmetric tensor field could be some external effective matter field, but its coupling has to be carefully tuned in real systems.

III Origin for non-conservation of energy momentum

- The most interesting and natural possibility for the symmetric tensor field: the gravitational field

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad \mathcal{O}(h_{\mu\nu}) \sim \mathcal{O}(k)$$

- Energy momentum is conserved covariantly $\nabla_\mu T^{\mu\nu} = 0$
- Expanding the covariant conservation equation to first order of $h_{\mu\nu}$

$$\partial_\mu \delta T^{\mu\nu} = -\frac{1}{2}\partial_\alpha h \delta T^{\alpha\nu} - \frac{1}{2}\eta^{\nu\beta}(2\partial_\mu h_{\alpha\beta} - \partial_\beta h_{\mu\alpha})\delta T^{\mu\alpha}$$

III Origin for non-conservation of energy momentum

- With the following nonzero components of $h_{\mu\nu}$

$$h_{tt} = h_{xx} = mx\,, \quad h_{tx} = h_{xt} = \frac{1}{2}mt(v_s^2+1)\,,$$
$$h_{ty} = h_{yt} = -\frac{1}{2}bv_s z\,, \quad h_{tz} = h_{zt} = \frac{1}{2}bv_s y\,.$$

infinite many possibilities for $h_{\mu\nu}$, here we pick a simple choice

- The covariant conservation equation gives the non-conservation terms needed

$$\partial_\mu \delta T^{\mu t} = m\delta T^{tx}\,, \quad \partial_\mu \delta T^{\mu x} = -mv_s^2 \delta T^{tt}$$
$$\partial_\mu \delta T^{\mu y} = bv_s \delta T^{tz}\,, \quad \partial_\mu \delta T^{\mu z} = -bv_s \delta T^{ty}$$

- How do we get this gravitational field $h_{\mu\nu}$?
- Surprisingly all Riemann tensors vanish for this metric!
- $h_{\mu\nu}$ could emerge from a coordinate transformation from the flat spacetime

$$\tilde{x}_\mu = x_\mu + \xi_\mu$$

$$\xi_\mu = \left(\frac{mxt}{2}, \quad \frac{mx^2}{4} + \frac{mt^2}{4} v_s^2, \quad -\frac{b}{4} v_s zt, \quad \frac{b}{4} v_s yt \right)$$

- In a specific non-inertial frame, we could observe hydrodynamic modes that are topologically protected even when they are topologically trivial in the original inertial frame.
- Another effect for accelerating frames in addition to the Unruh effect.

The non-inertial frame

- A rest observer in the new reference frame

$$d\tilde{x}^i = 0 \text{ for } i = 1,2,3$$

- Solving this equation, we have the movement of the rest observer in the original flat spacetime (at leading order in k)

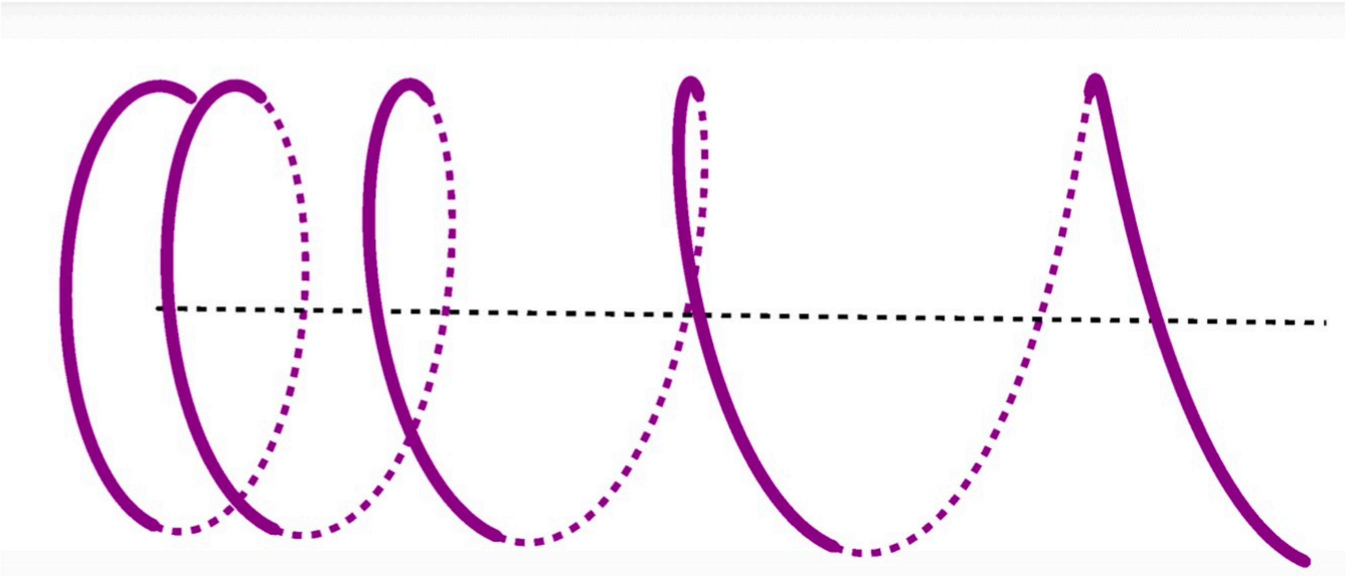
$$d\tilde{t} = dt, \; dx = -\frac{mv_s^2 t dt}{2}, \; dy = \frac{bv_s z dt}{4} \quad dz = -\frac{bv_s y dt}{4}$$

- Integrating these equations with appropriate boundary conditions, we have

$$y = R_0 \cos \tfrac{bv_s}{4} t \text{ and } z = -R_0 \sin \tfrac{bv_s}{4} t$$

The non-inertial frame

- The rest observer in the new reference frame:
- Rotating with a constant angular velocity $\omega_x = \frac{bv_s}{4}$ in the y-z plane
- Accelerating with a constant acceleration $a = -\frac{mv_s^2}{2}$ in the x direction



x direction

- A remark: with the coordinate transformation to the new frame, the constitutive equation has to be transformed, too, which could contribute to extra terms in the equation.

Accelerating observer, with the fluid rest in the inertial frame

- By considering these effects carefully by transformation the four velocity of the fluid to the non-inertial frame, extra terms all vanish and the spectrum does not change!

