The Global Anomaly Matching in the higher-dimensional $\mathbb{CP}^1$ Model

Takuya Furusawa (TITech/RIKEN)

TF & M. Hongo, PRB. 101, 155113 (2020)
(arXiv:2001.07373)

KEK-Keio-YITP joint seminar (online)., May., 2020

Outline

1. Introduction

2. $\mathscr{R} \times \mathrm{U}(1)_M$ anomaly in 2+1D

3. $\mathscr{R} \times (\mathbb{Z}_n)_M$ anomaly in 2+1D

4. 3+1D generalization & finite-temperature phase diagram

$\mathbb{CP}^1$ Model

$$\int |(d - ia)z|^2 + V(|z|^2) + \frac{1}{2g^2} da \wedge \star da$$

z: 2-component scalar, a:dynamical U(1) gauge field

(1+1)D $\mathbb{CP}^1$ model: asymptotic freedom

$\sim$ (3+1)D QCD

(Connected via compactification w/ twisted boundary condition)

Yamazaki (2017), Yamazaki, Yonekura (2017), Wan, Wang, Zheng (2018), Yamazaki, Yonekura (2019)

$\mathbb{CP}^1$ Model

$$\int |(d - ia)z|^2 + V(|z|^2) + \frac{1}{2g^2} da \wedge \star da$$

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Cond. mat. motivation: QFT for anti-ferromagnets

$$z^\dagger \sigma_\alpha z \sim n_\alpha$$

Gauge inv. combination ~ spin operator

$\mathbb{CP}^1$ Model

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Cond. mat. motivation: QFT for anti-ferromagnets

1+1D: Haldane conjecture Haldane (1983)

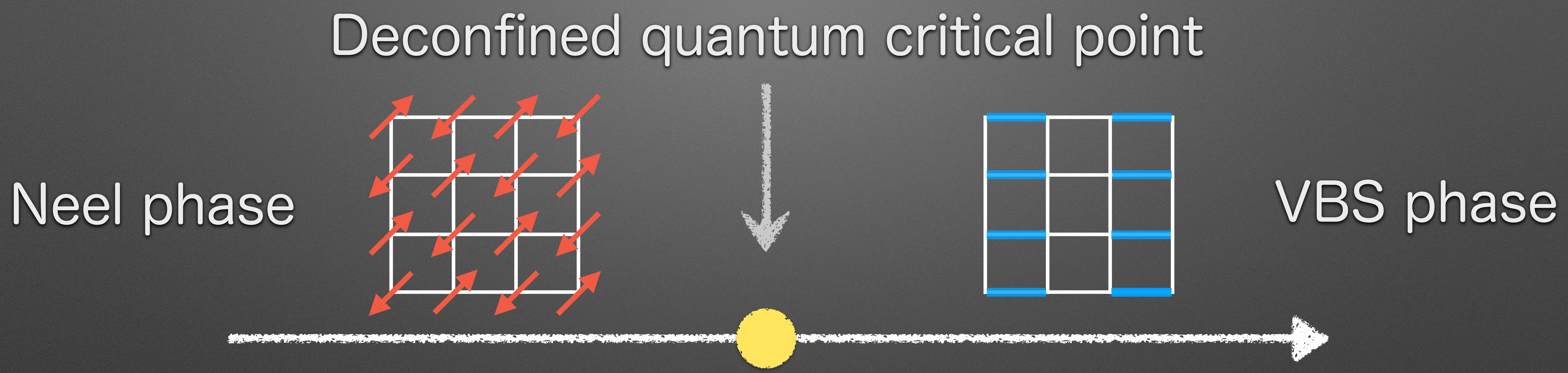
2+1D: Unconventional critical point

Deconfined quantum critical point (DQCP)

Senthil, Vishwanath, Balents, Sachdev, Fisher (2004)

3+1D: Neel - U(1) spin liquid transition

DQCP & Competing order



2+1D $\mathbb{CP}^1$ model

Competing order is explained by

't Hooft anomaly in $\mathbb{CP}^1$ model

Senthil, Vishwanath, Balents, Sachdev, Fisher (2004)

’t Hooft Anomaly

G: global symmetry, A: background gauge field

$$Z[A+\delta_\theta A]=Z[A]$$

’t Hooft (1980), Frishman, Schwimmer, Banks, Yankielowicz (1981), Coleman, Grossman (1982)

’t Hooft Anomaly

G: global symmetry, A: background gauge field

$$Z[A+\delta_\theta A] = Z[A]\underline{e^{i\mathscr{A}[\theta,A]}} \quad \left(\mathscr{A}[\theta,A] \neq \delta_\theta S[A]\right)$$

’t Hooft anomaly

Classic: Chiral symmetry in QCD

Recent: Discrete & higher-form symmetries

’t Hooft (1980), Frishman, Schwimmer, Banks, Yankielowicz (1981), Coleman, Grossman (1982)

Anomaly Matching Argument

The ’t Hooft anomaly is RG-invariant.

Anomaly at UV $\Rightarrow$ The same anomaly at IR

Consistency condition on IR behaviors!!

’t Hooft (1980), Frishman, Schwimmer, Banks, Yankielowicz (1981), Coleman, Grossman (1982)

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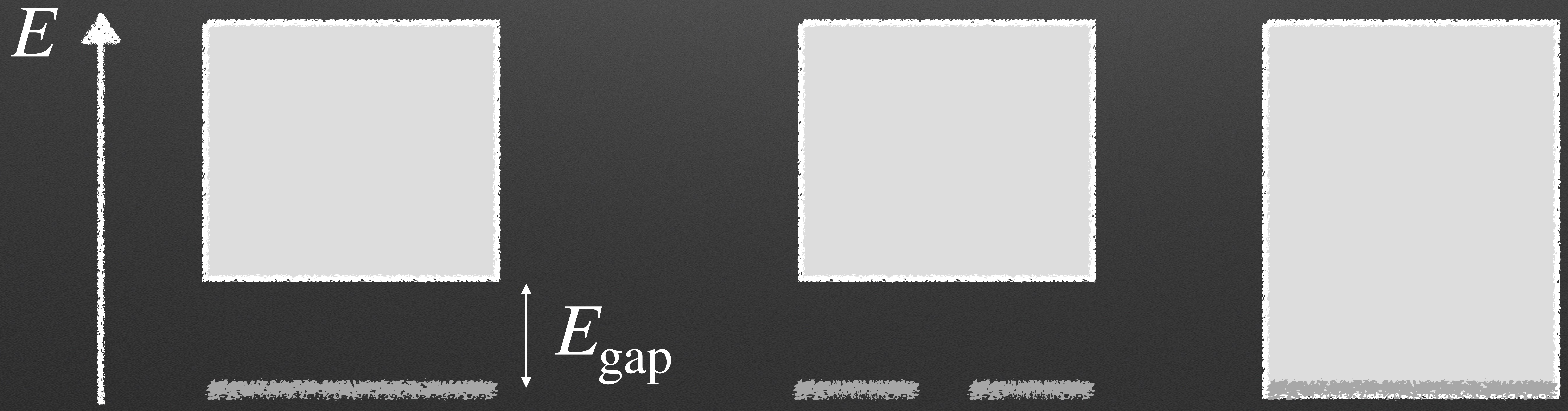
Unique gapped Degenerated Gapless



’t Hooft (1980), Frishman, Schwimmer, Banks, Yankielowicz (1981), Coleman, Grossman (1982)

Anomaly Matching Argument

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Global symmetries in $\mathbb{CP}^1$ model

(1) Flavor symmetry $\mathrm{SU}(2)/\mathbb{Z}_2 = \mathrm{SO}(3)_{\mathrm{spin}}$

$$z(x) \to Uz(x) \qquad U \in \mathrm{SU}(2)$$

$\mathbb{Z}_2 = \{\pm \mathbb{I}_2\}$ is redundant.

(2) Reflection symmetry (TR) $\mathscr{R}$

$$z(x) \to i\sigma^2 z^*(R_\mu \cdot x) \qquad \left(R_1 = \mathrm{diag}(-1,1,1)\right)$$

(3) Magnetic symmetry $\mathrm{U}(1)_M$

Anomalies in 2+1D $\mathbb{CP}^1$

Metlitski,Thorngren (2018),
Komargodski, Sulejmanpasic, Unsal (2018),
Komargodski, Sharon, Thorngren, Zhou, (2019).

Two mixed anomalies

$$[SU(2)/\mathbb{Z}_2]_F \times \mathcal{R} \times U(1)_M$$

Competing order in anti-ferromagnets

Anomalies in 2+1D $\mathbb{CP}^1$

Metlitski, Thorngren (2018),
Komargodski, Sulejmanpasic, Unsal (2018),
Komargodski, Sharon, Thorngren, Zhou, (2019).

Two mixed anomalies

$$[SU(2)/\mathbb{Z}_2]_F \times \mathcal{R} \times U(1)_M$$

(See the above Refs.)

This talk!! NEW

Furusawa, Hongo (2020)

Nontriviality w/o flavor symmetry!

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   & finite-temperature phase diagram

Magnetic symmetry $\mathrm{U}(1)_M$

Conserved current: $J^\mu_M = \frac{i}{2\pi}\epsilon^{\mu\nu\rho}\partial_\nu a_\rho \left(= \star \frac{i}{2\pi} da \right)$

Generator: $Q_M = \frac{i}{2\pi}\int_{\mathbb{R}^2} \epsilon^{ij}\partial_i a_j$

Magnetic flux

Charged object: Monopole instanton $\mathscr{M}$

Gauging $\mathscr{R} \times \mathrm{U}(1)_M$

$$\int |(d - ia)z|^2 + V(|z|^2) + \frac{1}{2g^2} da \wedge \star da + \frac{i}{2\pi} K \wedge da$$

Gauging $\mathrm{U}(1)_M$ = adding ____ via minimal coupling

Gauge field for $\mathscr{R}$?

Gauging $\mathscr{R} \times \mathrm{U}(1)_M$

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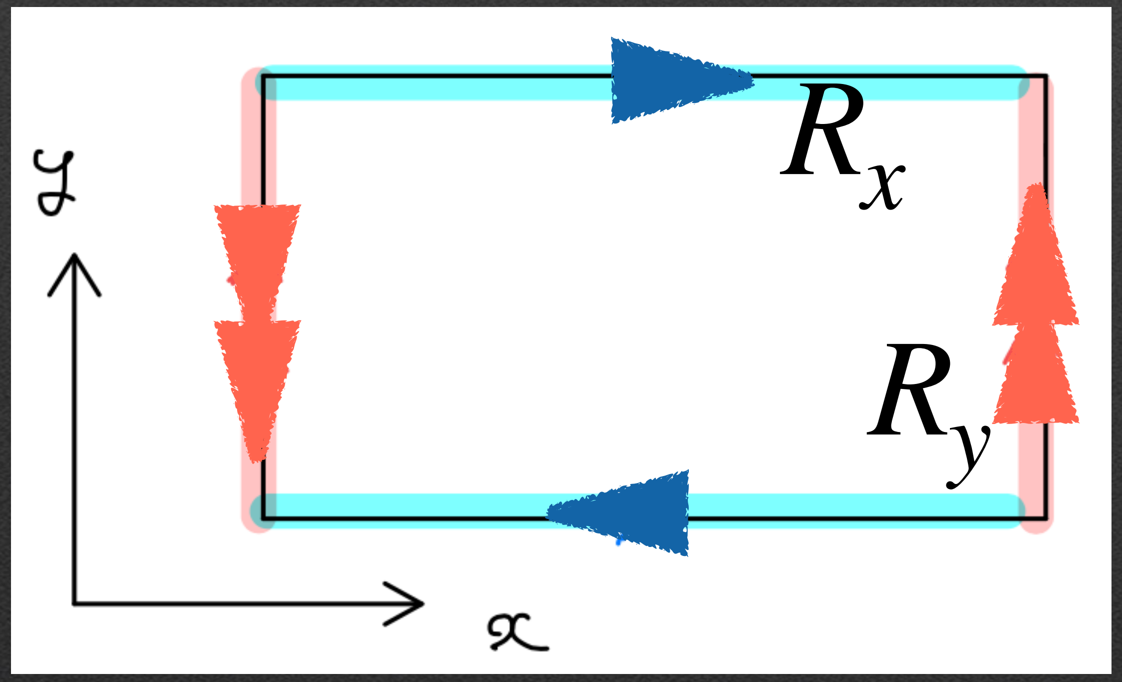
Gauging $\mathrm{U}(1)_M$ = adding ____ via minimal coupling

Gauge field for $\mathscr{R}$? = Twisted boundary condition by $\mathscr{R}$

$\mathbb{R}_\tau \times S^1_x \times S^1_y$



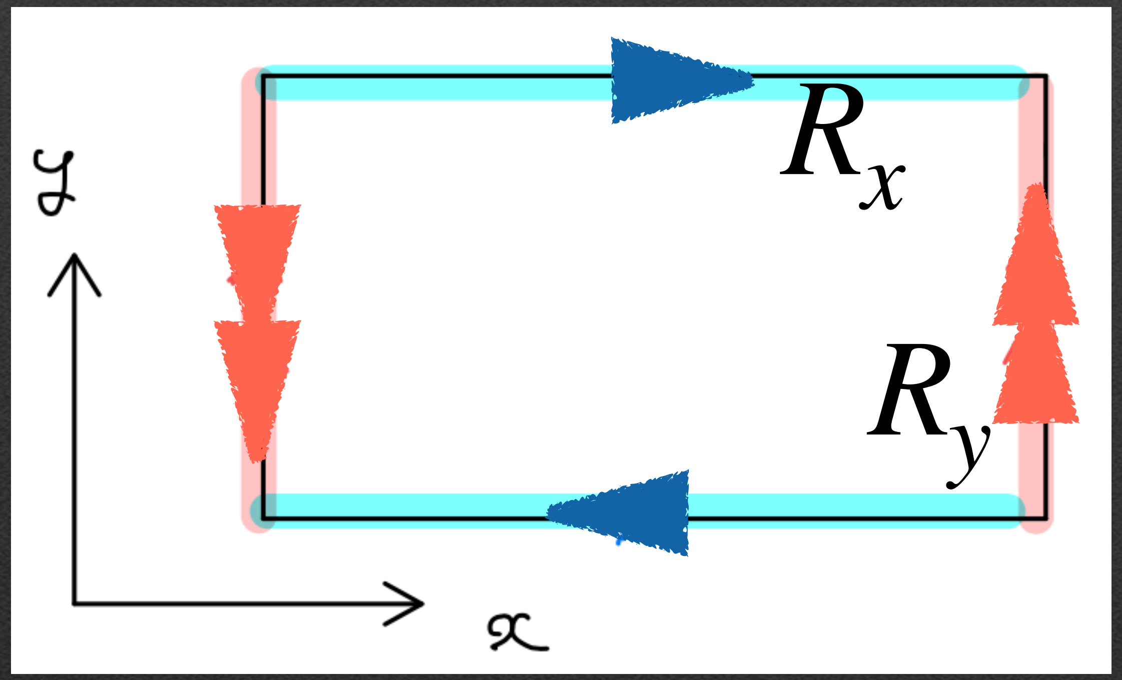
Gauging $\mathscr{R} \times \mathrm{U}(1)_M$

$$\int |(d - ia)z|^2 + V(|z|^2) + \frac{1}{2g^2} da \wedge \star da + \frac{i}{2\pi} K \wedge da$$

