From 3d dualities to hadron physics

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Introduction and Motivation

- Recently, there are exciting developments in QCD3 and its dualities.

[Aharony, Benini, Karch, Komargodski, Seiberg, Tong,…]

- For example,

[Komargodski, Seiberg ('17)]

$$SU(N)_k \text{ with } N_f \text{ fermions}$$

$$\Updownarrow \quad 2|k| < N_f < N_*$$

$$U(N_f/2 \pm k)_{\mp N} \text{ with } N_f \text{ scalars}$$

w/ some conditions

- In the phase of QCD3 w/ small m_f , the flavor sym. is broken, which looks similar to QCD4.

- Moreover, the sym. breaking is described by the Higgs phenomena in dual scalar theory.

➡ **Dual theory of hadrons?**

Dualities in 3d

[Peskin ('78), Dasgupta, Halperin ('81)]

- **A simplest one is particle-vortex duality, which is a duality between the Abelian-Higgs model and the XY-model.**

▸ **The Abelian-Higgs model:**

$$S_{\mathrm{AH}} = \int d^3x \left(-\frac{1}{4e^2} |f|^2 + |D\phi|^2 - m^2 |\phi|^2 - \frac{\lambda}{2} |\phi|^4 \right).$$

- **For $|m| \ll e^2$, the theory is strongly coupled in IR.**
- **a global sym. $U(1)_{\mathrm{top}}$ acts on monopole op. as**

$$U(1)_{\mathrm{top}} : \mathscr{M}(x) \mapsto e^{i\alpha} \mathscr{M}(x) .$$

- **there are two phases depending on m.**

- For $m^2 \gg e^4$, the scalar is decoupled.
- $U(1)$ gauge symmetry is unbroken

  $\iff$ Spontaneously broken $U(1)_{\rm top}$

  $\iff$ NG mode = photon
- The massive excitations that come from ϕ are logarithmically confined.

Gapless Coulomb phase

- **For $m^2 \ll - e^4$, the scalar condenses,**

$$|\phi|^2 = -\frac{m^2}{\lambda}.$$

- **$U(1)$ gauge symmetry is spontaneously broken**

$\Longleftrightarrow$ **Unbroken $U(1)_{\rm top}$**

$\Longleftrightarrow$ **Charged excitation = vortex**

Gapped Higgs phase

- **The XY-model:**

$$S_{\mathrm{XY}} = \int d^3x \left(|\partial\tilde{\phi}|^2 - \tilde{m}^2 |\tilde{\phi}|^2 - \frac{\tilde{\lambda}}{2} |\tilde{\phi}|^4 . \right)$$

  - **a global symmetry $U(1)$ acts on $\tilde{\phi}$ as**

$$U(1) : \tilde{\phi} \mapsto e^{i\alpha}\tilde{\phi} .$$

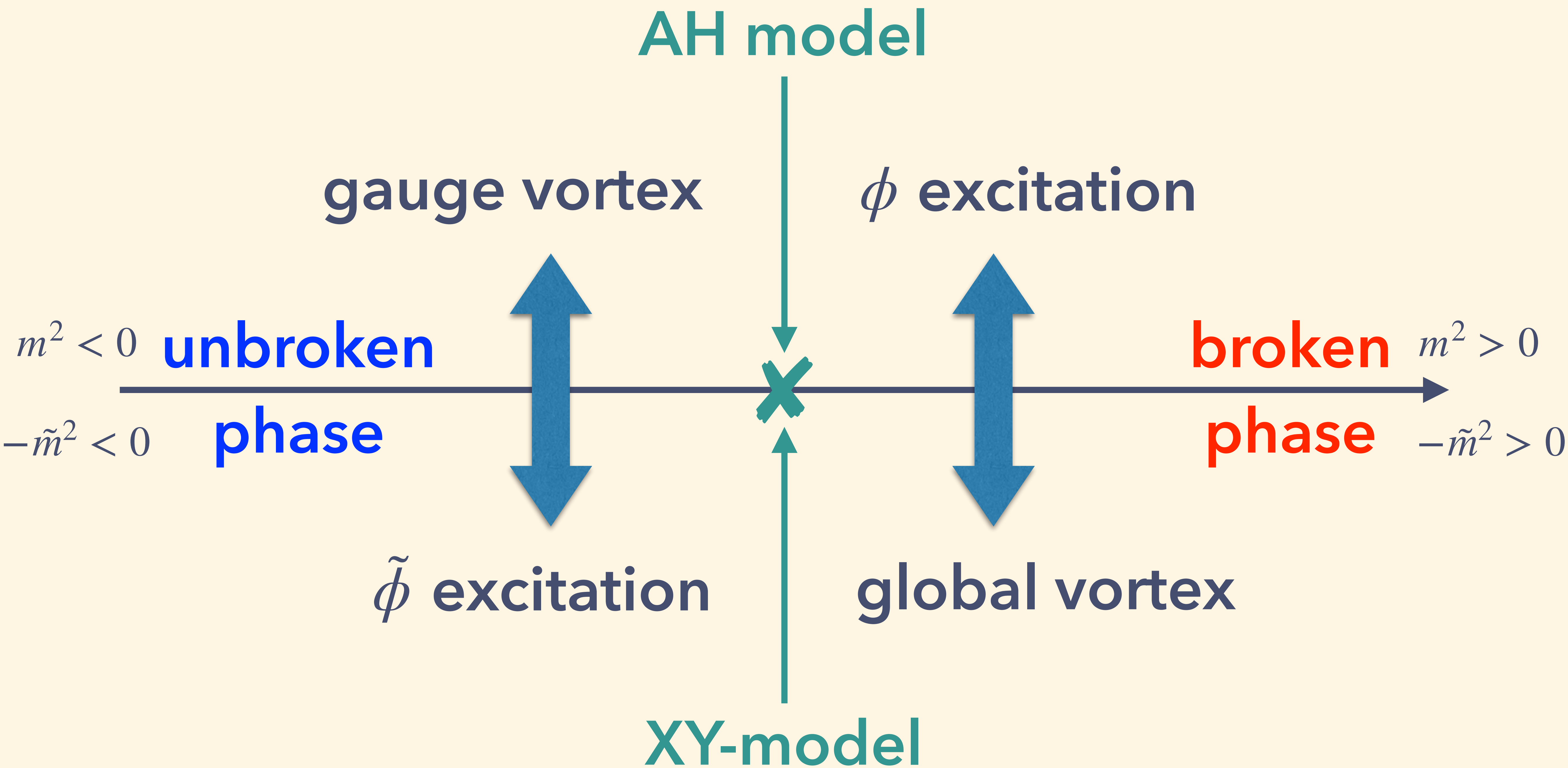
  - **there are also two phases depending on $\tilde{m}$.**

- For $\tilde{m}^2 > 0$, the phase is gapped.
- The excitations $\tilde{\phi}$ are massive and carry the charges under the unbroken global $U(1)$.

- For $\tilde{m}^2 < 0$, $\tilde{\phi}$ gets a VEV and the global $U(1)$ is broken.
- The NG mode is the angular component of $\tilde{\phi}$.
- A vortex and anti-vortex are logarithmically confined.

Particle-vortex duality

[Peskin ('78), Dasgupta, Halperin ('81)]



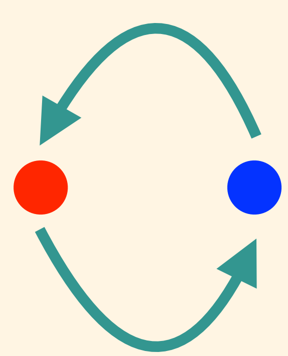
boson/fermion dualities
(3d bosonization)

- In 3d, we can write the Chern-Simons term:

$$S_{\mathrm{CS}} = \frac{k}{4\pi}\int d^3x\ \epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho,$$

where $k \in \mathbb{Z}$.

