Spreading of the Gamow-Teller Resonance in ^{90}Nb and ^{208}Bi

Nguyen Dinh Dang,[1,*] Akito Arima,[1] Toshio Suzuki,[2] and Shuhei Yamaji[1]
[1]*Cyclotron Laboratory, RIKEN, 2-1 Hirosawa, Wako, Saitama 351-01, Japan*
[2]*Department of Physics, College of Humanities and Sciences, Nihon University, Sakurajosui 3-25-40, Setagaya-ku, Tokyo 156, Japan*
(Received 25 October 1996)

Spreading properties of the Gamow-Teller resonance in ^{90}Nb and ^{208}Bi are studied within an approach including the $2p2h$ configuration mixing and the ground-state correlations beyond the random-phase approximation. The M3Y interaction and single-particle wave functions within the standard harmonic oscillator potential are used in numerical calculations. The single-particle energies around the Fermi surface are replaced with the empirical values or those given by a Woods-Saxon potential. Results obtained provide a reasonable account for recent experimental findings. The extension of the present approach to hot nuclei is also provided. [S0031-9007(97)03918-5]

PACS numbers: 24.30.Cz, 21.10.Pc, 21.60.Jz, 25.40.Kv

The giant Gamow-Teller resonance (GTR) was predicted in 1963 [1] to explain the hindrance of allowed unfavored β decays. It was first experimentally observed in 1975 in the $^{90}\mathrm{Zr}(p,n)$ reaction [2]. A measure of the total observed transition strength for the GTR is provided by the model-independent Ikeda sum rule [1], $S_{\beta_-} - S_{\beta_+} = 3(N - Z)$, which should be nearly exhausted by the transition strength summed over all Gamow-Teller (GT) states in the daughter nucleus $(Z + 1, N - 1)$. However, during the last 20 years experimental systematics over a wide range of the periodic table found only about 60% of the transition strength given by the sum rule. Even before the discovery of the GTR, some super-allowed β decays of light nuclei such as $^{39}\mathrm{Ca} \rightarrow {}^{39}\mathrm{K}$, $^{41}\mathrm{Sc} \rightarrow {}^{41}\mathrm{Ca}$ have been observed [3]. They are very much hindered down to 66% or 74% of their single-particle values. If they are squared, they amount only to 44% or 54% of their single-particle transition probabilities. In order to explain these quenchings, the importance of two-particle–one-hole $(2p1h)$ configuration mixing has been pointed out [4]. However, after the quenching of the GTR was confirmed, another mechanism was proposed. This is the $\Delta(1232)$ isobar-hole admixtures in the nuclear wave function [5]. Theoretical studies have shown that this effect seems to be small [4,6–8]. Instead of $2p1h$ admixtures, the $2p2h$ configuration mixing must be taken into account. If this mechanism plays an important role, the missing GT strength should be spread over the physical background below and above the GTR [8,9]. Bertsch and Hamamoto were the first, who showed in Ref. [10] the importance of $2p2h$ configuration mixing in spreading the GTR strength in ^{90}Zr up to 45 MeV. After Ref. [10] several models have been proposed to calculate the spreading of GTR due to mixing with $2p2h$ configurations [11–13]. Even though all of them recover the Ikeda sum rule in the energy interval below $\sim$50–60 MeV, the amount of strength in the GTR region as well as the shape of the strength function varies noticeably depending on models. The debate on GTR is still on the way to the final point.

Recently high-resolution measurements of the GTR in the $^{208}\mathrm{Pb}(^3\mathrm{He},t)^{208}\mathrm{Bi}$ reaction [14] and in the $^{90}\mathrm{Zr}(p,n)^{90}\mathrm{Nb}$ reaction [15] were carried out in the Research Center for Nuclear Physics (Osaka). In particular, the authors of Ref. [15] were able to extract, for the first time, the $L = 0$ cross section in the continuum, resulting in a long tail of GTR extending up to around 60 MeV, which seems to recover the missing part of the GTR strength. These data also confirm that the contribution from isobar-hole admixtures is less than 6% of the GTR sum rule. These new experimental results [14,15] have motivated the present work. We propose an approach to the damping of GTR based on the coupling of the GTR excitations in the random phase approximation (RPA) to $2p2h$ configurations via two-phonon states. Even though the way of constructing the $2p2h$ configuration mixing is similar to the one in Refs. [11,16], the essential difference is in the use of a two-body residual interaction in the form of the M3Y effective nucleon-nucleon force in our approach instead of a separable one. The novelty of this approach is that the effects of ground-state correlations beyond RPA and of the two-phonon backward processes are also included self-consistently for the first time. In contrast to the approaches in Ref. [11,13], we shall couple p_π-h_ν configurations also to very high-lying $2p2h$ (two-phonon) states. Finally we also carry out the calculations of the strength distribution of the GTR at nonzero temperature. It is important in various astrophysical scenarios, for example, in a supernova, when the Gamow-Teller strengths determine the electron capture rate and thus the dynamics of the early collapse.

