Exact form of the random phase approximation equation at finite temperature including the entropy effect

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The thermal random phase approximation (TRPA) equation is derived from the variational principle applied to the grand potential with an entropy term. This form of the TRPA equation is exact within the framework of the random phase approximation at finite temperature, whose matrix representation coincides with the stability matrix for the solution to the thermal Hartree-Fock-Bogoliubov equation. It is, however, shown that the γ-ray energy-dependence of the response function for a giant resonance built on a heated nucleus is not altered within a perturbation treatment of the entropy effect based on a simple microscopic model. Thus, an application of the TRPA formalism neglecting the entropy effect is justifiable as for giant resonance shape. The calculation employing a simple microscopic model shows that the increase of the Landau splitting of giant resonance levels with temperature is mainly attributed to the contributions of pp and hh configurations.

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I. INTRODUCTION

Recent experimental studies of the damping of giant resonances (GR's) built on heated nuclei [1–8] have supplied useful information for checking the theoretical models and methods, which have been successful in describing properties of the nuclear states in the yrast region and extensively applied to the highly excited states above the yrast line. As such an extension of microscopic formalism, the thermal random phase approximation (TRPA) equation [9–13] on top of the thermal Hartree-Fock-Bogoliubov (THFB) [14,15], or the thermal Hartree-Fock (THF) solution when the pairing correlation is not important, is expected to describe the temperature-dependent behavior of the hot GR. For this purpose, the formalism has to be guaranteed not only to smoothly tend to the ordinary RPA in the zero-temperature limit, but also to solve the problem of discrepancy between the TRPA matrix and the THFB stability matrix [11]. Therefore, in the first part of this paper (Sec. II), we derive in a comprehensive manner the exact TRPA equation from the grand potential including entropy term, which has been proposed only in a preliminary form in Ref. [16]. The entropy effect is taken into account by enlarging the dimension of the TRPA vector to include the shifts of occupation numbers of particles in addition to the original degrees of freedom of the TRPA amplitudes. However, we will see that a special device in our formalism prevents those shifts of occupation numbers from contributing to the norms of the TRPA eigenamplitudes. The exact TRPA equation will describe an increase of the inclination toward the instability of the mean field (i.e., the THFB) solution due to entropy effect. This instability is indicated by the occurrence of a vanishing TRPA eigenenergy at finite temperature. We present also explicit forms of the completeness condition and the finite temperature version of the energy-weighted sum rule (EWSR) [17].

In the second part of the paper, our interest is in the entropy effect and the temperature dependence of the GR width and centroid energy. In Sec. III we introduce a simplified microscopic model, and in Sec. IV we apply it to investigate (i) the entropy effect on the stability of the THFB solution, (ii) its effect on the GR shape, and (iii) the temperature dependence of the Landau splitting and the centroid energy. Then, we treat the contributions from the entropy to the GR spectra as perturbation. We will, however, see that such an effect turns out to be negligible unless the interaction strength is abnormally large. It is experimentally known that, in case of the giant dipole resonance (GDR) in a hot nucleus, its width at half maximum (i.e., the FWHM) increases rapidly with increasing temperature for Sn isotopes [1–6] and ^{208}Pb [7,8], and it seems to saturate at the temperature about $T>3\sim4$ MeV in the case of Sn isotopes. In Sec. V, we confirm, within the framework of our microscopic model, that the rapid increase of the Landau splitting with temperature is mainly due to the damping of the GR through the particle-particle (pp) and the hole-hole (hh) configurations. This is consistent with the theoretical expectation based on the ''*standard*'' TRPA equation in terms of the quasiparticle picture which automatically includes the $\alpha^\dagger\alpha$ terms in addition to the $\alpha^\dagger\alpha^\dagger$ and the $\alpha\alpha$ terms [9–13], and also with the phonon damping model (PDM) which takes into account the coupling of GDR phonon to the pp and hh configurations as the main mechanism of the width's increase and saturation [18,19]. We conclude the paper in Sec. VI.

II. DERIVATION OF THE EXACT TRPA EQUATION

A. Stability condition of the THFB solution

In order to elucidate the relation between the stability condition of the THFB solution and the TRPA equation, we

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derive an explicit form of the stability matrix. For an exact statistical operator $\hat{W}^{\mathrm{true}}$, the exact grand potential $\mathcal{F}^{\mathrm{true}}$ and entropy S^{true} are, respectively, given by

$$\mathcal{F}^{\mathrm{true}}=\langle \hat{H}'\rangle - TS^{\mathrm{true}} \tag{2.1}$$

and

$$S^{\mathrm{true}}=-k\,\mathrm{Tr}(\hat{W}^{\mathrm{true}}\ln \hat{W}^{\mathrm{true}}), \tag{2.2}$$

where $\hat{H}'$ is the auxiliary Hamiltonian for a finite system such as a nucleus

$$\hat{H}'=\hat{H}-\lambda_{\pi}\hat{Z}-\lambda_{\nu}\hat{N}-\omega_{\mathrm{rot}}\hat{J}_X . \tag{2.3}$$

Three constraints are required for the proton and the neutron numbers and the angular momentum

$$\langle \hat{Z}\rangle = Z, \quad \langle \hat{N}\rangle = N, \quad \langle \hat{J}_X\rangle = \sqrt{I(I+1)}, \tag{2.4}$$

which determine three Lagrange multipliers λ_{π}, λ_{ν}, and ω_{rot} introduced in Eq. (2.3). In Eq. (2.4) the ensemble average of an operator $\hat{O}$ is expressed as $\langle \hat{O}\rangle = \mathrm{Tr}(\hat{W}^{\mathrm{true}}\hat{O})$. In a practical problem, we replace $\hat{H}'$ including the full Hamiltonian simply by a bilinear form $\hat{H}^{\mathrm{eff}}$ expressed in terms of the quasiparticle operators $\{\alpha_{\mu},\alpha_{\mu}^{\dagger}\}$, and correspondingly, we replace the exact statistical operator $\hat{W}^{\mathrm{true}}$ by the trial operator

$$\hat{W}=\frac{\exp(-\beta\hat{H}^{\mathrm{eff}})}{\mathrm{Tr}\exp(-\beta\hat{H}^{\mathrm{eff}})}, \quad \hat{H}^{\mathrm{eff}}\equiv\sum_{\mu} E_{\mu}\alpha_{\mu}^{\dagger}\alpha_{\mu}, \tag{2.5}$$

where $\beta=1/kT$, and T is temperature and k the Boltzmann constant. The single-particle operators $\{c_k,c_k^{\dagger}\}$ are related to the quasiparticle operators $\{\alpha_{\mu},\alpha_{\mu}^{\dagger}\}$ through the generalized Bogoliubov transformation

$$c_k=\sum_{\mu} (u_{k\mu}\alpha_{\mu}+v_{k\mu}^{*}\alpha_{\mu}^{\dagger}), \tag{2.6a}$$

$$c_k^{\dagger}=\sum_{\mu} (v_{k\mu}\alpha_{\mu}+u_{k\mu}^{*}\alpha_{\mu}^{\dagger}). \tag{2.6b}$$

In addtition to a set of the generalized Bogoliubov transformation coefficients $\{u_{k\mu},v_{k\mu},u_{k\mu}^{*},v_{k\mu}^{*}\}$, the parameters E's introduced in Eq. (2.5) are also regarded as variational parameters. Based on the Peierls' inequality

$$\mathcal{F}^{\mathrm{true}}\leqslant\mathcal{F}=\mathrm{Tr}(\hat{W}\hat{H}')-TS, \quad S=-k\,\mathrm{Tr}(\hat{W}\ln\hat{W}), \tag{2.7}$$

which holds for any approximate grand potential $\mathcal{F}$ and entropy S defined in terms of the approximate statistical operator $\hat{W}$ as given by Eq. (2.5), we apply the variational principle $\delta\mathcal{F}=0$ to derive the thermal Hartree-Fock-Bogoliubov equation as well as a relation [14]

$$\langle \alpha_{\mu}^{\dagger}\alpha_{\nu}\rangle \equiv f_{\mu}\delta_{\mu\nu}=\frac{\delta_{\mu\nu}}{\exp(\beta E_{\mu})+1}. \tag{2.8}$$

