Chiral magnetic effect in the hadronic phase

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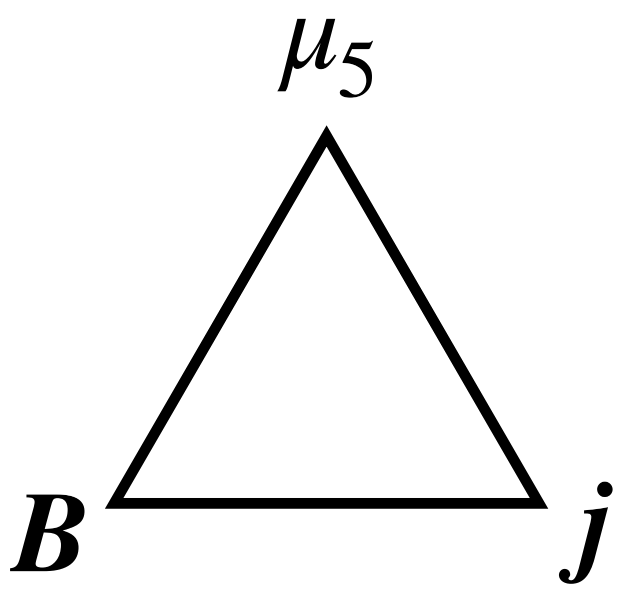
Summary:

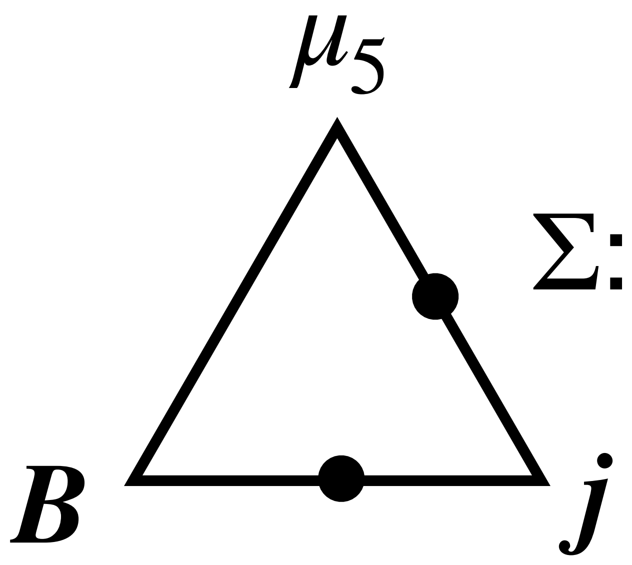
- Chiral magnetic effect in the hadronic phase involves pseudoscalar mesons.
- Form of the CME current in the hadronic phase is model-independent and higher-loop immune.
- This involvement of pseudoscalar mesons may decrease the CME strength.

Chiral phase



$$\boldsymbol{j} = \frac{e^2 N_c}{2\pi^2}\mu_5 \boldsymbol{B}\,\mathrm{tr}\left(Q^2\right)$$

Hadronic phase



Σ: mesons

$$\boldsymbol{j} = \frac{e^2 N_c}{2\pi^2}\mu_5 \boldsymbol{B}\,\mathrm{tr}\left(Q^2 + \frac{1}{6}[Q,\Sigma][Q,\Sigma^\dagger]\right)$$

Contents:

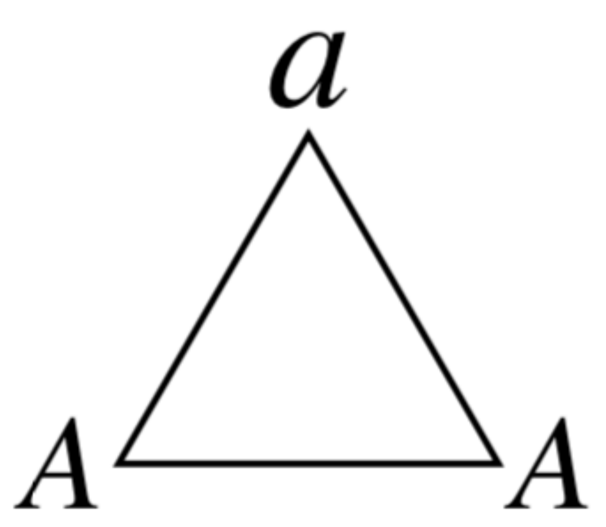
Chiral phase

Derivation:

- One derivation is through the effective action.
- Derivative expansion of the effective action and corresponding triangle diagram (Fig.) are:

$$S_{\rm eff} = -\,i \log \mathrm{Det}(i\not{D} - m) \qquad \left[iD_\mu = i\partial_\mu - eQA_\mu - \gamma_5 a_\mu\,, \quad a_\mu = (\mu_5, \mathbf{0})\right]$$

$$= \frac{e^2 N_c}{4\pi^2} \int d^4x \;\; a_\mu A_\nu \tilde{F}^{\mu\nu} \mathrm{tr}(Q^2) + \cdots$$