The usual RPA violates the Pauli principle by treating ph creation $B^\dagger_{ph}$ and annihilation B_{ph} operators as ideal bosons. A simple approach to correct for this deficiency has been proposed in Ref. [17]. Recently an improvement of this method has been made in Ref. [18]. In the present Letter this effect is included in the calculations of the GTR. The pp and hh pair operators can be expressed in terms of ph pair ones ($B^\dagger_{ph} \equiv a^\dagger_p a_h$) using the mapping

0031-9007/97/79(9)/1638(4)$10.00

procedure in Ref. [19]. The many-body Hamiltonian is then approximated in terms of only ph pair operators $B^{\dagger}_{ph}$ and B_{ph} as $H = H_0 + H_V$, where H_0 is diagonalized in RPA, while H_V is responsible for the configuration mixing beyond RPA. The *renormalized* phonon operator, generating the collective ph excitation is introduced in RPA as

$$Q^{\dagger}_{\nu} = \sum_{ph} D^{-1/2}_{ph}[X^{\nu}_{ph}B^{\dagger}_{ph} - Y^{\nu}_{ph}B_{ph}]. \quad (1)$$

The factor D_{ph} restores (up to second order in ph operators) the Pauli principle, which is broken within the quasiboson approximation. It characterizes the ground-state correlations beyond RPA, which we calculate by solving self-consistently a nonlinear RPA-like equation [18]. The case with $D_{ph} = 1$ means no ground-state correlations beyond RPA. This *renormalized* RPA-like equation has the same form as the conventional one with the two-body interaction multiplied by $(D_{ph}D_{p'h'})^{1/2}$. Its solution defines the energy ω_{ν} and the amplitudes X and Y of the ph excitation generated by the phonon operator in Eq. (1). The charge-exchange excitation is described by the p_{π}-h_{ν} $\{\mathcal{Q}^{\dagger}_{\alpha}, \mathcal{Q}_{\alpha}\}$ phonon operators defined from Eq. (1) with the p_{π} indices denoting proton particles and h_{ν} denotes neutron holes. The renormalized RPA-like equation for p_{π}-h_{ν} phonon excitations has the same form as the one for ph phonon excitations.

We study the fragmentation of GTR by considering the following two-time Green functions [20], which describe the following:

The one-phonon propagation.—

$$G^{--,+}_{\beta\gamma;\alpha'}(t - t') = \langle\langle Q_{\alpha}(t); Q^{\dagger}_{\alpha'}(t')\rangle\rangle, \quad (2)$$

$$\mathcal{G}^{-,+}_{\alpha;\alpha'}(t - t') = \langle\langle \mathcal{Q}_{\alpha}(t); \mathcal{Q}^{\dagger}_{\alpha'}(t')\rangle\rangle, \quad (3)$$

The mixing with two-phonon configurations.—

$$\mathcal{G}^{--,+}_{\beta\gamma;\alpha'}(t - t') = \langle\langle \mathcal{Q}_{\beta}(t)Q_{\gamma}(t); \mathcal{Q}^{\dagger}_{\alpha'}(t')\rangle\rangle, \quad (4)$$

and their backward processes described by $\mathcal{G}^{+,+}_{\alpha;\alpha'}(t - t')$ and $\mathcal{G}^{++,+}_{\beta\gamma;\alpha'}(t - t')$. The standard notation in Ref. [20] is used here to denote the two-time Green functions. The final set of the equations for the propagation of p_{π}-h_{ν} phonon with energy η is obtained in a matrix form as

