Pairing in Nuclear Matter

- General theory: Gorkov equations, BCS approximation
- In-medium interaction: Polarization effects
- nn pairing gaps in neutron matter
- np pairing in asymmetric nuclear matter
- Transition to Bose-Einstein (deuteron) condensation
- Application: Pairing gaps in neutron stars

Collaboration with

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**Motivation:**

- **Theoretical goal:**
  Microscopic calculation of pairing gaps from fundamental interaction (bare potential):

- **Experimental relevance:**
  - Electron systems
  - $T=1$ nn, pp pairing:
    - Pairing force in finite (halo) nuclei
    - Superfluidity in neutron stars: glitches, cooling
  - $T=0$ np pairing:
    - Relevance for finite nuclei ($N \approx Z$)?
    - Deuteron correlations, production
  - Pairing in trapped atomic gases
  - Color superconductivity
Superfluid Fermi Systems:

- **General Framework: Gorkov Equations:**

\[
G = G_0 + \Sigma + \Delta \\
F = \Sigma + \Delta
\]

Generalization of Dyson equation:
Gap function \( \Delta \) is analog of self-energy \( \Sigma \)

- **Gap Equation (4-dim):**

\[
\Sigma(k) = i \int \frac{d^4k'}{(2\pi)^4} \left\langle k, k' \mid T \mid k, k' \right\rangle G(k')
\]

\[
\Delta(k) = i \int \frac{d^4k'}{(2\pi)^4} \left\langle k, -k \mid \Gamma \mid k', -k' \right\rangle F(k')
\]

Irreducible interaction kernel

- **Simplest (BCS) approximation:** \( \Gamma = V \) (bare potential):

\[
\Sigma(k) = \sum_{k'} u_{k'}^2 \left\langle k, k' \mid V \mid k, k' \right\rangle_a
\]

\[
\Delta(k) = \sum_{k'} (uv)_{k'} \left\langle +k', -k' \mid V \right| +k, -k \left\rangle_a \left\langle k' \mid V \mid k \right\rangle
\]

Mean field approximation!
$^1S_0$ $nn$ Gap with and without Polarization Effects:

- **Free potential:**

\[ \Gamma = V \]

- **Including polarization:**

\[ \Gamma = V + \text{loop} + \ldots \]

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**Free s.p. spectrum**

- Paris
- Argonne $V_{14}$
- Argonne $V_{18}$
- CD-Bonn
- Nijmegen I
- Nijmegen II

**Including polarization**

- Chen et al., NPA 451, 509 (1986)
- Ainsworth et al., PLB 222, 173 (1989)
- Chen et al., NPA 555, 59 (1993)
- Wambach et al., NPA 555, 128 (1993)
- Schulze et al., PLB 375, 1 (1996)
- Schwenk et al., NPA 713, 191 (2003)

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"low density"
Beyond First Order: Babu-Brown Approach:


Too difficult to solve exactly:
Strong approximations necessary

Large uncertainty of results
- BCS results with bare $nn$ potentials:
  

- Not constrained by phase shifts above $k_F \approx 2 \text{ fm}^{-1}$
- Self-energy effects are large
- $P-F$ coupling is important
- Polarization effects are unknown
- TBF are important

(Schwenk & Friman, PRL 92: $\Delta_{3P_2} < 10^{-2} \text{ MeV}$)
Gaps in Neutron Star Matter:

EOS: BHF (V18 + UIX + NSC89)

Free s.p. spectrum

\[ V_{18} \]

\[ ^1S_0 \]

\[ ^3PF_2 \]

BHF s.p. spectrum

\[ V_{18} \]

\[ V_{18} + UIX \]

\( \Delta [\text{MeV}] \)

\( \rho_B [\text{fm}^{-3}] \)

Self-energy effects suppress gaps

TBF suppress \(^p p ^1S_0\) but strongly enhance \(^3PF_2\) gaps!