  - CS term is topological.
  - CS term contributes to the statistical phase:



$$= \quad \pm e^{i\pi/k}$$

- **conjecture: a bosonic theory with CS term is described by a fermionic theory, e.g.,**

$U(1)_1$ with ϕ $\Longleftrightarrow$ free ψ

$$\mathscr{L} = \frac{1}{4\pi} ada + |D_a \phi|^2 - |\phi|^4 \quad \Longleftrightarrow \quad \mathscr{L} = i\bar{\psi}\partial\!\!\!/ \psi$$

XY model $\Longleftrightarrow$ $U(1)_{-1/2}$ with ψ

$$\mathscr{L} = |\partial\phi|^2 - |\phi|^4 \quad \Longleftrightarrow \quad \mathscr{L} = i\bar{\psi} D\!\!\!/_a \psi$$

[Barkeshli, McGreevy ('12), Wang, Senthil, ('15), Metlitski, Vishwanath ('15), Karch, Tong ('16), Seiberg, Senthil, Wang, Witten ('16)]

- For non-Abelian QCD3,

Boson **Fermion**

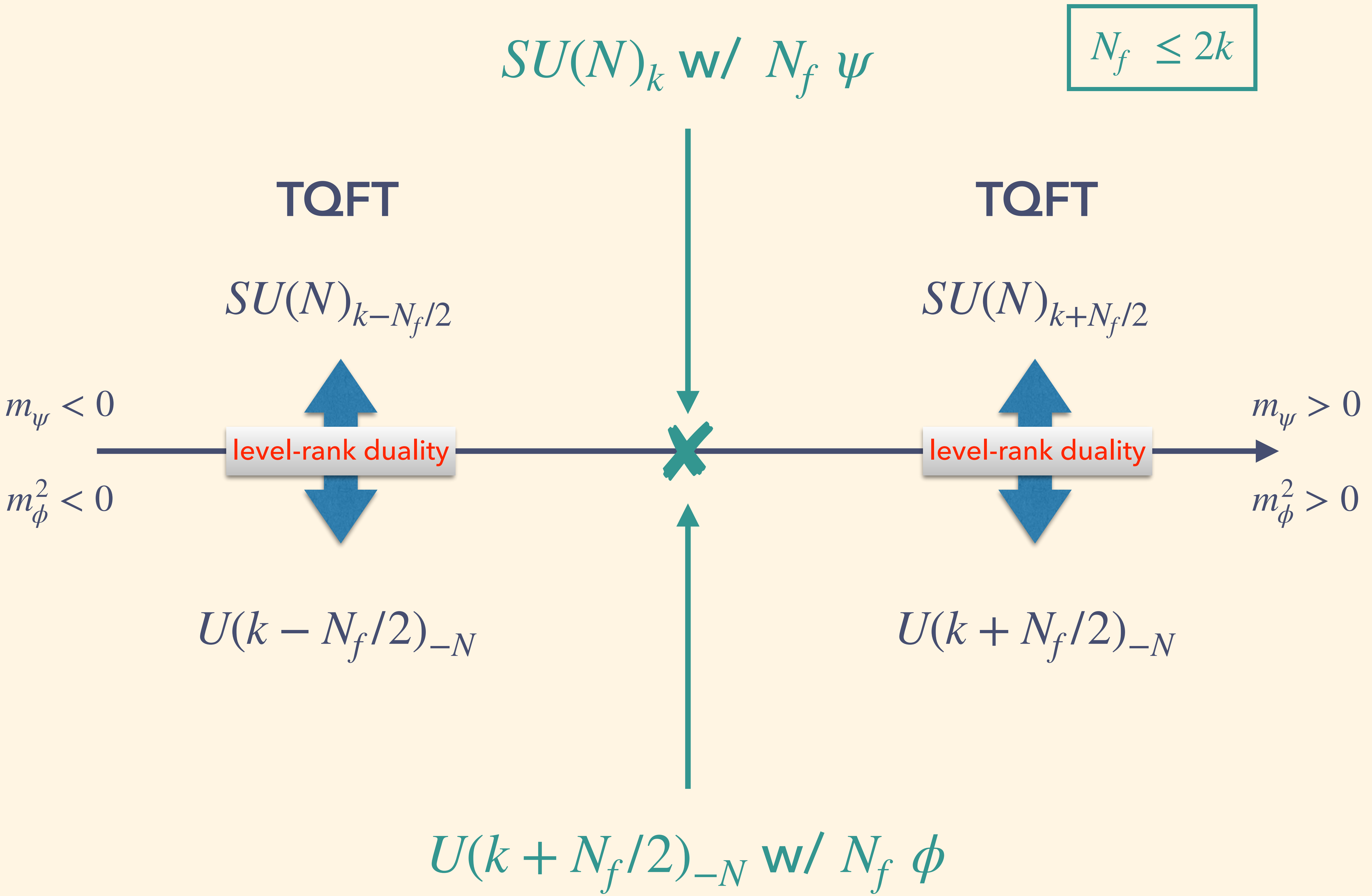
$$U(k+N_f/2)_{-N} \text{ with } N_f\ \phi \quad \overset{N_f \leq 2k}{\Longleftrightarrow} \quad SU(N)_k \text{ with } N_f\ \psi$$

$$SO(k+N_f/2)_{-N} \text{ with } N_f\ \phi \quad \overset{N_f \leq 2(k-1) \text{ if } N=2,\ N_f \leq 2k \text{ if } N>2}{\Longleftrightarrow} \quad SO(N)_k \text{ with } N_f\ \psi$$

$$Sp(k+N_f/2)_{-N} \text{ with } N_f\ \phi \quad \overset{N_f \leq 2k}{\Longleftrightarrow} \quad Sp(N)_k \text{ with } N_f\ \psi$$