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B}' & -\mathcal{A}' \end{pmatrix} = \eta \begin{pmatrix} \mathcal{G}^{-,+}(\eta) \\ \mathcal{G}^{+,+}(\eta) \end{pmatrix}. \quad (5)$$

The matrices $\mathcal{A}$, $\mathcal{B}$, $\mathcal{A}'$, and $\mathcal{B}'$ contain the self-energy parts, which are functions of two-body matrix elements $V^{\gamma}_{\alpha}(\beta) = \langle|\mathcal{Q}_{\beta}H_V[Q^{\dagger}_{\gamma} \otimes \mathcal{Q}^{\dagger}_{\alpha}]_{\beta}|\rangle$ ($|\rangle$ is the RPA ground-state of the parent even-even nucleus), the RPA p_{π}-h_{ν} phonon energy $\bar{\omega}_{\alpha}$, positive and negative poles $\pm(\bar{\omega}_{\beta} + \omega_{\gamma})$ (corresponding to two-phonon forward and backward processes, respectively), and also the factor $(1 + \bar{N}_{\beta} + N_{\gamma})$ with $\bar{N}_{\beta}$ and N_{γ} being the occupation numbers of $\mathcal{Q}$ and Q phonons in the ground-state. The matrix elements $V^{\gamma}_{\alpha}(\beta)$ are functions of the two-body interaction and the X and Y amplitudes. At finite temperature the average over the ground-state $|\rangle$ is replaced by the one over the thermal statistical ensemble. The phonon number $\bar{N}_{\beta}$ and N_{γ} can be then approximated by the Bose-Einstein statistics. The excitation energy η in the parent nucleus is defined as the solution of the determinant equation

$$\det\|\mathbf{\Pi}(\eta)\| \equiv \det\|[(\mathcal{A} - \eta\hat{\delta}) - \mathcal{B}(\mathcal{A}' + n\hat{\delta})^{-1}]\| = 0. \quad (6)$$

The response is calculated by inverting the matrix $\mathbf{\Pi}(\eta)$ without finding solutions η_i following a method developed in Ref. [16]. The strength function is proportional to the imaginary part of the response at complex energy $E = \eta + i\Delta$. The finite Δ plays the role of a smearing parameter in calculating the strength function. It can also account for coupling of $2p2h$ states to even more complicated configurations. In realistic calculations Δ is usually chosen to be ≤ 1 MeV. The present calculations are carried out with $\Delta = 1$ MeV, but our check has shown that the value of lowest moments of the strength function is rather stable against varying Δ from 0.1 up to 2 MeV.

We calculated the β_{-} strength distribution in ^{90}Nb and ^{208}Bi by coupling p_{π}-h_{ν} phonon excitations to natural parity $E\lambda^{\pi}$ (for ^{208}Bi) and unnatural parity $M\lambda^{\pi}$ phonon excitations (for ^{90}Nb) with $\lambda = \sim 1-5$. All the one-phonon states with energy below 40 MeV and two-phonon states with energy up to 60 MeV are included. The calculations in ^{208}Bi use the single-particle energies defined in the standard oscillator potential. The levels around the Fermi surface are replaced, however, with the empirical values. For the calculations in ^{90}Nb the information about the empirical single particle levels is not complete, so the single-particle spectrum defined from a Woods-Saxon potential [21] is used around the Fermi surface. We adopt the M3Y nucleon-nucleon force as effective interaction, whose parameters are given in Ref. [10,22]. The effects of ground-state correlations beyond RPA are found to be negligible in both nuclei (the factor D_{ph} is very close to 1). The calculated GTR strength functions for ^{90}Nb and ^{208}Bi are shown in Fig. 1. The calculated strength function in ^{90}Nb agrees better with the experimental one up to 40 MeV [Fig. 1(a)] as compared to the results in Ref. [12] (dotted curve). The peak around 10 MeV, which has been found to be very pronounced in Ref. [12], became much weaker in our calculations in agreement with the recent data. At higher excitation energy the calculated results underpredict the experimental tail [15]. It is worth noticing that the experimental strength function in Ref. [15] can be reproduced quite well by doubling the smearing parameter Δ to 2 MeV in our calculations, indicating the importance of mixing with configurations more complicated than $2p2h$ in ^{90}Nb. We also coupled the charge-exchange phonons, which generate the Gamow-Teller transitions in ^{90}Nb within the renormalized RPA, to the ph phonon states of natural parity. The change is found to be negligible as compared to

