# Electromagnetic Cross Sections of Double Giant Dipole Resonances in $^{136}$Xe and $^{208}$Pb within the Phonon Damping Model

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(Received 21 April 2000)

The electromagnetic cross sections of the double giant dipole resonances (DGDR) in $^{136}$Xe and $^{208}$Pb are calculated using the strength functions obtained within the phonon damping model. The parameters of the model have been selected to describe reasonably well the single giant dipole resonance in these nuclei. The results are found in an overall agreement with the recent experimental data for the DGDR cross sections in exclusive measurements at near-relativistic energies.

PACS numbers: 24.30.Cz, 21.10.Pc, 21.60.–n, 25.70.Lm

The recent observation of the double giant dipole resonances (DGDR) in heavy-ion collisions at near-relativistic energies [1–3] has been a challenge for the theoretical studies of multiphonon excitations. According to the independent-phonon (or harmonic) picture, a DGDR is considered as a two-dipole-phonon resonance, which is a (single-phonon) giant dipole resonance (GDR) built on top of another (single-phonon) GDR. As such, the DGDR parameters can be calculated by folding two independent GDRs [4]. As a result, the DGDR energy $E_{\mathrm{DGDR}}$ is found to be $2E_{\mathrm{GDR}}$ ($E_{\mathrm{GDR}}$ is the GDR energy), and the DGDR full width at the half-maximum (FWHM) $\Gamma_{\mathrm{DGDR}}$ is equal to $2\Gamma_{\mathrm{GDR}}$ ($\Gamma_{\mathrm{GDR}}$ is the GDR FWHM), if folding Lorentzian photoabsorption cross sections are used, or to $\sqrt{2}\,\Gamma_{\mathrm{GDR}}$, if Gaussians are folded. The experimentally extracted energy and width of DGDR differ slightly from these values. The energy shift $\Delta E \equiv 2E_{\mathrm{GDR}} - E_{\mathrm{DGDR}}$ is a few hundred keV for $^{136}$Xe, while a relation $\sqrt{2}\,\Gamma_{\mathrm{GDR}} \leq \Gamma_{\mathrm{DGDR}} \leq 2\Gamma_{\mathrm{GDR}}$ has been observed. Anharmonicities are usually referred to as a reason of these deviations from the independent-phonon picture. The most striking result is that the values of the experimentally extracted cross section of electromagnetic (EM) (or Coulomb) excitation for the DGDR are much larger than that given by the folding model. The enhancement is found to be around 178%–200% in the reactions with $^{136}$Xe projectiles at 700 MeV/$n$ kinetic energy [3], and around 133% using $^{208}$Pb projectiles at 640 MeV/$n$ kinetic energy, bombarding $^{208}$Pb target [2]. A number of theoretical approaches has been developed to study the multiphonon giant resonances. A detailed review of these approaches is given in [5]. Most of them can describe satisfactorily the energy and even the width of the DGDR, but a simultaneous description of both the observed GDR and DGDR cross sections is still an open question. Different theoretical models give so far the values of DGDR cross section, which are identical or quite close to that of the folding model, i.e., much smaller than the experimental values.

Recently, an approach to the multiphonon GDR in heavy nuclei has been proposed in Refs. [6] making use of the phonon damping model (PDM) [7,8]. This approach allows one to calculate microscopically the strength function of the multiphonon GDR using the same set of PDM parameters, which have been selected to describe well the GDR. The calculated DGDR energy and FWHM are found in reasonable agreement with the experimental data. It has been also found that there is a significant difference between the DGDR in open shell nuclei and the one in closed shell nuclei due to anharmonicities. The DGDR energy-weighted sum of strength in an open shell nucleus ($^{120}$Sn) is larger than that given by the folding model by around 30%, while the DGDR in a closed shell nucleus ($^{208}$Pb) is found to be rather harmonic.

In the present paper, we shall employ the strength functions for GDR and DGDR obtained within the PDM to calculate explicitly the cross sections of the DGDR in the relativistic Coulomb excitation of $^{136}$Xe projectile at 700 MeV/$n$ kinetic energy and of $^{208}$Pb projectile at 640 MeV/$n$ kinetic energy on $^{208}$Pb target and compare the results of calculations directly with the experimental data.

