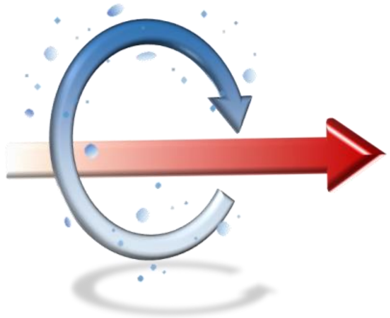
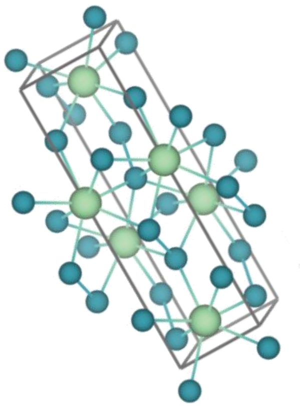




Anomalous Hydrodynamic Transport in Interacting Noncentrosymmetric Metals

Presenter : [1]Riki Toshio

[1]*Department of physics, Kyoto University, Japan*

Collaborators : [2]Kazuaki Takasan

[1]Norio Kawakami

[2]*Department of Physics, University of California, Berkeley, California*

arXiv:2005.03406 (2020).

Outline

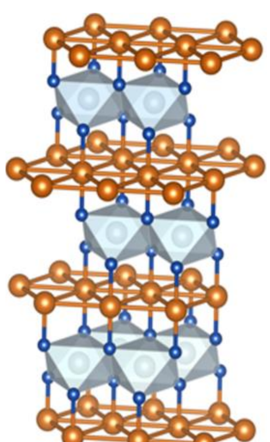
Target

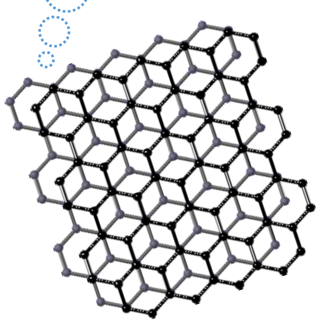
System *A novel class of matter* "*Hydrodynamic materials*"

Phenomena Unconventional **nonlinear** and **nonlocal** transport

I'll explain the detail later!!

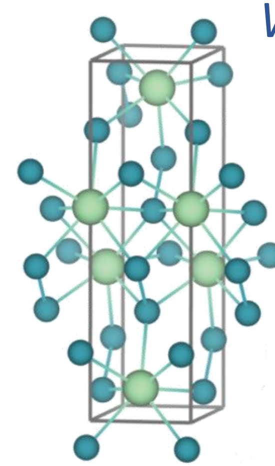
$PdCoO_2$





graphene

WP_2



Outline

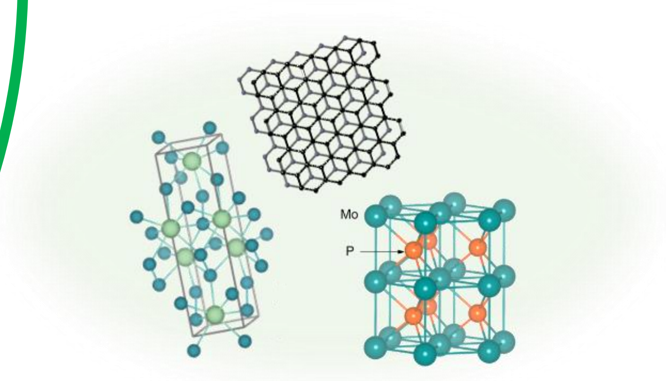
Target

System ***A novel class of matter*** “***Hydrodynamic materials***”

Phenomena Unconventional **nonlinear** and **nonlocal** transport

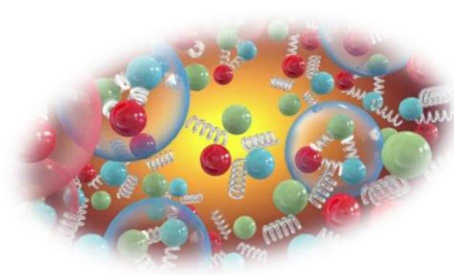
Result 1

Formulation of the hydrodynamic theory
for **noncentrosymmetric** hydrodynamic materials



Noncentrosymmetric Electron Fluids

Analogy!?



Chiral fluid

Outline

Target

<u>System</u> *A novel class of matter* "*<u>Hydrodynamic materials</u>*"

<u>Phenomena</u> Unconventional **<u>nonlinear</u>** and **<u>nonlocal</u>** transport

Result 1

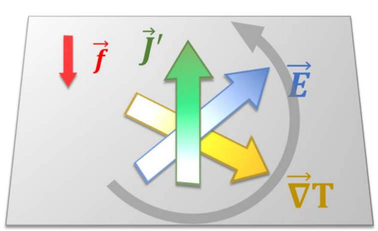
Formulation of the hydrodynamic theory

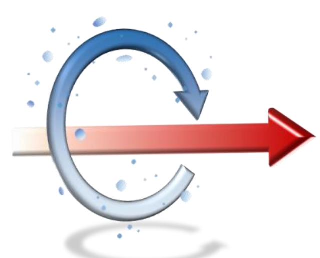
for **noncentrosymmetric** hydrodynamic materials

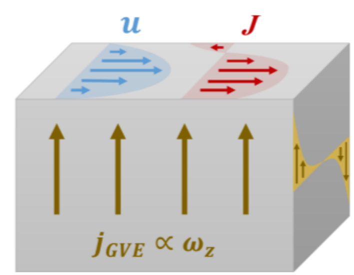
Result 2

Prediction of a variety of **novel anomalous transport**

- Current-induced anomalous thermal Hall effect
- Generalized vortical effect, asymmetric Poiseuille flow, *etc.*

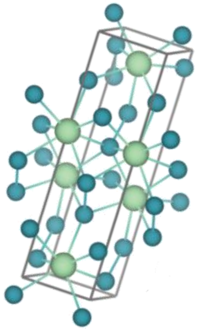






Introduction

~ What is "*hydrodynamic materials*" ~

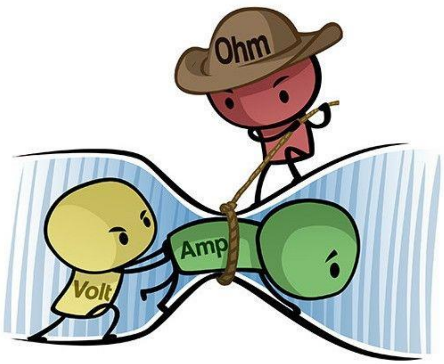


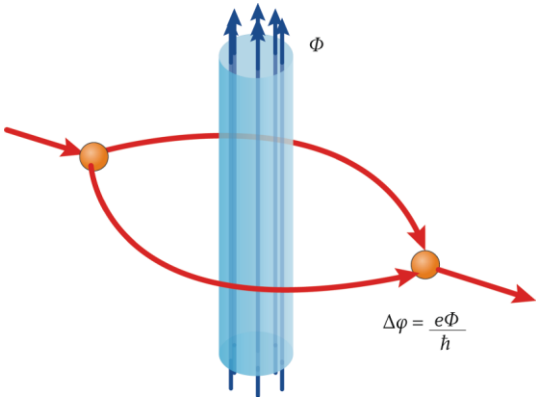
1. Hierarchy in electron dynamics

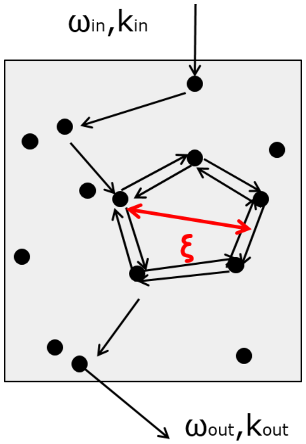
2. Experimental realization

3. Three important aspects of the electron hydrodynamics

Nonequilibrium phenomena in solids



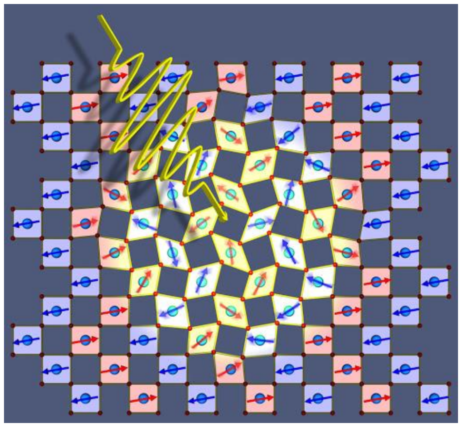


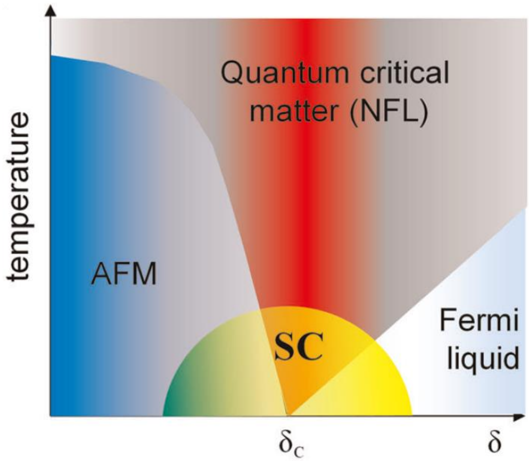


Ohm's law

Aharonov-Bohm effect

Anderson localization

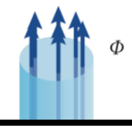


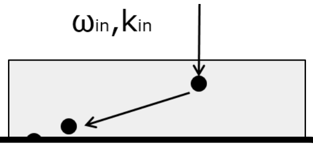


Light-induced phase transition

Anomalous transport near QCP

Nonequilibrium phenomena in solids



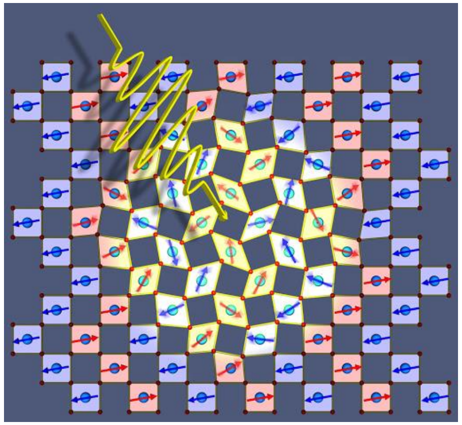


How can we extract a universal physics from these diverse and complex nonequilibrium phenomena?

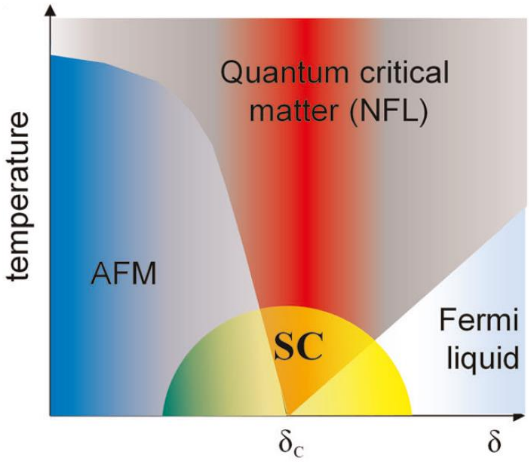
Ohm's law

Aharonov-Bohm effect

Anderson localization



Light-induced phase transition



Anomalous transport near QCP

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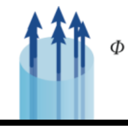

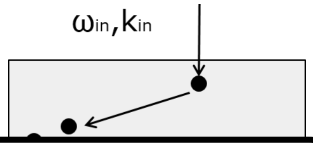

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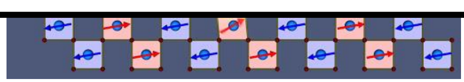
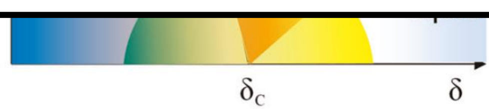




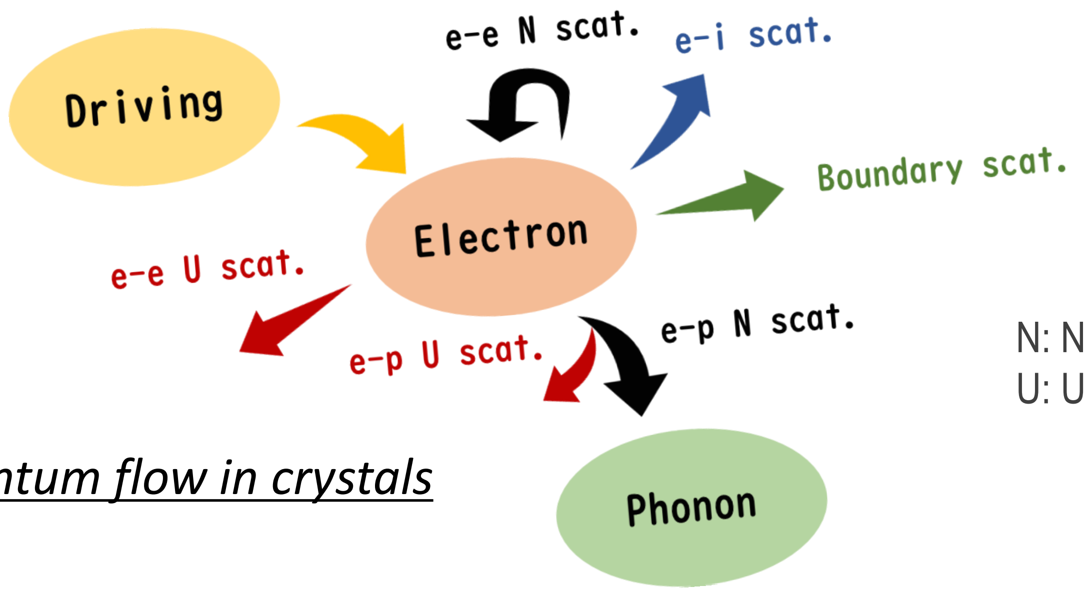
Hint

Spatio-Temporal Hierarchical structure in Electron Dynamics

Light-induced phase transition

Anomalous transport near QCP

3 *scales* to characterize the dynamics *in crystals*



Momentum flow in crystals

N: Normal
U: Umklapp

l_{MR}	=	Mean free path of momentum relaxing scattering	→	Impurity scattering, e-p scattering e-e Umklapp scattering etc…
l_{MC}	=	Mean free path of momentum conserving scattering	→	Normal e-e scattering
W	=	Characteristic length scale of the dynamics	→	System size, wavelength, etc

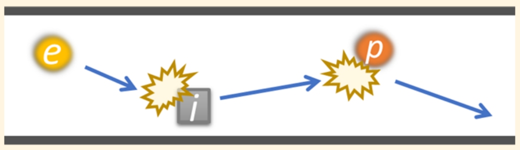
Hierarchy in electron dynamics

l_{MR} = Mean free path of momentum relaxing scattering	→	Impurity scattering, e-p scattering e-e Umklapp scattering etc...
l_{MC} = Mean free path of momentum conserving scattering	→	Normal e-e scattering
W = Characteristic length scale of the dynamics	→	System size, wavelength, etc

➡ the **minimal** scale determines the **effective** description *!!*

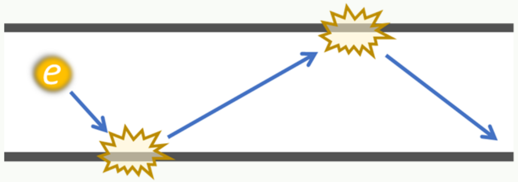
① Ohmic regime

$l_{MR} \ll l_{MC}, W$



② Ballistic regime

$W \ll l_{MC}, l_{MR}$



③ hydrodynamic regime

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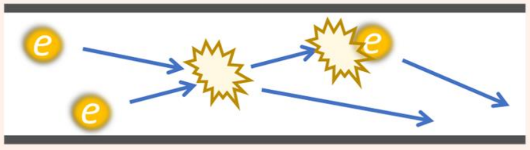
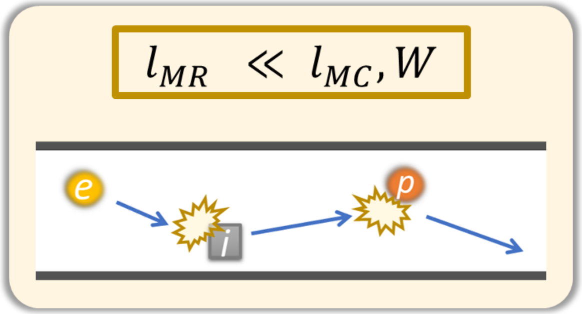
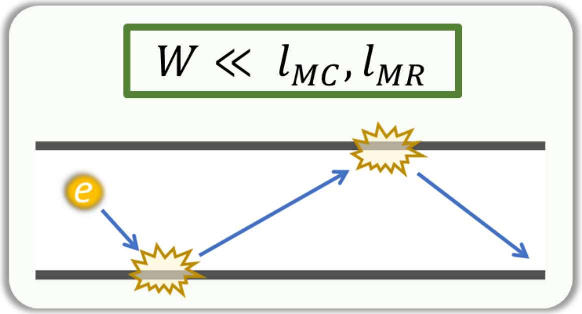




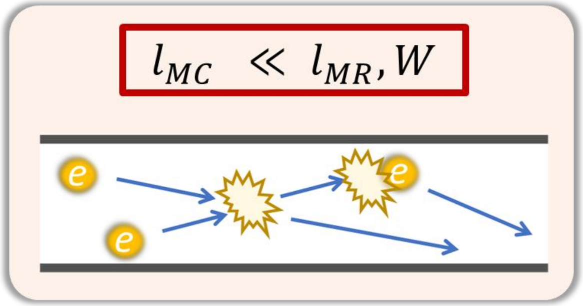
① Ohmic regime

Local & one-particle response



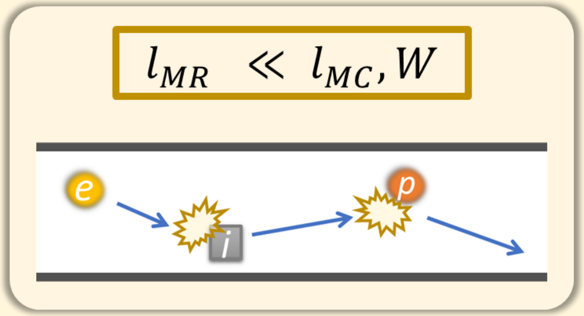
② Ballistic regime

One-particle response
with quantum coherence



③ hydrodynamic regime

Nonlocal/Nonlinear collective response



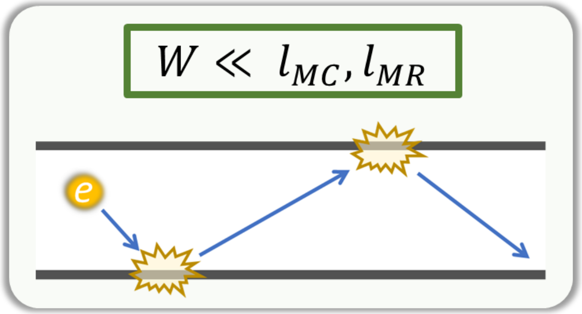
① Ohmic regime

Local & one-particle response

Particle

Realization typical metals/semiconductors

Example Ohm's law, Drude theory



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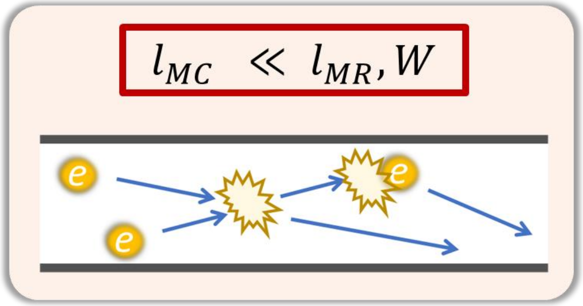

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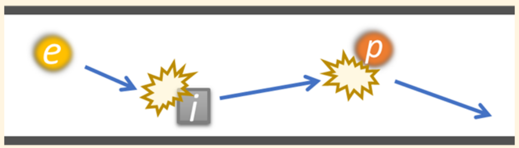
Nonlocal/Nonlinear collective response

$l_{MR} \ll l_{MC}, W$



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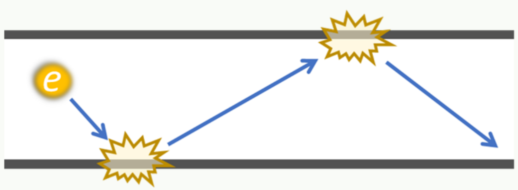


② Ballistic regime

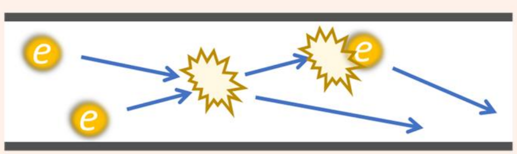
One-particle response with quantum coherence

Realization Mesoscopic systems

Example Aharonov-Bohm effect

Wave

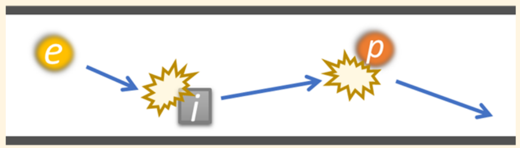
$l_{MC} \ll l_{MR}, W$



③ hydrodynamic regime

Nonlocal/Nonlinear collective response

$l_{MR} \ll l_{MC}, W$



① Ohmic regime

Local & one-particle response

Realization typical metals/semiconductors

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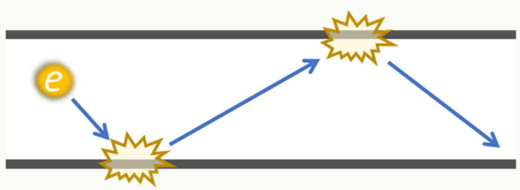

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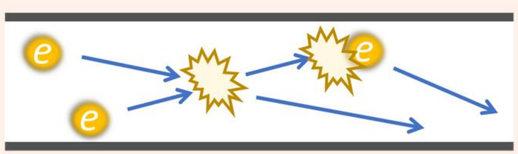
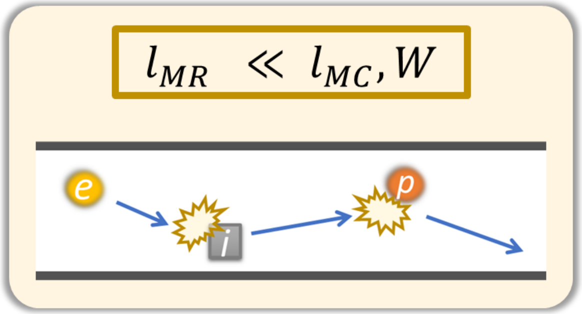


③ hydrodynamic regime

Nonlocal/Nonlinear collective response

➡ described by ”**electron hydrodynamics**”

Fluid



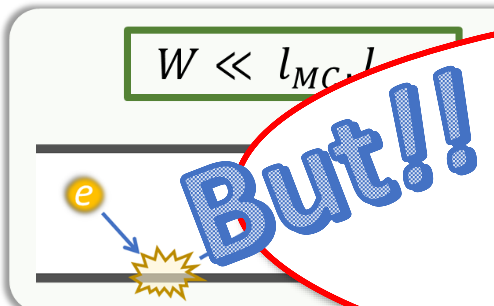
① Ohmic regime

Local & one-particle response

Realization typical metals/semiconductors

Example Ohm's law, Drude theory

Particle



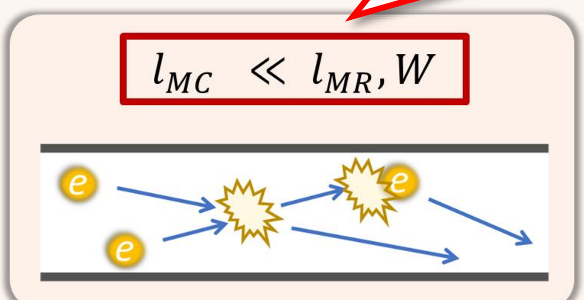
But!!