(Fig.) Corresponding diagram

$$\boldsymbol{j} = \frac{\delta S_{\rm eff}}{\delta \boldsymbol{A}} = \frac{e^2 N_c}{2\pi^2} \mu_5 \boldsymbol{B} \mathrm{tr}(Q^2)$$ **CME current in the chiral phase**

- *Note*: In order to tame the renormalization scheme dependence, one may impose a physical requirement that the effective action generates the canonical anomalous divergence,

$$\partial_\mu j^\mu = \frac{e^2 N_c}{16\pi^2} \left(F^R_{\mu\nu} \tilde{F}^{R\mu\nu} - F^{L\mu\nu} \tilde{F}^L_{\mu\nu}\right).$$

Contents:

Hadronic phase

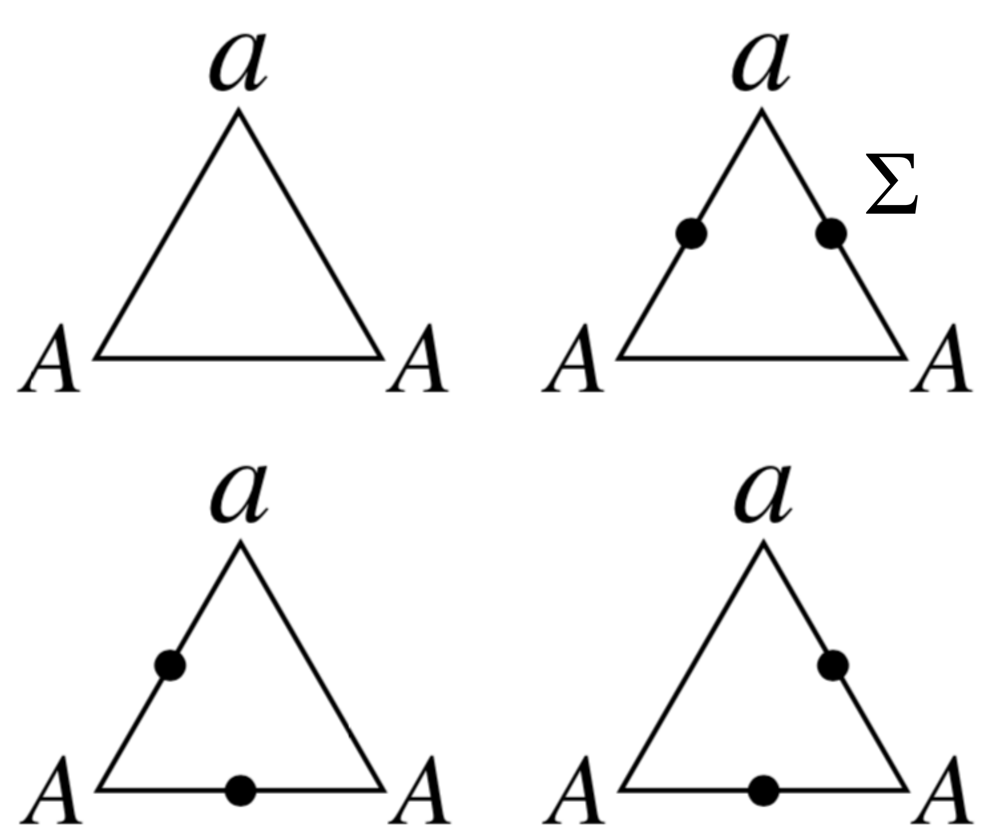
Derivation 1 (via a chiral effective model):

- We adopt a chiral effective model:

$$\mathscr{L} = \bar{q}(i\not{D} - gM)q\,. \qquad \left[M = P_R\Sigma + P_L\Sigma^\dagger\,, \quad \Sigma = \exp(i\pi^A\lambda^A/f_\pi)\right]$$

- Derivative expansion of the effective action and the corresponding triangle diagrams (Fig.) are:

$$\begin{aligned} S_{\rm eff} &= -\,i\log \mathrm{Det}(i\not{D} - gM) \\ &= -\,i\mathrm{Tr}\left(\gamma^5 \not{a}\frac{i\not{\partial} + gM^\dagger}{-\partial^2 - g^2} eQ\not{A}\frac{i\not{\partial} + gM^\dagger}{-\partial^2 - g^2} eQ\not{A}\frac{i\not{\partial} + gM^\dagger}{-\partial^2 - g^2}\right) + \cdots \\ &= \frac{e^2N_c}{4\pi^2}\int d^4x \;\; a_\mu A_\nu \tilde{F}^{\mu\nu}\;\mathrm{tr}\left(Q^2 + \frac{1}{6}[Q,\Sigma][Q,\Sigma^\dagger]\right) + \cdots\,. \end{aligned}$$



(Fig.) Corresponding diagrams

- The effective action involves pseudoscalar mesons as the diagrams illustrate.
- *Note*: In order to tame the renormalization scheme dependence, one may impose a physical requirement that the effective action reduces to that in the chiral phase for $\Sigma = 1$.