- It is also possible to have the fluid also accelerating: the spectrum does not change up to a rescaling of parameters m, b and vs.

Symmetry of the system

- To keep the exact form of these non-conservation terms, we need the isometry of the new spacetime metric.

- Isometry: coordinate transformation from the Poincare symmetry of the original flat spacetime

- Killing vectors

$$K_\mu = \sum_{i=0}^{3} a_i \chi_i + \sum_{i=1}^{6} c_i \theta_i$$

$$\chi_0 = \left(1 - \frac{mx}{2},\ \ -\frac{mt}{2},\ 0,\ 0\right)\ , \qquad \chi_1 = \left(\frac{mtv_s^2}{2},\ 1 + \frac{mx}{2},\ 0,\ 0\right)$$

$$\chi_2 = \left(-\frac{bzv_s}{4},\ 0,\ 1,\ \ -\frac{btv_s}{4}\right)\ , \qquad \chi_3 = \left(\frac{byv_s}{4},\ 0,\ \frac{btv_s}{4},\ 1\right)\ ,$$

$$\theta_1 = \left(-\frac{m(x^2 + v_s^2t^2)}{4} + x,\ \ -t - \frac{mtx}{2},\ 0,\ 0\right)\ ,$$

$$\theta_2 = \left((1 - \frac{mx}{2})y,\ \ -\frac{mty}{2},\ \ (-1 + \frac{mx}{2})t,\ \frac{bt^2v_s}{4}\right)\ ,$$

$$\theta_3 = \left((1 - \frac{mx}{2})z,\ \ -\frac{mtz}{2},\ \ -\frac{bt^2v_s}{4},\ \ (-1 + \frac{mx}{2})t\right)\ ,$$

$$\theta_4 = \left(\frac{mtyv_s^2}{2} + \frac{bxzv_s}{4},\ \ -\frac{btzv_s}{4} + y(1 + \frac{mx}{2}),\ \ -\frac{m(x^2 + v_s^2t^2)}{4} - x,\ \frac{btxv_s}{4}\right)$$

$$\theta_5 = \left(\frac{mtzv_s^2}{2} - \frac{bxyv_s}{4},\ \frac{btyv_s}{4} + z(1 + \frac{mx}{2}),\ \ -\frac{btxv_s}{4},\ \ -\frac{m(x^2 + v_s^2t^2)}{4} - x\right)$$

$$\theta_6 = \left(-\frac{bv_s(y^2 + z^2)}{4},\ 0,\ z,\ \ -y\right)\ .$$

The protecting symmetry

- It could be the whole symmetry of the isometry
- However, we only need the symmetry that forbids the m terms in the y and z directions. There could be extra b terms that change the value of b, which do not open the gap.
- The two Killing vectors for this symmetry are

$$\epsilon^{\mu} = a_y \chi_y + a_z \chi_z$$

$$\chi_y = \left(-\frac{bzv_s}{4},\ 0,\ 1,\ -\frac{btv_s}{4}\right)$$

$$\chi_z = \left(\frac{byv_s}{4},\ 0,\ \frac{byv_s}{4},\ 1\right)$$

This symmetry looks complicated, however, it is just the y and z translational symmetry in the inertial frame.

- Combined translational and boost symmetry in the y(z) and z(y) directions

Summary of the physical picture

- The non-conservation terms look ad-hoc; the protecting symmetry looks weird.
- Let us analyze the physical picture carefully.
- Hydrodynamic modes observed in the accelerating frame where the normal fluid is at rest in the inertial frame, i.e. the receiver accelerates while the fluid is at rest.
- In the accelerating frame, the non-conservation terms come out naturally due to the gravitational field, and the protecting symmetry comes out naturally as two generators of the isometry.
- We will have the covariant conservation of $T^{\mu\nu}$ in the accelerating frame as long as we have the conservation of $T^{\mu\nu}$ in the flat spacetime: the symmetry required is the y and z momentum conservation in the flat spacetime.
- All that is needed is the accelerating receiver who has to accelerate exactly in the way required and it is a natural accelerating frame of a helix.

Possible experimental realization

- Possible application: doubling of amplitudes at a finite k and w; stable under perturbations;

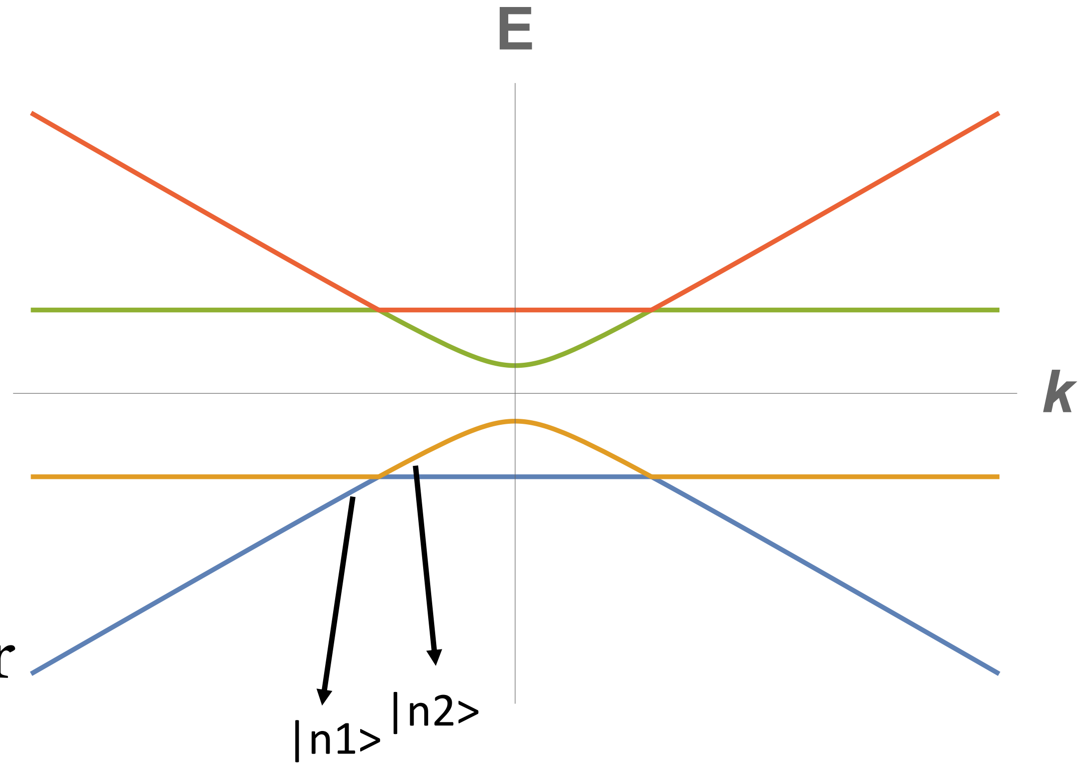
- Laboratory tests: accelerating the detector/observer for sound modes in a helix with small acceleration and angular velocity.