The parameter E_{μ} is interpreted as a quasiparticle energy since it is determined as an eigenvalue of the THFB equation. The variation of E_{μ} is related to that of the quasiparticle distribution function f_{μ} by

$$\delta f_{\mu}=-\beta f_{\mu}(1-f_{\mu})\delta E_{\mu}. \tag{2.9}$$

Furthermore, if we introduce the infinitesimal Thouless' transformation parameters $\{c_{\mu\nu},d_{\mu\nu}\}$ being related to the variations of the Bogoliubov transformation coefficients by [11]

$$\delta u_{k\mu}=(v^{*}c^{*})_{k\mu}+(ud)_{k\mu}, \tag{2.10a}$$

$$\delta v_{k\mu}=(u^{*}c^{*})_{k\mu}+(vd)_{k\mu}, \tag{2.10b}$$

then the stability condition of the THFB solution derived from the second order variation of $\mathcal{F}$ is expressed as

$$\delta^2\mathcal{F}=\frac{1}{2}\mathbf{V}^{\dagger}\mathbf{S}\mathbf{V}>0, \tag{2.11}$$

where

$$\mathbf{V}^{\dagger}\equiv(c_{\mu\nu}^{>*},c_{\mu\nu}^{>},d_{\mu\nu}^{*},\delta f_{\mu}),$$

$$c_{\mu\nu}^{>}\equiv c_{\mu\nu}=-c_{\nu\mu} \quad (\mu>\nu). \tag{2.12}$$

In Eq. (2.11) the stability matrix is defined as

$$\mathbf{S}\equiv\begin{pmatrix} A_{\mu\nu,\rho\sigma} & B_{\mu\nu,\rho\sigma} & C_{\mu\nu,\rho\sigma} & E_{\mu\nu,\sigma} \\ B_{\mu\nu,\rho\sigma}^{*} & A_{\mu\nu,\rho\sigma}^{*} & C_{\nu\mu,\sigma\rho}^{*} & E_{\mu\nu,\sigma}^{*} \\ C_{\rho\sigma\mu\nu}^{*} & C_{\sigma\rho,\nu\mu} & D_{\mu\nu,\rho\sigma} & E'_{\mu\nu,\sigma} \\ E_{\rho\sigma,\mu}^{*} & E_{\rho\sigma,\mu} & E'^{*}_{\sigma\rho,\mu} & F_{\mu,\sigma} \end{pmatrix}, \tag{2.13}$$

where the matrix elements are given by

$$\begin{aligned} A_{\mu\nu,\rho\sigma}&\equiv(E_{\mu}+E_{\nu})(\delta_{\mu\rho}\delta_{\nu\sigma}-\delta_{\mu\sigma}\delta_{\nu\rho})(1-f_{\mu}-f_{\nu}) \\ &\quad+4(H_{22})_{\mu\nu\rho\sigma}(1-f_{\mu}-f_{\nu})(1-f_{\rho}-f_{\sigma}), \\ B_{\mu\nu,\rho\sigma}&\equiv24(H_{40})_{\mu\nu,\rho\sigma}(1-f_{\mu}-f_{\nu})(1-f_{\rho}-f_{\sigma}), \\ C_{\mu\nu,\rho\sigma}&\equiv6(H_{31})_{\mu\nu\sigma\rho}(1-f_{\mu}-f_{\nu})(f_{\sigma}-f_{\rho}), \\ D_{\mu\nu,\rho\sigma}&\equiv(E_{\mu}-E_{\nu})(f_{\nu}-f_{\mu})\delta_{\mu\rho}\delta_{\nu\sigma} \\ &\quad+4(H_{22})_{\mu\sigma\nu\rho}(f_{\nu}-f_{\mu})(f_{\sigma}-f_{\rho}), \\ E_{\mu\nu,\sigma}&\equiv6(H_{31})_{\mu\nu,\sigma\sigma}(1-f_{\mu}-f_{\nu}), \\ E'_{\mu\nu,\sigma}&\equiv4(H_{22})_{\mu\sigma\nu\rho}(f_{\nu}-f_{\mu}), \\ F_{\mu,\sigma}&\equiv\frac{kT\delta_{\mu\sigma}}{f_{\mu}(1-f_{\mu})}+4(H_{22})_{\mu\sigma\mu\sigma}. \end{aligned} \tag{2.14}$$

In the above expressions, we have introduced the ordinary notations for the two-body terms in the Hamiltonian in the quasiparticle picture such as $\Sigma_{\mu\nu\rho\sigma}(H_{31})_{\mu\nu\rho\sigma}\alpha_\mu^\dagger\alpha_\nu^\dagger\alpha_\rho^\dagger\alpha_\sigma$, $\Sigma_{\mu\nu\rho\sigma}(H_{22})_{\mu\nu\rho\sigma}\alpha_\mu^\dagger\alpha_\nu^\dagger\alpha_\sigma\alpha_\rho$, etc.

It must be noticed that the stability matrix **S** is enlarged to include the additional matrix elements corresponding to δf_μ in the fourth row and the fourth column, i.e., $E_{\mu\nu,\sigma}$, $E'_{\mu\nu,\sigma}$, $E^*_{\mu\nu,\sigma}$, $E^{*'}_{\mu\nu,\sigma}$, and $F_{\mu,\sigma}$, while the 3×3 sector in the upper-left corner of **S** corresponds to the standard TRPA equation neglecting entropy effect [11].

B. Variational derivation of the TRPA equation

Keeping in mind a parallel with the THFB stability condition, we apply the variational principle again to derive the TRPA equation. We consider the nonunitary transformation

$$E_\mu \to E_\mu + \delta E_\mu \tag{2.15}$$

in addition to the unitary transformation of the trial statistical operator

$$\hat{W} \to \mathrm{e}^{iR}\hat{W}\mathrm{e}^{-iR}, \quad R = Q^\dagger + Q \tag{2.16}$$

with the TRPA operator in the quasiparticle picture defined by

$$Q^\dagger = \sum_{\mu>\nu}(X_{\mu\nu}\alpha_\mu^\dagger\alpha_\nu^\dagger - Y_{\mu\nu}\alpha_\nu\alpha_\mu) + \sum_{\mu\neq\nu} Z_{\mu\nu}\alpha_\mu^\dagger\alpha_\nu. \tag{2.17}$$

The third term in $Q^\dagger$ contributes only at finite temperature, and then plays an essential role in describing the rapid increase of the GR width with increasing temperature. When we consider the TRPA equation in the particle-hole (ph) picture, this term is expressed in terms of the combinations of pp and hh operators as will be done in the subsequent sections.

Regarding the TRPA collective motion about an equilibrium point determined by the mean field approximation (i.e., the THFB solution) as of small amplitude motion, we calculate the second order variation of the transformed grand potential with respect to $(X_{\mu\nu}, Y_{\mu\nu}, Z_{\mu\nu})$ and δf_μ to obtain

$$\delta^2\mathcal{F} = \frac{1}{2}\mathbf{X}^\dagger\mathbf{S}\mathbf{X},$$

$$\mathbf{X}^\dagger = (X^*_{\mu\nu} - Y_{\mu\nu}, Y^*_{\mu\nu} - X_{\mu\nu}, Z^*_{\mu\nu} + Z_{\nu\mu}, \delta f_\mu). \tag{2.18}$$

Here the matrix **S** is the same as the stability matrix that appeared in Eq. (2.11). This suggests the possibility of the extension of the TRPA equation to include also the contributions from the entropy term in the grand potential $\mathcal{F}$, but we need a special device to prevent δf_μ from contributing to the RPA amplitude. For this purpose we rewrite the expression $\mathbf{X}^\dagger\mathbf{S}\mathbf{X}$ as

$$\mathbf{X}^\dagger\mathbf{S}\mathbf{X} = (\mathbf{X}^{(1)} + \mathbf{X}^{(2)})^\dagger \boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}}(\mathbf{X}^{(1)} + \mathbf{X}^{(2)}), \tag{2.19}$$

which newly defines the extended form of the TRPA operator **O**. We have introduced the following notations:

$$\mathbf{X}^{(1)} \equiv \begin{pmatrix} \mathbf{V}^{(+)} \\ \delta\mathbf{f} \end{pmatrix}, \quad \mathbf{X}^{(2)} \equiv \begin{pmatrix} \mathbf{V}^{(-)} \\ \delta\mathbf{f} \end{pmatrix},$$

$$\boldsymbol{\mathcal{M}} \equiv \begin{pmatrix} \mathbf{M} & 0 \\ 0 & \sigma_2 \end{pmatrix} \tag{2.20}$$

with

$$\mathbf{V}^{(+)} = \begin{pmatrix} X_{\mu\nu} \\ Y_{\mu\nu} \\ Z_{\mu\nu} \end{pmatrix}, \quad \mathbf{V}^{(-)} = \begin{pmatrix} -Y^*_{\mu\nu} \\ -X^*_{\mu\nu} \\ Z^*_{\nu\mu} \end{pmatrix},$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \delta\mathbf{f} = \begin{pmatrix} \delta f_\mu \\ \delta f_{\bar{\mu}} \end{pmatrix}. \tag{2.21}$$