According to [4,9], we calculate the EM cross section $\sigma_{\mathrm{C}}$ of a relativistic projectile from the corresponding photoabsorption cross section $\sigma(E)$ and the photon spectral function $N(E)$ as

$$\sigma_{\mathrm{C}} \equiv \int \frac{d\sigma_{\mathrm{C}}}{dE}\,dE = \int N(E)\sigma(E)\,dE,$$
$$N(E) = 2\pi \int_{b_{\min}}^{\infty} e^{-m(b)} N(E,b) b\,db\,. \tag{1}$$

The expression for the spectrum $N(E,b)$ of virtual photons from a stationary target as seen by a projectile moving with a velocity $\beta = v/c$ at impact parameter $b$ is also given in [4,9]. The mean number of photons absorbed by the projectile is calculated as $m(b) = \int_{E_{\min}}^{\infty} N(E,b)\sigma(E)\,dE$.

  

For the GDR, the experimental photoabsorption cross section $\sigma_{\rm GDR}(E)$ is usually fitted in terms of a Lorentzian (Lor) [or Breit-Wigner (BW)] GDR strength function $S_{\rm GDR}^{\rm Lor/BW}(E)$ as

$$\sigma_{\rm GDR}(E) = c^{(1)} S_{\rm GDR}^{\rm Lor/BW}(E) E\,. \quad (2)$$

Instead of $S_{\rm GDR}^{\rm Lor/BW}(E)$, we use the microscopic GDR strength function $S_{\rm GDR}^{\rm PDM}(E)$ calculated within the PDM as

$$S_{\rm GDR}^{\rm PDM}(E) = \frac{1}{\pi} \frac{\gamma_{\rm GDR}(E)}{(E - E_{\rm GDR})^2 + [\gamma_{\rm GDR}(E)]^2}\,, \quad (3)$$

where $\gamma_{\rm GDR}(E)$ is the damping, which depends on the $\gamma$-ray energy $E$. The GDR energy $E_{\rm GDR}$ and the damping $\gamma_{\rm GDR}(E)$ are calculated microscopically within the PDM. The details are given in [7]. The GDR strength factor $c^{(1)}$ is defined so that the integrated cross section of the GDR satisfies the required experimental energy weighted sum of strength $(1 + \rho)$TRK, where TRK is the Thomas-Reiche-Kuhn sum rule equal to $60NZ/A$ (mb $\times$ MeV). Therefore

$$c^{(1)} = \frac{(1 + \rho)\mathrm{TRK}}{m_1^{(1)}}, \qquad m_1^{(1)} = \int_0^{E_{\rm max}} S_{\rm GDR}^{\rm PDM}(E) E\, dE\,. \quad (4)$$

The function $m_1^{(1)}$ is the first moment of the GDR. The factor $\rho$ takes into account the contribution of exchange forces and is in the range of $0.1 \sim 0.4$ depending on nuclei.

The DGDR strength function is calculated within the PDM as

$$S_{\rm DGDR}^{\rm PDM}(E) = \frac{2}{\pi} \frac{\gamma_{\rm DGDR}(E)}{(E - E_{\rm DGDR})^2 + [\gamma_{\rm DGDR}(E)]^2}\,, \quad (5)$$

where the DGDR energy $E_{\rm DGDR}$ and damping $\gamma_{\rm DGDR}(E)$ are calculated microscopically as in [6]. The DGDR cross section $\sigma_{\rm DGDR}(E)$ is calculated in a way similar to (2) as

$$\sigma_{\rm DGDR}(E) = c^{(2)} S_{\rm DGDR}^{\rm PDM}(E) E\,. \quad (6)$$

Since we want to describe the experimental data of the EM cross section based on our knowledge of the GDR, we define the DGDR strength factor $c^{(2)}$ in (6) as follows. Using (1) and the harmonic limit $S_{\rm DGDR(har)}^{\rm PDM}(E)$ of the DGDR strength function (5), which is obtained by folding two GDR strength functions (3) [see [7]], we write the formal expression of the harmonic limit $\sigma_{\rm C}^{(2)}(\text{har})$ of the EM cross section (1) for DGDR as

$$\sigma_{\rm C}^{(2)}(\text{har}) = \int \frac{d\sigma_{\rm C}^{(2)}(\text{har})}{dE}\, dE = c^{(2)} \int N_{\rm har}(E) S_{\rm DGDR(har)}^{\rm PDM}(E) E\,, \quad (7)$$