High conductivity or
strong correlation are required !

Unexplored regime… ! !

…onov-Bohm effect

Wave



③ hydrodynamic regime

Nonlocal/Nonlinear collective response

➡ described by "**electron hydrodynamics**"

Fluid

$l_{MR} \ll l_{MC}, W$

① Ohmic regime

Local & c… particle respons…

Rea… …uctor

E…

Particle

2016～

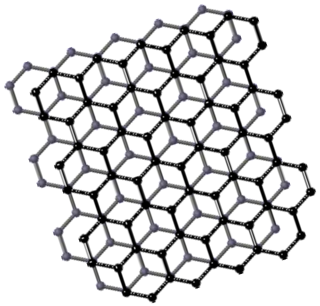
Realization of “hydrodynamic regime” has been reported very recently!!

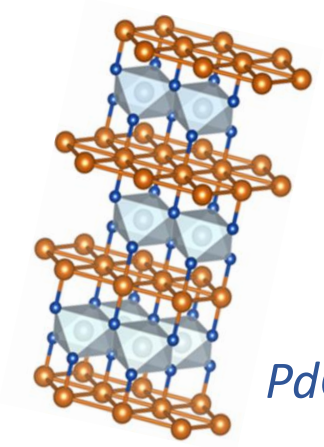
…l/Nonlinear …llective response

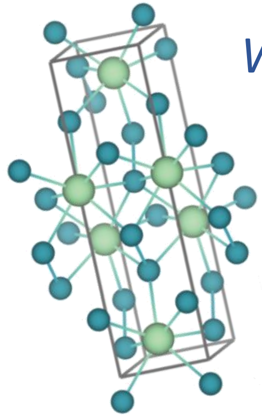
Fluid

➡ described by ”**electron hydrodynamics**”

Realizations in recent experiments







<u>Stages for realization</u>

① **Ultrapure** metal with **high conductivity**
（ex：Graphene, $PdCoO_2$, WP_2, MoP, GaAs quantum well）

② **Non-Fermi liquid** （ex：$Bi_2Sr_2CuO_6$）

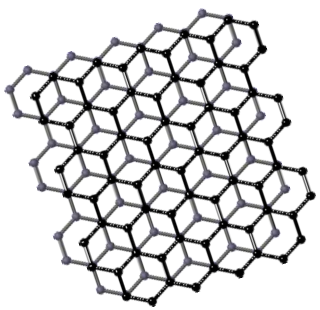
A. Amoretti, *et al.*, arXiv:1909.07991 (2019)

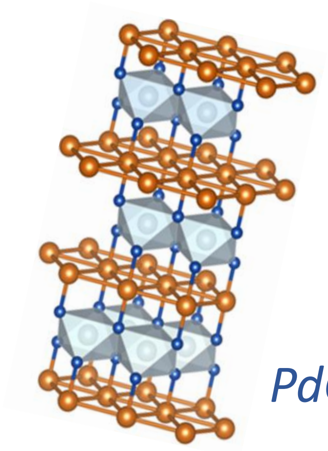
<u>Historical background</u>

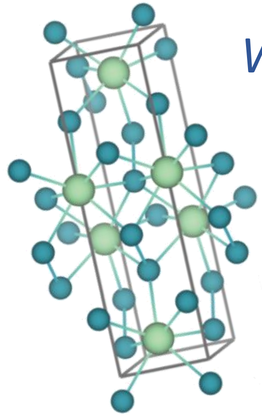
Various reports since 2016 …!!

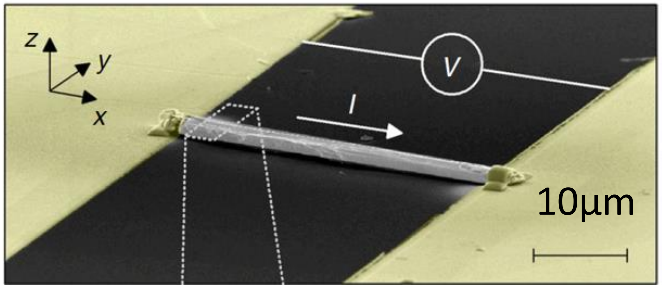
P. J. W. Moll, et al., Science **351** 1061 (2016).
R. K. Kumar, et al., Nature Physics **13** 1182 (2017).
J. Gooth, et al., Nature Communication **9** 4093 (2018).

Realizations in recent experiments









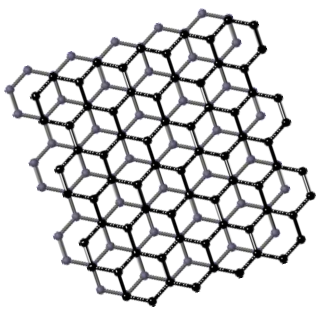
Microfabricate the sample in $\boldsymbol{\mu m}$ order

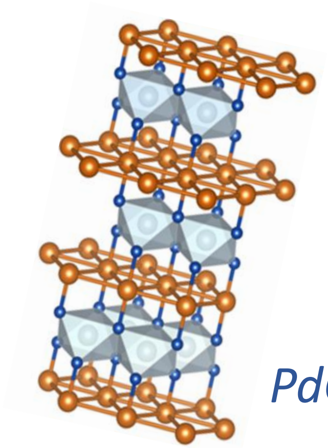
⇒ Measure the **resistivity $\boldsymbol{\rho(B,w)}$**

J. Gooth, et al., Nature Communication **9** 4093 (2018).

Realizations in recent experiments

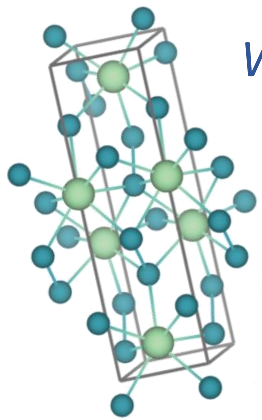
graphene

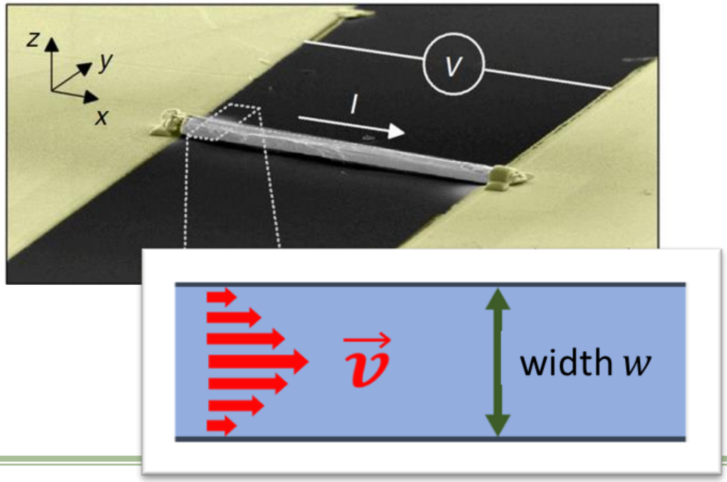




$PdCoO_2$

WP_2





Microfabricate the sample in ***µm*** order

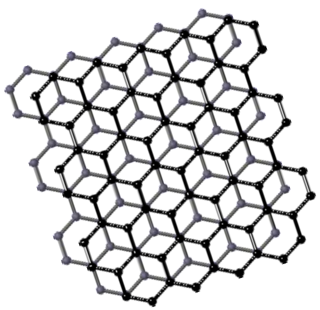
⇒ Measure the **resistivity $\rho(B, w)$**

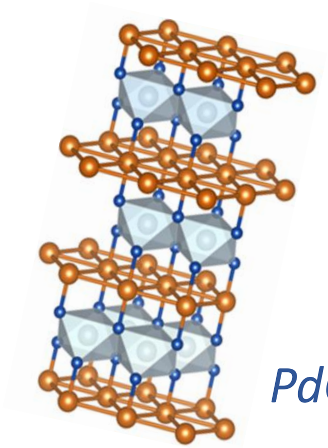
Poiseuille flow : $\rho \propto \eta(B), w^{-2}$

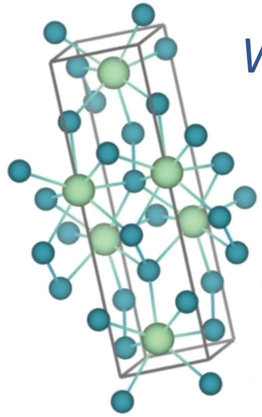
Derived from Navier-Stokes Eq.

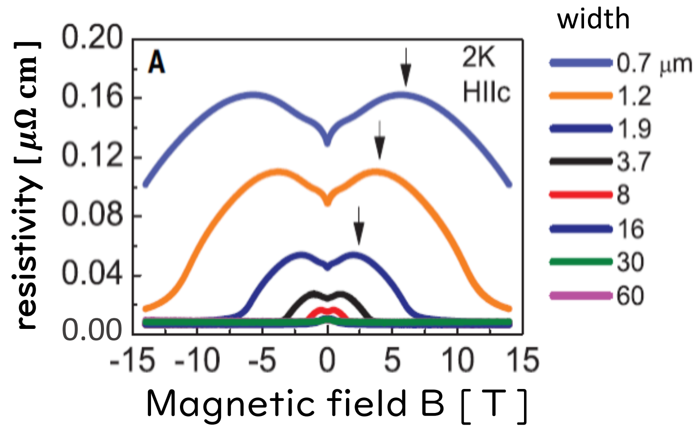
J. Gooth, et al., Nature Communication **9** 4093 (2018).

Realizations in recent experiments









B and W-dependence of resistivity

Microfabricate the sample in ***μm*** order

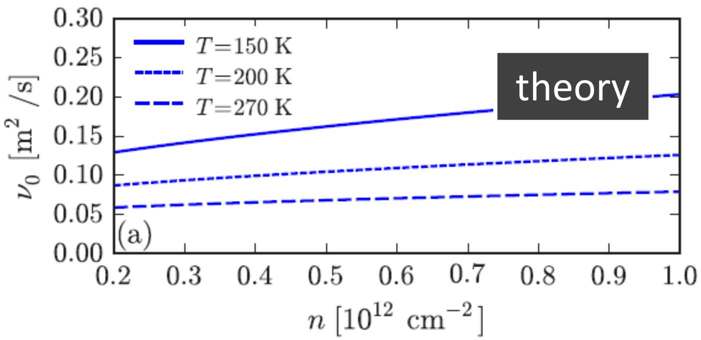
⇒ Measure the **resistivity $\rho(B, w)$**

Poiseuille flow : $\rho \propto \eta(B), w^{-2}$

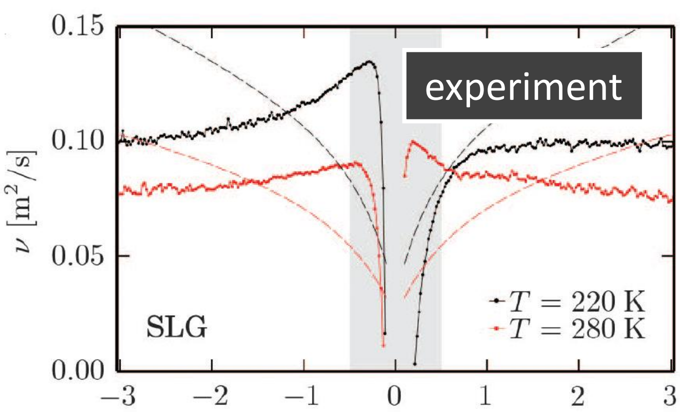
Derived from Navier-Stokes Eq.

P. J. W. Moll, et al., Science 351 1061 (2016).

Viscosity of Electron Fluids in Crystals



A. Principi, *et al.*, PRB **93**, 125410 (2016)



D. A. Bandurin, *et al.*, Science **351** 1055 (2016)

η : shear viscosity

$\nu = \eta/\rho$: kinematic viscosity

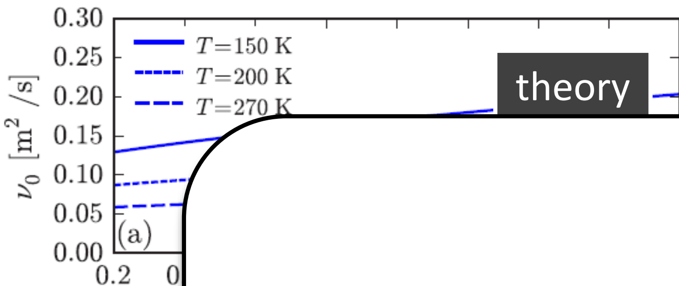
fluids (~300 K)	η [g/cm s]	ν $(cm^2\ s)$
Water	0.010	0.010
Air	0.00018	0.150
Alcohol	0.018	0.022
Glycerin	8.5	6.8
Mercury	0.0156	0.0012
Honey	18	13
Mayonnaise	80	80
Doped Graphene	$\sim 10^{-14}$ [g/s]	$\sim 10^3$

Consistent with theory and experiments

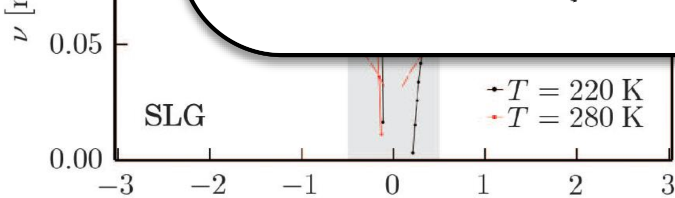
Viscosity of Electron Fluids in Crystals

η : shear viscosity

$\nu = \eta/\rho$: kinematic viscosity



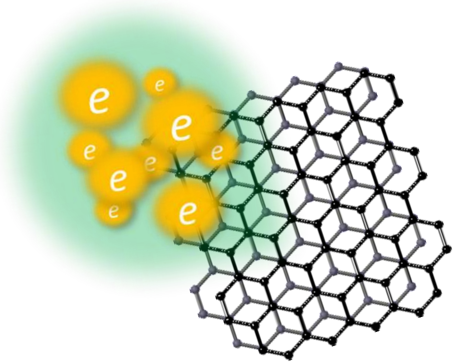
A. Prin



D. A. Bandurin, *et al.*, Science **351** 1055 (2016)

fluids (~300 K)	η [g/cm s]	ν (cm² s)
Doped Graphene	~10 [g/s]	~10^3

Kinetic viscosity



$\gg$

$\times 100$

Consistent with theory and experiments

Is the Reynolds number high? Or low?

Reynolds number : $R \equiv \dfrac{LV}{\nu}$

(L, V : characteristic length and velocity scale)

$R \gg 1$: *strongly nonlinear or turbulent flow*

$R \ll 1$: *viscous* flow

Current : $\boldsymbol{I = 2 \times 10^{-7}\ [A]}$

Particle density : $n = 10^{12}\ [cm^{-2}]$

$\longrightarrow$ $V \sim \dfrac{I}{enL} \simeq 10^{4}\ [cm \cdot s]$

Width : $L = 1\ [\mu m]$

Kinetic viscosity : $\nu = 10^{3}\ [cm^{2}s]$

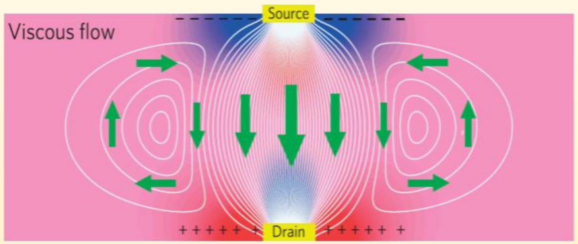
$\longrightarrow$ $R \sim 10^{-3} \ll 1$

$\Rightarrow$ *Viscosity-dominant regime*

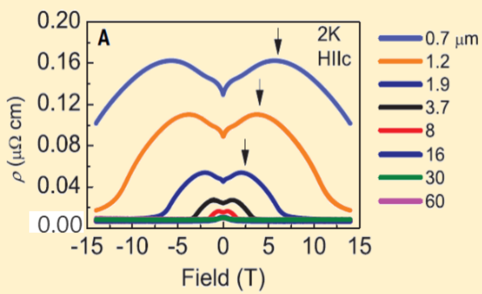
Unconventional transport

Correlation and Nonlocality, Nonlinearity

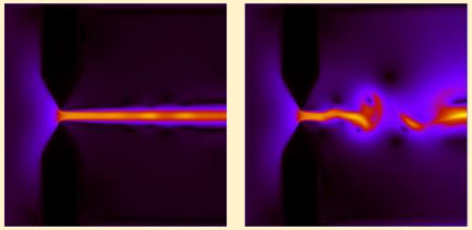
Nonlocal Transport



Viscosity Effects

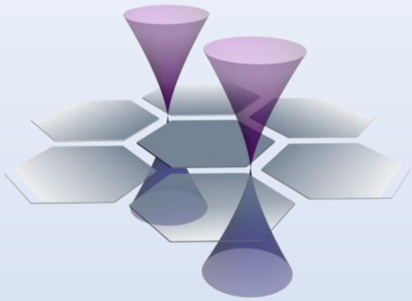


Instability and Turbulence



New Class of Fluids

Dirac/Weyl Fluids



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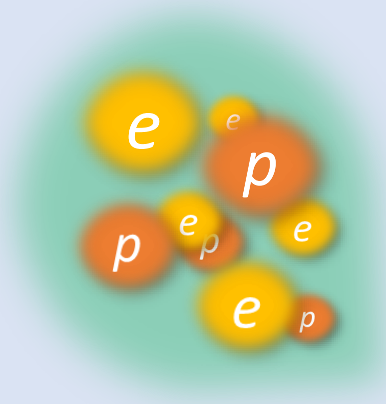


New Approach

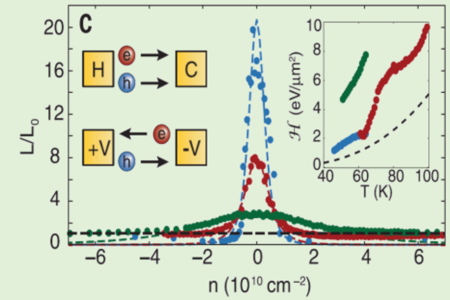
for understanding of Non-fermi liquid

AdS-CFT Bound

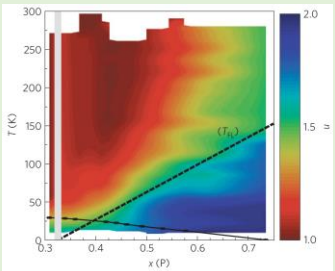
AdS/CFT bound

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

Breaking of WF law



Scaling of Resistivity in Strange Metals



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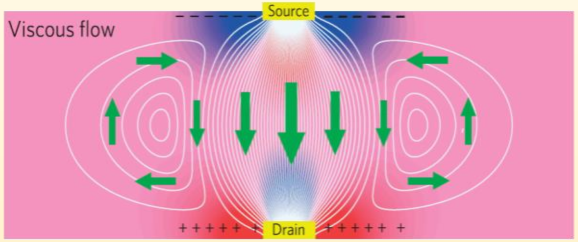