In Eq. (2.21), we classify all the quasiparticle states into two groups labeled μ and $\bar{\mu}$. Practically, there are many different ways of this classification. An example of such a classification is to refer to the eigenvalues $\pm i$ of the signature $e^{-i\pi\hat{J}_X}$. The matrix **M** defined as a part of $\boldsymbol{\mathcal{M}}$ in Eq. (2.20) is the TRPA metric given by [11,12]

$$\mathbf{M} = \begin{pmatrix} 1 - f_\mu - f_\nu & 0 & 0 \\ 0 & -(1 - f_\mu - f_\nu) & 0 \\ 0 & 0 & f_\nu - f_\mu \end{pmatrix}. \tag{2.22}$$

On the analogy with the stability condition of the TRPA solution [11,20]

$$\langle [Q,[H',Q]] \rangle = \langle [Q^\dagger,[H',Q^\dagger]] \rangle = 0 \tag{2.23}$$

and the normalization condition

$$\langle [Q,Q^\dagger] \rangle = 1, \tag{2.24}$$

we require

$$\mathbf{X}^{(1)\dagger}\boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}}\mathbf{X}^{(2)} = \mathbf{X}^{(2)\dagger}\boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}}\mathbf{X}^{(1)} = 0 \tag{2.25}$$

and

$$\mathbf{X}^{(1)\dagger}\boldsymbol{\mathcal{M}}\mathbf{X}^{(1)} = -\mathbf{X}^{(2)\dagger}\boldsymbol{\mathcal{M}}\mathbf{X}^{(2)} = 1. \tag{2.26}$$

These conditions are regarded as constraints in what follows. The grand potential is a functional of the trial statistical operator $\hat{W}$ which is a function of the parameters E_μ, i.e., $\mathcal{F} = \mathcal{F}[\hat{W}(E_\mu)]$. Applying simultaneously both the unitary transformation in Eq. (2.16) and the nonunitary transformation in Eq. (2.15) to $\mathcal{F}$, we consider a shift $\Delta\mathcal{F} = \mathcal{F}[e^{i\hat{R}}\hat{W}(E_\mu + \delta E_\mu)e^{-i\hat{R}}] - \mathcal{F}[\hat{W}(E_\mu)]$. Then, we find that $\Delta\mathcal{F}$ formally coincides with the second order variation $\delta^2\mathcal{F}$ given by Eq. (2.18) since the first order terms in $\hat{R}$ and δf_μ vanish in consequence of the THFB equation. Multiplying the quantities in Eqs. (2.25) and (2.26) by the Lagrange multipliers $a/2$, $b/2$, and ω, respectively, and subtracting those from $\Delta\mathcal{F}$, we put the variational requirement in the form

$$\delta\left(\Delta\mathcal{F}-\frac{a}{2}\mathbf{X}^{(1)\dagger}\boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}}\mathbf{X}^{(2)} -\frac{b}{2}\mathbf{X}^{(2)\dagger}\boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}}\mathbf{X}^{(1)}-\omega\mathbf{X}^{(1)\dagger}\boldsymbol{\mathcal{M}}\mathbf{X}^{(1)}\right)=0. \tag{2.27}$$

Performing the variations with respect to $X^{*}_{\mu\nu}$, $Y^{*}_{\mu\nu}$, $Z^{*}_{\mu\nu}$, and δE_{μ} (or δf_{μ}), we obtain a relation unified in the matrix form

$$(1-a)\boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}}\mathbf{X}^{(2)}+(1-b)\boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}}\mathbf{X}^{(1)} =(\omega\boldsymbol{\mathcal{M}}-\boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}})\mathbf{X}^{(1)}-(\omega\boldsymbol{\mathcal{M}}+\boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}})\mathbf{X}^{(2)}. \tag{2.28}$$

It must be noticed that, in this equation, the terms related to $\delta\mathbf{f}$ consist of different contributions, i.e., the term of the type $kT\delta_{\mu\sigma}/f_{\mu}(1-f_{\mu})$ arising from the second order variation of the entropy term $-TS$ in $\mathcal{F}$, the one of the type $4(H_{22})_{\mu\sigma\mu\sigma}$ from the ground state correlation at finite temperature, and the ones of the types $6(H_{31})_{\mu\nu\rho\rho}$ and $4(H_{22})_{\mu\rho\nu\rho}$ from $i\,\mathrm{Tr}(\hat{W}[\hat{H}',R])$, which vanishes if the nonunitary transformation in Eq. (2.15) is not applied. We will call the temperature effect described by these contributions simply by the entropy effect, hereafter.

Multiplying Eq. (2.28) by $\mathbf{X}^{(2)\dagger}$, or $\mathbf{X}^{(1)\dagger}$ from the left, and using two relations in Eq. (2.25), we derive the following two alternative equations:

$$(1-a)\mathbf{X}^{(2)\dagger}\boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}}\mathbf{X}^{(2)}=-\mathbf{X}^{(2)\dagger}(\omega\boldsymbol{\mathcal{M}}+\boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}})\mathbf{X}^{(2)}, \tag{2.29a}$$

$$(1-b)\mathbf{X}^{(1)\dagger}\boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}}\mathbf{X}^{(1)}=\mathbf{X}^{(1)\dagger}(\omega\boldsymbol{\mathcal{M}}-\boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}})\mathbf{X}^{(1)}. \tag{2.29b}$$

Here, we put a requirement that the right-hand sides (RHS) of the above equations vanish, i.e., ansatz

$$\mathbf{O}\boldsymbol{\mathcal{M}}\mathbf{X}^{(1)}=\mathbf{X}^{(1)}\omega,\quad \mathbf{O}\boldsymbol{\mathcal{M}}\mathbf{X}^{(2)}=\mathbf{X}^{(2)}(-\omega). \tag{2.30}$$

Due to this ansatz, the relations $a=b=1$ result from Eq. (2.29). Furthermore, we can show that the excitation energy of the system is certainly given by the eigenvalue of the first equation in Eq. (2.30) ω, i.e.,

$$\Delta\mathcal{F}=\frac{1}{2}(\mathbf{X}^{(1)\dagger}\boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}}\mathbf{X}^{(1)}+\mathbf{X}^{(2)\dagger}\boldsymbol{\mathcal{M}}\mathbf{O}\boldsymbol{\mathcal{M}}\mathbf{X}^{(2)}) =\frac{\omega}{2}(\mathbf{X}^{(1)\dagger}\boldsymbol{\mathcal{M}}\mathbf{X}^{(1)}-\mathbf{X}^{(2)\dagger}\boldsymbol{\mathcal{M}}\mathbf{X}^{(2)})=\omega. \tag{2.31}$$

This result justifies the ansatz in Eq. (2.30). Thus, our purpose of deriving an exact form of the RPA equation at finite temperature (TRPA) is attained. More explicitly the TRPA equation is expressed as

$$\begin{pmatrix}\boldsymbol{\Omega} & \mathbf{E}\\ \mathbf{E}^{\dagger} & \mathbf{F}\end{pmatrix}\begin{pmatrix}\mathbf{M} & 0\\ 0 & \boldsymbol{\sigma}_2\end{pmatrix}\begin{pmatrix}\mathbf{V}\\ \delta\mathbf{f}\end{pmatrix}=\begin{pmatrix}\mathbf{V}\\ \delta\mathbf{f}\end{pmatrix}\omega, \tag{2.32}$$

where

$$\boldsymbol{\Omega}\equiv\begin{pmatrix}\mathcal{A}_{\mu\nu\rho\sigma} & \mathcal{B}_{\mu\nu\rho\sigma} & \mathcal{C}_{\mu\nu\rho\sigma}\\ \mathcal{B}^{*}_{\mu\nu\rho\sigma} & \mathcal{A}^{*}_{\mu\nu\rho\sigma} & -\mathcal{C}^{*}_{\mu\nu\sigma\rho}\\ \mathcal{C}^{*}_{\rho\sigma\mu\nu} & -\mathcal{C}_{\rho\sigma\nu\mu} & \mathcal{D}_{\mu\nu\rho\sigma}\end{pmatrix}, \tag{2.33a}$$

$$\mathbf{E}\equiv\begin{pmatrix}3(H_{31})_{\mu\nu\sigma\sigma} & -3(H_{31})_{\mu\nu\overline{\sigma\sigma}}\\ 3(H_{31})^{*}_{\mu\nu\sigma\sigma} & -3(H_{31})^{*}_{\mu\nu\overline{\sigma\sigma}}\\ 2(H_{22})_{\mu\sigma\nu\sigma} & -2(H_{22})_{\mu\bar{\sigma}\nu\bar{\sigma}}\end{pmatrix}, \tag{2.33b}$$