- Direct detection of sound modes; indirect test of transport behavior;

- Implications to other topological materials, e.g. electronic systems.

IV topological invariants

- For symmetry protected topological states
- Topological invariants calculated at high symmetric points of the system ky=kz=0;
- |n1> and |n2> normal to each other: $\langle n1|n2\rangle=0$ undetermined Berry phase The singularity cannot become a trivial point by continuous change, unless after a topological phase transition



V. Transports, second order effects

- Transports: thermal conductivity
- m,b both have effects in thermal conductivity;

$$\kappa_{xx}(\omega, k_x) = -\frac{i\omega(\epsilon + P)}{T\left((k_x^2 + m^2)v_s^2 + i\frac{\eta}{\epsilon+P}\omega k_x^2 - \omega^2\right)},$$

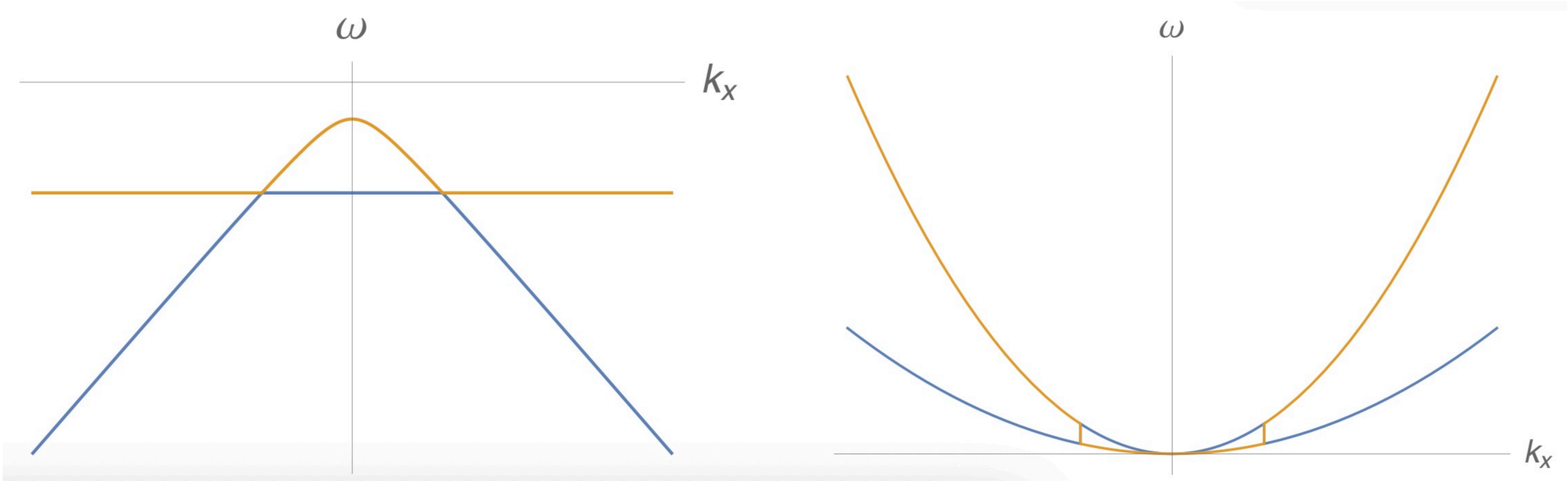
$$\kappa_{yy}(\omega, k_x) = \kappa_{zz}(\omega, k_x) = -\frac{k_x^2\eta + i\omega(\epsilon + P)}{T\left(b^2v_s^2 + (i\omega + \frac{\eta}{\epsilon+P}k_x^2)^2\right)}$$

$$\kappa_{yz}(\omega, k_x) = -\kappa_{zy}(\omega, k_x) = \frac{(\epsilon + P)bv_s}{T\left(b^2v_s^2 + (i\omega + \frac{\eta}{\epsilon+P}k_x^2)^2\right)}$$

- yz components become nonzero due to b terms; xx, yy, zz components do not diverge anymore due to b or m.

V. Imaginary parts: second order in k effects

- A jump in the imaginary part: indicating topological change?



VI. Holographic realization and Ward identities

- Strongly coupled hydrodynamic systems.
- Hydrodynamic modes-> gravitons
- Non-conservation of energy momentum: massive gravity?
- Another prescription for holographic realization of this system: holographic non-inertial reference frames, coordinate transformation from the original AdS/CFT correspondence
- First step to prove that it is indeed the holographic system needed: reproduce the Ward identities due to the energy momentum non-conservation terms

VI. Holographic realization and Ward identities

- Ward identities for the conserved energy momentum tensor

$$k_\mu(G^{\mu\nu\lambda\rho} - \eta^{\nu\lambda}\langle T^{\mu\rho}\rangle - \eta^{\nu\rho}\langle T^{\mu\lambda}\rangle - \eta^{\lambda\rho}\langle T^{\mu\nu}\rangle + \eta^{\mu\nu}\langle T^{\lambda\rho}\rangle) = 0$$

- With energy momentum non-conservation terms, the Ward identities become

$$k_\mu G^{\mu\nu,\lambda\rho}(k) + i\left[\Gamma^{(1)\mu}{}_{\mu\alpha}G^{\alpha\nu,\lambda\rho}(k) + \Gamma^{(1)\nu}{}_{\mu\alpha}G^{\mu\alpha,\lambda\rho}(k)\right] + \text{contact terms} = 0$$

VI. Holographic realization and Ward identities

- A new prescription to calculate holographic Ward identities without calculating all the components of the Green functions
- For perturbations of the metric $\delta g_{\mu\nu}(\vec{k})$, we denote the ten components $tt, tx, ty, tz, xx, xy, xz, yy, yz, zz$ as $\phi_i,\ i \in \{1..., 10\}$