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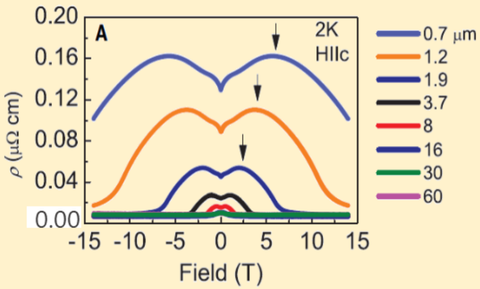

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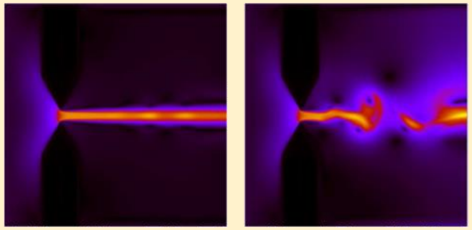

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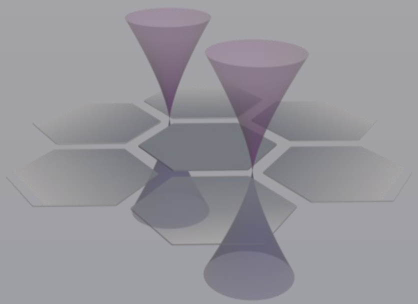

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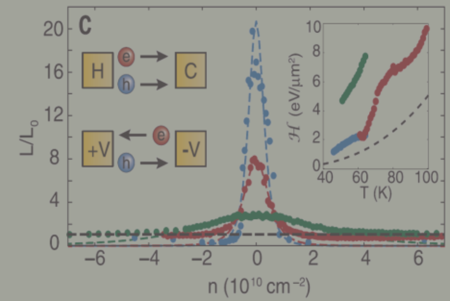

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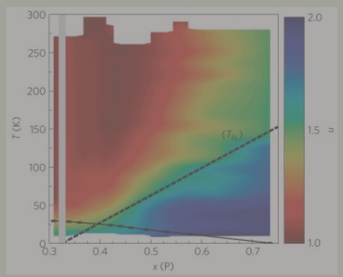

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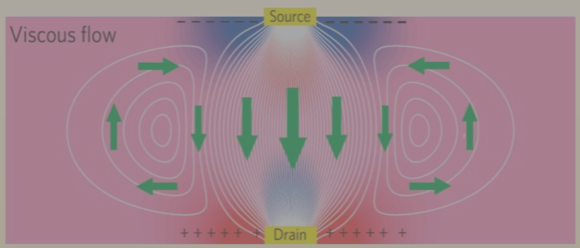

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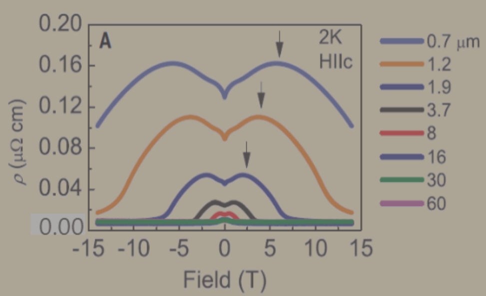

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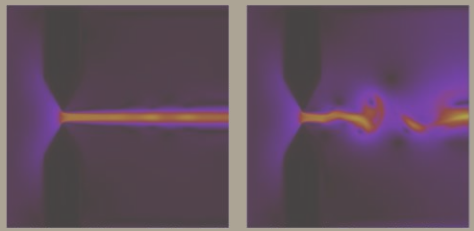

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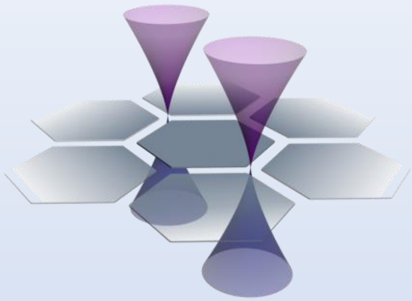

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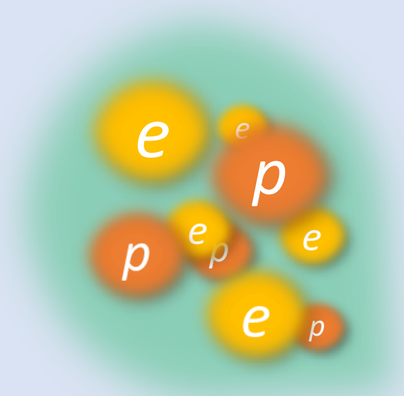

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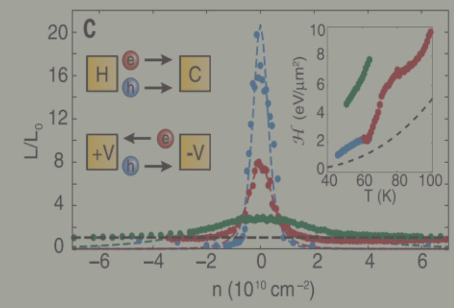

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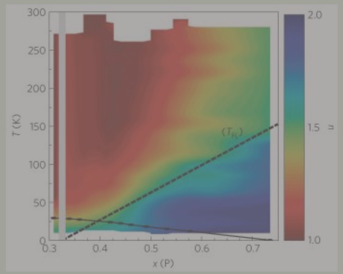

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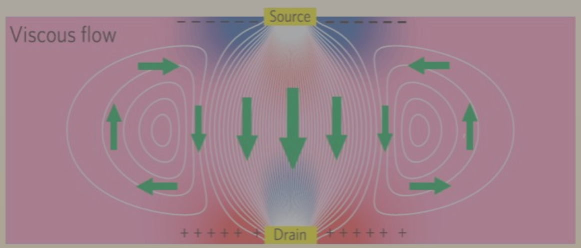

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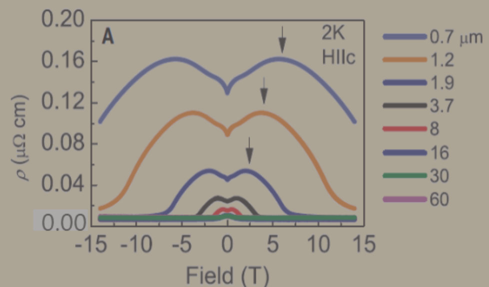

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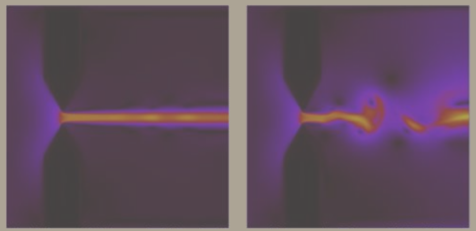

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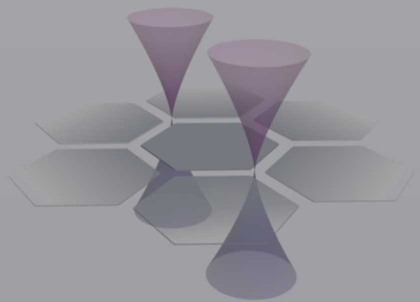

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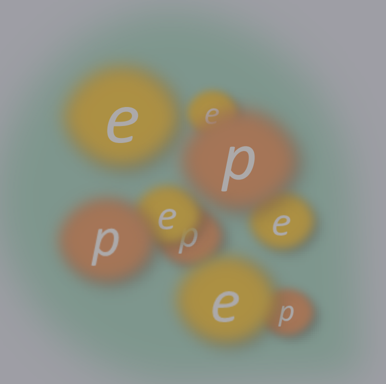

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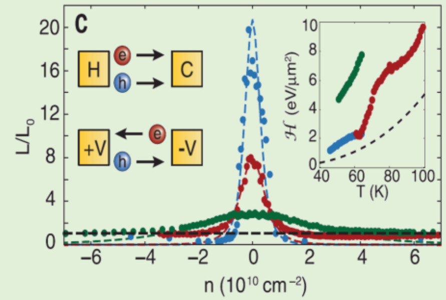

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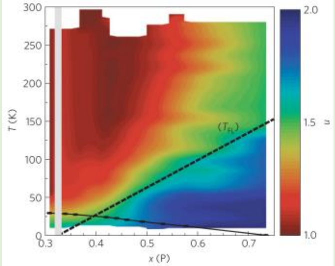

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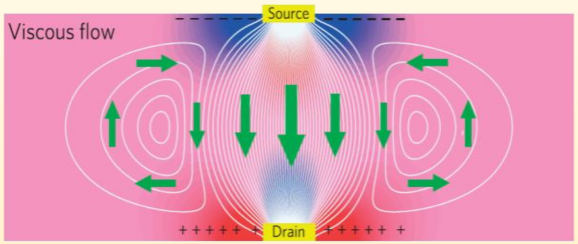

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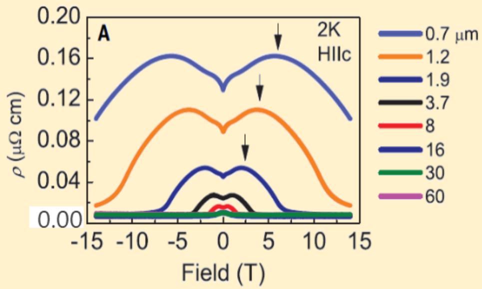

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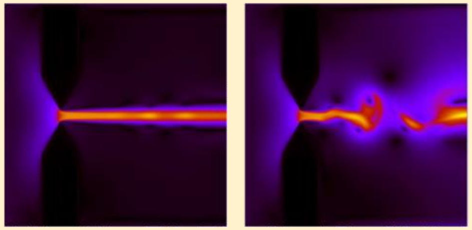

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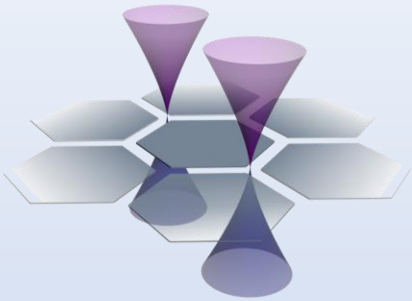

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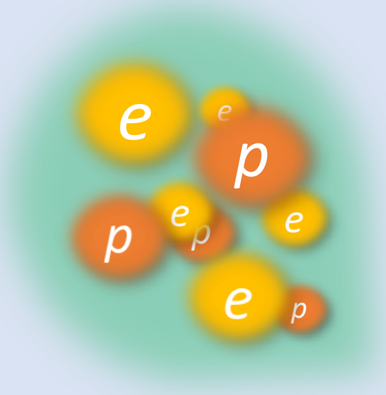

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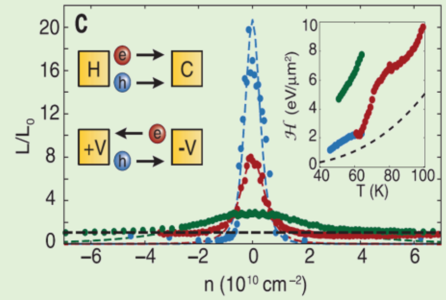

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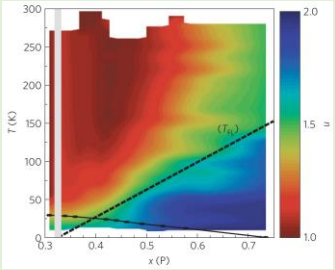

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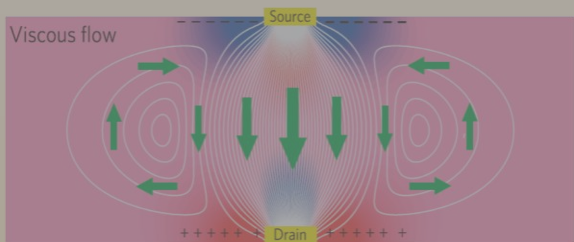

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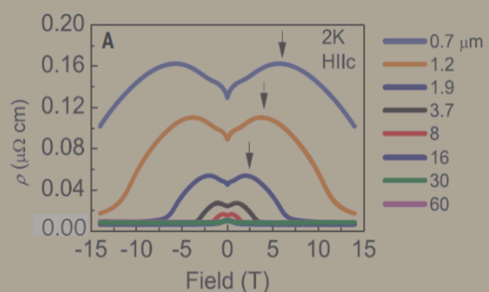

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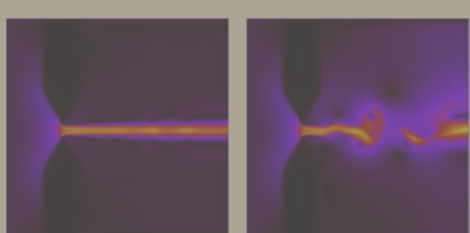

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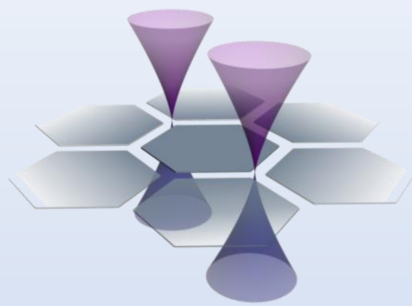

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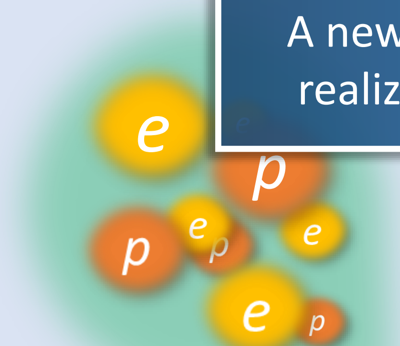

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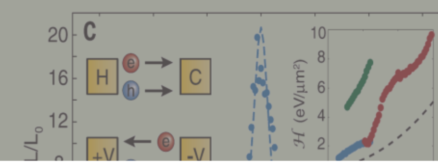

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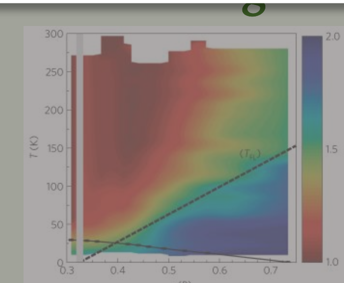


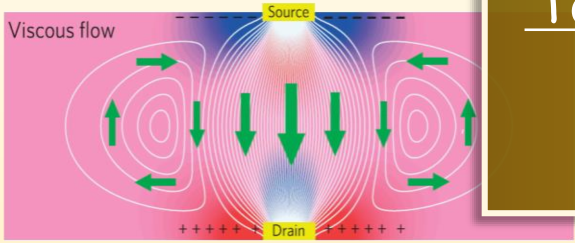
Today's Key Idea I

A new type of electron fluids realized in *noncentrosymmetric* metals

Unconventional transport

Correlation and Nonlocality, Nonlinearity

Nonlocal Transport



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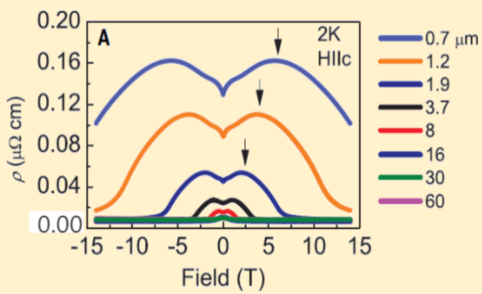

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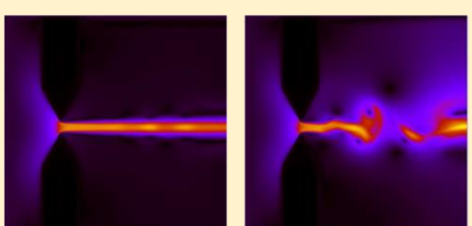

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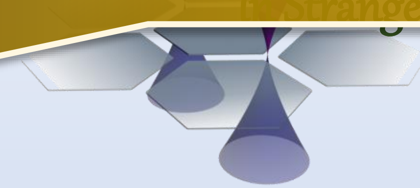

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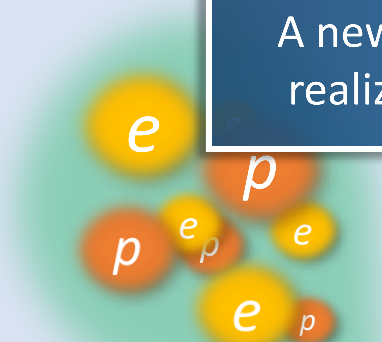

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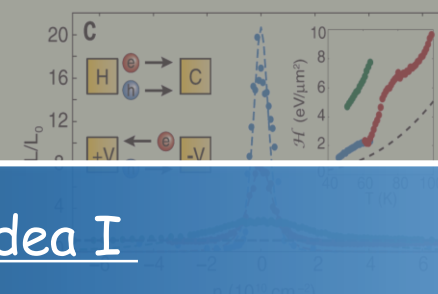
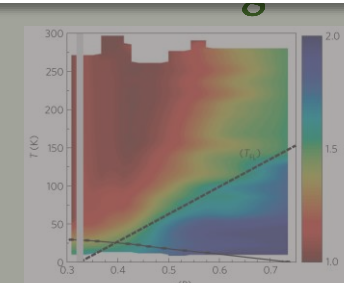




Today's Key Idea II

Novel *anomalous nonlocal* transport *due to the inversion breaking*

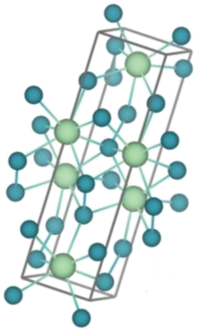
Today's Key Idea I

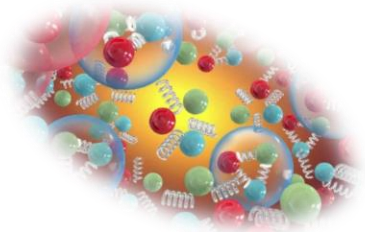
A new type of electron fluids realized in *noncentrosymmetric* metals

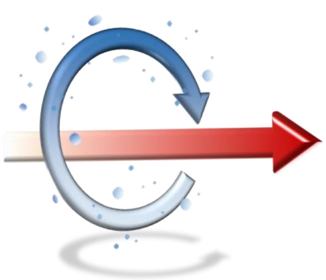
Research

～ Nonlocal and Nonlinear Anomalous Transport
In Noncentrosymmetric Hydrodynamic Materials ～

1. Our Question
2. Setting and Model
3. Results
4. Conclusion and Future Works





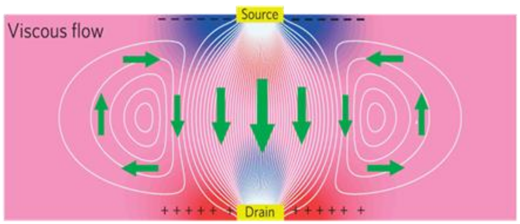


Question

Question What is the hydrodynamics peculiar to electron fluids **in crystals** ??

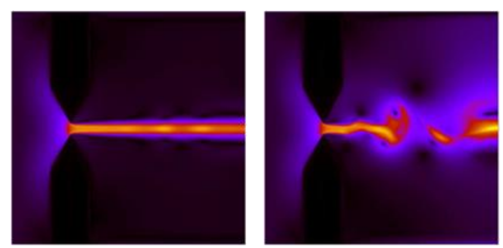
Conventional studies ⇒ not reflect the character of the fluids "in crystal"

(always assume the **isotropy and Galilei symmetry** on the fluids)



Current Backflow

L. Levitov and G. Falkovich, Nat. Phys. **12**, 672 (2016)

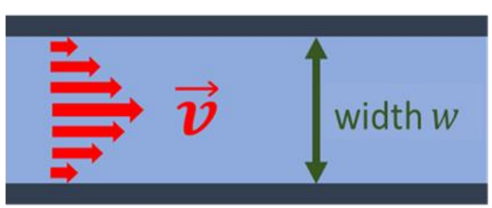


Preturbulent flow

M. Mendoza, *et al.*, PRL **106**, 156601 (2011)

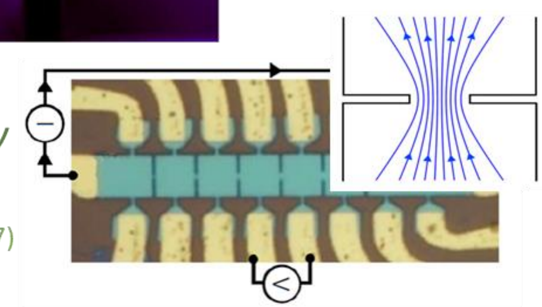
Poiseuille flow

P. J. W. Moll, et al., Science **351** 1061 (2016).
J. Gooth, et al., Nat. Comm. **9** 4093 (2018).



Superballistic flow

R. K. Kumar, *et al.*, Nat. Phys. **13**, 1182 (2017)



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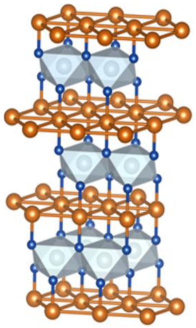
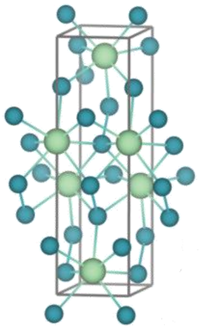
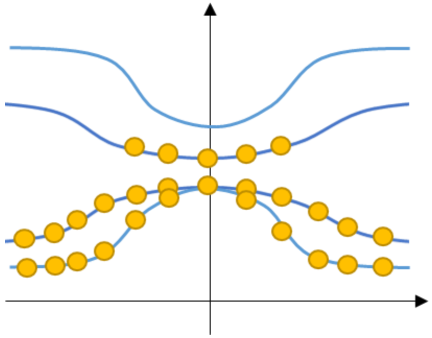
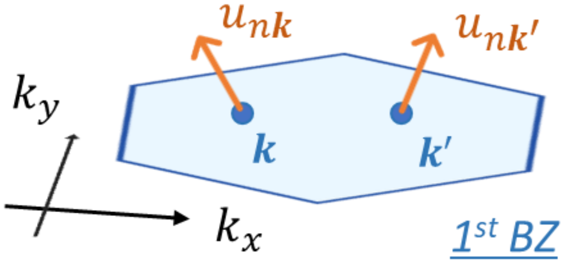
Q. What characterizes the fluids in crystals??