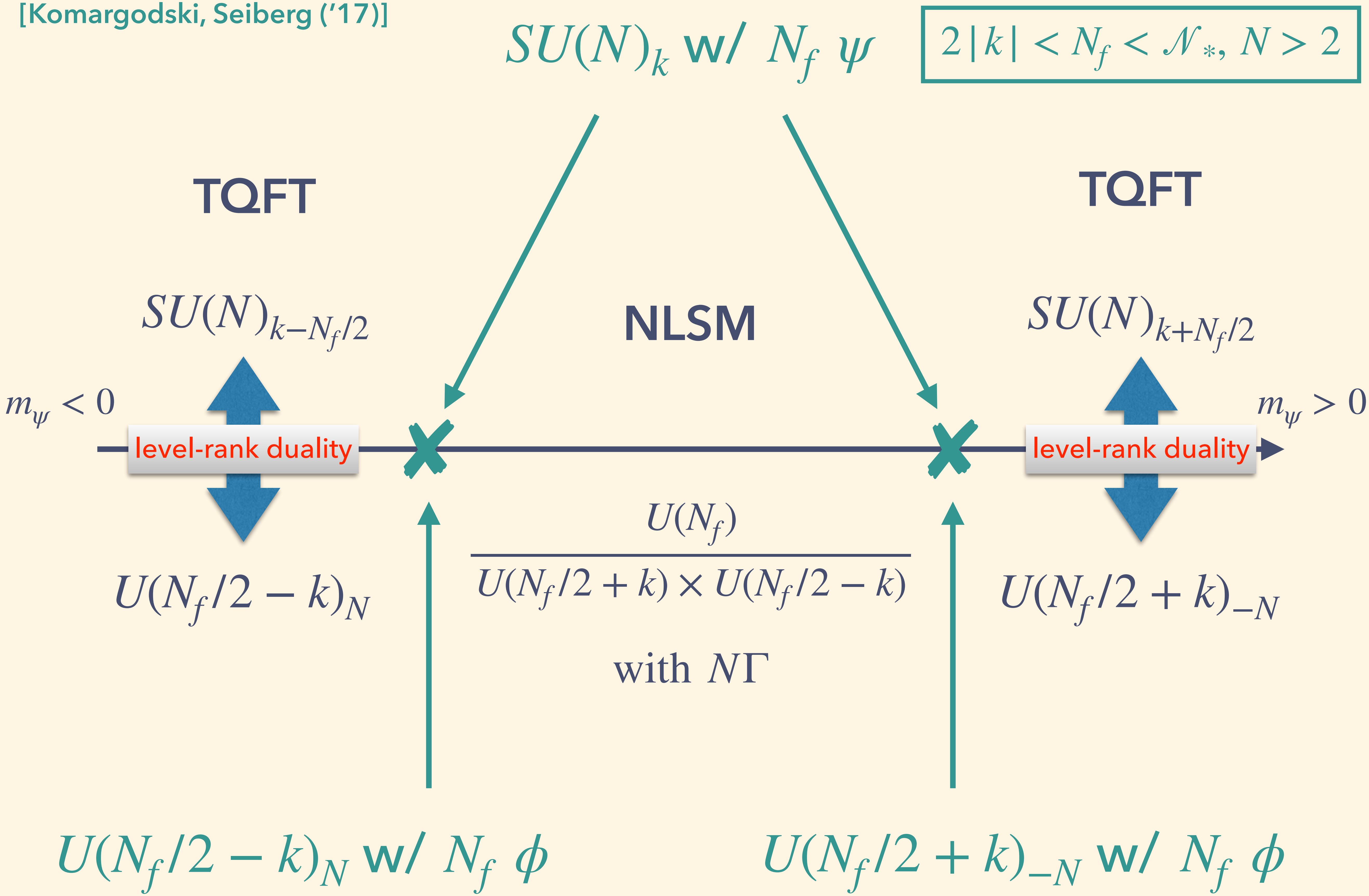
and their time-reversed versions.

[Aharony ('15), Hsin, Seiberg ('16), Aharony, Benini, Hsin, Seiberg ('16)]


$SU(N)_k$ w/ $N_f\ \psi$
$N_f \leq 2k$
TQFT
TQFT
$SU(N)_{k-N_f/2}$
$SU(N)_{k+N_f/2}$
$m_\psi < 0$
$m_\phi^2 < 0$
level-rank duality
level-rank duality
$m_\psi > 0$
$m_\phi^2 > 0$
$U(k - N_f/2)_{-N}$
$U(k + N_f/2)_{-N}$
$U(k + N_f/2)_{-N}$ w/ $N_f\ \phi$

N_f larger than 2k ??

[Komargodski, Seiberg ('17)]



Tests of the conjectures

[Nguyen, Sudbo ('99), Kajantie, Laine, Neuhaus, Rajantie, Rummukainen ('04)]

- lattice Monte Carlo simulation (for particle-vortex duality)

[Aharony ('15)]

- large N analyses
- flux attachments [Karch, Tong ('16)]
- from mirror symmetry [Kachru, Mulligan, Torroba, Wang ('16)]
- 't Hooft anomaly matching

[Benini, Hsin, Seiberg ('17), Komargodski, Seiberg ('17), Cordova, Hsin, Seiberg ('17)]

- embedding in string theory and/or holography

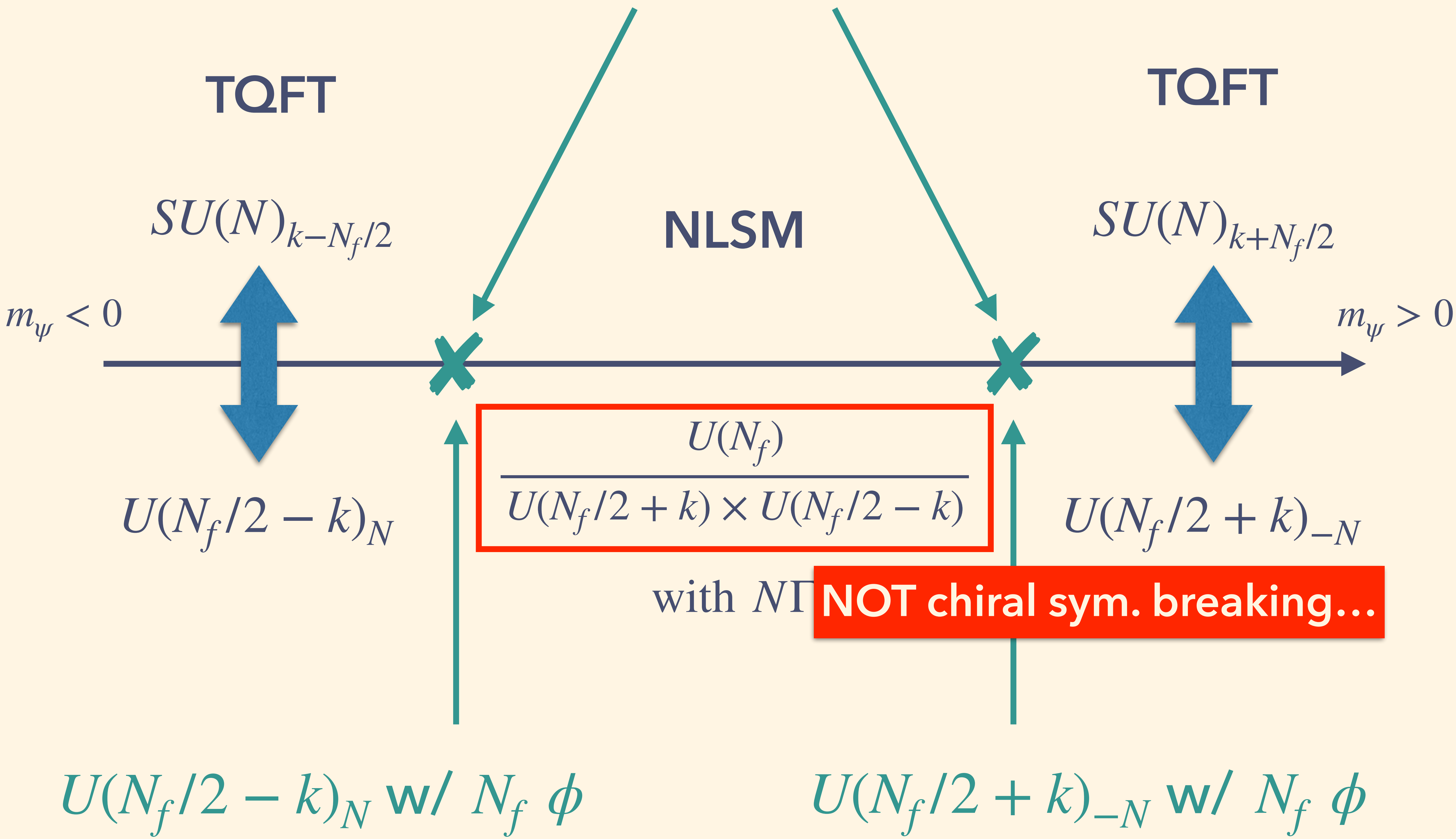
[Jensen, Karch ('17), Armoni, Niarchos ('17), Argurio, Bertolini, Bigazzi, Cotrone, Niro ('18), Aitken, Baumgartner, Karch ('18), Akhond, Armoni, Speziali ('19)]

toward hadron physics

[Komargodski, Seiberg ('17)]