  Fourier transformed to the momentum space in the t, x, y, z directions
- The action could be written as

$$S \supset \int \frac{dr d^4k}{(2\pi)^4} \left(W_1^{ij} \phi_i''(-\vec{k}) \phi_j(\vec{k}) + W_2^{ij} \phi_i'(-\vec{k}) \phi_j'(\vec{k}) + W_3^{ij} \phi_i'(-\vec{k}) \phi_j(\vec{k}) + W_4^{ij} \phi_i(-\vec{k}) \phi_j(\vec{k}) \right)$$

VI. Holographic realization and Ward identities

- Deriving equations of motion for this system and substituting the solutions into the action, we could the on-shell action.
- The on-shell action that is relevant to the Green functions:

$$S_{\text{on-shell}} \supset \int \frac{d^4k}{(2\pi)^4} W_2^{ij} \phi_i'(-\vec{k})\phi_j(\vec{k})\Big|_{r_h}^{r_b} + \cdots$$

- ⋯ are terms related to the contact terms

- Note that components with r could be viewed as constraint equations, which could be solved and substituted into the on-shell action.

VI. Holographic realization and Ward identities

- Holographic Ward identities--- diffeomorphism
- The action has to be composed of gauge invariant combinations
- All possible gauge invariant combinations:

$$\delta g_{\mu\nu} = \nabla_\mu \epsilon_\nu + \nabla_\nu \epsilon_\mu$$

$$Z_1 = \frac{\delta g_{xx}}{2k_x^2} + \frac{\delta g_{tx}}{\omega k_x} + \frac{\delta g_{tt}}{2\omega^2}, \quad Z_2 = \frac{\delta g_{yy}}{2k_y^2} + \frac{\delta g_{ty}}{\omega k_y} + \frac{\delta g_{tt}}{2\omega^2},$$

$$Z_3 = \frac{\delta g_{zz}}{2k_z^2} + \frac{\delta g_{tz}}{\omega k_z} + \frac{\delta g_{tt}}{2\omega^2}, \quad Z_4 = \frac{\delta g_{xx}}{2k_x^2} - \frac{\delta g_{xy}}{k_x k_y} + \frac{\delta g_{yy}}{2k_y^2},$$

$$Z_5 = \frac{\delta g_{xx}}{2k_x^2} - \frac{\delta g_{xz}}{k_x k_z} + \frac{\delta g_{zz}}{2k_z^2}, \quad Z_6 = \frac{\delta g_{yy}}{2k_y^2} - \frac{\delta g_{yz}}{k_y k_x} + \frac{\delta g_{zz}}{2k_z^2}$$

VI. Holographic realization and Ward identities

- The on-shell action should be

$$S \supset \int \frac{d^4k}{(2\pi)^4} G_{ij}(r) Z_i'(-\vec{k}) Z_j(\vec{k}) \Big|_{r_h}^{r_b}$$

- All 55 components of Green functions should be expressed using the 21 independent Gij functions.
- Eliminating all Gij's, we obtain 34 identities for holographic Green functions.
- 40 Ward identities need to be reproduced, 6 of which are independent that could be derived from other 34 identities
- They match to each other.

The holographic non-inertial frame

- The metric for the coordinate transformed AdS spacetime:

$$g_{\mu\nu}^{\text{bulk}} = g_{\mu\nu}^{AdS} + h_{\mu\nu}^{\text{bulk}}$$

$$h_{\mu\nu}^{\text{bulk}} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

- With the new metric, the form of the on-shell action would be different from the AdS one, nevertheless, it can still be written as sums of gauge invariant terms.

VI. Holographic realization and Ward identities

- New gauge invariant combinations

$$Z_1 = \frac{\delta g_{xx}}{2k_x^2} + \frac{\delta g_{tx}}{\omega k_x} + \frac{\delta g_{tt}}{2\omega^2} - \frac{im\delta g_{tt}}{4k_x\omega^2} + \frac{im\delta g_{tx}}{2k_x^2\omega} - \frac{imv_s^2\delta g_{xx}}{4k_x\omega^2}$$

$$Z_2 = \frac{\delta g_{yy}}{2k_y^2} + \frac{\delta g_{ty}}{\omega k_y} + \frac{\delta g_{tt}}{2\omega^2} - \frac{imv_s^2\delta g_{xx}}{4k_x\omega^2} - \frac{ibv_s\delta g_{zz}}{4k_yk_z\omega},$$

$$Z_3 = \frac{\delta g_{zz}}{2k_z^2} + \frac{\delta g_{tz}}{\omega k_z} + \frac{\delta g_{tt}}{2\omega^2} - \frac{imv_s^2\delta g_{xx}}{4k_x\omega^2} + \frac{ibv_s\delta g_{yy}}{4k_yk_z\omega},$$

$$Z_4 = \frac{\delta g_{xx}}{2k_x^2} - \frac{\delta g_{xy}}{k_xk_y} + \frac{\delta g_{yy}}{2k_y^2} - \frac{im\delta g_{xx}}{4k_x^3},$$

$$Z_5 = \frac{\delta g_{xx}}{2k_x^2} - \frac{\delta g_{xz}}{k_xk_z} + \frac{\delta g_{zz}}{2k_z^2},$$

$$Z_6 = \frac{\delta g_{yy}}{2k_y^2} - \frac{\delta g_{yz}}{k_yk_z} + \frac{\delta g_{zz}}{2k_z^2} - \frac{im\delta g_{xx}}{4k_x^3}.$$

Using the same method as the asymptotic AdS case, we could match the Ward identities from both sides

VI. Holographic realization and Ward identities

- This method for calculating holographic Ward identities could also be generalized to massive gravities.

- More to do:
- More details: hydrodynamics modes, Green functions;
- Other holographic realizations, massive gravity? External fields?