Question

Question   What is the hydrodynamics peculiar to electron fluids **in crystals** ??

Conventional studies ⇒ not reflect the character of the fluids “in crystal”

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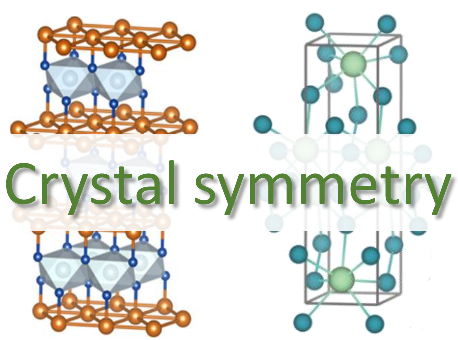

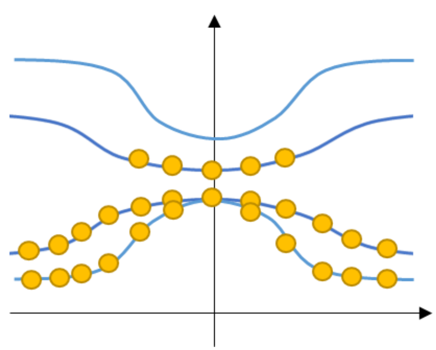

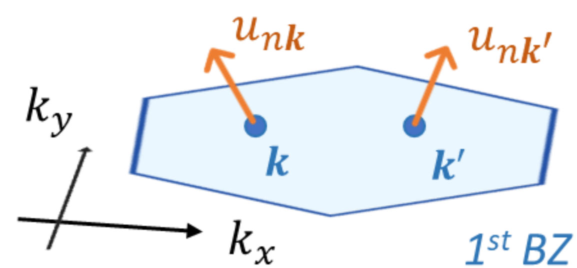

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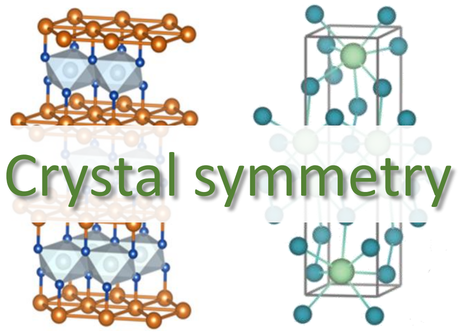

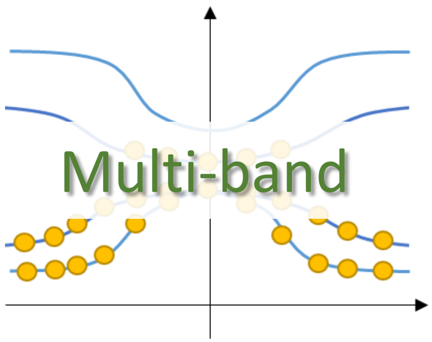

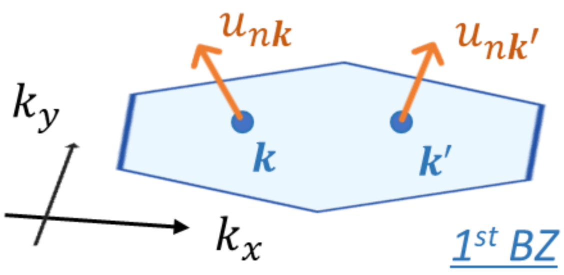

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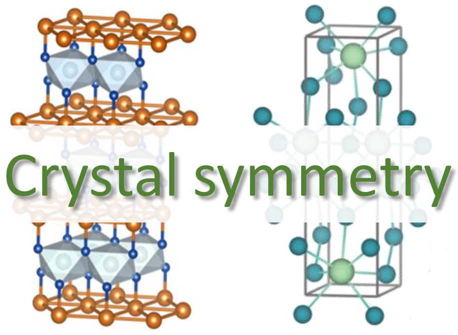
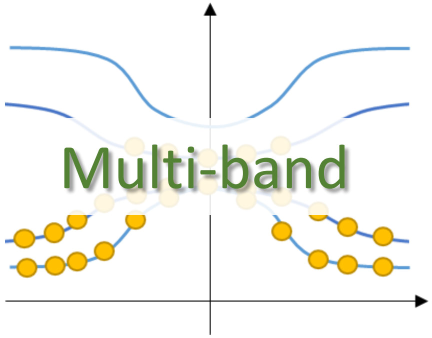
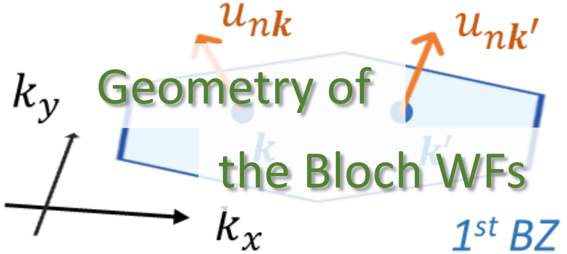
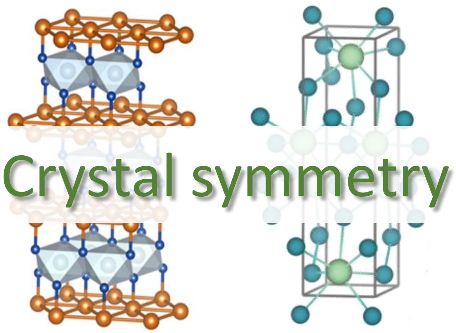
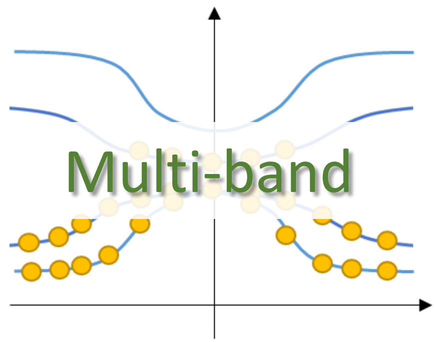


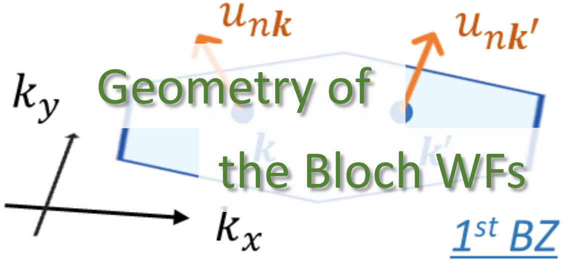




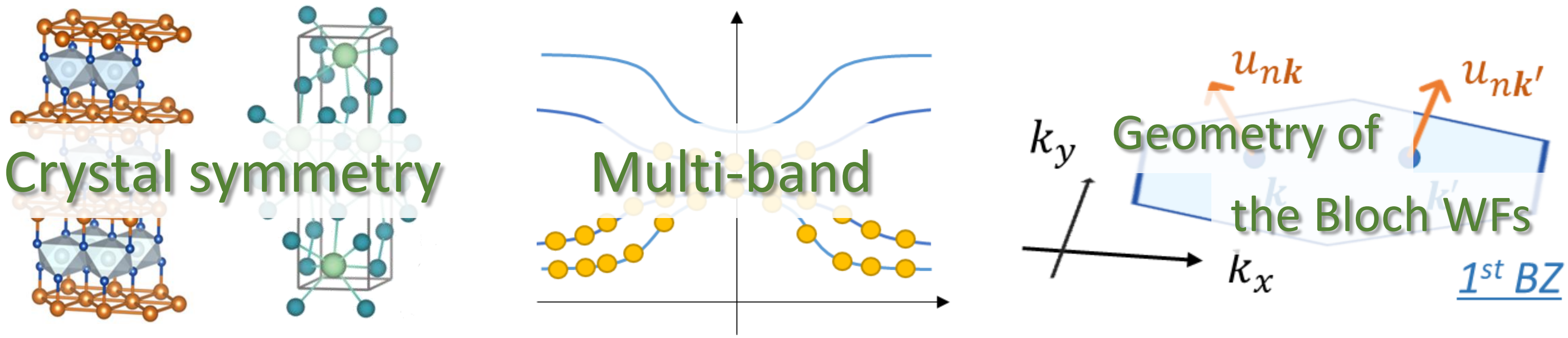
Abstract


Crystal symmetry


Multi-band


Geometry of
the Bloch WFs
u_{nk}
$u_{nk'}$
k_y
k_x
1^{st} BZ

Abstract



Generalized Euler Equation

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} + \frac{\boldsymbol{\nabla} p}{\rho} + \frac{e}{mn\hbar}\left[\frac{1}{m}\hat{C}(\boldsymbol{\nabla}\times\boldsymbol{E}) + \hat{F}\left(\boldsymbol{E}\times\frac{\boldsymbol{\nabla} T}{T}\right) + \widehat{D}(\boldsymbol{E}\times\boldsymbol{\nabla}\mu)\right] + \frac{e}{m}\boldsymbol{E} = -\frac{\boldsymbol{u}}{\tau_{mr}}$$

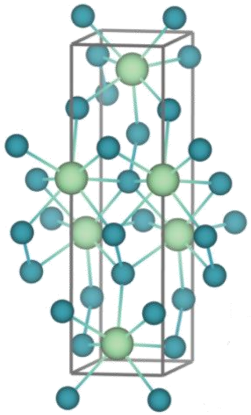
Renormalized to ***three geometrical tensors*** $\hat{C}, \hat{F}, \widehat{D}$

and cause ***various anomalous transports*** as ***additional driving forces*** !!

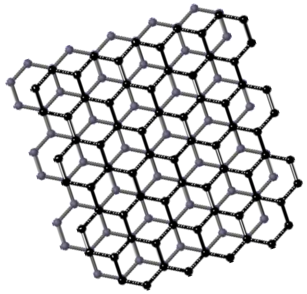
Model

Goal： Integration of **the electron hydrodynamic theory** with **the crystal symmetry** and **geometrical properties**.

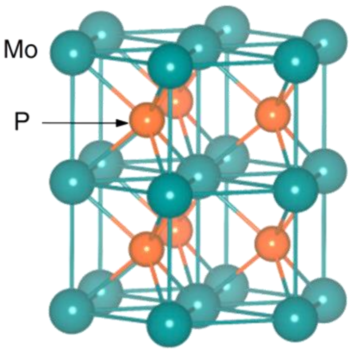
Target： **TRS and Noncentrosymmetric** Hydrodynamic materials



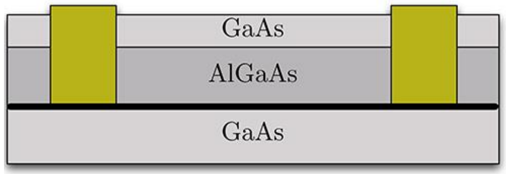
WP_2



Bilayer-graphene



MoP



GaAs quantum well

Model

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Local and **Linear** anomalous current (anomalous Hall or thermal Hall effect, *et al*) → ***prohibited*** ✕

Nonlocal and **Nonlinear** anomalous current → ***dominant*** ○

Di Xiao, *et al.*, Rev. Mod. Phys. **82**, 1959 (2010)

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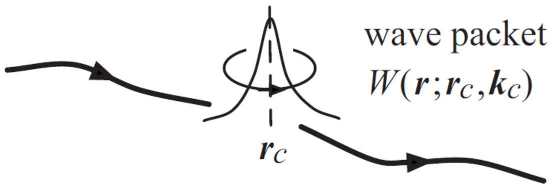
EOM for electron wave-packet

Anomalous velocity

$$\dot{\boldsymbol{r}}_c = \frac{1}{\hbar}\frac{\partial \varepsilon_n(\boldsymbol{k}_c)}{\partial \boldsymbol{k}_c} - \dot{\boldsymbol{k}}_c \times \boldsymbol{\Omega}\,(\boldsymbol{k}_c), \qquad \hbar\dot{\boldsymbol{k}}_c = -eE$$

Group velocity

Origin of various anomalous transport
（anomalous Hall effect, valley Hall effect, *et al*）



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$A_n(\boldsymbol{k}) \equiv i\langle u_{n\boldsymbol{k}}|\nabla u_{n\boldsymbol{k}}\rangle$: ***Berry connection***

$\Omega_{n,a}(\boldsymbol{k}) \equiv [\boldsymbol{\nabla}_{\boldsymbol{k}} \times A_n(\boldsymbol{k})]_a$: ***Berry curvature***

$$= i\epsilon_{abc}\sum_{m\neq n}\frac{\langle n|\partial_{k_b}\hat{H}|m\rangle\langle m|\partial_{k_a}\hat{H}|n\rangle}{(\varepsilon_n-\varepsilon_m)^2}$$

- geometry of Bloch w.f.
- multi-band properties

Model

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Boltzmann eq.

$$\frac{\partial f}{\partial t} + \dot{\boldsymbol{r}}_c \frac{\partial f}{\partial \boldsymbol{r}_c} + \dot{\boldsymbol{k}}_c \frac{\partial f}{\partial \boldsymbol{k}_c} = \mathcal{C}[f]$$

Local equilibrium

➡ Continuity eq. ➡ Generalized Euler eq.

Outline of derivation：continuity eq.

Boltzmann eq.

$$\frac{\partial f}{\partial t} + \dot{\boldsymbol{r}}_c \frac{\partial f}{\partial \boldsymbol{r}_c} + \dot{\boldsymbol{k}}_c \frac{\partial f}{\partial \boldsymbol{k}_c} = \mathcal{C}[f] \qquad (\mathcal{C}[f] = \mathcal{C}^{mc}[f] + \mathcal{C}^{mr}[f])$$

Multiply conserved quantity $X_i(\boldsymbol{k})$
and **Integrate** out $\boldsymbol{k}$

$$(X_i = n, \boldsymbol{p}, \epsilon)$$

Identity *between*
conserved quantity *and* ***M-C scat. term***

$$\int d\boldsymbol{k} X_i(\boldsymbol{k}) C^{mc}[f] = 0$$

$$\frac{\partial X_i(\boldsymbol{r})}{\partial t} + \nabla_j \Pi_{ij}(\boldsymbol{r}) = (in/outflow\ of\ X_i)$$

：continuity eq. of $X_i(\boldsymbol{r}, t)$

Outline of derivation：continuity eq.

For example, in the case of ***momentum*** $(X_i = p_i)$

Continuity Equation of Electron Momentum

$$\frac{\partial P_i}{\partial t} + \nabla_j \Pi_{ij} = -eE_i - \frac{P_i}{\tau_{mr}} \qquad \left(\mathcal{C}^{mr}[f] = \frac{f - f_0}{\tau_{mr}} \right)$$

where...

Momentum density :

$$\mathbf{P}(t, \boldsymbol{r}) = \boldsymbol{P}_\alpha(t, \boldsymbol{r}) = \sum_\alpha \int \boldsymbol{p} f_\alpha(t, \boldsymbol{r}, \boldsymbol{p}) \frac{d\boldsymbol{p}}{(2\pi\hbar)^d}$$

(α : valley index)

Momentum flux :

$$\hat{\Pi}(t, \boldsymbol{r}) \equiv \sum_\alpha \langle \boldsymbol{p} \boldsymbol{v}_\alpha \rangle_\alpha = \sum_\alpha \int \boldsymbol{p} \otimes \left(\frac{\partial \varepsilon_\alpha(\boldsymbol{p})}{\partial \boldsymbol{p}} + \frac{e\boldsymbol{E}}{\hbar} \times \Omega_\alpha(\boldsymbol{p}) \right) f_\alpha(t, \boldsymbol{r}, \boldsymbol{p}) \frac{d\boldsymbol{p}}{(2\pi\hbar)^d}$$

Assumption

Assumption ① Each conduction band can be approximated as ***parabolic*** : $\varepsilon(\boldsymbol{p}) = \dfrac{\boldsymbol{p}^2}{2m}$

⟶ ***Crystal anisotropy*** is reflected *only* through **Berry curvature** $\boldsymbol{\Omega}(\boldsymbol{p})$*!!*

(*approximately Galilean invariant*)

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(*approximately Galilean invariant*)

Assumption ② The electron system exists in ***the hydrodynamic regime*** $(l_{mc} \ll l_{mr}, W)$ and thus, $f(\boldsymbol{r}, \boldsymbol{p}, t)$ can be described with perturbative theory from ***local equilibrium distribution function*** $f_0^{local}(r, p, t)$:

$$f(\boldsymbol{r}, \boldsymbol{p}, t) = f_0^{local}(\boldsymbol{r}, \boldsymbol{p}, t) + \delta f_1 + \cdots \qquad (\, \delta f_1 \propto \mathcal{O}(l_{mc}/W) \,)$$

$$f_0^{local}(\boldsymbol{r}, \boldsymbol{p}, t) \equiv \left[1 + e^{\beta(\varepsilon(p) - \boldsymbol{u}\cdot\boldsymbol{p} - \mu)}\right]^{-1}$$

→ *Electron states can be described only by* ***hydrodynamic variables***!!

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⟶ ***Crystal anisotropy*** is reflected *only* through ***Berry curvature*** $\boldsymbol{\Omega}(\boldsymbol{p})$!!

(*approximately Galilean invariant*)

<u>Assumption</u> ② The electron system exists …

and thus, $f(\boldsymbol{r},\boldsymbol{p},t)$ can be describe…

local equilibrium state $f_0^{local}(r,p,t)$:

We consider the zeroth order approx.

→ *Hydro. equation for ideal fluids*

$$f(\boldsymbol{r},\boldsymbol{p},t) = f_0^{local}(\boldsymbol{r},\boldsymbol{p},t) + \delta f_1 + \cdots \qquad (\,\delta f_1 \propto \mathcal{O}(l_{mc}/W)\,)$$

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⟶ *Electron states can be described only by* ***hydrodynamic variables***!!

Result ①: Generalized Euler Equations

$\boldsymbol{u}$: velocity field
p: pressure
$\boldsymbol{E}$: electric field

Momentum flux

$$\Pi_{ij} = mnu_i u_j + P\delta_{ij} + \frac{e}{\hbar}\epsilon_{jkl} C_{il} E_k \quad + \mathcal{O}(E^2)$$

Conventional term — *Additional geometrical term*

where... $C_{il} = \sum_{\alpha} C_{il}^{\alpha}, \quad C_{il}^{\alpha} \equiv \int [d\boldsymbol{p}]\, p_i \Omega_{\alpha,l} f_{0\alpha}$

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Conventional term — Additional geometrical term

$$\boldsymbol{P} = mn\boldsymbol{u}$$

$$\frac{\partial P_i}{\partial t} + \nabla_j \Pi_{ij} = qE_i - \frac{P_i}{\tau_{mr}}$$

Generalized Euler Equation

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} + \frac{\boldsymbol{\nabla}p}{\rho} + \frac{e}{mn\hbar}\left[\frac{1}{m}\hat{C}(\boldsymbol{\nabla}\times\boldsymbol{E}) + \hat{F}\left(\boldsymbol{E}\times\frac{\boldsymbol{\nabla}T}{T}\right) + \hat{D}(\boldsymbol{E}\times\boldsymbol{\nabla}\mu)\right] + \frac{e}{m}\boldsymbol{E} = -\frac{\boldsymbol{u}}{\tau_{mr}}$$

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Geometrical coefficients

$$C_{il}^{\alpha} \equiv \int [d\boldsymbol{p}] p_i \Omega_{\alpha,l} f_{0\alpha}, \quad F_{il}^{\alpha} \equiv -\int [d\boldsymbol{p}] \varepsilon_\alpha \Omega_{\alpha,l} \frac{\partial f_{0\alpha}}{\partial p_i}, \quad D_{il}^{\alpha} \equiv -\int [d\boldsymbol{p}] \Omega_{\alpha,l} \frac{\partial f_{0\alpha}}{\partial p_i}$$

Reflect the ***symmetry*** and ***geometry*** in crystals

(Symmetry classification of C,F,D will be discussed later)

Result ① : Generalized Euler Equations

Generalized Euler Equation

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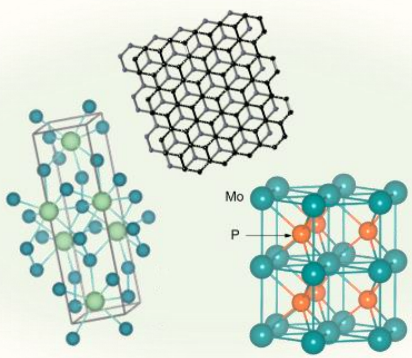
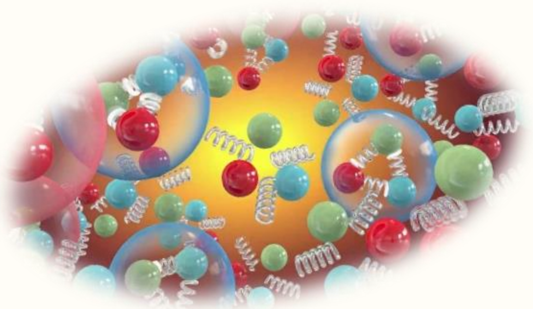


Noncentrosymmetic electron fluids

Analogy!?