$$\mathbf{F}\equiv\begin{pmatrix}\dfrac{kT\delta_{\overline{\mu\sigma}}}{4f_{\bar{\mu}}(1-f_{\bar{\mu}})}+(H_{22})_{\overline{\mu\sigma\mu\sigma}} & -(H_{22})_{\mu\bar{\sigma}\mu\bar{\sigma}}\\ -(H_{22})_{\bar{\mu}\sigma\bar{\mu}\sigma} & \dfrac{kT\delta_{\mu\sigma}}{4f_{\mu}(1-f_{\mu})}+(H_{22})_{\mu\sigma\mu\sigma}\end{pmatrix}, \tag{2.33c}$$

$$\mathbf{V}^{\mathrm{tr}}\equiv(X_{\mu\nu},Y_{\mu\nu},Z_{\mu\nu}),\quad \delta\mathbf{f}^{\mathrm{tr}}\equiv(\delta f_{\mu},\delta f_{\bar{\mu}}) \tag{2.33d}$$

with the definitions

$$\mathcal{A}_{\mu\nu\rho\sigma}=\frac{E_{\mu}+E_{\nu}}{1-f_{\rho}-f_{\sigma}}(\delta_{\mu\rho}\delta_{\nu\sigma}-\delta_{\mu\sigma}\delta_{\nu\rho})+(H_{22})_{\mu\sigma\nu\rho}, \tag{2.34a}$$

$$\mathcal{B}_{\mu\nu\rho\sigma}=-24(H_{40})_{\mu\nu\rho\sigma},\quad \mathcal{C}_{\mu\nu\rho\sigma}=6(H_{31})_{\mu\nu\rho\sigma}, \tag{2.34b}$$

$$\mathcal{D}_{\mu\nu\rho\sigma}=\frac{E_{\mu}-E_{\nu}}{f_{\nu}-f_{\mu}}\delta_{\mu\rho}\delta_{\nu\sigma}+4(H_{22})_{\mu\sigma\nu\rho}. \tag{2.34c}$$

The normalization condition in Eq. (2.26) becomes

$$\mathbf{X}^{\dagger}\boldsymbol{\mathcal{M}}\mathbf{X}=\mathbf{V}^{\dagger}\mathbf{M}\mathbf{V}=\langle[Q,Q^{\dagger}]\rangle=1, \tag{2.35}$$

which is irrelevant to the shift δE_{μ} (or δf_{μ}) as expected. This completes the derivation of the extended form of the TRPA equation [16].

For later convenience, we eliminate the shift of the occupation number $\delta\mathbf{f}$ from the TRPA equation in Eq. (2.32). Then, the TRPA equation is converted to

$$[\boldsymbol{\Omega}+\mathbf{E}\sigma_2(\omega-\mathbf{F}\sigma_2)^{-1}\mathbf{E}^{\dagger}]\mathbf{M}\mathbf{V}=\mathbf{V}\omega, \tag{2.36}$$

whose nth eigensolution $\{\omega_n,\mathbf{V}^{(n)}\}$ determines the shifts of occupation numbers in the single-particle levels under the influence of this excited collective mode

$$\delta\mathbf{f}^{(n)}=\delta\mathbf{f}^{(-n)}=(\omega_n-\mathbf{F}\sigma_2)^{-1}\mathbf{E}^{\dagger}\mathbf{M}\mathbf{V}^{(n)} \tag{2.37}$$

as well as the TRPA operators

$$Q^{(n)\dagger}=Q^{(-n)} =\sum_{\mu>\nu}(X^{(n)}_{\mu\nu}\alpha^{\dagger}_{\mu}\alpha^{\dagger}_{\nu}-Y^{(n)}_{\mu\nu}\alpha_{\nu}\alpha_{\mu})+\sum_{\mu\neq\nu}Z^{(n)}_{\mu\nu}\alpha^{\dagger}_{\mu}\alpha_{\nu}. \tag{2.38}$$

Some complication is unavoidable in solving the eigenvalue equation (2.36) since the eigenvalue ω appears in both sides of the equation.

In general, the THFB solution becomes unstable at the critical point where there occurs a vanishing TRPA eigenmode. Therefore, putting $\omega=0$ in Eq. (2.36), we obtain a general expression for the instability line (or the boundary of the THFB stability domain)

$$\det(\mathbf{\Omega}-\mathbf{E}\mathbf{F}^{-1}\mathbf{E}^{\dagger})=0, \tag{2.39}$$

which defines a functional relation between the temperature T and the relevant coupling strenghts of interactions. It can be inferred from this expression that the instability occurs at the temperature lower than the one predicted by the standard TRPA without the entropy effect, since the diagonal energy $E_{\mu}+E_{\nu}$ in the matrix $\mathbf{\Omega M}$ is partly cancelled by the diagonal contributions in the second term $-\mathbf{E}\mathbf{F}^{-1}\mathbf{E}^{\dagger}\mathbf{M}$. We will see in some detail that this is the case in the perturbation treatment of the entropy term for a simple model in the last part of Sec. IV.

C. Completeness condition and energy-weighted sum rule

From the hermiticity of the TRPA operator $\mathbf{O}$ and the TRPA metric $\boldsymbol{\mathcal{M}}$, the TRPA equation (2.32) requires

$$(\omega_m-\omega_n)(\mathbf{V}^{(m)\dagger},\delta\mathbf{f}^{(m)\mathrm{tr}})\boldsymbol{\mathcal{M}}\begin{pmatrix}\mathbf{V}^{(n)}\\ \delta\mathbf{f}^{(n)}\end{pmatrix}=0. \tag{2.40}$$

Thus, if there is no degeneracy in the TRPA energies ω's, the orthonormality relation is given by

$$(\mathbf{V}^{(m)\dagger},\delta\mathbf{f}^{(m)\mathrm{tr}})\boldsymbol{\mathcal{M}}\begin{pmatrix}\mathbf{V}^{(n)}\\ \delta\mathbf{f}^{(n)}\end{pmatrix}=\mathbf{V}^{(m)\dagger}\mathbf{M}\mathbf{V}^{(n)}+\delta\mathbf{f}^{(m)\mathrm{tr}}\sigma_2\delta\mathbf{f}^{(n)}$$
$$=\frac{\omega_n}{|\omega_n|}\delta_{mn}, \tag{2.41}$$

which represents both the normalization condition in Eq. (2.26) and the orthogonality relations in Eq. (2.25) altogether. Use of Eq. (2.37) together with the reality of $\delta\mathbf{f}$'s allows us to rewrite Eq. (2.41) as

$$\mathbf{V}^{(m)\dagger}\mathbf{M}[\mathbf{I}+\mathbf{E}(\omega_m-\sigma_2\mathbf{F})^{-1}\sigma_2(\omega_n-\mathbf{F}\sigma_2)^{-1}\mathbf{E}^{\dagger}\mathbf{M}]\mathbf{V}^{(n)}$$
$$=\frac{\omega_n}{|\omega_n|}\delta_{mn}, \tag{2.42}$$

where the second term on the left-hand side (LHS), which is identically zero for $m=n$, represents the modification due to the entropy effect.

Ensemble averages of the commutators among eigenoperators, $Q^{(n)}(=Q^{(-n)\dagger})$ $(n=\pm1,\pm2,\ldots)$, are given by

$$\langle[Q^{(m)},Q^{(n)\dagger}]\rangle=-\langle[Q^{(m)\dagger},Q^{(n)}]\rangle^{*}=\mathbf{V}^{(m)\dagger}\mathbf{M}\mathbf{V}^{(n)}$$
$$=\frac{\omega_n}{|\omega_n|}\delta_{mn}-\delta\mathbf{f}^{(m)\mathrm{tr}}\sigma_2\delta\mathbf{f}^{(n)}, \tag{2.43a}$$

$$\langle[Q^{(m)},Q^{(n)}]\rangle=-\langle[Q^{(m)\dagger},Q^{(n)\dagger}]\rangle^{*}=\mathbf{V}^{(m)\dagger}\mathbf{M}\mathbf{V}^{(-n)}$$
$$=-\delta\mathbf{f}^{(m)\mathrm{tr}}\sigma_2\delta\mathbf{f}^{(n)}. \tag{2.43b}$$