VII. Generalized systems with two sectors

- With two separately conserved hydrodynamic systems.
- Introducing weak interchange of energy and momentum between the two systems
- Start from the simplest case: two 1+1d systems each with an energy momentum tensor

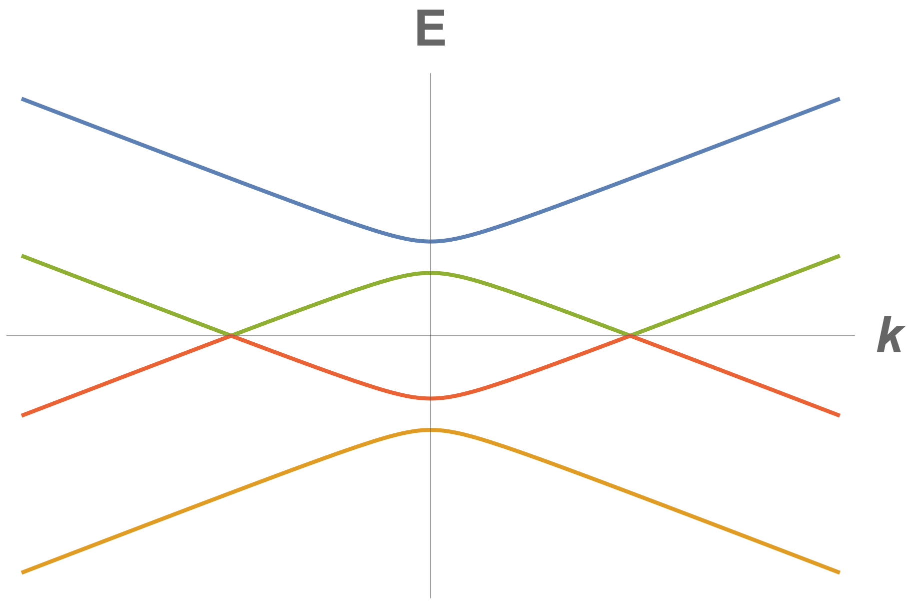
$$
\begin{aligned}
\partial_\mu \delta T_L^{\mu t} &= m_1 \delta T_L^{tx} + b_1 \delta T_R^{tt}\,, \\
\partial_\mu \delta T_L^{\mu x} &= -m_1 v_{sL}^2 \delta T_L^{tt} + b_1 \delta T_R^{tx}\,, \\
\partial_\mu \delta T_R^{\mu t} &= m_2 \delta T_R^{tx} - b_2 \delta T_L^{tt}\,, \\
\partial_\mu \delta T_R^{\mu x} &= -m_2 v_{sR}^2 \delta T_R^{tt} - b_2 \delta T_L^{tx}\,,
\end{aligned}
$$

VII. Generalized systems with two sectors

- A better version with two interacting energy momentum tensors in 2d+2d

$$H = \begin{pmatrix} 0 & k_x + im & ib & 0 \\ (k_x - im)v_s^2 & 0 & 0 & ib \\ -ib & 0 & 0 & k_x + im \\ 0 & -ib & (k_x - im)v_s^2 & 0 \end{pmatrix}$$

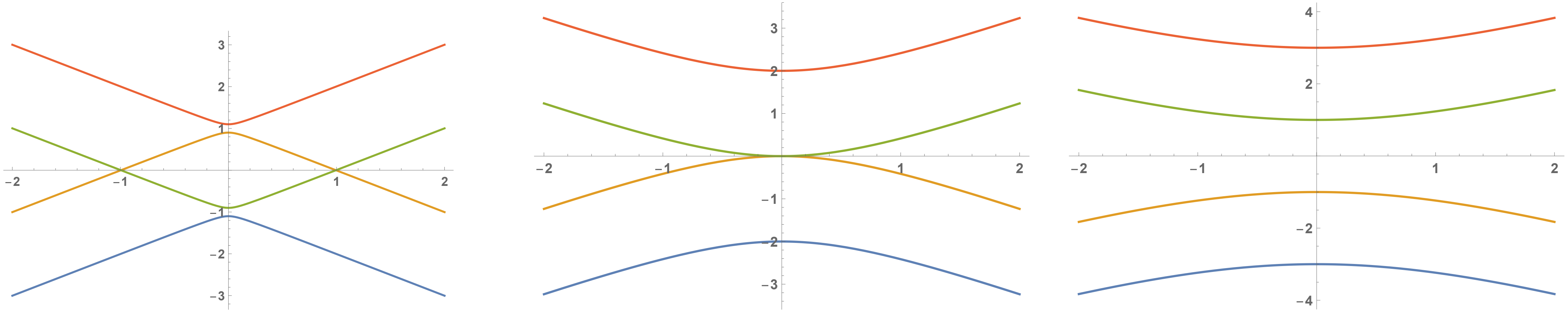
$$\omega = \pm b \pm \sqrt{m^2 + k_x^2} v_s$$



- It does not need to be protected by any symmetry

VII. Generalized systems with two sectors

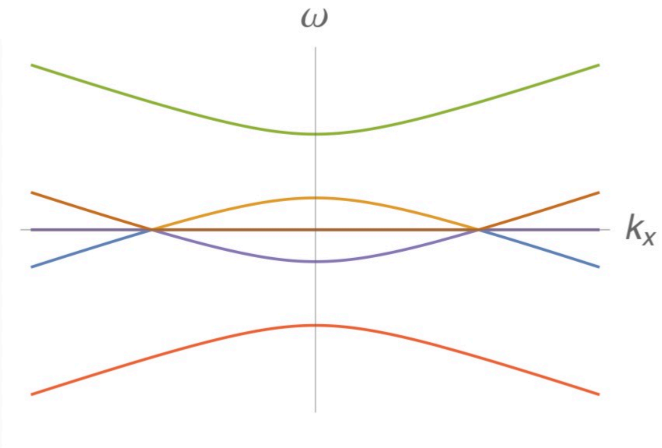
- (Topological) phase transition:

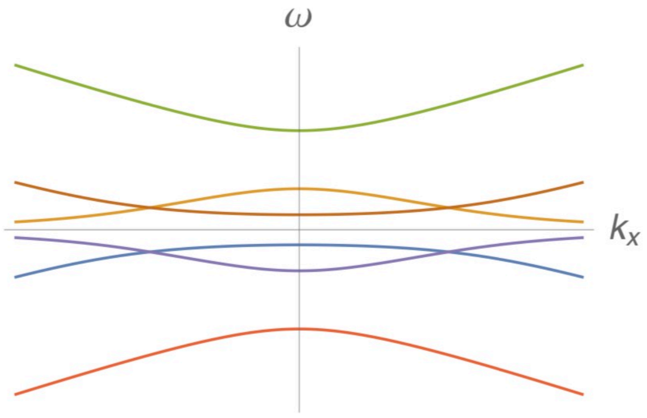


3d+3d: case I

Effective Hamiltonian $$H_{3D+3D,I} = \begin{pmatrix} 0 & k_x+im & k_y & ib & 0 & 0 \\ k_x-im & 0 & 0 & 0 & ib & 0 \\ k_y & 0 & 0 & 0 & 0 & 0 \\ -ib & 0 & 0 & 0 & k_x+im & k_y \\ 0 & -ib & 0 & k_x-im & 0 & 0 \\ 0 & 0 & 0 & k_y & 0 & 0 \end{pmatrix}$$