$SU(N)_k$ w/ $N_f\ \psi$

$2|k| < N_f < \mathcal{N}_*,\ N > 2$



- We add the explicit breaking terms

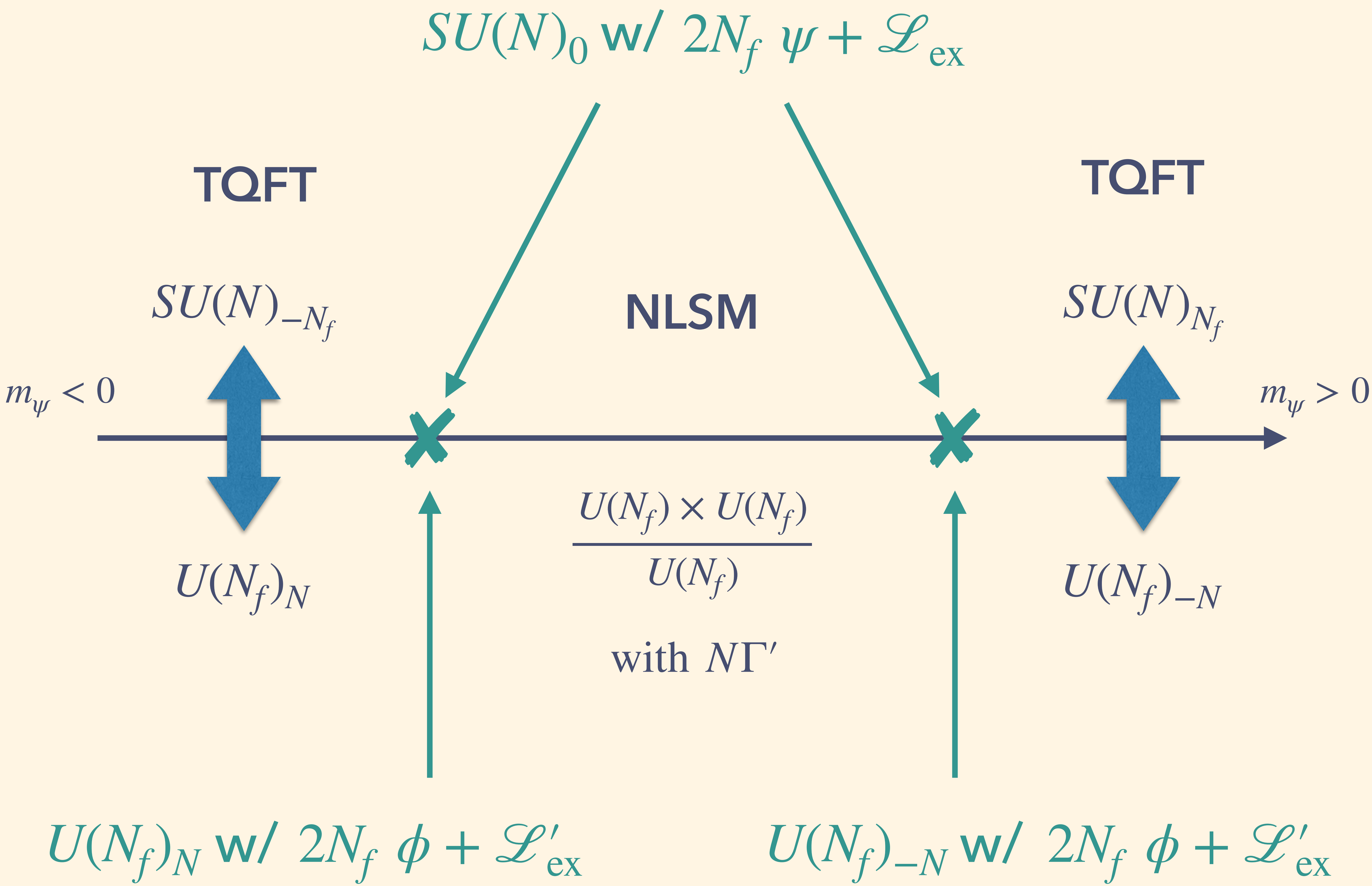
$$\mathscr{L}_{\mathrm{ex}} = -\bar{\psi} a_3 \psi + \bar{\tilde{\psi}} a_3 \tilde{\psi},$$

to $SU(N)_0$ with $(N_f + N_f)$ fermions.

- The theory in the broken phase is described by $(U(N_f) \times U(N_f))/U(N_f)$.

➡ **coincide with chiral symmetry breaking**

[NK, Kitano, Yankielowicz, Yokokura ('19)]



QCD_4

- **We start with QCD4 on $M_3 \times S^1$,**

$$S = \int_{M_3 \times S^1} \left[-\frac{1}{2g_4^2} \mathrm{Tr}\, |f|^2 + \frac{\theta(x_3)}{8\pi^2} \mathrm{Tr}\left(f^2\right) + i \sum_{i=1}^{N_f} \bar{\Psi}_i \, \not{D}_a \Psi_i \right],$$

where the θ winds around S1,

$$\int_{S^1} d\theta = 2\pi k$$

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where the θ winds around S1,

$$\int_{S^1} d\theta = 2\pi k = 2\pi N_f \,.$$

Effective theory
for small and large radius

- For small radius, $\Lambda_4 R \ll 1$, we can perform the KK decomposition.

- From the θ-term, we find the CS term,

$$\frac{1}{8\pi^2}\int_{M_3\times S^1}\theta\mathrm{Tr}(ff) = \frac{1}{8\pi^2}\int_{M_3\times S^1}\mathrm{Tr}\left(ada+\frac{2}{3}a^3\right)d\theta, \quad \mathrm{mod}\ 2\pi\,.$$

- There is a mass gap, but the low energy limit is the CS theory, $SU(N)_{N_f}$.

- **For large radius, $\Lambda_4 R \gg 1$, the low effective theory is given by**

$$S_{\rm eff} = \int_{M_3\times S^1} d^4x \left[f_\pi^2 \mathrm{Tr}\,|\,\partial_M U\,|^2 - \frac{m_\eta^2 f_\pi^2}{N_f} \left|\log(e^{-i\theta}\det U)\right|^2 + \cdots \right],$$

where $U = \exp(i\pi^a T^a + i\eta)$ **.**

- The EoM for η is

$$\frac{\partial^2}{\partial x_3^2}\eta = m_\eta^2\left(\eta - \frac{\theta}{N_f}\right),$$

➡ the η gets a winding,

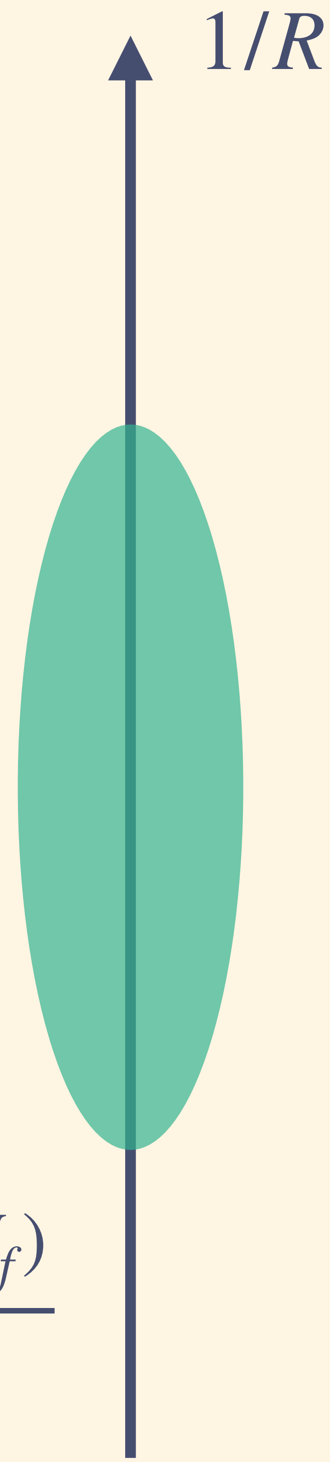
$$\eta(x_3 + 2\pi R) = \eta(x_3) + 2\pi\,.$$