where $N_{\rm har}(E)$ is calculated using the harmonic limit $\sigma_{\rm DGDR(har)}^{\rm PDM}(E)$ of (6) in (1) and $m(b)$. We require this cross section (7) to be equal to the one calculated by folding two GDR cross sections, namely

$$\sigma_{\rm C(f)}^{(2)} \equiv \int \frac{d\sigma_{\rm C(f)}^{(2)}}{dE}\, dE = \frac{1}{2} \int dE\, dE_1\, dE_2 N(E_1, E_2) \sigma_{\rm GDR}(E_1) \sigma_{\rm GDR}(E_2) \delta(E - E_1 - E_2)$$
$$= \frac{[c^{(1)}]^2}{\pi} \int dE\, dE_1\, dE_2 N(E_1, E_2) S_{\rm GDR}^{\rm PDM}(E_1) S_{\rm GDR}^{\rm PDM}(E_2) E_1 E_2 \varepsilon / [(E - E_1 - E_2)^2 + \varepsilon^2]\,, \quad (8)$$

where the representation $\delta(x) = [(x - i\varepsilon)^{-1} - (x + i\varepsilon)^{-1}]/(2\pi i)$ and the expression for $N(E_1, E_2)$ given in [9] are used. Equalizing the right-hand sides of (7) and (8), we define $c^{(2)}$. Knowing $c^{(2)}$, we can calculate the EM cross section $\sigma_{\rm C}^{(2)}$ of the DGDR from (7) using $S_{\rm DGDR}^{\rm PDM}(E)$ (5) instead of its harmonic limit.

The calculations employ the single-particle energies for $^{136}$Xe and $^{208}$Pb obtained within the Hartree-Fock (HF) method using the SGII interaction. The unperturbed phonon energy $\omega_q$ and the degenerate particle-hole interaction $F_{ph}$ are two parameters of PDM necessary for the present calculations. They are chosen so that the GDR energy $E_{\rm GDR}$ and FWHM $\Gamma_{\rm GDR}$, calculated within PDM, reproduce the experimentally extracted values (see the details in [6,7]). Given the HF single-particle spectra above, their values are $\omega_q = 15.59$ MeV, $F_{ph} = 0.15$ MeV for $^{136}$Xe, and $\omega_q = 13.49$ MeV, $F_{ph} = 0.1785$ MeV for $^{208}$Pb. Using them, we found $E_{\rm GDR} = 15.6$ MeV and $\Gamma_{\rm GDR} = 4.96$ MeV for $^{136}$Xe. For $^{208}$Pb, we found $E_{\rm GDR} = 13.5$ MeV and $\Gamma_{\rm GDR} = 4.04$ MeV. These values agree well with the corresponding experimental values. The GDR strength factor $c^{(1)}$ is found to be 155.314 mb [with $\rho = 0.15$ in (4)] for $^{136}$Xe, and 245.408 mb (with $\rho = 0.3$) for $^{208}$Pb. The values of the DGDR strength factor $c^{(2)}$, which are determined from $c^{(1)}$ and the GDR strength function as discussed above, are equal to 42.273 mb for $^{136}$Xe, and 97.974 mb for $^{208}$Pb. The smearing parameter $\varepsilon$ is chosen to be 0.2 MeV. We have verified that the sum rule relationship in [10] between the first moment of the DGDR in its harmonic limit and four times of the product of the GDR first and zero moments is satisfied with this value of $\varepsilon$. The minimum impact parameter $b_{\rm min}$ in (1) has been defined following [11] for $^{135}$Xe, or using a simple formula given in [4] for $^{208}$Pb.