Gauging $\mathrm{U}(1)_M$ = adding ______ via minimal coupling

Gauge field for $\mathscr{R}$? = Twisted boundary condition by $\mathscr{R}$

$\mathbb{R}_\tau \times S^1_x \times S^1_y$ $\mathbb{R}_\tau \times \mathbb{RP}^2_{xy}$



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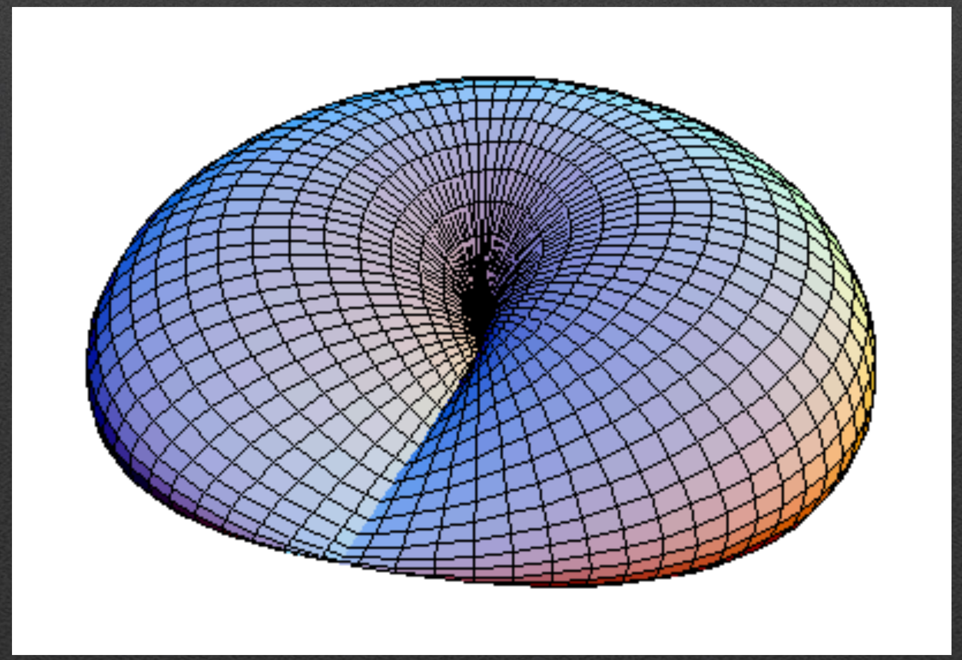

Gauging $\mathscr{R} \times \mathrm{U}(1)_M$

$$\int |(d - ia)z|^2 + V(|z|^2) + \frac{1}{2g^2} da \wedge \star da + \frac{i}{2\pi} K \wedge da$$

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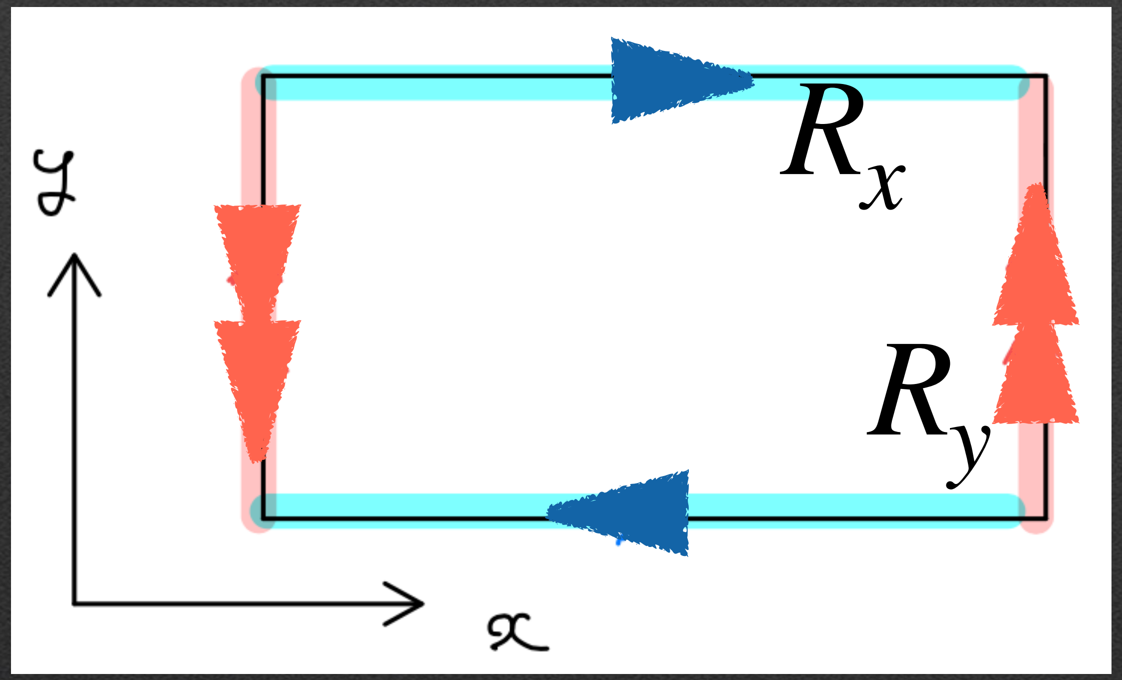
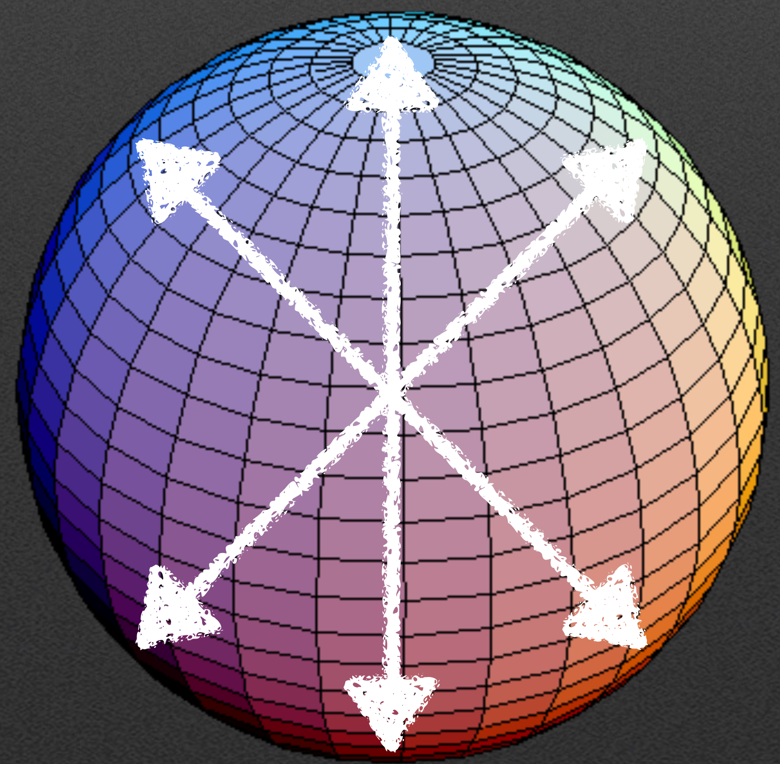


$\simeq$

$\mathbb{R}_\tau \times \mathbb{RP}^2_{xy}$



$$\int_{\mathbb{RP}^2} da = \pi$$

Half-monopole inside RP2

Gauging $\mathscr{R} \times \mathrm{U}(1)_M$

$$\int |(d - ia)z|^2 + V(|z|^2) + \frac{1}{2g^2} da \wedge \star da + \frac{i}{2\pi} K \wedge da$$

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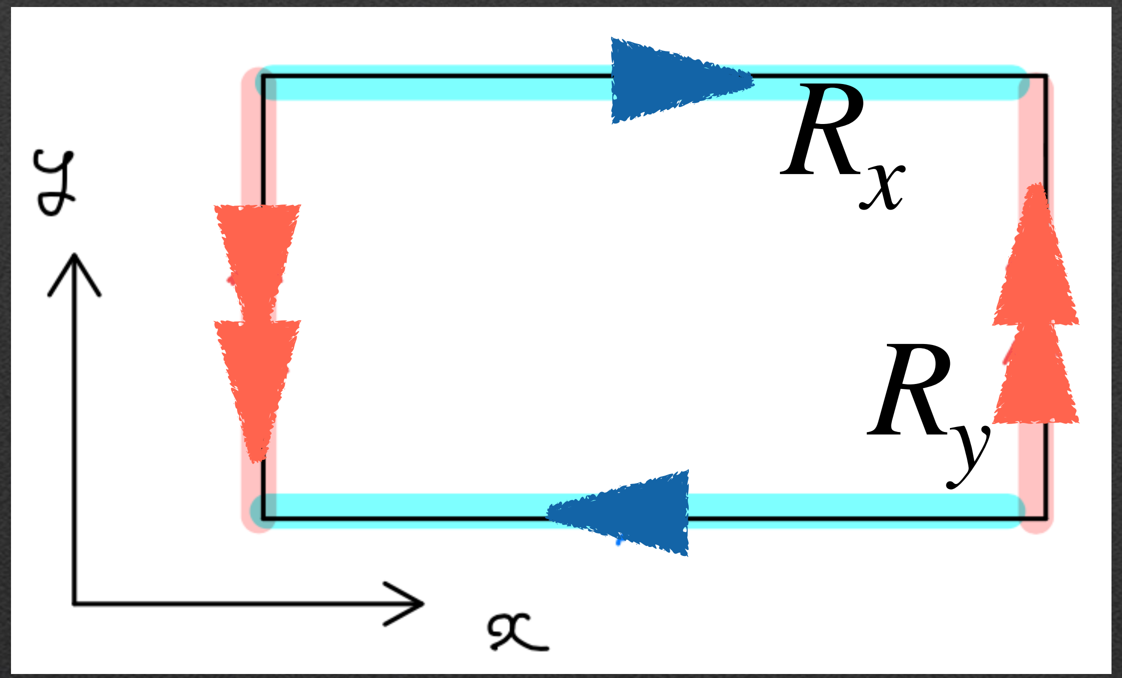
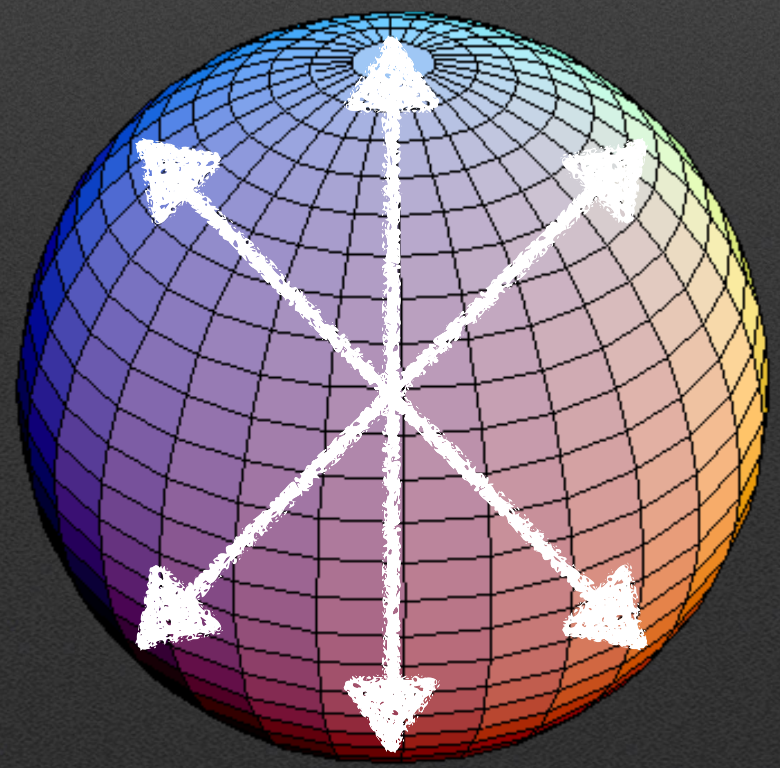


$\simeq$

$\mathbb{R}_\tau \times \mathbb{RP}^2_{xy}$



$$\int_{\mathbb{RP}^2} da = \pi = \int_{\mathbb{RP}^2} \pi w_2$$

2nd Stiefel-Whitney class

Sulejmanpasic, Tanizaki (2018),
Furusawa, Hongo (2020)

Inconsistency on $\mathbb{R}_\tau \times \mathbb{RP}^2_{xy}$

$$\int |(d - ia)z|^2 + V(|z|^2) + \frac{1}{2g^2} da \wedge \star da + \frac{i}{2\pi} K \wedge da$$

$\mathrm{U}(1)_M$ gauge transformation

$$K \to K + d\Lambda$$

$$Z_{\mathbb{CP}^1}[w_2, K] \to Z_{\mathbb{CP}^1}[w_2, K] e^{\frac{i}{2\pi}\int d\Lambda \wedge \pi w_2}$$

$\equiv i\pi$

$\mathscr{R} \times \mathrm{U}(1)_M$ anomaly

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Ingappablity w/o flavor symmetry

$\mathrm{SO}(3)_{\mathrm{spin}}$-breaking but $\mathscr{R}$-preserving perturbations:

Easy-plane　　Dzyaloshinskii-Moriya

$$V(n^\alpha) = \sum_{\alpha} \mu^\alpha n^\alpha n^\alpha + \sum_{\alpha,\beta,\gamma} \kappa_i^\alpha \epsilon^{\alpha\beta\gamma} n^\beta (\partial_i n^\gamma + (\kappa_i \times n)^\gamma) \,.$$

$$\left(n^\alpha = z^\dagger \sigma^\alpha z \right)$$

	$\mathrm{PSU}(2)_F$	$\mathcal{R}_1(\sim \mathcal{T})$	$\mathcal{R}_2$	$\mathcal{R}_3$	$\cdots$	$\mathcal{R}_D$	$\mathrm{U}(1)_M^{[D-3]}$
$\mu^{\alpha=x}$	$\to \mathrm{O}(2)_x$	○	○	○	$\cdots$	○	○
$\kappa_{i=2}^{\alpha=x}$	$\to \mathrm{O}(2)_x$	○	×	○	$\cdots$	○	○