- **Under the background where the η has winding, the 4d WZW term which couple to the external gauge fields includes 3d WZ term,**

$$S_{\rm WZW} \supset -\frac{N}{8\pi^2}\int_{M_3\times S^1} {\rm Tr}\left(AdA+\frac{2}{3}A^3\right)d\eta\,.$$

4d on S1

QCD4 on S1



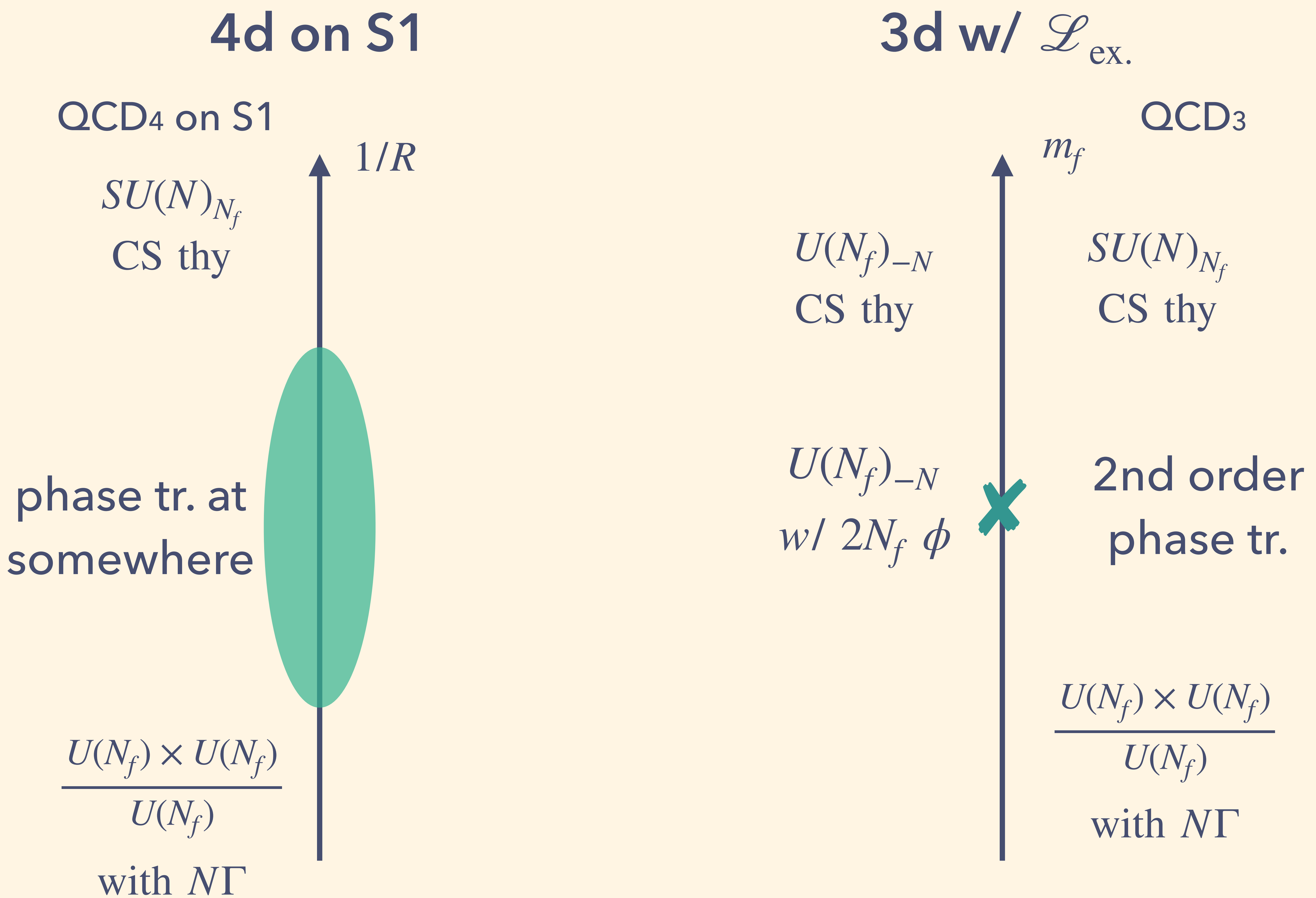
$SU(N)_{N_f}$

CS thy

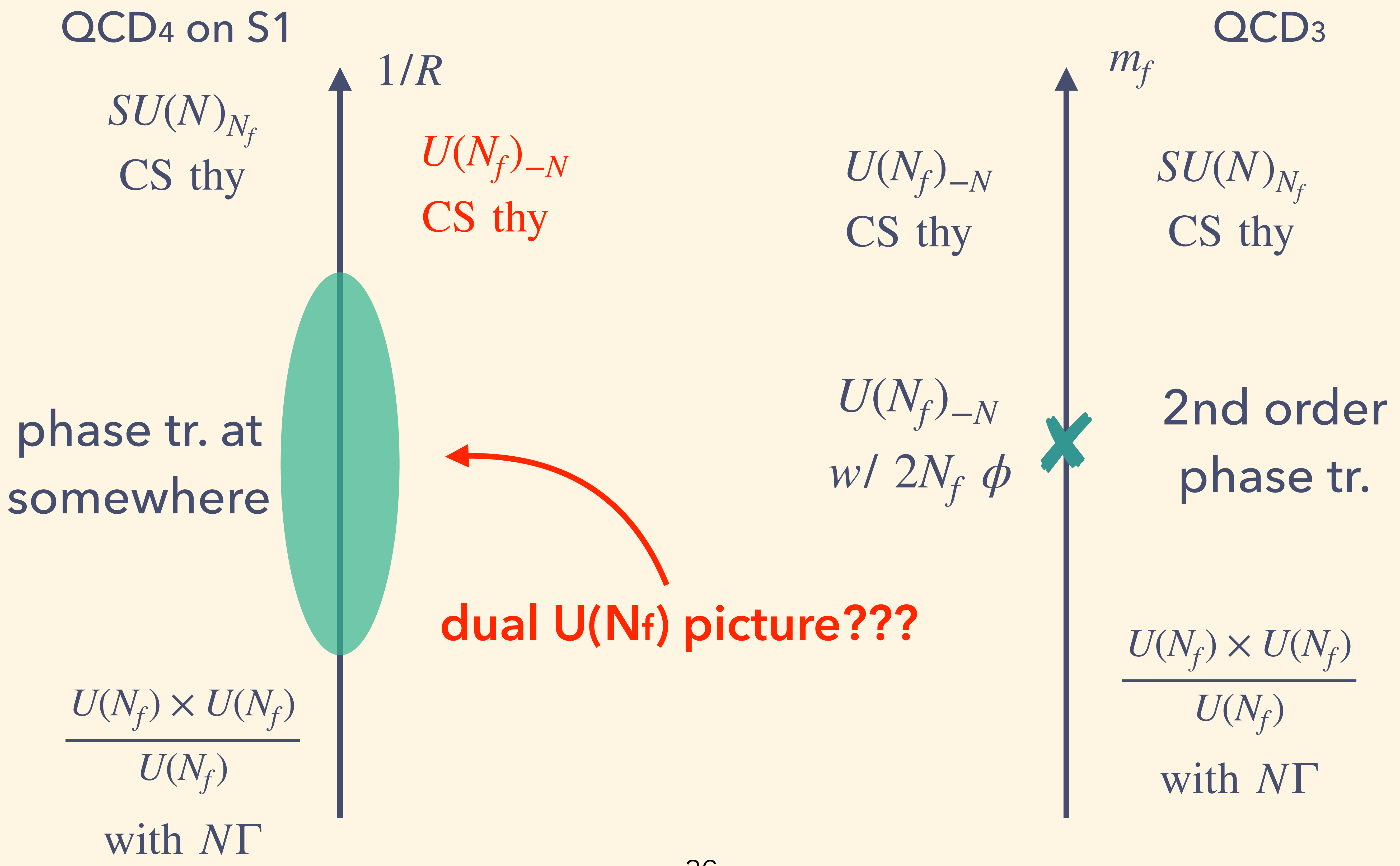
phase tr. at somewhere

$$\frac{U(N_f)\times U(N_f)}{U(N_f)}$$