Derivation 2 (via Wess-Zumino-Witten action):

- Wess-Zumino-Witten action gives rise to the same effective action.

$$S(A_L, A_R, \phi) = C\Big(\int_{D_5} \tfrac{1}{10} \mathrm{Tr}\{(g^{-1}\mathrm{d}g)^5\} - \int_{S_4} \tfrac{1}{2}\mathrm{Tr}\{(\phi^{-1}A_L^3\phi A_R - \phi A_R^3\phi^{-1}A_L) + \tfrac{1}{2}\phi^{-1}A_L\phi A_R\phi^{-1}A_L\phi A_R$$
$$+ (\phi^{-1}\mathrm{d}A_L A_L\phi A_R - \phi\mathrm{d}A_R A_R\phi^{-1}A_L) + (\mathrm{d}A_R\phi^{-1}A_L\phi A_R - \mathrm{d}A_L\phi A_R\phi^{-1}A_L)$$
$$+ (U_L A_L\phi A_R\phi^{-1}A_L - U_R A_R\phi^{-1}A_L\phi A_R) + (U_L\phi\mathrm{d}A_R\phi^{-1}A_L - U_R\phi^{-1}\mathrm{d}A_L\phi A_R)$$
$$+ (U_L^2\phi A_R\phi^{-1}A_L - U_R^2\phi^{-1}A_L\phi A_R) + [U_L(A_L\mathrm{d}A_L + \mathrm{d}A_L A_L) - U_R(\mathrm{d}A_R A_R + A_R\mathrm{d}A_R)]$$
$$+ U_L A_L^3 - U_R A_R^3 + \tfrac{1}{2}(-U_L A_L U_L A_L + U_R A_R U_R A_R) + (-U_L^3 A_L + U_R^3 A_R)\}\Big),$$

Wess-Zumino-Witten action*

$$S_{\text{eff}} = \frac{e^2 N_c}{4\pi^2} \int d^4x \;\; a_\mu A_\nu \tilde{F}^{\mu\nu} \,\mathrm{tr}\left(Q^2 + \frac{1}{6}[Q, \Sigma][Q, \Sigma^\dagger] \right) + \cdots \; .$$

- This derivation implies that the effective action is independent to microscopic details and higher-loop corrections.

* Source: Kawai-Tye 1984.

Hadronic phase

Result:

- The chiral magnetic current reads:

$$\boldsymbol{j} = \frac{\delta S_{\rm eff}}{\delta \boldsymbol{A}} = \frac{e^2 N_c}{2\pi^2}\mu_5 \boldsymbol{B}\,\mathrm{tr}\left(Q^2 + \frac{1}{6}[Q,\Sigma][Q,\Sigma^\dagger]\right)$$

CME current in the hadronic phase

- The current involves the pseudoscalar mesons.
- The functional form of the current is independent to microscopic details and higher-loop corrections.

Note:

- Taking the expectation value of the pseudoscalar mesons, the current reduces to the familiar form:

$$\langle \boldsymbol{j} \rangle = \kappa \frac{e^2 N_c}{2\pi^2}\mu_5 \boldsymbol{B}\,\mathrm{tr}(Q^2)\,, \quad \kappa \equiv \frac{1}{\mathrm{tr}(Q^2)}\left\langle \mathrm{tr}\left(Q^2 + \frac{1}{6}[Q,\Sigma][Q,\Sigma^\dagger]\right)\right\rangle .$$

- With a physical value $\boldsymbol{H} = \kappa \boldsymbol{B}$ adjusted, the form of the anomalous current is still protected.

Contents:

Strength

Dielectric constant:

- Now our interest is on the strength of the CME current:

$$\langle \boldsymbol{j} \rangle = \kappa \frac{e^2 N_c}{2\pi^2} \mu_5 \boldsymbol{B} \mathrm{tr}(Q^2)\,, \quad \kappa \equiv \frac{1}{\mathrm{tr}(Q^2)} \left\langle \mathrm{tr}\left(Q^2 + \frac{1}{6}[Q,\Sigma][Q,\Sigma^\dagger] \right) \right\rangle$$