No unique gapped ground state with $\mathscr{R} \times \mathrm{U}(1)_M$

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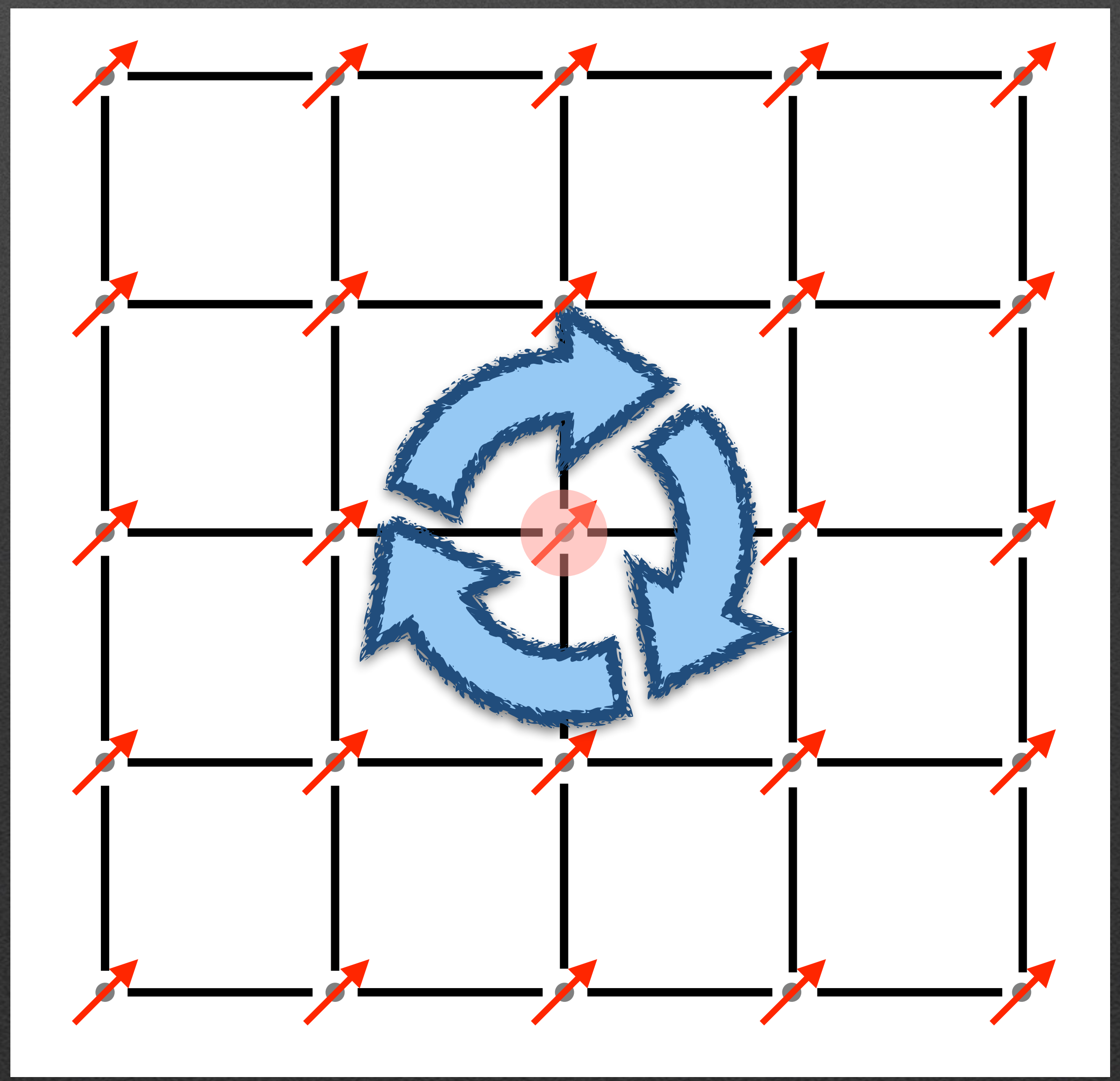
   & finite-temperature phase diagram

Discrete Magnetic symmetry

At the lattice scale,

Monopoles are charged under site-centered rotation because of the spin Berry phase. Haldane (1988)

Magnetic symmetry ~ Site-centered rotation



Square lattice

$\mathbb{Z}_{n=4}$

Discrete Magnetic symmetry

At the lattice scale,

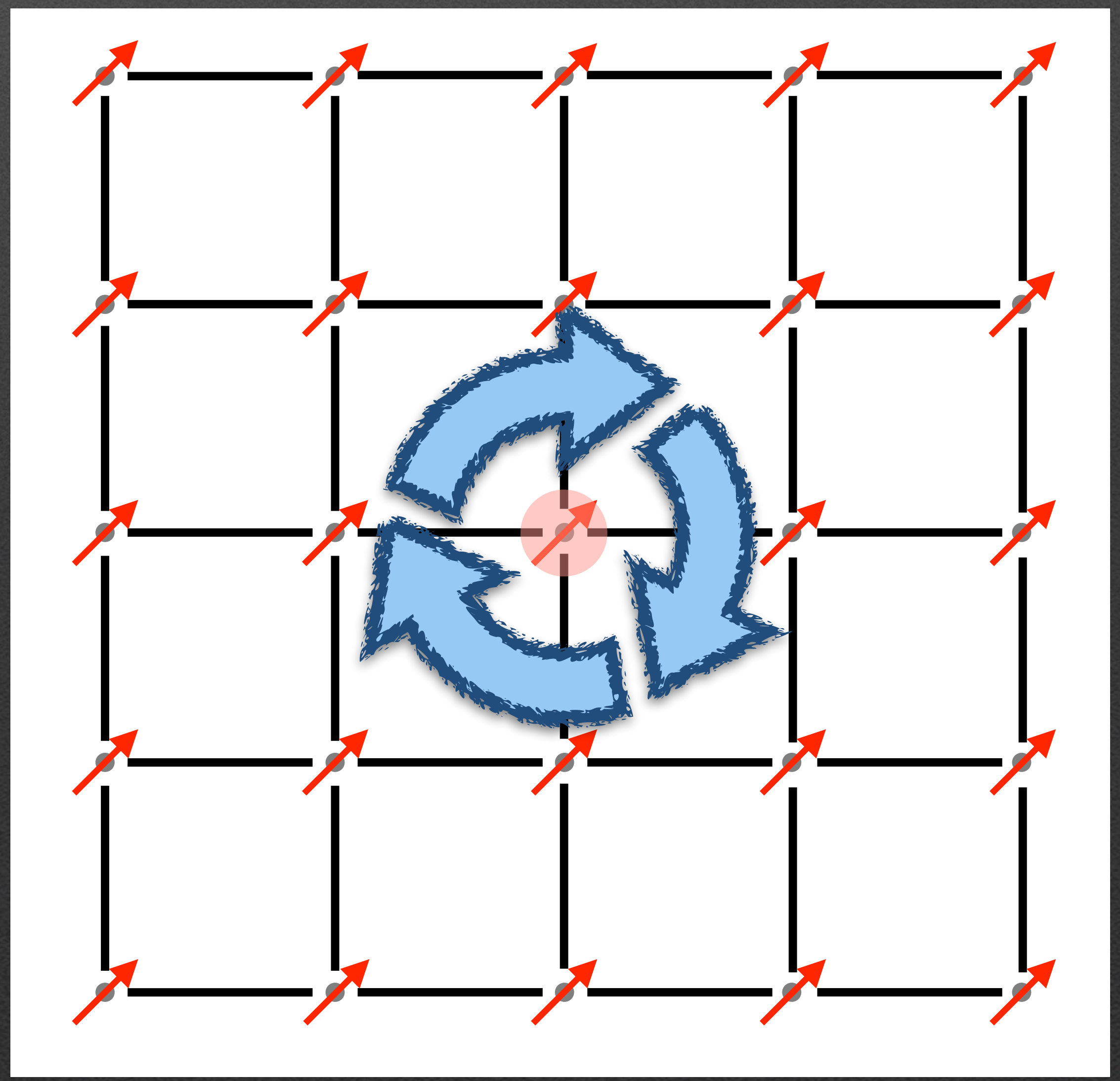
Skyrmions are charged under site-centered rotation because of the spin Berry phase.

Haldane (1988)

Magnetic symmetry ~ Site-centered rotation

$$\mathscr{M}^4 + \mathscr{M}^{-4}$$

4-monopole event



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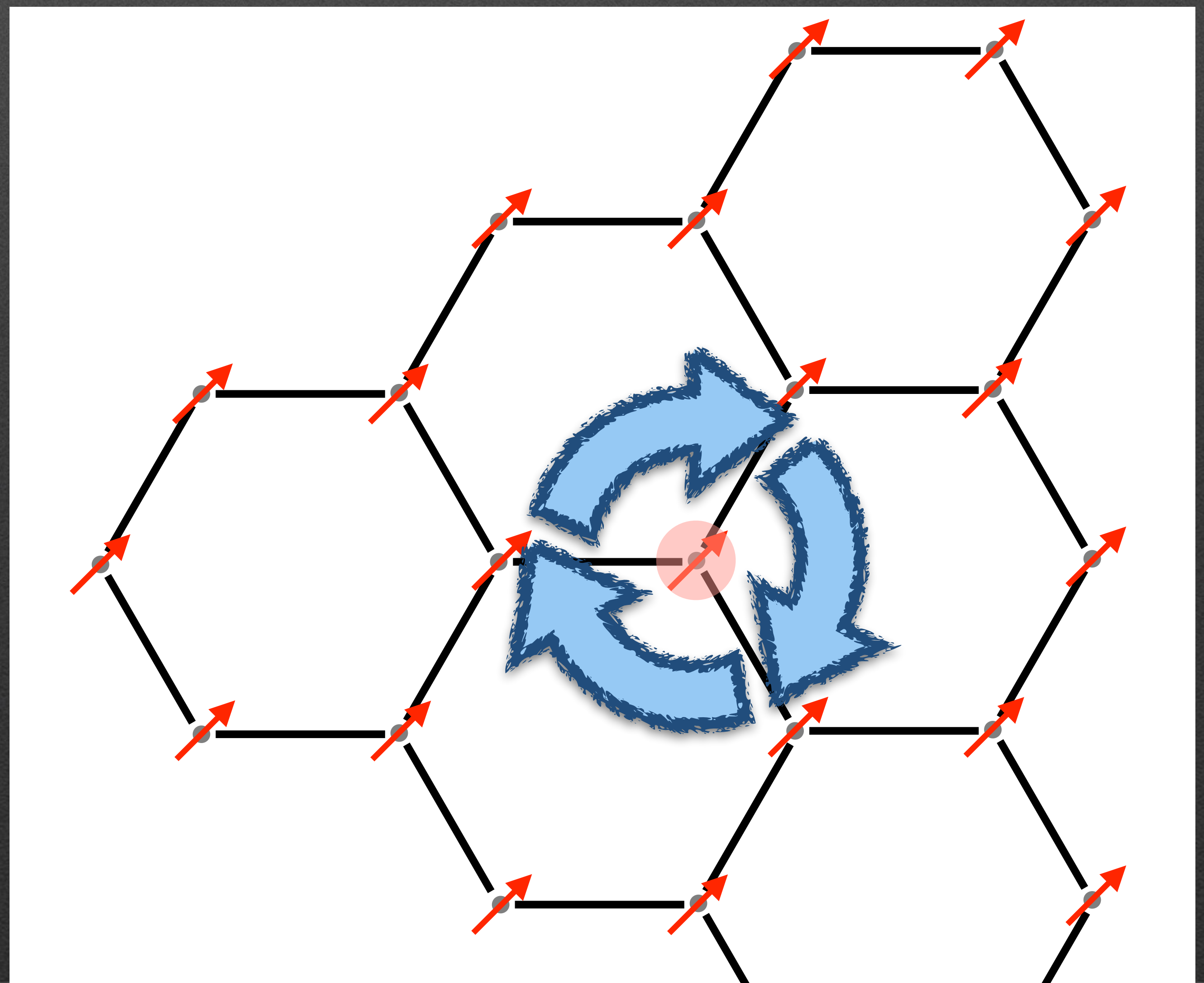
3-monopole event



Honeycomb lattice

$$\mathbb{Z}_{n=3}$$

Discrete Magnetic symmetry

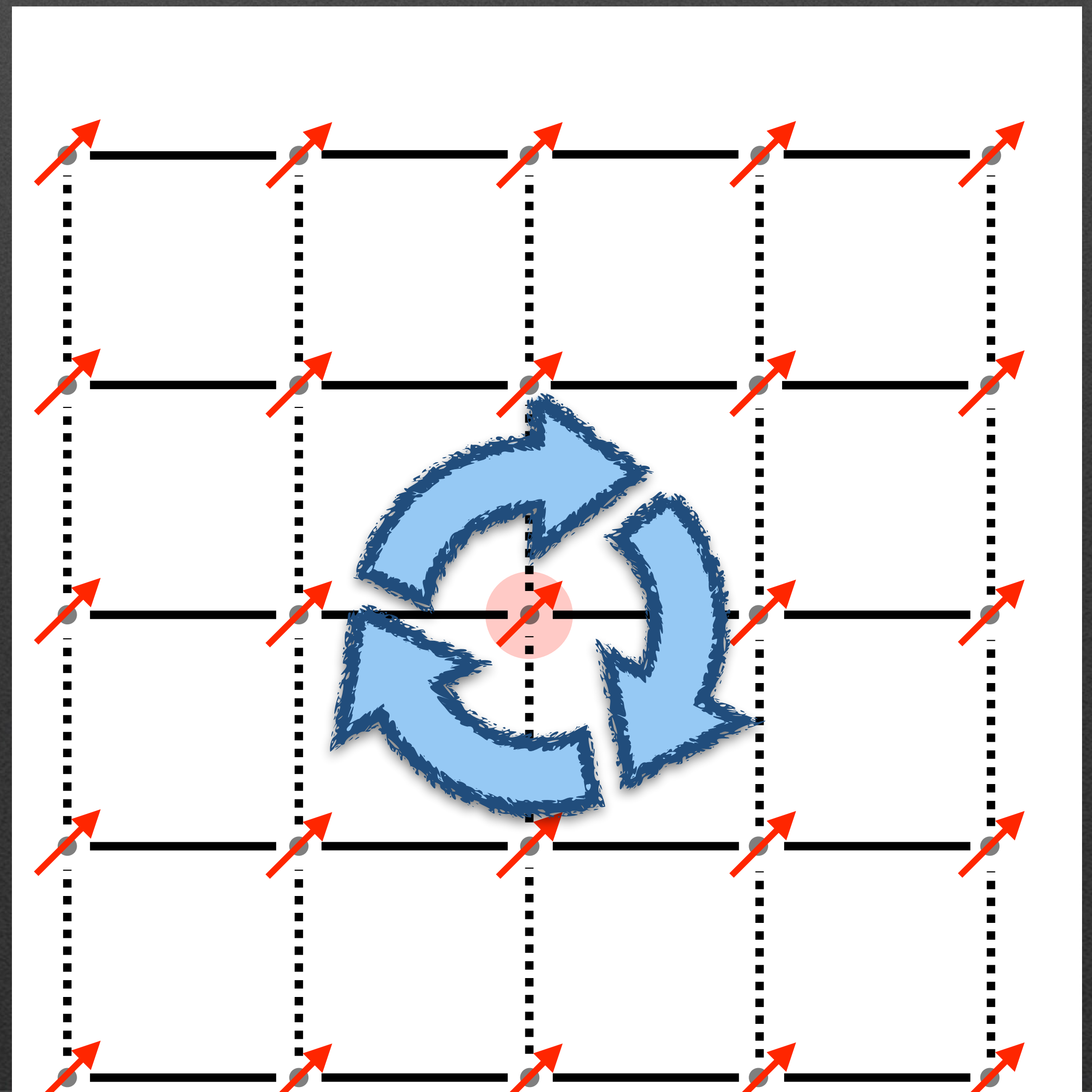
At the lattice scale,

Skyrmions are charged under site-centered rotation because of the spin Berry phase. Haldane (1988)

Magnetic symmetry ~ Site-centered rotation

$$\mathscr{M}^2 + \mathscr{M}^{-2}$$

2-monopole event



Rectangular lattice

$$\mathbb{Z}_{n=2}$$

$\mathscr{R} \times (\mathbb{Z}_n)_M$ Anomaly

Replace K by $\mathbb{Z}_n$ gauge field K_n. $\left(nK_n = dH\right)$

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$\mathscr{R} \times (\mathbb{Z}_n)_M$ Anomaly