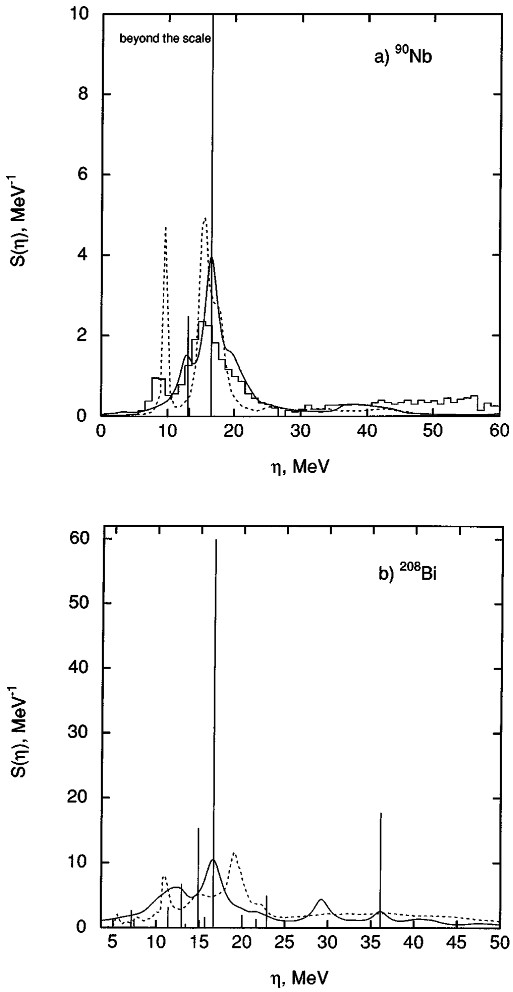

FIG. 1. Strength function of GTR in (a) ^{90}Nb and (b) ^{208}Bi. Energies are measured with respect to the ground states of ^{90}Zr and ^{208}Pb. The experimental strength distribution from Ref. [15] is shown by the histogram in (a). Impulses show the RPA results. Dotted curves refer to the results of Ref. [12] while solid curves represent our results.

the case with coupling to ph phonon states of unnatural parity.

The main peak of the GTR strength function in ^{209}Bi [Fig. 1(b)] is located at 16.6 MeV compared to the experimental value 19.2 MeV [14]. The oscillator levels far above the Fermi surface are obviously the reason for this deviation. As a matter of fact, an energy of GTR closer to the experimental value was achieved by reducing the standard oscillator strength by $\sim 4-5\%$. By doing so, we also improve the similarity between our results and the ones of Ref. [12]. However, we would like to keep the standard parameters of the potential and effective interaction unchanged throughout all calculations. The coupling to $2p2h$ states spreads the GTR up to around 60 MeV. The sum rule has been examined in three energy intervals: (i) below the GTR location, (ii) in the GTR location, and (iii) above it. The results are summarized in Table I, where the calculated values of the spreading width $\Gamma^{\downarrow}$ and the energy centroid $\bar{E}$ of the GTR in the resonance location are also shown. In the GTR region in ^{208}Bi the integrated strength amounts to 57% of the Ikeda sum rule compared to $60\% \pm 15\%$ found in experiments [14]. The tail in the energy region above the GTR amounts to 38% of the Ikeda sum rule, while there is 7.3% of the sum rule distributed in the lower-energy region (i). The spreading width $\Gamma^{\downarrow}$ is 3.87 MeV compared to the experimental value 3.72 MeV[1]. For ^{90}Nb we found around 30% of the Ikeda sum rule in the region of excitation energies higher than 22 MeV compared to around 40% extracted for the first time in experiments [15]. We argue that the nature of the continuum in this energy region still remains an open question, requiring further measurements. The spreading width and the energy centroid of the GTR in ^{90}Nb are $\Gamma^{\downarrow} \simeq 2.4$ MeV and $\bar{E} \simeq 16.8$ MeV, respectively. The calculated total β_- strength of GTR up to more than 60 MeV amounts to around 103% of the Ikeda sum rule in both nuclei. This is in agreement with the empirical observation on the contribution of the β_+ transitions of around $1.7\% \pm 0.2\%$ of the GTR sum rule. Inclusion of two-phonon backward processes made the GTR more collective in the resonance region. These effects are better seen in ^{208}Bi [see Table I(b)]. Further study with different choices of the residual interaction is highly desirable to see whether these effects interfere constructively or destructively with the damping of GTR. A test by switching off the tensor part of the M3Y interaction confirms that the tensor part is responsible for pushing the GTR distribution up to higher energy with a long high-lying tail [7,12]. Our calculations and those in Refs. [10,12] show that the use of a non-separable interaction including the tensor force in calculating the $1p1h$-$2p2h$ configuration mixing is decisive in achieving an adequate GTR strength distribution up to excitation energies of 60 MeV. For a comparison, we refer to Refs. [11,13]. The authors of Ref. [11] adopted a separable interaction, whose parameters are adjusted from nucleus to nucleus to reproduce the experimental resonance energy. The GTR strength below 30 MeV already exhausts the Ikeda sum rule. The GTR in ^{208}Bi calculated in Ref. [13] is concentrated in a narrow region around ($\sim 18-24$ MeV). The energy of the main peak of the GTR is higher than the experimental value by $\sim 2-4$ MeV, depending on the type of the Skyrme interaction in use. Both approaches in Refs. [11,13] give a GTR spreading width in ^{208}Bi, which is narrower than the experimental findings by around 20%.