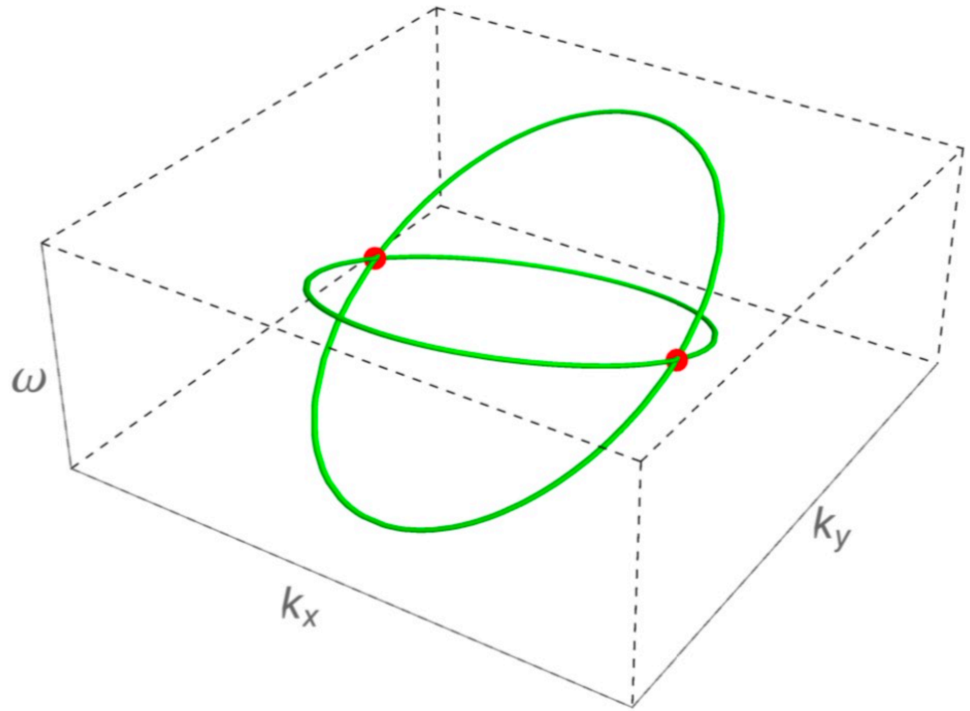
Spectrum



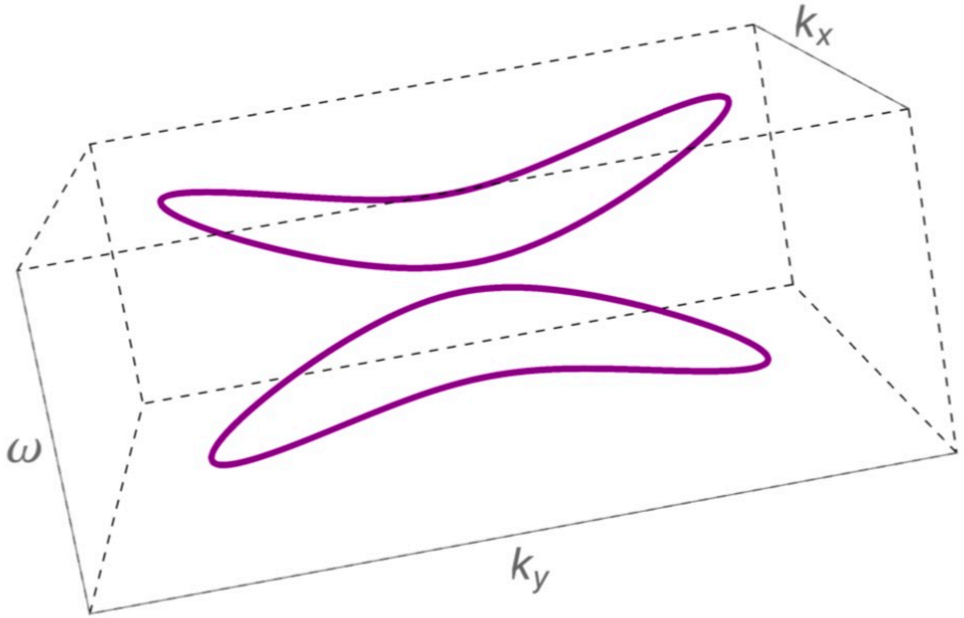


VII. Generalized systems with two sectors

- Crossing nodes



No m in the y direction



With m in the y direction

Two red points: symmetry protected; two circles, no need for symmetry protection

3d+3d case II

- Effective H

$$H_{3D+3D,II} = \begin{pmatrix} 0 & k_x+im & k_y+im & ib & 0 & 0 \\ k_x-im & 0 & 0 & 0 & ib & 0 \\ k_y-im & 0 & 0 & 0 & 0 & ib \\ -ib & 0 & 0 & 0 & k_x+im & k_y+im \\ 0 & -ib & 0 & k_x-im & 0 & 0 \\ 0 & 0 & -ib & k_y-im & 0 & 0 \end{pmatrix}$$