with $N\Gamma$


4d on S1
3d w/ $\mathscr{L}_{\text{ex.}}$
QCD4 on S1
$1/R$
$SU(N)_{N_f}$
CS thy
phase tr. at
somewhere
$\frac{U(N_f) \times U(N_f)}{U(N_f)}$
with $N\Gamma$
QCD3
m_f
$U(N_f)_{-N}$
CS thy
$SU(N)_{N_f}$
CS thy
$U(N_f)_{-N}$
w/ $2N_f$ ϕ
2nd order
phase tr.
$\frac{U(N_f) \times U(N_f)}{U(N_f)}$
with $N\Gamma$

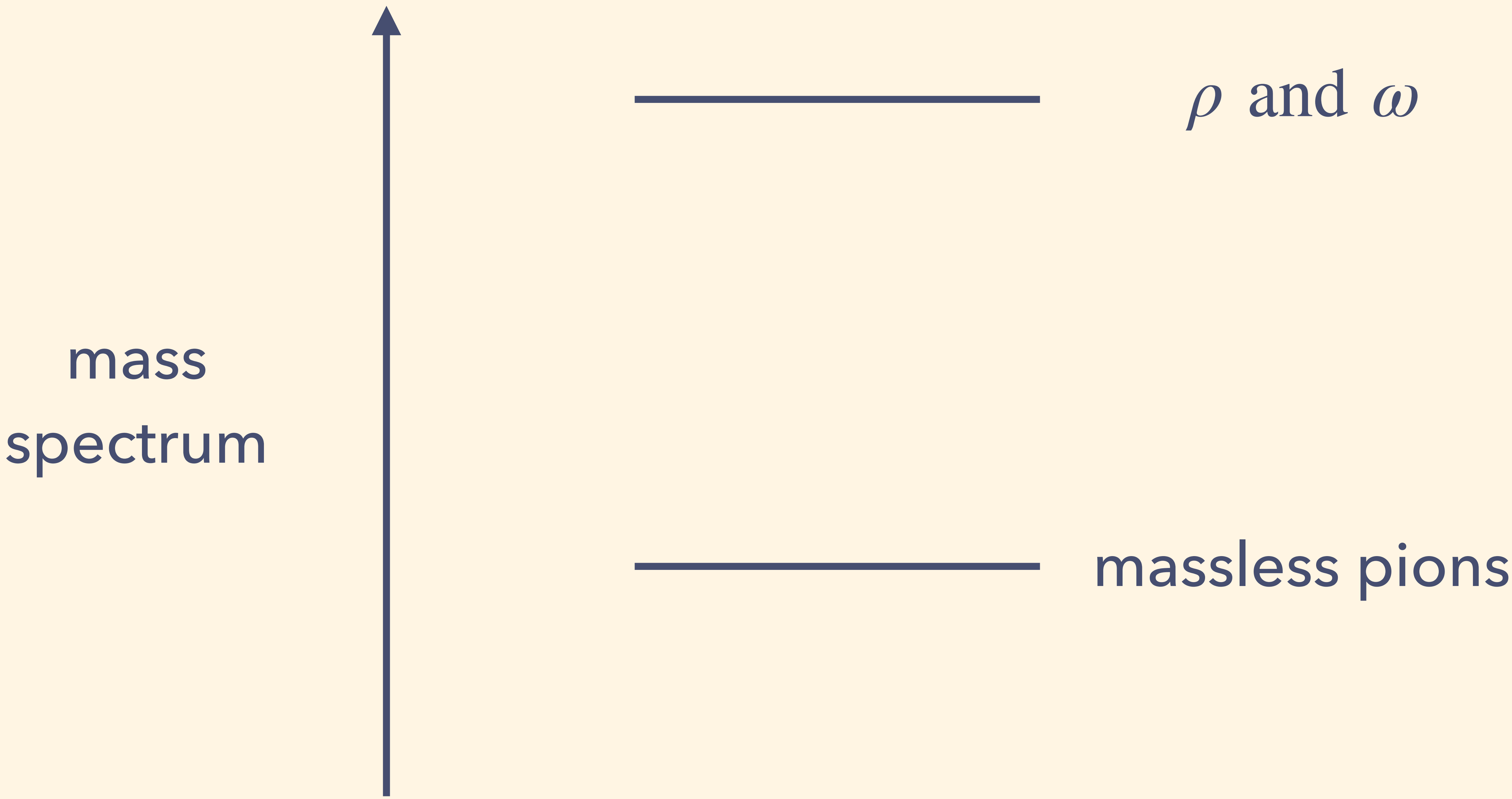
4d on S1 | 3d w/ $\mathscr{L}_{\text{ex.}}$