The calculations were also carried out at several temperatures up to $T = 6$ MeV. The strength in the GTR region in ^{208}Bi increases less than 10% at $T = 6$ MeV, while it decreases accordingly in the higher-energy tail. The spreading width $\Gamma^{\downarrow}$ is broadened slightly. The centroid energy in the interval (ii) moves downward by less than 0.3 MeV (at $T = 6$ MeV). The temperature dependence of the strength function of GTR in ^{90}Nb is negligible.

TABLE I. Average quantities extracted from the strength distribution of GTR in ^{90}Nb and ^{208}Bi. The amounts of strength summed up in three energy intervals (i) below ($< E_<$), (ii) between ($E_<, E_>$), and (iii) above ($> E_>$) the GTR location are denoted as $S_<$, S_{GTR}, and $S_>$, respectively. The percentage of strength with respect to the Ikeda sum rule is given in parentheses. The centroid energy $\bar{E}$ and spreading width $\Gamma^{\downarrow}$ are calculated in the interval ($E_<, E_>$). The values ($E_<, E_>$) are (12 MeV, 22 MeV) in ^{90}Nb, and (8 MeV, 25 MeV) in ^{208}Bi. (a) Results including two-phonon backward processes; (b) results neglecting these backward processes.

	$\bar{E}$ (Mev)	$\Gamma^{\downarrow}$ (MeV)	$S_<$	S_{GTR}	$S_>$
			^{90}Nb		
(a)	16.81	2.41	2.75 (9.17%)	19.06 (63.53%)	9.00 (30%)
(b)	16.81	2.41	2.68 (8.93%)	19.08 (63.60%)	8.997 (29.99%)
			^{208}Bi		
(a)	15.43	3.87	9.62 (7.3%)	75.32 (57.1%)	50.26 (38.1%)
(b)	15.28	3.85	10.73 (8.1%)	63.97 (48.5%)	62.14 (47.1%)

In conclusion, we have proposed a microscopic approach for the calculations of $1p1h$-$2p2h$ configuration mixing in the GTR. There are no free parameters in the present approach. The results of our calculations for ^{90}Nb and ^{208}Bi show that our formalism can provide a good account for recent experimental findings on the spreading properties of these nuclei. Our results reconfirm that one may not need recoursing to Δ isobar-hole admixtures for an explanation of the GTR quenching in agreement with the recent empirical observation [15]. Our formalism also shows that the GTR is rather stable against temperature up to $T = 6$ MeV in agreement with the usual assumption in astrophysical calculations [23].

Numerical calculations were carried out at the Computational Center of RIKEN. The authors are grateful to K. Takayanagi for providing the program code of matrix elements of M3Y interaction and RPA and N. Onishi for valuable discussions and comments. Thanks are also due to T. Wakasa and H. Sakai for discussions and for providing the experimental data of Ref. [15] before publication.

*Research fellow of Science and Technology Agency of Japan.

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