Replace K by $\mathbb{Z}_n$ gauge field K_n. $\left(nK_n = dH\right)$

$(\mathbb{Z}_n)_M$ gauge transformation: $K_n \to K_n + d\Lambda, \;\; H \to H + n\Lambda$.

$$Z_{\mathbb{CP}^1}[w_2, K_n] \to Z_{\mathbb{CP}^1}[w_2, K_n] e^{\frac{i}{2\pi}\int d\Lambda \wedge \pi w_2}$$

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$\mathcal{R} \times (\mathbb{Z}_n)_M$ Anomaly

Replace K by $\mathbb{Z}_n$ gauge field K_n. $\left(nK_n = dH\right)$

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$$Z_{\mathbb{CP}^1}[w_2, K_n] \to Z_{\mathbb{CP}^1}[w_2, K_n]e^{\frac{i}{2\pi}\int d\Lambda\wedge\pi w_2}$$

Local counter term:

$$\frac{i}{2\pi}\int dH \wedge \pi w_2 \to \frac{i}{2\pi}\int dH \wedge \pi w_2 + \frac{in}{2\pi}\int d\Lambda \wedge \pi w_2$$

$$\begin{cases} 0 & n : \text{even} \\ i\pi & n : \text{odd} \end{cases}$$

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$\mathscr{R} \times (\mathbb{Z}_n)_M$ Anomaly

Even n : anomaly present (square, rectangular, ⋯)

Odd n : anomaly absent (honeycomb, ⋯)

$$Z_{\mathbb{CP}^1}[w_2, K_n] \to Z_{\mathbb{CP}^1}[w_2, K_n] e^{\cdots}$$

Local counter term:

$$\frac{i}{2\pi}\int dH \wedge \pi w_2 \to \frac{i}{2\pi}\int dH \wedge \pi w_2 + \frac{in}{2\pi}\int d\Lambda \wedge \pi w_2$$

$$\begin{cases} 0 & n : \text{even} \\ i\pi & n : \text{odd} \end{cases}$$

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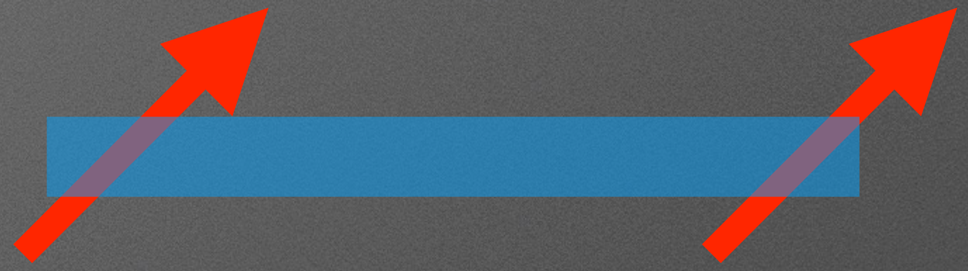
$\mathscr{R} \times (\mathbb{Z}_n)_M$ Anomaly on Lattice

Spin 1/2

$\mathscr{R} \sim \mathscr{T}$ anomaly

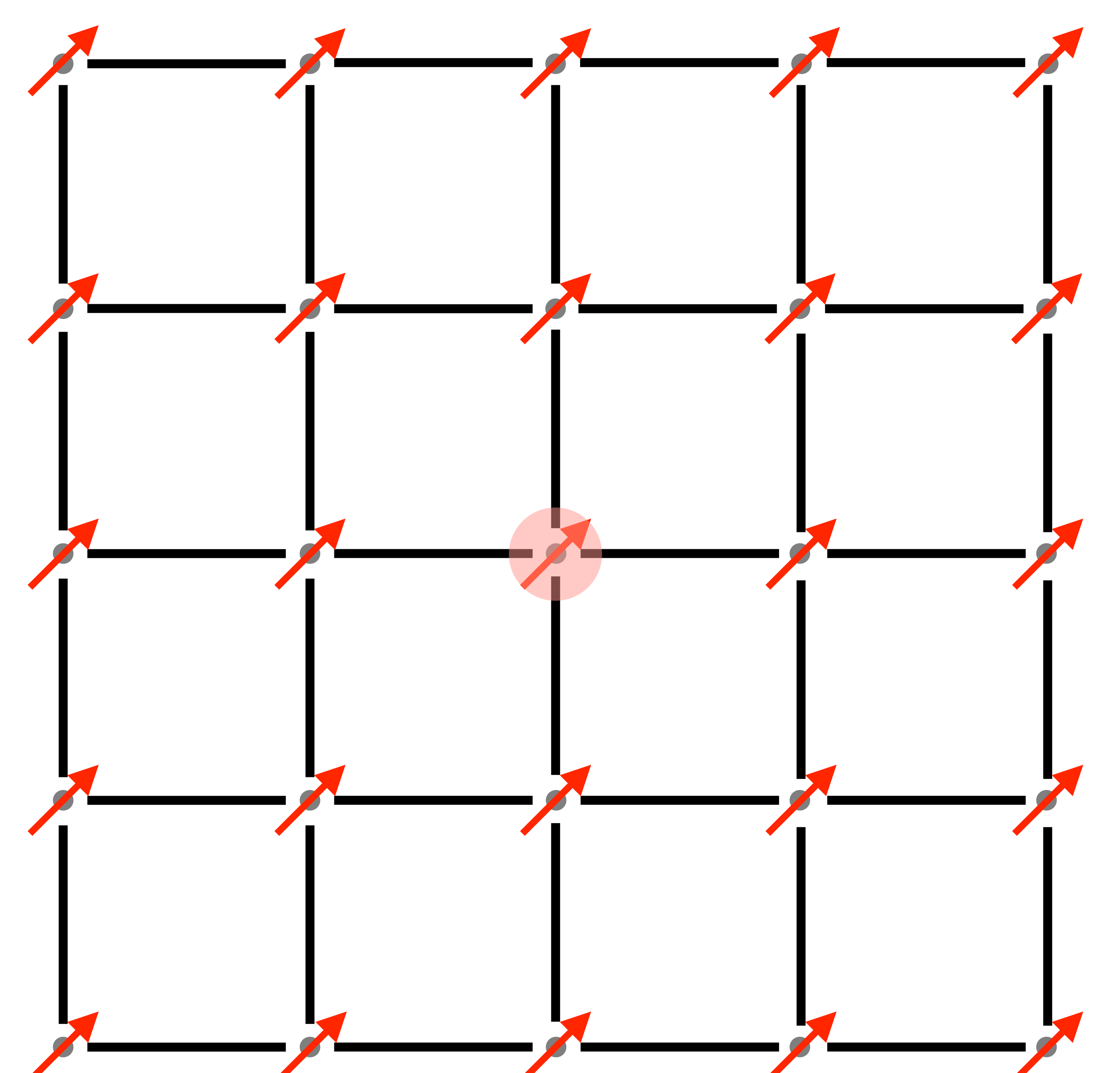
(Kramars doublet)

Haldane chain



1+1D Spin 1 system

No anomaly



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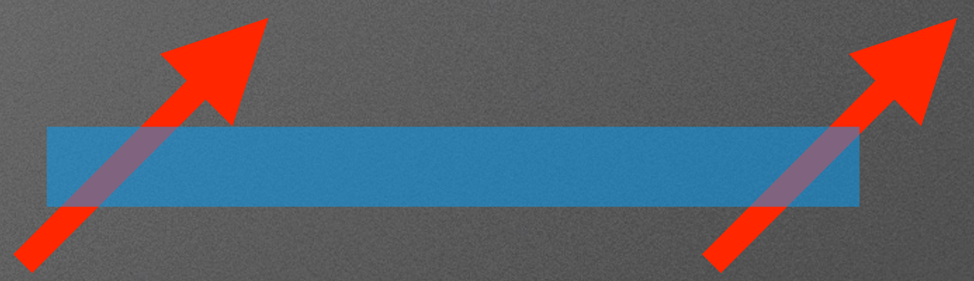

1+1D Spin 1 system

No anomaly

Can we construct the spin 1/2 systems without the anomaly??

Rule: We must keep the site-centered rotation.

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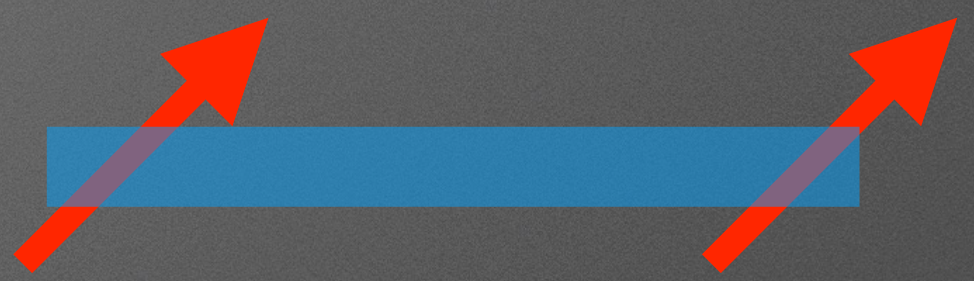
$\mathscr{R}\times(\mathbb{Z}_n)_M$ Anomaly on Lattice

Spin 1/2

$\mathscr{R}\sim\mathscr{T}$ anomaly

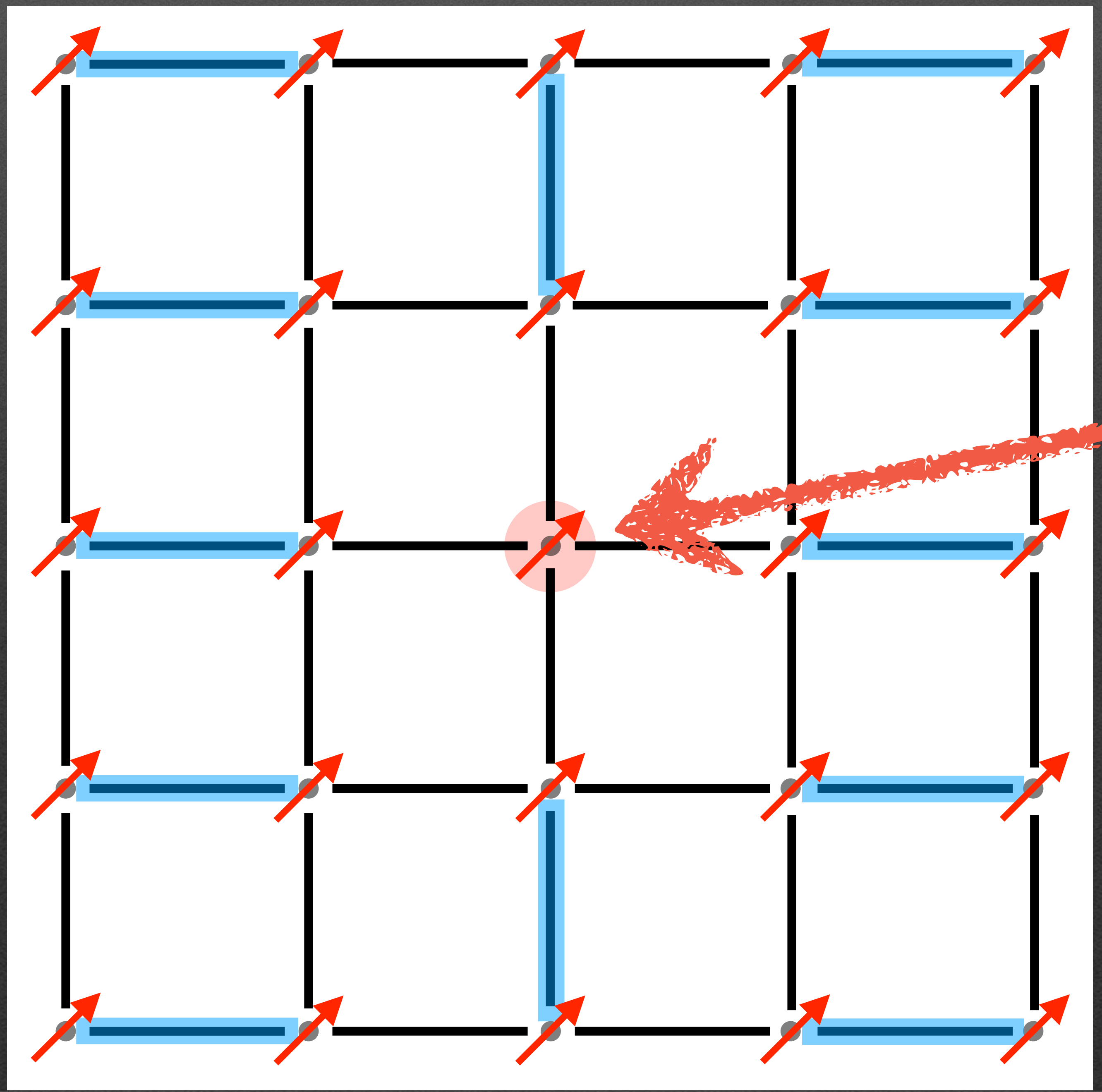
(Kramars doublet)

Haldane chain



1+1D Spin 1 system

No anomaly



Rotation center

Rectangular ($\mathbb{Z}_2$)

Anomalous

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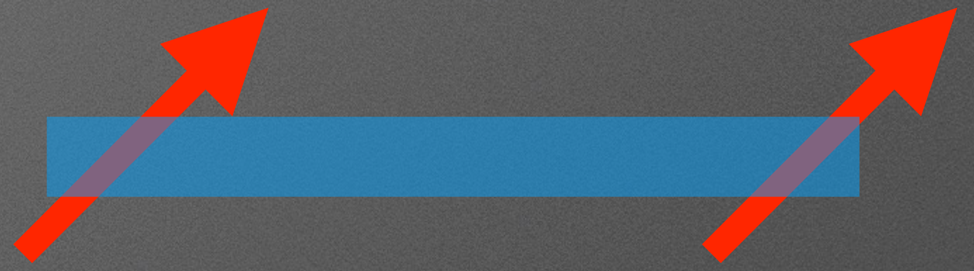
$\mathscr{R} \times (\mathbb{Z}_n)_M$ Anomaly on Lattice

Spin 1/2

$\mathscr{R} \sim \mathscr{T}$ anomaly

(Kramars doublet)