Quark-Gluon Plasma (Chiral fluid)

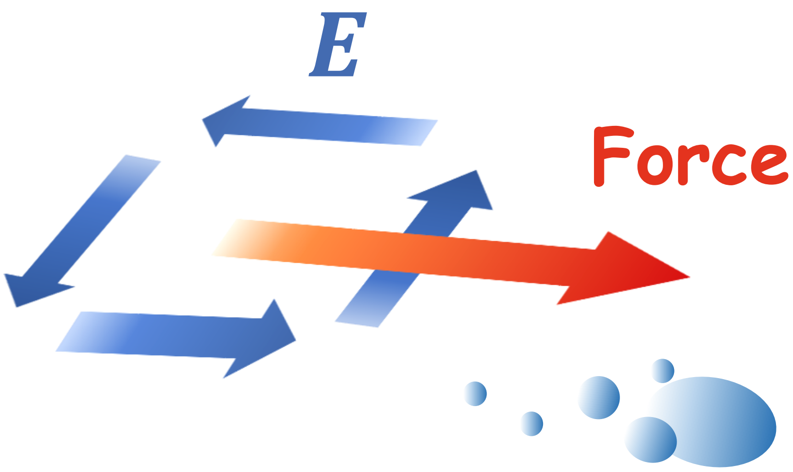
For example, Yoshimasa Hidaka, Shi Pu, and Di-Lun Yang, Phys. Rev. D **97**, 016004 (2018)

Result ① : Generalized Euler Equations

Generalized Euler Equation

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} + \frac{\boldsymbol{\nabla} p}{\rho} + \frac{e}{mn\hbar}\left[\frac{1}{m}\hat{C}(\boldsymbol{\nabla}\times\boldsymbol{E}) + \hat{F}\left(\boldsymbol{E}\times\frac{\boldsymbol{\nabla} T}{T}\right) + \widehat{D}(\boldsymbol{E}\times\boldsymbol{\nabla}\mu)\right] + \frac{e}{m}\boldsymbol{E} = -\frac{\boldsymbol{u}}{\tau_{mr}}$$

Unconventional Inverse Edelstein Effect (?)*



*As for this term, we need further discussion under magnetic field.

Result ①： Generalized Euler Equations

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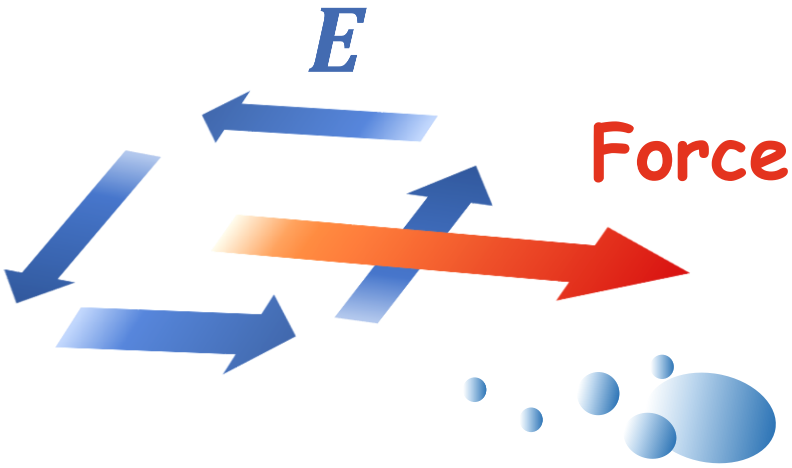
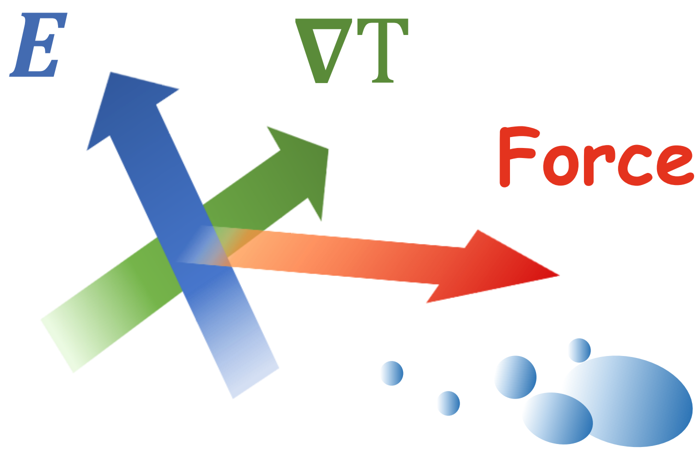
Unconventional Inverse Edelstein Effect (?)*

Nonlinear Anomalous Thermoelectric Effect





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e.g.) Nonlinear anomalous thermoelectric effect

Generalized Euler Equation

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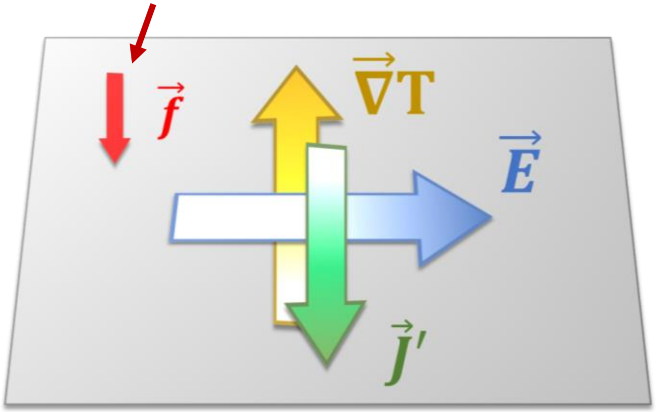
Nonlinear Anomalous Thermoelectric Effect

e.g.) 2D system under ***thermal gradient*** and ***electric field***

Perpendicular to mirror line in 2D plane



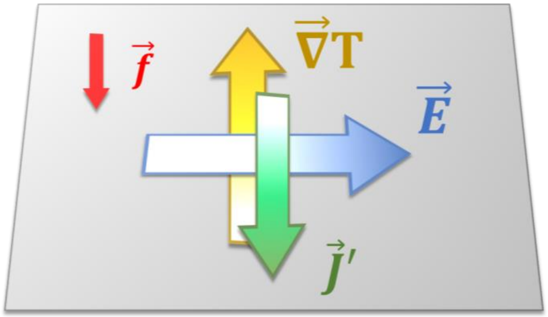
Nonlinear thermoelectric current：

$$J = \frac{e^2}{\hbar T}\frac{\tau_{mr}}{1+i\omega\tau_{mr}}[\boldsymbol{E}\times\boldsymbol{\nabla}T]_z\cdot\boldsymbol{f}$$

$$F_{ij} = f_i\delta_{jz}$$

e.g.) Nonlinear anomalous thermoelectric effect

e.g.) 2D system under ***thermal gradient*** and ***electric field***



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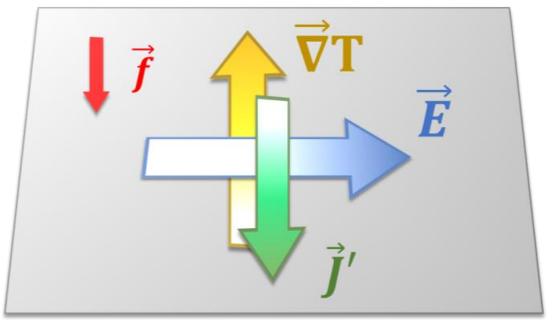
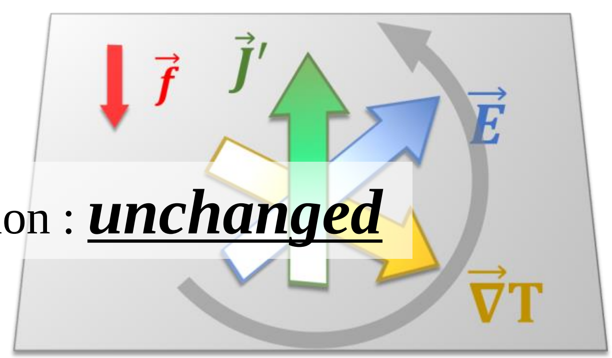


Simultaneous Rotation of E and ∇T



Current direction : ***<u>unchanged</u>***

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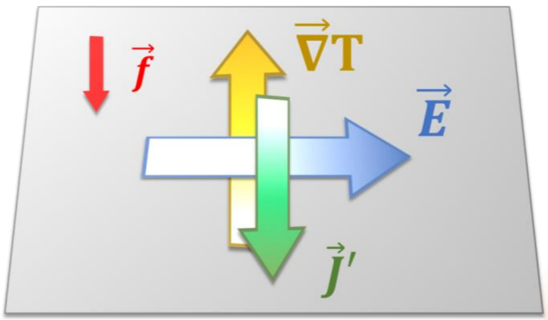

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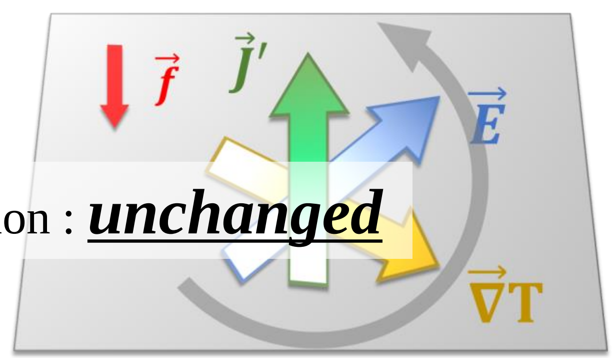
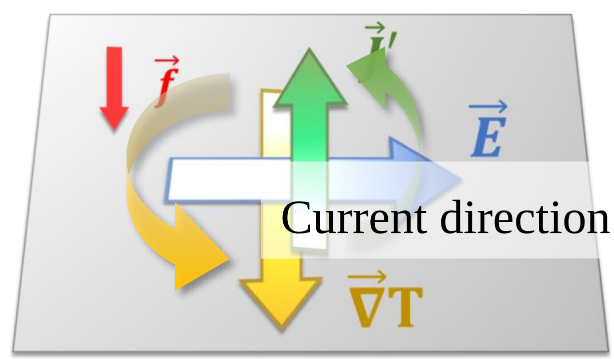
Simultaneous Rotation of E and ∇T

Switching of Either of ∇T or E



Current direction : ***unchanged***



Current direction : ***Flipped***

Result ②： Hydrodynamic description of electric current

Question *How can we translate $\boldsymbol{u}(\boldsymbol{r},t)$ into $\boldsymbol{j}(\boldsymbol{r},t)$??*

Generally, *"transport current"* is given as

$$\boldsymbol{j}(\boldsymbol{r},t) = (-e)\sum_{\alpha}\int [dp]\,\dot{\boldsymbol{r}}_{c,\alpha}\,f_\alpha + \boldsymbol{\nabla}\times\left(\sum_{\alpha}\int [dp]\boldsymbol{m}_\alpha f_\alpha\right) - \boldsymbol{\nabla}\times\boldsymbol{M}$$

where…

$$\boldsymbol{M}(\boldsymbol{r}) = \sum_{\alpha}\int [dp]\boldsymbol{m}_\alpha f_\alpha + \sum_{\alpha}\frac{1}{\beta}\int [d\boldsymbol{p}]\frac{e}{\hbar}\Omega_\alpha \cdot \log(1 + e^{-\beta(\varepsilon_\alpha - \mu)})$$

Total magnetization

$$\boldsymbol{m}_\alpha(\boldsymbol{k}) = -i\frac{e}{2\hbar}\langle \boldsymbol{\nabla}_{\boldsymbol{k}} u_{\alpha\boldsymbol{k}}| \times [\hat{H}(k) - \varepsilon_\alpha(k)] |\boldsymbol{\nabla}_{\boldsymbol{k}} u_{\alpha\boldsymbol{k}}\rangle$$

Orbital magnetic moment

Di Xiao, *et al.*, PRL **97,** 026603 (2006). / N. R. Cooper, *et al.*, PRB **55,** 2344 (1997).

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Correction due to self-rotation of WP *Magnetization current*

where…

$$\boldsymbol{M}(\boldsymbol{r}) = \sum_{\alpha}\int [d\boldsymbol{p}]\boldsymbol{m}_\alpha f_\alpha + \sum_{\alpha}\frac{1}{\beta}\int [d\boldsymbol{p}]\frac{e}{\hbar}\Omega_\alpha \cdot \log\left(1+e^{-\beta(\varepsilon_\alpha-\mu)}\right)$$

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$$\boldsymbol{j}(\boldsymbol{r},t) = (-e)\sum_{\alpha}\int [d\boldsymbol{p}]\,\dot{\boldsymbol{r}}_{c,\alpha}\, f_{\alpha} + \boldsymbol{\nabla}\times\left(\sum_{\alpha}\int [dp]\boldsymbol{m}_{\alpha} f_{\alpha}\right) - \boldsymbol{\nabla}\times\boldsymbol{M}$$

$f_{\alpha} \simeq f_{0\alpha}^{local}$

Hydrodynamic description of electric current

$$\boldsymbol{j}(\boldsymbol{r},t) = -en\boldsymbol{u} - \frac{e}{\hbar}\left[m(e\boldsymbol{E}+\boldsymbol{\nabla}\mu)\times\left({}^{t}\widehat{D}\boldsymbol{u}\right) + \boldsymbol{\nabla}\times\left({}^{t}\hat{C}\boldsymbol{u}\right) + m\left(\frac{\boldsymbol{\nabla}T}{T}\right)\times\left({}^{t}\hat{F}\boldsymbol{u}\right)\right]$$

Result ②： Hydrodynamic description of electric current

Hydrodynamic description of electric current

$$\boldsymbol{j}(\boldsymbol{r},t) = -en\boldsymbol{u} - \frac{e}{\hbar}\left[m(e\boldsymbol{E} + \boldsymbol{\nabla}\mu) \times \left({}^{t}\widehat{D}\boldsymbol{u}\right) + \boldsymbol{\nabla} \times \left({}^{t}\hat{C}\boldsymbol{u}\right) + m\left(\frac{\boldsymbol{\nabla}T}{T}\right) \times \left({}^{t}\hat{F}\boldsymbol{u}\right)\right]$$

Result ②： Hydrodynamic description of electric current

Hydrodynamic description of electric current

$$\boldsymbol{j}(\boldsymbol{r},t) = -en\boldsymbol{u} - \frac{e}{\hbar}\left[m(e\boldsymbol{E}+\boldsymbol{\nabla}\mu)\times\left({}^{t}\widehat{D}\boldsymbol{u}\right)+\boldsymbol{\nabla}\times\left({}^{t}\hat{C}\boldsymbol{u}\right)+m\left(\frac{\boldsymbol{\nabla}T}{T}\right)\times\left({}^{t}\hat{F}\boldsymbol{u}\right)\right]$$

Conventional relation under *Galilean invariance*

Result ②： Hydrodynamic description of electric current

Hydrodynamic description of electric current

$$\boldsymbol{j}(\boldsymbol{r},t) = -en\boldsymbol{u} - \frac{e}{\hbar}\left[m(e\boldsymbol{E} + \boldsymbol{\nabla}\mu) \times ({}^t\widehat{D}\boldsymbol{u}) + \boldsymbol{\nabla} \times ({}^t\hat{C}\boldsymbol{u}) + m\left(\frac{\boldsymbol{\nabla}T}{T}\right) \times ({}^t\hat{F}\boldsymbol{u}) \right]$$

Geometrical coefficients

$$D_{il}^{\alpha} \equiv -\int [d\boldsymbol{p}]\Omega_{\alpha,l}\frac{\partial f_{0\alpha}}{\partial p_i}, \quad C_{il}^{\alpha} \equiv \int [d\boldsymbol{p}]p_i\Omega_{\alpha,l}f_{0\alpha}, \quad F_{il}^{\alpha} \equiv -\int [d\boldsymbol{p}]\varepsilon_{\alpha}\Omega_{\alpha,l}\frac{\partial f_{0\alpha}}{\partial p_i}$$

Result ②： Hydrodynamic description of electric current

Hydrodynamic description of electric current

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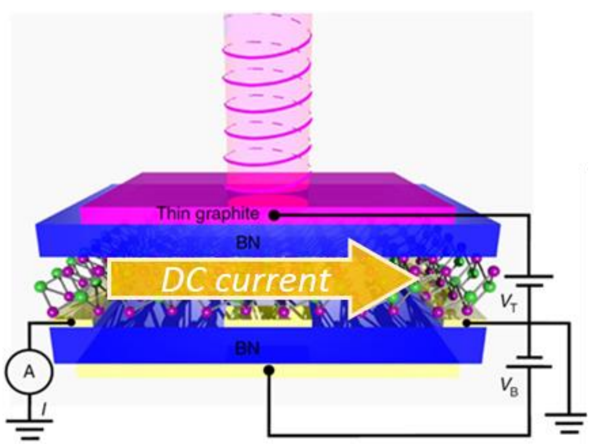
Quantum Nonlinear Hall Effect

$$C_{il}^{\alpha} \equiv \int [d\boldsymbol{p}] p_i \Omega_{\alpha,l} f_{0\alpha}\,, \qquad F_{il}^{\alpha} \equiv -\int [d\boldsymbol{p}] \varepsilon_{\alpha} \Omega_{\alpha,l} \frac{\partial f_{0\alpha}}{\partial p_i}$$



I. Sodemann and Liang Fu, PRL **115**, 216806 (2015).
Su-Yang Xu, *et al.*, Nat. Phys. 14, 900 (2018)

Result ②： Hydrodynamic description of electric current

Hydrodynamic description of electric current

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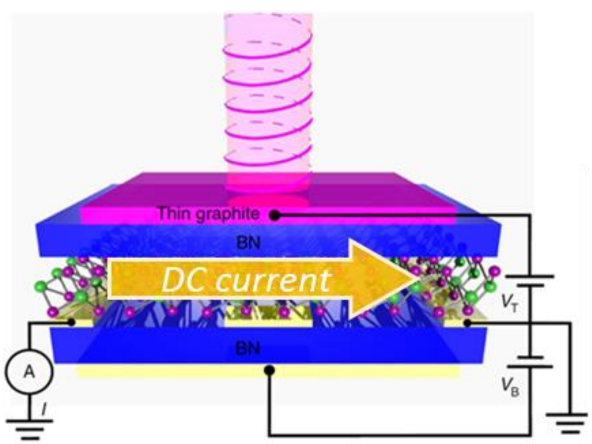
Quantum Nonlinear Hall Effect *Generalized Vortical Effect*

$$F_{il}^{\alpha} \equiv -\int [d\boldsymbol{p}]\varepsilon_{\alpha}\Omega_{\alpha,l}\frac{\partial f_{0\alpha}}{\partial p_i}$$



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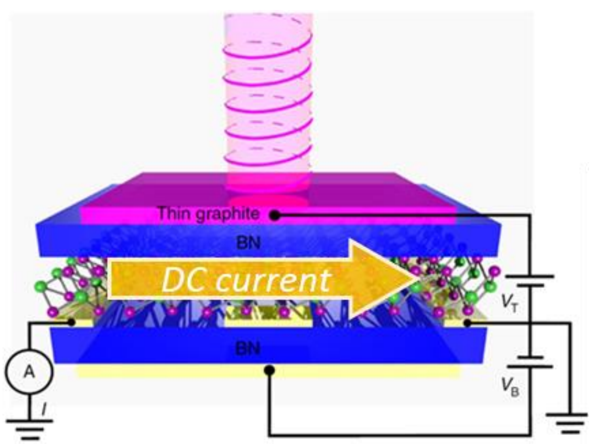
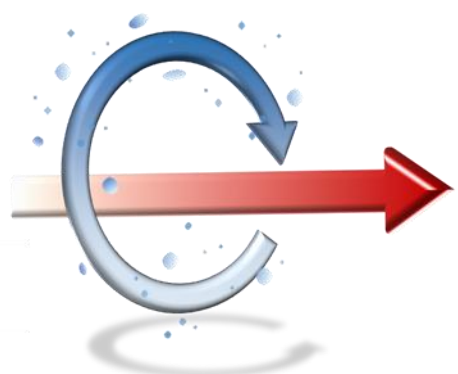
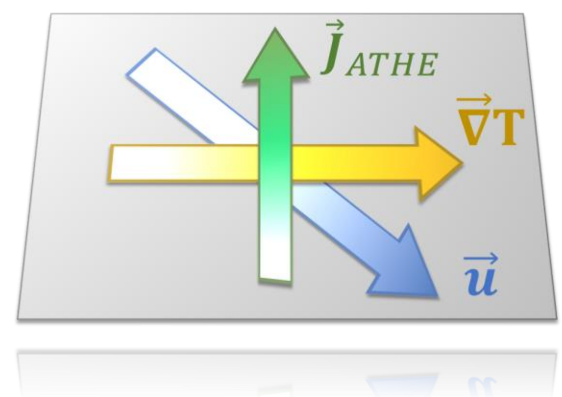
Quantum Nonlinear Hall Effect — *Generalized Vortical Effect* — *Current-induced anomalous Thermal Hall effect*