Note that $\delta\mathbf{f}^{(m)\mathrm{tr}}\sigma_2\delta\mathbf{f}^{(n)}=0$ for $m=n$ in the last expressions in Eqs. (2.43a), (2.43b). Any one-body transition (i.e., nondiagonal) operator $\hat{P}$ can be expanded in terms of $Q^{(n)\dagger}$ and $Q^{(n)}(=Q^{(-n)\dagger})$ as

$$\hat{P}=\sum_{n>0}(a_nQ^{(n)\dagger}+b_nQ^{(n)}), \tag{2.44}$$

whose expansion coefficients, a's and b's, are determined by a set of equations as follows:

$$\langle[Q^{(m)},\hat{P}]\rangle=\sum_{n>0}\{a_n\langle[Q^{(m)},Q^{(n)\dagger}]\rangle+b_n\langle[Q^{(m)},Q^{(n)}]\rangle\}$$
$$=a_m-\sum_{n>0}(a_n+b_n)\delta\mathbf{f}^{(m)\mathrm{tr}}\sigma_2\delta\mathbf{f}^{(n)}, \tag{2.45a}$$

$$\langle[Q^{(m)\dagger},\hat{P}]\rangle=\sum_{n>0}\{a_n\langle[Q^{(m)\dagger},Q^{(n)\dagger}]\rangle+b_n\langle[Q^{(m)\dagger},Q^{(n)}]\rangle\}$$
$$=-b_m-\sum_{n>0}(a_n+b_n)\delta\mathbf{f}^{(m)\mathrm{tr}}\sigma_2\delta\mathbf{f}^{(n)}. \tag{2.45b}$$

Solving Eq. (2.45) with respect to a's and b's, we get

$$a_n=\langle[Q^{(m)},\hat{P}]\rangle+\sum_{m>0}\langle[Q^{(m)}-Q^{(m)\dagger},\hat{P}]\rangle\delta\mathbf{f}^{(n)\mathrm{tr}}\sigma_2\delta\mathbf{f}^{(m)}, \tag{2.46a}$$

$$b_n=-\langle[Q^{(n)\dagger},\hat{P}]\rangle-\sum_{m>0}\langle[Q^{(m)}-Q^{(m)\dagger},\hat{P}]\rangle\delta\mathbf{f}^{(n)\mathrm{tr}}\sigma_2\delta\mathbf{f}^{(m)}. \tag{2.46b}$$

If we use these expressions for the coefficients in Eq. (2.44) and Eq. (2.37), we obtain an equation having $\hat{P}$ in both sides. Requiring that this equation is an identity for $\hat{P}$, we derive an extended expression for the completeness condition as

$$\sum_{n>0}\Bigg[\mathbf{V}^{(n)}\mathbf{V}^{(n)\dagger}\Bigg\{\mathbf{I}+\mathbf{M}\mathbf{E}(\omega_n-\sigma_2\mathbf{F})^{-1}\sigma_2$$
$$\times\sum_{m>0}(\omega_m-\mathbf{F}\sigma_2)^{-1}\mathbf{E}^{\dagger}\mathbf{M}\mathbf{V}^{(m)}(\mathbf{V}^{(m)\dagger}-\mathbf{V}^{(-m)\dagger})\Bigg\}$$
$$-\mathbf{V}^{(-n)}\mathbf{V}^{(-n)\dagger}\Bigg\{\mathbf{I}+\mathbf{M}\mathbf{E}(\omega_n-\sigma_2\mathbf{F})^{-1}\sigma_2$$
$$\times\sum_{m>0}(\omega_m-\mathbf{F}\sigma_2)^{-1}\mathbf{E}^{\dagger}\mathbf{M}\mathbf{V}^{(-m)}$$
$$\times(\mathbf{V}^{(-m)\dagger}-\mathbf{V}^{(m)\dagger})\Bigg\}\Bigg]\mathbf{M}=\mathbf{I}. \tag{2.47}$$

When the operator $\hat{P}$ is Hermitian, the relation $b_n=a_n^{*}$ holds. Then, making use of Eqs. (2.44), (2.43), and (2.37), we reduce the ensemble average of the double commutator between $\hat{P}$ and $\hat{H}$ as follows:

$$\frac{1}{2}\langle[\hat{P},[H,\hat{P}]]\rangle=\frac{1}{2}\sum_{n,m>0}(a_m^*\mathbf{V}^{(m)\dagger}+a_m\mathbf{V}^{(-m)\dagger})\times\mathbf{M\Omega M}(a_n\mathbf{V}^{(n)}+a_n^*\mathbf{V}^{(-n)})$$
$$=\sum_{n>0}\omega_n|a_n|^2+\frac{1}{2}\sum_{m\neq n(\mathrm{all})}a_m^*a_n\mathbf{V}^{(m)\dagger}\times\mathbf{ME}(\mathbf{F}-\omega_n\sigma_2)^{-1}\mathbf{E}^\dagger\mathbf{M\Omega V}^{(n)} \quad (2.48)$$

with

$$a_n=a_{-n}^*=\langle[Q^{(n)},\hat{P}]\rangle+\mathbf{V}^{(n)\dagger}\mathbf{ME}(\mathbf{F}-\omega_n\sigma_2)^{-1}\sigma_2\times\sum_{m(\neq n)}(\mathbf{F}-\omega_m\sigma_2)^{-1}\mathbf{E}^\dagger\mathbf{MV}^{(m)}\langle[Q^{(m)}-Q^{(m)\dagger},\hat{P}]\rangle. \quad (2.49)$$

The identity given by Eq. (2.48) provides an extension of the EWSR for a Hermitian one-body operator $\hat{P}$ [17] to the case of an excited system in a finite temperature. The first term in its final expression corresponds to an experimental EWS extending over the strength $|a_n|^2$ of the nth eigenmode, and its second term represents a correction attributed to the entropy effect. Since a diagonal element of the matrix **F** is proportional to $[\beta f_\mu(1-f_\mu)]^{-1}$ or $[\beta f_{\bar{\mu}}(1-f_{\bar{\mu}})]^{-1}$ as seen in Eq. (2.33c), it diverges in the zero-temperature limit. Thus, the diagonal elements of **F** dominate in the denominator of the matrix form $(\mathbf{F}-\omega_n\sigma_2)^{-1}$ in Eqs. (2.48) and (2.49). Therefore, the entropy effect represented by the second terms in the expressions of Eqs. (2.48) and (2.49) becomes less important at low temperatures.

For an arbitrary one-body operator $\hat{P}$, the LHS of Eq. (2.48) is in general not a constant as in a microscopic model which will be introduced in the next section. In case of GDR, the operator $\hat{P}$ is replaced by the electric dipole operator

$$\hat{D}=\frac{Z}{A}\sum_{n=1}^{N}x_n-\frac{N}{A}\sum_{p=1}^{Z}x_p, \quad (2.50)$$

where Z, N, and A stand for the numbers of neutrons, protons, and the sum of them, respectively. Since $\langle[\hat{D},[V,\hat{D}]]\rangle=0$ for the velocity-independent nuclear interaction potential V, we have

$$\frac{1}{2}\langle[\hat{D},[H,\hat{D}]]\rangle=\frac{1}{2M}\frac{NZ}{A}. \quad (2.51)$$

Thus, the identity in Eq. (2.48) gives an extension of the Thomas-Reiche-Kuhn sum rule to the GDR built on an excited nucleus at a finite temperature. In case of such an identity, it is expected that the increase of the contribution corresponding to the second term in the final expression of Eq. (2.48) is compensated by the decrease of the experimental EWS $\Sigma_{n>0}\omega_n|a_n|^2$ with increasing temperature.

III. MICROSCOPIC MODEL

In order to perform numerical analysis, we consider a simple microscopic model in which protons and neutrons are not discriminated; and angular momenta (or spins) and parities are completely ignored. We do not take into account the pairing correlations so that the particle-hole picture can be employed. In this model, we take L single-particle levels with an equal spacing ε_0 except for a shell gap Δ above the l_{gap}-th level. Two single-particle states labeled μ and $\bar{\mu}$ are degenerate in each single-particle level. Thus, the single-particle energies measured from the Fermi energy ε_F are given by

$$\varepsilon_l=(l-1)\varepsilon_0-\varepsilon_F \quad \text{for} \quad 1\leqslant l\leqslant l_{\mathrm{gap}},$$
$$\varepsilon_l=(l-2)\varepsilon_0+\Delta-\varepsilon_F \quad \text{for} \quad l_{\mathrm{gap}}<l\leqslant L. \quad (3.1)$$