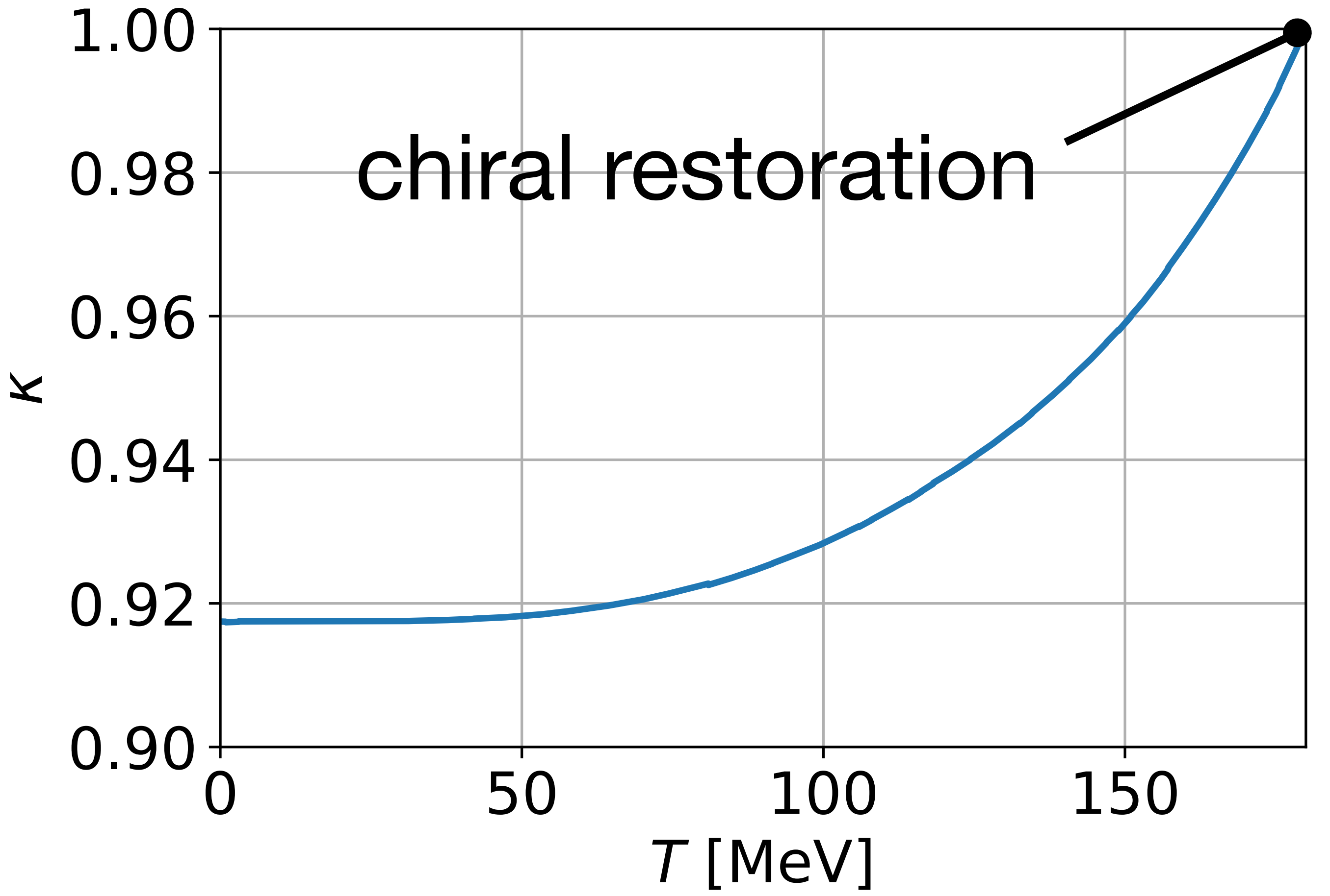
- Here κ is "dielectric constant" incorporating interactions with pseudoscalar mesons.

Strength:

- The dielectric constant as a function of T is analytically calculable for two-flavor free pion gas:

$$\kappa(T) = \frac{1}{5}(12 + 3e^{-2G} + 9e^{-G} - 18e^{-\frac{1}{2}G})\,, \quad G \equiv f_\pi^{-2} \langle \pi^A(x) \pi^A(x) \rangle\,.$$

- Mesonic medium reduces the current strength (Fig.).
- It is interesting to note that the beam energy scan programs have reported reduced CME signals for low beam energies for which the fireball may have a short lifetime until it hadronizes.



(Fig.) T dependence of dielectric constant

Contents:

Conclusion

Conclusion:

- CME in the hadronic phase involves interaction with pseudoscalar mesons.
- The functional form of the current is independent to microscopic details and higher-loop corrections.
- CME current could be weakened by pseudoscalar mesons as the two-flavor analysis implied.

Outlooks:

- Large multi-pion correlations may much more influence the strength of CME.
- Other chiral transports in the hadronic phase deserve further study.
  - E.g. Chiral separation effect is also modified, and thus chiral magnetic wave could be modified.
  - E.g. Chiral vortical effect may also be modified.