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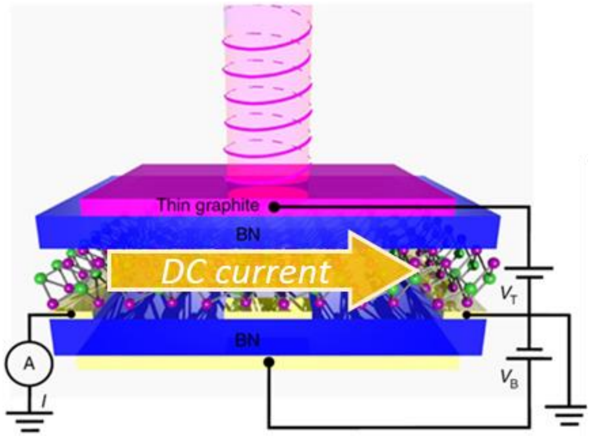
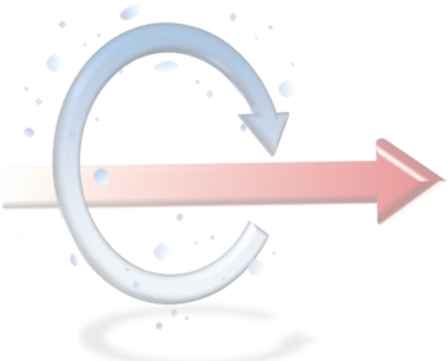
Quantum Nonlinear Hall Effect *Generalized Vortical Effect* *Current-induced anomalous Thermal Hall effect*





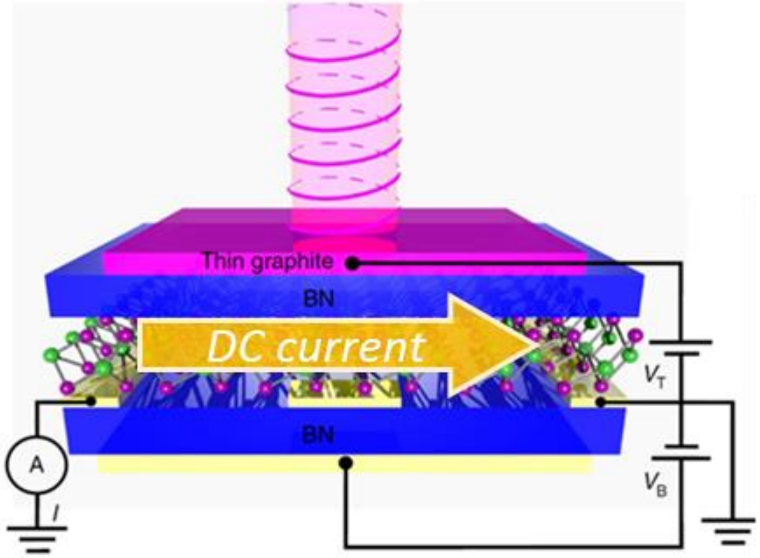
I. Sodemann and Liang Fu, PRL **115**, 216806 (2015).
Su-Yang Xu, *et al.*, Nat. Phys. 14, 900 (2018)

Result ③： Quantum Nonlinear Hall effect

<u>Assumption</u>

① Apply a **spatially uniform AC electric field** : $\boldsymbol{E} = \mathrm{Re}[\tilde{\boldsymbol{E}} e^{i\omega t}]$

② Consider the optical response **up to the <u>second order</u> of $\boldsymbol{E}$**



I. Sodemann and Liang Fu, PRL **115**, 216806 (2015).
Su-Yang Xu, *et al.*, Nat. Phys. 14, 900 (2018)

<u>The solution of Hydro. Eq.</u>

$$\boldsymbol{u}(t) = \mathrm{Re}\left[-\frac{e\tau_{mr}}{m(1+i\omega\tau_{mr})}\tilde{\boldsymbol{E}}\, e^{i\omega t}\right]$$

Result ③： Quantum Nonlinear Hall effect

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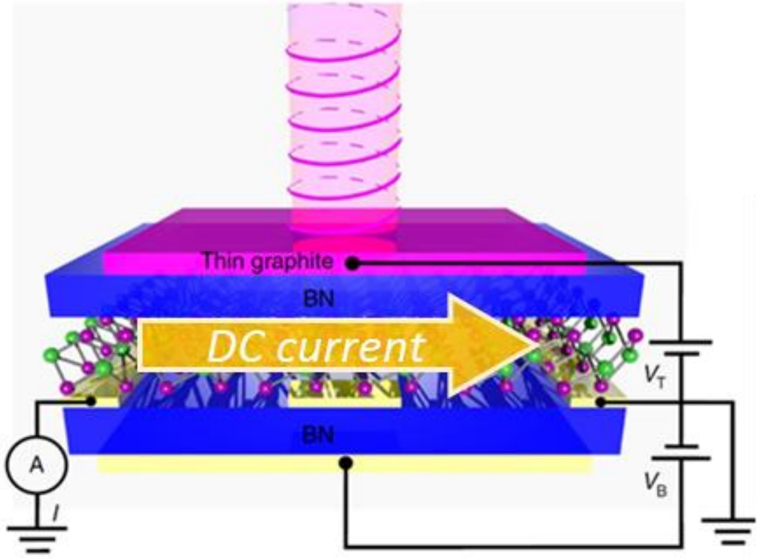

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$$\boldsymbol{u}(t) = \mathrm{Re}\left[-\frac{e\tau_{mr}}{m(1+i\omega\tau_{mr})}\tilde{\boldsymbol{E}}\, e^{i\omega t}\right]$$

$$j_i = Re[j_i^0 + j_i^{\omega} e^{i\omega t} + j_i^{2\omega} e^{2i\omega t}]$$

$$j_i^{\omega} = \sigma^{(1)}\tilde{E}_i, \quad j_i^0 = \sigma_{ijk}^{(2)}\tilde{E}_j\tilde{E}_k^* \quad j_i^{2\omega} = \sigma_{ijk}^{(2)}\tilde{E}_j\tilde{E}_k$$

$$\sigma^{(1)} = \frac{\sigma_D}{1+i\omega\tau_{mr}}$$

The Drude conductivity

Berry curvature dipole

$$\sigma_{ijk}^{(2)} = \epsilon_{ilk}\frac{e^3\tau_{mr}}{2(1+i\omega\tau_{mr})}\boldsymbol{D_{jl}}$$

Quantum Nonlinear Hall effect

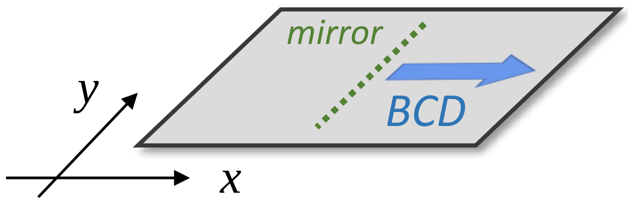
Result ③： Quantum Nonlinear Hall effect

Berry curvature dipole

Quantum Nonlinear Hall effect : $j_i^0 = \sigma_{ijk}^{(2)} \tilde{E}_j \tilde{E}_k^*$ $\quad \sigma_{ijk}^{(2)} = \epsilon_{ilk} \dfrac{e^3 \tau_{mr}}{2(1 + i\omega\tau_{mr})} \boldsymbol{D_{jl}}$

e.g.) 2D systems : $D_{jl} \propto \delta_{jx}\delta_{lz}$

J. E. Moore and J. Orenstein, PRL **105**, 026805 (2010)



$$Re[\, \boldsymbol{j}^{(\mathbf{0})}] \propto \frac{1}{1/\tau^2 + \omega^2}\left[\, i\omega\left(E_x E_y^* - E_y E_x^*\right)\hat{\boldsymbol{x}} + 1/\tau\left(E_x E_y^* + E_y E_x^*\right)\hat{\boldsymbol{x}} + |E_x|^2\,\hat{\boldsymbol{y}}\,\right]$$

Circular Photogalvanic

- Maximal for *circular* polarization
- Change the sign with helicity
- Allowed in *chiral* groups

Linear Photogalvanic

- Maximal for *linear* polarization
- Change the sign with direction
- Allowed in *noncentrosymmetric* materials

"Typical" Photogalvanic

- Independent on polarization
- Not change the sign

Result ③：Quantum Nonlinear Hall effect

$$Re\left[\boldsymbol{j}^{(0)}\right] \propto \frac{1}{1/\tau^2+\omega^2}\left[\, i\omega\left(E_xE_y^*-E_yE_x^*\right)\hat{\boldsymbol{x}} + 1/\tau\left(E_xE_y^*+E_yE_x^*\right)\hat{\boldsymbol{x}} + |E_x|^2\,\hat{\boldsymbol{y}}\,\right]$$

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- Maximal for *circular* polarization
- Change the sign with helicity
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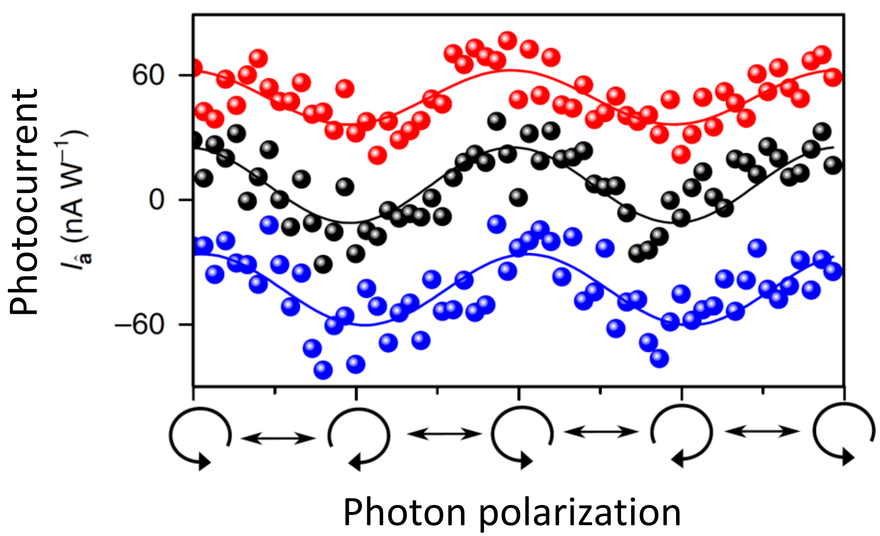
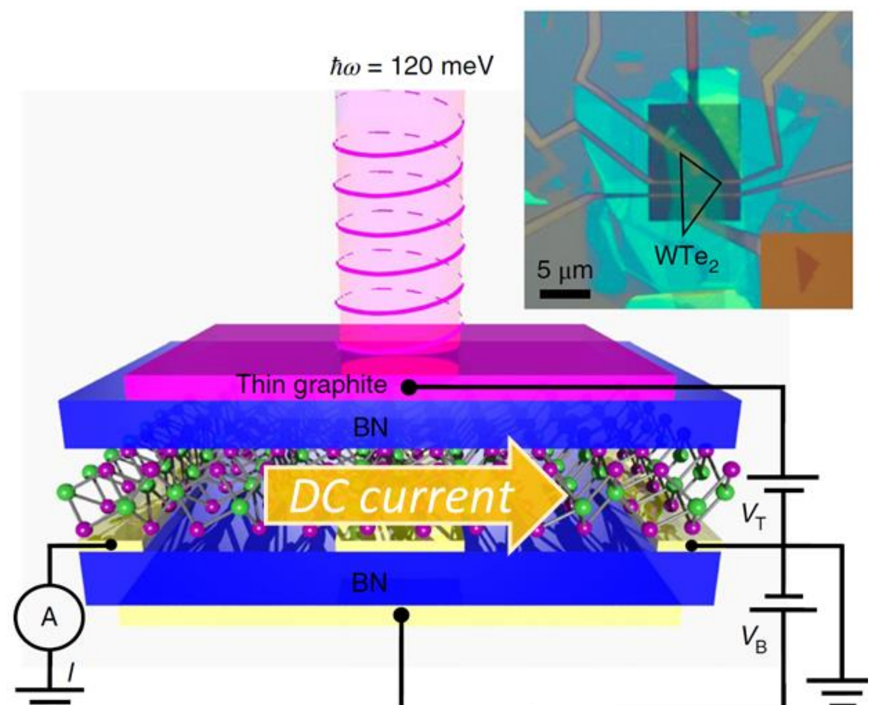
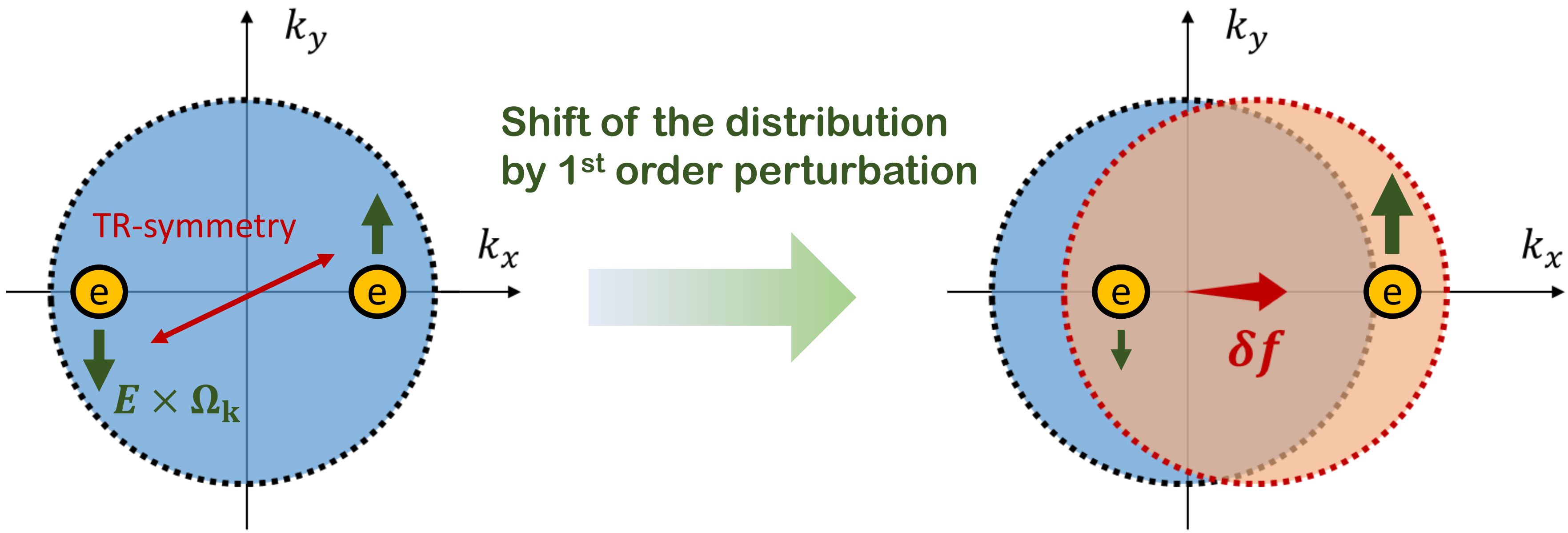




Su-Yang Xu, *et al.*, Nat. Phys. **14**, 900 (2018)

Result ③： Quantum Nonlinear Hall effect



equilibrium distribution

Compensation of anomalous currents due to TR symmetry!!

~~**Anomalous Hall effect**~~

Steady distribution under E

Net anomalous currents emerge by breaking of TR-symmetry!!

Quantum Nonlinear Hall effect

Result ④： Generalized Vortical Effect

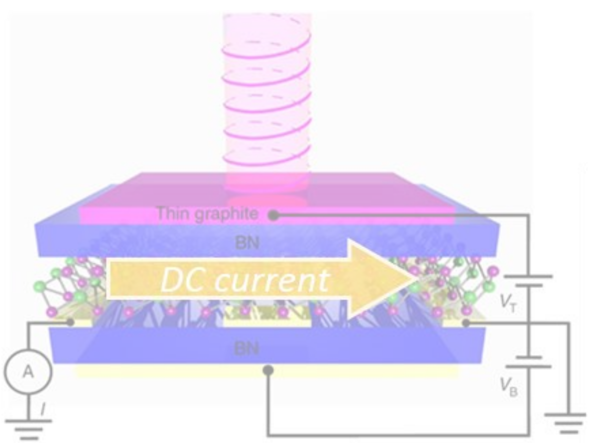
Hydrodynamic description of electric current

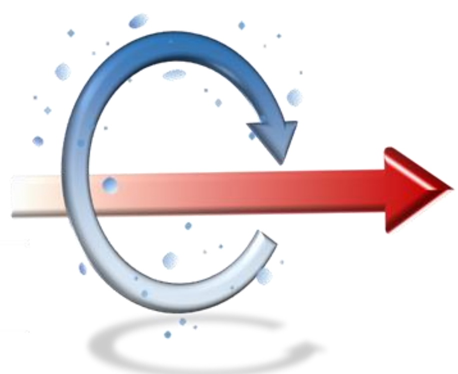
$$\boldsymbol{j}(\boldsymbol{r},t) = -en\boldsymbol{u} - \frac{e}{\hbar}\left[m(e\boldsymbol{E} + \boldsymbol{\nabla}\mu) \times ({}^t\hat{D}\boldsymbol{u}) + \boldsymbol{\nabla} \times ({}^t\hat{C}\boldsymbol{u}) + m\left(\frac{\boldsymbol{\nabla}T}{T}\right) \times ({}^t\hat{F}\boldsymbol{u}) \right]$$

Quantum Nonlinear Hall Effect

Generalized Vortical Effect

Current-induced anomalous Thermal Hall effect





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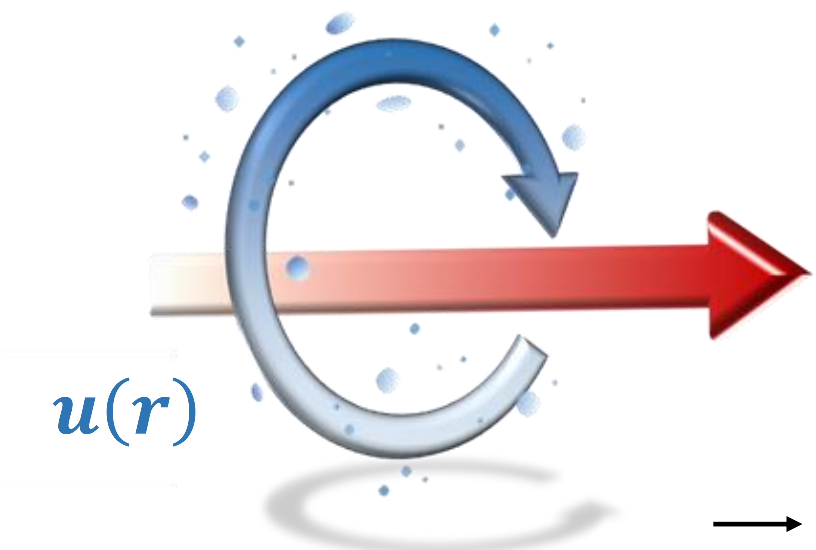
Result ④： Generalized Vortical Effect

Hydrodynamic description of electric current

$$\boldsymbol{j}(\boldsymbol{r},t) = -en\boldsymbol{u} - \frac{e}{\hbar}\left[m(e\boldsymbol{E} + \boldsymbol{\nabla}\mu) \times \left({}^t\hat{D}\boldsymbol{u}\right) + \boldsymbol{\nabla} \times \left({}^t\hat{C}\boldsymbol{u}\right) + m\left(\frac{\boldsymbol{\nabla}T}{T}\right) \times \left({}^t\hat{F}\boldsymbol{u}\right)\right]$$

Generalized Vortical Effect

= *velocity-gradient* induced anomalous current



$$\boldsymbol{j}_{GVE} \propto \boldsymbol{C}(\boldsymbol{\nabla} \times \boldsymbol{u})$$

$\equiv \boldsymbol{\omega}$: **vorticity**

⟶ *Realization of* ***Chiral vortical effect*** *in crystals!?*

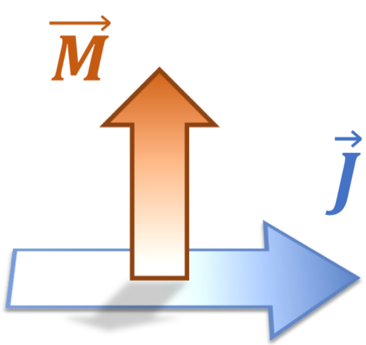
Result ④： Generalized Vortical Effect

Hydrodynamic description of electric current

$$\boldsymbol{j}(\boldsymbol{r},t) = -en\boldsymbol{u} - \frac{e}{\hbar}\left[m(e\boldsymbol{E} + \boldsymbol{\nabla}\mu) \times ({}^{t}\hat{D}\boldsymbol{u}) + \boldsymbol{\nabla} \times ({}^{t}\hat{C}\boldsymbol{u}) + m\left(\frac{\boldsymbol{\nabla}T}{T}\right) \times ({}^{t}\hat{F}\boldsymbol{u})\right]$$