Haldane chain

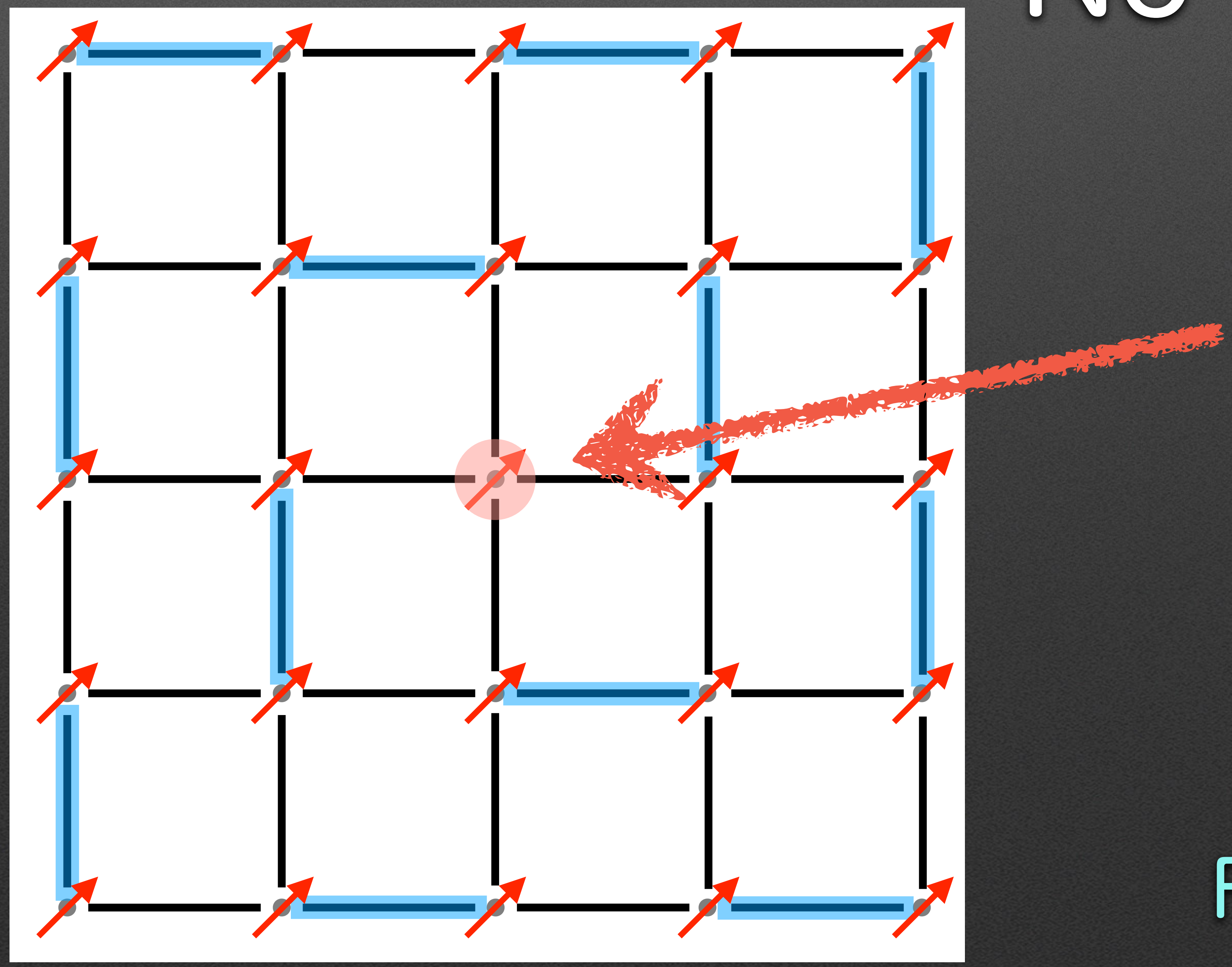


1+1D Spin 1 system

No anomaly

Square ($\mathbb{Z}_4$)

Anomalous



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$\mathscr{R} \times (\mathbb{Z}_n)_M$ Anomaly on Lattice

Spin 1/2

$\mathscr{R} \sim \mathscr{T}$ anomaly

(Kramars doubl

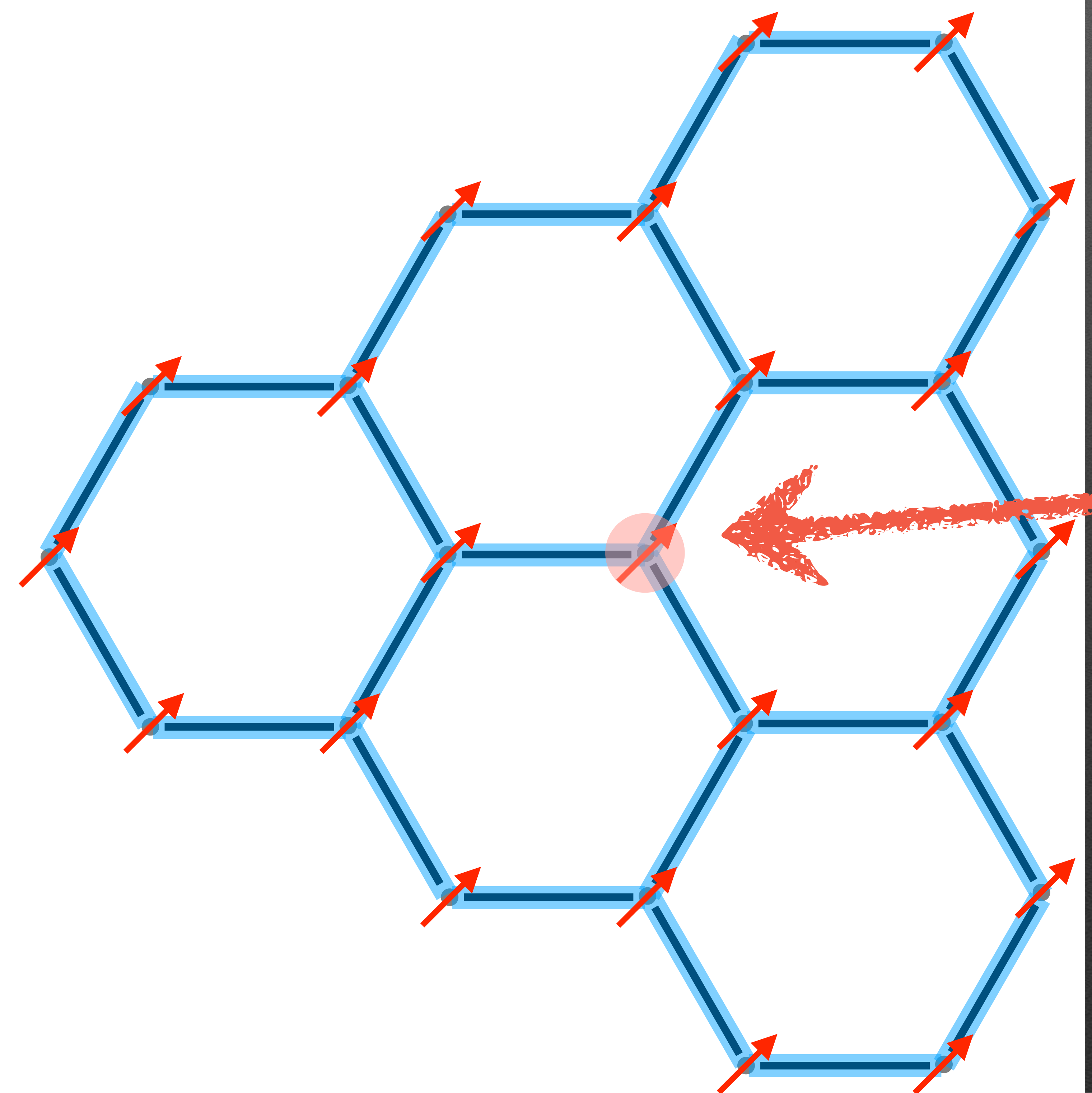
Haldane chain

1+1D Spin 1 system

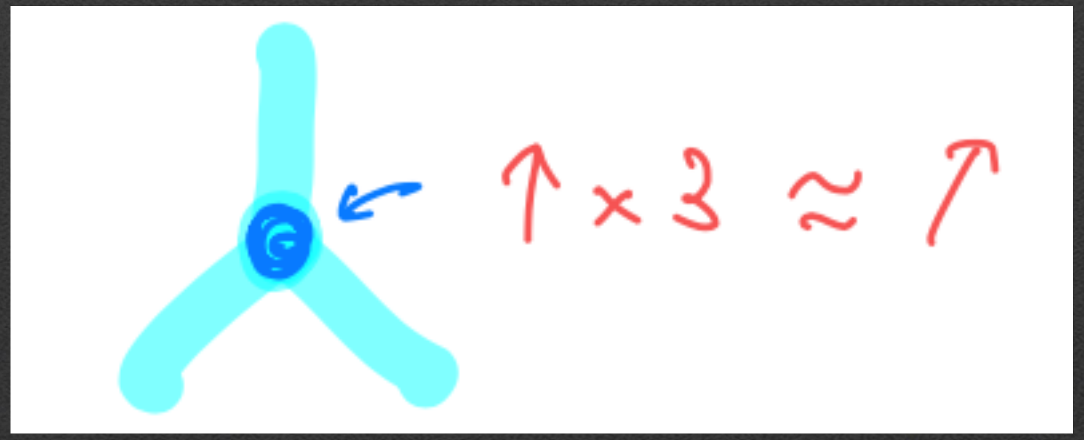
No anomaly

Honeycomb ($\mathbb{Z}_3$)

Not anomalous



Rotation center



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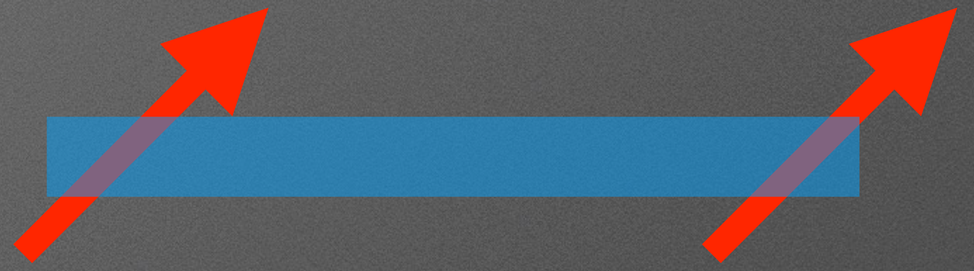
$\mathscr{R} \times (\mathbb{Z}_n)_M$ Anomaly on Lattice

Spin 1/2

Haldane chain

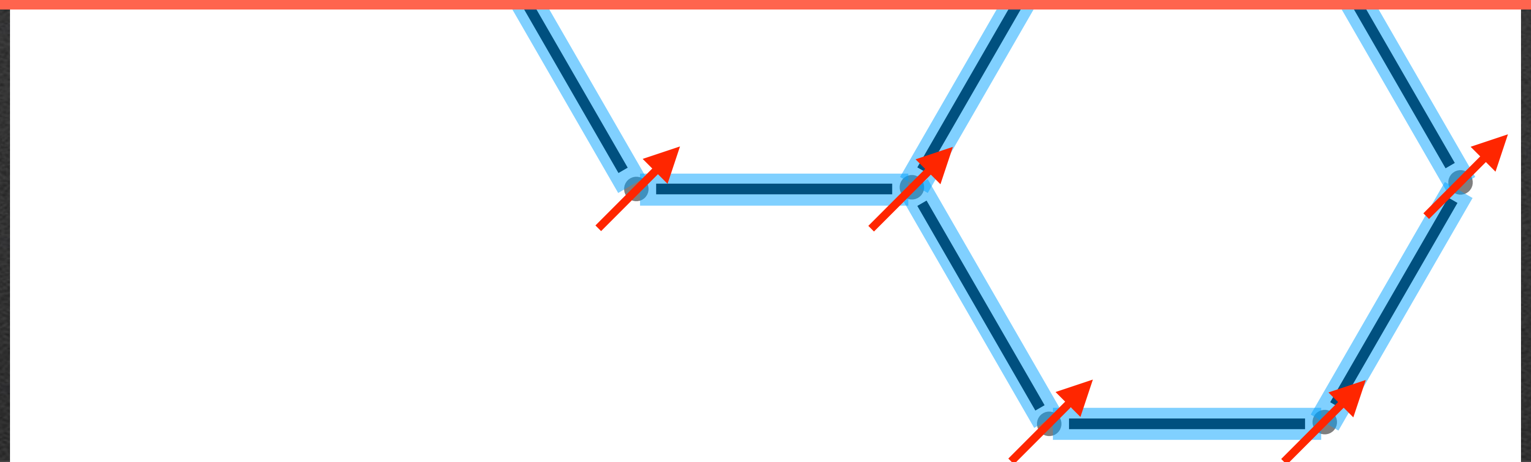


Consitent with $\mathscr{R} \times (\mathbb{Z}_n)_M$ anomaly.

It should be present at the lattice scale.

ter

Not anomalous



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Magnetic symmetry in 3+1D

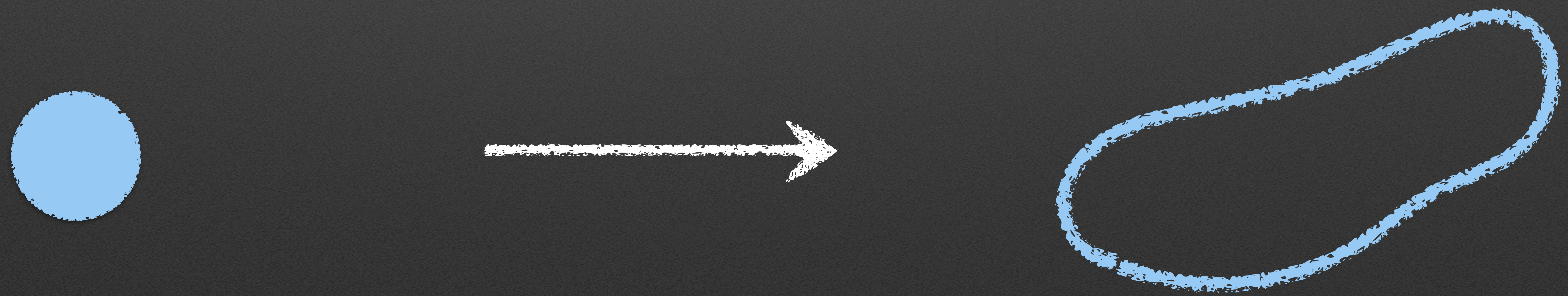
2+1D

$$J^{\mu}_{M} = \frac{i}{2\pi}\epsilon^{\mu\nu\rho}\partial_{\nu}a_{\rho} \longrightarrow J^{\mu\nu}_{M} = \frac{i}{2\pi}\epsilon^{\mu\nu\rho\eta}\partial_{\rho}a_{\eta}$$

3+1D $\mathrm{U}(1)^{[1]}_{M}$

Monopole

Magnetic loop ('t Hooft loop)



Gaiotto, Kapustin, Seiberg, Willett (2015)

Gauging $\mathscr{R} \times \mathrm{U}(1)_M^{[1]}$

$$\int |(d - ia)z|^2 + V(|z|^2) + \frac{1}{2g^2} da \wedge \star da + \frac{i}{2\pi} K \wedge da$$

(1)Adding ____ via minimal coupling

2-form gauge field

Gaiotto, Kapustin, Seiberg, Willett (2015)