Pairing in Asymmetric Matter:

- **Principal equations:**

\[
\Delta_{k'} = - \sum_k V_{kk'} \frac{\Delta_k}{2E_k} \left[ 1 - f(E_k^+) - f(E_k^-) \right]
\]

\[
\rho_1 + \rho_2 = \sum_k \left[ 1 - \frac{\epsilon_k}{E_k} \left[ 1 - f(E_k^+) - f(E_k^-) \right] \right]
\]

\[
\rho_1 - \rho_2 = \sum_k \left[ f(E_k^-) - f(E_k^+) \right]
\]

\[
\mu = \frac{(\mu_1 + \mu_2)}{2}, \quad \delta \mu = \frac{(\mu_1 - \mu_2)}{2}, \quad E_k^\pm = E_k \pm \delta \mu
\]

- **At zero temperature:** \(f(E_k^+) = 0\), \(f(E_k^-) = \theta(\delta \mu - E_k)\):

Unpaired particles concentrated in region around \(\mu\), Pauli-blocking the gap equation

Strong suppression of the gap with asymmetry

- **Solution in weak-coupling approximation \(\Delta \ll \mu\):**

\[
\frac{\Delta_\alpha}{\Delta_0} = \sqrt{1 - \frac{\alpha}{\alpha_{\text{max}}}}, \quad \alpha = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}
\]

\[
\alpha_{\text{max}} = \frac{3\Delta_0}{4\mu} = \frac{6}{e^2} \exp \left[ \frac{\pi}{2k_Fa} \right]
\]

- **Very small maximal asymmetry allowing pairing!**
Transition to Bose-Einstein Condensation:

- In case of strong attraction with a bound state:
  \[ \mu \xrightarrow{\rho \to 0} \mu_B = -\frac{E_B}{2} < 0 \]

- Two equations for \( \Delta_k \) and \( \mu \):
  \[ \Delta_{k'} = \sum_k (u v)_k V(k, k') \quad (u v)_k = \frac{-\Delta_k}{2E_k}, \quad E_k^2 = \epsilon_k^2 + \Delta_k^2 \]
  \[ \rho = 2 \sum_k v_k^2 \quad v_k^2 = \frac{1}{2} \left( 1 - \frac{\epsilon_k}{E_k} \right), \quad \epsilon_k = \frac{k^2}{2m} - \mu \]

Combination:

Schrödinger equation with Pauli-blocking \( \rightarrow \mu(\rho) \):

- weak coupling, \( \Delta \ll \mu \):
  - Cooper pairs:

- strong coupling, \( \mu < 0 \):
  - Bound states:

Interpretation: Formation and Bose-Einstein condensation of bound states below the Mott density

Example: n-p pairing in the \(^3SD_1\) channel \( \rightarrow \) deuteron . . .
Deuteron Condensation in Asymmetric Matter:

U. Lombardo, P. Nozières, P. Schuck, H.-J. Schulze, A. Sedrakian; PRC 46, 064314

- **Blocking windows:**

  \[ \mu \gg \Delta : \]

  Cooper pairs

  \[ \mu \to -E_D/2 : \]

  Bound states

- **Interpretation:** Bose-condensed deuterons and a Fermi sea of excess neutrons coexist. Pauli-blocking due to the excess neutrons becomes negligible in the dilute limit.

- **Analytical low-density result** \((E_D = 2.2 \text{ MeV})\):

  \[
  \frac{\Delta}{E_D}(\rho, \alpha) = \sqrt{\frac{\rho(1 - \alpha)}{\rho_\mu}}, \quad \rho_\mu \equiv \frac{(mE_D)^{3/2}}{2\pi} \approx 0.0020 \text{ fm}^{-3}
  \]

  Much stronger than weak-coupling pairing

  Superfluid at any asymmetry

- **Numerical calculation** (bare potential, free s.p. spectrum):

  \[ \alpha_{\text{max}}(\rho_0) \approx 0.2 \]

  \[ \rho_{\text{Mott}} \approx \rho_0/100 \]

  symmetric matter

  neutron matter
Summary

- BCS is never valid:
  - Low density ($k_F \ll 1/|a|$): $\Delta \xrightarrow{k_F \rightarrow 0} \Delta_{BCS}/(4\epsilon)^{1/3}$ approached from below!
  - Higher density: Polarization diagrams in pp and ph channels are important → Babu-Brown approach
  - Result: polarization suppresses the BCS $^1S_0$ gap

- Asymmetry destroys pairing rapidly: $\alpha_{\text{max}} = 3\Delta_0/4\mu$

- Transition to BE (deuteron) condensation:
  - Strong $^3S^1_D$ pairing at any asymmetry

- Future Problem: Reliable calculation at normal density:
  - Polarization interaction beyond first order
  - 4-dim gap equation

- Future Applications:
  - Gaps in neutron stars: cooling, glitches
  - Microscopic pairing forces in finite nuclei