An intuitive understanding

Orbital Edelstein effect



Current ⇒
Orbital Magnetization

Taiki Yoda, Takehito Yokoyama, Shuichi Murakami, Scientific Reports, **5**, 12024 (2015)

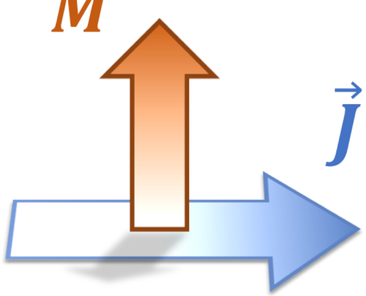
Result ④： Generalized Vortical Effect

Hydrodynamic description of electric current

$$\boldsymbol{j}(\boldsymbol{r},t) = -en\boldsymbol{u} - \frac{e}{\hbar}\left[m(e\boldsymbol{E}+\boldsymbol{\nabla}\mu)\times\left({}^t\hat{D}\boldsymbol{u}\right) + \boldsymbol{\nabla}\times\left({}^t\hat{C}\boldsymbol{u}\right) + m\left(\frac{\boldsymbol{\nabla}T}{T}\right)\times\left({}^t\hat{F}\boldsymbol{u}\right)\right]$$

An intuitive understanding

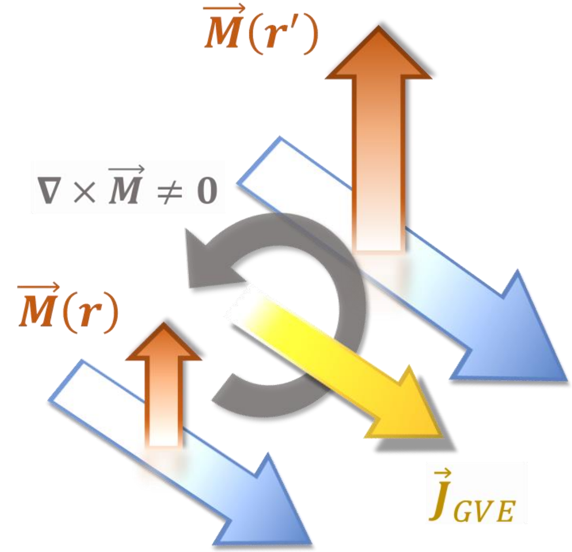
Orbital Edelstein effect



Current ⇒ Orbital Magnetization

Generalized vortical effect

nonuniform current
➡ nonuniform $\boldsymbol{M}(\boldsymbol{r})$
➡ magnetization current



Result ④： Generalized Vortical Effect

Hydrodynamic description of electric current

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3D system

$C_{ij} = C\delta_{ij}$

$$\boldsymbol{j} \propto C\boldsymbol{\omega}$$

$\hat{C} = diag(C, C, -2C)$

$$j_z \propto C\omega_z$$

$$j_x \propto -\frac{C}{2}[\omega_x + 6\lambda_{yz}]$$

$$j_y \propto -\frac{C}{2}[\omega_y + 6\lambda_{zx}]$$

$(\lambda_{ij} \equiv [\partial_i u_j + \partial_j u_i]/2 : strain\ rate\ tensor)$

2D system

$C_{ij} = C\delta_{ix}\delta_{jz}$

$$j_x \propto C\partial_y u_x$$

$$j_y \propto -C\partial_x u_x$$

Result ④： Generalized Vortical Effect

Hydrodynamic description of electric current

$$\boldsymbol{j}(\boldsymbol{r},t) = -en\boldsymbol{u} - \frac{e}{\hbar}\left[m(e\boldsymbol{E} + \boldsymbol{\nabla}\mu) \times ({}^t\hat{D}\boldsymbol{u}) + \boldsymbol{\nabla} \times ({}^t\hat{C}\boldsymbol{u}) + m\left(\frac{\boldsymbol{\nabla}T}{T}\right) \times ({}^t\hat{F}\boldsymbol{u})\right]$$

Constraint beyond the symmetry consideration

$$C \equiv \mathrm{Tr}[\hat{C}] = \int [d\boldsymbol{p}] p_i \Omega_{\alpha,i} f_{0\alpha}$$

Here, we used the assumption of *parabolic dispersion*

$$\propto \int [d\boldsymbol{p}] \frac{\partial \varepsilon}{\partial p_i} \Omega_{\alpha,i} f_{0\alpha}$$

$$= \int d\varepsilon\, f_{0\alpha}(\varepsilon) \oint_{\varepsilon_\alpha(\boldsymbol{k})=\varepsilon} (d\boldsymbol{S} \cdot \Omega_\alpha(\boldsymbol{k}))$$

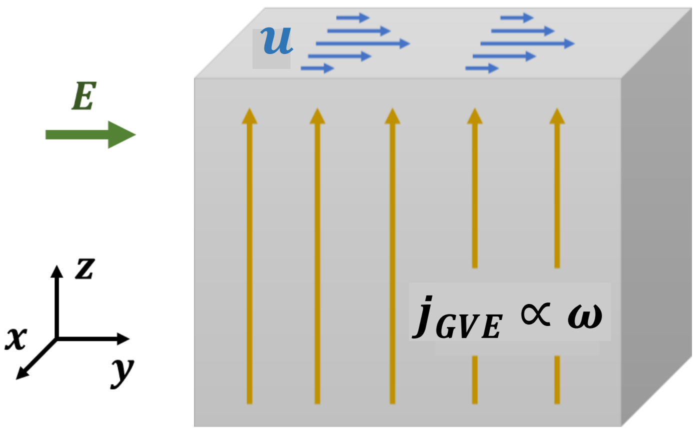
$$= \text{(total monopole charge in k-space)} = \mathbf{0}$$

Trace component of $\hat{C}$ is always zero even in chiral crystals*!!*

Result ④： Asymmetric Poiseuille flow

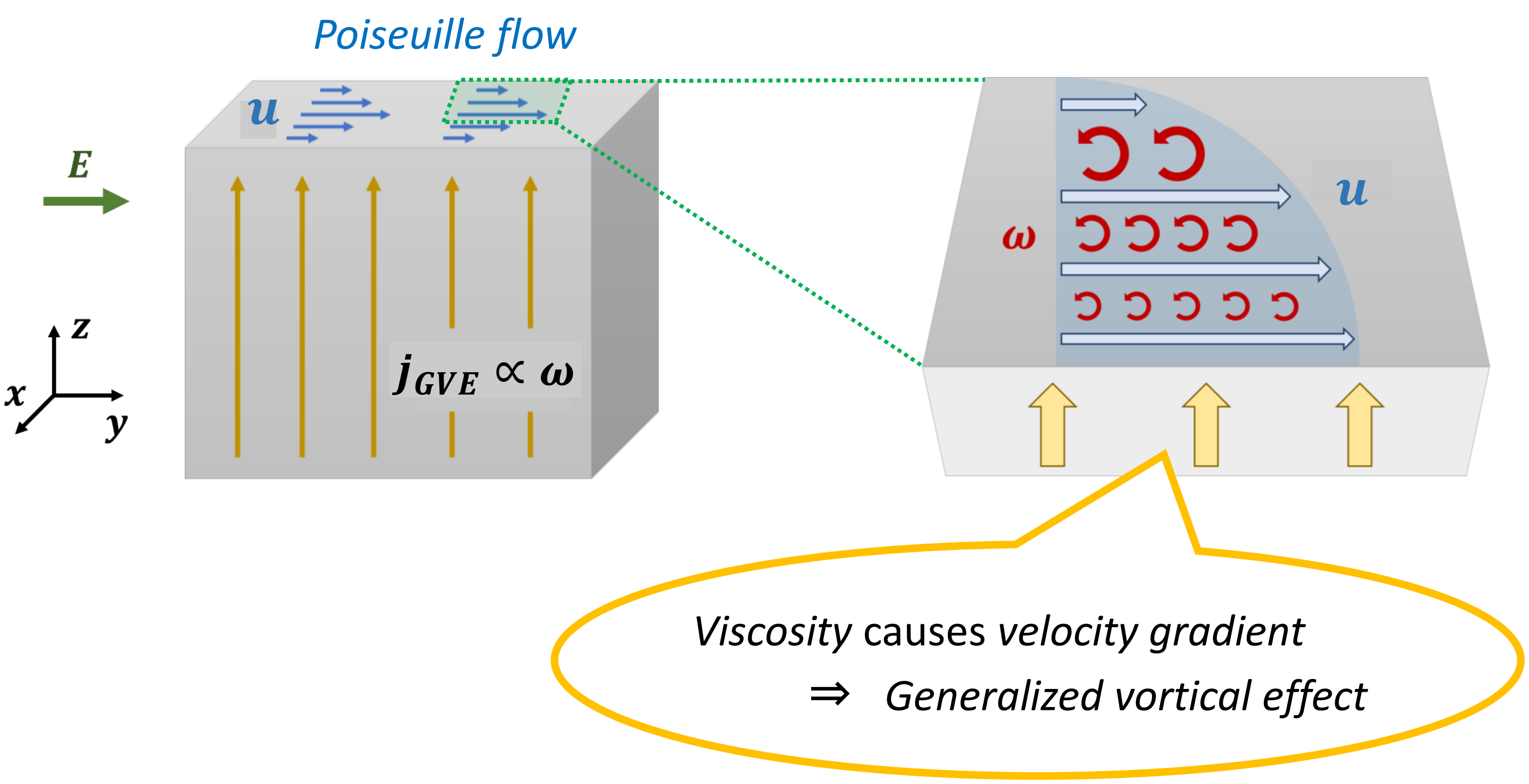
Application　*Asymmetric Poiseuille flow* and *Anomalous edge current*

Poiseuille flow



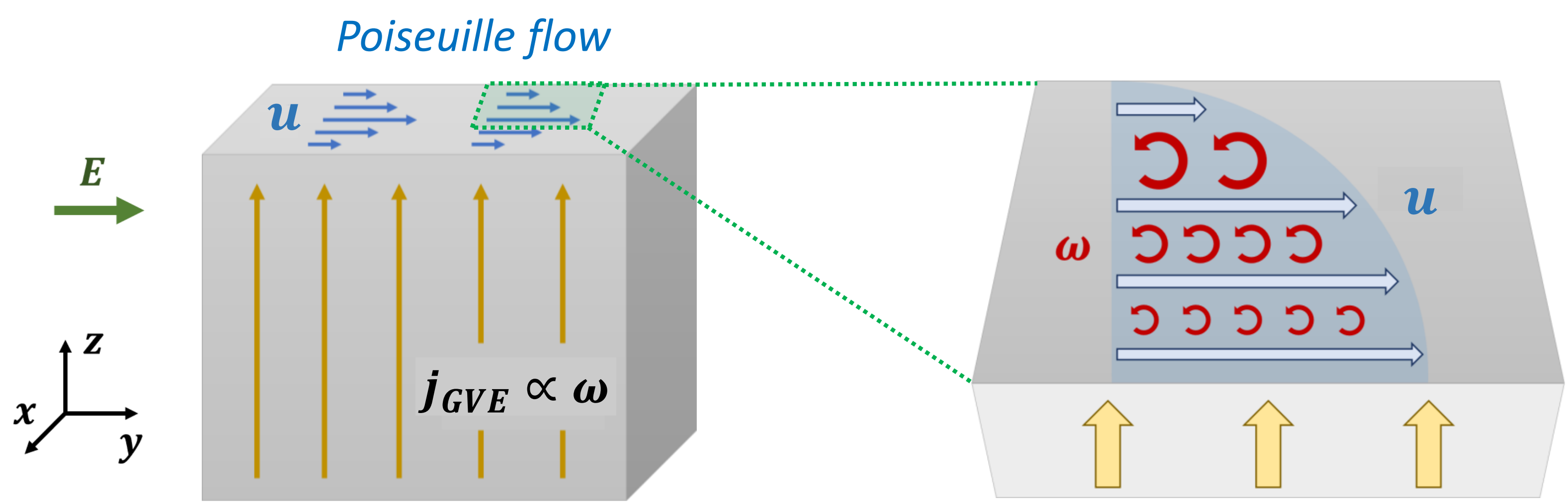
Result ④： Asymmetric Poiseuille flow

<u>Application</u> *Asymmetric Poiseuille flow* and *Anomalous edge current*



Result ④： Asymmetric Poiseuille flow

Application　*Asymmetric Poiseuille flow* and *Anomalous edge current*

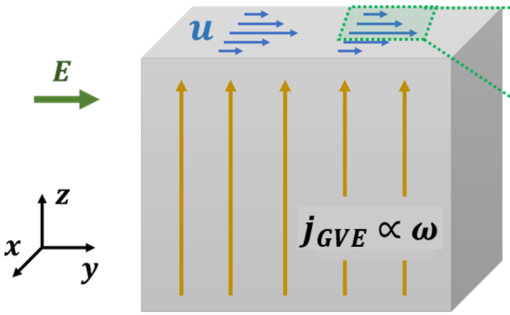


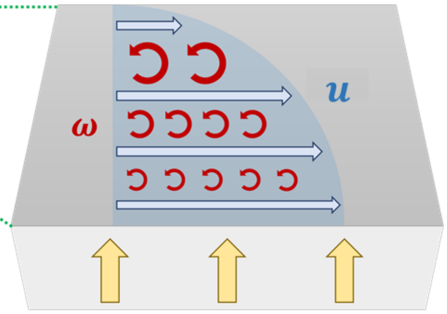
The solution of Hydro. Eq.

$$u_y(x) = \frac{e\tau_{mr}E}{m}\left[1 - \frac{\cosh(x/l)}{\cosh(w/2l)}\right] \longrightarrow \begin{cases} J_y = -enu_y + \frac{e}{\hbar}C_{zy}\omega_z \\ J_z = -\frac{e}{\hbar}C_{yy}\omega_z \end{cases}$$

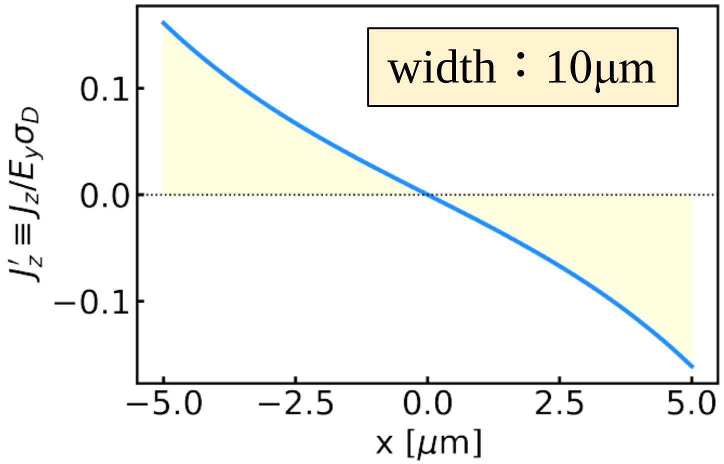
Result ④： Asymmetric Poiseuille flow

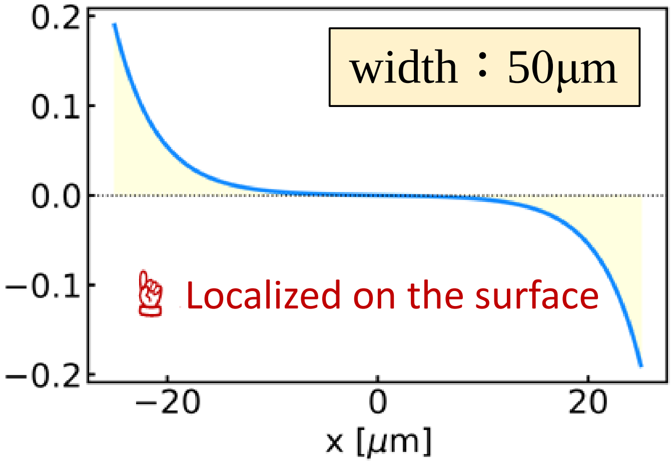
Application　*Asymmetric Poiseuille flow* and *Anomalous edge current*





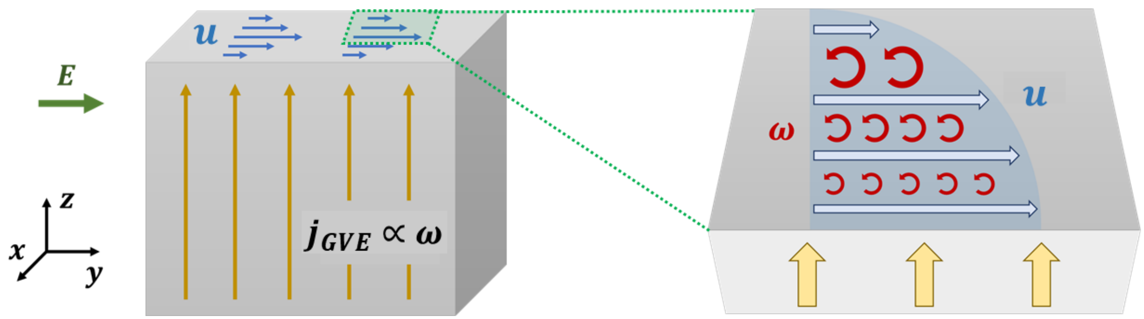
$$J_z = -\frac{e}{\hbar} C_{yy}\omega_z$$





Result ④： Asymmetric Poiseuille flow

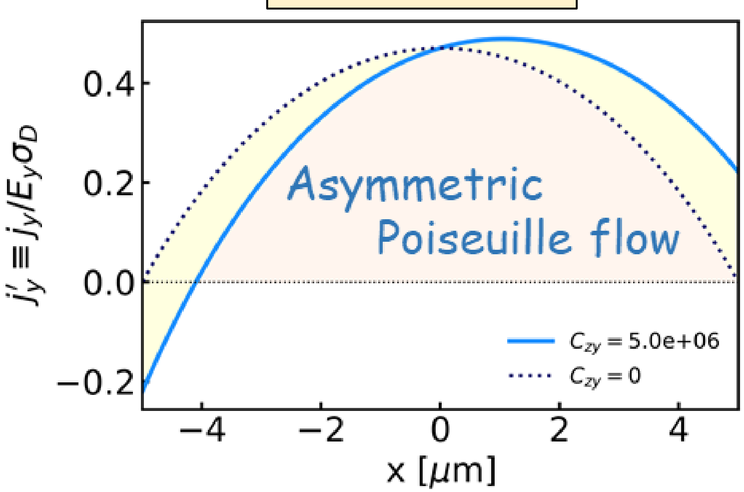
Application　*Asymmetric Poiseuille flow* and *Anomalous edge current*

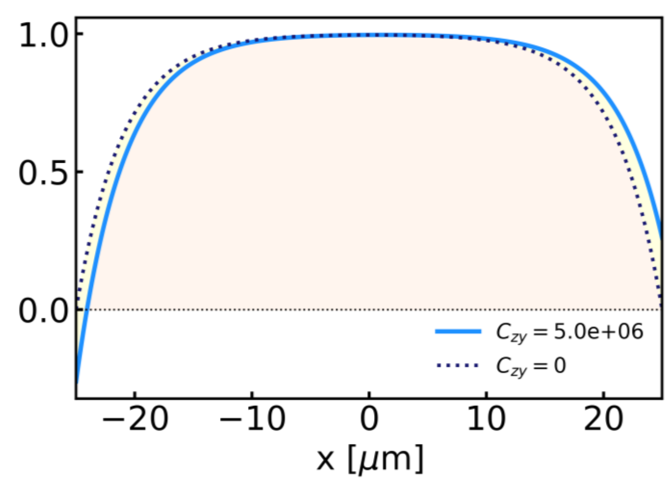


$$J_y = -enu_y + \frac{e}{\hbar} C_{zy}\omega_z$$

width：10μm

width：50μm





Result ⑤： Crystal symmetry and Realization

Geometrical coefficients $\hat{C}, \hat{F}, \hat{D}$

- Asymmetric component C_n, C_{nv}
- Symmetric component $C_n, D_n, T, O, C_{1v}, C_{2v}, S_4$
- Scalar component C_n, D_n, T, O

3D systems: MoP, WP_2, $PdCoO_2$

2D systems: (110)-GaAs, ML-Graphene, BL-Graphene, $ML\text{-}WTe_2$, $ML\text{-}MoS_2$

Material	point group	components	operations
MoP	D_{3h}	NO	—
WP_2	C_{2v}	A_{xy}, A_{yx}	—
$PdCoO_2$	D_{3d}	NO	—
(110)-GaAs	C_s	$\boldsymbol{A} \parallel \boldsymbol{m}$	—
ML-Graphene	D_{6h}	NO	—
BL-Graphene	$D_{3h} \to C_s$	$\boldsymbol{A} \parallel \boldsymbol{m}$	uniaxial strain
$ML\text{-}WTe_2$	C_s	$\boldsymbol{A} \parallel \boldsymbol{m}$	—
$ML\text{-}MoS_2$	$D_{3h} \to C_s$	$\boldsymbol{A} \parallel \boldsymbol{m}$	uniaxial strain