The peak energies $E_i$, FWHM $\Gamma_i$, and EM cross section $\sigma_{\rm C}^i$ for GDR ($i$ = GDR) and DGDR ($i$ = DGDR) calculated within PDM are shown in Table I in comparison with the experimental data for $^{136}$Xe [1,3] and $^{208}$Pb [2,5]. The integration of the differential cross section has been carried out within $0 \le E_\gamma \le 100$ MeV for GDR, and within 15 MeV $\le E_\gamma \le E_{\rm max}$ for DGDR with $E_{\rm max} = 45$ MeV for $^{136}$Xe, and 40 MeV for $^{208}$Pb. According to the selection of the strength factor $c^{(1)}$ discussed above, the GDR in $^{136}$Xe exhausts 115% of TRK compared to the

TABLE I. The energies $E_i$, FWHM $\Gamma_i$, and EM cross section $\sigma_C^i$ for GDR ($i$ = GDR) (a) and DGDR ($i$ = DGDR) (b), calculated within PDM (theory) in comparison with the experimental data (experiment) for $^{136}$Xe and $^{208}$Pb.

| a | $E_{GDR}$ (MeV) | | $\Gamma_{GDR}$ (MeV) | | $\sigma_C^{GDR}$ (mb) | |
|---|---|---|---|---|---|---|
| Nucleus | Theory | Experiment | Theory | Experiment | Theory | Experiment |
| $^{136}$Xe | 15.6 | 15.2 | 4.96 | 4.8 | 1676.28 | 1420(42) ± 100 |
| $^{208}$Pb | 13.5 | 13.4 | 4.04 | 4.0 | 3039.67 | 3280 ± 50 |

| b | $E_{DGDR}$ (MeV) | | $\Gamma_{DGDR}$ (MeV) | | $\sigma_C^{DGDR}$ (mb) | |
|---|---|---|---|---|---|---|
| Nucleus | Theory | Experiment | Theory | Experiment | Theory | Experiment |
| $^{136}$Xe | 29.2 | 28.3 ± 0.7 | 7.0 | 6.3 ± 1.6 | 159.33 | 164(85) ± 35 |
| $^{208}$Pb | 26.6 | 26.6 ± 0.8 | 6.3 | 6.3 ± 1.3 | 420.92 | 380 ± 40 |

experimental value of 105 ± 3.1 ± 8% of TRK obtained using the thin lead target [3]. In $^{208}$Pb, the GDR exhausts 130% of TRK with the selected value of $c^{(1)}$ compared to 122%–127% reported in [5]. As seen in Table I, all the calculated values are in reasonable agreement with the corresponding experimental values.

In order to shed some light on the issue of the enhancement of $\sigma_C^{DGDR}$ compared to that of the folding model, we show in Fig. 1 (solid lines) the EM differential cross sections $d\sigma_C^{DGDR}/dE$ calculated from (6) using the DGDR strength functions (5) within the PDM. The dotted lines are the results obtained by folding two identical GDR differential cross sections, which have been calculated using a Lorentzian with a width and a peak energy equal to the corresponding empirical values for the GDR in these nuclei. These folding results have been normalized so that the integrated folding cross sections are approximately equal to that of the folding model for $^{136}$Xe in [2,3], and for $^{208}$Pb in Fig. 6 of [5], respectively. We found that this can be achieved only if the Lorentzian GDR exhausts 105% of TRK for $^{136}$Xe ($\sigma_{C(f)}^{DGDR}$ = 90.28 mb compared to 92 mb in [3]), and 100% of TRK for $^{208}$Pb ($\sigma_{C(f)}^{DGDR}$ = 301.39 mb compared to the value of ~286 mb in [2]). The calculated values of the ratio $r \equiv \sigma_C^{DGDR}/\sigma_{C(f)}^{DGDR}$ amount to 1.77 for $^{136}$Xe, and 1.40 for $^{208}$Pb. They are comparable with the experimentally extracted values for this ratio, which are 1.78 for $^{136}$Xe (on the thin lead target) [3] and 1.33 for $^{208}$Pb [2]. In our opinion, this is the main reason of the large "enhancement" of the DGDR compared to the folding results [2,3,5] since the EM cross sections $\sigma_C^{DGDR}$, depicted by the solid curves, have been obtained using the strength factor $c^{(2)}$, which is calculated from $c^{(1)}$ using (8). The factor $c^{(1)}$ has been selected so that the GDR exhausts 115% and 130% of TRK for $^{136}$Xe and $^{208}$Pb, respectively, as has been discussed above. However, by renormalizing the Lorentzian so that the GDR exhausts 115% and 130% of TRK for $^{136}$Xe and $^{208}$Pb, respectively, we get the folding results $\sigma_{C(f)}^{DGDR}$ = 107.2 mb for $^{136}$Xe, and 532.91 mb for $^{208}$Pb. The calculated values of the ratio $r$ become 1.49 for $^{136}$Xe and only 0.79 for $^{208}$Pb, showing the inadequacy of folding two ideal Lorentzians to describe the EM cross sections of DGDR. Meanwhile, the results obtained through folding two GDR cross sections within PDM, which exhaust the same amounts of TRK (115% for $^{136}$Xe and 100% for $^{208}$Pb), yield the values of $\sigma_{C(f)}^{DGDR}$ equal to 121.74 mb for $^{136}$Xe, and 395.87 mb for $^{208}$Pb. The values of $r$ now amount to 1.31 for $^{136}$Xe, and 1.06 for $^{208}$Pb. Also in this case, the effect of anharmonicities on the DGDR in $^{136}$Xe, an open shell nucleus, is still much larger than that in the doubly closed shell nucleus $^{208}$Pb. These enhancements are consistent with those discussed previously in [6].