- Spectrum

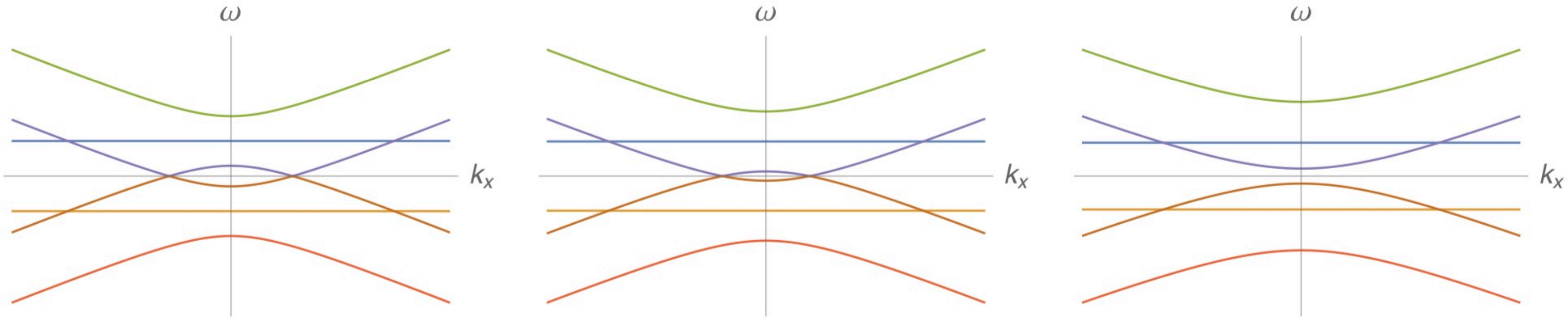


Figure 10: The spectrum of the modified hydrodynamics with (3.17) $H_{3D+3D,II}$ at $m < b$ and increasing from $k_y = 0$ to larger values from left to right.

Crossing nodes are three circles, no need for symmetry protection

4d+4d cases

- More complicated, qualitatively similar;

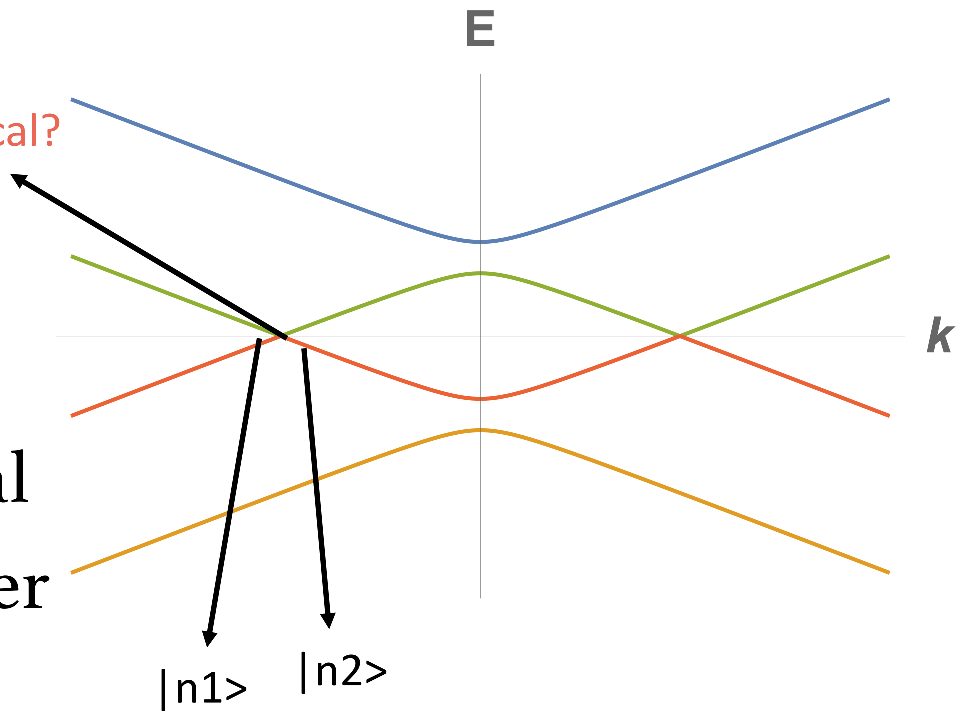
- With maximal b terms, no need for symmetry protection, while the crossing nodes are two dimensional spheres: co-dimension one surfaces;
- With fewer b terms, symmetry protected by the symmetry forbidding the m term in the direction with no b term: effectively co-dimension one in the calculation of topological invariants

Topological invariants

- For the 2d+2d case
- Only parameter: kx
- $|n_1\rangle$ and $|n_2\rangle$ normal to each other:

$\langle n_1|n_2\rangle=0$ undetermined Berry phase

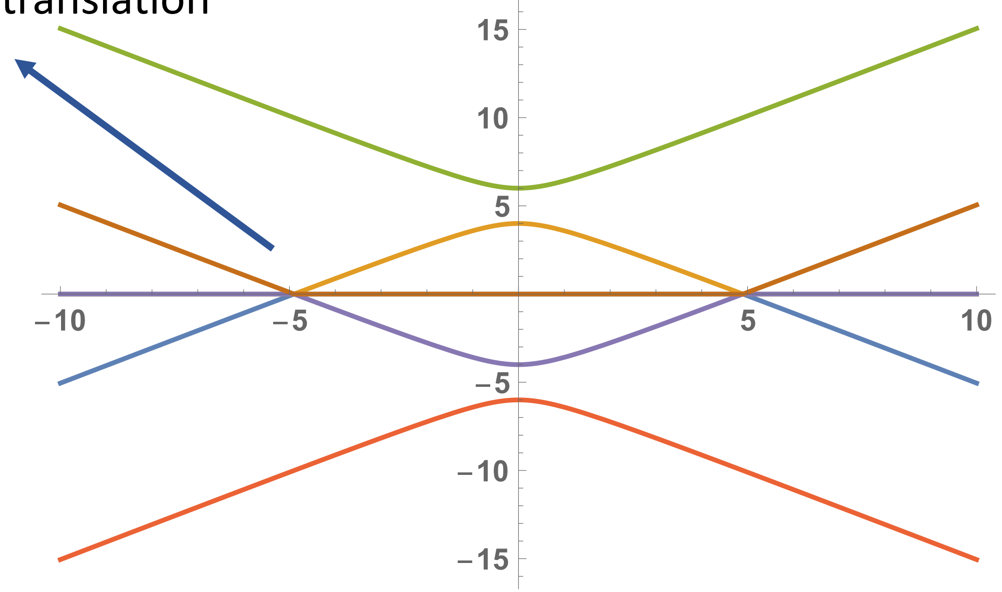
The singularity cannot become a trivial point by continuous change, unless after a topological phase transition