Our Hamiltonian with two-body interaction is expressed in terms of the particle operators $\{p_k,p_k^\dagger\}$ and the hole operators $\{h_k,h_k^\dagger\}$ as

$$\hat{H}=\sum_{k>F}\varepsilon_k p_k^\dagger p_k-\sum_{h<F}\varepsilon_k h_k^\dagger h_k+\frac{\chi}{2}:\hat{P}^2: \quad (3.2)$$

with

$$\hat{P}=\hat{P}^\dagger=\sum_{kl(k\neq l)}g_{kl}c_k^\dagger c_l, \quad (3.3)$$

where g_{kl} stands for the form factor representing the transition matrix element; $c_k=p_k$ for a particle state and $c_k=h_k^\dagger$ for a hole state. The Hamiltonian is formally expressed in terms of self-evident notations as follows:

$$\hat{H}=\sum_{p_1p_2}(H_{11})_{p_1p_2}p_1^\dagger p_2+\sum_{h_1h_2}(H_{11})_{h_1h_2}h_1^\dagger h_2+\sum_{p_1p_2h_1h_2}(H_{22})_{p_1h_1p_2h_2}p_1^\dagger h_1^\dagger h_2p_2$$
$$+\sum_{p_1p_2p_3p_4}(H_{22})_{p_1p_2p_3p_4}p_1^\dagger p_2^\dagger p_4p_3+\sum_{h_1h_2h_3h_4}(H_{22})_{h_1h_2h_3h_4}h_1^\dagger h_2^\dagger h_4h_3$$
$$+\sum_{p_1p_2p_3h_1}\{(H_{31})_{p_1p_2p_2h_1}p_1^\dagger p_2^\dagger h_1^\dagger p_2+(H_{31})^*_{p_1p_2p_3h_1}p_3^\dagger h_1p_2p_1\}$$
$$+\sum_{h_1h_2h_3p_1}\{(H_{31})_{h_1h_2h_3p_1}h_1^\dagger h_2^\dagger p_1^\dagger h_3+(H_{31})^*_{h_1h_2h_3p_1}h_3^\dagger p_1h_2h_1\}$$
$$+\sum_{p_1p_2h_1h_2}\{(H_{40})_{p_1p_2h_1h_2}p_1^\dagger p_2^\dagger h_2^\dagger h_1^\dagger+(H_{40})^*_{p_1p_2h_1h_2}h_1h_2p_2p_1\} \quad (3.4)$$

with

$$(H_{11})_{p_1p_2}=\varepsilon_{p_1}\delta_{p_1p_2},$$

$$(H_{11})_{h_1h_2}=-\varepsilon_{h_1}\delta_{h_1h_2},$$

$$(H_{22})_{p_1h_2p_2h_1}=v_{p_1h_2h_1p_2},$$

$$(H_{22})_{p_1p_2p_3p_4}=\frac{1}{4}v_{p_1p_2p_3p_4},$$

$$(H_{22})_{h_1h_2h_3h_4}=\frac{1}{4}v_{h_1h_2h_3h_4},$$

$$(H_{31})_{p_1p_2p_3h_1}=\frac{1}{4}v_{p_1p_2p_3h_1},$$

$$(H_{31})_{h_1h_2h_3p_1}=\frac{1}{4}v_{h_1h_2h_3p_1},$$

$$(H_{40})_{p_1p_2h_1h_2}=\frac{1}{4}v_{p_1p_2h_1h_2}, \tag{3.5}$$

and

$$v_{klmn}\equiv\chi(g_{km}g_{ln}-g_{kn}g_{lm}). \tag{3.6}$$

In order to introduce a smooth cutoff in energy for the above transition matrix elements defined in the finite single-particle model space, we assume a transition form factor given by

$$g_{kl}=e^{-[(\varepsilon_k-\varepsilon_l)/A]^2}-e^{-[(\varepsilon_k-\varepsilon_l)/B]^2}. \tag{3.7}$$

In the practice of numerical analysis carried out for nonrotating nuclei (i.e., $\omega_{\rm rot}=0$) in the subsequent sections, we ignore the temperature dependence of single-particle levels. The numerical values of five constants are chosen to be $\varepsilon_0=0.275$ MeV, $\Delta=6.0$ MeV, $\chi=0.1$ MeV, $A=6.8$ MeV, and $B=5.5$ MeV. Total number of levels is $L=80$, and the shell gap is placed between 36th and 37th levels (i.e., $l_{\rm gap}=36$).

IV. PERTURBATION DUE TO ENTROPY EFFECT

It is obvious that the second term in the LHS of Eq. (2.36) describes the entropy effect. Since this contribution is in the second order of the coupling constant, i.e., χ^2, the perturbation treatment is applicable to calculate the shift of the eigenvalue of collective excitation due to the entropy effect $\Delta\omega$. Introducing notation for the quantity

$$U\equiv{\rm Tr}(\hat{W}[\hat{P},Q^\dagger]), \tag{4.1}$$

which frequently appears in what follows, and an expression for the TRPA amplitude

$$\mathbf{V}^{\rm tr}=(X_{ph},Y_{ph},Z_{p_1p_2},Z_{h_1h_2})=\left(\frac{g_{ph}U}{\omega-\varepsilon_p+\varepsilon_h},\frac{-g_{ph}U}{\omega+\varepsilon_p-\varepsilon_h},\frac{g_{p_1p_2}U}{\omega-\varepsilon_{p_1}+\varepsilon_{p_2}},\frac{g_{h_1h_2}U}{\omega+\varepsilon_{h_1}-\varepsilon_{h_2}}\right), \tag{4.2}$$

we derive an explicit formula for the shift of the TRPA eigenenergy as

$$\begin{aligned}\Delta\omega\cong|U|^2\Bigg[&\sum_{ph}g_{ph}^2(n_h-n_p)^2\\&\times\left\{\frac{1}{|\omega-\varepsilon_p+\varepsilon_h|^2}+\frac{1}{|\omega+\varepsilon_p-\varepsilon_h|^2}\right\}\\&\times(\mathbf{E}(\omega\sigma_2-\mathbf{F})^{-1}\mathbf{E}^\dagger)_{ph,ph}\\&+\sum_{p_1p_2}\frac{g_{p_1p_2}^2(n_{p_2}-n_{p_1})^2}{|\omega-\varepsilon_{p_1}+\varepsilon_{p_2}|^2}(\mathbf{E}(\omega\sigma_2-\mathbf{F})^{-1}\mathbf{E}^\dagger)_{p_1p_2,p_1p_2}\\&+\sum_{h_1h_2}\frac{g_{h_1h_2}^2(n_{h_2}-n_{h_1})^2}{|\omega-\varepsilon_{h_1}+\varepsilon_{h_2}|^2}(\mathbf{E}(\omega\sigma_2-\mathbf{F})^{-1}\mathbf{E}^\dagger)_{h_1h_2,h_1h_2}\Bigg]\end{aligned} \tag{4.3}$$

together with the relation derived from the normalization condition in Eq. (2.35), i.e.,

$$\begin{aligned}|U|^{-2}=&\sum_{ph}g_{ph}^2(n_h-n_p)\\&\times\left\{\frac{1}{|\omega-\varepsilon_p+\varepsilon_h|^2}-\frac{1}{|\omega+\varepsilon_p-\varepsilon_h|^2}\right\}\\&+\sum_{p_1p_2}\frac{g_{p_1p_2}^2(n_{p_2}-n_{p_1})}{|\omega-\varepsilon_{p_1}+\varepsilon_{p_2}|^2}-\sum_{h_1h_2}\frac{g_{h_1h_2}^2(n_{h_2}-n_{h_1})}{|\omega-\varepsilon_{h_1}+\varepsilon_{h_2}|^2},\end{aligned} \tag{4.4}$$

where

$$n_k=\frac{1}{e^{\beta\varepsilon_k}+1} \tag{4.5}$$

is the single-particle occupation number. The explicit forms of the matrix elements appearing in Eq. (4.3) are given by

$$[\mathbf{E}(\omega\sigma_2-\mathbf{F})^{-1}\mathbf{E}^\dagger]_{ph,ph}=-\sum_{p_1}\frac{v_{p_1pp_1h}^2\gamma_{p_1}+v_{p_1p\bar{p}_1h}^2\gamma_{\bar{p}_1}}{1-(4\omega)^2\gamma_{p_1}\gamma_{\bar{p}_1}}-\sum_{h_1}\frac{v_{h_1ph_1h}^2\gamma_{h_1}+v_{h_1p\bar{h}_1h}^2\gamma_{\bar{h}_1}}{1-(4\omega)^2\gamma_{h_1}\gamma_{\bar{h}_1}},$$

$$(\mathbf{E}(\omega\sigma_2-\mathbf{F})^{-1}\mathbf{E}^\dagger)_{p_1p_2,p_1p_2}=-\sum_p \frac{v^2_{p_1pp_2p}\gamma_p+v^2_{p_1\bar{p}p_2\bar{p}}\gamma_{\bar{p}}}{1-(4\omega)^2\gamma_p\gamma_{\bar{p}}}-\sum_h \frac{v^2_{p_1hp_2h}\gamma_h+v^2_{p_1\bar{h}p_2\bar{h}}\gamma_{\bar{h}}}{1-(4\omega)^2\gamma_h\gamma_{\bar{h}}},$$

$$(\mathbf{E}(\omega\sigma_2-\mathbf{F})^{-1}\mathbf{E}^\dagger)_{h_1h_2,h_1h_2}=-\sum_p \frac{v^2_{h_1ph_2p}\gamma_p+v^2_{h_1\bar{p}h_2\bar{p}}\gamma_{\bar{p}}}{1-(4\omega)^2\gamma_p\gamma_{\bar{p}}}-\sum_h \frac{v^2_{h_1hh_2h}\gamma_h+v^2_{h_1\bar{h}h_2\bar{h}}\gamma_{\bar{h}}}{1-(4\omega)^2\gamma_h\gamma_{\bar{h}}} \tag{4.6}$$

with the definition

$$\gamma_k\equiv\beta n_k(1-n_k). \tag{4.7}$$

The temperature dependence of the Fermi energy (or the chemical potential) ε_F is determined by the self-consistent requirement for a given particle number N

$$2\sum_{k=1}^{L} n_k=N. \tag{4.8}$$

Thus, the Fermi energy moves with temperature when the distribution of single-particle levels is asymmetric with respect to ε_F at the temperature $T=0$. However, it will be shown in the subsequent section that its temperature dependence is slight only if we take into account enough number of single-particle levels.