Gauging $\mathscr{R} \times \mathrm{U}(1)_M^{[1]}$

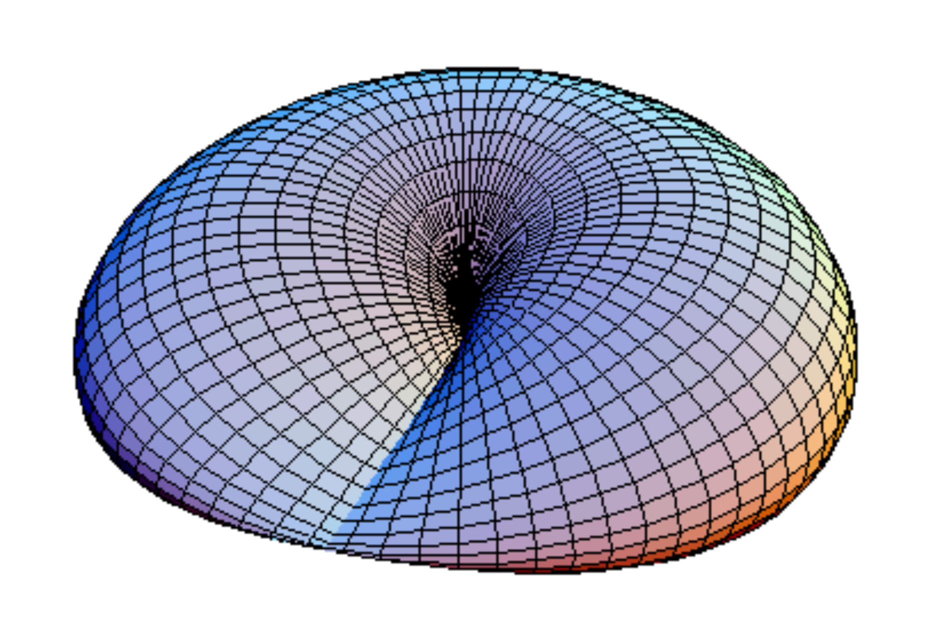
$$\int |(d - ia)z|^2 + V(|z|^2) + \frac{1}{2g^2} da \wedge \star da + \frac{i}{2\pi} K \wedge da$$

(1)Adding ______ via minimal coupling

2-form gauge field

(2)Twisted boundary condition by $\mathscr{R}$

$$\mathbb{R}_\tau \times \mathbb{R}_x \times \mathbb{RP}^2_{yz}$$



$$\int_{\mathbb{RP}^2} da = \int_{\mathbb{RP}^2} \pi w_2$$

Sulejmanpasic, Tanizaki (2018,) Furusawa, Hongo (2020)

Inconsistency on $\mathbb{R}_\tau \times \mathbb{R}_x \times \mathbb{RP}^2_{yz}$

$$\int |(d - ia)z|^2 + V(|z|^2) + \frac{1}{2g^2} da \wedge \star da + \frac{i}{2\pi} K \wedge da$$

$\mathrm{U}(1)^{[1]}_M$ gauge transformation

$$K \to K + d\Lambda$$

$$Z_{\mathbb{CP}^1}[w_2, K] \to Z_{\mathbb{CP}^1}[w_2, K] e^{\frac{i}{2\pi}\int d\Lambda \wedge \pi w_2}$$

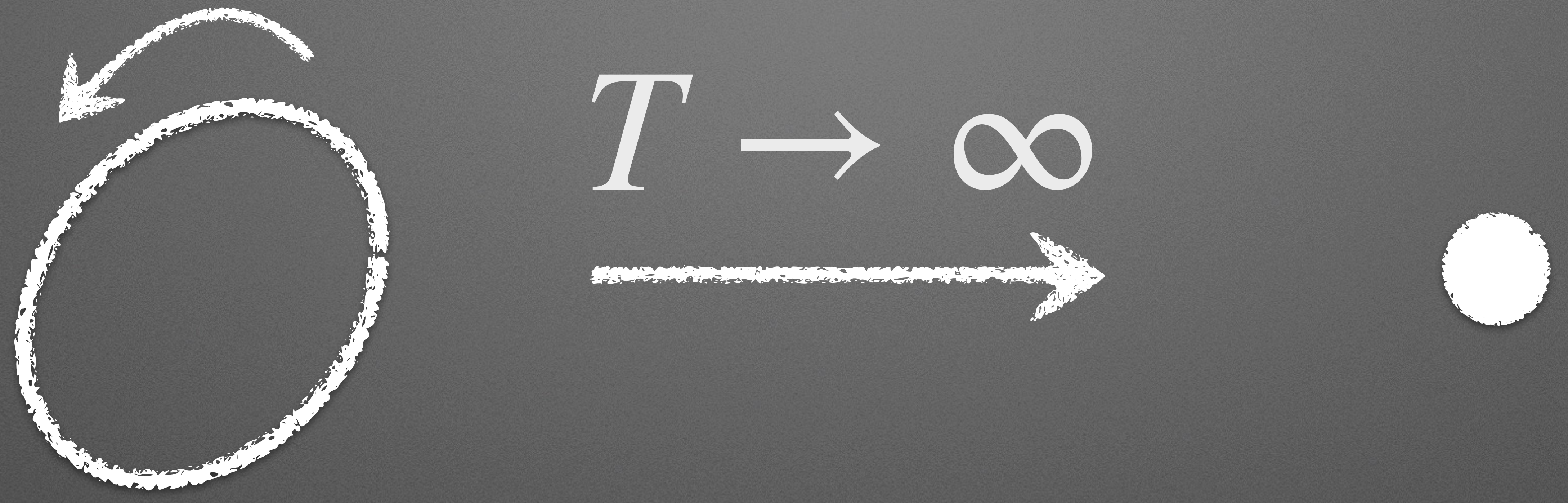
$\in i\pi\mathbb{Z}$

$\mathscr{R} \times \mathrm{U}(1)^{[1]}_M$ anomaly

Furusawa, Hongo (2020)

Fate of anomalies at finite T

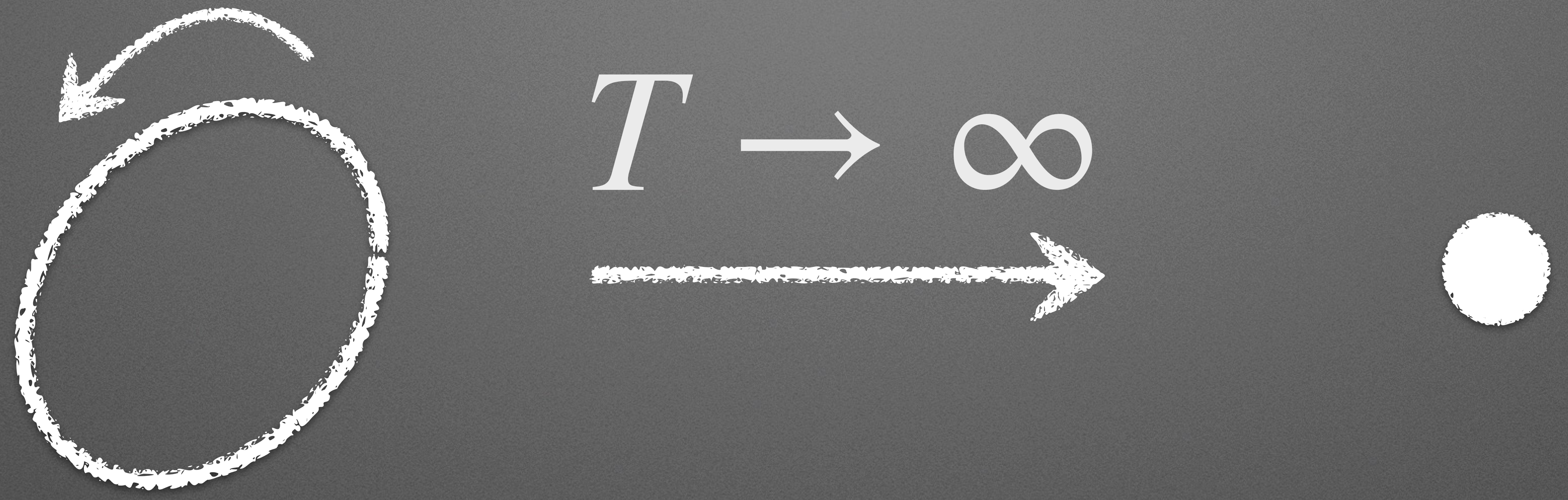
$$\beta = 1/T$$



Circle compactification.

Fate of anomalies at finite T

$\beta = 1/T$



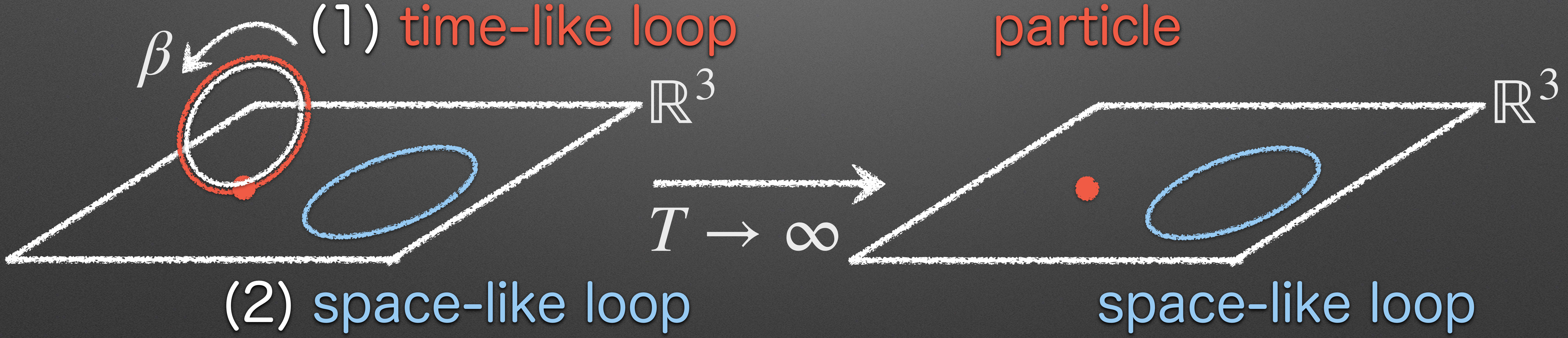
Anomalies w/ 0-form symm. $\Rightarrow$ Can be trivial.

Anomalies w/ 1-form symm. $\Rightarrow$ Must be nontrivial.

Gaiotto, Kapustin, Komargodski, Seiberg (2017), Komargodski, Sulejmanpasic, Unsal (2018)
Shimizu, Yonekura (2018), Tanizaki, Kikuchi, Misumi, Sakai (2018), Yonekura(2019), Furusawa, Hongo (2020)

1-form symmetry at finite T

At finite T, two types of magnetic loops



Decomposition of the gauge field: $K = \underline{K^{(2)}} + \frac{d\tau}{\beta} \wedge \underline{K^{(1)}}$

Inconsistency on $S^1_\tau \times \mathbb{R}_x \times \mathbb{RP}^2_{yz}$

$$\int |(d - ia)z|^2 + V(|z|^2) + \frac{1}{2g^2} da \wedge \star da + \frac{i}{2\pi} K \wedge da$$

Inconsistency on $S^1_\tau \times \mathbb{R}_x \times \mathbb{RP}^2_{yz}$

$$\int |(d - ia)z|^2 + V(|z|^2) + \frac{1}{2g^2} da \wedge \star da + \frac{i}{2\pi} K \wedge da$$

$\mathrm{U}(1)_M$ gauge transformations:

$$K^{(1)} \to K^{(1)} + d\Lambda^{(1)}$$

$$Z_{\mathbb{CP}^1}[w_2, K] \to Z_{\mathbb{CP}^1}[w_2, K] e^{\frac{i}{2\pi}\int d\Lambda^{(1)} \wedge \pi w_2}$$

$\mathscr{R} \times \mathrm{U}(1)_M$ anomaly at finite T

(See also our paper for $\mathrm{PSU}(N) \times \mathrm{U}(1)^{[1]}_M$ anomaly at finite T)

Finite-T phase diagram

Anomalies at finite T ⇒ No trivial phase at any T

Finite-T phase diagram

Anomalies at finite T ⇒ No trivial phase at any T

* Typical phases: Neel phase & U(1) spin liquid phase

(Higgs) (Coulomb)

SSB of $\mathscr{R}$ [or $SO(3)_{\rm spin}$]

⇒ vacuum degeneracy

[NG bosons]