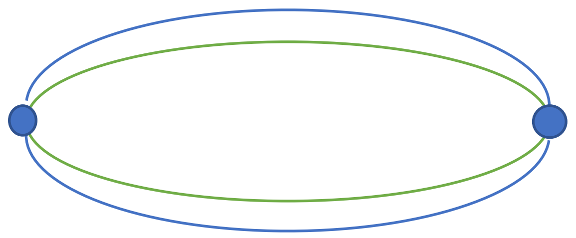
Topological invariants

- For the 3d+3d/4d+4d case: much more complicated
- symmetry protected by yz translation symmetries: calculated at high symmetric points: ky=kz=0
- The same as previous cases

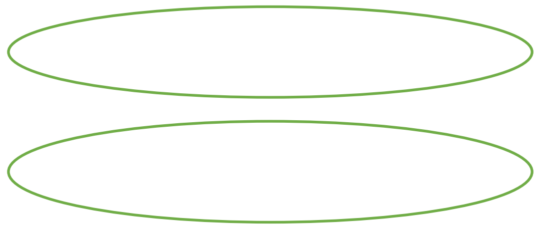
Protected by y translation



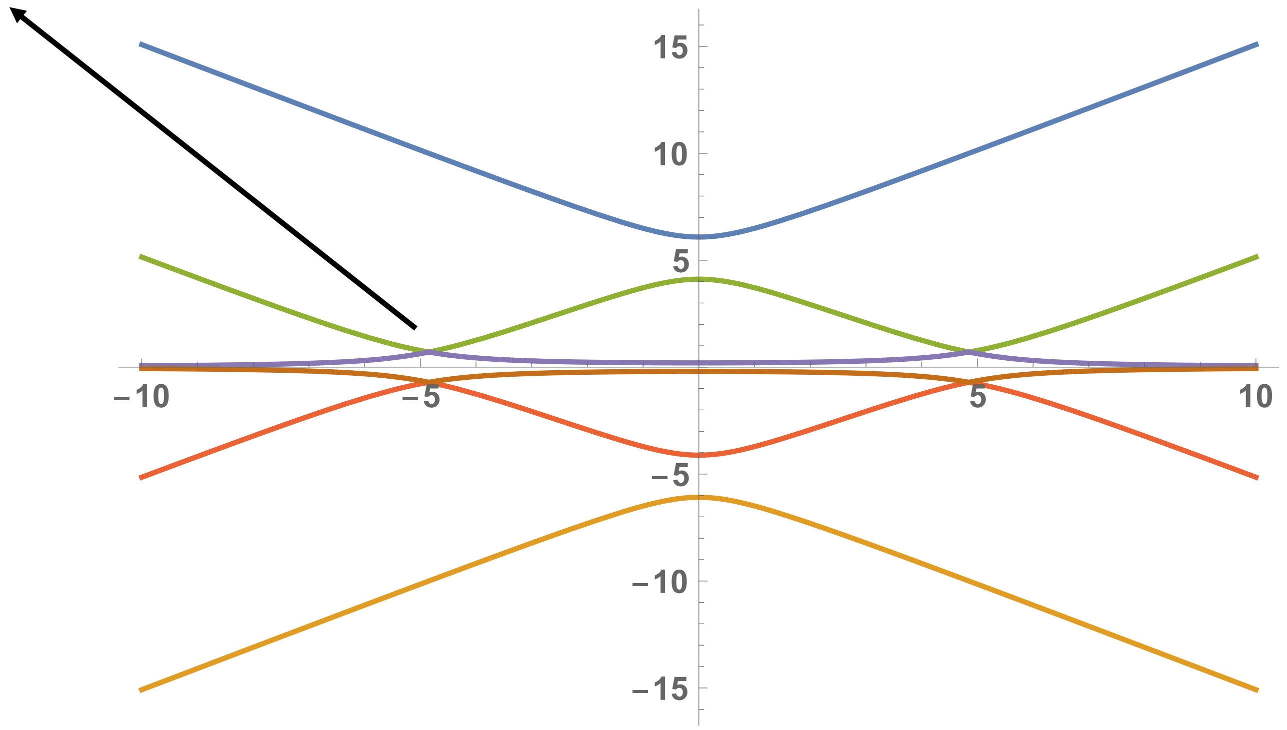
- Two cases:
- With y translation



- Without y translation



Not symmetry protected ones



Transport properties

- 2d+2d case: four components of thermal conductivity; m=0 finite;

$$\kappa_{LL}(\omega,k_x)=\kappa_{RR}(\omega,k_x)=-\frac{i\omega(\epsilon+P)\left(b^2+(k_x^2+m^2)v_s^2-\omega^2\right)}{T\left(b^4+\left((k_x^2+m^2)v_s^2-\omega^2\right)^2-2b^2\left((k_x^2+m^2)v_s^2+\omega^2\right)\right)}$$

$$\kappa_{LR}(\omega,k_x)=-\kappa_{RL}(\omega,k_x)=\frac{(\epsilon+P)\left(b^2-(k_x^2+m^2)v_s^2-\omega^2\right)}{T\left(b^4+\left((k_x^2+m^2)v_s^2-\omega^2\right)^2-2b^2\left((k_x^2+m^2)v_s^2+\omega^2\right)\right)}$$

- No second order effects

Summary

- Gapless topological modes in relativistic hydrodynamics
- Several possible realizations: 4d, 2d+2d, 3d+3d/4d+4d…
- Symmetry protected topological modes; phase transitions; topological invariants;
- Transport; second order effects; Holography
- The take-home message: *normal modes become (symmetry protected) topologically nontrivial gapless modes in a certain non-inertial reference frame: the frame of the accelerating observer moving in a helix; could be tested in laboratories;*
- A new effect for accelerating observes, in addition to the Unruh effect;

Open questions

- Next steps:
- Extra U(1) current;
- Holographic calculation of hydrodynamic modes
- Non-Hermitian, PT symmetry related?
- Fermionic topological systems, non-inertial frame? Preliminary results

Open questions:

- Gapped topological modes?
- Gapless modes with other kinds of topology?
- With U(1)*U(1) symmetry, more transports
- Possible experimental realizations?
- Non-relativistic systems?
- Holographic realizations from massive gravity?
- Two sector systems: possible non-inertial frames?
- Holographic realization for two sector systems?
- Relation with nontrivial topological modes in gravitational waves?

Thank you!