Within an approximation neglecting exchange terms and the entropy terms, the TRPA equation in the particle-hole picture becomes

$$\left[\frac{1}{\chi}-2\sum_{ph}\frac{g^2_{ph}(\varepsilon_p-\varepsilon_h)(n_h-n_p)}{\omega^2-(\varepsilon_p-\varepsilon_h)^2}-\sum_{p_1p_2}\frac{g^2_{p_1p_2}(n_{p_2}-n_{p_1})}{\omega-\varepsilon_{p_1}+\varepsilon_{p_2}}+\sum_{h_1h_2}\frac{g^2_{h_1h_2}(n_{h_2}-n_{h_1})}{\omega+\varepsilon_{h_1}-\varepsilon_{h_2}}\right]U=0. \tag{4.9}$$

Thus, the corresponding linear response function is given by

$$R=\frac{R_0}{1-\chi R_0} \tag{4.10}$$

with

$$R_0\equiv 2\sum_{ph}\frac{g^2_{ph}(\varepsilon_p-\varepsilon_h)(n_h-n_p)}{\omega^2-(\varepsilon_p-\varepsilon_h)^2}+\sum_{p_1p_2}\frac{g^2_{p_1p_2}(n_{p_2}-n_{p_1})}{\omega-\varepsilon_{p_1}+\varepsilon_{p_2}}-\sum_{h_1h_2}\frac{g^2_{h_1h_2}(n_{h_2}-n_{h_1})}{\omega+\varepsilon_{h_1}-\varepsilon_{h_2}}. \tag{4.11}$$

The first term on the RHS of this expression describes the contribution from the ph configuration, while the second and the third terms correspond to the contributions from the pp and the hh configurations, respectively. Then, the strength function of the GR is given by the imaginary part of R as

$$S(\omega)=-\frac{1}{\pi}\mathrm{Im}\,R=\frac{\mathrm{Im}\,R_0}{(1-\chi\,\mathrm{Re}\,R_0)^2+(\chi\,\mathrm{Im}\,R_0)^2}. \tag{4.12}$$

Finally, we consider the stability problem within the perturbation treatment. For the eigenenergy ω_0 of the standard TRPA equation without the entropy effect, $\mathbf{\Omega MV}=\mathbf{V}\omega_0$, the shift of the eigenenergy due to the entropy effect is given by putting $\omega=\omega_0$ in Eq. (4.3). Assuming that ω_0 is already very small and neglecting it in the denominators in Eqs. (4.3), (4.4), and (4.6), we get a shift caused by the entropy effect

$$\begin{aligned}\Delta\omega\cong-|U|^2\Bigg[&2\sum_{ph} g^2_{ph}\left(\frac{n_p-n_h}{\varepsilon_p-\varepsilon_h}\right)^2\left\{\sum_{p_1}(v^2_{p_1pp_1h}\gamma_{p_1}+v^2_{\bar{p}_1p\bar{p}_1h}\gamma_{\bar{p}_1})+\sum_{h_1}(v^2_{h_1ph_1h}\gamma_{h_1}+v^2_{\bar{h}_1p\bar{h}_1h}\gamma_{\bar{h}_1})\right\}\\&+\sum_{p_1p_2} g^2_{p_1p_2}\left(\frac{n_{p_1}-n_{p_2}}{\varepsilon_{p_1}-\varepsilon_{p_2}}\right)^2\left\{\sum_p(v^2_{p_1pp_2p}\gamma_p+v^2_{p_1\bar{p}p_2\bar{p}}\gamma_{\bar{p}})+\sum_h(v^2_{p_1hp_2h}\gamma_h+v^2_{p_1\bar{h}p_2\bar{h}}\gamma_{\bar{h}})\right\}\\&+\sum_{h_1h_2} g^2_{h_1h_2}\left(\frac{n_{h_1}-n_{h_2}}{\varepsilon_{h_1}-\varepsilon_{h_2}}\right)^2\left\{\sum_p(v^2_{h_1ph_2p}\gamma_p+v^2_{h_1\bar{p}h_2\bar{p}}\gamma_{\bar{p}})+\sum_h(v^2_{h_1hh_2h}\gamma_h+v^2_{h_1\bar{h}h_2\bar{h}}\gamma_{\bar{h}})\right\}\Bigg],\end{aligned} \tag{4.13}$$

which is a negative quantity. Thus, the exact TRPA equation is expected to describe the occurrence of the THFB instability at finite temperature caused by the entropy effect, though it is needed to solve exactly the TRPA equation (2.36) especially for the case of stronger coupling constant in order to study the detailed behavior of the instability line.

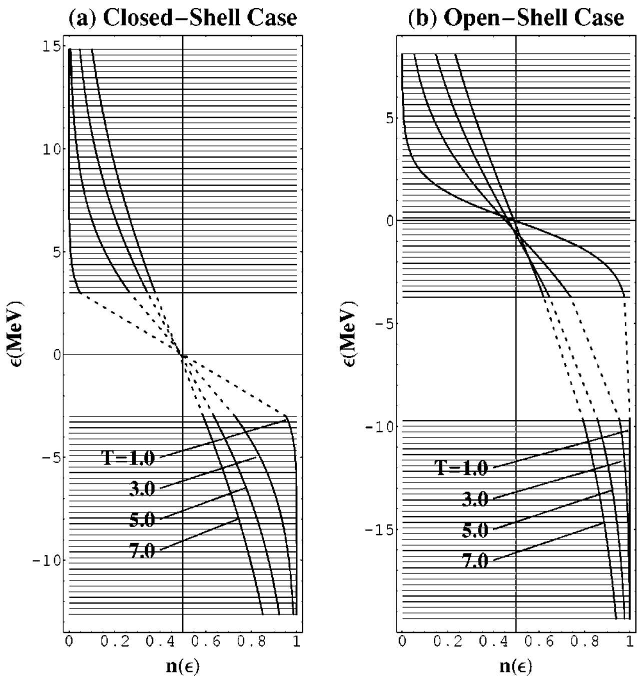

FIG. 1. Single-particle level scheme and the single-particle occupation numbers calculated at various temperatures for two cases (a) ''a closed-shell model nucleus'' (the left diagram) and (b) ''an open-shell model nucleus'' (the right diagram). In each diagram, the horizontal thin lines represent 80 single-particle levels with an equal spacing except for a gap between the 36th and the 37th levels; four curves with solid lines connected by the straight dashed lines through the gap region represent the single-particle occupation numbers calculated at four temperatures $T=1.0$, 3.0, 5.0, and 7.0 MeV as indicated in the diagram. In the right diagram for case (b), the crossing points of the four curves with a vertical straight line at $n=0.5$ demonstrate the temperature-dependent shift of the Fermi energy.

V. TEMPERATURE DEPENDENCE OF THE GR WIDTH

We carry out numerical analysis for two interesting cases (a) ''a closed-shell model nucleus'' whose Fermi energy is in the middle of the shell gap assumed to be between the 36th level and the 37th level in our model space and (b) ''an open-shell model nucleus'' whose Fermi energy is between the 50th level and the 51st level. With two diagrams for these cases (a) and (b) in Fig. 1, we show the single-particle level scheme in the present model by horizontal thin lines (i.e., 36 levels below and 44 levels above the gap) and the single-particle occupation numbers in these levels by four curves with solid lines calculated at the temperature $T=1.0$, 3.0, 5.0, and 7.0 MeV. The counterparts of each curve below and above the gap are connected by a straight dashed line through the gap region. Though the single-particle levels are assumed to be independent of temperature, the Fermi energy ε_F becomes temperature-dependent by the requirement in Eq. (4.8). The temperature-dependent shift of the Fermi energy starting from $\varepsilon_F=0$ at $T=0$ is negligible in the case (a), while it is explicitly shown by the shift of crossing points of the occupation number curves with a vertical straight line at $n(\varepsilon)=0.5$, though its extent is only within a few single-particle levels in the temperature range $T=0.0\sim 7.0$ MeV. The occupation numbers in higher levels indicate that the temperature of 5 MeV is already too high for the configuration space of the model.