SSB of $U(1)^{[1]}_M$

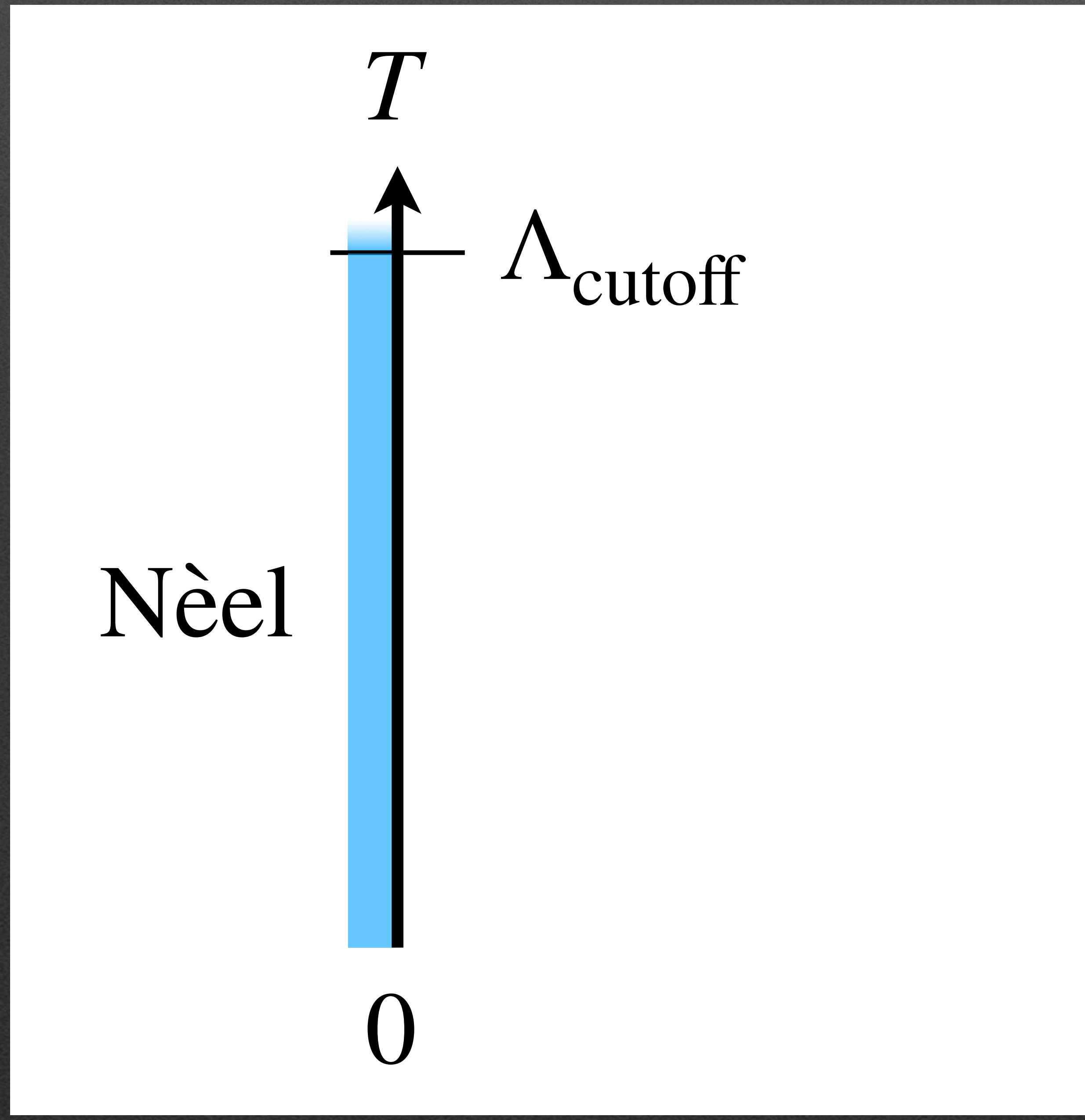
⇒ Dynamical gauge boson

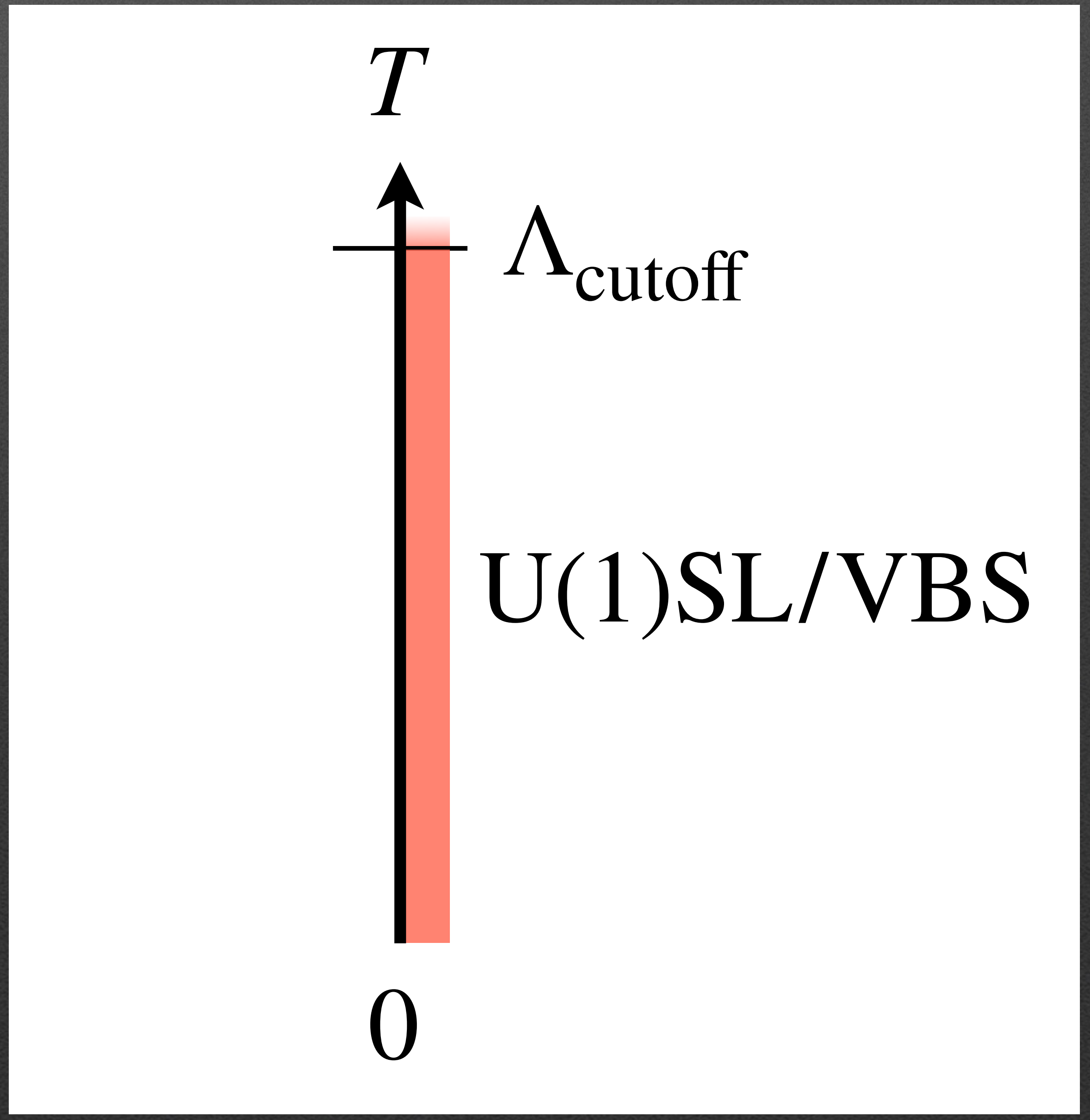
(c.f. Hidden-local symm.)

Finite-T phase diagram

Anomalies at finite T ⇒ No trivial phase at any T

* Typical phases: Neel phase (Higgs) & U(1) spin liquid phase (Coulomb)





Finite-T phase diagram

Anomalies at finite T ⇒ No trivial phase at any T

* Typical phases: Neel phase & $\mathrm{U(1)}$ spin liquid phase

(Higgs) (Coulomb)

Breaks at $T_{\mathrm{N\grave{e}el}}$

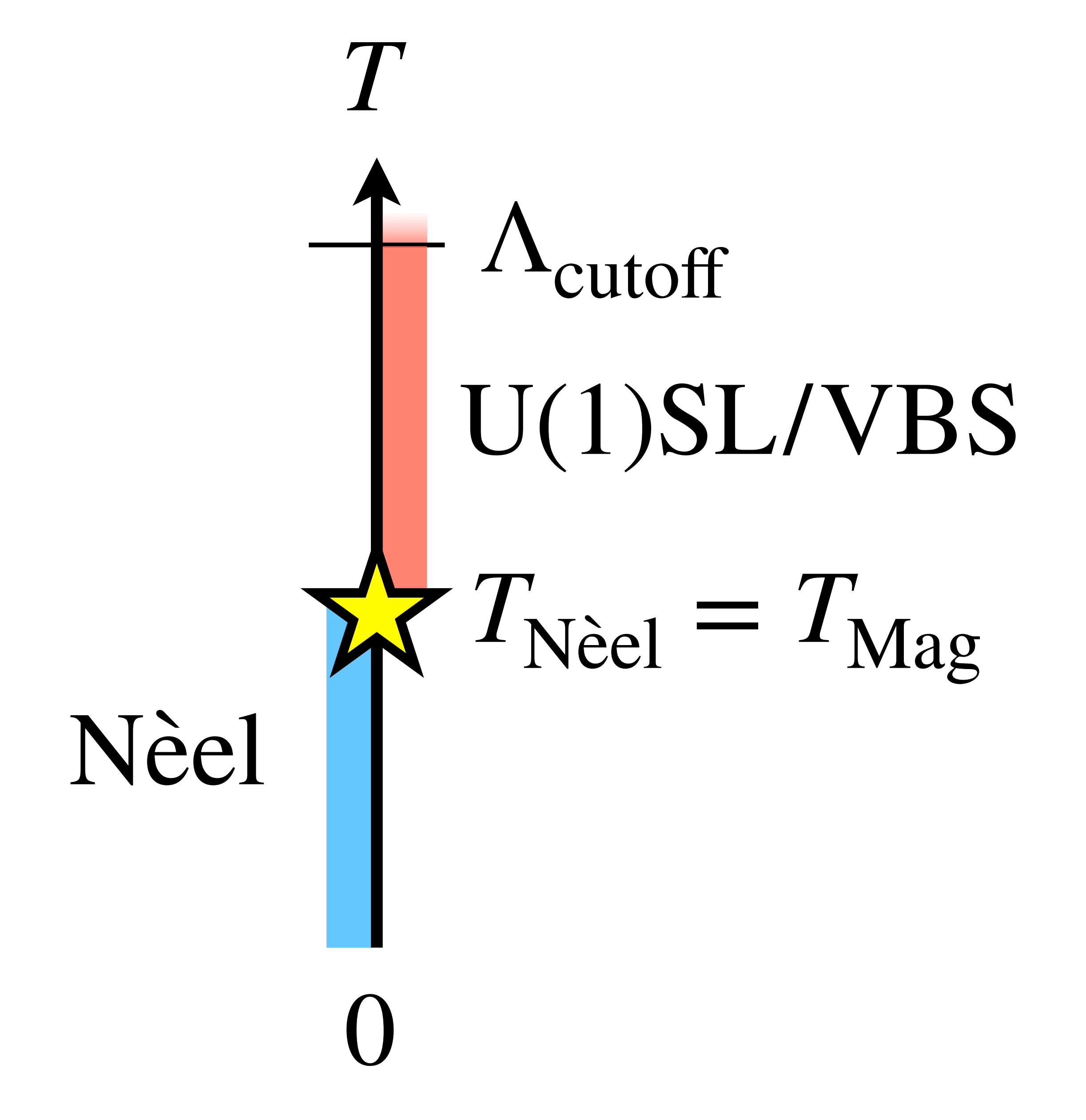
Realizes at $T_{\mathrm{Mag.}}$

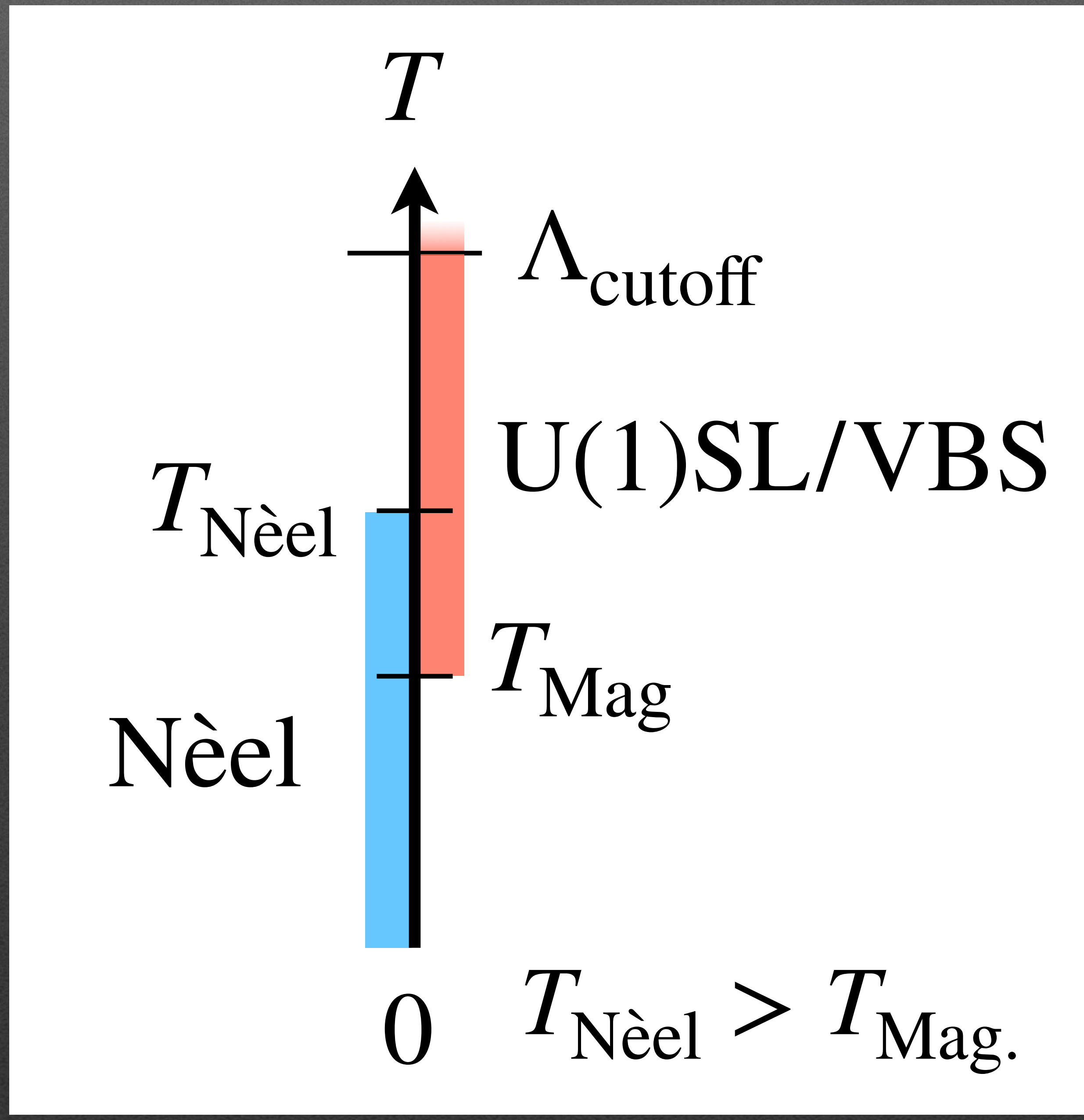
Caution: 3+1D $\mathbb{CP}^1$ model is cutoff theory w/ $\Lambda_{\mathrm{cutoff}}$.