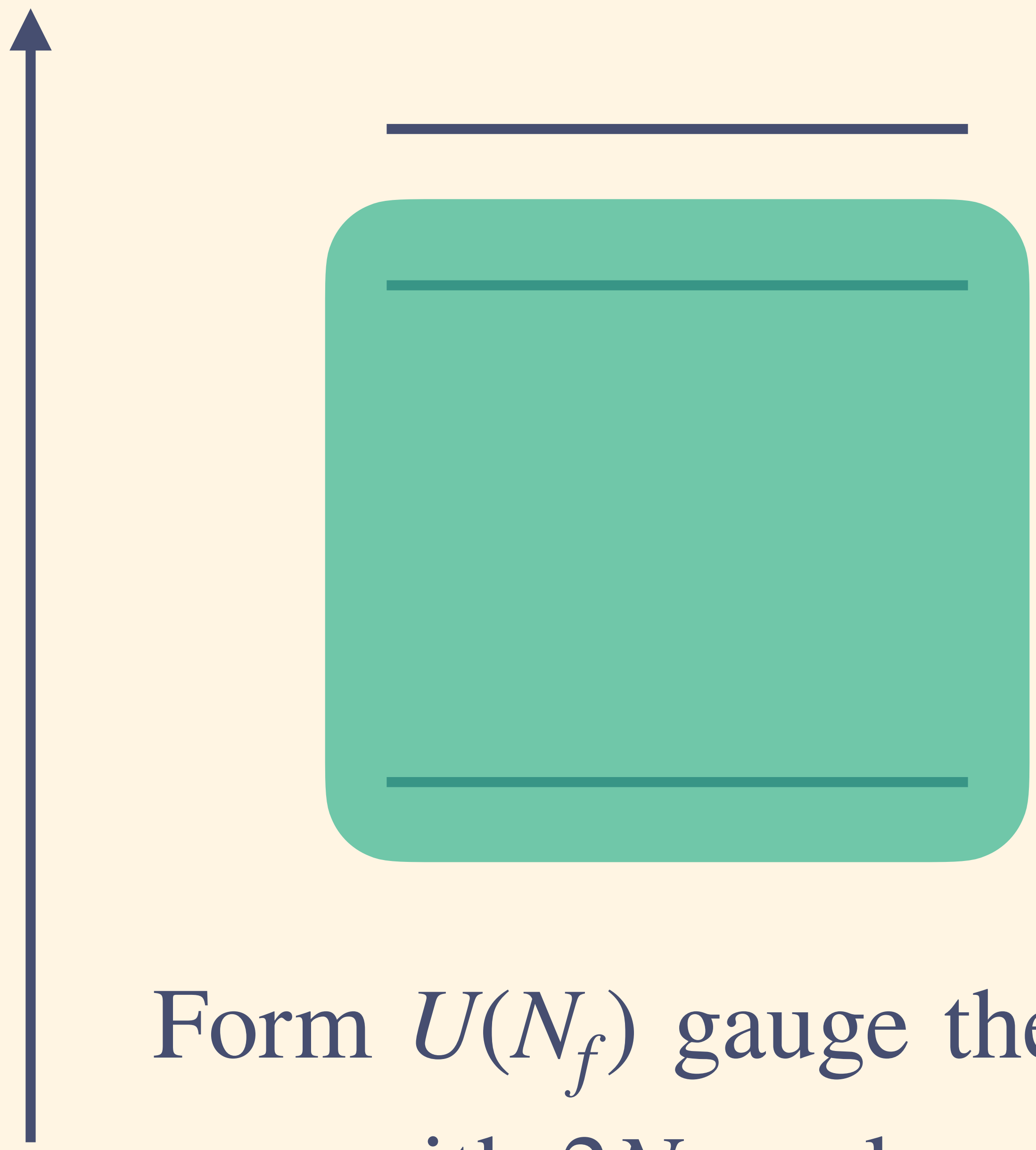
- The extension of the chiral Lagrangian to U(N_f) gauge theory is known to give a great success to describe the phenomenology of the vector mesons ρ and ω. [Bando, Kugo Uehara, Yamawaki, Yanagida ('85)]