Result ⑤： Crystal symmetry and Realization

3D systems

2D systems

Material	point group	components	operations
MoP	D_{3h}	NO	—
WP_2	C_{2v}	A_{xy}, A_{yx}	—
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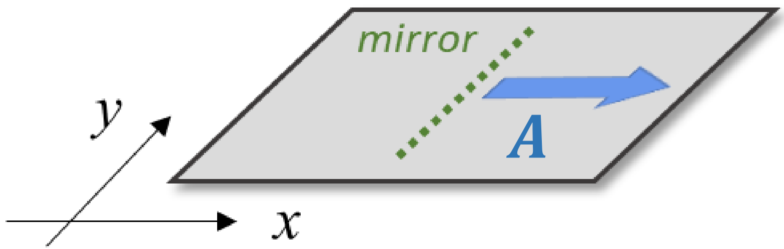
ML-WTe_2	C_s	$\boldsymbol{A} \parallel \boldsymbol{m}$	—
ML-MoS_2	$D_{3h} \to C_s$	$\boldsymbol{A} \parallel \boldsymbol{m}$	uniaxial strain

e.g.) 2D systems



These tensors become pseudo-vectors in the plane :

$$\hat{D} \to \boldsymbol{D}, \hat{C} \to \boldsymbol{C}, \hat{F} \to \boldsymbol{F}$$

→ Perpendicular to the mirror line

Result ⑤： Crystal symmetry and Realization

3D systems

2D systems

Material	point group	components	operations
MoP	D_{3h}	NO	—
WP_2	C_{2v}	A_{xy}, A_{yx}	—
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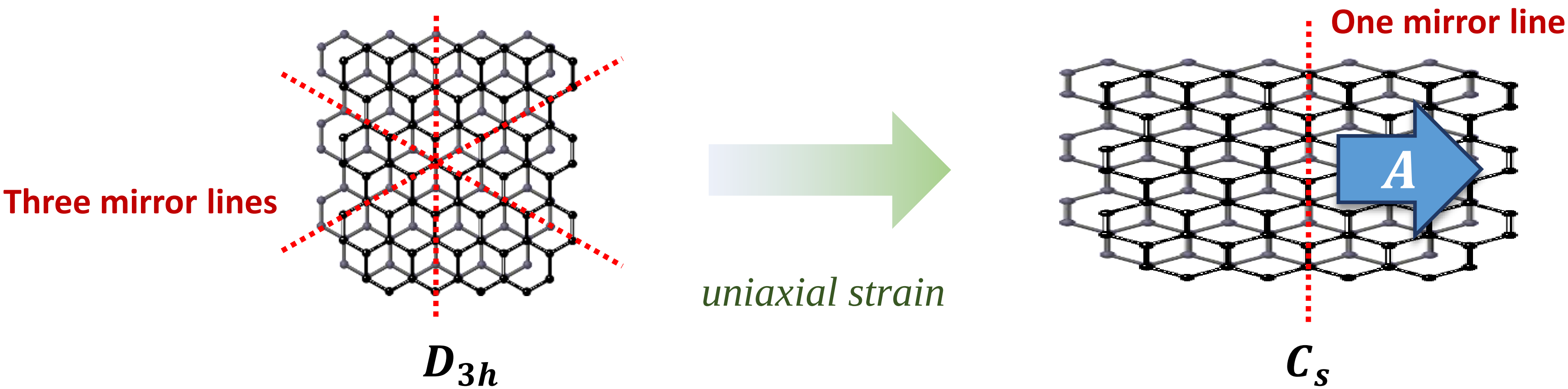
$ML-WTe_2$	C_s	$\boldsymbol{A} \parallel \boldsymbol{m}$	—
$ML-MoS_2$	$D_{3h} \to C_s$	$\boldsymbol{A} \parallel \boldsymbol{m}$	uniaxial strain

e.g.) Bilayer Graphene



Summary

Result 1

Formulation of the hydrodynamic theory

for **noncentrosymmetric** hydrodynamic materials

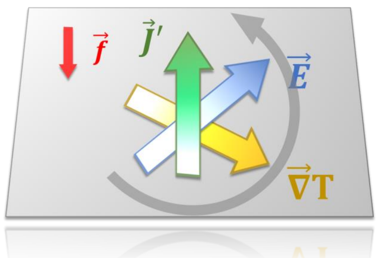
$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} + \frac{\boldsymbol{\nabla}p}{\rho} + \frac{e}{mn\hbar}\left[\frac{1}{m}\hat{C}(\boldsymbol{\nabla}\times\boldsymbol{E}) + \hat{F}\left(\boldsymbol{E}\times\frac{\boldsymbol{\nabla}T}{T}\right) + \hat{D}(\boldsymbol{E}\times\boldsymbol{\nabla}\mu)\right] + \frac{e}{m}\boldsymbol{E} = -\frac{\boldsymbol{u}}{\tau_{mr}}$$

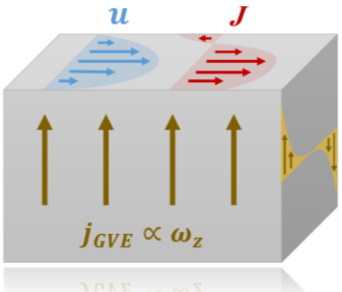
$$\boldsymbol{j}(\boldsymbol{r},t) = -en\boldsymbol{u} - \frac{e}{\hbar}\left[m(e\boldsymbol{E}+\boldsymbol{\nabla}\mu)\times({}^{t}\hat{D}\boldsymbol{u}) + \boldsymbol{\nabla}\times({}^{t}\hat{C}\boldsymbol{u}) + m\left(\frac{\boldsymbol{\nabla}T}{T}\right)\times({}^{t}\hat{F}\boldsymbol{u})\right]$$

Result 2

Prediction of a variety of **novel anomalous transport**

- Current-induced anomalous thermal Hall effect
- Generalized vortical effect, asymmetric Poiseuille flow, *etc.*





Supplemental Materials

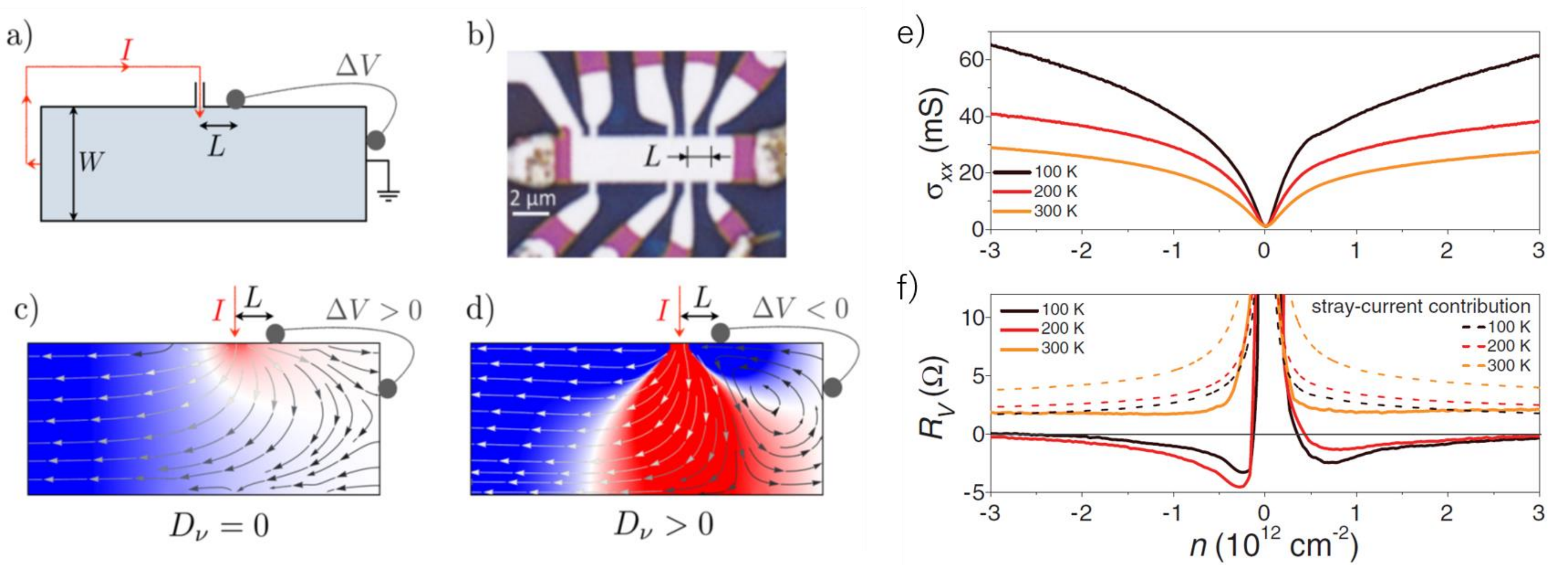
Parameters in some hydrodynamic materials

Materials	v_F [10^6m/s]	l_{mc} [μm]	τ_{mc} [ps]	l_{mr} [μm]	τ_{mr} [ps]
PdCoO2	7.5	2	2~3	20	20~30
Graphene	~1 [2.3]	0.1 - 0.3	0.1 – 0.3	<1	1
WP2		0.45	~1	100	~400
GaAs	0.33~0.41	1.4~2.8	3~5 (2~40K)	20~40	80~90

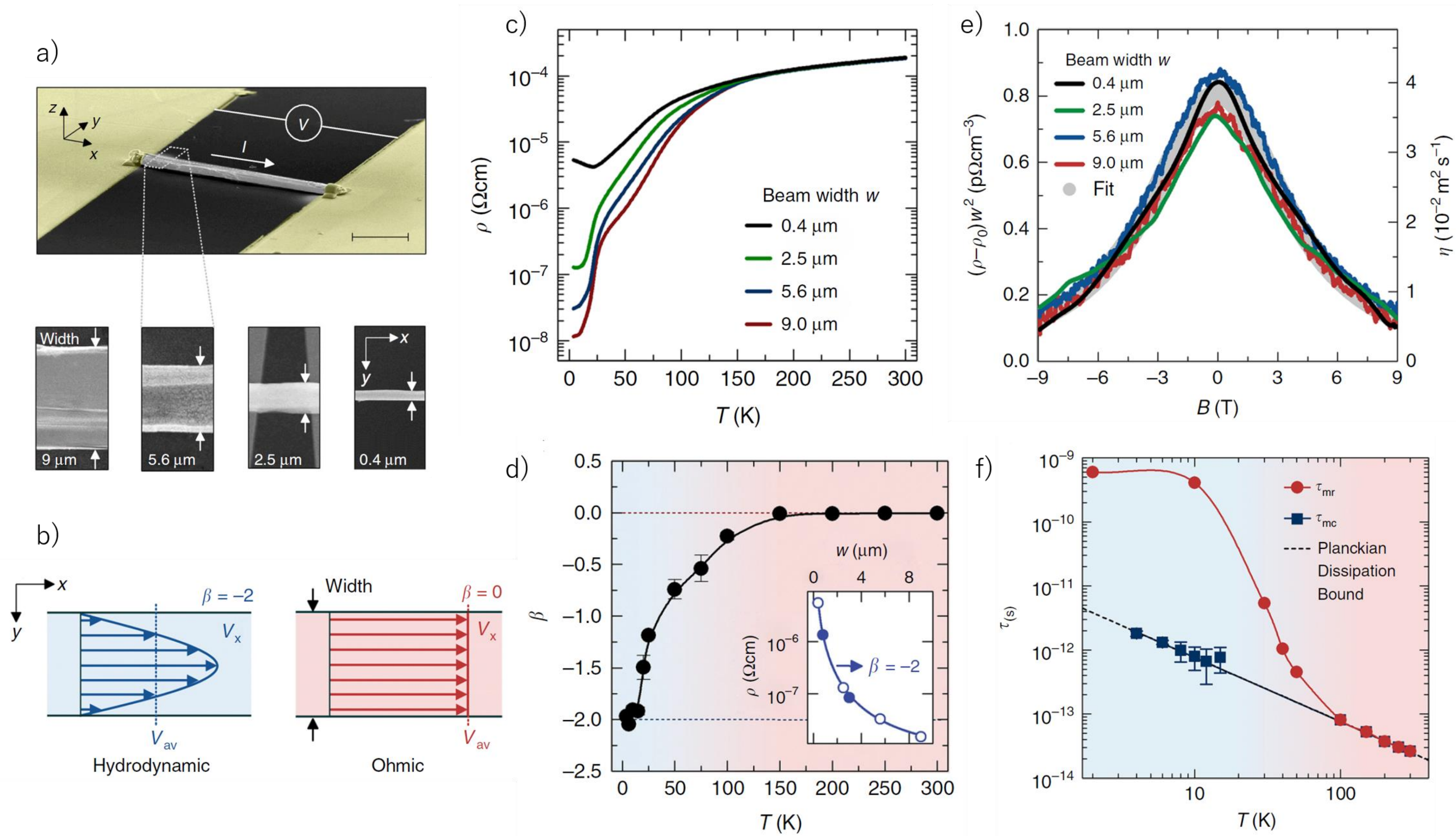
Materials	$T(K)$	$\nu\ (m^2/s)$	n
PdCoO2	2.0	3.0×10^{-2}くらい	$2.45 \times 10^{22} cm^{-3}$
graphene	100~200	1.0×10^{-1}	
WP2	4(~20)	3.8×10^{-2}	$2.9 \times 10^{21} cm^{-3}$
GaAs	1.4(~50?)	$1.2{\sim}3.0 \times 10^{-1}$	$6{\sim}9.1 \times 10^{11} cm^{-2}$

Nonlocal transport phenomena in Graphene

D. A. Bandurin et al, Science 351, 6277 (2016).



Observation of the Poiseuille flow in WP_2



Question ?

How and Why are the hydro. regime different from the conventional regime ??

Question

How and Why are the hydro. regime different from the conventional regime ??

Hint

Continuity equation of electron Momentum

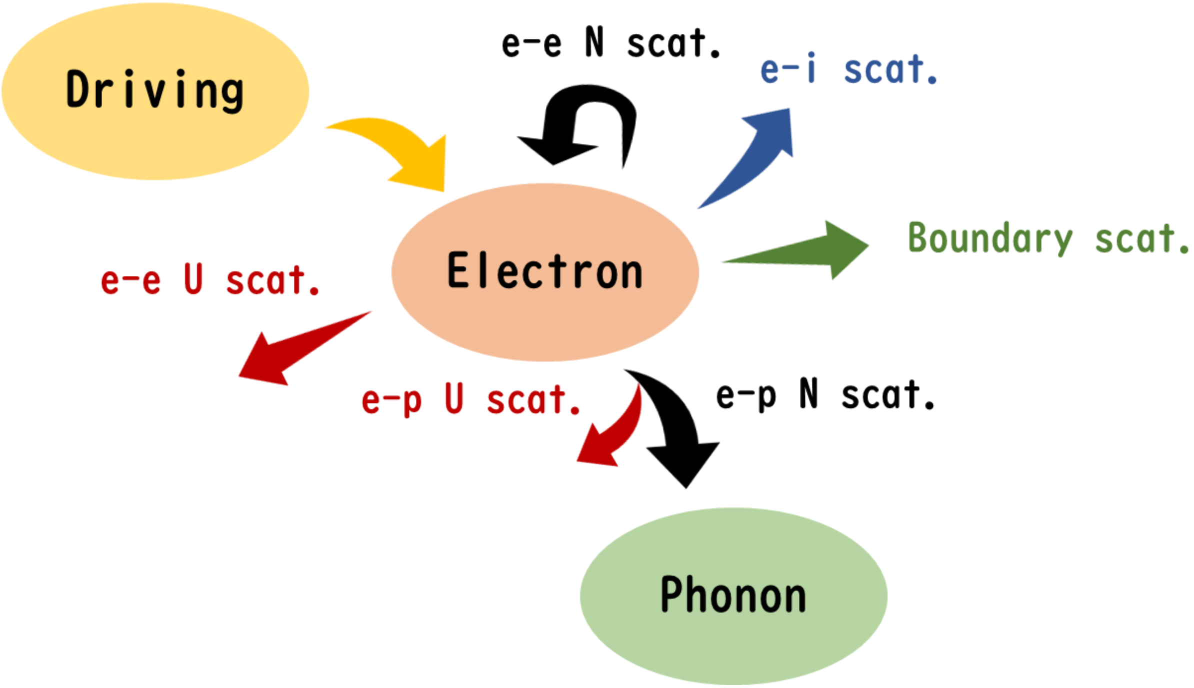
Continuity equation of Electron Momentum

General case :

$$\frac{\partial P_i}{\partial t} + \nabla_j \Pi_{ij} = qE_i - \frac{P_i}{\tau_{mr}}$$

Advection/Diffusion **Driving force**

Loss due to the translational symmetry breaking



Continuity equation of Electron Momentum

General case :

$$\frac{\partial P_i}{\partial t} + \nabla_j \Pi_{ij} = qE_i - \frac{P_i}{\tau_{mr}}$$

Advection/Diffusion **Driving force** **Loss** due to the translational symmetry breaking

classify

Ohmic regime :

$$\frac{\partial P_i}{\partial t} = qE_i - \frac{P_i}{\tau_{mr}}$$

Loss or friction

$l_{mr} \ll l_{mc}, W$

- ✓ **Friction** is **Dominant**!! $\Rightarrow$ **Local** transport
- ✓ Continuity eq. of $\boldsymbol{P}$ = **Drude eq.**

Continuity equation of Electron Momentum

General case :

$$\frac{\partial P_i}{\partial t} + \nabla_j \Pi_{ij} = qE_i - \frac{P_i}{\tau_{mr}}$$

Advection/Diffusion **Driving force**

Loss due to the translational symmetry breaking

classify

Hydrodynamic regime :

$$\frac{\partial P_i}{\partial t} = -\nabla_j \Pi_{ij} + qE_i - \frac{P_i}{\tau_{mr}}$$

Advection/Diffusion

$l_{mc} \ll l_{mr}, W$

Advection/Diffusion is important!! ⇒ ***Nonlocal*** transport

✓ Momentum flux can be expanded by gradient **∇** *!!*

Continuity eq. = **Hydrodynamic equations**

Continuity equation of Electron Momentum

Hydrodynamic regime :

$$\frac{\partial P_i}{\partial t} = -\nabla_j \Pi_{ij} + qE_i - \frac{P_i}{\tau_{mr}}$$

Nonlinear term	P-gradient term	Viscosity term
$(\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{v}$	$\boldsymbol{\nabla} P$	$\eta_{ijkl}\partial_j\partial_k v_l$

reversible (nonlinear term and P-gradient term)

irreversible (viscosity term)

Continuity equation of Electron Momentum

Hydrodynamic regime :

$$\frac{\partial P_i}{\partial t} = \underline{-\nabla_j \Pi_{ij}} + qE_i - \frac{P_i}{\tau_{mr}}$$

Nonlinear term	P-gradient term	Viscosity term
$(\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{v}$	$\boldsymbol{\nabla} P$	$\eta_{ijkl}\partial_j \partial_k v_l$

Nonlinear term and P-gradient term: *reversible*

Viscosity term: *irreversible*

- ***Nonlinear*** response
- ***Instability***
- ***Turbulence***

Continuity equation of Electron Momentum

Hydrodynamic regime :

$$\frac{\partial P_i}{\partial t} = \underline{-\nabla_j \Pi_{ij}} + qE_i - \frac{P_i}{\tau_{mr}}$$

Nonlinear term	P-gradient term	Viscosity term
$(\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{v}$	$\boldsymbol{\nabla} P$	$\eta_{ijkl}\partial_j \partial_k v_l$
reversible	reversible	irreversible

- ***Nonlinear*** response
- ***Instability***
- ***Turbulence***

- Coupling between density and momentum
- Dispersion relation of collective mode

Continuity equation of Electron Momentum

Hydrodynamic regime :

$$\frac{\partial P_i}{\partial t} = \underline{-\nabla_j \Pi_{ij}} + qE_i - \frac{P_i}{\tau_{mr}}$$

Nonlinear term	P-gradient term	Viscosity term
$(\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{v}$	$\boldsymbol{\nabla} P$	$\eta_{ijkl}\partial_j\partial_k v_l$

reversible (Nonlinear term, P-gradient term) — *irreversible* (Viscosity term)

- ***Nonlinear*** response
- ***Instability***
- ***Turbulence***

- Coupling between density and momentum
- Dispersion relation of collective mode

- ***Non-local*** linear response
- ***Finite size effect***
- Lifetime of collective mode

Continuity equation of Electron Momentum

Hydrodynamic regime :

$$\frac{\partial P_i}{\partial t} = \underline{-\nabla_j \Pi_{ij}} + qE_i - \frac{P_i}{\tau_{mr}}$$

No...

$\eta_{ijkl}\partial_j\partial_k v_l$

reversible

irreversible

Nonlocal/Nonlinear responses characterize the Hydro. Regime!!

- ***Nonlinear*** response
- ***Instability***
- ***Turbulence***

- Coupling between density and momentum
- Dispersion relation of collective mode

- ***Non-local*** linear response
- ***Finite size effect***
- Lifetime of collective mode