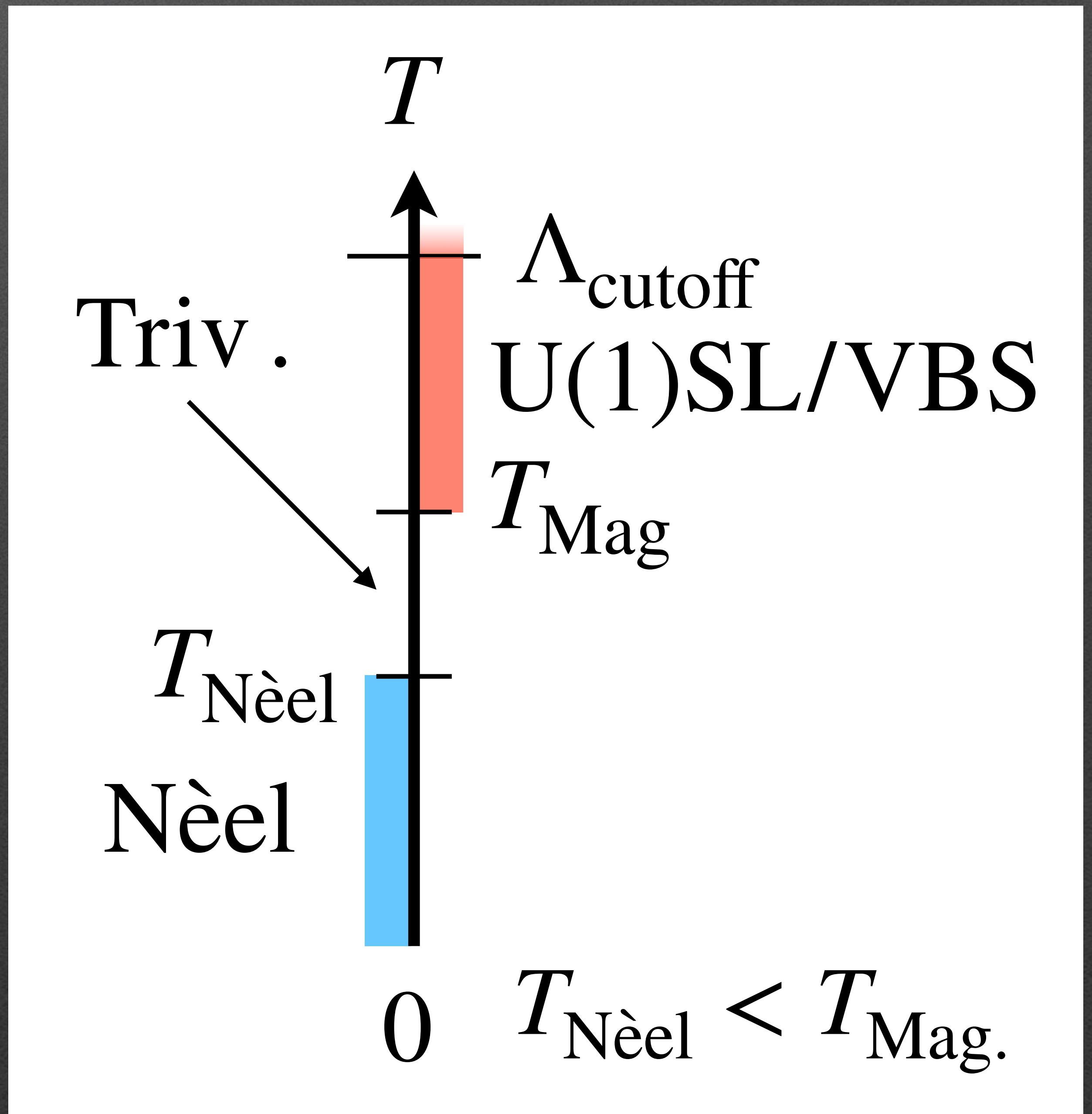
Finite-T phase diagram

Anomalies at finite T ⇒ No trivial phase at any T

* Typical phases: Neel phase & U(1) spin liquid phase





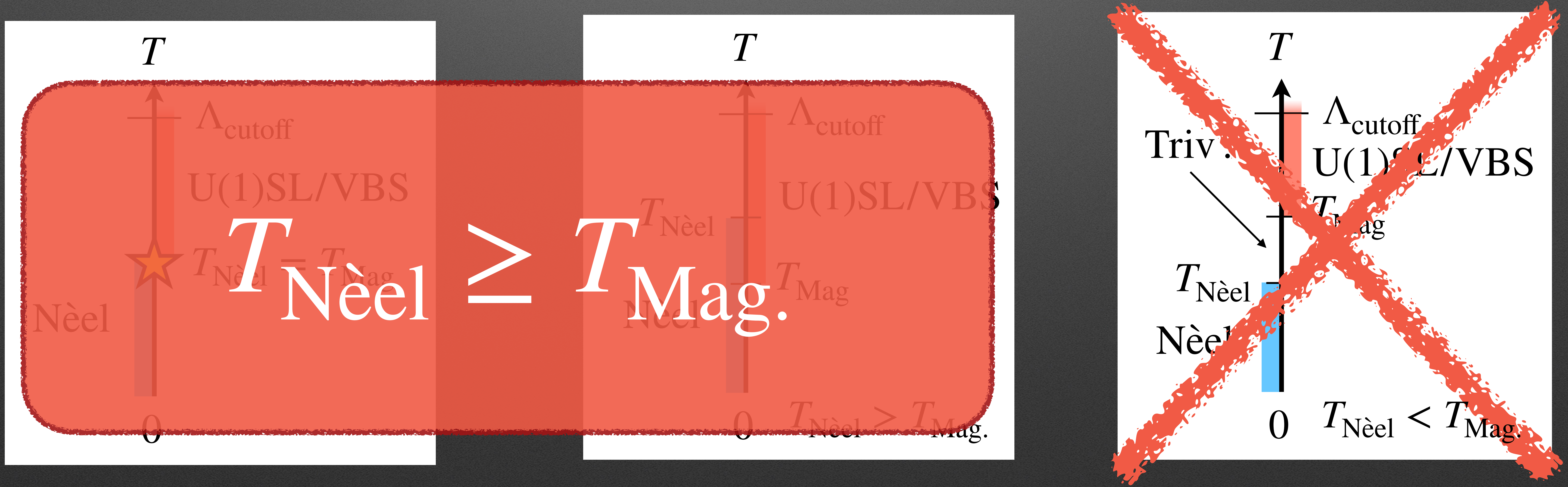


Caution: 3+1D $\mathbb{CP}^1$ model is cutoff theory w/ Λ_{cutoff}.

Finite-T phase diagram

Anomalies at finite T ⇒ No trivial phase at any T

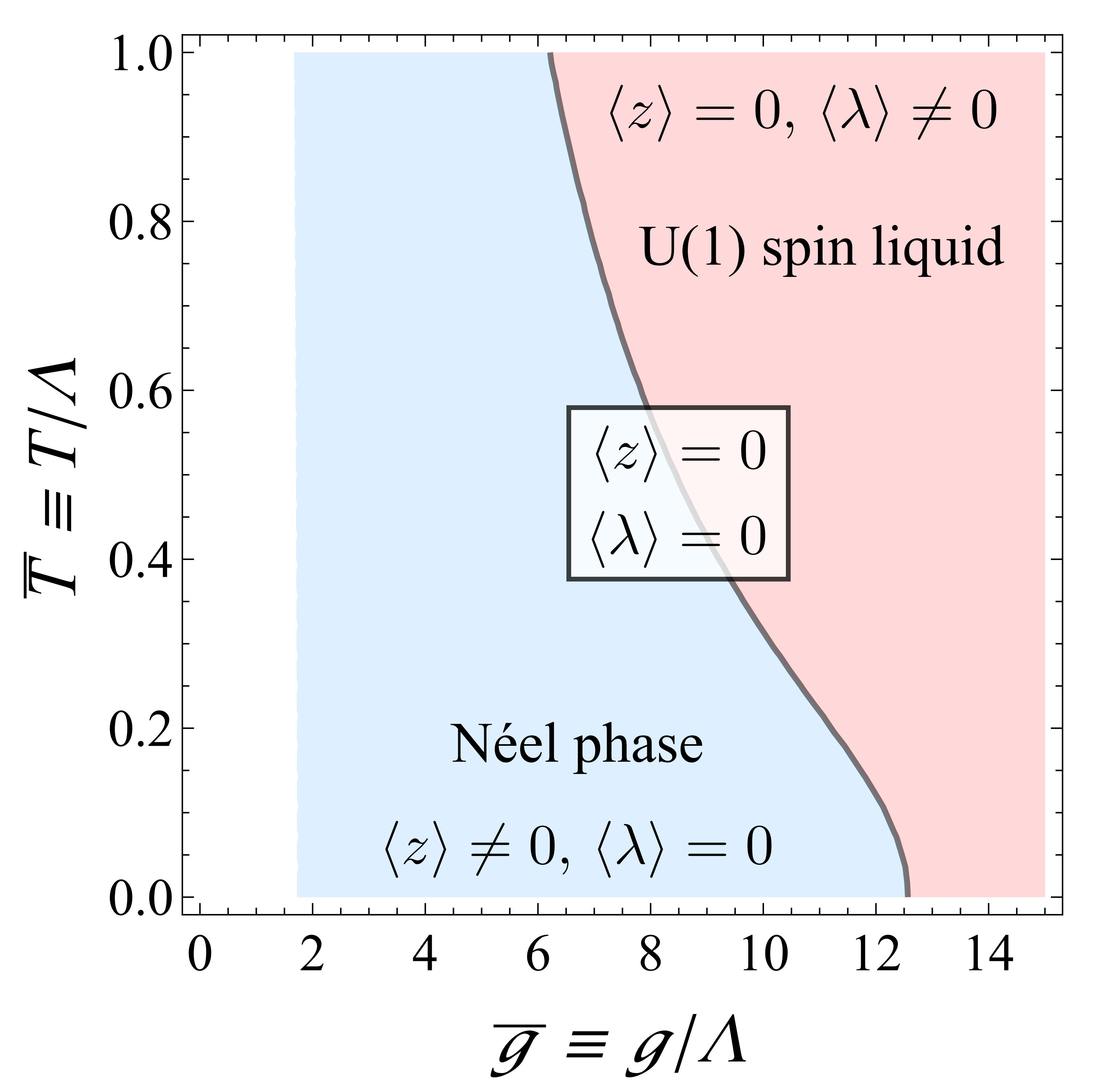
* Typical phases: Neel phase & U(1) spin liquid phase



Caution: 3+1D $\mathbb{CP}^1$ model is cutoff theory w/ Λ_{cutoff}.

Large-N analysis

Phase diagram in the Large-N limit



The $\mathscr{R} \times \mathrm{U}(1)_M^{[1]}$ anomaly for even N.

Consistent with anomaly matching!!

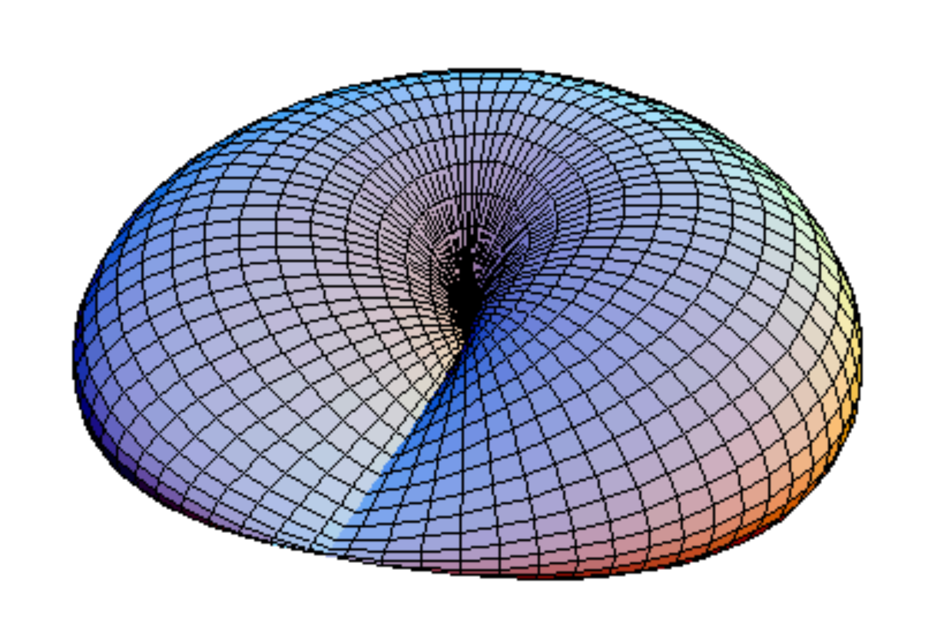
2nd order direct transition.

Summary

Anomalies in $\mathbb{CP}^1$ model in 2+1D & 3+1D are studied:

$\mathscr{R} \times \mathrm{U}(1)_M$ anomaly in $\mathbb{CP}^1$ model in 2+1D

⇒ Ingappability w/o flavor symmetry.



$\mathscr{R} \times (\mathbb{Z}_n)_M$ anomaly present for even n/ not for odd n.

1-form magnetic symmetry in 3+1D

⇒ $\mathscr{R} \times \mathrm{U}(1)_M$ anomalies in 3+1D

3+1D anomalies at any T.

⇒ Constraint on the finite-T phase diagram

$T_{\text{Nèel}} \geq T_{\text{Mag.}}$

Backup slides

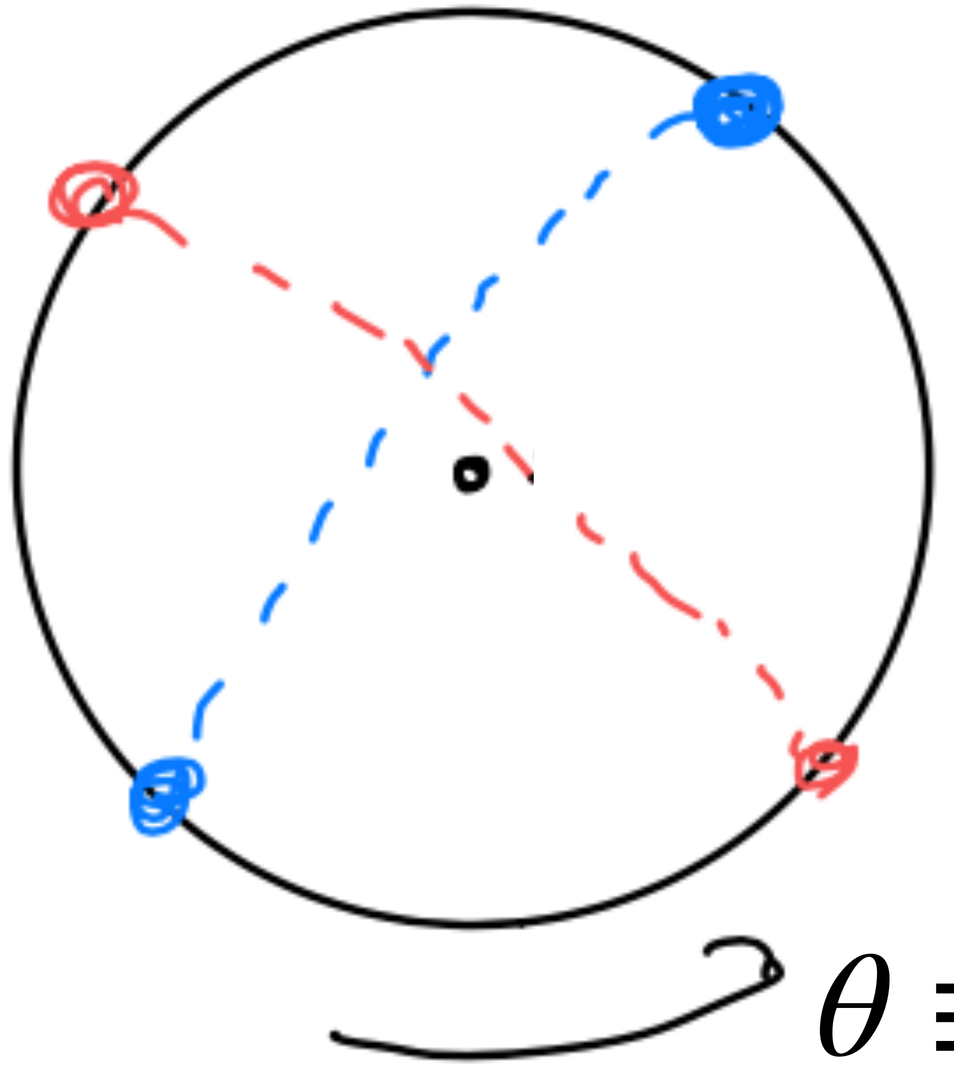
Dirac quantization on $\mathbb{RP}^2$

Boundary condition:

$$\phi(\theta+\pi) = (i\sigma^2)e^{i\eta(\theta)}\phi^*(\theta)$$

$$a_\theta(\theta+\pi) = -a_\theta(\theta) + \partial_\theta\eta(\theta)$$

$\mathbb{RP}^2 \simeq$



$\theta \equiv \theta + 2\pi$

A constraint on $\eta(\theta)$:

$$\phi(\theta+2\pi) = (i\sigma^2)e^{i\eta(\theta+\pi)}\phi^*(\theta+\pi)$$
$$= (i\sigma^2)e^{i\eta(\theta+\pi)}(i\sigma^2)e^{-i\eta(\theta)}\phi(\theta)$$

$$\eta(\theta+\pi) - \eta(\theta) \in \pi + 2\pi\mathbb{Z}$$

Fractional Dirac quantization:

$$\int_{\mathbb{RP}^2} da = \int_0^\pi [a_\theta(\theta+\pi) + a_\theta(\theta)] = \int_0^\pi \partial_\theta\eta(\theta) = \pi \pmod{2\pi}$$