candidates for U(N_f) gauge boson

- **At large radius,** $\Lambda_4 R \gg 1$,

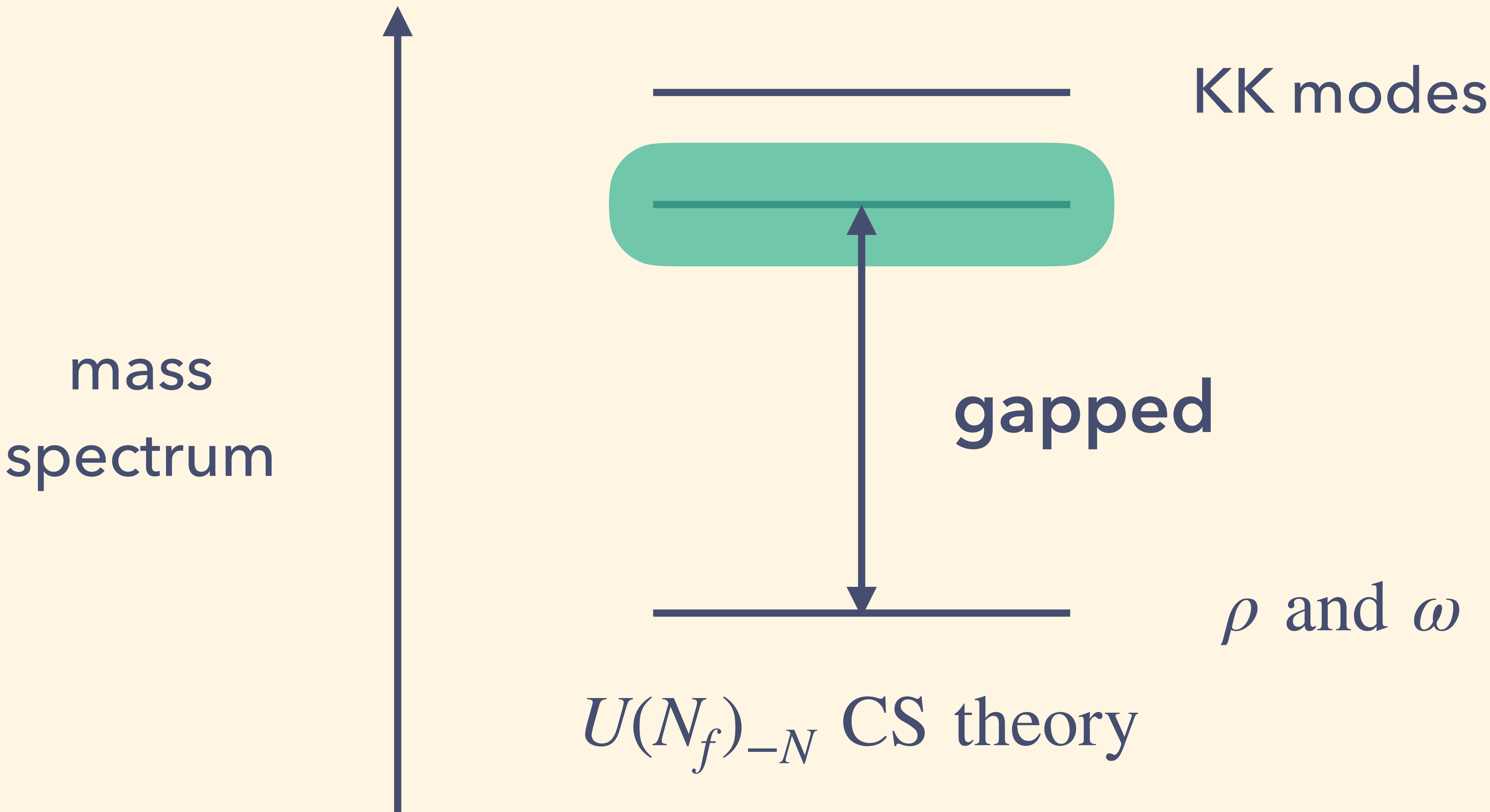


- **Close to critical radius, $\Lambda_4 R \sim 1$,**



Form $U(N_f)$ gauge theory
with $2N_f$ scalars

- **At small radius,** $\Lambda_4 R \ll 1$,

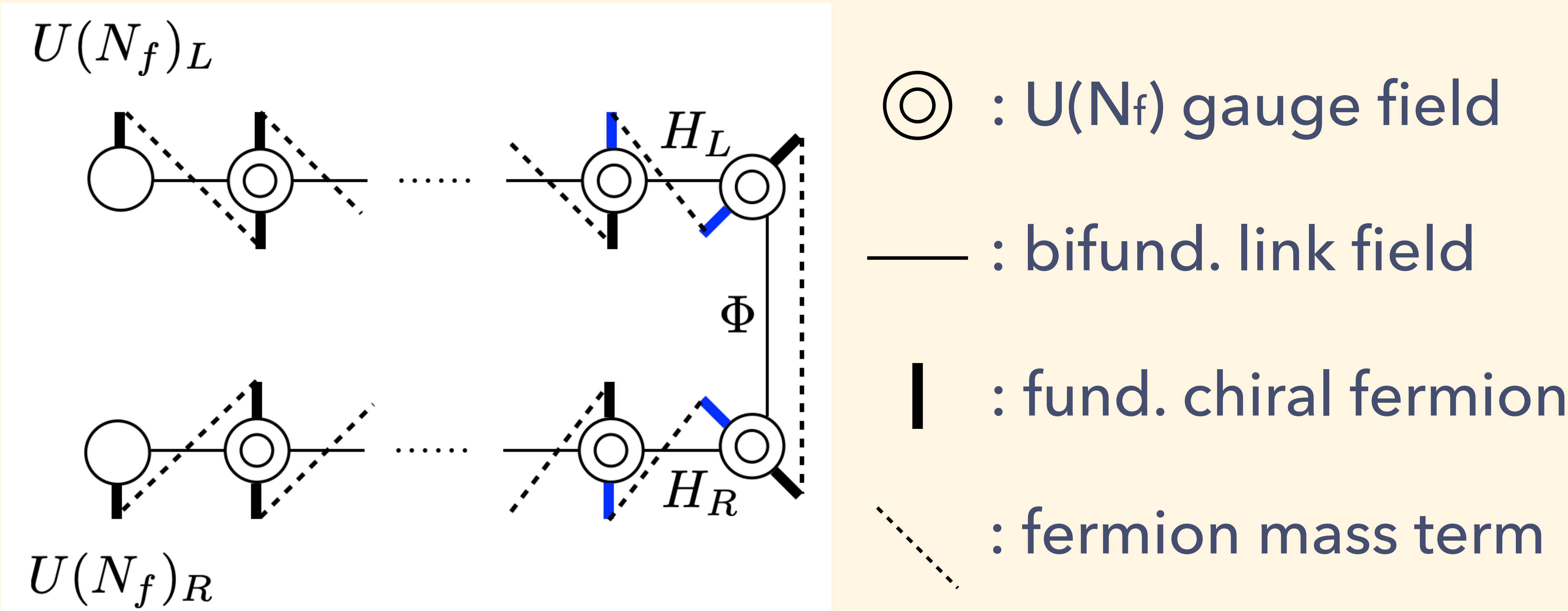


(chiral sym. is restored)

Holographic model

The quiver diagram of our model

[NK, Kitano, Yankielowicz, Yokokura ('19)]



- For $\langle H_{L,R} \rangle \neq 0$, describing π's and η.
- When $\langle H_{L,R} \rangle = 0$, the model becomes $U(N_f)_{-N}$.

Summary

In QCD_4 with winding θ, there is a phase transition between the large and small radius.

We conjectured the dual $U(N_f)$ description near the critical pt from 3d duality, and suggested the new picture of hadrons!!!

We proposed the holographic model of the dual theory that realizes our picture.

ばっくあっぷ

Conventions

- Dynamical gauge fields are given by lowercase letters $a_\mu, b_\mu, \ldots$; $A_\mu, B_\mu, \ldots$ represent non-dynamical fields.

- We represent the Lagrangian of QED with a CS term as

$$\mathscr{L} = -\frac{1}{4e^2} f_{\mu\nu} f^{\mu\nu} + i\bar{\psi}\gamma^\mu(\partial_\mu - i a_\mu)\psi - m\bar{\psi}\psi + \frac{k_{bare}}{4\pi}\epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

$$\equiv i\bar{\psi}\not{D}_a \psi + \frac{k_{bare}}{4\pi} a da,$$

where $k_{bare} \in \mathbb{Z}$.

- **When $|m| \gg e^2$, we can integrate out a fermion, and this shifts the bare CS level by $\mathrm{sgn}(m)/2$.**

- **In addition, the theory is regularized to preserve gauge invariance with a Pauli-Villars regulator, which shifts the bare CS level by $-1/2$.**

- **For this reason, we define the CS level k as**

$$k = k_{bare} - N_f/2,$$

where N_f is the number of the fermions.

- **For example, QED with $k_{bare} = 0$**

$$\mathscr{L} = i\bar{\psi}\not{D}_a\psi$$

is expressed as $U(1)_{-1/2} + \psi$.

- When the fermions have masses and they are integrated out, the CS level in the low energy theory is

$$k_{IR} = k + N_f \cdot \mathrm{sgn}(m)/2.$$

- Our labeling of theories with non-Abelian gauge groups is analogous.

The Lagrangian of the model

$$
\begin{aligned}
\mathscr{L} = & i\bar{q}^{(L)0}\gamma^{\mu}(\partial_{\mu} - iA_{\mu}^{(L)})P_L q^{(L)0} + i\bar{q}^{(R)0}\gamma^{\mu}(\partial_{\mu} - iA_{\mu}^{(R)})P_L q^{(R)0} \\
& + \sum_{i=1}^{n_L} i\bar{q}^{(L)i}\gamma^{\mu}(\partial_{\mu} - ib_{\mu}^{(L)i})q^{(L)i} + \sum_{i=1}^{n_R} i\bar{q}^{(R)i}\gamma^{\mu}(\partial_{\mu} - ib_{\mu}^{(R)i})q^{(R)i} \\
& - \sum_{i=1}^{n_L} \frac{1}{2g_i^{(L)2}} \mathrm{Tr}(f_{\mu\nu}^{(L)i} f^{(L)i\mu\nu}) - \sum_{i=1}^{n_R} \frac{1}{2g_i^{(R)2}} \mathrm{Tr}(f_{\mu\nu}^{(R)i} f^{(R)i\mu\nu}) \\
& + \mathrm{Tr}\,|\,\partial_{\mu}U_{01}^{(L)} - iA_{\mu}^{(L)}U_{01}^{(L)} + iU_{01}^{(L)}b_{\mu}^{(L)1}\,|^2 + \sum_{i=1}^{n_L-1} \mathrm{Tr}\,|\,\partial_{\mu}U_{i,i+1}^{(L)} - ib_{\mu}^{(L)i}U_{i,i+1}^{(L)} + iU_{i,i+1}^{(L)}b_{\mu}^{(L)i+1}\,|^2 \\
& + \mathrm{Tr}\,|\,\partial_{\mu}U_{01}^{(R)} - ib_{\mu}^{(R)1}U_{01}^{(R)} + iU_{01}^{(R)}A_{\mu}^{(R)}\,|^2 + \sum_{i=1}^{n_R-1} \mathrm{Tr}\,|\,\partial_{\mu}U_{i+1,i}^{(R)} - ib_{\mu}^{(R)i+1}U_{i+1,i}^{(L)} + iU_{i+1,i}^{(R)}b_{\mu}^{(R)i}\,|^2 \\
& + \mathrm{Tr}\,|\,\partial_{\mu}\Phi - ib_{\mu}^{(L)n_L}\Phi + i\Phi b_{\mu}^{(R)n_R}\,|^2 \\
& - \sum_{i=0}^{n_L-1} m_{i,i+1}^{(L)i+1}\bar{q}^{(L)i+1}U_{i,i+1}^{(L)\dagger}P_L q^{(L)i} - \sum_{i=0}^{n_R-1} m_{i+1,i}^{(R)}\bar{q}^{(R)i}U_{i+1,i}^{(R)\dagger}P_L q^{(R)i+1} + \mathrm{h.c.} \\
& - m_{\Phi}\bar{q}^{(R)n_R}\Phi^{\dagger}P_L q^{(L)n_L} + \mathrm{h.c.}
\end{aligned}
$$