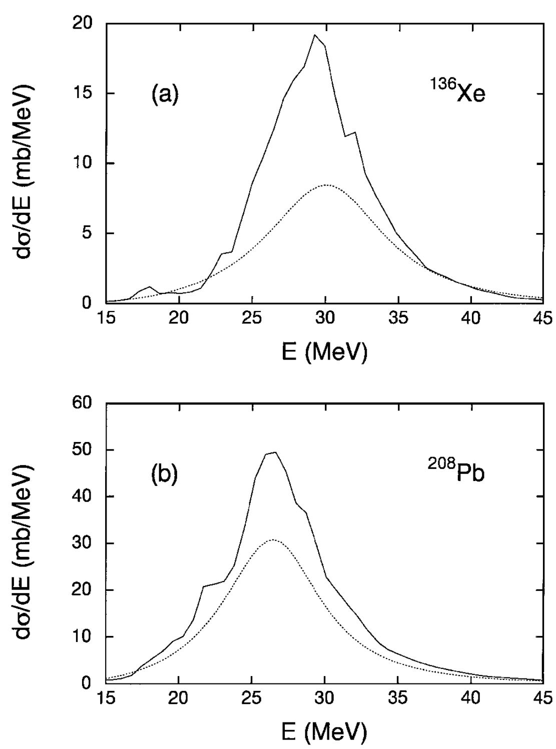


FIG. 1. EM differential cross sections $\frac{d}{dE}\sigma_C^{DGDR}$ for DGDR in $^{136}$Xe (a) and $^{208}$Pb (b). The solid curves are results obtained by using the PDM strength functions for DGDR. The dotted curves show the results of folding two Lorentzians that describe the GDR as discussed in the text.

For a meaningful comparison between the experimental differential cross sections of the DGDR with the theoretical ones within the same figure, the calculated cross-section distributions have to pass an "instrumental filter" prior to comparison with the experimental data [2]. It has been pointed out by Ritman *et al.* [12] that unfolding the detector response from the data increased the peak energy by 0.5 MeV and decreased the width by around 10% for the DGDR formed in the system 1A GeV $^{208}$Bi on $^{208}$Pb. We're now planning to incorporate the DGDR differential cross sections calculated within the PDM in the data analysis of [2,3] to carry out such a comparison. The results will be reported elsewhere.

In conclusion, we have used the DGDR strength functions obtained within the PDM to calculate explicitly the EM cross sections, and also the peak energy and FWHM of the DGDR formed in relativistic heavy-ion collisions. The calculations use a set of the PDM parameters that yields a good description of the experimentally extracted GDR parameters and cross sections. The results obtained agree reasonably well with the experimental systematic for DGDR in $^{136}$Xe and $^{208}$Pb. To our knowledge, this is the first time that theory can reproduce simultaneously both the GDR and DGDR EM cross sections. We have also pointed out the inadequacy of using two ideal Lorentzians for the GDR cross sections in the folding model to describe the EM cross section of the DGDR. Since the EM cross section is sensitive to the fine structure of the strength function, the microscopic GDR cross sections, such as those of PDM, must be used instead. This reduces significantly the large discrepancy between the folding results and the experimental systematic for DGDR. The remaining enhancement is due to the anharmonicity, which is stronger in open shell nuclei.

The authors are grateful to T. Suzuki of Nihon University for valuable assistance.

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