The perturbation result for the gamma-ray energy shift in Eq. (4.3) gives simply a shift of the γ-ray energy, i.e., $\omega \rightarrow \omega+\Delta\omega$, without changing the functional form of the strength function $S(\omega)$ given by Eq. (4.12). The shift $\Delta\omega$ calculated by the formula in Eq. (4.13) turns out to be at most of the order of 0.1 keV for the above temperature range and the set of parameters given below Eq. (3.7). Therefore, the GR shape is not modified by the entropy effect provided that the interaction strength is not too large so that such an effect can be dealt with as a perturbation. Then, the overall distribution of the GR strengths is well approximated by the solution to the TRPA equation neglecting completely the entropy effect.

Utilizing the expression for the strength function in Eq. (4.12), we carry out numerical analysis for the above two cases (a) and (b). In the practice of numerical analysis, we assume a fictitious finite imaginary part called ''the escape width,'' $\gamma_{\mathrm{esc}}=0.5$ MeV, only for the purpose of smearing the functional behavior of the strength function. Therefore, we replace ω by $\omega+i\gamma_{\mathrm{esc}}$ in the denominator of each term in the function R_0.

In order to investigate the effect of the pp and the hh configurations on the Landau splitting of the collective RPA levels, we compare the giant resonance shapes calculated including these configurations in addition to the ordinary ph configuration with the one calculated only with the ph configurations up to the temperature $T=7.0$ MeV for both cases (a) (four panels on the left) and (b) (four panels on the right) in Fig. 2. In both cases we observe a clear trend that the pp and hh configurations contribute much to increase the absolute values of the resonance strengths $S(\omega)$. At the temperature $T=1.0$ MeV, the difference between two strengths calculated with and without the pp and the hh contributions is slight in the closed-shell nucleus, while its difference is already seen in case of the open-shell nucleus. This is due to the fact that the Fermi energy is in a region of larger single-particle level density and the single-particle energy measured from the Fermi energy is smaller, so that the distortion of the Fermi distribution starts from lower temperatures. As a result, the excitations via the pp and the hh configurations can start already from lower temperatures in the open-shell case.

Comparing the temperature dependence of the resonance shapes including the pp and the hh contributions between two cases (a) and (b), we recognize that the decrease of resonance strength is more rapid in the open-shell case, and the centroid energies slowly decrease with temperature in both cases. The latter shift is at most 2.0 MeV in both cases. The increase of the resonance widths (i.e., the Landau splittings) is seen only in the resonance curves including the pp and the hh contributions in both cases, while the resonance widths without those configurations do not change with temperature and keep almost constant values in both cases. As our model is within the RPA, the absolute value of full width

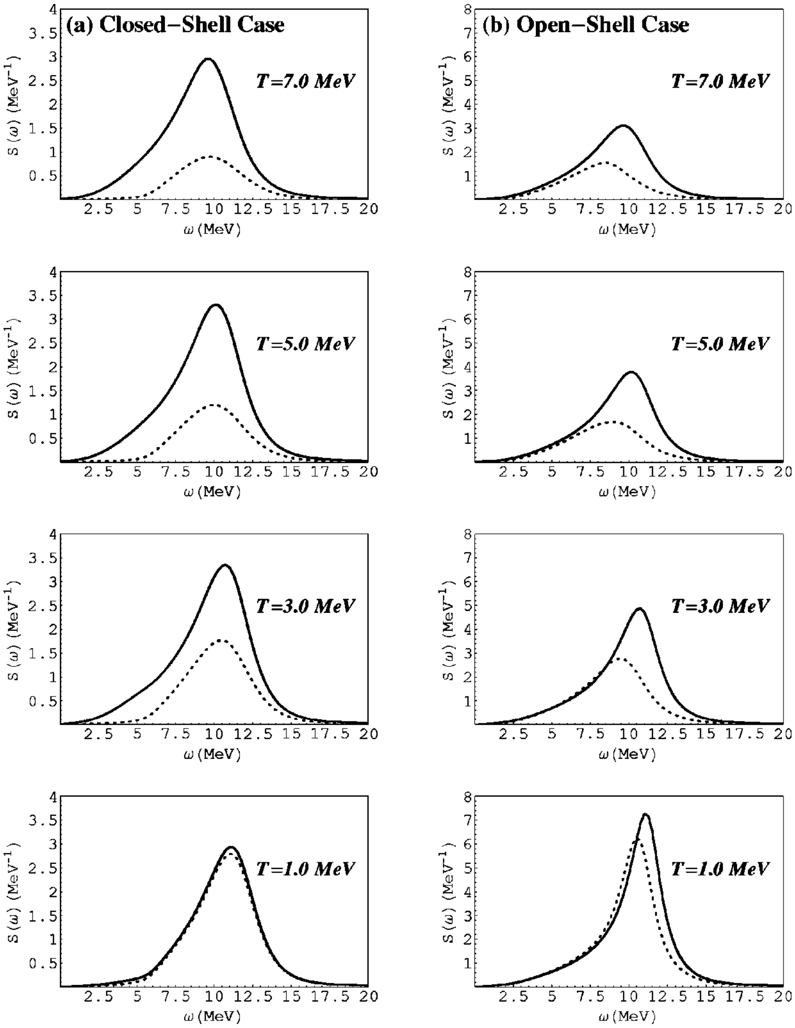

FIG. 2. Temperature dependence of the strength functions of giant resonance calculated for two cases (a) ''a closed-shell model nucleus'' (four panels on the left) and (b) ''an open-shell model nucleus'' (four panels on the right). In each panel a solid line represents the standard TRPA result which includes the damping via the pp and the hh configurations in addition to the ph configuration, and a dashed line the one including only the ph configuration. In each case, four panels correspond to the results calculated at the temperatures $T=1.0$, 3.0, 5.0, and 7.0 MeV, respectively.

is not as large as that of GDR, and its change with temperature is not so rapid. Needless to say, a detailed way how rapidly the resonance strength changes with temperature depends upon microscopic models.

VI. CONCLUSION

In the present paper we have shown detailed steps for the variational derivation of the exact thermal RPA (TRPA) equation whose matrix representation precisely corresponds to the thermal HFB (THFB) stability matrix. This equation describes an interplay between the collective excitation and the entropy effect. It must be noticed that the extended parts of the TRPA matrix, i.e., the matrices $\mathbf{E}$, $\mathbf{E}^{\dagger}$, and $\mathbf{F}$ in Eq. (2.32), represent the temperature effect arising from the ground state correlations modified by the existing collective mode as well as the entropy effect. If these effects are meant simply by the entropy effect, the TRPA equation correctly describes such an entropy effect that causes the instability of the THFB state already at finite temperature. However, the giant resonance shape is not much affected by the entropy effect unless the coupling of the collective modes to particle-hole configurations is strong enough, since the entropy effect is described in terms of the second order in the coupling constant. Based on the EWSR extended to the finite temperature given by Eq. (2.47), we can point out a possibility that, in case of the GDR, the increasing contributions arising from the entropy effect (i.e., the second term in the RHS) compensates the decreasing EWS with temperature (i.e., the first term in the same expression) so that the sum of both contributions gives a value which does not depend on temperature.

Making use of a simple microscopic model, we have studied the effect of the pp and the hh configurations on the Landau splitting of the GR. We find that these configurations play decisive roles in the temperature-dependent phenomena of the GR built on a hot nucleus. The distortion of the Fermi distribution due to temperature effect allows the damping of GR via the pp and hh configurations and the Landau splitting width of the GR increases with increasing the temperature. Since this general mechanism works also in the damping of the GR via four quasiparticle configurations, it can be inferred that the increase of the spreading width is controlled mainly by the $pppp$, $ppph$, $phhh$, and $hhhh$ configurations. This physical picture is consistent with the theoretical expectation based on the standard TRPA in the quasiparticle picture [9–13] and the recent approaches with the phonon damping model (PDM) [18,19]. It should be remarked that the change of the mean field with temperature, due to the change of pairing correlation and deformation which are not considered in the present model, are important at low temperatures [10]. Therefore, for the purpose of investigating the temperature dependence of the GR phenomena, it is the most desirable to perform for realistic nuclei the TRPA calculations on top of the self-consistent solution to the thermal HFB equation.

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